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Article

The Models of Primary Particles

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Abstract: If we assume that:

a. The four fundamental forces of nature are independent waves without rest mass, and their speeds are constant in a vacuum, just like light.
b. Light or electromagnetic waves and gravity are comparable in structure. The weak and strong interactions are similar in structure.
c. Light and the weak interaction have the same speed c_L with spin number +1 or -1.
d. Gravity and the strong interaction have the same speed c_G without spin.
e. The primary particles, namely electrons (or positrons), electron neutrinos, and dark neutrinos in this paper, are made of the above four waves.

We can find and describe some fundamental characteristics of the primary particles (e.g., their sizes, energies, and interactions) and introduce new attractive results from them (e.g., the source of the Pauli exclusion principle, the solution of the Einstein-Podolsky-Rosen paradox, and c_G slightly faster than c_L).

Keywords: models of particles; electrons; electron neutrinos; dark matter; sizes and energies of particles; interactions between two particles; Pauli exclusion principle; the solution of the Einstein-Podolsky-Rosen paradox; speed of gravity

1. Introduction

We know intimately the term "atom" which comes from ancient Greek and means "uncuttable" or translated as "indivisible." In the early 19th century, the scientist John Dalton introduced the modern definition of an atom to characterize chemical elements. It was discovered that Dalton's atoms were not actually indivisible about a century later. An atom is made up of three basic types of subatomic particles: electrons, protons, and neutrons, which occupy the tiny space inside an atom. Protons and neutrons form the nucleus, which contains most of the mass of an atom. Electrons are the lightest charged particles in nature and revolve around the nucleus of an atom. An electron is seemingly indivisible yet. So far, we have not split an electron into two or more smaller particles. We just annihilate the positive and negative electrons. A free neutron is unstable and decays into a proton, an electron and a neutrino. A free proton, on the other hand, is stable, and consists of two up quarks and one down quark in the modern Standard Model. Another question is whether a quark can be cut into smaller parts or whether the matter is infinitely divisible.

This paper tries to answer the above questions from a different perspective. What happens when light or electromagnetic, gravitational and other waves make up the primary particles that are the fundamental elements of matter? The assumptions are then derived:

a. Light or electromagnetic waves, the weak interaction, gravity, and the strong interaction are independent waves without rest mass. However, their structures are different.
b. Light and gravity can be described by the wave equation with the field strength $\hat{\mathbf{E}}$ and speed c.

$$\nabla^2\hat{\mathbf{E}} = \frac{1}{c^2}\frac{\partial^2\hat{\mathbf{E}}}{\partial t^2} \quad (1)$$

c. The weak and strong interactions can be described by the 4-dimensional Laplace equation with field strength $\hat{\mathbf{E}}'$ and speed c'.

$$\nabla^2\hat{\mathbf{E}}' = -\frac{1}{c'^2}\frac{\partial^2\hat{\mathbf{E}}'}{\partial t^2} \quad (2)$$

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d. According to the electroweak theory, light and the weak interaction have the same speed c_L with spin number +1 or –1.

e. Gravity and the strong interaction have the same speed c_G without spin, and c_G is constant in a vacuum.

f. The primary particles, which are electrons, electron neutrinos, and dark neutrinos in this paper, are made of the four waves mentioned above.

2. The Formation of Primary Particles

Suppose that the birth of primary particles can be divided into the following two parts:

a. Light and the weak interaction couple together (hereafter referred to as the E-W couple) when they have the same spin number and the second-order partial derivatives of their fields with respect to time $\frac{\partial^2 \hat{\mathbf{E}}}{\partial t^2}$ are equal. Gravity and the strong interaction also couple together (hereafter referred to as the G-S couple) when the second-order partial derivatives of their fields with respect to time $\frac{\partial^2 \hat{\mathbf{E}}'}{\partial t^2}$ are equal. So we have

$$c_L^2 \nabla^2 (\hat{\mathbf{E}}_e + \hat{\mathbf{E}}_w) = 0\,, \qquad (3)$$

and

$$c_G^2 \nabla^2 (\hat{\mathbf{E}}_G + \hat{\mathbf{E}}_S) = 0\,. \qquad (4)$$

Where $\hat{\mathbf{E}}_e$, $\hat{\mathbf{E}}_w$, $\hat{\mathbf{E}}_G$, and $\hat{\mathbf{E}}_S$ are electric, weak interaction, gravitational, and strong interaction fields.

b. The original spins of the light and the weak interaction convert the polarity of the electric and weak charges when an E-W couple is formed.

c. It makes a primary particle when two coupled waves attract each other and shrink to a tiny sphere. One E-W couple and one G-S couple produce an electron or a positron whose charge property depends on the original spin of the E-W couple. Dark neutrinos are composed of two G-S couples. Two E-W couples with different original spin compress themselves into an electron neutrino. But they cannot attract each other if they have the same original spin.

Thus, in the spherical coordinate system, we have the uniform Laplace equation for equations (3), (4)

$$c^2\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \hat{E}}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial \hat{E}}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 \hat{E}}{\partial\varphi^2}\right]=0, \qquad (5)$$

whose general solution is

$$\hat{E}=\begin{cases}\dfrac{1}{c^2}\displaystyle\sum_{j=0}^{+\infty}\sum_{k=0}^{j}\left(A_j r^j+\frac{B_j}{r^{j+1}}\right)\mathrm{P}_j^k(\cos\theta)e^{-ik\varphi}\\[2ex] \dfrac{1}{c^2}\displaystyle\sum_{j=0}^{+\infty}\sum_{k=0}^{j}\left(A_j r^j+\frac{B_j}{r^{j+1}}\right)\mathrm{P}_j^k(\cos\theta)e^{ik\varphi}\end{cases}, \qquad (6)$$

where A_j and B_j are constants, $\mathrm{P}_j^k(\cos\theta)$ are associated Legendre polynomials, j and k are integers, j = 0, 1, 2, 3, …, $k \le j$, and j is called the degree of associated Legendre polynomials.

3. The Fields and Binding Energies

We can now derive the electric, gravitational, weak interaction, and strong interaction fields $\hat{E}$ based on equation (6) and existing physical laws and data.

It is reasonable that equation (6) can be transformed into a pair of conjugate solutions.

$$\hat{E}=\frac{1}{c^2}\left(A_j r^j+\frac{B_j}{r^{l+1}}\right)\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{-ik\varphi}$$
$$\hat{E}^*=\frac{1}{c^2}\left(A_j r^j+\frac{B_j}{r^{j+1}}\right)\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{ik\varphi} \qquad (7)$$

Clearly, there is

$$[\hat{E}_a+\hat{E}_b]^*=\hat{E}_a^*+\hat{E}_b^* \qquad (8)$$

where the subscripts a and b denote electric, gravitational, weak interaction, or strong interaction.

And we let

$$(\hat{E}^*)^*=\hat{E} \qquad (9)$$

The field $\hat{E}$ may be split into the macroscopic item $\frac{1}{c^2}\left(A_j r^j+\frac{B_j}{r^{j+1}}\right)$ and the quantum factors $\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{-ik\varphi}$, $\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{ik\varphi}$. The degree of associated Legendre polynomials j rules properties of the field $\hat{E}$ because it is not only an exponent of r in the macroscopic item but also impacts forms of the quantum factors.

Comparing Equation (7) to Gauss's law of electrostatics and Newton's law of gravity, the electric field $\hat{E}_e$ is

$$\hat{E}_e=\pm\frac{1}{c_L^2}\frac{B_1}{r^2}\sum_{k=0}^{-1}\mathrm{P}_1^k(\cos\theta)e^{-ik\varphi}=\pm\frac{\hat{q}}{c_L^2 r^2}(\cos\theta+\sin\theta e^{-i\varphi})$$
$$\hat{E}_e^*=\pm\frac{1}{c_L^2}\frac{B_1}{r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{ik\varphi}=\pm\frac{\hat{q}}{c_L^2 r^2}(\cos\theta+\sin\theta e^{i\varphi}) \quad , \qquad (10)$$

and the gravitational field $\hat{E}_G$ is

$$\hat{E}_G=-\frac{1}{c_G^2}\frac{B_1}{r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{-ik\varphi}=-\frac{\hat{m}}{c_G^2 r^2}(\cos\theta+\sin\theta e^{-i\varphi})$$
$$\hat{E}_G^*=-\frac{1}{c_G^2}\frac{B_1}{r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{ik\varphi}=-\frac{\hat{m}}{c_G^2 r^2}(\cos\theta+\sin\theta e^{i\varphi}) \quad , \qquad (11)$$

where '–' means attractive interaction, and '+' means repulsive interaction, as usual in this paper, $\hat{q}$ is the mathematical electric charge, and $\hat{m}$ is the mathematical mass or the gravitational charge.

The weak interaction has an intensity of a similar magnitude to the electromagnetic force at very short distances (around 10^{-18} meters), but this starts to decrease exponentially with increasing distance. Its effective range is about 10^{-17} to 10^{-16} meters[1–3]. All of the above can help us to determine the weak interaction field $\hat{E}_w$ from equation (7). $\hat{E}_w$ is therefore supposed to be equal to

$$\hat{E}_w=\begin{cases}\pm\frac{1}{c_L^2}\frac{B_1}{r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{-ik\varphi}=\pm\frac{\hat{w}}{c_L^2 r^2}(\cos\theta+\sin\theta e^{-i\varphi}) & r\le R_{cw}\\ \pm\frac{1}{c_L^2}\frac{B_m}{r^{m+1}}\sum_{k=0}^{m}\mathrm{P}_m^k(\cos\theta)e^{-ik\varphi}=\pm\frac{\hat{w}R_{cw}^{m-1}}{c_L^2 r^{m+1}}\sum_{k=0}^{m}\mathrm{P}_m^k(\cos\theta)e^{-ik\varphi} & r>R_{cw}\end{cases}$$
$$\hat{E}_w^*=\begin{cases}\pm\frac{1}{c_L^2}\frac{B_1}{r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{ik\varphi}=\pm\frac{\hat{w}}{c_L^2 r^2}(\cos\theta+\sin\theta e^{i\varphi}) & r\le R_{cw}\\ \pm\frac{1}{c_L^2}\frac{B_m}{r^{m+1}}\sum_{k=0}^{m}\mathrm{P}_m^k(\cos\theta)e^{ik\varphi}=\pm\frac{\hat{w}R_{cw}^{m-1}}{c_L^2 r^{m+1}}\sum_{k=0}^{m}\mathrm{P}_m^k(\cos\theta)e^{ik\varphi} & r>R_{cw}\end{cases} \qquad (12)$$

where $\hat{w}$ is the mathematical weak charge, R_{cw} is the critical radius of the weak interaction, and m is an integer greater than 1.

The strong force is a short-range interaction (around 10^{-15} meters) similar to the weak force. But its range is more complex than that of the weak force. At distances comparable to the diameter of a proton, it is approximately 100 times as strong as the electromagnetic force. At smaller distances, however, it becomes weaker. In particle physics, this effect is known as asymptotic freedom[4–6]. Furthermore, it is supposed that the fields of the four fundamental forces have a unified form in a very small region. Hence, Equation (7) can be translated into the strong interaction field $\hat{E}_S$

$$\hat{E}_S=\begin{cases}-\dfrac{B_1}{c_G^2r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{-ik\varphi}=-\dfrac{\hat{s}}{c_G^2r^2}(\cos\theta+\sin\theta e^{-i\varphi}) & r\le R_{cS1}\\ -\dfrac{A_1r}{c_G^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{-ik\varphi}=-\dfrac{\hat{s}r}{c_G^2R_{cS1}^3}(\cos\theta+\sin\theta e^{-i\varphi}) & R_{cS1}<r\le R_{cS2}\\ -\dfrac{1}{c_G^2}\dfrac{B_n}{r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{-ik\varphi}=-\dfrac{\hat{s}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{-ik\varphi} & r>R_{cS2}\end{cases}$$

$$\hat{E}_S^*=\begin{cases}-\dfrac{B_1}{c_G^2r^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{ik\varphi}=-\dfrac{\hat{s}}{c_G^2r^2}(\cos\theta+\sin\theta e^{i\varphi}) & r\le R_{cS1}\\ -\dfrac{A_1r}{c_G^2}\sum_{k=0}^{1}\mathrm{P}_1^k(\cos\theta)e^{ik\varphi}=-\dfrac{\hat{s}r}{c_G^2R_{cS1}^3}(\cos\theta+\sin\theta e^{i\varphi}) & R_{cS1}<r\le R_{cS2}\\ -\dfrac{1}{c_G^2}\dfrac{B_n}{r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{ik\varphi}=-\dfrac{\hat{s}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{ik\varphi} & r>R_{cS2}\end{cases} \tag{13}$$

where $\hat{s}$ is the mathematical strong charge, R_{cS1} and R_{cS2} are the 1st and 2nd critical radii of the strong interaction, and n is an integer greater than 1.

Further, it is assumed that $R_{cS1} \ll R_{cw} < R_{cS2}$ and $m \neq n$.

Now we turn to determine the energy. The energy density of the wave equation (1) is given by $\left|\hat{E}\right|^2$, and the Equations (1) and (2) are Lorentz invariant. So we hope that the equation of energy is also Lorentz invariant, and let the binding energy $E_{a\wedge b}$ of the four fundamental fields be

$$E_{a\wedge b}\propto\frac{1}{t_2-t_1}\iiint_V\int_{ct_1}^{ct_2}\left|\hat{E}_a\hat{E}_b^*\right|dxdydzdct \quad . \tag{14}$$

Note that $\hat{E}_a$ and $\hat{E}_b^*$ are independent of time, and there are

$$\int_0^{2\pi}e^{im\varphi}e^{-in\varphi}d\varphi=2\pi\delta_{mn} \tag{15}$$

and

$$\int_0^{\pi}\mathrm{P}_j^m(\cos\theta)\,\mathrm{P}_k^m(\cos\theta)\sin\theta d\theta=\frac{(j+m)!}{(j-m)!}\frac{2}{2j+1}\delta_{jk}=\mathrm{N}_j^m\,\delta_{jk} \quad . \tag{16}$$

When we use light as a measurement medium to determine the energy as usual, we can translate Equation (14) into the spherical coordinate system with the optical medium, which is

$$E_{a\wedge b}=\frac{1}{(t_2-t_1)2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\int_{c_Lt_1}^{c_Lt_2}\left|\hat{E}_a\hat{E}_b^*\right|r^2\sin\theta drd\theta d\varphi dc_Lt$$
$$=\frac{c_L^2}{2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left|\hat{E}_a\hat{E}_b^*\right|r^2\sin\theta drd\theta d\varphi \tag{17}$$

where R is the radius at which two fields begin to interact with each other.

The general energy expression can be calculated when Equation (7) is substituted into Equation (17).

$$E_{a\wedge b}=\frac{c_L^2}{2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left|\hat{E}_a\hat{E}_b^*\right|r^2\sin\theta drd\theta d\varphi$$
$$=\frac{c_L^2}{2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left[\frac{1}{c_a^2}\left|A_{aj}r^j+\frac{B_{aj}}{r^{j+1}}\right|\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{-ik\varphi}\right.$$
$$\left.\times\frac{1}{c_b^2}\left|A_{bj}r^j+\frac{B_{bj}}{r^{j+1}}\right|\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{ik\varphi}\right]r^2\sin\theta drd\theta d\varphi$$
$$=\frac{c_L^2}{2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left[\frac{1}{c_b^2}\left|A_{bj}r^j+\frac{B_{bj}}{r^{j+1}}\right|\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{-ik\varphi}\right.$$
$$\left.\times\frac{1}{c_a^2}\left|A_{aj}r^j+\frac{B_{aj}}{r^{j+1}}\right|\sum_{k=0}^{j}\mathrm{P}_j^k(\cos\theta)e^{ik\varphi}\right]r^2\sin\theta drd\theta d\varphi$$
$$=\frac{c_L^2}{2\pi\sum_{k=0}^{j}\mathrm{N}_j^k}\int_R^{\infty}\int_0^{\pi}\int_0^{2\pi}\left|\hat{E}_b\hat{E}_a^*\right|r^2\sin\theta drd\theta d\varphi$$
$$=E_{b\wedge a} \tag{18}$$
$$=\frac{c_L^2}{c_a^2c_b^2}\int_R^{\infty}\left|A_{aj}r^j+\frac{B_{aj}}{r^{j+1}}\right|\left|A_{bj}r^j+\frac{B_{bj}}{r^{j+1}}\right|r^2dr$$

Based on Equation (18) and associated with Equations (10) to (13), we first compute the self-binding energy of the four fields. The self-binding energy of the four fields $E_{e\wedge e}$, $E_{G\wedge G}$, $E_{w\wedge w}$, and $E_{S\wedge S}$ are

$$E_{e\wedge e}=c_L^2\int_r^{\infty}\frac{\hat{q}^2}{c_L^4r^2}dr=\frac{\hat{q}^2}{c_L^2r}\quad, \tag{19}$$

$$E_{G\wedge G}=c_L^2\int_r^{\infty}\frac{\hat{m}^2}{c_G^4r^2}dr=\frac{c_L^2\hat{m}^2}{c_G^4r}\quad, \tag{20}$$

$$E_{w\wedge w}=\begin{cases}c_L^2\int_r^{R_{cw}}\frac{\hat{w}^2}{c_L^4r^2}dr+c_L^2\int_{R_{cw}}^{\infty}\frac{\hat{w}^2R_{cw}^{2m-2}}{c_L^4r^{2m}}dr=\frac{\hat{w}^2}{c_L^2}\left[\frac{1}{r}-\frac{2m-2}{(2m-1)R_{cw}}\right] & r\le R_{cw}\\ c_L^2\int_r^{\infty}\frac{\hat{w}^2R_{cw}^{2m-2}}{c_L^4r^{2m}}dr=\frac{\hat{w}^2R_{cw}^{2m-2}}{c_L^2(2m-1)r^{2m-1}} & r>R_{cw}\end{cases} \tag{21}$$

and

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$$E_{S\wedge S}=\begin{cases} c_L^2\int_r^{R_{cS1}}\frac{\hat{s}^2}{c_G^4 r^2}dr+c_L^2\int_{R_{cS1}}^{R_{cS2}}\frac{\hat{s}^2 r^4}{c_G^4 R_{cS1}^6}dr+c_L^2\int_{R_{cS2}}^{\infty}\frac{\hat{s}^2 R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6 r^{2n}}dr \\ =\frac{c_L^2\hat{s}^2}{c_G^4}\left[\frac{1}{r}-\frac{6}{5R_{cS1}}+\frac{2(n+2)R_{cS2}^5}{5(2n-1)R_{cS1}^6}\right] & r\le R_{cS1} \\ c_L^2\int_r^{R_{cS2}}\frac{\hat{s}^2 r^4}{c_G^4 R_{cS1}^6}dr+c_L^2\int_{R_{cS2}}^{\infty}\frac{\hat{s}^2 R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6 r^{2n}}dr \\ =\frac{c_L^2\hat{s}^2}{c_G^4}\left[\frac{1}{5R_{cS1}^6}\left(R_{cS2}^5-r^5\right)+\frac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\right] & R_{cS1}<r\le R_{cS2} \\ c_L^2\int_r^{\infty}\frac{\hat{s}^2 R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6 r^{2n}}dr=\frac{c_L^2\hat{s}^2}{c_G^4}\times\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6 r^{2n-1}} & r>R_{cS2} \end{cases}.\quad(22)$$

The binding energy of the four fields, such as $E_{e\wedge G}$, $E_{e\wedge w}$, $E_{e\wedge S}$, etc. are

$$E_{e\wedge G}=c_L^2\int_r^{\infty}\frac{\hat{q}}{c_L^2 r^2}\frac{\hat{m}}{c_G^2 r^2}r^2dr=\frac{\hat{q}\hat{m}}{c_G^2 r}\quad,\quad(23)$$

$$E_{e\wedge w}=\begin{cases} c_L^2\int_r^{R_{cw}}\frac{\hat{q}}{c_L^2 r^2}\frac{\hat{w}}{c_L^2 r^2}r^2dr+0=\frac{\hat{q}\hat{w}}{c_L^2}\left(\frac{1}{r}-\frac{1}{R_{cw}}\right) & r\le R_{cw} \\ 0 & r>R_{cw} \end{cases},(24)$$

$$E_{e\wedge S}=\begin{cases} c_L^2\int_r^{R_{cS1}}\frac{\hat{q}}{c_L^2 r^2}\frac{\hat{s}}{c_G^2 r^2}r^2dr+c_L^2\int_{R_{cS1}}^{R_{cS2}}\frac{\hat{q}}{c_L^2 r^2}\frac{\hat{s}r}{c_G^2 R_{cS1}^3}r^2dr+0=\frac{\hat{q}\hat{s}}{c_G^2}\left(\frac{1}{r}-\frac{3}{2R_{cS1}}+\frac{R_{cS2}^2}{2R_{cS1}^3}\right) & r\le R_{cS1} \\ c_L^2\int_r^{R_{cS2}}\frac{\hat{q}}{c_L^2 r^2}\frac{\hat{s}r}{c_G^2 R_{cS1}^3}r^2dr+0=\frac{\hat{q}\hat{s}}{2c_G^2 R_{cS1}^3}\left(R_{cS2}^2-r^2\right) & R_{cS1}<r\le R_{cS2} \\ 0 & r>R_{cS2} \end{cases},\quad(25)$$

$$E_{G\wedge w}=\begin{cases} c_L^2\int_r^{R_{cw}}\frac{\hat{m}}{c_G^2 r^2}\frac{\hat{w}}{c_L^2 r^2}r^2dr+0=\frac{\hat{m}\hat{w}}{c_G^2}\left(\frac{1}{r}-\frac{1}{R_{cw}}\right) & r\le R_{cw} \\ 0 & r>R_{cw} \end{cases},\quad(26)$$

$$E_{G\wedge S}=\begin{cases} c_L^2\int_r^{R_{cS1}}\frac{\hat{m}}{c_G^2 r^2}\frac{\hat{s}}{c_G^2 r^2}r^2dr+c_L^2\int_{R_{cS1}}^{R_{cS2}}\frac{\hat{m}}{c_G^2 r^2}\frac{\hat{s}r}{c_G^2 R_{cS1}^3}r^2dr+0=\frac{c_L^2\hat{m}\hat{s}}{c_G^4}\left(\frac{1}{r}-\frac{3}{2R_{cS1}}+\frac{R_{cS2}^2}{2R_{cS1}^3}\right) & r\le R_{cS1} \\ c_L^2\int_r^{R_{cS2}}\frac{\hat{m}}{c_G^2 r^2}\frac{\hat{s}r}{c_G^2 R_{cS1}^3}r^2dr+0=\frac{c_L^2\hat{m}\hat{s}}{2c_G^4 R_{cS1}^3}\left(R_{cS2}^2-r^2\right) & R_{cS1}<r\le R_{cS2} \\ 0 & r>R_{cS2} \end{cases},\quad(27)$$

and

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$$E_{w \wedge S} = \begin{cases} c_L^2 \int_r^{R_{cS1}} \frac{\hat{w}}{c_L^2 r^2} \frac{\hat{s}}{c_G^2 r^2} r^2 dr + c_L^2 \int_{R_{cS1}}^{R_{cw}} \frac{\hat{w}}{c_L^2 r^2} \frac{\hat{s}r}{c_G^2 R_{cS1}^3} r^2 dr + 0 = \frac{\hat{w}\hat{s}}{c_G^2}\left(\frac{1}{r} - \frac{3}{2R_{cS1}} + \frac{R_{cw}^2}{2R_{cS1}^3}\right) & r \le R_{cS1} \\ c_L^2 \int_r^{R_{cw}} \frac{\hat{w}}{c_L^2 r^2} \frac{\hat{s}r}{c_G^2 R_{cS1}^3} r^2 dr + 0 = \frac{\hat{w}\hat{s}}{2c_G^2 R_{cS1}^3}\left(R_{cw}^2 - r^2\right) & R_{cS1} < r \le R_{cw} \\ 0 & r > R_{cw} \end{cases}$$

. (28)

For the convenience of subsequent calculations, we require the results of our defined energy to be consistent with those of the conventional physical method. Comparing Equations (19) and (20) with the electric and gravitational potential energy formulas, it is easy to see that the relations of the mathematical electric charge $\hat{q}$ and the mathematical mass $\hat{m}$ to the electric charge q and the mass m are as follows

$$\hat{q} = c_L \sqrt{k} q \tag{29}$$

and

$$\hat{m} = \frac{c_G^2 \sqrt{G} m}{c_L} \tag{30}$$

where k is the Coulomb constant, and G is the gravitational constant.

4. The Structures of Primary Particles

A primary particle looks like a tiny spheroidal balloon with two envelopes (Figure 1). Each of them is formed by an E-W couple or a G-S couple. The envelopes can characterize as:

a. The whole binding energy of the coupled waves concentrates on the envelopes.
b. The macroscopic items of combined field strengths of the two coupled waves are equal on the envelopes. Outside the envelopes, the coupled waves become two independent static fields. But there are no fields inside the envelopes.
c. The size of the envelope, that is the size of a primary particle, depends on the critical radius of the weak or strong interaction.
d. The two envelopes have the same inherent frequency ν_{in}, although this is not mathematically required.
e. The degree of the associated Legendre polynomials j is the same on the two envelopes.
f. The behaviors of the two envelopes obey the Self-Conjugate Mechanism, which requires that one occupies the surface of $\sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) e^{-ik\varphi}$ and the other must take up $\sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) e^{ik\varphi}$, or they are conjugate to each other.

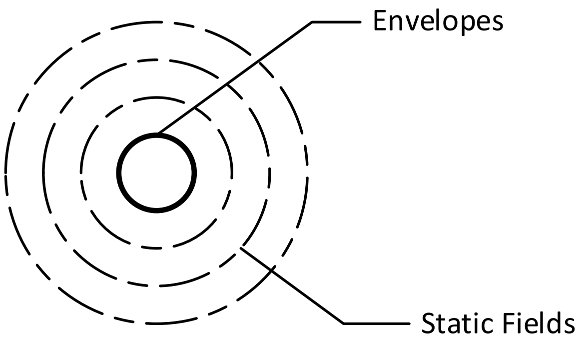

Figure 1. Schematic structures of a primary particle.

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Hence, the binding energy of a primary particle E_{pri} can be generally described as

$$\begin{aligned} E_{pri} &= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_R^\infty \int_0^\pi \int_0^{2\pi} \left| [\hat{E}_a + \hat{E}_b][\hat{E}_c + \hat{E}_d]^* \right| r^2 \sin\theta dr d\theta d\varphi \\ &= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_R^\infty \int_0^\pi \int_0^{2\pi} \left| \hat{E}_a\hat{E}_c^* + \hat{E}_a\hat{E}_d^* + \hat{E}_b\hat{E}_c^* + \hat{E}_b\hat{E}_d^* \right| r^2 \sin\theta dr d\theta d\varphi \quad , \\ &= E_{a\wedge c} + E_{a\wedge d} + E_{b\wedge c} + E_{b\wedge d} \end{aligned} \tag{31}$$

based on Equation (17), where the subscripts a to d denote the electric, gravitational, weak interaction, or strong interaction.

E_{pri} is clearly equivalent to the rest mass. The total energy of a primary particle comprises the binding energy or the rest mass and the energy in static fields.

4.1. An Electron Neutrino

An electron neutrino is composed of two E-W couples with different original spin. In order to explore its structure, these assumptions should be adopted:

a. Its radius r_{e_v} is equal to the critical radius of the weak interaction R_{cw}.
b. The charges in Equations (10) and (12) are equal and minimal for an electron neutrino, i.e., $\hat{q}_{e\_v} = \hat{w}_{e\_v}$, if $\hat{q}_{e\_v}$ and $\hat{w}_{e\_v}$ are the mathematical electric charge and the mathematical weak charge of an electron neutrino.

Integrating Equations (10), (12), and the above characters, we have two field equations on envelopes of an electron neutrino $\hat{E}_{E,\, e\_v}$

$$\hat{E}_{E,\, e\_v} \begin{cases} \left[\hat{E}_e + \hat{E}_w\right]_{r=R_{cw}} = \dfrac{\hat{q}_{e\_v} + \hat{w}_{e\_v}}{c_L^2 R_{cw}^2} (\cos\theta + \sin\theta e^{-i\varphi}) \\ -\left[\hat{E}_e + \hat{E}_w\right]_{r=R_{cw}}^* = -\dfrac{\hat{q}_{e\_v} + \hat{w}_{e\_v}}{c_L^2 R_{cw}^2} (\cos\theta + \sin\theta e^{i\varphi}) \end{cases} \tag{32}$$

where '–' only indicates that two E-W couples are attracted to each other on the envelopes.

Based on Equation (31) and associated Equations (19), (21), and (24), we can easily compute the binding energy of an electron neutrino E_{e_v}.

$$\begin{aligned} E_{e\_v} &= \left[E_{e\wedge e} + 2E_{e\wedge w} + E_{w\wedge w}\right]_{r=R_{cw}} \\ &= \left[\frac{\hat{q}_{e\_v}^2}{c_L^2 r} + \frac{2\hat{q}_{e\_v}\hat{w}_{e\_v}}{c_L^2}\left(\frac{1}{r} - \frac{1}{R_{cw}}\right) + \frac{\hat{w}_{e\_v}^2}{c_L^2 r}\left(\frac{1}{r} - \frac{2m-2}{(2m-1)R_{cw}}\right)\right]_{r=R_{cw}} \\ &= \frac{1}{c_L^2 R_{cw}}\left(\hat{q}_{e\_v}^2 + \frac{\hat{w}_{e\_v}^2}{2m-1}\right) = \frac{2m}{(2m-1)R_{cw}} \frac{\hat{q}_{e\_v}^2}{c_L^2} = \frac{2m}{(2m-1)R_{cw}} \frac{\hat{w}_{e\_v}^2}{c_L^2} \end{aligned} \tag{33}$$

Combining the envelopes' characters b. with Equations (10), (12), and (32), the fields around an electron neutrino $\hat{E}_{e\_v}$ can be directly written as

$$\hat{E}_{e\_v}\Big|_{r>R_{cw}} \begin{cases} \left[\hat{E}_{e-e\_v} + \hat{E}_{w-e\_v}\right] = \pm\dfrac{\hat{q}_{e\_v}}{c_L^2 r^2}(\cos\theta + \sin\theta e^{-i\varphi}) \pm \dfrac{\hat{w}_{e\_v} R_{cw}^{m-1}}{c_L^2 r^{m+1}} \sum_{k=0}^{m} \mathrm{P}_m^k(\cos\theta) e^{-ik\varphi} \\ \left[\hat{E}_{e-e\_v} + \hat{E}_{w-e\_v}\right]^* = \mp\dfrac{\hat{q}_{e\_v}}{c_L^2 r^2}(\cos\theta + \sin\theta e^{i\varphi}) \mp \dfrac{\hat{w}_{e\_v} R_{cw}^{m-1}}{c_L^2 r^{m+1}} \sum_{k=0}^{m} \mathrm{P}_m^k(\cos\theta) e^{ik\varphi} \end{cases} \quad r > R_{cw}$$

. (34)

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4.2. Dark Neutrinos

Two G-S couples make a dark neutrino. However, the strong interaction field has two critical radii, so there are two types of dark neutrinos, and they are named Dark I and Dark II. Similar to Section 0, it is assumed that:

a. The sizes of Dark I and II are equal to the 1st and 2nd critical radii of the strong interaction.
b. Dark I and II have the same mathematical mass and the same mathematical strong charge.
c. The mathematical strong charge $\hat{s}_{D\_\nu}$ is minimal and equal to the mathematical mass $\hat{m}_{D\_\nu}$, i.e., $\hat{m}_{D\_\nu} = \hat{s}_{D\_\nu}$.

Replicating the process of the previous section, we have the fields of a Dark I on the envelopes $\hat{E}_{E,\,D\_\nu I}$

$$\hat{E}_{E,\,D\_\nu I}\begin{cases}\left[\hat{E}_G+\hat{E}_S\right]_{r=R_{cS1}} &= \dfrac{\hat{m}_{D\_\nu}+\hat{s}_{D\_\nu}}{c_G^2 R_{cS1}^2}\left(\cos\theta+\sin\theta e^{-i\varphi}\right)\\ -\left[\hat{E}_G+\hat{E}_S\right]^*_{r=R_{cS1}} &= -\dfrac{\hat{m}_{D\_\nu}+\hat{s}_{D\_\nu}}{c_G^2 R_{cS1}^2}\left(\cos\theta+\sin\theta e^{i\varphi}\right)\end{cases}, \quad (35)$$

where '−' only means that two G-S couples are attracted to each other on the envelopes.

Based on Equation (31) and associated Equations (20), (22), and (27), we can compute the binding energy of a Dark I $E_{D\_\nu I}$

$$\begin{aligned}E_{D\_\nu I} &= \left[E_{G\wedge G}+2E_{G\wedge S}+E_{S\wedge S}\right]_{r=R_{cS1}}\\ &= \frac{c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_\nu}^2}{r}+\hat{m}_{D\_\nu}\hat{s}_{D\_\nu}\left(\frac{1}{r}-\frac{3}{2R_{cS1}}+\frac{R_{cS2}^2}{2R_{cS1}^3}\right)+\hat{s}_{D\_\nu}^2\left(\frac{1}{r}-\frac{6}{5R_{cS1}}+\frac{2(n+2)R_{cS2}^5}{5(2n-1)R_{cS1}^6}\right)\right]_{r=R_{cS1}}\\ &\approx \frac{2(n+2)R_{cS2}^5}{5(2n-1)R_{cS1}^6}\frac{c_L^2\hat{s}_{D\_\nu}^2}{c_G^4} = \frac{2(n+2)R_{cS2}^5}{5(2n-1)R_{cS1}^6}\frac{c_L^2\hat{m}_{D\_\nu}^2}{c_G^4}\end{aligned}$$

. (36)

To get the fields of a Dark II on the envelopes $\hat{E}_{E,\,D\_\nu II}$ and the binding energy of a Dark II $E_{D\_\nu II}$, we imitate the last process and have

$$\hat{E}_{E,\,D\_\nu II}\begin{cases}\left[\hat{E}_G+\hat{E}_S\right]_{r=R_{cS2}} &= \dfrac{1}{c_G^2}\left(\dfrac{\hat{m}_{D\_\nu}}{R_{cS2}^2}+\dfrac{R_{cS2}}{R_{cS1}^3}\hat{s}_{D\_\nu}\right)\left(\cos\theta+\sin\theta e^{-i\varphi}\right)\\ -\left[\hat{E}_G+\hat{E}_S\right]^*_{r=R_{cS2}} &= -\dfrac{1}{c_G^2}\left(\dfrac{\hat{m}_{D\_\nu}}{R_{cS2}^2}+\dfrac{R_{cS2}}{R_{cS1}^3}\hat{s}_{D\_\nu}\right)\left(\cos\theta+\sin\theta e^{i\varphi}\right)\end{cases}, (37)$$

and

$$\begin{aligned}E_{D\_\nu II} &= \left[E_{G-G}+2E_{G-S}+E_{S-S}\right]_{r=R_{cS2}}\\ &= \frac{c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_\nu}^2}{r}+\frac{\hat{s}_{D\_\nu}^2}{5R_{cS1}^6}\left(R_{cS2}^5-r^5\right)+\frac{R_{cS2}^5\hat{s}_{D\_\nu}^2}{(2n-1)R_{cS1}^6}\right]_{r=R_{cS2}}\\ &\approx \frac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\frac{c_L^2\hat{s}_{D\_\nu}^2}{c_G^4} = \frac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\frac{c_L^2\hat{m}_{D\_\nu}^2}{c_G^4}\\ &\approx \frac{5}{2(n+2)}E_{D\_\nu I}\end{aligned}. \quad (38)$$

Following the computations of the particle external field in the previous section, we can obtain the fields around a Dark I $\hat{E}_{D\_\nu I}$

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$$\hat{E}_{D\_vI}\Big|_{r>R_{cS1}}\begin{cases}\left[\hat{E}_{G-D\_vI}+\hat{E}_{S-D\_vI}\right]=\begin{cases}\hat{E}_{G-D\_vI}-\dfrac{\hat{s}_{D\_v}r}{c_G^2R_{cS1}^3}(\cos\theta+\sin\theta e^{-i\varphi}) & R_{cS1}<r\le R_{cS2}\\ \hat{E}_{G-D\_vI}-\dfrac{\hat{s}_{D\_v}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{-ik\varphi} & r>R_{cS2}\\ \text{where } \hat{E}_{G-D\_vI}=-\dfrac{\hat{m}_{D\_v}}{c_G^2r^2}(\cos\theta+\sin\theta e^{-i\varphi}) & \end{cases}\\ \left[\hat{E}_{G-D\_vI}+\hat{E}_{S-D\_vI}\right]^*=\begin{cases}\hat{E}^*_{G-D\_vI}-\dfrac{\hat{s}_{D\_v}r}{c_G^2R_{cS1}^3}(\cos\theta+\sin\theta e^{i\varphi}) & R_{cS1}<r\le R_{cS2}\\ \hat{E}^*_{G-D\_vI}-\dfrac{\hat{s}_{D\_v}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{ik\varphi} & r>R_{cS2}\\ \text{where } \hat{E}^*_{G-D\_vI}=-\dfrac{\hat{m}_{D\_v}}{c_G^2R_{cS1}^2}(\cos\theta+\sin\theta e^{i\varphi}) & \end{cases}\end{cases}$$

, (39)

and the fields around a Dark II $\hat{E}_{D\_vII}$

$$\hat{E}_{D\_vII}\Big|_{r>R_{cS2}}\begin{cases}\left[\hat{E}_{G-D\_vII}+\hat{E}_{S-D\_vII}\right]=-\dfrac{\hat{m}_{D\_v}}{c_G^2r^2}(\cos\theta+\sin\theta e^{-i\varphi})\\ \qquad -\dfrac{\hat{s}_{D\_v}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{-ik\varphi}\\ \left[\hat{E}_{G-D\_vII}+\hat{E}_{S-D\_vII}\right]^*=-\dfrac{\hat{m}_{D\_v}}{c_G^2R_{cS1}^2}(\cos\theta+\sin\theta e^{i\varphi})\\ \qquad -\dfrac{\hat{s}_{D\_v}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{ik\varphi}\end{cases} \quad r>R_{cS2} \quad .(40)$$

Comparing Equations (36) with (38) reveals that, as far as measurements, namely energy, are concerned, there is little difference between Dark I and Dark II. However, their volumes are significant differences in the microscopic domain. There should only be Dark IIs in most cases following the principle of energy minimization.

4.3. An Electron or A Positron

Electrons and positrons have the same structure. We will not distinguish significantly between electrons and positrons in the subsequent descriptions and computations. One E-W couple and one G-S couple attract each other to form an electron or a positron, so its structure is the most complex in primary particles. Following the assumptions about dark neutrinos, it is supposed that:

a. The radius of an electron r_e equals the critical radius of the weak interaction R_{cw}, although there are three critical radii for the weak and strong interactions.
b. The mathematical electric charge $\hat{q}_e$ and the mathematical weak charge $\hat{w}_e$ are equal, i.e., $\hat{q}_e=\hat{w}_e$.
c. The mathematical strong charge $\hat{s}_e$ is minimal, i.e., $\hat{s}_e=\hat{s}_{D\_v}$.

Referring to the way we did in the previous sections, we can obtain the field of an electron on the envelopes $\hat{E}_{E,e}$ and the binding energy of an electron E_e.

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$$\hat{E}_{E,e}\begin{cases}\left[\hat{E}_e+\hat{E}_w\right]_{r=R_{cw}}=\dfrac{\hat{q}_e+\hat{w}_e}{c_L^2R_{cw}^2}(\cos\theta+\sin\theta e^{-i\varphi})\\ -\left[\hat{E}_G+\hat{E}_S\right]^*_{r=R_{cw}}=-\dfrac{1}{c_G^2}\left(\dfrac{\hat{m}_e}{R_{cw}^2}+\dfrac{\hat{s}_eR_{cw}}{R_{cS1}^3}\right)(\cos\theta+\sin\theta e^{i\varphi})\\ \dfrac{\hat{q}_e+\hat{w}_e}{c_L^2R_{cw}^2}=\dfrac{1}{c_G^2}\left(\dfrac{\hat{m}_e}{R_{cw}^2}+\dfrac{\hat{s}_eR_{cw}}{R_{cS1}^3}\right)\end{cases}, \quad (41)$$

and

$$\begin{aligned}E_e&=\left[E_{e\wedge G}+E_{e\wedge S}+E_{G\wedge w}+E_{w\wedge S}\right]_{r=R_{cw}}\\ &=\left[\frac{\hat{q}_e\hat{m}_e}{c_G^2r}+\frac{\hat{q}_e\hat{s}_e}{2c_G^2R_{cS1}^3}\left(R_{cS2}^2-r^2\right)\right]_{r=R_{cw}}\\ &=\frac{\hat{q}_e\hat{m}_e}{c_G^2R_{cw}}+\frac{\hat{q}_e\hat{s}_e}{2c_G^2R_{cS1}^3}\left(R_{cS2}^2-R_{cw}^2\right)\end{aligned}. \quad (42)$$

According to the previous assumptions $R_{cS1} << R_{cw} < R_{cS2}$, $\hat{q}_e=\hat{w}_e$, and $\hat{s}_e=\hat{s}_{D\_v}=\hat{m}_{D\_v}$, we have

$$\frac{2\hat{q}_e}{c_L^2R_{cw}^2}=\frac{2\hat{w}_e}{c_L^2R_{cw}^2}\approx\frac{\hat{s}_{D\_v}R_{cw}}{c_G^2R_{cS1}^3}=\frac{\hat{m}_{D\_v}R_{cw}}{c_G^2R_{cS1}^3} \quad (43)$$

and

$$E_e\approx\frac{\hat{q}_e\hat{s}_{D\_v}}{2c_G^2R_{cS1}^3}\left(R_{cS2}^2-R_{cw}^2\right)=\frac{q_e^2}{c_L^2R_{cw}}\left(\frac{R_{cS2}^2}{R_{cw}^2}-1\right). \quad (44)$$

Now we directly give the result for fields around an electron $\hat{E}_e$.

$$\hat{E}_e\Big|_{r>R_{cw}}\begin{cases}\left[\hat{E}_{e-e}+\hat{E}_{w-e}\right]=\pm\dfrac{\hat{q}_e}{c_L^2r^2}(\cos\theta+\sin\theta e^{-i\phi})\pm\dfrac{\hat{w}_e}{c_L^2r^2}(\cos\theta+\sin\theta e^{-i\varphi}) & r>R_{cw}\\ \left[\hat{E}_{G-e}+\hat{E}_{S-e}\right]^*=\begin{cases}-\dfrac{\hat{m}_e}{c_G^2r^2}(\cos\theta+\sin\theta e^{i\phi})-\dfrac{\hat{s}_er}{c_G^2R_{cS1}^3}(\cos\theta+\sin\theta e^{i\varphi}) & R_{cw}<r\le R_{cS2}\\ -\dfrac{\hat{m}_e}{c_G^2r^2}(\cos\theta+\sin\theta e^{i\phi})-\dfrac{\hat{s}_{D\_v}R_{cS2}^{n+2}}{c_G^2R_{cS1}^3r^{n+1}}\sum_{k=0}^{n}\mathrm{P}_n^k(\cos\theta)e^{ik\varphi} & r>R_{cS2}\end{cases}\end{cases}$$

(45)

Next, the examination of Equations (41) and (42) reveals that the 2nd critical radius of the strong interaction R_{cS2} should be the geometric characterization parameter of the G-S envelope rather than the 1st critical radius of the strong interaction R_{cS1}. It is further assumed that R_{cw} and R_{cS2} are proportional to the wavelengths of the E-W and the G-S couples, respectively, i.e., $R_{cw}=\xi\lambda_{E-W}$, $R_{cS2}=\xi\lambda_{G-S}$. Thus, we can write directly with the envelopes characters d.

$$\frac{R_{cS2}}{R_{cw}}=\frac{\lambda_{G-S}}{\lambda_{E-W}}=\frac{c_G}{c_L}, \quad (46)$$

which shows that the speed of gravity c_G is faster than the speed of light c_L when the above equation compares with Equation (42).

5. The Interactions Between Two Primary Particles

Imagine that two static primary particles are initially rested on each other and then separated by a repulsive force over a distance of l. Particles I and II have potential and kinetic energy in the separated state (Figure 2). In the initial state, particles I and II are equivalent in that they share a common center of the sphere (Figure 2) because there are no fields inside the envelopes of the primary particles.

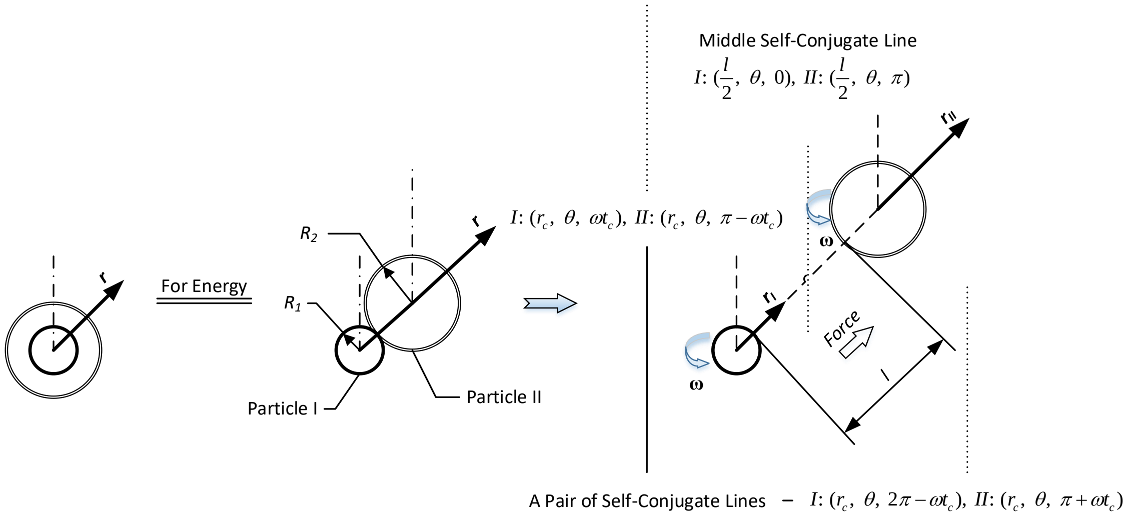

Figure 2. Energy conversion between two primary particles.

According to the law of conservation of energy, the initial external energy of particles I and II, or the total energy in their external fields, is equal to the sum of potential energy and non-potential energy (including kinetic energy, magnetic energy for electrons, etc.) after their separation. Referring to Equation (17), the potential energy of the two primary particles $E_{P_1 \wedge P_2}$ should be associated with the following equation

$$E_{P_1 \wedge P_2} \propto \frac{1}{(t_2 - t_1)2\pi \sum_{k=0}^{j} \mathrm{N}_j^k} \times \left(\int_{R_c}^{\infty}\int_0^{\pi}\int_0^{2\pi}\int_{c_L t_1}^{c_L t_2} \left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi dc_L t - \int_{R_c}^{l}\int_0^{\pi}\int_0^{2\pi}\int_{c_L t_1}^{c_L t_2} \left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi dc_L t \right)$$
$$= \frac{1}{(t_2 - t_1)2\pi \sum_{k=0}^{j} \mathrm{N}_j^k} \int_{l}^{\infty}\int_0^{\pi}\int_0^{2\pi}\int_{c_L t_1}^{c_L t_2} \left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi dc_L t$$
$$= \frac{c_L^2}{2\pi \sum_{k=0}^{j} \mathrm{N}_j^k} \int_{l}^{\infty}\int_0^{\pi}\int_0^{2\pi} \left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi dc_L t \qquad l \text{ is constant} \tag{47}$$

where $\hat{E}_{P_1}$ and $\hat{E}_{P_2}$ are the external fields of primary particles I and II, l is the distance between the two particles (Figure 2), R_c is the larger of the two particles' radii, and $\int_{R_c}^{l}\int_0^{\pi}\int_0^{2\pi}\int_{c_L t_1}^{c_L t_2} \left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi dc_L t$ converts to the non-potential energy.

From Equations (19) and (20), the integration result of Equation (47) in the macroscopic world is consistent with the expressions for electric potential energy and gravitational potential energy, then Equation (47) must still hold in the microscopic world. It is therefore assumed that the Self-Conjugate Mechanism between two interacting primary particles remains everywhere. The Self-Conjugate

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Mechanism makes two sets of fields around one particle conjugate to two sets of fields around another particle, depending on the rotation of two particles. In other words, two primary particles have achieved the self-conjugacy after rotating one cycle with angular velocity **ω**. There are self-conjugate lines in pairs, and it is further presumed that in their respective spherical coordinate systems, one is at $I\!: (r_c,\ \theta,\ \omega t_c)$ for particle I and $II\!: (r_c,\ \theta,\ \pi-\omega t_c)$ for particle II, and another is at $I\!: (r_c,\ \theta,\ 2\pi-\omega t_c)$ for particle I and $II\!: (r_c,\ \theta,\ \pi+\omega t_c)$ for particle II. Obviously, these lines are in the opposite position meanwhile $r_I = r_{II} = r$, $\theta_I = \theta_{II} = \theta$ (Figure 2). Thus, at a self-conjugate line, taking the $\mathbf{r}_I$ coordinate system of the particle I as a reference (Figure 2), and using Equation (7) , Equation (47) can be rewritten as

$$
\begin{aligned}
E_{P_1 \wedge P_2} &= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_l^\infty \int_0^\pi \int_0^{2\pi} \left| \hat{E}_{P_1}(\mathbf{r}_I) \hat{E}_{P_2}^*(\mathbf{r}_{II}+\mathbf{l}) \right| r_I^2 \sin\theta_I dr_I d\theta_I d\varphi_I \\
&= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_l^\infty \int_0^\pi \int_{2\eta\pi}^{2\eta\pi+2\pi} \left| \hat{E}_{P_1}(r,\ \theta,\ \omega t_c) \hat{E}_{P_2}^*(r,\ \theta,\ \pi-\omega t_c) \right| r^2 \sin\theta dr d\theta d(\omega t_c) \\
&= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_l^\infty \int_0^\pi \left[\hat{E}_{P_1}(r) \sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) \right] \left[\hat{E}_{P_2}(r) \sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) \right] r^2 \sin\theta dr d\theta \int_{2\eta\pi}^{2\eta\pi+2\pi} e^{im(\omega t_c)} e^{-in(\pi-\omega t_c)} d(\omega t_c) \\
&= \frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_l^\infty \int_0^\pi \left[\hat{E}_{P_1}(r) \sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) \right] \left[\hat{E}_{P_2}(r) \sum_{k=0}^{j} \mathrm{P}_j^k(\cos\theta) \right] r^2 \sin\theta dr d\theta \times \\
&\begin{cases} \int_{2\eta\pi}^{2\eta\pi+2\pi} e^{im(\omega t_c)} e^{-in(\pi+\omega t_c)} d(\omega t_c) = -2\pi\delta_{mn} & \text{Opposite rotational directions} \\ \int_{2\eta\pi}^{2\eta\pi+2\pi} e^{im(\omega t_c)} e^{-in(\pi-\omega t_c)} d(\omega t_c) = 0 & \text{Same rotational direction} \end{cases} \\
&= \begin{cases} -\frac{c_L^2}{2\pi\sum_{k=0}^{j} \mathrm{N}_j^k} \int_l^\infty \int_0^\pi \int_0^{2\pi} \left| \hat{E}_{P_1} \hat{E}_{P_2}^* \right| r^2 \sin\theta dr d\theta d\varphi & \text{Opposite rotational directions} \\ 0 & \text{Same rotational direction} \end{cases}
\end{aligned}
\tag{48}
$$

,

where **l** is the vector of the distance l, t_c is the time of conjugation of two primary particles, and $\eta = 0$, 1, 2, …, η. Here, the negative sign only indicates that the energy spreads out in this process. In other words, Equation (48) should have the positive sign when two primary particles move in the opposite mode of Figure 2, i.e., two rested on each other particles are attracted at the initial distance of l and approach each other, since the $\mathbf{r}_{II}$ coordinate system is reversed in Figure 2, which means that the energy collects. Later, we will continue to use Equation (48) and ignore the negative sign, without distinguishing whether the energy is dissipated or accumulated in the motion of two primary particles.

Thus we can follow the results from the previous chapters when there is the potential energy between two primary particles, and the distance of l remains constant. There are only two conjugate forms between the two particles because each of the two particles consists of two coupled waves. Therefore, Equation (48) can be translated into

$$E_{P_1\wedge P_2} = \frac{c_L^2}{2\pi\sum_{k=0}^{j} N_j^k}\int_l^{\infty}\int_0^{\pi}\int_0^{2\pi}\left|\hat{E}_{P_1}\hat{E}_{P_2}^*\right| r^2 \sin\theta dr d\theta d\varphi = \frac{c_L^2}{2\pi\sum_{k=0}^{j} N_j^k}\times$$
$$\int_l^{\infty}\int_0^{\pi}\int_0^{2\pi}\left|\pm[\hat{E}_{a-P_1}+\hat{E}_{b-P_1}][\hat{E}_{c-P_2}+\hat{E}_{d-P_2}]^* \pm[\hat{E}_{w-P_1}+\hat{E}_{x-P_1}][\hat{E}_{y-P_2}+\hat{E}_{z-P_2}]^*\right| r^2 \sin\theta dr d\theta d\varphi$$
$$= \left[E_{a-P_1\wedge c-P_2}+E_{a-P_1\wedge d-P_2}+E_{b-P_1\wedge c-P_2}+E_{b-P_1\wedge d-P_2}\right]_{r=l+R_c}$$
$$+\left[E_{w-P_1\wedge y-P_2}+E_{w-P_1\wedge z-P_2}+E_{x-P_1\wedge y-P_2}+E_{x-P_1\wedge z-P_2}\right]_{r=l+R_c} \quad (49)$$

where the subscripts a to d, and w to z denote the electric, gravitational, weak interaction, or strong interaction.

According to Newtonian mechanics, the work done is equal to the potential energy when primary particles I and II move relative to each other. Reversing the Newtonian definition of work, i.e., $\mathbf{F} = \nabla W = \nabla E_{P_1\wedge P_2}$, the force between two primary particles $F_{P_1\wedge P_2}$ is therefore

$$F_{P_1\wedge P_2} = \pm\frac{dE_{P_1\wedge P_2}}{dl} \quad . \quad (50)$$

From Equation (48) it can be seen that two self-conjugate primary particles have potential energy or force when they rotate in opposite directions, while they have zero potential energy or rest when they rotate in the same direction. Clearly there are two modes of opposite rotation for two self-conjugate primary particles, which shows that each primary particle has spin values of $\pm\frac{1}{2}$.

The result of Equation (49) correlates the conjugate forms of all four fundamental waves, making subsequent calculations based on it lengthy. Therefore, in the following equations, only the terms associated with the mathematical electric charge $\hat{q}$ and the mathematical mass $\hat{m}$ are shown to make the results concise, since the previous assumption was that $\hat{q}_{e\_v} = \hat{w}_{e\_v}$, $\hat{q}_e = \hat{w}_e$, $\hat{s}_e = \hat{s}_{D\_v} = \hat{m}_{D\_v}$. The assumption $R_{cS1} << R_{cw} < R_{cS2}$ is also used to further simplify the results of the later equations.

5.1. Two Particles of the Same Type

Start by computing the interaction between two electron neutrinos. Combining Equations (19), (21), and (49), we have the potential energy $E_{e\_v\wedge e\_v}$

$$\begin{aligned}E_{e\_v\wedge e\_v} &= 2\left[E_{e-e\_v\wedge e-e\_v}+2E_{e-e\_v\wedge w-e\_v}+E_{w-e\_v\wedge w-e\_v}\right]_{r=l+R_{cw}}\\ &= 2\left[\frac{\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})}+0+\frac{R_{cw}^{2m-2}\hat{w}_{e\_v}^2}{c_L^2(2m-1)(l+R_{cw})^{2m-1}}\right]\\ &= \frac{2\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})}\left[1+\frac{R_{cw}^{2m-2}}{(2m-1)(l+R_{cw})^{2m-2}}\right]\\ &\approx \frac{2\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})} \qquad l \ge R_{cw}\end{aligned} \quad . \quad (51)$$

Since two equations (34) have attractive and repulsive states under the Self-Conjugate Mechanism, from equation (50), the force of two electron neutrinos $F_{e\_v\wedge e\_v}$ is

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$$F_{e\_v\wedge e\_v}=\pm 2\left[\frac{\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})^2}+\frac{R_{cw}^{2m-2}\hat{w}_{e\_v}^2}{c_L^2(l+R_{cw})^{2m}}\right]=\pm\frac{2\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})^2}\left[1+\frac{R_{cw}^{2m-2}}{c_L^2(l+R_{cw})^{2m-2}}\right]\approx\pm\frac{2\hat{q}_{e\_v}^2}{c_L^2(l+R_{cw})^2}\quad l\geq R_{cw}\quad, (52)$$

where the sign '–' or '+' depends on the self-conjugate forms between two electron neutrinos, and the "–" or "+" should be random.

Association equation (49) with equations (20), (22), (27), and (39), we can compute the potential energy between two Dark Is $E_{D\_vI\wedge D\_vI}$

$$E_{D\_vI\wedge D\_vI}=2\left[E_{G-D\_vI\wedge G-D\_vI}+2E_{G-D\_vI\wedge S-D\_vI}+E_{S-D\_vI\wedge S-D\_vI}\right]_{r=l+R_{cS1}}$$

$$=\begin{cases}\frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{l+R_{cS1}}+\frac{\hat{m}_{D\_v}\hat{s}_{D\_v}}{R_{cS1}^3}\left(R_{cS2}^2-(l+R_{cS1})^2\right)+\frac{\hat{s}_{D\_v}^2}{R_{cS1}^6}\left(\frac{R_{cS2}^5-(l+R_{cS1})^5}{5}+\frac{R_{cS2}^5}{2n-1}\right)\right]\\ =\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}\left[\frac{R_{cS1}^6}{l+R_{cS1}}+R_{cS1}^3\left(R_{cS2}^2-(l+R_{cS1})^2\right)+\frac{R_{cS2}^5-(l+R_{cS1})^5}{5}+\frac{R_{cS2}^5}{2n-1}\right]\\ \approx\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}\left[\frac{R_{cS2}^5-(l+R_{cS1})^5}{5}+\frac{R_{cS2}^5}{2n-1}\right] & 0\leq l\leq R_{cS2}-R_{cS1}\\ \frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{l+R_{cS1}}+0+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6}\frac{\hat{s}_{D\_v}^2}{(l+R_{cS1})^{2n-1}}\right]\\ =\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4}\left[\frac{1}{l+R_{cS1}}+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6(l+R_{cS1})^{2n-1}}\right]\\ =\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS1})}\left[1+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6(l+R_{cS1})^{2n-2}}\right] & l>R_{cS2}-R_{cS1}\end{cases}$$

, (53)

and the force between two Dark Is $F_{D\_vI\wedge D\_vI}$

$$F_{D\_vI\wedge D\_vI}=\begin{cases}-\frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{(l+R_{cS1})^2}+\frac{2\hat{m}_{D\_v}\hat{s}_{D\_v}}{R_{cS1}^3}(l+R_{cS1})+\frac{\hat{s}_{D\_v}^2}{R_{cS1}^6}(l+R_{cS1})^4\right]\\ =-\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS1})^2}\left[1+\frac{2(l+R_{cS1})^3}{R_{cS1}^3}+\frac{(l+R_{cS1})^6}{R_{cS1}^6}\right] & 0\leq l\leq R_{cS2}-R_{cS1}\\ -\frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{(l+R_{cS1})^2}+0+\frac{R_{cS2}^{2n+4}}{R_{cS1}^6}\frac{\hat{s}_{D\_v}^2}{(l+R_{cS1})^{2n}}\right]\\ =-\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS1})^2}\left[1+\frac{R_{cS2}^{2n+4}}{R_{cS1}^6(l+R_{cS1})^{2n-2}}\right] & l>R_{cS2}-R_{cS1}\end{cases}.$$

(54)

Duplicating the last process yields the potential energy between two Dark IIs $E_{D\_vII\wedge D\_vII}$

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$$\begin{aligned} E_{D\_vII\wedge D\_vII} &= 2\left[E_{G-D\_vII\wedge G-D\_vII} + 2E_{G-D\_vII\wedge S-D\_vII} + E_{S-D\_vII\wedge S-D\_vII}\right]_{r=l+R_{cS2}} \\ &= \frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{l+R_{cS2}} + 0 + \frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6}\frac{\hat{s}_{D\_v}^2}{(l+R_{cS2})^{2n-1}}\right], \\ &= \frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS2})}\left[1+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6(l+R_{cS2})^{2n-2}}\right] \end{aligned} \quad (55)$$

and the force between two Dark IIs $F_{D\_vII\wedge D\_vII}$

$$\begin{aligned} F_{D\_vII\wedge D\_vII} &= -\frac{2c_L^2}{c_G^4}\left[\frac{\hat{m}_{D\_v}^2}{(l+R_{cS2})^2} + \frac{R_{cS2}^{2n+4}}{R_{cS1}^6}\frac{\hat{s}_{D\_v}^2}{(l+R_{cS2})^{2n}}\right] \\ &= -\frac{2c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS2})^2}\left[1+\frac{R_{cS2}^{2n+4}}{R_{cS1}^6(l+R_{cS2})^{2n-2}}\right] \end{aligned}. \quad (56)$$

The potential energy between two electrons $E_{e\wedge e}$ has two forms because an electron is composed of one E-W couple and one G-S couple. Same as electron neutrinos, the two forms should be random and rely on the self-conjugate forms between two electrons. Combining equations (19) to (28) and (49), we can compute the two forms of the potential energy. One is $E_{e\wedge eI}$ when the two E-W couples are conjugate, and the two G-S couples are conjugate.

$$\begin{aligned} E_{e\wedge eI} &= \left[E_{e-e\wedge e-e} + 2E_{e-e\wedge w-e} + E_{w-e\wedge w-e} + E_{G-e\wedge G-e} + 2E_{G-e\wedge S-e} + E_{S-e\wedge S-e}\right]_{r=l+R_{cw}} \\ &= \begin{cases} \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})} + 0 + \dfrac{R_{cw}^{2m-2}\hat{w}_e^2}{c_L^2(2m-1)(l+R_{cw})^{2m-1}} + \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})} + \dfrac{c_L^2\hat{m}_e\hat{s}_e}{c_G^4R_{cS1}^3}\left[R_{cS2}^2-(l+R_{cw})^2\right] \\ \quad + \dfrac{c_L^2\hat{s}_e^2}{c_G^4}\left[\dfrac{1}{5R_{cS1}^6}\left(R_{cS2}^5-(l+R_{cw})^5\right) + \dfrac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\right] & 0 \le l \le R_{cS2}-R_{cw} \\ \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})} + \dfrac{R_{cw}^{2m-2}\hat{w}_e^2}{c_L^2(2m-1)(l+R_{cw})^{2m-1}} + \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})} \\ \quad + \dfrac{c_L^2R_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6}\dfrac{\hat{s}_e^2}{(l+R_{cw})^{2n-1}} & l > R_{cS2}-R_{cw} \end{cases} \\ &= \begin{cases} \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})}\left[1+\dfrac{R_{cw}^{2m-2}}{c_L^2(2m-1)(l+R_{cw})^{2m-2}}\right] + \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})} + \dfrac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4R_{cS1}^3}\left[R_{cS2}^2-(l+R_{cw})^2\right] \\ \quad + \dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4}\left[\dfrac{1}{5R_{cS1}^6}\left(R_{cS2}^5-(l+R_{cw})^5\right) + \dfrac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\right] & 0 \le l \le R_{cS2}-R_{cw} \\ \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})}\left[1+\dfrac{R_{cw}^{2m-2}}{c_L^2(2m-1)(l+R_{cw})^{2m-2}}\right] + \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})} \\ \quad + \dfrac{c_L^2R_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6}\dfrac{\hat{m}_{D\_v}^2}{(l+R_{cw})^{2n-1}} & l > R_{cS2}-R_{cw} \end{cases} \\ &\approx \frac{\hat{q}_e^2}{c_L^2(l+R_{cw})} + \frac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})} \qquad l \text{ is large} \end{aligned} \quad (57)$$

Another is $E_{e\wedge eII}$ when the two E-W couples are conjugate to the two G-S couples.

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$$E_{e\wedge eII} = 2\left[E_{e-e\wedge G-e} + E_{e-e\wedge S-e} + E_{w-e\wedge G-e} + E_{w-e\wedge S-e}\right]_{r=l+R_{cw}}$$
$$= \begin{cases} \dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})} + \dfrac{\hat{q}_e\hat{s}_e}{c_G^2R_{cS1}^3}\left[R_{cS2}^2 - (l+R_{cw})^2\right] + 0 + 0 & 0 \le l \le R_{cS2} - R_{cw} \\ \dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})} + 0 + 0 + 0 & l > R_{cS2} - R_{cw} \end{cases} \qquad (58)$$
$$= \begin{cases} \dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})} + \dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2R_{cS1}}\left[R_{cS2}^2 - (l+R_{cw})^2\right] & 0 \le l \le R_{cS2} - R_{cw} \\ \dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})} & l > R_{cS2} - R_{cw} \end{cases}$$

Derivation of the last two equations can yield the forces between two electrons in both forms that are

$$F_{e\wedge eI} = \begin{cases} \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})^2} + \dfrac{R_{cw}^{2m-2}\hat{w}_e^2}{c_L^2(l+R_{cw})^{2m}} - \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})^2} - \dfrac{2c_L^2\hat{m}_e\hat{s}_e}{c_G^4R_{cS1}^3}(l+R_{cw}) \\ -\dfrac{c_L^2\hat{s}_e^2}{c_G^4R_{cS1}^6}(l+R_{cw})^4 & 0 \le l \le R_{cS2} - R_{cw} \\ \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})^2} + \dfrac{R_{cw}^{2m-2}\hat{w}_e^2}{c_L^2(l+R_{cw})^{2m}} - \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})^2} - \dfrac{c_L^2R_{cS2}^{2n+4}}{c_G^4R_{cS1}^6}\dfrac{\hat{s}_e^2}{(l+R_{cw})^{2n}} & l > R_{cS2} - R_{cw} \end{cases}$$
$$= \begin{cases} \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})^2}\left[1 + \dfrac{R_{cw}^{2m-2}}{(l+R_{cw})^{2m-2}}\right] - \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})^2} - \dfrac{2c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4R_{cS1}^3}(l+R_{cw}) \\ -\dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}(l+R_{cw})^4 & 0 \le l \le R_{cS2} - R_{cw} \\ \dfrac{\hat{q}_e^2}{c_L^2(l+R_{cw})^2}\left[1 + \dfrac{R_{cw}^{2m-2}}{(l+R_{cw})^{2m-2}}\right] - \dfrac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})^2} - \dfrac{c_L^2R_{cS2}^{2n+4}}{c_G^4R_{cS1}^6}\dfrac{\hat{m}_{D\_v}^2}{(l+R_{cw})^{2n}} & l > R_{cS2} - R_{cw} \end{cases}$$
$$\approx \frac{\hat{q}_e^2}{c_L^2(l+R_{cw})^2} - \frac{c_L^2\hat{m}_e^2}{c_G^4(l+R_{cw})^2} \qquad l \text{ is large}$$

(59)

and

$$F_{e\wedge eII} = \begin{cases} -\dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})^2} - \dfrac{2\hat{q}_e\hat{s}_e}{c_G^2R_{cS1}^3}(l+R_{cw}) & 0 \le l \le R_{cS2} - R_{cw} \\ -\dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})^2} & l > R_{cS2} - R_{cw} \end{cases}$$
$$= \begin{cases} -\dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})^2} - \dfrac{2\hat{q}_e\hat{m}_{D\_v}}{c_G^2R_{cS1}^3}(l+R_{cw}) & 0 \le l \le R_{cS2} - R_{cw} \\ -\dfrac{2\hat{q}_e\hat{m}_e}{c_G^2(l+R_{cw})^2} & l > R_{cS2} - R_{cw} \end{cases} \qquad . \qquad (60)$$

Comparing equations (57) with (59) and (58) with (60) shows that the potential energy and the force between two electrons are very different in the two self-conjugate forms, with the smaller one close to zero.

5.2. Two Particles of the Different Type

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Similar to the last section, this one starts by computing the interaction of an electron neutrino with another primary particle. And note that the electron charge-to-mass ratio is 1.76×10^{11}C/kg[7], so the mathematical electric charge-to-mass ratio of electrons is $\frac{c_L^2\sqrt{k}}{c_G^2\sqrt{G}}\frac{q_e}{m_e}=\frac{c_L^2\sqrt{8.99\times10^9}}{c_G^2\sqrt{6.67\times10^{-11}}}\times1.76\times10^{11}=2.04\times10^{21}\frac{c_L^2}{c_G^2}$ from equations (29) and (30). This also means that the mathematical electric charge of an electron $\hat{q}_e$ is much larger than the mathematical mass of an electron $\hat{m}_e$, despite the different definitions of electron mass and mathematical mass of an electron. Therefore, in the subsequent calculation results, the terms relating to $\hat{q}_e$ are retained, and the terms relating to $\hat{m}_e$ are omitted.

Combining Equations (23) to (28), (49), and (50), the potential energy between an electron neutrino and a Dark I $E_{e\_v\wedge D\_vI}$ is

$$E_{e\_v\wedge D\_vI}=2\left[E_{e-e\_v\wedge G-D\_vI}+E_{e-e\_v\wedge S-D\_vI}+E_{w-e\_v\wedge G-D\_vI}+E_{w-e\_v\wedge S-D\_vI}\right]_{r=l+R_{cw}}$$
$$=\begin{cases}\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+\frac{\hat{q}_{e\_v}\hat{s}_{D\_v}}{c_G^2R_{cS1}^3}\left(R_{cS2}^2-(l+R_{cw})^2\right)+0+0 & 0\le l\le R_{cS2}-R_{cw}\\ \frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+0+0+0 & l>R_{cS2}-R_{cw}\end{cases},$$
$$=\begin{cases}\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}\left[1+\frac{R_{cS2}^2(l+R_{cw})-(l+R_{cw})^3}{2R_{cS1}^3}\right] & 0\le l\le R_{cS2}-R_{cw}\\ \frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})} & l>R_{cS2}-R_{cw}\end{cases}$$
(61)

and the force between an electron neutrino and a Dark I $F_{e\_v\wedge D\_vI}$ is

$$F_{e\_v\wedge D\_vI}=\begin{cases}-\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}-\frac{2\hat{q}_{e\_v}\hat{s}_{D\_v}}{c_G^2R_{cS1}^3}(l+R_{cw}) & 0\le l\le R_{cS2}-R_{cw}\\ -\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2} & l>R_{cS2}-R_{cw}\end{cases}$$
$$=\begin{cases}-\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}\left[1-\frac{(l+R_{cw})^3}{R_{cS1}^3}\right] & 0\le l\le R_{cS2}-R_{cw}\\ -\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2} & l>R_{cS2}-R_{cw}\end{cases}.(62)$$

Repeating the previous processes gives the potential energy between an electron neutrino and a Dark II $E_{e\_v\wedge D\_vII}$

$$E_{e\_v\wedge D\_vII}=2\left[E_{e-e\_v\wedge G-D\_vII}+E_{e-e\_v\wedge S-D\_vII}+E_{w-e\_v\wedge G-D\_vII}+E_{w-e\_v\wedge S-D\_vII}\right]_{r=l+R_{cS2}}$$
$$=\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cS2})}+0+0+0 \quad ,(63)$$
$$=\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cS2})}$$

the force between an electron neutrino and a Dark I $F_{e\_v\wedge D\_vII}$

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$$F_{e\_v\wedge D\_vII} = -\frac{2\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2(l+R_{cS2})^2} \quad , \qquad (64)$$

the potential energy between an electron neutrino and an electron $E_{e\_v\wedge e}$

$$E_{e\_v\wedge e} = \left[E_{e-e\_v\wedge e-e} + E_{e-e\_v\wedge w-e} + E_{w-e\_v\wedge e-e} + E_{w-e\_v\wedge w-e}\right]_{r=l+R_{cw}}$$
$$+\left[E_{e-e\_v\wedge G-e} + E_{e-e\_v\wedge S-e} + E_{w-e\_v\wedge G-e} + E_{w-e\_v\wedge S-e}\right]_{r=l+R_{cw}}$$
$$= \begin{cases} \frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})} + 0 + 0 + \frac{R_{cw}^{2m-2}\hat{w}_{e\_v}\hat{w}_e}{c_L^2(2m-1)(l+R_{cw})^{2m-1}} \\ + \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})} + \frac{\hat{q}_{e\_v}\hat{s}_e}{2c_G^2 R_{cS1}^3}\left(R_{cS2}^2 - (l+R_{cw})^2\right) + 0 + 0 & 0 \le l \le R_{cS2} - R_{cw} \\ \frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})} + 0 + 0 + \frac{R_{cw}^{2m-2}\hat{w}_{e\_v}\hat{w}_e}{c_L^2(2m-1)(l+R_{cw})^{2m-1}} + \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})} + 0 + 0 + 0 & l > R_{cS2} - R_{cw} \end{cases}$$
$$= \begin{cases} \frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})}\left[1 + \frac{R_{cw}^{2m-2}}{(2m-1)(l+R_{cw})^{2m-2}}\right] + \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})} \\ + \frac{\hat{q}_{e\_v}\hat{m}_{D\_v}}{2c_G^2 R_{cS1}^3}\left(R_{cS2}^2 - (l+R_{cw})^2\right) & 0 \le l \le R_{cS2} - R_{cw} \\ \frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})}\left[1 + \frac{R_{cw}^{2m-2}}{(2m-1)(l+R_{cw})^{2m-2}}\right] & l > R_{cS2} - R_{cw} \end{cases}$$
$$\approx \frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})} \qquad l \ge R_{cw}$$
$$, \qquad (65)$$

and the force between an electron neutrino and an electron $F_{e\_v\wedge e}$

$$F_{e\_v\wedge e} = \begin{cases} \pm\frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})^2} \pm \frac{R_{cw}^{2m-2}\hat{w}_{e\_v}\hat{w}_e}{c_L^2(l+R_{cw})^{2m}} - \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})^2} - \frac{\hat{q}_{e\_v}\hat{s}_e}{c_G^2 R_{cS1}^3}(l+R_{cw}) & 0 \le l \le R_{cS2} - R_{cw} \\ \pm\frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})^2} \pm \frac{R_{cw}^{2m-2}\hat{w}_{e\_v}\hat{w}_e}{c_L^2(l+R_{cw})^{2m}} - \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})^2} & l > R_{cS2} - R_{cw} \end{cases}$$
$$= \begin{cases} \pm\frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})^2}\left[1 + \frac{R_{cw}^{2m-2}}{(l+R_{cw})^{2m-2}}\right] - \frac{\hat{q}_{e\_v}\hat{m}_e}{c_G^2(l+R_{cw})^2} - \frac{\hat{q}_{e\_v}\hat{m}_{D\_v}}{c_G^2 R_{cS1}^3}(l+R_{cw}) & 0 \le l \le R_{cS2} - R_{cw} \\ \pm\frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})^2}\left[1 + \frac{R_{cw}^{2m-2}}{(l+R_{cw})^{2m-2}}\right] & l > R_{cS2} - R_{cw} \end{cases}$$
$$\approx \pm\frac{\hat{q}_{e\_v}\hat{q}_e}{c_L^2(l+R_{cw})^2} \qquad l \ge R_{cw}$$
$$, \qquad (66)$$

where the sign '–' or '+' randomizes in equation (66).

It is next computed that the potential energies and forces between the two types of dark neutrinos and between each of them and an electron. Based on equations (20), (22), (27), (49), and (50), the potential energy between a Dark I and a Dark II $E_{D\_vI\wedge D\_vII}$ is

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$$
\begin{aligned}
E_{D\_vI\wedge D\_vII} &= 2\left[E_{G-D\_vI\wedge G-D\_vII} + E_{G-D\_vI\wedge S-D\_vII} + E_{S-D\_vI\wedge G-D\_vII} + E_{S-D\_vI\wedge S-D\_vII}\right]_{r=l+R_{cS2}} \\
&= 2\left[\frac{c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})} + 0 + 0 + \frac{c_L^2 R_{cS2}^{2n+4}}{c_G^4 (2n-1) R_{cS1}^6} \frac{\hat{s}_{D\_v}^2}{(l+R_{cS2})^{2n-1}}\right] \\
&= \frac{2c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})}\left[1 + \frac{R_{cS2}^{2n+4}}{(2n-1) R_{cS1}^6 (l+R_{cS2})^{2n-2}}\right]
\end{aligned}
$$

, (67)

and the force between a Dark I and a Dark II $F_{D\_vI\wedge D\_vII}$ is

$$
\begin{aligned}
F_{D\_vI\wedge D\_vII} &= -\frac{2c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})^2} - \frac{2c_L^2 R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6} \frac{\hat{s}_{D\_v}^2}{(l+R_{cS2})^{2n}} \\
&= -\frac{2c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})^2}\left[1 + \frac{R_{cS2}^{2n+4}}{R_{cS1}^6 (l+R_{cS2})^{2n-2}}\right]
\end{aligned}
\quad . \qquad (68)
$$

Based on equations (23) to (28), (43), (49), and (50), the potential energy between a Dark I and an electron $E_{D\_vI\wedge e}$ is

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$$E_{D\_vI\wedge e}=\left[E_{G-D\_vI\wedge e-e}+E_{G-D\_vI\wedge w-e}+E_{S-D\_vI\wedge e-e}+E_{S-D\_vI\wedge w-e}\right]_{r=l+R_{cw}}$$
$$+\left[E_{G-D\_vI\wedge G-e}+E_{G-D\_vI\wedge S-e}+E_{S-D\_vI\wedge G-e}+E_{S-D\_vI\wedge S-e}\right]_{r=l+R_{cw}}$$
$$=\begin{cases}\frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+0+\frac{\hat{q}_e\hat{s}_{D\_v}}{2c_G^2R_{cS1}^3}\left(R_{cS2}^2-(l+R_{cw})^2\right)+0+\frac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})} \\ +\frac{c_L^2\hat{m}_{D\_v}\hat{s}_e}{2c_G^4R_{cS1}^3}\left(R_{cS2}^2-(l+R_{cw})^2\right)+\frac{c_L^2\hat{m}_e\hat{s}_{D\_v}}{2c_G^4R_{cS1}^3}\left(R_{cS2}^2-(l+R_{cw})^2\right) \\ +\frac{c_L^2\hat{s}_{D\_v}\hat{s}_e}{c_G^4}\left[\frac{1}{5R_{cS1}^6}\left(R_{cS2}^5-(l+R_{cw})^5\right)+\frac{R_{cS2}^5}{(2n-1)R_{cS1}^6}\right] & 0\geq l\geq R_{cS2}-R_{cw} \\ \frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+0+0+0+\frac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})}+0+0+\frac{c_L^2\hat{s}_{D\_v}\hat{s}_eR_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6(l+R_{cw})^{2n-1}} & l>R_{cS2}-R_{cw}\end{cases}$$
$$=\begin{cases}\frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2}\left[\frac{1}{l+R_{cw}}+\frac{R_{cS2}^2-(l+R_{cw})^2}{2R_{cS1}^3}\right]+\frac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4}\left[\frac{1}{l+R_{cw}}+\frac{R_{cS2}^2-(l+R_{cw})^2}{2R_{cS1}^3}\right] \\ +\frac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^3}\left[\frac{R_{cS2}^2-(l+R_{cw})^2}{2}+\frac{R_{cS2}^5-(l+R_{cw})^5}{5R_{cS1}^3}+\frac{R_{cS2}^5}{(2n-1)R_{cS1}^3}\right] & 0\geq l\geq R_{cS2}-R_{cw} \\ \frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+\frac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})}+\frac{c_L^2\hat{m}_{D\_v}^2R_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6(l+R_{cw})^{2n-1}} & l>R_{cS2}-R_{cw}\end{cases}$$
$$\approx\begin{cases}\frac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}\left[\frac{R_{cS2}^5-(l+R_{cw})^5}{5}+\frac{R_{cS2}^5}{2n-1}\right] & 0\geq l\geq R_{cS2}-R_{cw} \\ \frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})}+\frac{c_L^2\hat{m}_{D\_v}^2R_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6(l+R_{cw})^{2n-1}} & l>R_{cS2}-R_{cw}\end{cases}$$
$$\approx\begin{cases}\frac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}\left[\frac{R_{cS2}^5-(l+R_{cw})^5}{5}+\frac{R_{cS2}^5}{2n-1}\right] & 0\geq l\geq R_{cS2}-R_{cw} \\ \frac{c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cw})}\left[\frac{R_{cw}^3}{2R_{cS1}^3}+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6(l+R_{cw})^{2n-2}}\right] & l>R_{cS2}-R_{cw}\end{cases}$$
, (69)

the force between a Dark I and an electron $F_{D\_vI\wedge e}$ is

$$F_{D\_vI\wedge e}=\begin{cases}-\dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}-\dfrac{\hat{q}_e\hat{s}_{D\_v}}{c_G^2R_{cS1}^3}(l+R_{cw})-\dfrac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})^2} & \\ -\dfrac{c_L^2\hat{m}_{D\_v}\hat{s}_e}{c_G^4R_{cS1}^3}(l+R_{cw})-\dfrac{c_L^2\hat{m}_e\hat{s}_{D\_v}}{c_G^4R_{cS1}^3}(l+R_{cw})-\dfrac{c_L^2\hat{s}_{D\_v}\hat{s}_e}{c_G^4R_{cS1}^6}(l+R_{cw})^4 & 0\geq l\geq R_{cS2}-R_{cw}\\ -\dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}-\dfrac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})^2}-\dfrac{c_L^2\hat{s}_{D\_v}\hat{s}_eR_{cS2}^{2n+4}}{c_G^4R_{cS1}^6(l+R_{cw})^{2n}} & l>R_{cS2}-R_{cw}\end{cases}$$

$$=\begin{cases}-\dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}\left[1+\dfrac{(l+R_{cw})^3}{R_{cS1}^3}\right]-\dfrac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})^2}\left[1+\dfrac{(l+R_{cw})^3}{R_{cS1}^3}\right] & \\ -\dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^3}(l+R_{cw})\left[1+\dfrac{(l+R_{cw})^3}{R_{cS1}^3}\right] & 0\geq l\geq R_{cS2}-R_{cw}\\ -\dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}-\dfrac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cw})^2}-\dfrac{c_L^2\hat{m}_{D\_v}^2R_{cS2}^{2n+4}}{c_G^4R_{cS1}^6(l+R_{cw})^{2n}} & l>R_{cS2}-R_{cw}\end{cases}$$

$$\approx\begin{cases}-\dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}(l+R_{cw})^4 & 0\geq l\geq R_{cS2}-R_{cw}\\ -\dfrac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cw})^2}-\dfrac{c_L^2\hat{m}_{D\_v}^2R_{cS2}^{2n+4}}{c_G^4R_{cS1}^6(l+R_{cw})^{2n}} & l>R_{cS2}-R_{cw}\end{cases}$$

$$\approx\begin{cases}-\dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4R_{cS1}^6}(l+R_{cw})^4 & 0\geq l\geq R_{cS2}-R_{cw}\\ -\dfrac{c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cw})^2}\left[\dfrac{R_{cw}^3}{2R_{cS1}^3}+\dfrac{R_{cS2}^{2n+4}}{R_{cS1}^6(l+R_{cw})^{2n-2}}\right] & l>R_{cS2}-R_{cw}\end{cases}$$

, (70)

the potential energy between a Dark II and an electron $E_{D\_vII\wedge e}$ is

$$\begin{aligned}E_{D\_vII\wedge e}&=\left[E_{G-D\_vII\wedge e-e}+E_{G-D\_vII\wedge w-e}+E_{S-D\_vII\wedge e-e}+E_{S-D\_vII\wedge w-e}\right]_{r=l+R_{cS2}}\\&+\left[E_{G-D\_vII\wedge G-e}+E_{G-D\_vII\wedge S-e}+E_{S-D\_vII\wedge G-e}+E_{S-D\_vII\wedge S-e}\right]_{r=l+R_{cS2}}\\&=\frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cS2})}+0+0+0+\frac{c_L^2\hat{m}_e\hat{m}_{D\_v}}{c_G^4(l+R_{cS2})}+0+0+\frac{c_L^2\hat{s}_{D\_v}\hat{s}_eR_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6(l+R_{cS2})^{2n-1}}\\&\approx\frac{\hat{q}_e\hat{m}_{D\_v}}{c_G^2(l+R_{cS2})}+\frac{c_L^2\hat{m}_{D\_v}^2R_{cS2}^{2n+4}}{c_G^4(2n-1)R_{cS1}^6(l+R_{cS2})^{2n-1}}\\&\approx\frac{c_L^2\hat{m}_{D\_v}^2}{c_G^4(l+R_{cS2})}\left[\frac{R_{cw}^3}{2R_{cS1}^3}+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6(l+R_{cS2})^{2n-2}}\right]\end{aligned},$$

(71)

and the force between a Dark II and an electron $F_{D\_vII\wedge e}$ is

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$$
\begin{aligned}
F_{D\_vII\wedge e} &= -\frac{\hat{q}_e \hat{m}_{D\_v}}{c_G^2 (l+R_{cS2})^2} - \frac{c_L^2 \hat{m}_e \hat{m}_{D\_v}}{c_G^4 (l+R_{cS2})^2} - \frac{c_L^2 \hat{s}_{D\_v} \hat{s}_e R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6 (l+R_{cS2})^{2n}} \\
&\approx -\frac{\hat{q}_e \hat{m}_{D\_v}}{c_G^2 (l+R_{cS2})^2} - \frac{c_L^2 \hat{m}_{D\_v}^2 R_{cS2}^{2n+4}}{c_G^4 R_{cS1}^6 (l+R_{cS2})^{2n}} \\
&\approx -\frac{c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})^2} \left[\frac{R_{cw}^3}{2R_{cS1}^3} + \frac{R_{cS2}^{2n+4}}{R_{cS1}^6 (l+R_{cS2})^{2n-2}} \right]
\end{aligned}
. \quad (72)
$$

6. The Structure Values of Primary Particles

In this chapter, we attempt to incorporate the existing physical data into the computational results of the previous chapters in order to obtain the structural values of primary particles. Nowadays, we are fully aware of the characteristics of electricity, such as the charge, the potential energy, and the field, at both the macroscopic and microscopic levels. We can be confident that the available measurements reflect the characteristics of the electric charge and no other factors. Thus, comparing equation (45) with Gauss's law of electrostatics and combining equation (29), it is easy to see that the relationship between the mathematical electric charge of an electron $\hat{q}_e$ and an electron charge q_e in the present physical data.

$$\hat{q}_e = c_L \sqrt{k} q_e \quad (73)$$

It is somewhat difficult to obtain the charges of gravity and the strong force because gravity is the weakest of the four fundamental interactions, and the strong force is a short-range interaction. In the models of this paper, although the gravitational and inertial masses are different, it can be hypothesized that Newton's law of gravity is largely accurate because only electron neutrinos in the model do not have gravitational waves and because electron neutrinos have much less binding energy or rest mass. We focus on protons ---- stable, heavy subatomic particles ---- to explore these charges. A proton is supposed to consist of one Dark II and one positron, because a combination of three or more primary particles must have one or more pairs of primary particles rotating in the same direction or in the rest state. In other words, the stability of the combination decreases as the number of primary particles increases. From Equations (71) and (72), a proton could have the structure shown in Figure 3, and the two primary particles are close together, i.e., $l = 0$.

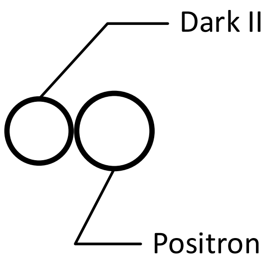

Figure 3. Schematic structure of a proton.

It is further supposed that the non-potential energies of the two primary particles are negligibly small compared with their binding energies and potential energy. Hence, the energy of a proton E_p equals

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$$\begin{aligned}E_p &= E_{D\_vII} + E_e + E_{D\_vII\wedge e}\Big|_{l=0} \\ &\approx \frac{c_L^2 R_{cS2}^5 \hat{m}_{D\_v}^2}{(2n-1)c_G^4 R_{cS1}^6} + E_e + \frac{c_L^2 \hat{m}_{D\_v}^2}{c_G^4 (l+R_{cS2})}\left[\frac{R_{cw}^3}{2R_{cS1}^3} + \frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6 (l+R_{cS2})^{2n-2}}\right]_{l=0} \\ &\approx \frac{2c_L^2 R_{cS2}^5 \hat{m}_{D\_v}^2}{(2n-1)c_G^4 R_{cS1}^6} \approx 2E_{D\_vII}\end{aligned} \tag{74}$$

when Equations (38) and (71) are subsequently substituted into the above equation, and the energy of an electron E_e is neglected in the above equation because E_p is 1836 times E_e (E_p = 938MeV, E_e = 0.511MeV)[7]. Furthermore,

$$\hat{m}_{D\_v} = \frac{c_G^2 \sqrt{G}}{2c_L} m_p \tag{75}$$

can be obtained from Equations (30) and (40), where m_p is the mass of a proton.

Next, by combining Equations (43), (44), and (73) to (75), we have

$$\begin{cases} \dfrac{R_{cw}^3}{R_{cS1}^3} = \dfrac{4\sqrt{kq_e}}{\sqrt{G}m_p} \\ \dfrac{\sqrt{kq_e}\sqrt{G}m_p}{4} \dfrac{R_{cS2}^2 - R_{cw}^2}{R_{cS1}^3} = E_e \\ \dfrac{Gm_p^2}{2(2n-1)} \dfrac{R_{cS2}^5}{R_{cS1}^6} = E_p \end{cases} \tag{76}$$

that we can solve and yield

$$\left[\frac{2(2n-1)E_p}{Gm_p^2}\right]^{\frac{2}{5}} R_{cS1}^{\frac{2}{5}} - \frac{4E_e}{\sqrt{kq_e}\sqrt{G}m_p} R_{cS1} = \left[\frac{4\sqrt{kq_e}}{\sqrt{G}m_p}\right]^{\frac{2}{3}}$$

$$\left[\frac{2(2n-1)\times 938.272\times 10^6 \times 1.60218\times 10^{-19}}{6.67430\times 10^{-11}\times (1.67262\times 10^{-27})^2}\right]^{\frac{2}{5}} R_{cS1}^{\frac{2}{5}}$$

$$-\frac{4\times 510.999\times 10^3 \times 1.60218\times 10^{-19}}{\sqrt{8.98755\times 10^9}\times 1.60218\times 10^{-19}\times \sqrt{6.67430\times 10^{-11}}\times 1.67262\times 10^{-27}} R_{cS1} .$$

$$= \left[\frac{4\sqrt{8.98755\times 10^9}\times 1.60218\times 10^{-19}}{\sqrt{6.67430\times 10^{-11}}\times 1.67262\times 10^{-27}}\right]^{\frac{2}{3}}$$

$$\left[(2n-1)R_{cS1}\right]^{\frac{2}{5}} - 3.2758\times 10^{11} R_{cS1} = 5.6137\times 10^{-10}$$

(77)[7]

Evidently, R_{cS1} is very small from the above equation. Hence

$$\begin{aligned}&\left[(2n-1)R_{cS1}\right]^{\frac{2}{5}} \approx 5.6137\times 10^{-10} \\ &R_{cS1} = \frac{7.47\times 10^{-24}}{2n-1}\ (\text{m}) = \frac{7.47\times 10^{-9}}{2n-1}\ (\text{fm})\end{aligned} . \tag{78}$$

Next

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$$R_{cw}=\left(\frac{4\sqrt{kq_e}}{\sqrt{Gm_p}}\right)^{\frac{1}{3}}R_{cS1}=\left(\frac{4\sqrt{8.98755\times10^{9}}\times1.60218\times10^{-19}}{\sqrt{6.67430\times10^{-11}}\times1.67262\times10^{-27}}\right)^{\frac{1}{3}}R_{cS1} \quad . \tag{79}$$
$$=1.6444\times10^{6}R_{cS1}=\frac{1.23\times10^{-17}}{2n-1}\ (\mathrm{m})=\frac{1.23\times10^{-2}}{2n-1}\ (\mathrm{fm})$$

Since in the solution of R_{cS1} from equation (76), we actually make $\left[\frac{2(2n-1)E_p}{3Gm_p^2}\right]^{\frac{2}{5}}R_{cS1}^{\frac{2}{5}}=\left[\frac{4\sqrt{kq_e}}{\sqrt{Gm_p}}\right]^{\frac{2}{3}}$, i.e., $\frac{2(2n-1)E_p}{Gm_p^2}R_{cS1}=\left[\frac{4\sqrt{kq_e}}{\sqrt{Gm_p}}\right]^{\frac{5}{3}}$, from which we can derive $\frac{R_{cS2}^5}{R_{cS1}^5}=\left[\frac{4\sqrt{kq_e}}{\sqrt{Gm_p}}\right]^{\frac{5}{3}}$ from $\frac{Gm_p^2}{2(2n-1)}\frac{R_{cS2}^5}{R_{cS1}^6}=E_p$ and tell ourselves that R_{cS2} and R_{cw} are almost equal from equation (79). This computational procedure is not precise enough, so we use $\frac{\sqrt{kq_e}\sqrt{Gm_p}}{4}\frac{R_{cS2}^2-R_{cw}^2}{R_{cS1}^3}=E_e$ in equation (76) and the above expressions to find R_{cS2}

$$R_{cS2}=\sqrt{\frac{4E_e}{\sqrt{kq_e}\sqrt{Gm_p}}R_{cS1}^3+R_{cw}^2}\approx R_{cw}\left[1+\frac{2E_e}{\sqrt{kq_e}\sqrt{Gm_p}}\times\frac{R_{cS1}^2}{R_{cw}^2}\times R_{cS1}\right]$$
$$=R_{cw}\left[1+\frac{2E_e}{\sqrt{kq_e}\sqrt{Gm_p}}\times\left(\frac{\sqrt{Gm_p}}{4\sqrt{kq_e}}\right)^{\frac{2}{3}}\times\frac{Gm_p^2}{2(2n-1)E_p}\left(\frac{4\sqrt{kq_e}}{\sqrt{Gm_p}}\right)^{\frac{5}{3}}\right]$$
$$=R_{cw}\left[1+\frac{1}{2n-1}\frac{4E_e}{E_p}\right]=R_{cw}\left[1+\frac{1}{2n-1}\frac{4\times510.999\times10^{3}}{938.272\times10^{6}}\right] \quad . \tag{80}$$
$$=\left(1+\frac{3.20\times10^{-3}}{2n-1}\right)R_{cw}$$
$$=\left(\frac{1.23\times10^{-2}}{2n-1}\right)\left(1+\frac{3.20\times10^{-3}}{2n-1}\right)\ (\mathrm{fm})$$

Based on equation (74), the binding energy of a Dark II E_{D_vII} is

$$E_{D\_vII}\approx\frac{E_p}{2}=\frac{938.272}{2}=469\ (\mathrm{MeV}) \quad . \tag{81}$$

From equation (38), the binding energy of a Dark I E_{D_vI} is

$$E_{D\_vI}\approx\frac{2(n+2)}{5}E_{D\_vII}=\frac{(n+2)E_p}{5}=(n+2)\times\frac{938.272}{5} \quad . \tag{82}$$
$$=(n+2)\times188\ (\mathrm{MeV})$$

However, I have not found a way to calculate the energy of an electron neutrino E_{e_v}, which also leads to an inability to determine the tiny difference between gravitational mass and inertial mass, especially when $\hat{m}_e=\frac{c_G^2\sqrt{Gm_e}}{c_L}$.

It can be determined that $m > n$ since $R_{cS2}\approx R_{cw}$ and the effective range of the weak interaction is smaller than the one of the strong interaction. I would further suggest that $n = 3$ and $m = 5$ because they seem to fit the current knowledge of the strong and weak interactions, and 1, 3, 5 is a tiny, pretty odd series. Of course, the exact number of m, n can only be found experimentally.

The speed of gravity c_G is easily obtained from equations (46) and (80)

$$c_G = \frac{R_{cS2}}{R_{cw}} c_L = \left(1 + \frac{3.20\times10^{-3}}{2n-1}\right) c_L \quad . \qquad (83)$$

Moreover, the energy definition Equations (14) and (17) reveal that regardless of the speed of light c_L or the speed of gravity c_G, there is an invariant that is the rest mass m_0. Therefore, we can obtain the relation of momentum and energy between the two measurement media of light and gravitational waves

$$\frac{1}{c^2}\left(p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2}\right) = \frac{1}{c'^2}\left(p_x'^2 + p_y'^2 + p_z'^2 - \frac{E'^2}{c'^2}\right) \qquad (84)$$

from the relationship between momentum and energy in Special Relativity $p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} = -m_0^2c^2$. Equation (84) directly derives the relation of space-time between the two measurement media

$$\frac{1}{c^2}\left(dx^2 + dy^2 + dz^2 - c^2dt^2\right) = \frac{1}{c'^2}\left(dx'^2 + dy'^2 + dz'^2 - c^2dt'^2\right) \quad . \qquad (85)$$

So the proper time τ is the same in all reference frames with the two measurement media.

7. Conclusions and Discussion

In this paper, field and energy equations for the strong and weak interactions are derived based on a set of fundamental assumptions that are consistent with existing knowledge. These equations are used to determine the structures of the primary particles and to analyze the interactions between two primary particles. The characteristic parameters of the primary particles (Table I) are calculated from the physical constants.

Table I. Characteristic Parameters of the Primary Particles.

Particle Name	Binding Energy* (MeV)	Elementary Charge (e)	Spin	Radius* (fm)	Determinants of Radius
Electron/ Positron	0.511 (Known quantity)	±1	$\pm\frac{1}{2}$	$\frac{1.23\times10^{-2}}{2n-1}$	Critical radius of the weak interaction R_{cw}
Electron Neutrino	N.A.	0			
Dark I	$(n+2)\times188$			$\frac{7.47\times10^{-9}}{2n-1}$	1st critical radius of the strong interaction R_{cS1}
Dark II	469			$\left(\frac{1.23\times10^{-2}}{2n-1}\right)\left(1+\frac{3.20\times10^{-3}}{2n-1}\right)$	2nd critical radius of the strong interaction R_{cS2}

* ---- n = 2, 3, 4, …… (Undetermined, suggested by 3).

Moreover, we can derive the matrix $\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$ when we rewrite the quantum factors in equations (32), (35), (37), and (41) as $\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & \cos\theta \end{pmatrix}$ and multiply it by the Pauli matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. $\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$ has well-known results in quantum mechanics with eigenvalues of ±1 and eigenvectors of $\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\varphi} \end{pmatrix}$, $\begin{pmatrix} \sin\frac{\theta}{2}e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$, i.e., spin values $\pm\frac{1}{2}$. Thus the conclusions of this paper are compatible with existing physical-mathematical methods.

Furthermore, the interactive analysis reveals that:

a. Two self-conjugate primary particles have potential energy or force when they rotate in opposite directions. However, they have zero potential energy or rest when they spin in the same direction. This is one of the foundations of the Pauli exclusion principle.
b. Dark Is have the asymptotic freedom characteristic, but following the principle of energy minimization, there should be only Dark IIs in most cases.
c. The force between two electrons has two values, one large and one small, if the two electrons have potential energy or force. This is random.
d. It is also random that two electron neutrinos or an electron neutrino and an electron (or a positron) attract or repel each other. Because of this, electron neutrinos are a weak destabilizer in the nucleus, and even though the binding energy of electron neutrinos is the smallest of the primary particles, no evidence of electron neutrino destruction has yet been found.
e. Primary particles behave like perfect tiny spheres in terms of energies and interactions, but they also look like uneven minuscule spheres in external fields. Which is the reality of a primary particle? Observation or mathematics? The answer should be that "the Moon is always there, doesn't matter we see it or not", however, the Moon is changed when we see it.

In interactive analysis, the Self-Conjugate Mechanism plays a key role in the microscopic domain. Consider the case where external actions change the spin directions of particles in a combination of two primary particles from the initial opposite directions to the same direction, and then the combination is broken up. In other words, the two primary particles entangle after the separation. Obviously, the distance of quantum entanglement is finite, not infinite as is currently believed, because the strength of the fields around a primary particle decreases with distance. The transmission speed of quantum entanglement is the speed of light or the speed of gravity. In this case, Einstein is right.

Moreover, the speed of gravity c_G is slightly greater than the speed of light c_L $\left(\frac{c_G}{c_L} = 1 + \frac{3.20\times10^{-3}}{2n-1}\right)$, and the rest mass m_0, the proper time τ are the invariants with the two measuring media of light and gravity. Gravity may have similar quantum properties, such as $E_G = h_G \nu_G$. What are the quantum properties of the weak and strong interactions?

It can further be hypothesized that a dark matter particle is created when the positron in a proton is replaced by a Dark II. Therefore, the energy of a dark matter particle E_{dm} is

$$\begin{aligned} E_{dm} &= 2E_{D\_\nu II} + E_{D\_\nu II \wedge D\_\nu II}\Big|_{l=0} \\ &\approx \frac{2c_L^2 R_{cS2}^5 \hat{m}_{D\_\nu}^2}{(2n-1)c_G^4 R_{cS1}^6} + \frac{2c_L^2 \hat{m}_{D\_\nu}^2}{c_G^4 (l+R_{cS2})}\left[1+\frac{R_{cS2}^{2n+4}}{(2n-1)R_{cS1}^6 (l+R_{cS2})^{2n-2}}\right]_{l=0}, \\ &\approx \frac{4c_L^2 R_{cS2}^5 \hat{m}_{D\_\nu}^2}{(2n-1)c_G^4 R_{cS1}^6} \approx 4E_{D\_\nu II} = 2E_p \end{aligned} \tag{86}$$

and the mathematical mass of a dark matter particle $\hat{m}_{dm}$ is

$$\hat{m}_{dm} = 4\hat{m}_{D\_\nu} = \frac{2c_G^2\sqrt{G}}{c_L} m_p \tag{87}$$

from equations (38) and (55). Equations (74), (75), (86), and (87) remind us once again that the difference between gravitational mass and inertial mass is almost negligible for a single Dark II or a

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combination of double Dark IIs. Certainly, it is also reasonable to assume that a combination of two Dark IIs has been annihilated by itself, and that there are only free-state Dark IIs as dark matter in our cosmic, since no particle heavier than a proton is as stable as a proton. Or, are we listening to a concerto for Dark II solos and duets, and conventional matter in our universe today?

Positrons and negative protons are so rare in our universe that Dark IIs only confine positrons, which means that there is a law of nature that causes a dark neutrino and a positron to rotate in opposite directions, and a dark neutrino and an electron to rotate in the same direction.

Let us now imagine what will happen if all the primary particles in our universe are gathered together, crushed until they are destroyed at one point. This consequence should be similar to the electron-positron annihilation. There will be only the four fundamental waves with extremely high energy in our universe at that moment, the scene of the Big Bang.

It is worth noting that this paper cannot answer what the self-conjugate angular velocity ω of a primary particle is, and whether ω is stable or variable.

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