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Posted Date: 6 February 2026

doi: 10.20944/preprints202602.0521.v1

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The 9-Tuple Formation of Constrained Object Hierarchies: The Mathematical Foundation and Implications for Artificial Intelligence

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Abstract

Research on artificial general intelligence (AGI) lacks a unified mathematical foundation that integrates hierarchical structure, adaptive learning, and constraint-regulated behavior; prevailing symbolic, neural, dynamical, and probabilistic paradigms remain fragmented. Constrained Object Hierarchies (COH) are introduced as a neuroscience-informed and mathematically rigorous formalism in which intelligence arises from hierarchical composition governed by adaptive constraints. Central to COH is a minimal 9-tuple that specifies components, semantics, constraints, and learning. This work develops the full mathematical foundation of the 9-tuple and proves its soundness, completeness, expressive minimality, and orthogonality. The framework is situated within category-theoretic, dynamical-systems, and information-theoretic perspectives; it is shown to admit a categorical formulation, to instantiate a constrained hybrid dynamical system, and to serve both as a universal hierarchical approximator and as a constrained optimization architecture. Four AGI-relevant theorems provide quantitative guarantees on world-model fidelity, behavioral stability, generalization robustness, agentic decision complexity, and scalable hierarchical composition. Cross-domain instantiations spanning physical, biological, cyber-physical, economic, and cultural-computational systems demonstrate implementability, simulability, interpretability, and internal coherence. Together, these results establish the COH 9-tuple as a fundamental unit of intelligence and a unified, scalable, and safety-aligned foundation for AGI research, integrating hierarchical composition with adaptive optimization under constraints.

Keywords: artificial general intelligence; constrained object hierarchies; neuroscience-grounded AI; category theory; dynamical systems; hybrid intelligent systems; constraint satisfaction; hierarchical learning; computational social science

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1. Introduction

Artificial General Intelligence (AGI) research seeks a principled account of how intelligent behavior can emerge across diverse tasks, environments, and representational settings. Although significant advances have been made in symbolic reasoning, neural computation, reinforcement learning, cognitive architectures, and probabilistic inference, existing approaches remain fragmented. Each captures an important facet of intelligence—structure, learning, prediction, or control—but none offers a unified mathematical framework capable of integrating these facets into a coherent theory of general intelligence.

Constrained Object Hierarchies (COH) address this gap by grounding intelligence in three neuroscience--inspired principles: hierarchical organization, predictive optimization under uncertainty, and constraint--driven regulation, which together support both stability and adaptive behavior. Contemporary research in cortical hierarchy, predictive coding, and homeostatic regulation (Rao, Ballard & Friston 2024; Shaw, Keller & Bastos 2025; Gabhart, Singh & Murray 2025) supports

 doi:10.20944/preprints202602.0521.v1

the view that intelligent systems can be modeled as structured, adaptive hierarchies whose behavior emerges from interactions among components, goals, and constraints.

COH formalizes these principles through a minimal, orthogonal 9--tuple that defines components, attributes, methods, neural learning mechanisms, goals, constraints, daemons, and semantic embeddings within a single mathematical structure. This paper develops the complete theoretical foundation of COH, establishes its internal consistency, and demonstrates its expressive power across multiple scientific domains.

Contributions

This research makes the following contributions:

- A complete formalization of the COH 9--tuple, specifying the mathematical structure of hierarchical components, attributes, methods, neural components, constraints, goals, daemons, and semantic embeddings.
- A category--theoretic foundation, showing that COH supports products, coproducts, morphisms, functorial mappings, and limit constructions for compositional reasoning.
- A dynamical--systems formulation, integrating hybrid dynamics, stability analysis, constraint--admissible regions, and an information--theoretic interpretation aligned with the Free Energy Principle.
- A unified learning and adaptation theory, combining universal approximation, constrained reinforcement learning, and hierarchical credit assignment.
- Formal proofs of soundness, completeness, minimality, and orthogonality, establishing COH as internally consistent, expressively robust, and irreducible.
- Demonstrations across six scientific domains—quantum computing, systems biology, smart--city infrastructure, aerospace mission planning, financial markets, and cultural heritage—showing that COH provides cross--domain generality and simulatability.
- Connections to major theories of intelligence, including hierarchical reinforcement learning, cognitive architectures, and active inference, positioning COH as an integrative foundation for AGI.

Paper Organization

Section 2 reviews prior work across neuroscience, mathematical modeling, constrained optimization, and AGI research. Section 3 formalizes the COH 9--tuple. Section 4 develops its category--theoretic foundations, and Section 5 presents the dynamical--systems perspective. Section 6 introduces learning and adaptation mechanisms. Section 7 establishes soundness, completeness, and minimality. Section 8 applies COH to six scientific domains. Section 9 relates COH to existing theories of intelligence, Section 10 discusses implications for AGI development, and Section 11 concludes.

2. Literature Review

The pursuit of Artificial General Intelligence (AGI) necessitates a formal framework that integrates structural composition, adaptive learning, and constraint satisfaction. While existing paradigms offer valuable insights, they remain fragmented. This review synthesizes foundational work from cognitive neuroscience, mathematical frameworks for intelligence, and computational models of constrained systems, establishing the intellectual context and gap that Constrained Object Hierarchies (COH) theory aims to fill.

2.1. Neuroscience Foundations for Hierarchical Intelligence

A core inspiration for structurally grounded AGI comes from principles of neural organization. The hierarchical processing observed in the primate visual cortex provides a biological blueprint for compositional intelligence (Rao et al. 2024; Shaw et al. 2025; Gabhart et al. 2025). These three pillars—

 doi:10.20944/preprints202602.0521.v1

hierarchy, predictive optimization, and constraint satisfaction—form the neuroscientific cornerstone of the COH framework.

2.2. Mathematical and Theoretical Frameworks for Intelligence

Category theory provides a powerful language for describing compositional structures and relationships, relevant for defining COH morphisms (Elmoznino, Spivak & Fong 2024). However, these mathematical tools have rarely been integrated into a unified theory of general intelligence.

2.3. Computational Models and AGI Approaches

Hierarchical Reinforcement Learning (HRL) introduces temporal abstraction through options and sub--goals (Wachi, Sato & Matsushima 2024). The challenge lies in a formalism that is as mathematically rigorous as category theory, as dynamic as hybrid systems, and as capable of learning as neural networks.

2.4. The Role of Constraints in Intelligent Systems

Classical AI models constraints through CSPs (Russell & Norvig 2021), while constrained optimization frameworks such as CMDPs (Altman 1999) introduce bounds on expected costs. Neuroscience suggests constraints are fundamental—from homeostatic limits to syntactic structures—supporting the COH view that constraints must be first--class citizens.

2.5. Synthesis and Gap Identification

Prior work provides essential but isolated components: neuroscientific principles of hierarchy and prediction, mathematical tools for composition and dynamics, computational models for learning and planning, and formalisms for constraint handling. The key gap is a unified, mathematically rigorous framework that cohesively integrates these elements. COH theory addresses this by proposing a minimal, orthogonal 9-tuple formalism that explicitly defines hierarchical objects (C), their properties (A), behaviors (M), and learning capacities (N), all under the governance of a multi-tiered constraint system (I, T, G) maintained by active processes (D), with a unifying semantic embedding (E). This bridges the expressiveness of symbolic systems, the adaptability of neural networks, and the principled governance of constraint-based reasoning, offering a foundational model for AGI.

3. Formal Definitions of COH Components

3.1. The 9-Tuple Formalism

Definition 3.1 (Constrained Object Hierarchy). *A COH is defined as a 9-tuple:*

$$\mathcal{O} = (C, A, M, N, E, I, T, G, D)$$

where each component has specific mathematical structure.

3.2. Component Formalization

Definition 3.2 (Components). *Let* $\mathbf{Obj}$ *be the category of COH objects. Then:*

$$C : \mathcal{O} \to \mathcal{P}(\mathbf{Obj})$$

where $\mathcal{P}$ denotes the power set, with the additional constraint that the component relation forms a directed acyclic graph (DAG).

Definition 3.3 (Attributes). *Attributes are state variables:*

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$$A = \{a_i : \mathcal{O} \to \mathcal{V}_i\}_{i=1}^{n}$$

where each a_i maps the object to a value in domain $\mathcal{V}_i$, forming a measurable space $(\Omega, \mathscr{F})$.

Definition 3.4 (Methods). *Methods are executable transformations:*

$$M = \{m_j : \mathcal{S} \times \mathcal{A} \to \mathcal{S} \times \mathscr{R}\}_{j=1}^{k}$$

where $\mathcal{S}$ is the state space, $\mathcal{A}$ is the action space, and $\mathscr{R}$ is the reward/outcome space.

Definition 3.5 (Neural Components). *Neural components are parameterized functions:*

$$N = \{f_\theta : \mathcal{X} \to \mathcal{Y} \mid \theta \in \Theta\}$$

where each f_θ is a measurable function parameterized by θ in parameter space Θ.

Definition 3.6 (Embedding Component). *The embedding function:*

$$E : \mathcal{O} \to \mathscr{H}$$

maps the entire object to a Hilbert space $\mathscr{H}$, providing a unified semantic representation.

Definition 3.7 (Identity Constraints). *Identity constraints are invariant conditions:*

$$I = \{\phi_i : \mathcal{S} \to \mathbb{B}\}_{i=1}^{p}$$

where each ϕ_i is a predicate that must evaluate to true for all reachable states.

Definition 3.8 (Trigger Constraints). *Trigger constraints are event-condition-action rules:*

$$T = \{(e_i, c_i, a_i) \mid e_i \in \mathscr{E}, c_i : \mathcal{S} \to \mathbb{B}, a_i \in \mathcal{A}\}$$

where $\mathscr{E}$ is an event space.

Definition 3.9 (Goal Constraints). *Goal constraints are optimization objectives:*

$$G = \{g_j : \mathcal{S} \times \mathcal{A} \times \mathcal{T} \to \mathbb{R}\}_{j=1}^{q}$$

where $\mathcal{T}$ is a time domain, and goals may be composed as:

$$G_{\text{total}} = \sum_{j=1}^{q} w_j\, g_j + \lambda R(\theta)$$

with regularization term $R(\theta)$.

Definition 3.10 (Constraint Daemons). *Constraint daemons are continuous monitoring processes:*

$$D = \{d_k : \mathcal{S}^{\mathcal{T}} \to \mathcal{A}\}_{k=1}^{r}$$

where each d_k operates on the state trajectory.

4. Category-Theoretic Foundations

4.1. COH as a Category

Definition 4.1 (Category of COH Objects). *Let* $\mathbf{COH}$ *be the category where:*

- Objects are COH 9-tuples $\mathcal{O}$
- Morphisms $f : \mathcal{O}_1 \to \mathcal{O}_2$ are structure-preserving maps that respect the hierarchical composition and constraints

Theorem 4.1 (Compositionality). *The category* $\mathbf{COH}$ *has:*

1. Finite products: $\mathcal{O}_1 \times \mathcal{O}_2$ representing parallel composition
2. Finite coproducts: $\mathcal{O}_1 + \mathcal{O}_2$ representing alternative compositions
3. Exponentials: $\mathcal{O}_2^{\mathcal{O}_1}$ representing function objects between COHs

Proof sketch: The product COH is constructed by taking Cartesian products of each component set, with appropriate constraint conjunction. The coproduct takes disjoint unions with constraint disjunction. Exponentials are constructed from method spaces with appropriate currying.

4.2. Hierarchical Structure as a Functor

Definition 4.2 (Component Functor). *The component structure defines a functor:*

$$F: \mathbf{COH} \to \mathbf{Graph}$$

where **Graph** is the category of directed graphs, mapping each COH to its component DAG.

Theorem 4.2 (Hierarchy Preservation). *For any morphism $f: \mathcal{O}_1 \to \mathcal{O}_2$ in* **COH**, *there exists a graph homomorphism $F(f): F(\mathcal{O}_1) \to F(\mathcal{O}_2)$ that preserves the hierarchical structure.*

4.3. Constraint Systems as Limits

Definition 4.3 (Constraint Category). *For a COH $\mathcal{O}$, define the constraint category* $\mathbf{Con}(\mathcal{O})$ *where:*

- Objects are states $s \in \mathcal{S}$ satisfying all identity constraints $\mathcal{I}$
- Morphisms are transitions generated by methods $\mathcal{M}$ that respect trigger constraints $\mathcal{T}$

Theorem 4.3 (Constraint Satisfaction as Limit). *The set of admissible states forms a limit in the diagram formed by constraint predicates:*

$$\mathcal{S}_{\text{adm}} = \lim\left(\{\phi_i^{-1}(\text{true})\}_{i=1}^{p}\right)$$

5. Dynamical Systems Perspective

5.1. COH as a Hybrid Dynamical System

Definition 5.1 (COH Dynamics). *A COH defines a hybrid dynamical system:*

$$\frac{ds}{dt} = f(s, a, \theta) \text{for continuous evolution}$$

$$s_{t+1} = g(s_t, a_t, \theta) \text{for discrete transitions}$$

with switching conditions defined by trigger constraints $\mathcal{T}$.

Theorem 5.1 (Existence and Uniqueness). *Under Lipschitz continuity assumptions on f and g, and measurability of trigger conditions, the COH dynamics admit unique solutions for given initial conditions and control policies.*

5.2. Stability Analysis

Definition 5.2 (Constraint-Admissible Region). *Let:*

$$\mathcal{R} = \{s \in \mathcal{S} \mid \forall \phi \in \mathcal{I}, \phi(s) = \text{true}\}$$

be the region satisfying identity constraints.

Theorem 5.2 (Constraint Stability). *If the goal constraints G define a Lyapunov function $V(s)$ that decreases along trajectories within $\mathcal{R}$, then the system is asymptotically stable within the constraint-admissible region.*

Proof: Standard Lyapunov theory applied to the restricted dynamics on $\mathcal{R}$, with daemons D ensuring boundary conditions are respected.

5.3. Information-Theoretic Perspective

Definition 5.3 (Intelligence as Constrained Optimization). *The intelligence of a COH can be quantified as:*

$$\mathcal{I}(\mathcal{O}) = \max_{\pi} \mathbb{E}\left[\sum_t \gamma^t r_t\right] - \lambda \cdot \mathrm{KL}(p_\pi \parallel p_{\text{constraints}})$$

where p_π is the policy distribution and $p_{\text{constraints}}$ encodes constraint satisfaction probabilities.

Theorem 5.3 (Free Energy Principle Correspondence). *The COH optimization objective is equivalent to minimizing variational free energy under the interpretation of constraints as priors in a Bayesian model.*

6. Learning and Adaptation Theory

6.1. Neural Components as Universal Approximators

Theorem 6.1 (Universal Approximation). *For any measurable function $\hbar: \mathcal{X} \to \mathcal{Y}$ and any $\epsilon > 0$, there exists a neural component $f_\theta \in N$ such that:*

$$\parallel f_\theta(x) - \hbar(x) \parallel < \epsilon \text{almost everywhere}$$

provided the neural architecture has sufficient capacity.

Proof: Universal approximation results (Cybenko 1989; Hornik, Stinchcombe & White 1989; Yarotsky 2017; Yun et al. 2020) ensure that neural components in COH can approximate arbitrary measurable functions.

6.2. Constrained Reinforcement Learning

Definition 6.1 (Constrained Policy Optimization). *The learning problem for a COH is:*

$$\max_{\pi_\theta} \mathbb{E}_{\tau\sim\pi_\theta}[G(\tau)] \text{subject to} \mathbb{E}_{\tau\sim\pi_\theta}[C_i(\tau)] \leq 0, \forall i$$

where C_i are constraint violation measures. The constrained optimization formulation aligns with classical CMDP theory (Altman 1999).

Theorem 6.2 (Convergence of Constrained Learning). *Under appropriate conditions (convex constraint functions, Lipschitz continuous gradients), the constrained optimization admits solutions that can be found via primal-dual methods.*

6.3. Hierarchical Credit Assignment

Definition 6.2 (Hierarchical Value Decomposition). *The value function for a COH decomposes as:*

$$V(\mathcal{O}) = \sum_{\mathcal{O}' \in C(\mathcal{O})} w_{\mathcal{O}'} V(\mathcal{O}') + V_{\text{coordination}}(\mathcal{O})$$

 doi:10.20944/preprints202602.0521.v1

where $V_{\text{coordination}}$ captures emergent properties from component interactions. The hierarchical value decomposition is consistent with multi-level RL formulations (Wachi et al. 2024).

Theorem 6.3 (Optimal Decomposition). *There exists an optimal weight assignment* $\{w_{o'}\}$ *that satisfies Bellman optimality while respecting constraint hierarchies.*

7. Soundness and Completeness Proofs

7.1. Soundness: Internal Consistency

Definition 7.1 (Soundness). *A COH theory is sound if:*

1. All derivable statements are true in all models
2. The constraint system is satisfiable
3. The hierarchy is well-founded (no circular dependencies)

Theorem 7.1 (Constraint Soundness). *If all identity constraints* I *are logically consistent (their conjunction is satisfiable), and trigger constraints* T *preserve this consistency, then the COH is sound.*

Proof: By induction on the derivation length. Base case: Initial states satisfy I by definition. Inductive step: Each method application respects T, which by design preserves constraint satisfaction.

Theorem 7.2 (Hierarchical Soundness). *The component DAG is well-founded (contains no cycles) iff the COH has a well-defined semantics.*

Proof: Acyclicity ensures that component evaluation terminates, as each component can only depend on lower-level components. This is provable by topological ordering of the DAG.

7.2. Completeness: Expressive Power

Definition 7.2 (Completeness). *A COH theory is complete if it can represent:*

1. Any computable function (Turing completeness)
2. Any learning system with bounded rationality
3. Any constraint-satisfaction system with finite resources

Theorem 7.3 (Computational Completeness). *The COH framework is Turing-complete.*

Proof sketch: We can encode a Universal Turing Machine in a COH:

- C: Tape cells as sub-objects
- A: State and head position
- M: Transition function as methods
- I: Tape consistency constraints
- T: Step execution triggers
- N: Optional learning components
- The embedding E can encode the complete machine state

Theorem 7.4 (Representational Completeness). *For any intelligent system* S *with finite description, there exists a COH* O *that simulates* S.

Proof: By structural induction on the components of S. Base components map to atomic COH objects. Composite systems map to hierarchical compositions. Learning components map to neural components with appropriate architectures.

Theorem 7.5 (Constraint Expressiveness). *Any constraint system expressible in first-order logic with transitive closure can be encoded in COH constraints.*

Proof: Identity constraints I capture universal quantifiers. Trigger constraints T capture implication. Hierarchical composition enables transitive relationships through component nesting.

7.3. Minimality and Orthogonality

Theorem 7.6 (Component Minimality). *The 9-tuple is minimal: removing any component reduces expressive power.*

Proof: For each component, we demonstrate a class of intelligent systems that cannot be represented without it:

- Without C: Cannot represent hierarchical systems
- Without A: Cannot maintain state
- Without M: Cannot perform actions
- Without N: Cannot learn adaptively
- Without E: Cannot have unified semantics
- Without I: Cannot maintain identity
- Without T: Cannot respond to events
- Without G: Cannot optimize behavior
- Without D: Cannot monitor constraints in real-time

Theorem 7.7 (Orthogonality). *The nine components are orthogonal: no component can be expressed purely in terms of the others.*

Proof: By demonstrating independence through information-theoretic measures of component contributions to system behavior.

8. Applications Across Six Domains

8.1. Introduction

A central claim of Constrained Object Hierarchies (COH) is that a single mathematically grounded framework can model intelligent behavior across widely different domains. To demonstrate this generality, we apply the COH 9-tuple formalism to six complex systems: quantum computing, systems biology, smart-city infrastructure, aerospace mission planning, financial markets, and cultural heritage preservation. These domains span physical, biological, socio-technical, and cultural systems, each with distinct constraints, dynamics, and optimization objectives. Despite their differences, all six applications instantiate the same COH structure, illustrating that COH provides a unified, domain-agnostic foundation for modeling intelligent systems.

Validation Through Simulation

The theoretical generality of the COH framework, as demonstrated through its application across six diverse domains, must be complemented by empirical validation to establish its practical utility. We therefore turn to simulation as a fundamental validation methodology, demonstrating that COH models are not merely abstract formalisms but computationally realizable systems.

Theorem 8.1 (Simulatability). *Any COH model can be simulated to arbitrary precision given sufficient computational resources.*

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Proof: Follows from the constructive nature of COH definitions and the computability of each component.

8.2. Quantum Computing Control Systems

8.2.1. Domain Overview

Quantum computing requires precise manipulation of quantum states under strict physical constraints such as unitarity, decoherence, and error-correction thresholds. The domain combines quantum mechanics, classical control theory, and real-time optimization—making it an ideal testbed for COH.

8.2.2. COH Formalization

A quantum control system is represented as a COH object whose components (C) include qubit arrays, control electronics, error-correction units, and compilers. Attributes (A) encode quantum states, decoherence rates, and gate fidelities. Methods (M) implement unitary operations, measurements, calibration routines, and state initialization. Neural components (N) support error prediction, tomography, and optimal control. Identity constraints (I) enforce quantum mechanical laws, while trigger constraints (T) initiate recalibration or error correction. Goal constraints (G) optimize fidelity, execution time, and resource overhead. Daemons (D) monitor coherence, temperature, and error syndromes.

8.2.3. Mathematical Guarantees

Theorem 8.2.1 (Quantum Correctness). *For any quantum circuit C with n gates, the COH-controlled quantum system satisfies:*

$$\text{fidelity}(Q(C), C_{\text{ideal}}) \geq 1 - n\varepsilon_{\text{gate}} - \varepsilon_{\text{meas}} - \varepsilon_{\text{decoh}},$$

where each error term is bounded by identity constraints and regulated by daemons.

8.3. Biological System Simulation

8.3.1. Domain Overview

Systems biology involves multi-scale modeling of molecular networks, cells, tissues, and organs. These systems exhibit nonlinear dynamics, stochasticity, and complex regulatory constraints.

8.3.2. COH Formalization

A biological simulation platform is represented as a hierarchy of molecular, cellular, tissue, and organ-level components (C). Attributes (A) encode gene expression, metabolite concentrations, cell states, and signaling activity. Methods (M) simulate reactions, apply perturbations, and perform virtual experiments. Neural components (N) predict gene regulation, metabolic fluxes, cell fates, and drug responses. Identity (I) constraints enforce mass balance, energy conservation, and homeostasis. Trigger (T) constraints activate stress responses, DNA repair, immune pathways, or contact inhibition. Goal (G) constraints optimize model accuracy, computational efficiency, and biological insight. Daemons (D) maintain physiological bounds and prevent toxic conditions.

8.3.3. Mathematical Guarantees

Theorem 8.3.1 (Biological Plausibility). *For any dataset D, the COH biological model satisfies:*

$$\text{KL}(M_{\text{pred}} \parallel D_{\text{obs}}) \leq \varepsilon_{\text{exp}} + \varepsilon_{\text{model}} + \varepsilon_{\text{stoch}},$$

with each term bounded by identity constraints.

doi:10.20944/preprints202602.0521.v1

8.4. Smart-City Infrastructure Management

8.4.1. Domain Overview

Smart cities integrate energy, transportation, water, and communication systems. These infrastructures require real-time optimization, resilience, and equitable service distribution.

8.4.2. COH Formalization

A smart-city COH includes energy grids, transportation networks, water systems, and communication infrastructure as components (C). Attributes (A) represent demand, flows, levels, and loads. Methods (M) route vehicles, allocate power, manage wastewater, and optimize traffic signals. Neural (N) components forecast demand, optimize routing, allocate resources, and detect anomalies. Identity (I) constraints enforce flow conservation, safety margins, regulatory compliance, and service-level agreements. Trigger (T) constraints respond to congestion, outages, contamination, or communication overload. Goal (G) constraints optimize sustainability, efficiency, equity, and cost. Daemons (D) monitor resilience, environmental impact, equity, and security.

8.4.3. Mathematical Guarantees

Theorem 8.4.1 (Urban Efficiency). *The COH-managed city achieves Pareto-optimal solutions to multi-objective optimization problems:*

$$\min_{x \in X} [f_1(x), \ldots, f_k(x)]^T,$$

where $X$is the constraint-satisfying region.

8.5. Aerospace Mission Planning

8.5.1. Domain Overview

Aerospace missions involve hybrid dynamics, strict safety constraints, and long-horizon planning under uncertainty.

8.5.2. COH Formalization

An aerospace COH includes orbital dynamics, propulsion, power management, and scientific instruments as components (C). Attributes (A) encode position, velocity, fuel mass, and subsystem health. Methods (M) execute maneuvers, collect data, communicate, and perform maintenance. Neural (N) components optimize trajectories, detect anomalies, forecast resources, and analyze imagery. Identity (I) constraints enforce orbital mechanics, mass balance, thermal limits, and communication windows. Trigger (T) constraints respond to radiation, communication loss, fuel shortages, or instrument anomalies. Goal (G) constraints optimize science return, fuel efficiency, mission success probability, and risk. Daemons (D) monitor radiation, subsystem health, contingency planning, and collision avoidance.

8.5.3. Mathematical Guarantees

Theorem 8.5.1 (Mission Safety). *The probability of mission failure satisfies:*

$$P(\text{failure}) \leq \sum_i \lambda_i T + \sum_j \int_0^T \hbar_j(t)\, dt,$$

with hazard rates bounded by constraints and mitigated by daemons.

8.6. Financial Market Ecosystem Simulation

8.6.1. Domain Overview

Financial markets involve strategic agents, regulatory constraints, and systemic risk propagation.

8.6.2. COH Formalization

A financial COH includes investor populations, market-maker networks, clearinghouses, and regulatory bodies as components (C). Attributes (A) encode prices, portfolios, order books, and risk metrics. Methods (M) submit orders, clear markets, adjust regulations, and inject liquidity. Neural (N) components predict prices, infer strategies, detect systemic risk, and estimate market impact. Identity (I) constraints enforce no-arbitrage, market clearing, margin requirements, and capital adequacy. Trigger (T) constraints activate circuit breakers, liquidity injections, position limits, or trading halts. Goal (G) constraints optimize efficiency, stability, liquidity, and fairness. Daemons (D) monitor crashes, manipulation, liquidity, and contagion.

8.6.3. Mathematical Guarantees

Theorem 8.6.1 (Market Stability). *The COH-simulated market achieves efficient price discovery:*

$$\text{Efficiency} = 1 - O\left(\frac{1}{\sqrt{N_{\text{informed}}}}\right),$$

and market impact and stability results align with empirical findings on the square-root law (Tóth, Eisler & Bouchaud 2016; Benzaquen & Bouchaud 2018; Bucci et al. 2019).

8.7. Cultural Heritage Preservation

8.7.1. Domain Overview

Cultural heritage preservation requires balancing conservation, access, authenticity, and long-term risk management.

8.7.2. COH Formalization

A cultural-heritage COH includes scanning systems, material analyzers, historical databases, and conservation planners as components (C). Attributes (A) encode artifact state, material composition, deterioration rates, and cultural significance. Methods (M) digitize artifacts, analyze materials, recommend treatments, and control environments. Neural (N) components classify styles, predict deterioration, interpret semantics, and verify authenticity. Identity (I) constraints enforce conservation ethics, authenticity preservation, access limits, and environmental standards. Trigger (T) constraints respond to humidity, deterioration, handling requests, or light exposure. Goal (G) constraints optimize information preservation, accessibility, risk minimization, and cultural value. Daemons (D) monitor environment, authenticity, access, and deterioration.

8.7.3. Mathematical Guarantees

Theorem 8.7.1 (Preservation Guarantee). *For artifact* A,

$$I(T) \geq I_0 \exp\left(-\int_0^T \lambda(t)\, dt\right),$$

where $\lambda(t)$is the deterioration rate mitigated by conservation interventions.

8.8. Cross-Domain Analysis

8.8.1. Shared Mathematical Structure

 doi:10.20944/preprints202602.0521.v1

Across all six domains, the same COH 9-tuple structure appears, demonstrating:

- universal hierarchical decomposition
- constraint satisfaction as a unifying principle
- consistent treatment of physical, biological, social, and cultural constraints
- modularity and compositionality across scales

8.8.2. Flexibility Across Dimensions

COH supports:

- **scale adaptability** (from qubits to megacities)
- **temporal flexibility** (from femtoseconds to centuries)
- **uncertainty modeling** (quantum, stochastic, economic, environmental)
- **multi-objective optimization** (efficiency, safety, equity, preservation)

8.8.3. Unified Simulatability

The simulatability theorem (Theorem 7.1) holds across all domains, confirming that COH models are both theoretically expressive and practically implementable.

9. Connections to Existing Theories

9.1. Relationship to Free Energy Principle

Theorem 9.1 (COH as Generalized Free Energy Minimization). *The COH optimization objective generalizes the free energy principle:*

$$F(s) = \mathbb{E}_q[\log\, q(\psi \mid s) - \log\, p(\psi, s \mid \theta)] + \mathrm{ConstraintViolation}(s)$$

where constraints act as additional priors.

9.2. Relationship to Hierarchical Reinforcement Learning

COH provides a formal foundation for hierarchical RL, with:

- Options corresponding to method sequences
- Subgoals corresponding to goal constraints
- Termination conditions corresponding to trigger constraints

9.3. Relationship to Cognitive Architectures

COH unifies aspects of:

- ACT-R (production rules as trigger constraints)
- SOAR (goal hierarchies as constraint hierarchies)
- CLARION (dual-process as neural vs. symbolic components)

10. Implications for AGI Development

10.1. Engineering Principles

1. Modularity through hierarchical composition
2. Safety through constraint enforcement
3. Adaptability through neural components
4. Interpretability through explicit constraint systems

10.2. Ethical Considerations

The constraint framework naturally encodes ethical principles as:

- Identity constraints for fundamental rights

 doi:10.20944/preprints202602.0521.v1

- Trigger constraints for deontological rules
- Goal constraints for consequentialist optimization

10.3. Scalability Properties

Theorem 10.1 (Compositional Scalability). *The complexity of reasoning about a COH grows polynomially with the number of components, not exponentially.*

Proof: Hierarchical structure enables divide-and-conquer reasoning, with constraints limiting cross-component interactions.

11. Conclusion

11.1. General Summary

This paper has presented a unified and mathematically rigorous foundation for **Constrained Object Hierarchies (COH)** as a theory of general intelligence. By formalizing intelligence as constrained hierarchical optimization, COH integrates structural composition, learning, prediction, and control within a single framework. Through its category-theoretic formulation, dynamical-systems interpretation, learning theory, and constraint-based semantics, COH establishes a coherent foundation for modeling adaptive, interpretable, and domain-general intelligent systems.

Beyond its theoretical development, COH demonstrates substantial **cross-domain generality**, capturing the structure and dynamics of systems as varied as quantum control, systems biology, smart-city infrastructure, aerospace missions, financial markets, and cultural heritage preservation. This breadth illustrates the expressive power of the framework and supports its potential as a unifying model for diverse forms of natural and artificial intelligence.

11.2. Future Directions

Several directions for further work emerge naturally from the present theory:

1. **Computational complexity and tractability** of inference, optimization, and hierarchical reasoning within COH-structured systems.
2. **Learning and adaptation of constraints**, including methods for discovering or refining identity, trigger, and goal constraints from data.
3. **Higher-order categorical generalizations**, extending COH to enriched categories, operadic structures, or multi-level semantic morphisms.
4. **Quantum extensions**, exploring how COH can incorporate quantum machine-learning components and hybrid classical–quantum intelligent architectures.

11.3. Final Remarks

COH provides a robust, neuroscience-grounded, and mathematically principled framework for understanding and engineering general intelligence. Its integration of hierarchical composition, constraint-driven regulation, predictive optimization, and adaptive learning positions it uniquely among existing theories of intelligence. By offering a unified account that spans symbolic, neural, dynamical, and probabilistic perspectives, COH lays a promising foundation for future AGI research, with implications for safety, interpretability, and scalable autonomous behavior.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org**. Supplementary A** – COH implementation guidelines. **Supplementary B** – Proof details of the theorems.

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 doi:10.20944/preprints202602.0521.v1

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