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Conceptual Problems in Bell's Inequality and Quantum Entanglement

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Abstract: The description of the microscopic world in quantum mechanics is very different from that in classical physics, and there are some points of view that are contrary to intuition and logic. The first is the problem of reality, quantum mechanics believes the behavior of micro particles is random and jumping. The second is the loss of certainty, the conjugate physical variables of a system cannot be determined synchronously, they satisfy the Heisenberg uncertainty principle. The third is the non-local correlation. The measurement of one particle in the quantum entanglement pair will influence the state of the other entangled particle simultaneously. In this paper, some concepts related to quantum entanglement, such as EPR correlation, quantum entanglement correlation function, Bell's inequality and so on, are analyzed in detail. Analysis shows that the mystery and confusion in quantum theory may be caused by the logical problems in its basic framework. Bell's inequality is only a mathematical theorem, but its physical meaning is actually unclear. The Bell state of quantum entangled pair may not satisfy the dynamic equation of quantum theory, so it cannot describe the true state of microscopic particles. In this paper, the correct correlation functions of spin entanglement pair and photonic entanglement pair are strictly derived according to normal logic. Quantum theory is a more fundamental theory than classical mechanics, and they may be not equal relation in logic. However, there are still some unreasonable contents in the framework of quantum theory, which need to be improved. In order to disclose the real relationship between quantum theory and classical mechanics, we propose some experiments which provide intuitionistic teaching materials for the new interpretation of quantum theory.

Keywords: Quantum mechanics interpretation; mathematical foundation of quantum mechanics; EPR correlation; Bohm's hidden variable theory; quantum entanglement; Bell's inequality; quantum correlation function; Schrödinger equation; Heisenberg uncertainty relation

I. A BRIEF HISTORICAL REVIEW

The description of the microscopic particles in quantum mechanics is very different from that in classical physics. The coordinates of an electron in atom are uncertain, and the electron stays at a place according to probability. A physical quantity is uncertain before it is measured. The wave function of the measured particle collapses to an eigenstate of the corresponding operator according to a certain probability. The most distinctive feature of the micro-world is uncertainty, observation and measurement results only obey statistical law. Conjugate physical quantities satisfy the Heisenberg uncertainty relation and cannot be accurately measured at the same time, such as position and momentum of a particle cannot be determined simultaneously.

There are many famous remarks about the loss of certainty and reality. For examples, Einstein oppugned Bohr, "Does the moon exist only when someone is looking at it?" The Schrödinger's cat in the black box is in a superposed state of both life and death. Feynman said, "I think I can safely say, nobody understands quantum mechanics". Niels Bohr, one of the founders of quantum theory, pointed out, "If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet."

As for the philosophical interpretation of quantum theory, two opposing camps were formed from the beginning, represented by Einstein and Bohr. The first encounter between the two factions was at the fifth Solvay Conference in 1927. It was an unprecedented summit in physics, with 17 of the 29 participants winning the Nobel Prize in physics. Einstein's main points of questioning quantum theory are three aspects: determinacy, reality and locality. Einstein always adheres to that, a complete physical theory should be of realism and causality. With regard to certainty, he said, God does not play dice. Reality holds that the existence in the world is independent of human observation. Locality means that there is no interaction at a distance between two points far away from each other.

In 1935, Einstein and his assistants Boris Podolsky and Nathan Rosen(EPR) at the Princeton Institute of Advanced Studies examined the completeness of quantum mechanics in depth[1]. EPR's paper puts forward a viewpoint that a satisfactory physical theory should not only be consistent with the experimental results, but also that every element of the physical system must have a corresponding concept in the theory, that is, the theoretical description should be complete. Quantum mechanics holds that the physical system is described by a state function, each physical quantity corresponds to an Hermitian operator,

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and the corresponding physical quantity is determined only when the state function is the eigenstate of an operator. If the two operators are not commutable, the state function cannot be the common essential state of the two operators at the same time, so at least one physical quantity is uncertain.

In EPR paper, the correlation between two particles after decoupling is discussed. The wave function of the correlated particles is at a mixed state before it is measured, If the physical quantity of the first particle is measured as P_1, the wave function of the second particle will also be at the corresponding eigenstate, so that the corresponding P_2 also has a definite value. If the conjugate variable Q_1 of the first particle is measured, the corresponding variable Q_2 of the second particle also has a definite value.

EPR believes that there is no interaction between the two particles after they are separated, but by measuring the physical quantity of the first particle, we can accurately know the physical quantity of the other particle, and both conjugate physical quantities have definite values. This contradicts the basic principle of quantum mechanics, so the description of reality in quantum mechanics is incomplete. The correlation phenomenon between two particles leaving away was called quantum entanglement, but Einstein himself never believed in the existence of this correlation. Later, David Bohm changed the momentum correlation of particle pairs in EPR paper to spin correlation which is more easy to test[2, 3].

In order to coordinate the contradiction between classical mechanics and quantum mechanics, David Bohm proposed an explanation of hidden variables theory about quantum mechanics in 1952, which linked the classical orbit of particles to the probability distribution of wave function[4, 5]. The main points of his explanation are as follows (for simplicity, we take $\hbar = c = 1$ as unit). Let $\psi(\vec{x}, t)$ be the solution of the time-dependent Schrödinger equation of a particle, that is,

$$i\partial_t\psi(\vec{x}, t) = (\frac{1}{2m}\hat{p}^2 + V)\psi, \tag{1}$$

where $\hat{p} = -i\nabla$ is the momentum operator of the particle. $\psi(\vec{x}, t)$ is expressed in the form of absolute value and corresponding phase as follows

$$\psi(\vec{x}, t) = R(\vec{x}, t)e^{iS(\vec{x},t)}. \tag{2}$$

Calculation shows that the phase function $S(\vec{x}, t)$ satisfies the generalized Hamilton-Jacobi equation in classical mechanics

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V + \hbar^2 Q = 0, \qquad Q \equiv -\frac{\Delta R}{2mR}, \tag{3}$$

In which Q is the quantum potential introduced by Bohm. Let $\hbar \to 0$, then S satisfies the classical Hamilton-Jacobi equation. If the relation $\vec{p} = \nabla S$ in Hamilton-Jacobi theory is

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introduced, by solving the momentum equation of the particle

$$\vec{p} = m\frac{d\vec{x}}{dt} = \nabla S(\vec{x}, t), \qquad \vec{x}(0) = \vec{x}_0, \tag{4}$$

we get a classical trajectory of the particle $\vec{x}(\vec{x}_0, t)$. Because we don't know the initial position $\vec{x}_0$ of particles exactly, one can only use the sets of these trajectories to explain the probability distribution of particle motion.

$$\rho(\vec{x}, t_0) = |\psi(\vec{x}, t_0)|^2 = R(\vec{x}, t_0)^2. \tag{5}$$

This is the classical explanation for quantum mechanics made by Bohm, which links quantum mechanics to the classical mechanics by (4) and Hamilton-Jacobi equations.

At the technical level, the book "Mathematical foundations of quantum Mechanics" published by J. von Neumann is a milestone[6]. According to the operator theory on Hilbert space, this book provides a strict mathematical framework for non-relativistic quantum mechanics, and proves the equivalence between matrix mechanics and wave mechanics. By introducing the concept of density matrix, the statistical theory of quantum mechanics is established. According to the uncertain characteristics of quantum theory, J. von Neumann proved that a realistic hidden variable theory is impossible.

John Stewart Bell carefully checked von Neumann's proof of the impossibility of hidden variables and found that there is a loophole in the correspondence between mathematics and physics. von Neumann, in his proof, used an assumption that "the average of the sum of the two observables is equal to the sum of the average of the observables"[3, 7]. Bell points out that if the two observables are conjugate variables, that is, when they obey the uncertainty principle in quantum mechanics, this conclusion is incorrect. Bell pointed out von Neumann's mistake in 1965, but as early as 1935, Grete Hermann, a little-known German mathematician, also pointed out this mistake. Grete Herman was the first student of the famous mathematician Emmy Nöther at Göttingen University. She made important contributions to the mathematical and philosophical foundation of quantum mechanics in her early days. However, in their rebuttal, whether the conjugate variables can be summable and whether the sum has physical meaning were not explained. After intensive study of the theory of hidden variables and the EPR paradox, Bell derived a famous inequality, whose purpose is to establish an experimental criterion for locality theory and quantum correlation[8].

Bell's discovery thus shifted Einstein and Bohr's debate from epistemology to the realm of experimental physics. Within a few years, Bell's inequalities were adapted to a practical scheme[9]. The first experiments were carried out in 1972 at the University of California,

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Berkeley[10], and at Harvard[11, 12], then in 1976 at Texas A&M[13]. Later, it was found that there were some loopholes in these experiments, and some improved schemes were put forward. Some more reliable experiments were carried out, but the experimental results violate Bell's inequality[14–16].

In the following sections, we directly move into the theme of this article, where we examine concretely the physical meaning of the Bell inequality and derive the correlation functions of quantum entanglement. The interpretation of quantum theory involves more philosophical viewpoints and fundamental theories, we put it in the last section to discuss in detail. This paper is completed in the context of the special edition "axiomatic approaches to quantum mechanics" of "Entropy", so we discuss some basic issues of quantum theory. I would like to thank the support and encouragement from the editors and consultants.

II. THE PHYSICAL MEANING OF BELL'S INEQUALITY?

If I do not consider the specific physical meaning of the inequality at first, Bell and CHSH actually proved the following mathematical theorem[9].

Theorem *Assume*

1° *For* $\lambda \in \mathbb{R}^n$, *the function* $\rho(\lambda)$ *satisfies*

$$\rho(\lambda) \geq 0, \qquad \int_{\mathbb{R}^n} \rho d\lambda = 1. \tag{6}$$

2° *For any two unit direction vectors* $\mathbf{a}$ *and* $\mathbf{b}$, *the functions* $A(\mathbf{a}, \lambda), B(\mathbf{b}, \lambda)$ *satisfy*

$$|A(\mathbf{a}, \lambda)| \leq 1, \qquad |B(\mathbf{b}, \lambda)| \leq 1. \tag{7}$$

Define the correlation function between the two directions as

$$E(\mathbf{a}, \mathbf{b}) \equiv \int_{\mathbb{R}^n} A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \rho(\lambda) d\lambda, \tag{8}$$

Then we have the following inequality

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) + E(\mathbf{a}', \mathbf{b}')| \leq 2. \tag{9}$$

Now we examine the above theorem by corresponding to physical processes. I. What is the physical meaning of the function $\rho(\lambda)$, and does it correspond to specific particles and physical processes? Each term in the left hand of (9) corresponds to different particle and different process, but in proof of this inequality, several calculations similar to $\int A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) A(\mathbf{a}', \lambda) B(\mathbf{b}', \lambda) \rho(\lambda) d\lambda$ are also used. The proof is valid only if $\rho(\lambda)$ used in

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all calculations are the same function. This is a universal function that contains everything, whose requirement is equivalent to the theory of everything, and in which the so-called hidden variable λ is not clear whether it is related to specific particles and processes. If the existence of the universal function and its physical meaning cannot be explained clearly, the physical meaning of the Bell's inequality is also unclear.

II. Let's examine a specific example. Suppose there is a coupling positron-electron pair with a total spin of 0. For $t \geq 0$, they are moving away relatively after decoupling, and their spins are opposite. Assuming the electron has coordinate $\vec{x}$ and moves to $\vec{X}$, and the positron has coordinate $\vec{y}$ and moves to $\vec{Y}$. Their wave function is given by $\psi(t, \vec{x}, \vec{y})$. Record the spin of the electron measured at time $t = t_a$, place $\vec{X}$ and direction $\mathbf{a}$ as $A(\mathbf{a}, \vec{X}, t_a)\frac{\hbar}{2}$, and measured the spin of the positron at $t = t_b$, place $\vec{Y}$ and direction $\mathbf{b}$ as $B(\mathbf{b}, \vec{Y}, t_b)\frac{\hbar}{2}$.

Because we only care about spin of particle now, the coordinates $\lambda = (\vec{x}, \vec{y})$ can be regarded as part of the hidden variable according to the Bohm's point of view, and the probability density function is given by $\rho(\lambda) = |\psi(t, \vec{x}, \vec{y})|^2$. These are the usual concepts of quantum mechanics, and they obviously satisfy the conditions 1° and 2° of the theorem. The time parameter t is introduced here because all measurements must have time to complete, but Bell's proof uses a density function of $\rho(\lambda)$ without time, so the proof implies the requirement of simultaneity[18]. However, the four measurements in the Bell's inequality cannot be synchronized at all, and the functions $\rho(\lambda) = |\psi(t, \vec{x}, \vec{y})|^2$ for different particle pairs are also different. Considering the physical process, the physical meaning of (9) is not clear, so the Bell's inequality cannot explain the spin correlation of entangled particle pairs.

III. We clearly assume that the spin of the entangled pair is opposite before the measurement, but Bell, when defining (8), says that the two measurements are independent. We should replace $\rho(\lambda)$ by $\{\rho(\lambda, \vec{S}_x, \vec{S}_y, t)|\vec{S}_x = -\vec{S}_y\}$ to meet the actual situation, and it is logical to calculate the spin correlation function by calculating the conditional probability. Therefore, the physical meaning of corresponding concepts in (8) is not clear.

IV. Spin of a particle is a very sensitive physical quantity. Only an infinitesimal energy is needed to drive the spin to follow the magnetic field. The above measured results are not the actual spin values of the particles before the measurement, but the spin values polarized by the magnetic field of instrument. These values already have little to do with the spin values before the measurement.

To sum up, we find that the Bell's inequality is only a strange mathematical theorem, which has not clear corresponding relationship with the physical process that Bell wants to explain. The physical content it represents is actually unclear. There are also a lot of studies that question Bell's inequality from other aspects[18–26].

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III. SPIN CORRELATION FUNCTION OF ENTANGLED PARTICLE PAIR

The above analysis shows that, the physical meaning represented by Bell's inequality is actually unclear. Now let's check the calculation of spin correlation function of entangled particles in quantum mechanics[3, 27]. As in the previous section, denoting the particle has coordinate $\vec{x}$ and moves to $\vec{X}$, and the other has coordinate $\vec{y}$ and moves to $\vec{Y}$. Their wave function is given by $\psi(t, \vec{x}, \vec{y})$. Record the spin of the particle measured at $t = t_a$, place $\vec{X}$ and dirction $\mathbf{a}$ as $A(\mathbf{a}, \vec{X}, t_a)\frac{\hbar}{2}$, and measured at $t = t_b$, place $\vec{Y}$ and dirction $\mathbf{b}$ as $B(\mathbf{b}, \vec{Y}, t_b)\frac{\hbar}{2}$.

The following is the expression of quantum theory, and the wave function matrix of this entangled particle pair is represented as 'Bell state'

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow(1)\rangle \otimes |\downarrow(2)\rangle - |\downarrow(1)\rangle \otimes |\uparrow(2)\rangle]. \tag{10}$$

The expected value of quantum mechanics for the correlation function $E(\mathbf{a}, \mathbf{b})$ is given by

$$\langle E(\mathbf{a}, \mathbf{b})\rangle_\psi = \langle\psi|\hat{\sigma} \cdot \mathbf{a} \otimes \hat{\sigma} \cdot \mathbf{b}|\psi\rangle. \tag{11}$$

However, there are the following problems in the calculation of the above formula:

I. From the point of view of quantum theory, the spin direction of the entangled particles is uncertain until it is measured, and the statistical expectations should be calculated averagely in all directions. However, in (10), the state functions of entangled pairs are polarized, and the direction can only be along the direction of $\pm z$, only up or down cannot be determined and the probability is 50% each.

II. In (10) and (11), the direct product is Kronecker product $A \otimes B \equiv (a_{kl}B)$. Obviously, two entangled particles $(|1\rangle, |2\rangle)$ are completely symmetrical, which particle is the first one or the second one is completely designated. Similarly, the two measuring instruments $(\mathbf{a}, \mathbf{b})$ are also completely symmetrical. But the calculating result of the direct product is neither symmetrical nor antisymmetrical to $(|1\rangle, |2\rangle)$ or $(\mathbf{a}, \mathbf{b})$, which is clearly illogical in theory. How about a few more entangled particles?

III. It is clear that there is also a time-dependent problem in calculating statistical expectations in (11). Although the final result of quantum theory

$$\langle E(\mathbf{a}, \mathbf{b})\rangle_\psi = -\cos(\mathbf{a}, \mathbf{b}) \tag{12}$$

is correct, the result was only obtained by guess, because the concept of direct product for multi-particle systems is not reasonably defined. The original intention of EPR's paper is to question the unclear concepts of quantum theory, but the problem is still not solved.

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Now we deal with this problem according to normal logic, and calculate the expected value of the correlation function reasonably. For the description of particle spin, to use Clifford algebra is more convenient, and the calculation is more standard and simple. The element in three-dimensional Euclidean space is given by

$$d\mathbf{x} = \sigma_a dx^a = \sigma^a dx_a, \tag{13}$$

in which the generators σ_a ($a \in \{1, 2, 3\}$) meet the $C\ell_{3,0}$ Clifford algebra

$$\sigma_a\sigma_b + \sigma_b\sigma_a = 2\delta_{ab}, \quad \sigma^a\sigma^b + \sigma^b\sigma^a = 2\delta^{ab}, \quad \sigma_a = \delta_{ab}\sigma^b.$$

The simplest representation of these generators is the Pauli matrices.

$$\sigma = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}.$$

They form a natural representation of the vector basis of the 3-d Euclidean space. The basis of the complete Clifford algebra is given by

$$I \in \Lambda^0, \quad \sigma_a \in \Lambda^1, \quad \sigma_{ab} \equiv \sigma_a \wedge \sigma_b = i\epsilon_{abc}\sigma^c \in \Lambda^2, \quad \sigma_{abc} = i\epsilon_{abc}I \in \Lambda^3.$$

They correspond to scalar, vector, pseudo vector and pseudo scalar respectively. Clifford algebra naturally converts geometric calculation into algebra operation, according to which all 3-d Euclidean geometry and field theory can be derived, and the conclusion can be simply extended to the case of curved space[28].

Any unit vector $\hat{r}$ in the spherical coordinate system can be represented as

$$\hat{r} = \hat{x}^a\sigma_a = \sin\theta\cos\varphi\sigma_1 + \sin\theta\sin\varphi\sigma_2 + \cos\theta\sigma_3. \tag{14}$$

The spin operator $\hat{S}_r$ along the direction of $\hat{r}$ can be represented by matrix form

$$\hat{S}_r = \frac{\hbar}{2}\hat{r} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}. \tag{15}$$

Originally, $\hat{S} \in \Lambda^2$ corresponds to a pseudo vector, for the sake of simplicity, we ignore the factor $i\epsilon_{abc}$. It is easy to check the eigenvalues of $\hat{S}_r$ are $\pm\frac{1}{2}\hbar$ and the corresponding characteristic vectors are given by

$$|\uparrow\rangle_r = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\varphi} \end{pmatrix}, \qquad |\downarrow\rangle_r = \begin{pmatrix} \sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}. \tag{16}$$

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Therefore, before the spin of the particle is polarized, the correct form of the state vector represented by (10) should be

$$|\psi\rangle_r = |\uparrow(1)\rangle_r \otimes |\downarrow(2)\rangle_r. \tag{17}$$

The spin direction of a particle is just the direction of the symmetry axis of the wave function. Because the electromagnetic interaction has dissipation effect for a particle at the non-energy eigenstate, the steady state wave function of each charged particle cannot be a combination of several energy eigenstates. Therefore, representations such as (10) are contrary to logical and physical laws and cannot be realistic. The expected value of the correlation function $E(\mathbf{a},\mathbf{b})$ should be

$$\begin{aligned}\langle E(\mathbf{a},\mathbf{b})\rangle &= g\frac{1}{4\pi}\cdot\frac{4}{\hbar^2}\int_\Omega \langle\psi|\hat{S}_1\cdot\mathbf{a}\otimes\hat{S}_2\cdot\mathbf{b}|\psi\rangle_r \sin\theta d\theta d\varphi \\ &= -\frac{g}{4\pi}\int_\Omega \langle\psi|\hat{r}\cdot\mathbf{a}\otimes\hat{r}\cdot\mathbf{b}|\psi\rangle_r \sin\theta d\theta d\varphi,\end{aligned} \tag{18}$$

Where Ω represents the unit sphere, and the integral is the expected value of quantum mechanics, which is the pure geometric projection average. The above formula uses the conditions $\vec{S}_1 = -\vec{S}_2$. g is a measuring factor, which reflects the dynamic effect of the measuring instrument on the particle spin, that is, the polarization of the magnetic field on the particle spin.

In accordance with the logical rules, the calculation of correlation function should be under the conditional probability, which is equivalent to the calculation of the direct product in the above formula is just ordinary multiplication, and the spin projection calculation of two particles is separated as follows,

$$\begin{aligned}\langle E(\mathbf{a},\mathbf{b})\rangle &= -\frac{g}{4\pi}\int_\Omega \langle\uparrow|\hat{r}\cdot\mathbf{a}|\uparrow\rangle_r\cdot\langle\downarrow|\hat{r}\cdot\mathbf{b}|\downarrow\rangle_r \sin\theta d\theta d\varphi \\ &= -\frac{g}{4\pi}\int_\Omega (a_1\sin\theta\cos\varphi + a_2\sin\theta\sin\varphi + a_3\cos\theta)\cdot \\ &\qquad (b_1\sin\theta\cos\varphi + b_2\sin\theta\sin\varphi + b_3\cos\theta)\sin\theta d\theta d\varphi \\ &= -\frac{1}{3}g\mathbf{a}\cdot\mathbf{b} = -\frac{1}{3}g\cos(\mathbf{a},\mathbf{b}).\end{aligned} \tag{19}$$

If $g = 3$, (19) is the same as (12).

$g = 3$ has a profound physical meaning and reflects the polarization of particle spin by the instrument. An extremely weak magnetic field can turn the original spin in the other two vertical directions into the direction of the magnetic field. The projection calculation of quantum theory obviously does not include this dynamic effect of measuring instruments on particles, and such effect cannot also be simply explained by the uncertainty relation.

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(19) shows that the correlation of spins of the entangled particle pair does not have any mysterious content that goes beyond the classical concept and the normal logic, so the previous dispute may be caused by incorrect correspondence between concept and physical reality.

IV. POLARIZATION CORRELATION OF ENTANGLED PHOTONS

Let the angles between the two direction vectors $\mathbf{a}, \mathbf{b}$ and the x axis be a and b, respectively, then $\bar{a} = a + \frac{\pi}{2}$, $\bar{b} = b + \frac{\pi}{2}$ are the angles of two vertical directions. Let $|a\rangle$ and $|b\rangle$ represent the states of polarization of photons in $\mathbf{a}$ and $\mathbf{b}$ directions respectively, in the form of a matrix we have

$$|a\rangle = \begin{pmatrix} \cos a \\ \sin a \end{pmatrix}, \qquad |b\rangle = \begin{pmatrix} \cos b \\ \sin b \end{pmatrix}.$$

The calculation of state function for tangled photon pair in the published literature is given by[3, 27]

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|x(1)\rangle \otimes |x(2)\rangle + |y(1)\rangle \otimes |y(2)\rangle], \tag{20}$$

in which

$$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

is the line polarization state functions in the direction of x and y respectively. (20) obviously does not represent the state of all associated photon pairs. For example, if the angle between the polarization direction of a photon and the x-axis is $0 \leq \varphi < \pi$, the polarization state functions of this photon and the photon with perpendicular polarization direction are respectively given by

$$|\varphi\rangle = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}, \qquad |\varphi_\perp\rangle = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}. \tag{21}$$

Therefore, before the measurement, the state function of the entangled polarized photon pair should be

$$|\psi\rangle = |\varphi(1)\rangle \otimes |\varphi(2)\rangle, \qquad (0 \leq \varphi < \pi). \tag{22}$$

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Denote the operators of projection onto the direction of **a** and **b** respectively by

$$\hat{P}_a = |a\rangle\langle a| = \begin{pmatrix} \cos^2 a & \cos a \sin a \\ \cos a \sin a & \sin^2 a \end{pmatrix},$$
$$\hat{P}_b = |b\rangle\langle b| = \begin{pmatrix} \cos^2 b & \cos b \sin b \\ \cos b \sin b & \sin^2 b \end{pmatrix}.$$

The projection expectation function is given by

$$\begin{aligned} R(a,b) &\equiv \frac{1}{\pi}\int_0^{\pi} \langle\psi|\hat{P}_a \otimes \hat{P}_b|\psi\rangle d\varphi \\ &= \frac{1}{\pi}\int_0^{\pi} \langle\varphi(1)|\hat{P}_a|\varphi(1)\rangle \cdot \langle\varphi(2)|\hat{P}_b|\varphi(2)\rangle d\varphi \\ &= \frac{1}{4} + \frac{1}{8}\cos[2(a-b)]. \end{aligned} \tag{23}$$

Then the expected value of quantum mechanics for the correlation functions of the two measurement directions is as follows:

$$\begin{aligned} \langle E(a,b)\rangle_\psi &= \frac{k}{\pi}\int_0^{\pi} \langle\psi|(\hat{P}_a - \hat{P}_{\bar{a}}) \otimes (\hat{P}_b - \hat{P}_{\bar{b}})|\psi\rangle d\varphi \\ &= k[R(a,b) + R(\bar{a},\bar{b}) - R(a,\bar{b}) - R(\bar{a},b)] \\ &= \frac{1}{2}k\cos[2(a-b)], \end{aligned} \tag{24}$$

where k is the measuring factor, reflecting the dynamic effect of the polarizer on the measurement results. If $k = 2$, (24) is the same as the experimental result[27].

V. DISCUSSION AND SUGGESTION

The mathematical framework of modern quantum mechanics is mainly the following four basic assumptions[6, 29]:

A1. *Any isolated physical system has a Hilbert space as the state vector space, and the state of the system is completely described by a unit state vector.*

A2. *The evolution of a closed quantum system can be characterized by a unitary transformation of a state vector, that is, from t_0 to t we have $|\psi'\rangle = U(t,t_0)|\psi\rangle$.*

A3. *An observable quantity corresponds to an Hermitian operator M, the eigenvalue m of operator M is a possible result of the experimental measurement of observable quantity, and the modular square of the projection from the state vector $|\psi\rangle$ to the eigenvector $|m\rangle$ is the probability of measuring the value of m, that is,*

$$p(m) = \langle\psi|\mathbf{M}_m|\psi\rangle, \qquad \mathbf{M}_m \equiv |m\rangle\langle m|, \qquad \sum_{\forall m}\mathbf{M}_m = I. \tag{25}$$

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A4. *The state space of a composite physical system is the tensor product of the state space of each subsystem. If the state function of the k-th subsystem is* $|\psi_k\rangle$*, the state function of the composite system is given by* $|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \cdots \otimes |\psi_n\rangle$.

As mathematical axioms of linear operators on a Hilbert space, the above framework may be elegant and effective. However, as the physical theory that emphasizes the correspondence between theory and reality, the above framework and concept are broad, vague and fallible. The misunderstanding and debate about quantum mechanics are related to the abstraction and ambiguity of the framework. In what follows, we analyze the defects of the above framework on the specific issues already involved in this paper. From these analyses, we can also see the similarities and differences between the academic styles and requirements of mathematics and physics.

1. In order to be clear, the evolution law of a physical system must be described by specific dynamics, as is the case with all classical theories such as Newton mechanics, electrodynamics and general relativity. But the above framework of quantum theory has lost this scientific tradition, which is the main cause of the confusion. The hypothesis A2 is used to characterize the evolution process of the system, but it is only said that there is a unitary transformation of a state vector. However, the specific information about this unitary operator is not given clearly, the concept is similar to the "Tao" in Chinese philosophy, so this hypothesis is too vague.

   Then, can all unitary transformations describe physical systems? Of course not, so this assumption is too broad. But is it certain that all microsystems can be characterized by a linear unitary transformation? Nor is it, because the evolution operator is nonlinear for an elementary particle with nonlinear potential, this assumption is too grudging in this case. Mathematical axiom system pursues abstraction, generality, purity and compatibility, but physical principles pursue the accuracy of description, the simplicity of expression, the clarity corresponding to reality, and of course, logical compatibility. Then, what assumptions should we choose to describe the evolution of a system? In the author's opinion, in the non-relativistic case, it should be the Schrödinger equation, and if the potential contains spin-magnetic coupling term, the dynamic equation should be the following Pauli's equation[30, 31],

$$i\partial_t \Psi = \mathbf{H}\Psi, \qquad \mathbf{H} = \sum_{k=1}^{N} \mathbf{H}_k + \frac{1}{2}\sum_{k=1}^{N}\sum_{l\neq k} \mathbf{P}_{kl}, \tag{26}$$

   in which $\mathbf{H}$ is the total Hamiltonian operator of the system, $\mathbf{H}_k$ is the energy operator of the k-th particle in external fields, $\mathbf{P}_{kl}$ is the interaction potential between k-th and

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l-th particles. In the case of N electrons we have

$$\mathbf{H}_k = \frac{1}{2m}\hat{p}_k^2 + e\Phi - \frac{e}{m}\vec{S}_k \cdot \vec{B}, \qquad \mathbf{P}_{kl} = \frac{e^2}{4\pi r_{kl}}.$$

The total state function is defined by

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \cdots \otimes |\psi_N\rangle, \qquad \psi_k = \begin{pmatrix} \phi_{k1} \\ \phi_{k2} \end{pmatrix}.$$

Because the Hamiltonian operator is Hermitian, the state function $|\Psi\rangle$, as the solution of Schrödinger equation, satisfies the normalization condition. So, the evolution of the state function satisfies $|\Psi(t)\rangle = U(t, t_0)|\Psi(t_0)\rangle$, which is indeed equivalent to the unitary transformation of a unit vector. However, at this time, the transformation operator $U(t, t_0)$ is an operator determined by $\mathbf{H}$, not a general linear operator in mathematical sense. Since $\mathbf{H}$ is equivalent to the total energy of the system, it has the clearest physical meaning and correspondence to reality. It is natural choice to adopt (26) as the basic principle of quantum mechanics in the case of non-relativistic theory, which is the same as the classical mechanics using Newton's second law as the basic principle.

2. In the case of non relativity, if (26) is used as the basic dynamic equation, then the assumption A1 is superfluous, because the solution of Schrödinger equation or Pauli equation is equivalent to a unit vector in Hilbert space, the assumption A1 is actually a theorem. In addition, the assumption A4 is also superfluous, because the solution of Schrödinger equation of composite system is just like this. The detailed derivation of this conclusion were given in [30–34], and this result is also used in the calculation of (18) and (19). In the above framework, only the assumption A3 is reasonable to some extent, which provides the physical interpretation of the solution to Schrödinger equation and the corresponding rules between theory and measurement. This point has long been discussed in the literature [30]: "The only essentially new concept of the quantum theory imposed on the classical field theory is the hypothesis that an initial state $|A\rangle$ evolving into a final state $|B\rangle$ is undetermined in some cases and the probability is in proportion to $|\langle B|A\rangle|^2$. However $\langle B|A\rangle$ is the projection of $|B\rangle$ on $|A\rangle$, and $1 - |\langle B|A\rangle|^2$ is a measurement of distance between the two vectors, so this interpretation is really reasonable in logic." Thus it can be seen that the above framework A1∼A4 may be not very suitable as a physical theory.

   Besides, the Schrödinger equation (26) has an important advantage, if an operator $\mathbf{A}$ can commutate with the Hamiltonian $\mathbf{H}$, then they have common eigenstates, which

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is the condition to solve the equation by separating variables. If the Hamiltonian $\mathbf{H}$ can be decomposed into two commutating Hermitian operators $\mathbf{H} = \mathbf{A} + \mathbf{B}$, and satisfy $[\mathbf{A}, \mathbf{B}] = 0$, then the equation (26) can be decomposed into two independent Schrödinger equations,

$$i\partial_t \Psi_A = \mathbf{A}\Psi_A, \qquad i\partial_t \Psi_B = \mathbf{B}\Psi_B.$$

This decomposition is the condition that quantum mechanics can analyze some properties of the subsystems separately, such as the coupling effect of spin and magnetic field.

3. The above framework is misleading to researchers. For example, in EPR's paper [1], let the eigenvalues of the physical quantity A acting on system I be $a_1, a_2, a_3, \cdots$, and the corresponding eigenfunctions $u_1(x_1), u_2(x_1), u_3(x_1), \cdots$. Then the wave function of the composite system of two particles is given by

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1), \tag{27}$$

where x_2 is the coordinate of the particle II, $\psi_n(x_2)$ is just coefficients of the series expansion of wave function $\Psi(x_1, x_2)$. If we measure the particle I of the correlated system obtain a physical quantity $A = a_k$, then the state function of the composite system will reduce to a pure state $\Psi = \psi_k(x_2)u_k(x_1)$.

However, at first, the coefficient of $\psi_n(x_2)$ is not necessarily the eigenstate of A acting on particle II, and $\Psi(x_1, x_2)$ is also not necessarily the solution to Schrödinger equation. For two particles moving away relatively, the solution of the Schrödinger equation should be time-dependent, that is, the coefficients in the series (27) should be time-dependent $\psi_k = \psi_k(t, x_2)$. Therefore, according to Schrödinger equation, when the particle I is measured, the particle II does not necessarily collapse to eigenstate. Secondly, if the particles are charged, the electromagnetic interaction has dissipative effect on the non-energy eigenstate, so the non-energy eigenstate is unstable, which has energy exchange with the environment. In this case the system is in the **quantum process**[33]. The wave function can only steadily stay at the energy eigenstate, i.e., the **particle state**, but cannot stay at the mixed state depicted by (27). Similarly, (10) is a mixed state rather than an energy eigenstate, so it is not a solution to Schrödinger equation. If we do not refer to the Schrödinger equation, but only refer to the framework A1∼A4, these problems will be overlooked and concealed.

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There is nothing mysterious about the explanation of probability. For particles in metastable state, the wave function takes an energy eigenstate or 'particle state'. It is the critical point of the energy functional. At this state, the Schrödinger equation is very sensitive to external interference, and at what time the wave function will evolve into which possible eigenstate is indeed uncertain. This situation is similar to a small ball at the top of a smooth sphere, what time and which direction the small ball will roll off are uncertain. However, the 'quantum process' is a deterministic process, and the quantum transition is transient.

4. Newton mechanics for N particles is only a $6N$ dimensional dynamic system, while Schrödinger equation (26) is infinite dimensional, so Schrödinger equation contains far more information than momentum, coordinates and so on. With regard to the relationship between quantum theory and classical mechanics, the correct procedure should be to explain classical mechanics by Schrödinger equation. This procedure requires a logical conceptual system and rigorous reasoning. It is not difficult to solve these problems, see the literature [31–34] for details.

   However, the current interpretation of quantum mechanics is not feasible. For example, quantum mechanics holds that particle momentum is the eigenvalue of momentum operator, but in general, a solution to Schrödinger equation is impossible to be the eigenfunction of $i\nabla$. On the other hand, according to Nöther's theorem, the energy and momentum of particles correspond to the translation invariance of spacetime, and the momentum of any system must exist accurately. This leads to contradiction. In addition, the traditional view that classical mechanics is the limit of quantum mechanics as $\hbar \to 0$ is also improper, how can a basic physical constant take a limit? The real reason should be that when the moving scale of a particle is much larger than its own scale, then its motion can be described by classical mechanics. If a particle goes inside an atom, it has to be described by wave function. In fact, the macro body is also the case. In the solar system, the treatment of the earth as a point particle is accurate, but to study the earth's movement near the earth, it must be treated as continuous medium. Of course, Schrödinger equation (26) itself has limitations, and it can only describe the motion and properties of electrons in atoms and molecules. For more complex cases, it is not enough. For example, in the case of high-speed motion, the relativity effect must be taken into account; the nonlinear potentials are obviously contained in the elementary particles, and so on. In these cases, nonlinear Dirac equation must be used. Although this is another problem, all the researchers

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should keep these boundaries in mind.

5. For Schrödinger equation (26), any reversible and differentiable 1-1 corresponding transformation of the wave function Ψ, such as (2), is mathematically equivalent to the original representation. This is the same as the legal coordinate transformation in the general theory of relativity, and an improper representation of the wave function will only make the problem more complicated. So the Bohm's explanation of hidden variables for quantum mechanics is superfluous. Wave function is the basic property of an elementary particle. Like the electromagnetic field of a charge, there is nothing more important for elementary particles than these fields. The classical concepts such as coordinates, momentum and energy of a particle can be defined strictly by wave function, and the electric charge, the strong charge and so on are only some interaction coefficients in the dynamic equation[31–34]. The microscopic particle is firstly a 4 dimensional field, then a particle in macroscopic view.

6. With respect to the relationship between quantum theory and classical mechanics, some experiments can be designed to illustrate. These experiments should be related to the wave properties and particle properties simultaneously. The experiment to test the Einstein's mass-energy relation is this type[35]. Here we propose again a simpler experiment. In the Stern-Gerlach experiment, assume the particles move along y-axis. The magnetic field to measure the magnetic moment of particles is substantially uniform along this direction, but it is symmetrical with respect to z-axis and non-uniform in the z direction, so that the particle beam is split into two beams in the z direction. This confirms that the spin of an electron is $\pm\frac{1}{2}$.

   If we set up a rotating magnetic field distribution along the y-axis, that is, we convert the magnets in multi-stage Stern-Gerlach experiment into a continuous and spiral one, and set a movable display screen perpendicular to the y-axis. When the particle beam passes through the magnetic field along the y-axis, it also leaves two bright spots on the screen. If the screen moves slowly along the y-axis, we should see two bright spots rotating around the y-axis. This proves that the spin of the particles follows continuously the force line of magnetic field, rather than jumping as described by quantum theory.

[1] Einstein A, Podolsky B, Rosen N., *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Phys. Rev., 1935, 47: 777-780

[2] D. Bohm, *Quantum theory* , New York, Prentice-Hall, Inc., 1951.

[3] Genovese M . *Research on hidden variable theories: A review of recent progresses*. Physics Reports, 2005, 413(6):319-396.

[4] D. Bohm, *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables*. I & II, Phys. Rev. 85 (1952) 166, 180.

[5] D. Bohm, B.J. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*, Routledge, London, 1993.

[6] J. von Neuman, *Mathematical foundations of quantum Mechanics*, Princeton Univ. Press, Princenton, 1955.

[7] Bell J S. *On the Problem of Hidden Variables in Quantum Mechanics*. Review of Modern Physics, 1966, 38(3):447-452.

[8] J. S. Bell, *On the Einstein-Podolsky-Rosen Paradox*, Physics 1, 195 (1964)

[9] Clauser J F, Horne M A, Shimony A and Holt R A, *Proposed Experiment to Test Local Hidden-Variable Theories*. Physical Review Letters, 1971, 23(15):880-884.

[10] S. J. Freedman, J. F. Clauser, *Experimental Test of Local Hidden-Variable Theories*, Phys. Rev. Lett. 28, 938 (1972).

[11] F. M. Pipkin, *Atomic Physics Tests of the Basic Concepts in Quantum Mechanics*, Adv. At. Mol. Phys. 14, 281 (1979);

[12] J. F. Clauser, *Experimental Investigation of a Polarization Correlation Anomaly*, Phys. Rev. Lett. 36, 1223 (1976).

[13] E. S. Fry, R. C. Thompson, *Experimental Test of Local Hidden-Variable Theories*, Phys. Rev. Lett. 37, 465 (1976).

[14] A. Khrennikov, I. Basieva, *Towards Experiments to Test Violation of the Original Bell Inequality*, Entropy 2018, 20, 280; doi:10.3390/e2004028

[15] J. F. Clauser, A. Shimony, *Bell's Theorem. Experimental Tests and Implications*, Rep. Prog. Phys. 41, 1881 (1978).

[16] A. Aspect, J. Dalibard, G. Roger, *Experimental Test of Bell's Inequalities Using Time- Varying Analyzers*, Phys. Rev. Lett. 49, 1804 (1982).

[17] A. Aspect, *Closing the Door on Einstein and Bohr's Quantum Debate*. Physics, 2015, 8:123.

[18] Hess K , Philipp W, *A possible loophole in the theorem of Bell*, Proceedings of the National

Academy of Sciences, 2001, 98(25):14224-14227.

[19] Lochak G, *Has Bell's inequality a general meaning for hidden-variable theories?*, Foundations of Physics, 1976, 6(2):173-184.

[20] Carl H. Brans. *Bell's theorem does not eliminate fully causal hidden variables*, International Journal of Theoretical Physics, 1988, 27(2):219-226.

[21] V. Pankovic, *The impossibility of experimental testing of the hidden variables theory*, Physics Letters A, 1989, 137(4-5):158-160.

[22] Squires E J. *On an alleged "proof" of the quantum probability law*, Physics Letters A, 1990, 145(2-3):67-68.

[23] M. Kupczynski, *EPR Paradox , Locality and Completeness of Quantum Theory*, 2007 AIP Conf. Proc vol 962 274, arXiv:0710.3510

[24] A. Khrennikov, *Bell-Boole Inequality: Nonlocality or Probabilistic Incompatibility of Random Variables?*, Entropy, (2008) 10, 19-32. https://doi.org/10.3390/entropy-e10020019

[25] Avis D, Fischer P, Hilbert A, Khrennikov A, *Single, Complete, Probability Spaces Consistent With EPR-Bohm-Bell Experimental Data*, 2009 Found. Prob. and Physics 750 294-301,

[26] A. Khrennikov, *Bell's inequality: Physics meets Probability*, Information Science 179, N. 5, 492-504 (2009)

[27] K. H. Zhao, W. Y. Luo, *New Concept Physics Tutorial·Quantum Physics(2nd ed, chap. VI, sect. II)*, High Education Press, 2008(in Chinese).

[28] Y. Q. Gu, *Space-Time Geometry and Some Applications of Clifford Algebra in Physics*, Advances in Applied Clifford Algebras, 2018, 28(4):79.

[29] Nielson M A, Chuang I L, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, 2000

[30] Y. Q. Gu, *The Electromagnetic Potential Among Nonrelativistic Electrons*, Advances in Applied Clifford Algebras, **9(1)**, 61-79(1999)

[31] Y. Q. Gu, *Clifford Algebra, Lorentz Transformation and Unified Field Theory*, Adv. Appl. Clifford Algebras **28(2)**:37, (2018)

[32] Y. Q. Gu, *New Approach to N-body Relativistic Quantum Mechanics*, Int. J. Mod. Phys. A22:2007-2020(2007), arXiv:hep-th/0610153

[33] Y. Q. Gu, *Local Lorentz Transformation and Mass-Energy Relation of Spinor*, Physics Essays Vol. 31: 1-6(2018). arXiv:hep-th/0701030

[34] Y. Q. Gu, *The Simplification of Spinor Connection and Classical Approximation*, arXiv:gr-qc/0610001

[35] Y. Q. Gu, *Test of Einstein's Mass-Energy Relation*, Appl. Phys. Res. Vol. 10, No. 1, 1-4(2018).

 doi:10.20944/preprints202002.0128.v1