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Article

A Wavelet Quality of Life Multi-index and the Impact of Social Media during the Pandemic and the Crisis

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Abstract: In the present paper, we investigate the impact of the timescale factor on the quality of life index behavior on specific time intervals characterized by the presence of socio-economic, political and/or health severe movements such as pandemics and crises. We essentially aim to show that effectively the quality of life measuring based on a single index in the existing studies may be described more adequately by a variable index due to the social, political, economic and also health environment. The variability discovered is expressed by the existence and the estimation of a multi-index instead of a single one relatively to many factors. Our focus is mainly on the effect of the COVID-19 pandemics on the quality of life. Our model is applied empirically on a sample corresponding to Saudi Arabia as a case of study during the period from January 1990 to December 2021 as main period effected by the COVID-19 pandemic. The sample is based on social media conversations and texts discussing and describing the satisfaction about the quality of life. The study confirms effectively that the role of the timescale factor is more described when considering a multi-index rather than measurement on the whole time interval.

Keywords: Quality of life index; Pandemics, Social media; Mathematical models.

MSC: 62B10, 62B15, 65T60.

JEL Classification: C02, C6, C63, I1, I3, I31, I38.

1. Introduction and literature review

In the society and the literature, there are many methods and indices to evaluate the well-being or what we call the Quality of life (QoL) index. In fact the several methods applied to evaluate the well-being prove the hardness of measuring the QoL accurately and completely. The subjectivity and intangibility of the QoL index evaluation includes healthy, economic, and also social well-beings ([2–4]). In social sciences framework, for example, the quantification of the factors included is not easy and depends on the society, maybe the religion, the climate, etc. Generally, the most used methods are based on questionnaires and binary variables, where the qualitative and generally non quantitative variables are converted to quantitative ones to apply the quantitative models and methods such as statistics. [5–11])

Quality of life is a concept used to evaluate and/or to predict the satisfaction and/or the possibility to be a future satisfactory place or time for well being or well living. In the literature, many studies have been done for the aim of understanding and evaluating the QoL index. However, the majority of existing studies are characterized by their

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qualitative aspect, subjectivity, and also uncertainty due to the lack of an exact or even an approximating quantitative measure in the model. Factors like emotion, joy, stress, sadness, anger, have been applied.

Comparisons to fixed scales and thresholds have been also used, a person is well living if the model results in a QoL value or estimation greater that or better than the threshold value. These minimal values may be related to many factors such as the income, fertility, productivity, education level, health, wealth, materialistic goods. (see [2–4,36,37]).

In many human societies, the QoL is somehow a collective index, where living in a whole large family is already dominant, or at least the family support, and the family ties are still dominant and represent an important and fundamental factor in the stability of society. We may speak for example about religious ties types such as Muslim societies. In KSA, the case study in the present work, individuals feel to be happy and/or well living if the whole family is connected, and each other takes care about relatives.

In other countries where there are different religions, races, beliefs, the QoL depends on security and safety. We may cite here the case of India, south African countries, South America, and even in the USA, where the spread of crime is high. These differences between the nature of the societies, religions, races, traditions ... etc are also other factors like liveability, the place of birth, the place of childhood, family situation and stability, the rate of divorce, the rate of marriage, the satisfaction with the infrastructure. Economic point of view may include factors like the government taxation, government aid, sponsorship programs, freedom and political rights, the rate of employment, and racism. [44–47]. Finally, in sociology, there are also other factors that affect the QoL such as basic literacy, infant mortality, life expectancy ([9,10,38–43]).

Besides, the QoL index may be influenced by the global activities in the society and related markets especially local ones. Indeed, investing will help to open opportunities for local labors to be employed, and thus reduces the rate of unemployment in the society. Local investments themselves may be extended next to reach more regional, continental as well as international levels. This internationalization allows the country to be industrial and exporting more than importing. This will in turn need a qualified labor and educated society, which are factors in the heart of the QoL improving. (See [2,5–8,11,37–43,48–50]).

More backgrounds, discussions, theories as well as conclusions on QoL may be found in [4,12,44–47,52–54] and the references therein.

In several recent studies, the time-scale factor has been proved to be a main factor in studying and understanding the quality of life behaviour and index measuring (See for example [1]). One of the important factor and phenomena that has a great influence on the quality of life may be cite the appearance of pandemics such as the last COVID-19. This pandemic has confirmed that effectively many indices in the society are sensible to the time factor. COVID-19 compared to many cases of pandemics may be considered as a long pandemic, that has and remains affecting the society indices such as the QoL. Indeed, among the factors affecting the daily life, the well-being and the QoL index, the pandemics play a great role and have a strong influence on both individuals and societies. In the last period, the humanity has been affected by the COVID-19 from many points of view and many levels. Workers have lost their jobs, which affected negatively their life. Others suffered from many diseases such as cancer, diabetes, and there are no proof till now that COVID-19 is not the real cause exciting these diseases. The causality between cancer and COVID for example is already a subject to be investigated.

Besides, one of the main sources of information which may inform us nowadays about both concepts of QoL and pandemics is the social media. Since the rapid and challenging developments in technology inducing advancements and also challenges in internet, many social media websites and platforms have been put in hold for public to exchange information about real life events such as pandemics and their influence on our lives.

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The purpose of the present paper lies in this whole topic, and aims to understand the link between the well-being or the QoL index and the impact of pandemics such as the COVID-19 in the framework of social media and the time scale factor.

In the literature, there are several works investigating social sciences generally and involving quantitative measures, such as time series, entropy, fractals and multifractals, chaos, fluctuation analysis, Markov chains, wavelets, etc. In this context, the time-scaling concept permits assessing the dynamical complexity across temporal scales (See [21–33]). However, the literature is still reduced concerning the quantitative models and measures of the QoL. In [1], the authors re-considered a modified version of the mathematical model introduced in [5] to investigate the QoL index. In the present work, we continue to investigate the QOl measurement model in [1] in the framework of a pandemic situation, and derive a QoL multi-index of QoL instead of a single one. The proposed mathematical model depends on the following variables or factors,

- the Gross Domestic Product,
- the Gross National Income,
- the Gross Savings,
- the Unemployment rate,
- the Energy Consumption per Capita,
- the Death Rate,
- the Life Expectancy,
- the Education Index.

Indeed, compared to [5], the model investigated in [1] involved the time-scale to detect the time-wise behavior of the QoL index. One of the main conclusions pointed out in [1] is effectively the time-wise behavior of the QoL index, which appeared to be a non uniform measure as assumed in the existing studies. It instead depends strongly on the situation of the society as well as individuals according to the time-scale. We thus thought that a motivated study will be justified by a multi-index which depends and differs according to sub-periods instead of a whole time interval. In our opinion, the new model or the new study will integrate the movement of the society, and inform about what period is best for well-being, the eventual causes and factors, and also to forecast the ability for a country or region to be really a suitable future destination as a top living city.

The sample of data to be applied is due to Saudi Arabia case, and based on annual values of the variables raised above traded on the period 1991 to 2021. This is a remarkable period characterized by many political, economic and financial movements such as first and second Gulf wars (Iraq wars), Qatar embargo, Yemen war, Syria situation, NEOM project, 2030 KSA vision, the Arab spring, and lastly the COVID-19 pandemic. These facts are strong factors affecting the social, economic as well as political situation. The results will therefore be good basis for understanding current and future situations and may be thus bases of future decisions.

In the next section (section 2), focuses will concern the mathematical model as a quantitative way to evaluate the quality of life. A multiscale wavelet method is applied as in [1] to show the influence of involving the time scale into the model. Section 3 is concerned to the development of our empirical results based on a sample of QoL measures traded on Saudi Arabia case study during December 1, 2019 to December 31, 2021, as a top period for the COVID-19 spreading as well as its effects and consequences. Section 4 is a conclusion, and finally, section A is a brief presentation of the wavelet toolkit.

2. Development of the QoL Mathematical Model

In this section we propose to develop a mathematical model for measuring the QoL index. The starting step begins by considering the recent models developed in [5] based essentially on the economic well-being, and next modified in [1] applied on a time period of time $\mathbb{T} = [0, T]$, $T > 0$. Denote

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- LEB the Life Expectancy at Birth;
- LEI the Life Expectancy Index;
- $DAEd$ the Degree of Access to Education;
- MYS the Mean Years of Schooling;
- $MYSI$ the Mean Years of Schooling Index;
- EYS the Expected Years of Schooling;
- $EYSI$ the Expected Years of Schooling Index;
- RI Revenue Indicator;
- $GNIpc$ Gross National Income at purchasing power parity per capita.

We know that ([1,5–8,37,39,53,54])

$$LEI = \frac{LEB - 20}{65}, \quad MYSI = \frac{MYS}{15}, \quad EYSI = \frac{EYS}{18},$$

$$DAEd = \frac{MYSI + EYSI}{2} \quad \text{and} \quad RI = \frac{\ln(GNIpc) - \ln 100}{\ln(75) - \ln 100}.$$

The QoL index at a time instant $t \in \mathbb{T}$ is

$$HDI(t) = \sqrt[3]{LEI \times DAEd \times RI},$$

Denote next

- V_1: Gross Domestic Product (in Billions of USD dollars);
- V_2: Gross National Income (in Billions of USD dollars);
- V_3: Gross Savings (in Billions of USD dollars);
- V_4: Unemployment Rate;
- V_5: Electricity Consumption Kwh per Capita;
- V_6: Death Rate per 1000 people;
- V_7: Education spending Index;
- V_8: Life Expectancy,

and $\overline{V}$ the arithmetic mean of V_i, $i = 1, \ldots, 8$. In the existing studies such as [6–8,53,54], the QoL index HDI is evaluated on the fixed whole period $[0, T]$, as

$$HDI_m = \frac{1}{T^2} \int_{[0,T]^2} HDI(t)\overline{V}(s)dtds. \tag{1}$$

In the present work we aim to investigate the time scale influence on the QoL index measuring. Indeed, notice that the last formula (1) induces a global index on the comple period of study, and forgets about the timescale and the time-wise and/or instantaneous changes. An extending idea has been developed in [1] in which a wavelet multiscale QoL index has been measured according to a time scale $j \in \mathbb{N}$, for which

- The scale 0 corresponds to a period of 1 year dynamics;
- The scale 1 corresponds to a period of 2 years dynamics;
- The scale 2 corresponds to a period of 4 years dynamics;
- The scale $j \geq 0$ corresponds to a period of $2^j = 2^{j+1} - 2^j$ years dynamics,

by computing for each scale j a j-level HDI index, evaluated as

$$HDI_m(j) = \frac{1}{2^{2j}} \int_{2^j}^{2^{j+1}} HDI(t)dt \, . \int_{2^j}^{2^{j+1}} \overline{V}(t)dt. \tag{2}$$

Notice that in both models (1) and (2), we integrate a global index $HDI(t)$ and a global mean value $\overline{V}(t)$ which generates an under- or over-estimation of the index HDI_m and/or its modified multiscale variant $HDI_m(j)$. This is due to the fact that both integrands $HDI(t)$ and $\overline{V}(t)$ did not depend on j.

Our idea in the present modeling form consists in integrating more the factor of the time-scale in the evaluation of the QoL index by involving the details of the integrated

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indices by means of their wavelet decomposition and thus obtaining a more adapted multiscale index. We thus consider for each timescale j, a j-QoL index by considering a multi-index

$$MHDI_j = \frac{1}{2^{2j}} \int_{2^j}^{2^{j+1}} HDI(j,t)dt \,.\, \int_{2^j}^{2^{j+1}} \overline{V}(j,t)dt, \tag{3}$$

where the $HDI(j,t)$ is the equivalent of the index $HDI(t)$ relative to the period due to the scale j ($2^j = 2^{j+1} - 2^j$ years dynamics), and similarly, $\overline{V}(j,t)$ is the equivalent of $\overline{V}(t)$ due to the same correspondence. The upper 'M' in $MHDI_j$ is the abbreviation of Multi.

We will see later in the empirical part that this modified formula permits to detect more efficiently the influence of the time-scale factor in the evaluation of the QoL index, and allows to really follow the impact of the severe changes in the society on the well-being of the society itself as a good well-being place for citizens and residents lives as well as a good destination for foreigners. Among the severe movements we mention the last COVID-19, the Arab spring, the wars, the financial crises, energy, and climate changes.

3. Data, Empirical Results and Discussion

In the present section, we aim to apply the multiscale mathematical model developed above to discuss the situation of the QoL measure in Saudi Arabia, based on special data extracted from the social media, which reflects somehow the opinion of both citizens, residents, as well other interested peoples to such a region such as investors from the economic and energy sectors. We essentially focused on the period 1990–2021, resulting in $32 = 2^5$ years. As a consequence, the maximum level will be fixed to $J_{max} = 5$. The choice of the period for data sample is not in fact random, it is however related strongly to the main subject/title of the present work. Indeed, this period is characterized by the presence of many phenomena that affect strongly and directly the QoL and either many other indices in the society such as economic, financial, political and also social indices. These phenomena may be resumed at

- The worldwide financial crisis at 2007–2009, and its propagation effects by the next,
- The previous corona virus SARS-2002 which starts essentially from Saudi Arabia, and touched many other regions and countries,
- The last corona virus COVID-19 that affected all human societies and extended its impact for a long time. The propagation waves of its effects are still existing until now. This epidemic was one of the most severe epidemics in the modern era, whether in the number of injuries and deaths, as well as the negative consequences on the economy and other fields.
- The impact of the second IRAQ Gulf war on all the region of the Middle East countries. We recall that this region especially GCC countries is the biggest or largest source of energy such as petroleum in the world.
- The second Gulf war at Yemen which affected directly and strongly on the economy, the marketing, the commerce, even international, due to the maritime trade lanes in the Red Sea, and also on socio-political relations between the countries and their neighbours.
- The movement of nationalisation of quasi all jobs as a main goal of the 2030-vision program of Saudi Arabia, which has appeared many shortcomings especially in the qualification of national labors. This fact, even if it is not clear how it affects on individuals, it impacts strongly on the global indices such as the QoL one on long periods, and thus on the future of the country which aims to be best destination for well living for citizens, residents, as well as foreign investors.

In Table 1 below, we provide the values of the variables gathered.

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Year	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
1990	117.6	119.9	17.87	6.52	4.018	5.113	14.7	72.46
1991	132.2	139.4	11.35	7.06	4.122	4.907	14.71	72.49
1992	137.1	146.9	22.92	7.2	4.397	4.701	14.72	72.52
1993	133.0	142.9	20.62	6.85	4.745	4.495	14.57	72.56
1994	135.2	140.7	20.81	6.41	4.385	4.369	16.81	72.6
1995	143.3	145.6	28.39	6.41	4.93	4.243	16.84	72.66
1996	158.7	152.9	35.15	5.55	4.968	4.117	15.25	72.71
1997	166.0	159.7	37.78	5.15	5.128	3.991	12.3	72.78
1998	146.8	158.5	24.61	4.72	5.471	3.865	23.98	72.85
1999	161.7	154.5	41.43	4.35	5.613	3.813	23.33	72.92
2000	189.5	168.4	57.34	4.57	5.665	3.761	17.84	72.94
2001	184.1	180.6	51.61	4.62	5.947	3.709	20.9	72.96
2002	189.6	182.5	55.5	5.27	6.049	3.657	21.29	72.98
2003	215.8	212.9	75.14	5.56	6.501	3.605	21.46	73.01
2004	258.7	255.5	109.0	5.82	6.352	3.592	20.19	73.1
2005	328.5	301.4	159.8	6.05	6.615	3.579	19.29	73.19
2006	376.9	347.0	182.8	6.25	6.843	3.566	21.6	73.28
2007	416.0	398.0	203.5	5.73	6.943	3.553	20.78	73.37
2008	519.8	477.9	274.2	5.08	7.203	3.54	19.26	73.46
2009	429.1	476.7	157.0	5.38	7.477	3.514	19.35	73.64
2010	528.2	514.2	230.1	5.55	7.975	3.488	19.05	73.83
2011	671.2	590.3	339.6	5.77	8.015	3.461	18.84	74.01
2012	736.0	700.8	360.1	5.52	8.503	3.435	18.62	74.2
2013	746.7	764.4	333.1	5.57	8.785	3.409	18.41	74.38
2014	756.3	790.0	294.4	5.72	9.401	3.421	18.19	74.48
2015	654.3	752.2	166.9	5.59	9.485	3.433	18.07	74.59
2016	644.9	701.2	175.6	5.65	9.333	3.446	24.95	74.69
2017	688.6	661.8	209.1	5.89	9.151	3.458	22.26	74.8
2018	816.6	736.8	287.3	6.04	8.954	3.47	19.36	74.9
2019	803.6	795.8	272.4	5.67	8.434	3.513	19.07	75.06
2020	703.4	775.5	172.8	7.45	8.263	3.557	19.06	75.22
2021	833.5	786.6	231.6	7.35	8.254	3.6	18.79	75.37

Table 1: The observed (real) variables' values,

Next, we provide the wavelet multiscale analysis of each variable at the level 5, using herewith the well-known Daubechies wavelet Db8 (See [35]). Recall that for a statistical series (t_i, X_i), $i = 1, 2, \ldots, N$, its wavelet decomposition at a level J is

$$X_J = \sum_k a_{J_0,k}\varphi_{J_0,k}(t) + \sum_{j=J_0+1}^{J}\sum_k d_{j,k}\psi_{j,k}(t), \tag{4}$$

Next, to apply the idea of a multi-index for the QoL measure, we provide in Tables 2–9 the wavelet decomposition at the level $J = 5$ for all the variables V_i, $i = 1, ..., 8$. The first column in each table represents the wavelet approximation component A_5 relative to each variable. The remaining columns are due to the detail components D_1, D_2, D_3, D_4 an D_5 for the same variables. Besides, to illustrate more the time factor dependence in these variables, we provide the graphical illustrations of the wavelet decomposition of the eight variables V_i, $i = 1, 2, ..., 8$ at the level of decomposition $J = 5$ in Figures 1–8.

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A_5	D_5	D_4	D_3	D_2	D_1
212.0	-4.929	-2.944	3.499	4.454	-94.41
224.1	5.041	0.7893	1.556	2.569	-101.8
236.5	2.332	5.046	0.5782	0.3787	-107.8
249.3	-4.628	3.092	-0.3091	-2.232	-112.3
262.5	0.4727	-5.178	-1.471	-5.737	-115.4
276.0	1.813	-3.758	-2.206	-11.18	-117.4
289.9	-0.2411	8.75	-2.488	-19.04	-118.3
304.2	2.517	8.487	-2.026	-29.14	-118.1
318.7	-6.961	-6.727	-0.7123	-40.85	-116.7
333.4	2.498	-8.253	0.4658	-52.54	-113.9
348.3	6.808	5.651	0.8994	-62.62	-109.5
363.3	-8.62	2.933	0.626	-70.37	-103.7
378.3	0.5103	-17.68	-0.6442	-74.43	-96.47
393.4	7.034	-20.78	-3.123	-72.93	-87.75
408.4	-2.21	1.412	-6.39	-64.8	-77.65
423.3	3.048	27.46	-9.457	-49.77	-66.12
438.1	-13.26	44.28	-10.45	-28.49	-53.26
452.8	-17.91	30.08	-6.168	-3.472	-39.34
467.3	70.72	-21.03	4.485	22.84	-24.53
481.7	-49.93	-61.02	18.15	49.1	-8.891
495.9	-19.19	-58.51	29.75	72.79	7.511
509.8	32.38	-17.23	31.19	90.46	24.66
523.4	2.529	49.9	17.85	99.79	42.48
536.8	-24.31	78.44	-4.517	99.65	60.6
549.9	30.54	34.85	-27.87	90.29	78.67
562.7	-18.61	-20.54	-40.31	74.74	96.28
575.3	-13.3	-52.33	-33.36	55.69	112.9
587.7	0.5029	-47.18	-14.15	33.57	128.1
599.9	51.3	4.814	8.731	10.42	141.4
612.1	-10.94	36.4	24.7	-10.51	151.9
624.2	-91.1	13.84	24.55	-27.2	159.1
636.3	61.99	-4.762	15.29	-37.97	162.7

Table 2: The wavelet decomposition of the variable V_1 at the level 5.

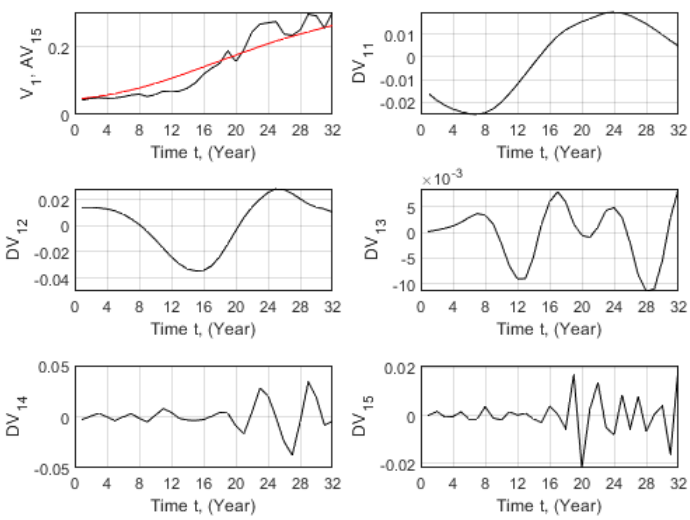

Figure 1. The wavelet decomposition of V_1 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
210.0	-6.386	-5.774	0.3202	10.45	-88.74
221.9	6.046	0.8948	-1.719	8.563	-96.28
234.2	3.015	8.394	-2.159	6.034	-102.6
246.9	-4.643	7.531	-1.865	2.703	-107.6
259.8	-0.342	-4.034	-1.446	-1.872	-111.4
273.2	3.119	-7.156	-0.8977	-8.548	-114.1
286.9	-2.555	2.752	-0.6423	-17.7	-115.8
301.0	-0.5185	5.075	-0.2676	-29.1	-116.5
315.3	3.773	-2.941	0.4459	-42.11	-115.9
329.9	-2.814	-4.213	0.8133	-55.19	-114.0
344.6	-0.8985	1.43	0.6339	-66.82	-110.6
359.4	2.742	0.07601	0.0059	-76.09	-105.6
374.3	-3.163	-7.095	-0.9726	-81.5	-99.11
389.3	2.48	-5.121	-1.639	-81.01	-91.05
404.2	0.9156	6.786	-1.565	-73.39	-81.45
419.0	-1.008	13.11	-1.071	-58.39	-70.27
433.7	-2.642	10.52	-0.3658	-36.65	-57.59
448.3	-5.453	9.083	0.2842	-10.63	-43.64
462.8	20.7	5.091	0.632	17.21	-28.63
477.2	-15.81	-18.73	1.086	45.5	-12.61
491.4	-4.026	-51.17	2.018	71.63	4.313
505.3	6.611	-39.28	3.659	91.93	22.05
519.0	7.245	24.22	5.958	103.9	40.47
532.4	-9.702	68.24	7.858	106.4	59.21
545.6	0.5791	58.61	7.873	99.44	77.89
558.5	-2.435	9.792	4.078	86.1	96.11
571.2	12.1	-60.35	-4.135	68.93	113.4
583.8	-10.23	-74.85	-14.15	48.07	129.2
596.1	-1.066	-4.529	-21.97	25.27	142.9
608.4	8.813	42.85	-21.65	3.51	153.9
620.7	-7.951	26.69	-9.973	-15.4	161.4
633.0	3.15	6.661	8.321	-29.71	165.2

Table 3: The wavelet decomposition of the variable V_2 at the level 5.

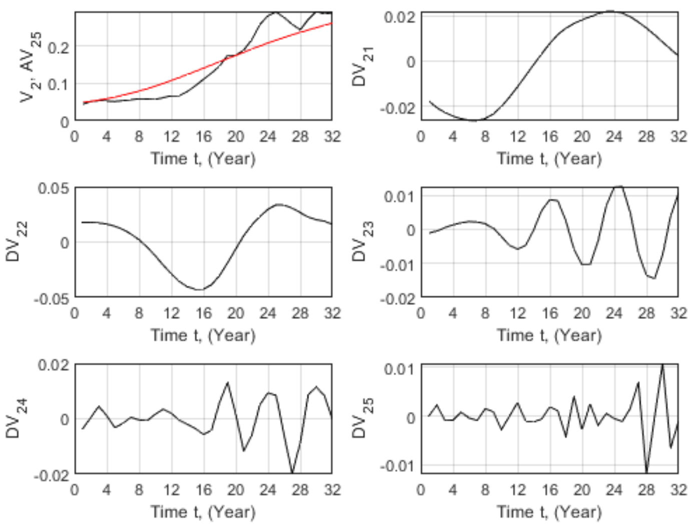

Figure 2. The wavelet decomposition of V_2 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
62.67	3.054	-1.146	3.665	6.745	-57.12
68.26	-4.836	-0.1951	3.279	2.76	-57.92
73.97	3.333	1.257	3.43	-1.689	-57.38
79.78	-1.104	0.6631	3.328	-6.504	-55.54
85.67	-1.574	-1.546	2.499	-11.71	-52.53
91.67	1.635	0.1056	1.204	-17.55	-48.67
97.75	0.8262	5.39	-0.5901	-24.05	-44.18
103.9	1.873	4.527	-2.408	-30.97	-39.16
110.1	-7.321	-2.957	-3.66	-37.91	-33.66
116.3	4.154	-3.178	-4.176	-44.05	-27.64
122.5	4.49	3.913	-3.844	-48.58	-21.09
128.5	-7.202	-1.37	-3.012	-51.02	-14.3
134.5	0.38	-18.77	-2.451	-50.63	-7.5
140.3	5.751	-20.29	-3.265	-46.47	-0.8843
145.9	0.01354	1.557	-5.764	-38.05	5.324
151.4	3.178	27.96	-8.385	-25.41	11.07
156.6	-16.92	44.81	-8.832	-9.119	16.27
161.6	-14.16	29.68	-3.713	9.336	20.74
166.4	68.82	-22.07	8.27	28.33	24.43
171.0	-50.27	-60.6	22.92	46.63	27.41
175.2	-16.93	-54.49	34.24	62.26	29.79
179.2	33.11	-10.76	33.53	72.58	31.9
182.9	-2.716	54.25	15.91	75.8	33.93
186.3	-21.38	72.4	-11.51	71.37	35.85
189.5	31.29	15.01	-38.66	59.73	37.56
192.4	-21.69	-34.98	-50.87	43.18	38.89
195.0	-5.097	-39.73	-38.22	24.05	39.66
197.4	1.393	-22.68	-10.14	3.219	39.95
199.6	29.64	14.18	21.32	-17.13	39.74
201.7	-4.57	29.59	40.32	-33.56	38.9
203.7	-57.81	-0.9872	34.35	-43.88	37.39
205.6	40.29	-16.55	14.05	-46.94	35.15

Table 4: The wavelet decomposition of the variable V_3 at the level 5.

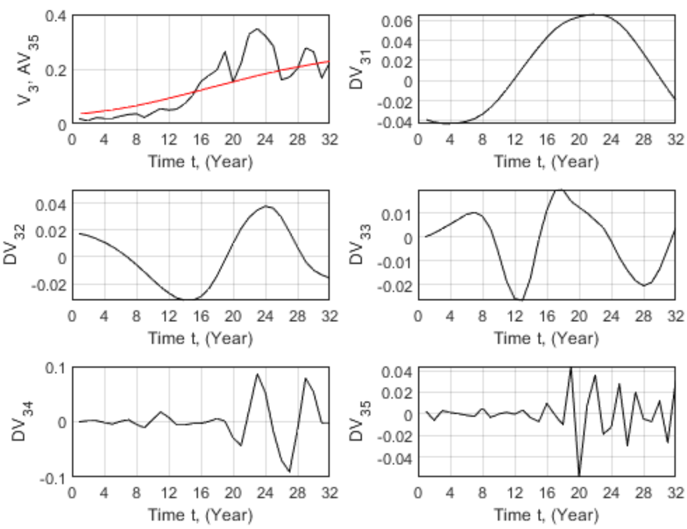

Figure 3. The wavelet decomposition of V_3 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
5.755	-0.2132	-0.1939	-0.171	0.8494	0.4935
5.754	0.1824	0.06113	-0.06148	0.6622	0.4619
5.753	0.1668	0.2619	0.1818	0.4239	0.4126
5.753	-0.1402	0.2648	0.4815	0.1445	0.3463
5.754	-0.1929	0.001286	0.7247	-0.1426	0.2657
5.755	0.2865	-0.1809	0.7425	-0.3692	0.176
5.757	-0.122	-0.109	0.4355	-0.4931	0.0813
5.76	-0.0051	-0.0075	-0.0779	-0.5038	-0.0159
5.764	0.0398	0.0686	-0.6296	-0.4092	-0.1138
5.769	-0.0789	0.1095	-0.9713	-0.2658	-0.2125
5.775	0.1148	0.0299	-0.9187	-0.1204	-0.3112
5.783	-0.1094	-0.084	-0.6013	0.0359	-0.4042
5.792	0.0954	-0.1475	-0.17	0.1865	-0.4868
5.803	-0.0405	-0.138	0.1946	0.2952	-0.5546
5.816	-0.0509	-0.0099	0.3208	0.3472	-0.603
5.831	-0.0031	0.2044	0.3291	0.3201	-0.631
5.847	0.1511	0.3586	0.3145	0.2164	-0.6377
5.865	-0.0628	0.1609	0.2937	0.0940	-0.6211
5.886	-0.1935	-0.3516	0.3319	-0.0105	-0.5819
5.908	0.2204	-0.4635	0.3423	-0.1053	-0.5217
5.932	-0.0259	0.0068	0.2634	-0.1822	-0.4442
5.958	-0.02871	0.2743	0.1588	-0.2359	-0.3568
5.986	-0.0950	0.1268	0.0359	-0.269	-0.2652
6.017	0.0513	0.0201	-0.0884	-0.2586	-0.1709
6.049	0.1428	-0.0083	-0.1849	-0.2027	-0.0754
6.082	-0.0663	-0.0206	-0.2911	-0.1365	0.0222
6.118	-0.1502	0.0504	-0.412	-0.0779	0.122
6.155	0.0502	-0.0125	-0.4944	-0.0262	0.2183
6.193	0.2576	-0.235	-0.4953	0.0131	0.3068
6.232	-0.4768	-0.1615	-0.3542	0.04571	0.3849
6.272	0.4704	0.2291	-0.0467	0.0766	0.4489
6.312	-0.1918	0.3004	0.3403	0.0907	0.4981

Table 5: The wavelet decomposition of the variable V_4 at the level 5.

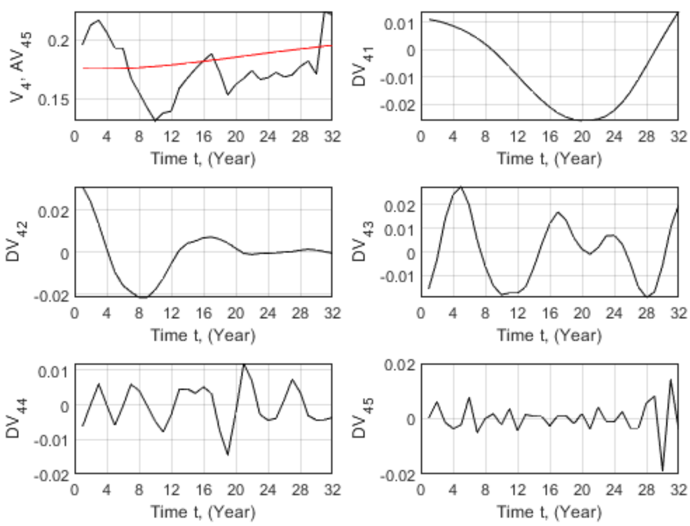

Figure 4. The wavelet decomposition of V_4 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
5.457	0.0236	-0.1228	-0.0011	-0.3559	-0.983
5.545	-0.0247	-0.0110	-0.0575	-0.3255	-1.004
5.634	-0.0168	0.1388	-0.0828	-0.2737	-1.003
5.726	0.1348	0.1621	-0.0951	-0.2031	-0.9794
5.819	-0.2586	-0.0074	-0.1041	-0.1273	-0.9365
5.914	0.192	-0.1191	-0.0990	-0.0790	-0.8792
6.012	-0.0013	-0.0722	-0.0817	-0.0769	-0.8114
6.11	-0.0806	-0.0008	-0.0426	-0.1236	-0.7347
6.21	0.0367	0.0625	0.0239	-0.2126	-0.65
6.311	0.0083	0.0616	0.1008	-0.3127	-0.556
6.411	-0.0300	-0.0416	0.1759	-0.3981	-0.4524
6.511	0.0756	-0.0647	0.2398	-0.4708	-0.344
6.61	-0.1211	0.0417	0.2734	-0.5204	-0.2347
6.708	0.1249	0.0667	0.2566	-0.527	-0.1277
6.803	-0.0999	-0.0213	0.1794	-0.4827	-0.0267
6.897	0.0342	-0.0539	0.0476	-0.3773	0.0674
6.988	0.0507	-0.0161	-0.1183	-0.2149	0.1536
7.076	-0.0519	-0.0040	-0.2771	-0.0290	0.2287
7.162	-0.0062	-0.0088	-0.3917	0.1554	0.292
7.245	-0.0248	0.0289	-0.4519	0.3348	0.3445
7.325	0.126	0.0789	-0.4388	0.4949	0.3885
7.402	-0.1426	0.0346	-0.3278	0.6196	0.4295
7.475	0.0693	-0.0898	-0.122	0.6999	0.4711
7.544	-0.0460	-0.0972	0.1513	0.7206	0.5126
7.61	0.0882	0.0415	0.4286	0.6805	0.5526
7.672	-0.0499	0.0853	0.5869	0.6028	0.588
7.731	-0.0563	-0.0121	0.5511	0.5032	0.6159
7.788	0.0260	-0.0451	0.3626	0.3821	0.6376
7.842	0.114	0.0106	0.0855	0.2487	0.653
7.895	-0.1005	0.0257	-0.1583	0.1127	0.6596
7.947	-0.0511	-0.0002	-0.2692	-0.0197	0.6565
7.998	0.0550	-0.0097	-0.2971	-0.1348	0.6428

Table 6: The wavelet decomposition of the variable V_5 at the level 5.

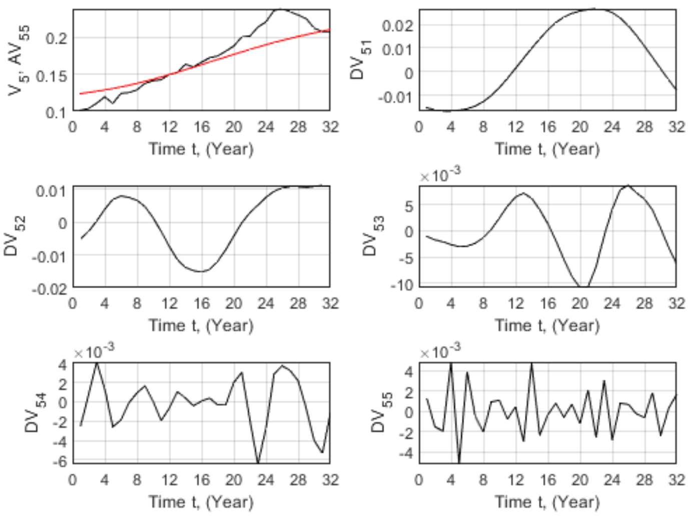

Figure 5. The wavelet decomposition of V_5 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
4.098	0.03	0.0971	0.1058	0.3855	0.3968
4.075	-0.0316	0.0055	0.1314	0.3335	0.3932
4.052	-0.0008	-0.1036	0.1167	0.2583	0.3787
4.028	0.0102	-0.1296	0.0707	0.162	0.3537
4.004	0.0023	-0.0218	0.0097	0.0553	0.3195
3.98	-0.0097	0.0678	-0.0380	-0.0358	0.279
3.955	0.0078	0.0668	-0.0515	-0.0959	0.2345
3.931	0.0032	0.0354	-0.0446	-0.1209	0.1872
3.906	-0.0155	-0.0191	-0.0309	-0.1129	0.1378
3.881	0.0128	-0.0495	-0.0253	-0.0920	0.0858
3.857	-0.0005	-0.0122	-0.0415	-0.0736	0.0316
3.833	-0.0068	0.0175	-0.0616	-0.0510	-0.0221
3.81	0.0060	0.0098	-0.0699	-0.0257	-0.07287
3.787	-0.0027	0.0051	-0.0599	-0.0062	-0.1184
3.766	0.0001	0.0007	-0.0236	0.0066	-0.1568
3.745	0.0003	-0.0088	0.0252	0.0048	-0.1873
3.725	0.0002	-0.0076	0.0696	-0.0119	-0.2096
3.707	-0.0013	-0.0022	0.0978	-0.0258	-0.2221
3.689	0.0029	0.0026	0.0980	-0.0278	-0.225
3.673	-0.0012	0.0079	0.0762	-0.0230	-0.219
3.658	-0.0026	0.0089	0.0425	-0.0129	-0.206
3.644	0.0007	0.0021	0.0067	-0.0036	-0.1894
3.632	0.0053	-0.0089	-0.0204	-0.0011	-0.172
3.621	-0.0056	-0.0114	-0.0423	0.0013	-0.1541
3.612	0.0009	-0.0009	-0.0601	0.0055	-0.1359
3.603	0.0004	0.0086	-0.0682	0.0053	-0.1164
3.596	0.0010	0.0097	-0.0661	0.0002	-0.0949
3.59	0.0002	0.0016	-0.0504	-0.0097	-0.0736
3.584	-0.0029	-0.0129	-0.0214	-0.0235	-0.0537
3.58	0.0051	-0.0116	0.0088	-0.0334	-0.0356
3.576	-0.0060	0.0099	0.0326	-0.0349	-0.0202
3.572	0.0015	0.0155	0.0492	-0.0305	-0.0074

Table 7: The wavelet decomposition of the variable V_6 at the level 5.

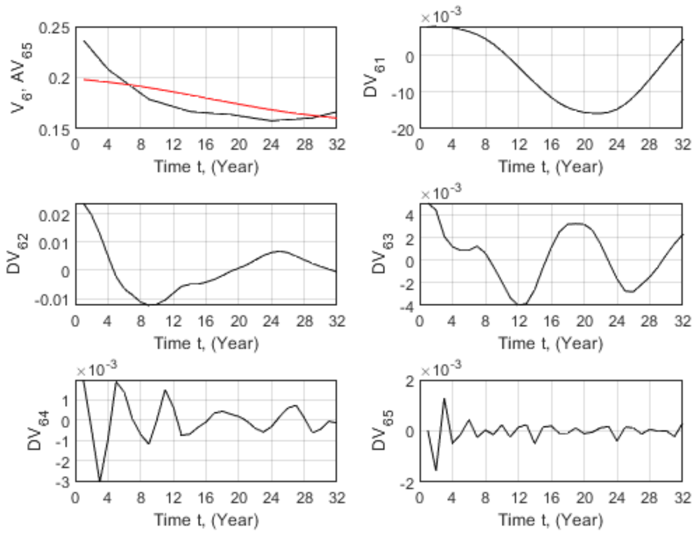

Figure 6. The wavelet decomposition of V_6 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
17.78	-0.2478	0.0066	0.9162	-2.027	-1.733
17.87	0.0756	-0.3356	0.6649	-1.873	-1.694
17.96	0.7765	-0.9767	0.1615	-1.596	-1.605
18.05	-0.3726	0.1015	-0.5418	-1.198	-1.469
18.14	-0.6416	2.581	-1.262	-0.7154	-1.292
18.23	-0.1044	1.593	-1.537	-0.2548	-1.088
18.32	1.722	-2.927	-1.123	0.1197	-0.866
18.42	-2.559	-3.082	-0.2329	0.3892	-0.6328
18.51	2.397	2.074	0.8347	0.5551	-0.3925
18.6	-0.4761	3.122	1.536	0.6884	-0.1437
18.69	-2.018	-1.259	1.476	0.836	0.1116
18.78	2.137	-2.283	0.9551	0.948	0.3596
18.87	-0.3739	0.9328	0.2708	1.003	0.5884
18.95	-0.5686	1.547	-0.2536	0.9958	0.7873
19.03	0.3417	-0.7338	-0.3014	0.9071	0.9452
19.11	-0.2855	-1.19	-0.1677	0.7676	1.059
19.18	0.5855	0.2262	-0.1048	0.5892	1.128
19.24	-0.3072	0.5654	-0.195	0.3305	1.145
19.3	-0.4391	-0.1268	-0.5787	-0.0088	1.111
19.36	0.4521	-0.0956	-1.009	-0.3869	1.031
19.41	0.1261	0.5752	-1.218	-0.7556	0.9138
19.45	-0.0881	0.8198	-1.097	-1.026	0.7773
19.49	-0.2382	0.3922	-0.5312	-1.131	0.6357
19.53	0.0282	-0.8434	0.2935	-1.086	0.492
19.55	0.4305	-2.326	1.091	-0.9072	0.3482
19.58	-1.255	-1.394	1.571	-0.6277	0.201
19.59	2.076	2.028	1.501	-0.2967	0.0489
19.61	-1.206	2.86	1.018	0.0777	-0.0956
19.62	-0.7284	-0.0863	0.3274	0.4555	-0.2236
19.62	1.006	-1.654	-0.3178	0.7449	-0.3308
19.63	0.2471	-0.6237	-0.6728	0.8923	-0.4104
19.63	-0.4842	0.0653	-0.8444	0.8831	-0.4614

Table 8: The wavelet decomposition of the variable V_7 at the level 5.

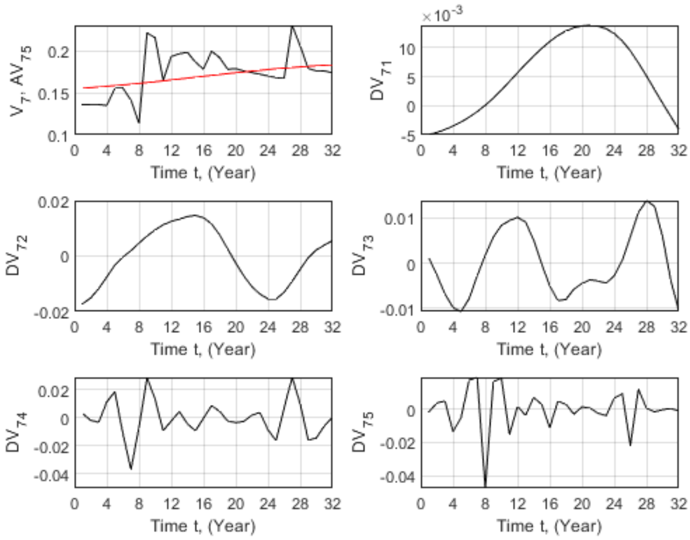

Figure 7. The wavelet decomposition of V_7 at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
72.9	-0.0051	-0.0082	0.0172	-0.148	-0.2949
72.94	0.0055	-0.0029	0.0010	-0.1228	-0.3292
72.98	0.0013	0.0048	-0.0160	-0.0886	-0.3604
73.02	-0.0012	0.0095	-0.0342	-0.0467	-0.3883
73.06	-0.0053	0.0062	-0.0502	-0.0023	-0.4126
73.11	0.0057	-0.0014	-0.0532	0.0335	-0.4338
73.16	-0.0009	-0.0085	-0.0385	0.0539	-0.4517
73.2	0.0010	-0.0061	-0.0099	0.0573	-0.466
73.25	-0.0048	0.0073	0.0257	0.0448	-0.4762
73.3	0.0060	0.0134	0.0528	0.0253	-0.4811
73.36	-0.0045	0.0036	0.0600	0.0060	-0.48
73.41	0.0002	-0.0119	0.0520	-0.0152	-0.4725
73.46	0.0055	-0.0246	0.0333	-0.0365	-0.4583
73.51	-0.0063	-0.0174	0.0090	-0.0530	-0.4366
73.57	0.0006	0.0143	-0.0131	-0.0631	-0.4073
73.62	-0.0001	0.035	-0.0354	-0.0630	-0.37
73.68	0.0063	0.0271	-0.0548	-0.0525	-0.3248
73.73	0.0019	0.0051	-0.0592	-0.0395	-0.2725
73.79	-0.0198	-0.0236	-0.0441	-0.0286	-0.2138
73.85	0.0130	-0.0400	-0.0126	-0.0173	-0.1491
73.9	0.0110	-0.0273	0.0282	-0.0052	-0.0789
73.96	-0.0114	-0.0011	0.0587	0.0093	-0.0040
74.01	-0.0079	0.0251	0.0652	0.0284	0.0747
74.07	0.014	0.0383	0.0538	0.0480	0.1558
74.13	-0.0052	0.0266	0.0288	0.0664	0.2378
74.18	0.0008	0.0058	-0.0026	0.0857	0.3193
74.24	-0.0058	-0.0136	-0.0302	0.1051	0.3984
74.29	0.0063	-0.0371	-0.0557	0.1222	0.473
74.35	0.0009	-0.048	-0.0737	0.134	0.5403
74.4	0.0057	-0.0121	-0.0670	0.1341	0.5974
74.46	-0.0221	0.0554	-0.0308	0.1185	0.6416
74.51	0.0105	0.0599	0.0256	0.0888	0.6718

Table 9: The wavelet decomposition of the variable V_8 at the level 5.

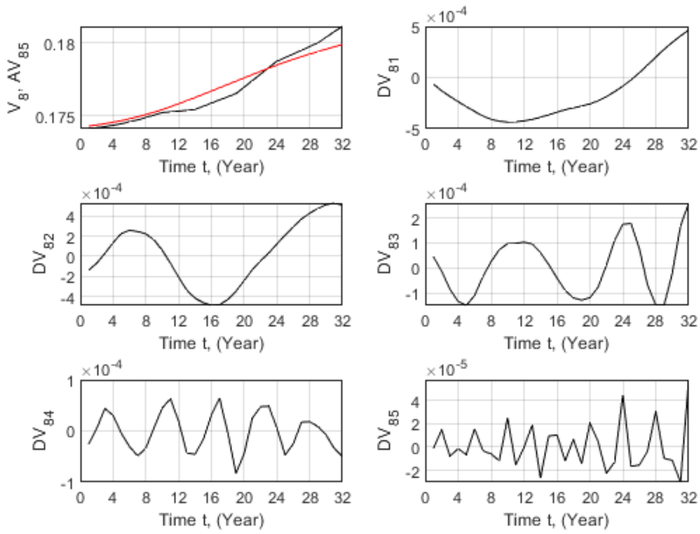

Figure 8. The wavelet decomposition of V_8 at the level 5.

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Regarding Table 1 and Figure 1 illustrating the different components due to the wavelet decomposition of the GDP variable V_1, we notice a global increasing aspect reflected on the wavelet approximation component A_5 shown in the top left-hand sub-figure and the first column of the table. However, this global shape hides behind it the real behavior of this variable which when going in its macroscopic state shows a the short horizons a decreasing behavior at the beginning of the time period. This decrease is in fact the real and logic aspect as in the 1990's, the GCC region has known a very hard and devastating event due to the Iraq's invasion of Kuwait in 1990. The first country which has been affected by this event was the Kingdom of Saudi Arabia which immediately hosted Kuwaiti refugees as well as their royal family governing the country. Besides, due to the UN decision on Kuwait liberation, troops from many countries have been gathered on the Saudi Arabia territory until the liberation next year. This event have affected strongly the situation in the kingdom from many points of view such as economic due to huge amount of money was spent for the liberation. Besides, social relations has been affected due to the strong relations and the common historical links between the people there. The USA troops for example left the country in 2003. This year itself (2003) was pointed out by the terrorist attacks in May and November 2003 on the oil infrastructure of the kingdom allowing US troops to return again to the region. This decreasing aspect is also show in other variables especially in Figure 2 and Table 3 relative to the variable V_2 due to the Gross National Income, the variable V_4 due to the unemployment rate illustrated in Figure 4 and Table 5, next, Figure 6 and Table 7 relative to the death rate, and finally in Figure 8 and Table 9 for the life expectancy. However, the variable V_3 due to the Gross Savings shows in Figure 3 and Table 4 an increasing aspect at the starting of the period at short horizons, and starts to decrease at medium and high horizons, with the appearance of clear perturbations (fluctuations) at the high levels or scales of time. The variables V_5 relative to the Energy consumption, V_7 relative to the Education however show a different starting behavior with an increasing aspect especially at short horizons, and becomes decreasing and perturbed (fluctuated) at high horizons.

Among the cause evoked previously, these behaviors may be naturally explained by many initiatives taken by the Saudi government in some special periods. Indeed, the kingdom has implemented several social and economic initiatives, including expanding employment and social opportunities for women especially, which attracted foreign investment, increasing the role of the private sector in the economy, and discouraging businesses from hiring foreign workers during the period 2002 to 2015 especially. Since 2015, women start to contribute in political life and governments, which encourages some liberal behavior in the society. Besides, a strong event that unfortunately appeared in the same period since 2015, and negatively affected again the quality of life from the economic point of view is due to a second war broken out at the south frontiers of the Kingdom which is directly engaged in it by its military campaign to restore the legitimate government of Yemen against some opposite parties.

Besides, and although the country passed from many critical moments and events, it remains a leading producer of oil and natural gas and holds about 17% of the world's proven oil reserves as of 2020. Furthermore, the kingdom continues to pursue economic reform and diversification.

At the end of the period, the COVID19 pandemic spreads in all the world affecting thus the situation from all points of view, especially health, economy, and also social.

On another side, Saudi Arabia has made significant progress in the case of schooling Index and efforts to improve learning outcomes and strengthening education. The education scale and progress affected positively human capabilities, which constitutes one of the most important goals of Vision 2030. The success is a consequence of the big investments in education made by the leadership. This is illustrated by the variable V_7 which is globally increasing reminiscent of some crashes especially at the end of the time period which may be due to the impact of the COVID19. Recall that Saudi Arabia is a very vast area, and an important number of citizens live in small villages and small collections far from the big cities, and thus the quarantine condition along the COVID19 period obliged a number of students to not follows uniformly their studies.

However, a critical look at the figures show that the global shape did not describe the real and hidden behavior of the variable in all cases and all timescales. In some case, we observe decreasing behavior at short horizons followed by a different behavior at next or different horizons. By means of quality of life vocabulary, this means that the situation is not stable, and a single measure on the whole time interval may not be adequate to describe the reality. We have to stop at each extreme value of the variables or at equivalently consider each timescale separately and compute a relative QoL index. This will permit to describe at real time the link between the QoL and the event(s) occurring in that time. Related to the case study, especially the period chosen, this is a natural consequence of many critical phenomena that happened in this period. Living with the consequences of the pandemic, the Yemen war which becomes to be a real nightmare, the new Russian/Ukrainian war, Climate problems are causes added to the history of events negatively impacting the well being. According the UN reports, Saudi Arabia is ranked 35th on a total number of 191 countries for 2022, decreasing thus by 5 places relatively to its previous rank. In the so-called G7 countries, Saudi arabia takes the 10th place with a most progress during 2019–2021.

In the remaining part, we investigate the multi-index measurement of the QoL as introduced in the previous section. To illustrate the timescale behaviour of the QoL index, we provide in Table 10 and Figure 9 the wavelet decomposition of the observed HDI index at the level 5.

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A_5	D_5	D_4	D_3	D_2	D_1
0.7387	-0.0019	-0.0027	-0.0013	-0.0095	-0.0261
0.7414	0.00204	0.00007	-0.0024	-0.0085	-0.027
0.7442	0.0005	0.0030	-0.0025	-0.0070	-0.0283
0.7471	-0.0012	0.0035	-0.0020	-0.0047	-0.0287
0.7500	0.00008	0.0002	-0.0012	-0.0024	-0.0286
0.7531	0.0006	-0.0021	-0.0005	-0.0008	-0.0282
0.7562	-0.0006	-0.0014	-0.0003	-0.0003	-0.0275
0.7594	-0.0001	-0.0003	-0.00005	-0.0012	-0.0266
0.7626	0.0010	0.0005	0.0004	-0.0032	-0.0254
0.7659	-0.0010	0.0013	0.0011	-0.0055	-0.0238
0.7692	0.0004	0.0008	0.0021	-0.0076	-0.0219
0.7725	0.00035	-0.0007	0.0031	-0.0094	-0.0198
0.7759	-0.0010	-0.0023	0.0036	-0.0108	-0.0174
0.7792	0.0004	-0.0018	0.0034	-0.0113	-0.0150
0.7825	0.0010	0.0012	0.0021	-0.0106	-0.0122
0.7858	-0.0010	0.0031	0.00020	-0.0086	-0.0094
0.7890	-0.0003	0.0022	-0.0021	-0.0052	-0.0063
0.7922	-0.0003	0.0002	-0.0041	-0.0013	-0.0037
0.7954	0.0024	-0.0021	-0.0052	0.0024	-0.0008
0.7985	-0.0020	-0.0031	-0.0055	0.0062	0.0020
0.8015	-0.0006	-0.0015	-0.0048	0.0095	0.0048
0.8045	0.0010	0.0004	-0.0028	0.0122	0.0078
0.8075	0.0010	0.0014	0.0002	0.0141	0.0112
0.8103	-0.0003	0.0022	0.0040	0.0150	0.0140
0.8131	-0.0020	0.0022	0.0071	0.0145	0.0170
0.8158	0.0010	0.0007	0.0081	0.0133	0.0200
0.8184	0.0015	-0.0014	0.0058	0.0118	0.0228
0.8210	-0.0024	-0.0030	0.0013	0.0100	0.0253
0.8236	0.0021	-0.0027	-0.0040	0.0073	0.0276
0.8261	0.0009	-0.0001	-0.0069	0.0047	0.0293
0.8286	-0.0047	0.0034	-0.0058	0.0018	0.0305
0.8311	0.0023	0.0034	-0.0021	-0.0010	0.0311

Table 10: The wavelet decomposition of the observed HDI index at the level 5.

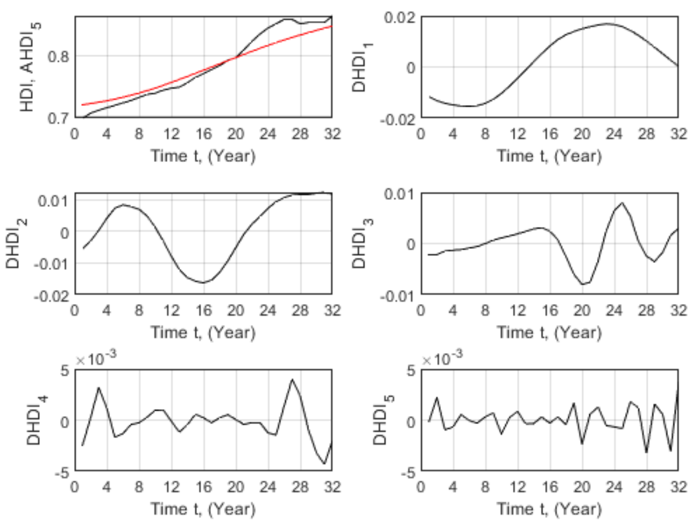

Figure 9. The graphs of the wavelet decomposition of the observed HDI index at the level 5.

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Notice from Table 10 and Figure 9, that the QoL measured by means of a single view presents an increasing global shape, which is immediately controverted by means of the wavelet detail components even at short horizons where a decreasing starting appears clearly. These contradictions described in fluctuations at high horizons confirm that effectively our idea about being a multi-index rather than a single index applicable for any period without taking into consideration the time scale factor seems to be more adequate. Table 11 below provides the resulting QoL HDI multi-index issued from our new/modified model (3).

$MHDI_1$	$MHDI_2$	$MHDI_3$	$MHDI_4$	$MHDI_5$
0.1707	0.1694	0.1688	0.1683	0.1588
0.1714	0.1696	0.1690	0.1697	0.1601
0.1724	0.1702	0.1695	0.1704	0.1603
0.1735	0.1709	0.1702	0.1708	0.1603
0.1744	0.1716	0.1709	0.1711	0.1605
0.1749	0.1719	0.1711	0.1714	0.1611
0.1750	0.1719	0.1710	0.1716	0.1620
0.1750	0.1719	0.1710	0.1719	0.1632
0.1748	0.1719	0.1709	0.1722	0.1646
0.1742	0.1716	0.1706	0.1718	0.1658
0.1734	0.1713	0.1703	0.1719	0.1678
0.1726	0.1710	0.1699	0.1716	0.1695
0.1719	0.1708	0.1697	0.1710	0.1708
0.1715	0.1710	0.1698	0.1715	0.1728
0.1715	0.1715	0.1703	0.1724	0.1751
0.1717	0.1721	0.1709	0.1727	0.1767
0.1721	0.1730	0.1716	0.1731	0.1785
0.1729	0.1740	0.1726	0.1734	0.1800
0.1740	0.1751	0.1737	0.1743	0.1820
0.1752	0.1764	0.1749	0.1741	0.1824
0.1764	0.1776	0.1761	0.1759	0.1843
0.1773	0.1785	0.1769	0.1780	0.1858
0.1782	0.1792	0.1776	0.1796	0.1866
0.1790	0.1798	0.1781	0.1808	0.1874
0.1797	0.1801	0.1784	0.1814	0.1881
0.1803	0.1801	0.1784	0.1821	0.1900
0.1807	0.1799	0.1781	0.1812	0.1909
0.1810	0.1795	0.1777	0.1788	0.1900
0.1813	0.1790	0.1771	0.1784	0.1906
0.1816	0.1786	0.1767	0.1776	0.1902
0.1819	0.1781	0.1762	0.1768	0.1893
0.1820	0.1775	0.1755	0.1786	0.1916

Table 11: The multi-index HDI due to the time-scale model (3).

Next, to more illustrate the variability of the QoL index HDI according to the time-scale j, we plotted the graphs illustrating firstly in Figures 10 the QoL HDI index due to the last old model (2) ([1]), and secondly the QoL multi-index model due to (3) and computed in Table 11 in Figures 11.

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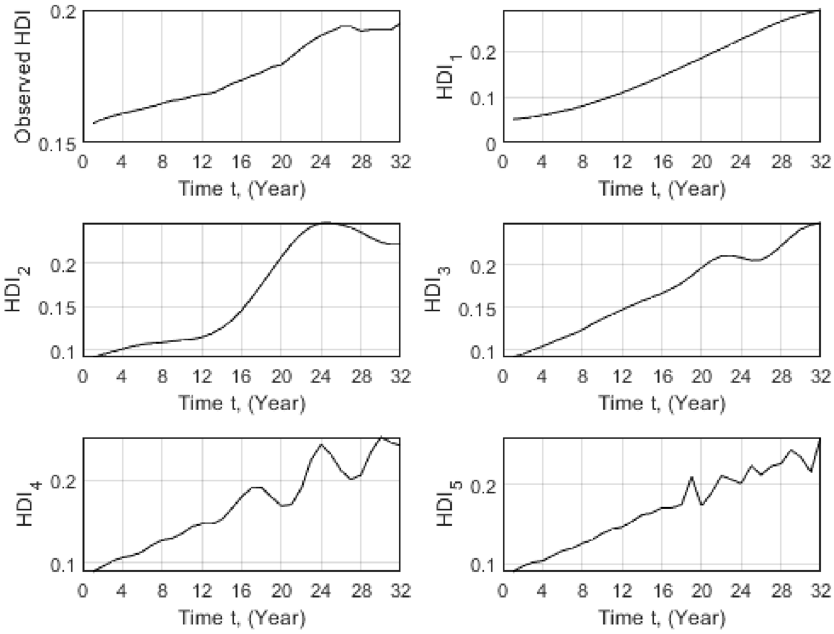

Figure 10. The QoL index due to the old model (2) [1].

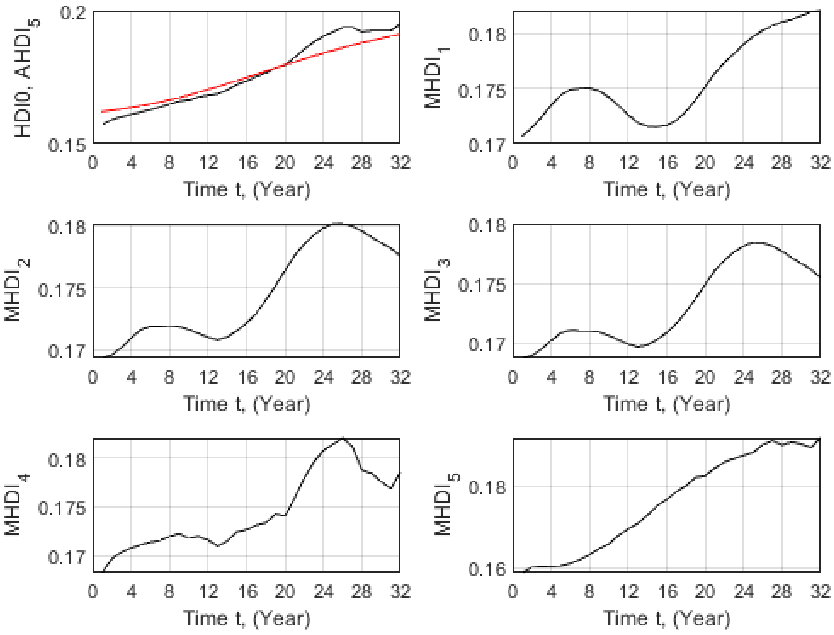

Figure 11. The QoL multi-index due to (3).

The graphs illustrate clearly the difference between the different QoL indices according to the time-scale, as there is no global invariant shape. In Figure 10 the shapes of the different multi-scale indices computed according to the horizon $j = 1, 2, \ldots, 5$ show quietly the same behavior, which looks like an increasing index, which tends to be linear on a big piece of the time interval. This behavior is broken down at high levels by the appearance again of the multi-index shape. As the timescale gets up the prediction of the QoL index becomes uncertain, which confirms that it should be computed according to sub-periods as separate indices (multi-index) rather than the same single formulation

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along the whole time interval. Figure 11 illustrates clearly a four shapes at short levels, which persists even for medium and quasi high horizons. At the end of the period the model predicts an increase of the QoL index, indicating that at far future the kingdom may be a good destination for well living. This fact may be explained strongly by the 2030-vision plan and programs which aims among many other goals to prepare suitable economic, health as well as educational and social destinations via many investment programs.

4. Conclusion

The present paper is devoted to the development or the improvement of some mathematical quantitative models for the estimation of the so-called QoL index. The main idea reposes on the assumption that QoL index is a timescale index which depends strongly on the time factor. Therefore, a suitable choice of time intervals along the whole period of study may describe the real situation adequately. In the present paper, we applied the wavelet theory as a mathematical tool sophisticated to quantify the socio-economic, political and/or health factors to measure the a QoL multi-index. The choice of time sub-intervals is basically related to the severe movements and events that affect the well being when they occur. We proved that effectively the quality of life measuring based on a single index in the existing studies may be described more adequately by a variable index due to the social, political, economic and also health environment. The new model is applied empirically on a sample corresponding to Saudi Arabia as a case of study during specific period.and based on social media treatment of the QoL.

The results join existing studies involving timescale factor to quantify and describe social indices, and on another side, confirm the adequacy of multiscale-in-time models, and the idea of being multi-index more than a single measure for the QoL. Many factors are strongly involved in the measuring of the QoL such as services availability, health care, security and safety, dignity, education, communication, participation in decision making, etc.

To resume, our present paper concerned the investigation of the multiscale behavior of the so-called quality of life index by means of a quantitative mathematical model based on wavelet multiscale method. We showed that the quality of life may be effectively well and adequately estimated in the form of a multi-index rather than a single measure on the whole time interval. The main idea uses the wavelet timescale decomposition of time series. The findings confirm that the single index evaluated in the existing studies via a single mathematical formulation may be fragmented into multi indices due to many factors such as the multifractality and/or the volatility of the factors applied in the formulation. The study is applied on a special data sample traded also on special time period characterized by the presence of many sever movements and events such as econo-financial crises, pandemics, ... The findings show that wavelet multifractal models are good modelers in these extreme cases.

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Appendix A. Wavelet Processing Toolkit

Wavelets have been introduced few decades ago in mathematical theory, although their discovery was related to applications in petroleum extraction ([13–17,35]).

A wavelet is simply a short wave function oscillating such as the Fourier modes, but with high frequency and small support, which we call in wavelet theory localization in time-frequency and/or time-space.

To analyze a statistical (time, financial, etc.) series, we have to compute the so-called wavelet transform, a two-parameter quantity evaluated by the correlation type (a convolution) of the analyzed series with translated-dilated copies of one fixed wavelet known as the mother wavelet, which is a special function that should satisfy at least an admissibility assumption as

$$\mathcal{A}_\psi = \int_{\mathbb{R}} \frac{|\widehat{\psi}(\xi)|^2}{|\xi|} d\xi < \infty. \tag{A1}$$

The wavelet analysis appeared originally in the theoretical form as a refinement and also terminology to Fourier and harmonic analysis is general. Therefore, it associates to the analyzed function a type of transform known as the wavelet transform or exactly the continuous wavelet transform obtained by a convolution product of the analyzed function with special copies of a source function known as the mother wavelet. More precisely, denote for $s > 0$ and $u \in \mathbb{R}$ fixed

$$\psi_{s,u}(x) = \frac{1}{\sqrt{s}} \psi(\frac{x-u}{s}). \tag{A2}$$

For a function $F \in L^2(\mathbb{R})$, its continuous wavelet transform at the scale s and the position u is defined by

$$\mathcal{WT}_{s,u}(F) = \int_{-\infty}^{\infty} F(t)\psi_{s,u}(t)dt. \tag{A3}$$

The parameters s and u have many nominations according to the context of use of the wavelet transform. s is known as a frequency, scale, or a dilation or compression parameter. u is known also as the translation parameter. When covering all the real values of s and u, we obtain the so-called time-frequency or time-space domain. This transform is called continuous because of the nature of the parameters s and u that may operate on all the space $(0,\infty) \times \mathbb{R}$. It holds in wavelet theory that the function F may be reproduced by means of its continuous wavelet transform in an L^2-sense and analogously as in Fourier analysis via the L^2-equality

$$F(t) = \frac{1}{\mathcal{A}_\psi} \int \int_{\mathbb{R}} \mathcal{WT}_{u,s}(F) \psi\left(\frac{x-u}{s}\right) \frac{dsdu}{s^2}. \tag{A4}$$

We conclude that the wavelet transform operates according two parameters: the parameter s permitting to compress or to dilate the graph of ψ, which allows in turn to reach the high/low magnitude fluctuations in the signal, and the parameter u, which permits translating the graph of ψ to localize or to approach the local fluctuations.

The discrete wavelet transform is a variant of the continuous one evaluated on a discrete grid for the parameters s and u. The most known one is the dyadic, while there is no essential difference between the discrete grids. In the dyadic case, we restrict on the set $\{(s,u) = (2^{-j}, k2^{-j})\,;\; j,k \in \mathbb{Z}\}$. In this case, the translated–dilated copies are defined by

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k). \tag{A5}$$

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The continuous wavelet transform $\mathcal{WT}_{s,u}$ will be called the discrete wavelet transform, which is denoted usually by $d_{j,k}$. The reconstruction formula (A4) will be evaluated via a discrete representation as

$$F(t) = \sum_{j,k} d_{j,k}(F)\psi_{j,k}(t), \tag{A6}$$

known as the wavelet series of the function F.

In the discrete case such as statistical series and discrete signals, the wavelet transform of a series $X(t)$ (known also as the discrete wavelet transform (DWT)) is obtained by correlation-type (discrete convolution)

$$d_{j,k}(X) = \sum_{n} X(n)\psi_{j,k}(n), \tag{A7}$$

known also as the wavelet coefficient or detail coefficient at the level or the scale j and the position k. In wavelet theory, it is proved that any series $X(t)$ may be decomposed in a series form as

$$X(t) = \sum_{j,k} d_{j,k}(X)\psi_{j,k}(t), \tag{A8}$$

known as the wavelet series or the wavelet decomposition of $X(t)$, and it guarantees a complete reconstruction formula of the original series $X(t)$ ([13–20,34,35]).

The greatest advantage of this decomposition is the fact that it allows splitting the data into different horizons known as levels. Each level is associated to a component of the series, and it makes itself a refinement of the preceding one, which we call in wavelet theory the concept of multi-resolution. Denote for $j \in \mathbb{Z}$, $W_j = spann(\psi_{j,k}\ ;\ k \in \mathbb{Z})$ (known as the detail space at the level j), and $V_j = \overset{\perp}{\bigoplus_{l \leq j}} W_l$ (known as the approximation space at the level j). We get an orthogonal decomposition $V_j = V_{j-1} \overset{\perp}{\bigoplus} W_{j-1}$. This permits splitting the wavelet decomposition above as

$$X(t) = \sum_{j \leq J_{\min},k} d_{j,k}(X)\psi_{j,k}(t) + \sum_{j > J_{\min},k} d_{j,k}(X)\psi_{j,k}(t), \tag{A9}$$

relative to a fixed integer $J_{\min} \in \mathbb{Z}$. For $j \in \mathbb{Z}$, the component

$$A_j(X(t)) = \sum_{l \leq j,k} d_{l,k}(X)\psi_{l,k}(t) \tag{A10}$$

belongs to V_j, and it is called the approximation of $X(t)$ at the level j. It describes the global behavior, the trend, or the shape of $X(t)$. The component

$$D_j(X(t)) = \sum_{k} d_{j,k}(X)\psi_{j,k}(t) \tag{A11}$$

belongs to the space W_j, and it is called the detail component of $X(t)$ at the level j. It reflects the higher frequency oscillations or the fine-scale deviations of the series near its trend. As a consequence, the wavelet decomposition of $X(t)$ in (A9) is a superposition as

$$X(t) = A_{J_{\min}}(X(t)) + D_{J_{\min}+1}(X(t)) + D_{J_{\min}+2}(X(t)) + \ldots. \tag{A12}$$

A second main advantage of wavelet theory is the reduction in computing the coefficients needed in the decomposition. Indeed, there exists a function φ (known as the scaling function or the father wavelet) characterized by the so-called two-scale relation

$$\varphi = \sum_{k \in \mathbb{Z}} h_k \varphi_{1,k}, \tag{A13}$$

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and which is related to the function ψ by

$$\psi = \sum_{k\in\mathbb{Z}} g_k \varphi_{1,k}, \tag{A14}$$

where

$$h_k = \int_{-\infty}^{+\infty} \varphi(t)\varphi_{1,k}(t)dt, \text{ and } g_k = (-1)^k h_{1-k}. \tag{A15}$$

It holds that $V_j = spann(\varphi_{j,k}\ ;\ k \in \mathbb{Z})$, where the $\varphi_{j,k}$ values are defined similarly to the $\psi_{j,k}$ in (A5). The component $A_j(X(t))$ is therefore written as

$$A_j(X(t)) = \sum_k a_{j,k}(X)\varphi_{j,k}(t), \tag{A16}$$

where the coefficients $a_{j,k}(X)$ (known as the approximation or scaling coefficients of $X(t)$) are evaluated as the $d_{j,k}(X)$ by replacing the function ψ by φ. The relation (A13) permits computing the level decomposition from each other as

$$a_{j,k}(X) = \sum_{l\in\mathbb{Z}} h_l a_{j+1,l+2k}(X), \tag{A17}$$

$$d_{j,k}(X) = \sum_{l\in\mathbb{Z}} g_l a_{j+1,l+2k}(X), \tag{A18}$$

and

$$a_{j+1,k}(X) = \sum_l h_{l-2k} a_{j,l}(X) + \sum_l g_{l-2k} d_{j,l}(X). \tag{A19}$$

The sequence $H = (h_k)_k$ is called the discrete wavelet low-pass filter, and the sequence $G = (g_k)_k$ is the discrete wavelet high-pass filter.

The truncation of the last decomposition in (A12) in a practical finite level $J > J_{\min} \in \mathbb{Z}$ gives the so-called J-level finite wavelet decomposition of $X(t)$ as

$$S_J = A_{J_0}(S) + \sum_{J_0<j\le J} D_j(S). \tag{A20}$$

The lower index $J_{\min}$ is in fact more flexible, and it is usually chosen to be 0. The choice of J is always critical, and it is related to the eventual error estimates requested. See [13–17,35] for more details.

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