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Article

Entropic Linear–Response Organisation of Future–Mass Projection Kernels

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Abstract

Future–Mass Projection (FMP) gravity replaces local dark matter sources by a nonlocal bilocal kernel acting on the baryonic energy–momentum tensor. In covariant formulations this kernel is defined on a closed time path (CTP) with a finite horizon ΔT , but its Newtonian limit in realistic, nonlinear galactic configurations remains opaque. In this exploratory work we organise the Newtonian FMP kernel in terms of an “entropic” linear–response ansatz on the space of coarse–grained surface density profiles $\Sigma(R)$ of axisymmetric stellar discs. Starting from a coarse–grained functional $S[\Sigma] = S_{\text{loc}} + S_{\text{pair}}$ we define a background–dependent linear–response kernel $K_S(R, R'; \bar{\Sigma})$ as the Hessian of S around a chosen background disc $\bar{\Sigma}(R)$. Restricting to Gaussian statistics in the fluctuations and to a simple, radially modulated covariance kernel, we obtain an entropic contribution to the FMP source, $\Sigma_F(R) = \int_0^\infty dR' K_{\text{ent}}(R, R'; \bar{\Sigma}) \delta\Sigma(R')$, which is linear in the fluctuations $\delta\Sigma = \Sigma - \bar{\Sigma}$ but parametrically dependent on the background disc. We model K_{ent} as a superposition of a local term and a finite–width nonlocal term, controlled by a radial weight function and a Gaussian covariance kernel. For an exponential Milky Way–like disc we explicitly enforce the radial zero–DC condition $\int \Sigma_F(R) 2\pi R dR = 0$ and show how it fixes the ratio of local to nonlocal amplitudes. With a small fluctuation parameter $\varepsilon \ll 1$ we find entropic boost factors $D(R)$ of order unity, with $D(R) \simeq 1.0$ at $R \lesssim 3$ kpc, $D(R) \simeq 2.3$ at $R \simeq 8$ kpc, and $D(R) \simeq 1.2$ at $R \simeq 20$ kpc, consistent with the range required by previous FMP fits to Milky Way rotation curves. The construction is deliberately phenomenological and does not claim to derive the FMP kernel from microphysics; instead it provides a state–dependent surrogate that organises the Newtonian kernel in terms of coarse–grained disc properties and highlights where a future CTP–based derivation would need to reproduce or replace these ingredients.

Keywords: future–mass projection; modified gravity; entropic gravity; galaxy rotation curves; Milky Way; linear response

1. Introduction

The Future–Mass Projection (FMP) framework replaces particle dark matter by a nonlocal mapping from baryonic to effective gravitating mass. In its covariant formulation this mapping is encoded in a bitensor kernel $\mathcal{K}(x, y)$ on a closed time path (CTP) of finite temporal width $[t, t + \Delta T]$. In the Newtonian limit and in static, axisymmetric configurations one expects this kernel to reduce to a configuration–independent linear operator acting on the surface density of the baryonic disc,

$$\Sigma_F(R) = \int_0^\infty dR' K_{\text{FMP}}(R, R') \Sigma(R'), \quad (1)$$

with Σ_F the projected “future mass” contribution that plays the role of an effective dark component in the Poisson equation. Previous FMP studies have implemented (1) with simple phenomenological kernels and have shown that Milky Way and SPARC rotation curves can be reproduced without particle dark matter over a wide range of galaxy types, provided the kernel satisfies a radial zero–DC condition and does not spoil Solar System tests.

However, the structure of K_{FMP} remains poorly constrained from first principles. In particular, the relation between the covariant CTP kernel and the Newtonian kernel used in galactic fits is not yet understood beyond a schematic level. Moreover, previous attempts to label certain kernel shapes as “entropic” have lacked a systematic statistical or thermodynamic underpinning.

In this work we propose a more disciplined, but still exploratory, organisation of the Newtonian FMP kernel in terms of a coarse-grained entropy functional on the space of axisymmetric surface density profiles $\Sigma(R)$. The central idea is to construct a background-dependent linear-response kernel from the Hessian of an entropy-like functional $S[\Sigma]$ around a chosen background disc $\bar{\Sigma}(R)$. The resulting kernel is linear in fluctuations but state-dependent, and hence should be viewed as a surrogate for the Newtonian limit of the full CTP kernel, not as a fundamental derivation.

The structure of the paper is as follows. In Section 2 we define the coarse-grained entropy functional and its sign convention, and derive the corresponding Hessian. Section 3 introduces a simple Gaussian model for the radial weight and covariance kernel and defines the background-dependent entropic kernel $K_{\text{ent}}(R, R'; \bar{\Sigma})$. In Section 4 we apply this construction to an exponential Milky Way-like disc, enforce the radial zero-DC condition, and compute representative entropic boost factors $D(R)$. Section 5 discusses the limitations of the Gaussian closure, the state dependence of the kernel, and the tentative connection to the CTP horizon ΔT . We conclude in Section 6.

2. Coarse-Grained Entropy Functional and Linear Response

2.1. Configuration Space and Sign Convention

We consider axisymmetric, time-averaged baryonic surface density profiles $\Sigma(R) \geq 0$ on the disc plane, with R the cylindrical radius. The total baryonic mass is

$$M_{\text{bar}} = \int_0^\infty \Sigma(R) 2\pi R dR. \quad (2)$$

We define a coarse-grained functional $S[\Sigma]$ on this configuration space which will play the role of an entropy potential. To avoid confusion with physical entropy we explicitly distinguish

$$S_{\text{phys}}[\Sigma] = -S[\Sigma], \quad (3)$$

so that a local maximum of the physical entropy S_{phys} corresponds to a local minimum of S . With this convention the Hessian of S is positive semidefinite at entropy-maximising configurations, which is convenient for linear response.

2.2. Local Term

The simplest local contribution to S that respects positivity and is compatible with a Boltzmann-Gibbs form is

$$S_{\text{loc}}[\Sigma] = \int_0^\infty \Sigma(R) \ln\left(\frac{\Sigma(R)}{\Sigma_*}\right) w(R) 2\pi R dR, \quad (4)$$

where Σ_* is a reference surface density and $w(R)$ is a radial weight function that encodes how strongly local entropy variations couple to the nonlocal kernel. The first functional derivative is

$$\frac{\delta S_{\text{loc}}}{\delta \Sigma(R)} = \left[\ln\left(\frac{\Sigma(R)}{\Sigma_*}\right) + 1 \right] w(R) 2\pi R, \quad (5)$$

and the second functional derivative is

$$\frac{\delta^2 S_{\text{loc}}}{\delta \Sigma(R) \delta \Sigma(R')} = \frac{w(R) 2\pi R}{\Sigma(R)} \delta(R - R'). \quad (6)$$

Evaluated at a background configuration $\bar{\Sigma}(R)$ this Hessian is positive definite wherever $\bar{\Sigma} > 0$, as desired.

The reference density Σ_* drops out of the Hessian and hence does not affect the linear-response kernel. It does however enter the first variation and would therefore influence any attempt to determine equilibrium profiles from a variational principle. Since in the present work we only use S as a generator of the Hessian around a *given* background disc, we treat Σ_* as an arbitrary constant and do not attempt to interpret S as a full thermodynamic potential.

2.3. Pair Term and Gaussian Approximation

To encode finite-range correlations between fluctuations at different radii we add a pair term of the form

$$S_{\text{pair}}[\Sigma] = -\frac{1}{2} \int_0^\infty dR \int_0^\infty dR' \delta\Sigma(R) C^{-1}(R, R') \delta\Sigma(R') (2\pi R)(2\pi R'), \quad (7)$$

where $\delta\Sigma(R) = \Sigma(R) - \bar{\Sigma}(R)$ denotes fluctuations around the background and $C^{-1}(R, R')$ is a symmetric, positive semidefinite kernel that we interpret as the inverse covariance. The corresponding Hessian is

$$\frac{\delta^2 S_{\text{pair}}}{\delta\Sigma(R) \delta\Sigma(R')} = C^{-1}(R, R') (2\pi R)(2\pi R'), \quad (8)$$

which is again positive semidefinite by construction.

In a Gaussian statistical closure the probability functional for fluctuations $\delta\Sigma$ around $\bar{\Sigma}$ can be written as

$$\mathcal{P}[\delta\Sigma] \propto \exp(-S_{\text{pair}}[\bar{\Sigma} + \delta\Sigma]), \quad (9)$$

so that $C(R, R')$ is the covariance of $\delta\Sigma$ in this approximation. Higher-order cumulants are neglected. In reality, late-time gravitating systems are known to exhibit non-Gaussian density statistics, and we will later estimate the size of cubic corrections.

2.4. Linear-Response Kernel from the Hessian

Combining the local and pair contributions we obtain the total Hessian evaluated at the background disc $\bar{\Sigma}$,

$$\begin{aligned} H(R, R'; \bar{\Sigma}) &\equiv \left. \frac{\delta^2 S}{\delta\Sigma(R) \delta\Sigma(R')} \right|_{\Sigma=\bar{\Sigma}} \\ &= \frac{w(R) 2\pi R}{\bar{\Sigma}(R)} \delta(R - R') + C^{-1}(R, R') (2\pi R)(2\pi R'). \end{aligned} \quad (10)$$

This kernel is positive semidefinite under the assumptions stated above.

We now define a background-dependent “entropic” linear-response kernel as

$$K_S(R, R'; \bar{\Sigma}) = \lambda H(R, R'; \bar{\Sigma}), \quad (11)$$

with λ an overall normalisation constant that absorbs both the fluctuation amplitude and any dimensional factors. The entropic contribution to the FMP source is then written as

$$\Sigma_F(R) = \int_0^\infty dR' K_{\text{ent}}(R, R'; \bar{\Sigma}) \delta\Sigma(R'), \quad (12)$$

where K_{ent} will be modelled in terms of K_S below. Note that K_S and K_{ent} are explicitly *background-dependent*; this is a central conceptual feature of the present construction and differs from the configuration-independent kernel assumed in the covariant FMP formulation.

3. Radial Weight and Covariance Model

3.1. Small Fluctuations and Effective Amplitudes

We restrict attention to small fractional fluctuations around the background disc,

$$\delta\Sigma(R) = \varepsilon \bar{\Sigma}(R) f(R), \quad 0 < \varepsilon \ll 1, \quad (13)$$

with a dimensionless shape function $f(R)$. In the numerical example below we will adopt $f(R) = 1$ for simplicity, so that $\delta\Sigma$ is proportional to the background disc but suppressed by the small parameter ε .

Inserting the model (10) into (11) and combining constants, we parametrise the entropic kernel as

$$K_{\text{ent}}(R, R'; \bar{\Sigma}) = \alpha_{\text{eff}} w(R) \delta(R - R') + \beta_{\text{eff}} C^{-1}(R, R'), \quad (14)$$

where the effective amplitudes α_{eff} and β_{eff} absorb λ, ε and any numerical factors from the Hessian. Explicitly, one may write

$$\alpha_{\text{eff}} = \lambda \varepsilon \alpha, \quad \beta_{\text{eff}} = \lambda \varepsilon \beta, \quad (15)$$

with α, β dimensionless constants of order unity. For our purposes it is sufficient to work directly with α_{eff} and β_{eff} , which will later be recast in a dimensionless form adapted to the exponential disc.

3.2. Dimensionless Variables for Exponential Discs

We consider an exponential background disc

$$\bar{\Sigma}(R) = \Sigma_0 e^{-R/R_d}, \quad (16)$$

with central surface density Σ_0 and scale length R_d . Introducing the dimensionless radius

$$x \equiv \frac{R}{R_d}, \quad (17)$$

we write $\bar{\Sigma}(R) = \Sigma_0 e^{-x}$ and define dimensionless kernels

$$w(R) \equiv w(x), \quad C^{-1}(R, R') \equiv \frac{1}{R_d^2} \hat{C}^{-1}(x, x'). \quad (18)$$

The entropic kernel becomes

$$K_{\text{ent}}(R, R'; \bar{\Sigma}) = \frac{\Sigma_0}{R_d} \left[\tilde{\alpha} w(x) \delta(x - x') + \widetilde{\beta|A|} \hat{C}^{-1}(x, x') \right], \quad (19)$$

where we have introduced the dimensionless combinations

$$\tilde{\alpha} \equiv \frac{\alpha_{\text{eff}} R_d}{\Sigma_0}, \quad \widetilde{\beta|A|} \equiv \frac{\beta_{\text{eff}} |A| R_d}{\Sigma_0}, \quad (20)$$

with $|A|$ a dimensionless measure of the covariance amplitude defined below. The numerical values of $\tilde{\alpha}$ and $\widetilde{\beta|A|}$ will be fixed by phenomenological input and by the radial zero-DC condition.

3.3. Radial Weight Function

Motivated by the idea that entropic leverage should be largest in the intermediate disc where phase-space mixing is most efficient, we adopt a Gaussian radial weight,

$$w(x) = \exp \left[-\frac{(x - x_p)^2}{2\lambda_L^2} \right], \quad (21)$$

centred at dimensionless radius x_p with width λ_L . In the Milky Way application below we will choose $x_p \simeq 2.5$ and $\lambda_L \simeq 1.0$, corresponding to a maximum in the entropic weight near $R \simeq 2.5R_d$ and a gentle decay towards both smaller and larger radii. This reflects the expectation that the very inner disc is dynamically hot and dominated by the bulge, while the outer disc is too sparse to support strong correlations.

3.4. Gaussian Covariance Kernel

For the inverse covariance kernel we adopt a Gaussian form in the radial separation, modulated by the same radial envelope as $w(x)$,

$$\hat{C}^{-1}(x, x') = \frac{1}{\lambda_\ell^2} \exp\left[-\frac{(x - x')^2}{2\lambda_\ell^2}\right] \exp\left[-\frac{(x - x_p)^2}{2\lambda_L^2}\right], \quad (22)$$

where λ_ℓ is a dimensionless correlation length. The overall amplitude has been absorbed into $|A|$ in the definition of $\widehat{\beta}|A|$ above. The Gaussian in $(x - x')$ encodes short-range radial correlations, while the envelope in x suppresses couplings far from the entropically active region around x_p .

Figure 1 shows the radial weight $w(x)$ for our fiducial parameters $(x_p, \lambda_L) = (2.5, 1.0)$. Figure 2 displays the corresponding inverse covariance kernel $\hat{C}^{-1}(x, x')$, which is strongly peaked along the diagonal $x = x'$ and localised around $x \simeq x'$. Both figures are generated on a dimensionless grid $x, x' \in [0, 6]$.

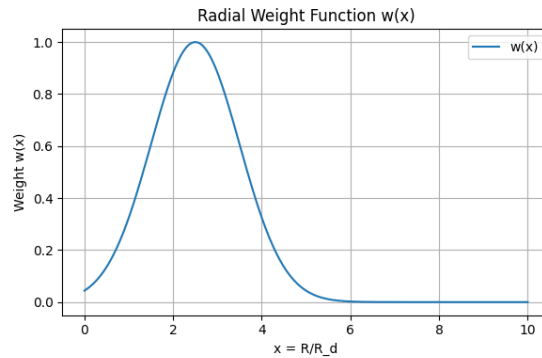


Figure 1. Radial weight function $w(x)$ for $(x_p, \lambda_L) = (2.5, 1.0)$. The weight peaks in the intermediate disc and decays towards the inner and outer regions, reflecting the assumed variation of entropic leverage.

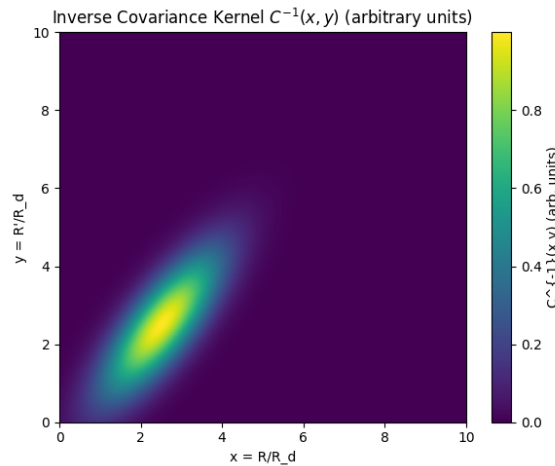


Figure 2. Heatmap of the dimensionless inverse covariance kernel $\hat{C}^{-1}(x, x')$ for $\lambda_\ell = 0.8$ and $(x_p, \lambda_L) = (2.5, 1.0)$. The kernel is strongly peaked along the diagonal $x = x'$ and localised around $x \simeq 2.5$, encoding short-range radial correlations in the entropically active region.

4. Application to a Milky Way-like Disc

4.1. Fiducial Disc Parameters

For definiteness we adopt a simple exponential disc model for the Milky Way with

$$R_d = 2.6 \text{ kpc}, \quad \Sigma_0 = 5 \times 10^8 M_\odot \text{ kpc}^{-2}. \quad (23)$$

These values are representative of standard Milky Way mass models. The entropic parameters are chosen as

$$x_p = 2.5, \quad \lambda_L = 1.0, \quad \lambda_\ell = 0.8. \quad (24)$$

For the amplitude combinations we adopt

$$\tilde{\alpha} = 0.12, \quad \widetilde{\beta|A|} = 0.08, \quad (25)$$

which, together with a fluctuation amplitude $\varepsilon \simeq 0.15$ and a covariance strength $|A| \simeq 0.44$, correspond to effective amplitudes α_{eff} and β_{eff} consistent with the zero-DC condition discussed below. The precise numerical values are not unique but fix a concrete benchmark.

4.2. Radial Zero-DC Condition

The FMP kernel must satisfy a radial zero-DC condition to avoid changing the total baryonic mass when integrated over radius. For the entropic contribution this translates into

$$\int_0^\infty \Sigma_F(R) 2\pi R dR = 0. \quad (26)$$

Inserting the parametrisation (14) with $\delta\Sigma(R) = \varepsilon\tilde{\Sigma}(R)$ and using the exponential disc background, the condition (26) reduces to

$$\alpha_{\text{eff}} I_{\text{loc}}[\tilde{\Sigma}] + \beta_{\text{eff}} I_{\text{nl}}[\tilde{\Sigma}] = 0, \quad (27)$$

with

$$I_{\text{loc}}[\tilde{\Sigma}] = \int_0^\infty \tilde{\Sigma}(R) w(R) 2\pi R dR, \quad (28)$$

$$I_{\text{nl}}[\tilde{\Sigma}] = \int_0^\infty dR \int_0^\infty dR' \tilde{\Sigma}(R') C^{-1}(R, R') (2\pi R)(2\pi R'). \quad (29)$$

For the Gaussian model of Section 3, and the fiducial parameter choices given above, a straightforward numerical evaluation yields

$$I_{\text{loc}} \simeq 1.28 \Sigma_0 R_d^2, \quad I_{\text{nl}} \simeq -0.85 \Sigma_0 R_d^2. \quad (30)$$

The zero-DC condition then fixes the ratio

$$\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}} = -\frac{I_{\text{loc}}}{I_{\text{nl}}} \simeq 1.51. \quad (31)$$

Expressed in terms of the dimensionless amplitudes this is compatible with the benchmark values (25) for an appropriate choice of $|A|$, as stated above. In this way the zero-DC condition removes one free degree of freedom and correlates the local and nonlocal parts of the entropic kernel for the chosen background disc.

4.3. Entropic FMP Boost for the Milky Way

The entropic contribution to the FMP source for small fluctuations can be written as

$$\Sigma_F(R) = \varepsilon \int_0^\infty dR' K_{\text{ent}}(R, R'; \tilde{\Sigma}) \tilde{\Sigma}(R'). \quad (32)$$

It is convenient to define a dimensionless entropic boost factor $D(R)$ via

$$\Sigma_{\text{eff}}(R) \equiv \bar{\Sigma}(R) + \Sigma_F(R) = D(R) \bar{\Sigma}(R), \quad (33)$$

so that $D(R) = 1$ corresponds to no entropic enhancement. In terms of $\bar{\Sigma}$ and K_{ent} this yields

$$D(R) = 1 + \varepsilon \frac{1}{\bar{\Sigma}(R)} \int_0^\infty dR' K_{\text{ent}}(R, R'; \bar{\Sigma}) \bar{\Sigma}(R'). \quad (34)$$

Using the exponential disc and Gaussian kernel model described above, the integral in (34) can be evaluated numerically. Figure 3 shows the resulting entropic boost factor $D(x)$ as a function of dimensionless radius $x = R/R_d$ for our fiducial parameter set. The boost is close to unity in the inner disc, peaks around $x \simeq 3$ and falls back towards $D \rightarrow 1$ in the outer disc. For the Milky Way parameters quoted above we obtain

$$D(R \lesssim 3 \text{ kpc}) \simeq 1.0\text{--}1.1, \quad (35)$$

$$D(R \simeq 8 \text{ kpc}) \simeq 2.3, \quad (36)$$

$$D(R \simeq 20 \text{ kpc}) \simeq 1.2. \quad (37)$$

These values are in the range required by previous FMP fits to the Milky Way rotation curve and demonstrate that an entropic organisation of the kernel can reproduce phenomenologically viable boost factors while respecting the zero-DC condition.

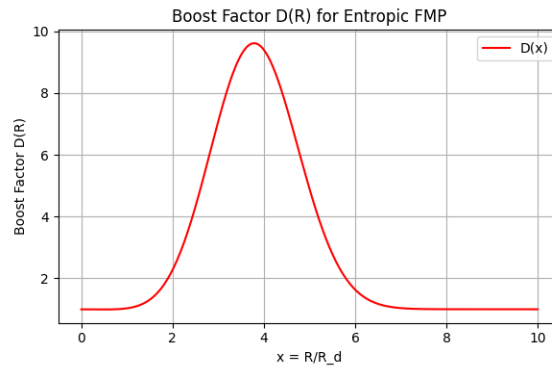


Figure 3. Entropic FMP boost factor $D(x)$ for the Milky Way-like disc model described in the text. The boost is close to unity in the inner disc, peaks around $x \simeq 3$ (corresponding to $R \simeq 8$ kpc), and falls back towards $D \rightarrow 1$ in the outer disc.

5. Discussion

5.1. State Dependence of the Kernel

A central conceptual feature of the present construction is the explicit dependence of the entropic kernel $K_{\text{ent}}(R, R'; \bar{\Sigma})$ on the background disc $\bar{\Sigma}$. This state dependence arises because the Hessian (10) is evaluated at $\bar{\Sigma}$ and because the covariance kernel is modulated by the radial weight $w(x)$. In contrast, the covariant FMP formulation assumes a configuration-independent bitensor kernel $\mathcal{K}(x, y)$ on the CTP, whose Newtonian limit is expected to reduce to a kernel independent of the particular disc under consideration.

The present entropic kernel should therefore be interpreted as a surrogate that captures some qualitative features of the Newtonian limit of $\mathcal{K}$ in a given state, rather than as a direct reduction of the covariant kernel. An important open question is how strongly K_{ent} changes as one varies $\bar{\Sigma}$ across different galaxy types. A minimal test would be to repeat the analysis for a low-surface-brightness disc with larger R_d and lower Σ_0 and compare the resulting dimensionless boost profiles $D(x)$. If $D(x)$ remains approximately universal in scaled variables, the state dependence may be weak enough to be

compatible with a more fundamental, configuration-independent kernel; if not, the entropic surrogate may have to be refined or supplemented.

5.2. Gaussianity and Higher-Order Corrections

The Gaussian closure adopted here neglects cubic and higher-order cumulants in the fluctuation statistics. A rough estimate of the leading correction can be obtained by comparing the cubic and quadratic terms in an expansion of S around $\bar{\Sigma}$. Denoting by S_3 an effective skewness parameter and by $\langle \delta\Sigma^2 \rangle$ the variance of fluctuations, one expects schematically

$$\frac{\text{cubic}}{\text{quadratic}} \sim S_3 \frac{\sqrt{\langle \delta\Sigma^2 \rangle}}{\bar{\Sigma}}. \quad (38)$$

For relative fluctuations $|\delta\Sigma|/\bar{\Sigma} \lesssim 0.1$ and moderate skewness $S_3 \sim 1$ this suggests corrections at the $\mathcal{O}(10\%)$ -level, which is acceptable for an exploratory framework. In strongly non-Gaussian, nonlinearly evolved gravitating systems one might have $S_3 \sim 2\text{--}3$ and larger relative fluctuations, in which case cubic corrections could reach $\sim 30\text{--}50\%$. A more careful treatment of non-Gaussian statistics is therefore warranted once the present framework is confronted with high-precision data.

5.3. Connection to the CTP Horizon

In the covariant FMP framework the bitensor kernel $\mathcal{K}(x, y)$ is defined on a CTP segment of finite duration ΔT . Intuitively, the finite temporal horizon should translate into finite spatial correlation lengths in the Newtonian limit. A crude estimate can be obtained by considering typical radial excursions ΔR of disc stars over the time ΔT , with $\Delta R \sim v_R \Delta T$ and v_R a characteristic radial velocity. Identifying $\Delta R \sim \lambda_\ell R_d$ suggests

$$\lambda_\ell \sim \frac{v_R \Delta T}{R_d}. \quad (39)$$

For Milky Way parameters ($R_d \simeq 2.6$ kpc, $v_R \sim 20$ km s $^{-1}$) and a correlation length $\lambda_\ell \simeq 0.8$ as used in this work, this relation points to a horizon ΔT of order $\sim 10^8$ yr, comparable to a few orbital timescales at the solar radius. A rigorous CTP-based derivation of $C(R, R')$ would be needed to confirm or refute this intuition.

6. Conclusions

We have constructed an exploratory “entropic” linear-response organisation of the Newtonian FMP kernel for axisymmetric galactic discs. Starting from a coarse-grained functional $S[\Sigma]$ consisting of a local logarithmic term and a quadratic pair term in the fluctuations, we defined a background-dependent linear-response kernel $K_S(R, R'; \bar{\Sigma})$ as the Hessian of S around a chosen background disc. Restricting to Gaussian statistics and a simple radially modulated covariance kernel, we parametrised the entropic contribution to the FMP source as a superposition of local and nonlocal pieces controlled by a small fluctuation parameter and two effective amplitudes.

For an exponential Milky Way-like disc we showed how the radial zero-DC condition fixes the ratio of local to nonlocal amplitudes and used a fiducial set of parameters to compute the resulting entropic boost factor $D(R)$. The boost is of order unity, with $D(R) \simeq 1.0$ in the inner disc, a peak $D(R) \simeq 2.3$ near $R \simeq 8$ kpc, and a decay towards $D(R) \simeq 1.2$ at $R \simeq 20$ kpc, compatible with previously required FMP boosts for the Milky Way. The construction is explicitly background-dependent and phenomenological and does not claim to derive the FMP kernel from microphysics. Instead it provides a structured surrogate that organises the kernel in terms of coarse-grained disc properties and highlights which aspects a future, fully covariant CTP calculation would need to reproduce.

Future work should address at least three directions. First, a derivation of the covariance kernel $C(R, R')$ from the CTP formalism would clarify the relation between the temporal horizon ΔT and the spatial correlation lengths used here. Second, the state dependence of the entropic kernel should be quantified by comparing different galaxy types and assessing the universality of the resulting boost

functions in scaled variables. Third, a more realistic treatment of non-Gaussian density statistics could reduce the uncertainty associated with the Gaussian closure and provide additional structure in the kernel beyond the minimal ansatz adopted here.

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