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Article

Relative-Entropy Variational Principle for Semiclassical Gravity with Finite-Resolution Boundaries

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Abstract

We propose a causal-diamond formulation of semiclassical gravity where a finite-resolution boundary regulator supplies the edge structure for a local Wheeler–DeWitt description. Because the diffeomorphism-invariant Hilbert space does not factorize, each diamond is equipped with a boundary-completed algebra $\mathfrak{A}_O$, ensuring the reduced state ρ_O and reference family $\sigma_O[\Lambda]$ are defined on identical degrees of freedom. Dynamics are defined by an informational principle: the relative-entropy functional quantifies the mismatch $S_{\text{rel}}(\rho_O \parallel \sigma_O[\Lambda])$ between data and reference. By discretizing gravitational stiffness (implemented as a finite response capacity of the boundary completion at the diamond waist) rather than the metric degrees of freedom, the variational principle is rendered well-defined. In the modular/KMS regime, the vacuum is at entanglement equilibrium, and the leading dynamics reduce to linear response governed by the Hessian of relative entropy (Kubo–Mori metric). This Hessian decouples tensor, vector and scalar deformations, recovering Einstein stiffness, Yang–Mills susceptibilities and mass gaps. Finite-resolution open updating induces a canonical reduction of non-abelian symmetry data to commuting Cartan phases, yielding a natural three-stage generation hierarchy. A quasi-local heat-kernel expansion maps the near-equilibrium response to a matching-scale EFT, identifying the leading R^2 saturation operator and accommodating a spinorial transport structure. Topological edge-mode counting fixes the effective internal degeneracy N ; combined with Newton's constant G , this calibrates the matching scale $M_s \sim 3 \times 10^{13}$ GeV. At M_s , the framework yields analytic relations for couplings and mass ratios as functions of discrete group-theoretic data of the boundary completion. Identifying M_s with stiffness saturation places the high-curvature regime in a plateau universality class with r at the 10^{-3} level. The architecture yields correlated, falsifiable targets governed by a single scale.

Keywords: semiclassical gravity; spectral action; quantum gravity; causal diamond; general relativity; holographic principle; emergent gravity; quantum coherence; quantum information; Bekenstein–Hawking entropy; thermodynamic gravity; noncommutative geometry (NCG); coherency screen; Kubo–Mori metric; Yang–Mills; GKSL; CPTP; lindblad; KMS; quantum Fisher; modular flow; modular update; fine-structure constant; lie filtration; relative-entropy; spectral tilt; tensor ratio; scalaron mass; cosmological constant; dark energy density; MOND scale; Higgs mass; top mass; Top Yukawa; Higgs quartic; Tau lifetime

1. Conceptual Foundations

We propose a finite-resolution framework for defining local quantum subsystems in semiclassical gravity, organized around the Wheeler–DeWitt (WDW) constraint [1]:

$$H\Psi = 0$$

where the global state Ψ is constrained rather than time-evolved and the operator H encodes the gravitational constraints and diffeomorphism invariance.

To ensure background independence, we adopt this relational framework in a weak sense: dynamics is treated as local statistical inference on causal diamonds $O(p, q)$ [2], which serve as the

fundamental units of accessible correlations [3]. In the small-diamond regime, modular flow supplies the intrinsic local clock [4,5]. Geometry does not evolve in external time but is inferred from entropic boundary correlations [6–13]. This inference uses relative entropy as a mismatch functional between the actual boundary-completed state and its geometric reference.

The minimal architecture consists of a causal diamond with boundary completion storing the edge data required for a subregion algebra [14]. Topological quantization yields an effective internal degeneracy N and, upon calibration to Newton's constant G , a matching-scale resolution M_s .

Discrete minimality/naturalness selections are tagged [M#]. Unity normalizations (order-one factors set to 1) are tagged [U#]. Consistency requirements, regime assumptions and truncations are stated explicitly in the text and are not tagged.

1.1. Axiomatic Basis: Minimal Architecture

We are not starting from a preferred lattice or field content; instead, we approach quantum gravity as a problem of local inference. We adopt the following minimal ingredients to define subsystems, clocks and regulated observables without background dependence:

Principle A: Operational Regions

Locality is defined operationally by the minimal covariant unit: a finite causal diamond [2]. This geometry uniquely fixes the boundary interface where subsystem-defining information resides [14].

P1 (Non-factorization): The diffeomorphism-invariant physical Hilbert space cannot factorize across spatial subregions ($\mathcal{H}_{\text{phys}} \neq \mathcal{H}_A \otimes \mathcal{H}_B$) [14]. Explicit boundary data are therefore required to define interiority and a unique subregion algebra (consistency requirement).

P2 (Causal Diamonds): A finite experiment is a closed query–response loop between two events p (initiation) and q (registration), with $q \in J^+(p)$. In an operational picture, one may take p and q as timelike-separated events on an observer's worldline, marking the start of a probe and the irreversible recording of an outcome. The causal diamond $O(p, q) \equiv J^+(p) \cap J^-(q)$ [2] is the observer's causal workspace: it is the unique spacetime region containing all degrees of freedom that can be influenced by the query at p and still influence the response at q . This fixes the operational region covariantly without choosing a foliation. For minimality, we restrict to the simply connected local closure: the maximal spatial slice is a 3-ball B^3 with boundary cut S^2 ; nontrivial topology introduces extra discrete labels and is therefore excluded by the minimal architecture premise [M1].

P3 (Boundary Completion): Since the physical Hilbert space does not factorize across regions (P1), a well-defined subregion algebra $\mathfrak{A}_O$ requires boundary completion. We implement this algebraically by adjoining edge observables that generate a nontrivial center $\mathcal{Z}(\mathfrak{A}_O)$, encoding the gluing or charge data [14,15] (consistency requirement).

Principle B: Finite-Resolution Modular Information

A boundary-defined subsystem requires finite capacity to avoid unbounded information density [16]. In the small-diamond/KMS regime, modular flow supplies the intrinsic clock of this boundary algebra [4,5], rendering time local and emergent.

P4 (Modular Locality): In the small-diamond (local Rindler) regime, the vacuum state restricted to $\mathfrak{A}_O$ is, to leading order, KMS with respect to a geometric modular flow: the modular Hamiltonian is well-approximated by the local boost generator (Bisognano–Wichmann/Unruh). Scale separation $\delta \ll \ell_O \ll R_{\text{curv}}$ controls this approximation; we do not assume modular locality outside this regime. The dimensionless modular parameter τ provides an intrinsic ordering of correlations (local clock) for subsystem; with standard boost normalization, KMS periodicity is 2π [4,5,17,18].

P5 (Finite Resolution): A boundary-defined subsystem requires finite operational capacity. To prevent an unbounded density of states and ensure a stable restriction to the local algebra $\mathfrak{A}_O$, we impose a physical regulator δ (vacuum impedance) and define the fundamental resolution scale $M_s \equiv \delta^{-1}$ (consistency requirement). This cutoff renders the boundary a finite-capacity completion, yielding a well-defined reduced description consistent with the Bekenstein bound [16,19,20].

Principle C: Topological Quantization and Minimal Transport

Consistency forces boundary charge sectors to be topologically quantized rather than continuously tunable [21]. Minimal isotropic discrete transport then fixes the transport rule [22], with smooth forces emerging as the effective continuum description.

P6 (Gauge Topology): We model boundary charge sectors by a bulk Chern–Simons theory whose boundary reduction is a WZW current algebra. On a bounded region, Gauss-law constraints generate edge currents as the gauge-invariant completion of the subregion algebra. The associated level $k \in \mathbb{Z}$ quantizes these currents, replacing continuous field normalization with a discrete integer. We adopt a CS/WZW edge-current realization as the minimal quantized boundary current algebra; its integer level k sets the fundamental stiffness unit and fixes the matching-scale inverse coupling $\alpha^{-1}(M_s)$ in linear response [M2].

P7 (Discrete Transport Structure) (Router): Minimal local boundary architecture in 3D is constrained by two requirements: Lattice connectivity: A local transport layer must exist and be isotropic in three spatial dimensions, with inversion symmetry (no built-in handedness) (consistency requirement). We realize this with the minimal inversion-symmetric spanning set $\{\pm x, \pm y, \pm z\}$, representing three independent antipodal pairs, fixing minimal coordination $z = 6$ (ports) and an octahedral (L_1) transport rule; other choices either break inversion/parity or introduce redundant modes requiring extra projections [M3]. This implies three independent spatial transport axes, over which isotropic accessibility counts sum linearly. Discrete transport is implemented in local orthonormal frames, as a global smooth tangent frame on S^2 is obstructed (Hairy Ball Theorem) [22–24] (consistency requirement). Matter content: To admit local spinorial matter content, the boundary algebra must lift local rotations from $SO(3)$ to its universal cover $SU(2)$ to support projective representations (consistency requirement for spinorial matter). Since $SU(2) \simeq S^3$, this selects an S^3 factor as the minimal internal representation. This spin-structure extension unavoidably introduces a minimal $\mathbb{Z}_2$ grading associated with the double cover, which we represent topologically as a $\mathbb{Z}_2$ twist sector [M4].

Corollary (Transport Axis Symmetry $|\Gamma|$): The nearest-neighbour coordination implies a transport axis redundancy of order $|\Gamma| = 2^3 = 8$, corresponding to independent sign configurations of the three transport axes. In commuting regimes, these sign labels act as redundant axis labels, quotienting the phase volume by $|\Gamma| = 8$.

Corollary (Tip Defect Parameter κ): Near the tips (p, q) of the causal diamond, the cross-section becomes sub-resolution (P5), forcing a geometric mismatch between smooth modular flow and discrete connectivity. We parameterize this irreducible overhead by a dimensionless scalar κ .

Corollary (Pixel): At finite resolution L_s , a pixel is one distinguishable spatial location (boundary patch) on the diamond waist, characterized by coupled spectral data $(\mathfrak{A}, \mathcal{H}, D)$. The local gauge algebra $\mathfrak{A}$ (P6) acts on the internal degrees of freedom $\mathcal{H}$ (P7) through the transport D .

1.2. Boundary Architecture, Resolution and Connectivity

Because the physical Hilbert space does not factorize across regions (P1) [14,25], defining a local subsystem requires boundary completion via edge modes [14,15] (independent of AdS/CFT). A cutoff surface at proper distance δ is therefore not optional; it is the minimal structure required to define an interior subregion algebra. Residing near the S^2 bifurcation surface, this regulated boundary is an active structure that distinguishes and updates states when excitations cross the region, registering these crossings as discrete update events (null arrivals) along the null generators.

A query–response loop between two events p and q defines the causal diamond $O(p, q)$ (P2), which is bounded by a null surface (lightlike boundary) [2]. The finite-resolution boundary regulator (screen) is the boundary completion on the causal diamond bifurcation surface (maximal-area waist), topologically S^2 , defined covariantly for each operational window rather than as a fixed background surface. The boundary is not a fixed object in the vacuum, but a relational boundary. Its stability across experiments arises from the consistency of overlapping diamonds.

Flux convention

The boundary interface is the waist S^2 defined by diamond topology (Axiom P2). When we translate a boundary response coefficient into standard gauge parameters $\alpha \equiv g^2/4\pi$, we apply the usual 4π flux normalization associated with unit-sphere interface.

Finite Resolution and UV Scales

We implement this via a stretched-horizon regulator at proper distance δ from the waist [26], with the resolution scale $M_s \equiv \delta^{-1}$ defined in P5 and $L_s \equiv M_s^{-1}$. In the wedge limit, the vacuum state is KMS with respect to the boost generator (Bisognano–Wichmann) [4]. At the regulator boundary, proper time t relates to this dimensionless modular parameter τ via the redshift factor $t \approx \delta\tau$ [5]. Identifying the minimum resolvable proper time with the resolution scale ($t_{\min} \equiv \delta$) naturally forces the modular cutoff to unity (one resolved modular update): $\varepsilon \equiv \tau_{\min} = 1$. Any variation of ε would correspond to shifting the cutoff surface, not tuning a free parameter.

A finite UV cutoff implies principal-branch ambiguity (aliasing): any continuum field description below L_s is a reconstruction, not additional physical distinguishability. This resolution limit means the continuous S^2 waist acts operationally as a finite set of distinguishable pixels [19].

Effective internal degeneracy (N): A dimensionless capacity factor (internal degrees of freedom) per pixel for internal response sectors (spin, charge, edge modes), defined by the vacuum entropy budget $N \equiv e^{S_{\text{vac}}}$. N is an effective degeneracy (log-capacity exponentiated), not a literal Hilbert-space dimension. Since the $\mathbb{Z}_2$ twist of P7 is treated as structural overhead, the addressable internal degrees of freedom are $n_{\text{ch}} \equiv N/\sqrt{2}$ [M5]. In linear response, the stiffness is extensive in the number of active internal sectors per pixel, yielding a large- N separation between macroscopic gravitational stiffness and intensive single-channel cutoff (like parallel strands in a cable) [27,28].

Assuming a uniform resolution scale across internal matter channels [M6], the per-update energy granularity is given as $E_{\text{pix}} \equiv M_s/n_{\text{ch}}$, an intensive energy unit for single-pixel saturation.

Lattice Connectivity and Coordination

Following Axiom P7, minimal isotropic discrete transport in three spatial dimensions is generated by the signed basis $\{\pm x, \pm y, \pm z\}$, implying coordination $z = 6$. Connectivity decomposes into lateral adjacency on the boundary surface and transverse coupling to the bulk. This structure induces a geometric conflict: the discrete connectivity defines a step-counting metric with an L_1 -type unit ball (octahedral neighborhood), whereas the continuum isotropic limit corresponds to an L_2 ball (sphere). This mismatch imposes an unavoidable digitization cost. While negligible at scales $\gg L_s$, this conflict becomes significant when the transverse size approaches the resolution scale near the diamond tips, necessitating a Regge-type deficit correction [29].

Implication for Dynamics

With $(O, \mathfrak{A}_O, \delta)$ fixed, we vary the reference $\sigma_O[g]$ and minimize $S_{\text{rel}}(\rho_O \parallel \sigma_O[g])$. This relative-entropy functional [10,30] is required because it vanishes when the reference matches the data and is monotonic under restriction, ensuring compatible descriptions on diamond overlaps (Axiom P3). Because the reference family lives on this fixed algebra, the effective internal degeneracy N is determined by the boundary topology. Furthermore, the fixed regulator organizes the local response into a controlled small-perturbation expansion via the heat-kernel spectral method [31]. Chapter 2 formulates the corresponding variational functional and shows that its Hessian decouples the response into orthogonal sectors governing the leading macroscopic dynamics.

Because the reference family lives on this fixed algebra, the effective internal degeneracy N is not a tunable parameter but is strictly determined by the boundary topology. Furthermore, the presence of the fixed boundary regulator allows the local response to be organized into a controlled small-perturbation expansion (via the heat-kernel spectral method [31]), naturally bridging the gap between information geometry and EFT.

1.3. Topological Budget: Effective Internal Degeneracy

With the boundary architecture set, we determine the depth N by summing the additive capacity costs implied by the minimal construction (P2–P5, P7). We distinguish three independent contributions (assuming minimality): the bulk surface topology (S^2), the discretization overhead at the diamond tips and the matter-sector twist sector.

Additive capacity costs (entropy S) map to a multiplicative channel multiplicity N via $N \equiv \exp(\sum_i S_i)$. Since N represents an effective capacity rather than a literal integer eigenvalue, non-integer values are physically admissible.

Bulk Topology (S_{bulk})

The minimal boundary-completion log-capacity is an additive, diffeomorphism-invariant, scale-free functional of the cut S^2 (P2) [32]. In two dimensions, the canonical minimal choice is the Euler characteristic density (Gauss-Bonnet [33]) [M7]:

$$S_{\text{bulk}} \equiv \oint_{S^2} R^{(2)} dA = 4\pi\chi(S^2) = 8\pi.$$

This value is fixed topologically by $\chi(S^2) = 2$ via Gauss-Bonnet [33] and is metric independent. By Axioms P2 and P3, the boundary completion uses a closed cut for algebraic gluing. Note that a hemisphere (4π) would be an open interface with boundary circle, and could not serve as the required closed gluing surface.

Tip Defect Parameter (κ)

The tips (p, q) are geometric bottlenecks where null generators meet and the transverse cross-section approaches the resolution scale L_s (Axiom P5). In this sub-resolution regime, smooth L_2 -isotropic modular flow must be implemented by the discrete transport structure (Axiom P7), so each modular cycle necessarily incurs a finite number of lattice coordination events (discrete reorientations).

Representing an L_2 flow with an L_1 step-transport primitive therefore carries an unavoidable digitization cost: an irreducible mismatch overhead [4,30]. We define κ as the resulting dimensionless impedance overhead per modular cycle. In the local KMS regime, the modular flow has period 2π (Axiom P4), and minimal 3D isotropy partitions transport into three independent antipodal axes $\{\pm x, \pm y, \pm z\}$ (Axiom P7). Normalizing for one unavoidable junction per axis per cycle fixes [M8]:

$$\kappa = \frac{1}{3 \times 2\pi} = \frac{1}{6\pi}$$

The tip overhead enters additively as a bookkeeping cost for maintaining smooth modular ordering across a discrete junction; it is not a reduction of the channel count. Given the axiomatic constraints on modular periodicity (P4), resolution limit (P5) and lattice connectivity (P7), this factor is a rigid geometric necessity rather than a tunable regulator.

Twist Contribution (S_{twist})

Among standard topological realizations of a nontrivial $\mathbb{Z}_2$ twist sector (P7), the minimal quantum dimension is $d_\sigma = \sqrt{2}$ [34] (alternative non-minimal choices ($d_\sigma > \sqrt{2}$) would strictly increase the effective degeneracy N). Adopting this minimum, the additive term to the vacuum entropy budget (a half-bit cost) is [M9]:

$$S_{\text{twist}} = \ln d_\sigma = \ln \sqrt{2}.$$

Larger non-Abelian twist sectors are excluded by the minimality premise (P7), as the spatial spinor lift strictly requires only the $\mathbb{Z}_2$ extension.

Resulting Effective Internal Degeneracy (N)

Summing bulk, tip and twist contributions yields the total vacuum entropy:

$$S_{\text{vac}} = S_{\text{bulk}} + \kappa + S_{\text{twist}} = 8\pi + \frac{1}{6\pi} + \ln \sqrt{2}.$$

Exponentiating this sum determines the topology-locked channel multiplicity per pixel:

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.23 \times 10^{11}.$$

Thus N quantifies the effective depth per pixel implied by the minimal boundary topology, tip overhead and spinor-lift sector. These degrees of freedom are localized to the boundary completion and need not introduce additional light propagating fields in the infrared.

Aggregate Defect Measure (Ω_{def})

We also define an aggregate coordination overhead per pixel, Ω_{def} . Given coordination $z = 6$, the total defect budget per pixel is:

$$\Omega_{\text{def}} \equiv z\kappa = \frac{6}{6\pi} = \frac{1}{\pi} \approx 0.318.$$

We use Ω_{def} strictly as a compact proxy for persistent per-pixel coordination overhead; it is not, by itself, a standalone cosmological identification.

Parameter Lock and Consistency

The bulk contribution $S_{\text{bulk}} = \oint_{S^2} R^{(2)} dA = 8\pi$ is fixed by Gauss–Bonnet [33] and the gluing requirements of boundary completion (P2–P3). Alternative normalizations (e.g., a 4π hemisphere or $z \neq 6$) would violate the closed gluing requirement (P2–P3) or the minimal parity-symmetric transport (P7). As a consistency check, κ matches the normalized Regge deficit $\delta_{\text{def}}/(2\pi)^2$ for a minimal curvature bottleneck [29].

With the effective internal degeneracy N fixed by these geometric contributions, the remaining step is to calibrate the resolution scale M_s using Newton’s constant.

1.4. Constitutive Relation and Calibration

The constitutive relation links the discrete internal degrees of freedom to the macroscopic stiffness of spacetime. We use the reduced Planck mass $M_P \equiv (8\pi G)^{-1/2}$ throughout.

Constitutive Stiffness Relation

In the Einstein–Hilbert action [35], the coefficient of curvature is the inverse gravitational constant. In the linear-response regime, the stiffness of the boundary scales with the number of effective channels [27,28]. Dimensional analysis at resolution scale M_s implies a single-channel resolution scale $\sim M_s^2$. Summing over N_{eff} channels yields [M10]:

$$M_P^2 = \sum_{a=1}^{N_{\text{eff}}} M_s^2 = N_{\text{eff}} M_s^2, \quad N_{\text{eff}} \equiv \kappa N.$$

The factor κ represents the coupling efficiency over the modular cycle, limited by sub-resolution lattice coordination at the tips; it is fixed by the coordination normalization (not tuned to match G). This allows us to recover the Einstein–Hilbert stiffness by summing over finite channels, rather than discretizing a manifold or quantizing the graviton. This spectral reconstruction of the action shares a conceptual lineage with the spectral action principle [36] and mirrors the species-bound logic, where

gravitational strength is diluted by the number of active degrees of freedom [28]. Chapter 2 will derive this structure from the Hessian of a relative-entropy variational principle.

Numerical Calibration

We calibrate the single-channel UV cutoff M_s by matching this constitutive relation to the measured Newton constant G . We first work with the reduced Planck mass to match the standard normalization of the Einstein-Hilbert action:

$$M_P \equiv (8\pi G)^{-1/2} \simeq 2.435 \times 10^{18} \text{ GeV}.$$

Substituting the constitutive law $M_P^2 = N_{\text{eff}} M_s^2$ and solving for the resolution scale yields:

$$M_s = \frac{M_P}{\sqrt{N_{\text{eff}}}} \approx 3.02 \times 10^{13} \text{ GeV}.$$

M_s is the fundamental resolution scale implied by δ . It represents the physical resolution limit of the algebra (sharp focus) and consequently serves as the exact universal matching scale where gravitational stiffness, gauge couplings and mass gaps decouple and can be derived analytically from discrete topological data, as detailed in Appendix A through D.

This derived scale $M_s \sim 10^{13} \text{ GeV}$ coincides with the phenomenological intermediate scale required by neutrino seesaw mechanisms [37] and plateau inflation [38].

Scale Inversion (L_P vs L_s)

The constitutive law implies the relation between the derived Planck length L_P and the physical UV cutoff $L_s = M_s^{-1}$:

$$L_P = L_s / \sqrt{N_{\text{eff}}}.$$

In this framework L_s is the finite resolution scale, while L_P is the effective stiffness scale implied by parallelization. Coarsening the vacuum geometry to the finite scale L_s removes the need to take $L_P \rightarrow 0$ as a physical resolution scale without sacrificing the macroscopic rigidity required by GR.

Consistency Check: Area Law

We verify that this calibration preserves the Bekenstein–Hawking entropy structure [39,40]. For a horizon of area A , the standard entropy (in natural units) is $S_{\text{BH}} = A/4G$. Substituting $1/G = 8\pi M_P^2$:

$$S_{\text{BH}} = 2\pi M_P^2 A = 2\pi (N_{\text{eff}} M_s^2) A = 2\pi N_{\text{surf}} N_{\text{eff}},$$

where $N_{\text{surf}} \equiv A/L_s^2 = AM_s^2$ is the number of resolvable boundary pixels on a waist of area A (set purely by geometry); the entropy factors as pixels $\times$ effective depth. Converting to bits ($S/\ln 2$) and taking $A_{\text{cosm}} \sim 4\pi H_0^{-2}$ gives $\sim 10^{122}$ bits.

Although the result follows algebraically from the constitutive lock $M_P^2 = N_{\text{eff}} M_s^2$, it implies that the standard horizon entropy is realized as pixel count (N_{surf}) times effective depth (N_{eff}). The required information density is achieved via channel multiplicity, rather than by shrinking the pixel size to L_P . This recovers the Bekenstein–Hawking scaling using the finite resolution scale L_s , without requiring the boundary resolution itself to be L_P .

Separation of Scales

This calibration links the scale M_P to the resolution scale M_s through the derived depth N . With M_s fixed, we define a per-pixel energy budget:

$$E_{\text{pix}} \equiv M_s / n_{\text{ch}}, \quad n_{\text{ch}} \equiv N / \sqrt{2},$$

where the $\sqrt{2}$ factor treats the twist sector as structural overhead rather than data-carrying matter content, so n_{ch} counts addressable internal degrees of freedom.

1.5. Emergent Time

In a Wheeler–DeWitt setting ($H\Psi = 0$), time is relational and defined by conditioning on an internal clock (Page–Wootters) [3]. For a partition $\mathcal{H}_{\text{tot}} = \mathcal{H}_C \otimes \mathcal{H}_R$, the conditional state is:

$$|\psi(t)\rangle_R = \langle t|_C |\Psi\rangle.$$

In the small-diamond regime (P4), the restricted vacuum is approximately KMS and the associated modular flow provides a canonical local ordering of correlations [4,5]. We take the modular parameter τ as this intrinsic ordering variable. Proper time t arises only after coarse-graining many resolved modular updates; at the cutoff surface one has $t \simeq \delta \tau$.

In the operator-algebraic viewpoint, the modular automorphism group defines intrinsic evolution on the algebra. Semiclassical Wheeler–DeWitt solutions admit a WKB form $\Psi[g] \approx \mathcal{A}[g] e^{iS_{\text{eff}}[g]}$; when modular flow is geometric, the phase functional S_{eff} motivates the relative-entropy variational principle introduced in Chapter 2.

Time is therefore diamond-local: the operational clock is the ordering of distinguishable boundary updates at resolution M_s . Discreteness enters only at the cutoff scale and coarse-graining yields an effectively continuous time parameter on observational scales.

With the dimensionless parameters (N, κ) fixed by boundary geometry and the resolution scale M_s anchored to G , these become the primary fixed inputs for the dynamics in Chapter 2.

2. Relative-Entropy Functional and Dynamics

Dynamics on a causal diamond emerge not as the time-evolution of a global Wheeler–DeWitt state [1], but as local statistical inference. The reduced boundary state ρ_O is the operational data. Geometry and other background fields enter only through a semiclassical reference family $\sigma_O[\Lambda]$ defined on the same boundary-completed algebra $\mathfrak{A}_O$.

Related entanglement-equilibrium approaches recover the tensor sector under locality assumptions [6–8,10–13,41,42]. Here, because the operational subsystem is defined on a fixed, finite-resolution boundary-completed algebra, the variational problem is posed at the definite matching scale M_s .

The dynamics are defined by minimizing the informational mismatch $I[\Lambda; O] = S_{\text{rel}}(\rho_O \| \sigma_O[\Lambda])$ at fixed $(O, \mathfrak{A}_O, \delta)$ [30,43,44], wherein its relative-entropy Hessian simultaneously decouples the macroscopic tensor stiffness, the vector (gauge) and scalar (mass) susceptibilities.

2.1. Entropic Variational Principle and Operational Domain

We fix the container $(O, \mathfrak{A}_O, \delta)$ with proper-distance regulator $\delta = M_s^{-1}$ ($\delta \ll \ell_O \ll R_{\text{curv}}$), varying only the reference family $\sigma_O[\Lambda]$ on the fixed algebra $\mathfrak{A}_O$. We define the variational functional [30]:

$$I[\Lambda; O] := S_{\text{rel}}(\rho_O \| \sigma_O[\Lambda]).$$

$S_{\text{rel}} \geq 0$ is monotone under restriction, enforcing the overlap consistency required by boundary completion (Axiom P3) [30,44–46]. On the fixed algebra $\mathfrak{A}_O$, Jaynes’ principle selects the maximum-entropy exponential reference family compatible with the chosen sources Λ [43,44]:

$$\sigma_O[\Lambda] = \frac{e^{-K_O[\Lambda]}}{\text{Tr}(e^{-K_O[\Lambda]})}.$$

The modular generator $K_O[\Lambda]$ couples the local background multiplet Λ to the boundary operator multiplet O , defining deformations via the inner product:

$$\delta K = \int d^4x \sqrt{-g} \delta \Lambda(x) \cdot O(x).$$

In the KMS regime, relative-entropy takes the modular free-energy form [30]. Because the vacuum σ_0 is an equilibrium state, the first law of entanglement ($\delta S_{\text{ent}} = \delta \langle K \rangle$) ensures the variational functional I is stationary at the reference background Λ_0 :

$$S_{\text{rel}}(\rho \| \sigma) = \Delta \langle K_\sigma \rangle - \Delta S \quad (K_\sigma = -\log \sigma), \quad \delta I|_{\Lambda_0} = 0.$$

With linear driving forces strictly vanishing, the leading restoring dynamics arise from the second variation (the Hessian), mirroring a harmonic elastic response.

Remark (variational domain): We vary the reference background $g \mapsto \sigma_O[g]$ at fixed container $(O, \mathfrak{A}_O, \delta)$. In the modular KMS regime, the reference state takes the form $\sigma_O[g] = e^{-K_O[g]} / \text{Tr}(e^{-K_O[g]})$ on the same boundary-completed algebra. Metric variations $\delta g_{\mu\nu}$ therefore deform the modular generator $K_O[g]$ and correlators on $\mathfrak{A}_O$ without altering the underlying degrees of freedom. The causal diamond O is fixed relationally by the endpoints (p, q) rather than a coordinate box. Admissible variations preserve the regulator placement at a fixed proper distance $\delta = M_s^{-1}$ from the waist.

Fixing δ pushes the diamond tips below resolution under modular flow, recovering the tip defect parameter κ and locking gravitational stiffness to the same substrate scale as leakage strength.

2.2. Hessian Bridge, Spectral Extraction and EFT Coefficients

Local background fields $\Lambda = (g_{\mu\nu}, A_\mu, \varphi)$ couple to the composite operator $\mathcal{O} = (T^{\mu\nu}, J^\mu, M)$. At leading derivative order, vacuum symmetries (Lorentz invariance, charge neutrality, isotropy) and Ward identities suppress mixing, block-diagonalizing the response into orthogonal tensor, vector and scalar sectors [35,46,47].

To fix normalizations unambiguously, the source–operator pairing in component form is (the factor $1/2$ reflects standard metric–source convention):

$$\delta K = \int d^4x \sqrt{-g} \left(\frac{1}{2} \delta g_{\mu\nu} T^{\mu\nu} + \delta A_\mu J^\mu + \delta \varphi M \right).$$

Expanding the variational functional around the KMS state Λ_0 determines the effective dynamics:

$$I[\Lambda_0 + \delta\Lambda] = I[\Lambda_0] + \int d^4x \delta\Lambda(x) \cdot \left. \frac{\delta I}{\delta \Lambda(x)} \right|_{\Lambda_0} + \frac{1}{2} \iint d^4x d^4y \delta\Lambda(x) \cdot G(x, y) \cdot \delta\Lambda(y) + R_{\geq 3}[\delta\Lambda].$$

At equilibrium, the state is $\rho_O = \sigma_0$ on the resolved algebra, yielding $I[\Lambda_0] = S_{\text{rel}}(\sigma_0 \| \sigma_0) = 0$. The entanglement first law guarantees stationarity at Λ_0 . For small deformations $\|\delta K\| \ll 1$, the remainder $R_{\geq 3}[\delta\Lambda]$ is suppressed, so the leading response is quadratic ($\delta^2 I$), governed by the Hessian:

$$G(x, y) := \left. \frac{\delta^2 I}{\delta \Lambda(x) \delta \Lambda(y)} \right|_{\Lambda_0}.$$

This Hessian coincides with the Kubo–Mori (quantum Fisher) metric, which captures the connected two-point response of the operators in the KMS vacuum [17,30,48,49]. Physically, $G(x, y)$ acts as an elastic modulus: it converts small deformations of the reference model into a quadratic cost, with its eigen-directions defining the independent stiffness modes at long wavelengths.

Scale separation $L_s \ll \ell_O \ll R_{\text{curv}}$ ensures the response kernel $G(x, y)$ is quasi-local, reducing the non-local double integral to a local derivative expansion at $p \ll M_s$:

$$G_{aa}(x, y) \simeq \left[c_a^{(0)} + c_a^{(2)} \nabla^2 + \dots \right] \frac{\delta^{(4)}(x - y)}{\sqrt{-g}}.$$

With this quasi-local form, the double integral $\iint \delta\Lambda \cdot G \cdot \delta\Lambda$ reduces to $\int d^4x \sqrt{-g}$ acting on a local derivative series [31,35].

Orthogonality and quasi-locality establish a Hessian mapping. We expand the quasi-local kernel in derivatives at $p \ll M_s$, identify the unique lowest-dimension local invariant consistent with sector symmetries (diffeomorphisms for T , gauge for J , scalar Lorentz for M) and read off its coefficient at M_s . This translates information-theoretic response kernels directly into canonical EFT invariants:

$$\begin{aligned} \delta^2 I &\simeq \underbrace{\frac{1}{2} \iint \delta g G_{TT} \delta g}_{\Rightarrow \int d^4x \sqrt{-g} \frac{M_p^2}{2} R} + \underbrace{\frac{1}{2} \iint \delta A G_{JJ} \delta A}_{\Rightarrow \int d^4x \sqrt{-g} \frac{1}{4g^2} F^2} + \underbrace{\frac{1}{2} \iint \delta \varphi G_{MM} \delta \varphi}_{\Rightarrow \int d^4x \sqrt{-g} m \bar{\psi} \psi} \end{aligned}$$

The top line represents the non-local quadratic form defined by the Hessian kernel; the bottom line its local EFT equivalent.

In the near-KMS small-diamond regime, stationarity at Λ_0 isolates the quadratic Kubo–Mori response $\delta^2 I = \frac{1}{2} \iint \delta \Lambda \cdot G \cdot \delta \Lambda$. Scale separation $L_s \ll \ell_O \ll R_{\text{curv}}$ implies $G(x, y)$ is quasi-local, yielding a local EFT functional via derivative expansion at $p \ll M_s$. We evaluate this local functional using a regulated spectral trace. This trace acts as a computational device for the near-equilibrium response via heat-kernel methods, rather than an exact non-perturbative identity. Defining a positive operator $D_{\mathcal{A}}^2(g, A, \dots)$ on the boundary-completed structure and applying a smooth window f to suppress modes above M_s [36,50,51]:

$$I \approx \text{Tr} f \left(\frac{D_{\mathcal{A}}^2}{M_s^2} \right).$$

Discrete connectivity and matter content fix the transport operator $D_{\mathcal{A}}$. At macroscopic scales $\gg L_s$, it approximates a continuum Dirac/Laplace operator, allowing standard heat-kernel asymptotics. The trace acts as an effective representation of the mode-summed Kubo–Mori response. The cutoff is the physical operational resolution scale M_s fixed by the regulated boundary, not a formal UV regulator [51,52].

To extract scale-dependent EFT coefficients from this trace, we express the window f via Laplace transform:

$$f(x) = \int_0^\infty dt \tilde{f}(t) e^{-tx} \quad \Rightarrow \quad \text{Tr} f \left(\frac{D_{\mathcal{A}}^2}{M_s^2} \right) = \int_0^\infty dt \tilde{f}(t) \text{Tr} e^{-t D_{\mathcal{A}}^2 / M_s^2}.$$

This maps the trace into a tractable sum of local, high-energy geometric invariants (a_k) via the short-time asymptotic expansion $\text{Tr} e^{-t D^2} \sim \sum_k a_k t^{(k-4)/2}$. Inserting this expansion yields the finite EFT coefficients c_k :

$$\text{Tr} f(D^2/M_s^2) \sim \sum_k a_k \left(\int_0^\infty dt t^{(k-4)/2} \tilde{f}(t) \right) M_s^{4-k}, \quad c_k = a_k \int_0^\infty dt t^{(k-4)/2} \tilde{f}(t).$$

In a finite-resolution boundary theory, the heat-kernel coefficients a_k naturally include boundary, interface and defect contributions induced by boundary completion and discrete transport [31,53]. At low derivative order, locality and symmetry fix the operator basis:

$$\text{Tr} f(D_{\mathcal{A}}^2/M_s^2) \sim \int d^4x \sqrt{-g} [c_0 M_s^4 + c_2 M_s^2 R - c_F F_{\mu\nu}^2 + c_{R^2} R^2 + \dots].$$

These coefficients represent response properties defined at M_s . Below M_s , standard EFT renormalization running proceeds normally.

2.3. Coupling Factorization and Spectral–Volume Correspondence

Quasi-locality enforces a strict volume–coupling factorization. For an internal gauge field on an operational domain $\mathcal{D}$, the integrated quadratic cost factors cleanly:

$$\int_{\mathcal{D}} d^4x \sqrt{-g} \frac{1}{4g_{\text{YM}}^2} F^2 \quad \Rightarrow \quad \text{Vol}_4(\mathcal{D}) \cdot (1/g_{\text{YM}}^2) \cdot \langle F^2 \rangle_{\mathcal{D}}.$$

This makes explicit the extensive vs. intensive separation: the Einstein term scales with the number of participating sectors, while $1/g^2$ and mass parameters are intrinsic susceptibilities at M_s (Appendix C).

Next, it establishes a spectral–volume correspondence. The trace is evaluated over the minimal compact operational history. In the small-diamond/KMS regime, modular time has a period of 2π , yielding a thermal circle S^1 [4,46]. Identifying the spatial part as two B^3 halves glued across the waist, $B^3 \cup_{S^2} B^3 \simeq S^3$, the minimal compact manifold for trace execution is $S^3 \times S^1$ [31].

On this manifold, the effective four-volume can be read off from the leading heat-kernel coefficient as a mode count at resolution M_s :

$$\text{Vol}_4(\mathcal{D}) = \frac{1}{M_s^4} \frac{\text{Tr}_{\mathcal{D}} \mathbf{1}_{\mathfrak{A}}}{v_{\text{int}}}.$$

Here, v_{int} is the internal degeneracy of the algebra. It is a rigid constant of the boundary architecture, not a tunable parameter. This provides a mode-counting expression for Vol_4 at resolution M_s , used later to estimate accessibility volumes Ω (Appendix D).

2.4. Tensor Sector: Gravity as Extensive Stiffness

Applying the Hessian mapping to $\delta g_{\mu\nu}$, the tensor block G_{TT} measures the stiffness of the geometric pixel under modular flow. In the low-curvature regime $|R| \ll M_s^2$, linear response around the modular vacuum governs the dynamics [6]. In the small-diamond KMS regime, the modular generator for metric deformations universally couples $\delta g_{\mu\nu}$ to the stress tensor $T^{\mu\nu}$ (via the conformal Killing vector preserving the diamond), supplemented by a boundary completion term required on bounded regions. This completion is fixed by diffeomorphism invariance; its variation reproduces the geometric entropy functional of the cut (area/Wald-type) [4,14,16,39].

With first-order variations vanishing at equilibrium, this geometric entropy functional drives the second-order response [7,54], selecting the Einstein universality class at leading derivative order. Evaluating the quasi-local expansion, the leading two-derivative diffeomorphism invariant at the matching-scale M_s is $\int d^4x \sqrt{-g} R$, with coefficient $M_P^2/2$ [35].

The tensor entry therefore isolates the Einstein stiffness coefficient M_P^2 . The stiffness is strictly extensive in the number of coherently participating channels, fixing the exported lock at the matching-scale:

$$M_P^2 = N_{\text{eff}} M_s^2.$$

2.5. Vector Sector: Gauge as Intrinsic Susceptibility

Applying the Hessian mapping to δA_μ , the vector block G_{JJ} measures the intrinsic susceptibility of boundary current. A weak background connection couples to the conserved edge current J^μ required by the CS/WZW structure [21,55] through the modular generator:

$$\delta K[A] = \int d^4x \sqrt{-g} A_\mu(x) J^\mu(x).$$

Using the G_{JJ} block, Ward identities enforce gauge invariance, isolating $F_{\mu\nu} F^{\mu\nu}$ as the leading two-derivative local operator. The matching-scale coupling $g(M_s)$ is read off as the coefficient of the canonical term $-\frac{1}{4} F^2$ in the quasi-local derivative expansion.

Finite resolution fixes the resolved modular interval $\tau \in [\varepsilon, 2\pi - \varepsilon]$. In the modular/KMS regime, the edge-current correlator on the modular circle takes the universal form $\langle J(\tau) J(0) \rangle = \frac{k}{4} \sin^{-2}(\tau/2)$ [56], where $k \in \mathbb{Z}$ is the WZW level fixed by gauge topology. Integrating over the resolved modular window $\tau \in [\varepsilon, 2\pi - \varepsilon]$ explicitly factors the raw susceptibility into an intensive topological core and a geometric kinematic integral:

$$\chi_{\text{pix}} = \int_{\varepsilon}^{2\pi-\varepsilon} d\tau \langle J(\tau) J(0) \rangle = \frac{k}{4} \underbrace{\int_{\varepsilon}^{2\pi-\varepsilon} d\tau \sin^{-2}(\tau/2)}_{I_{\text{mod}}(\varepsilon)} = k \cot(\varepsilon/2).$$

Factoring out the pure geometric kinematics I_{mod} isolates the normalized intrinsic susceptibility $\tilde{\chi}_{\text{pix}} = k/4$. To translate this into the four-dimensional coupling coefficient, we apply the volume-coupling factorization defined earlier. The canonical matching rule is:

$$\frac{1}{g^2(M_s)} \equiv \mathcal{N}_J \tilde{\chi}_{\text{pix}}, \quad \alpha^{-1}(M_s) \equiv \frac{4\pi}{g^2(M_s)}, \quad \mathcal{N}_J = 4.$$

The normalization factor $\mathcal{N}_J = 4$ is fixed once by the S^2 flux convention and the edge-current normalization, seamlessly yielding the matching-scale inverse coupling:

$$\alpha^{-1}(M_s) = 4\pi k.$$

This strictly intensive coefficient depends on k and the fixed definition ($\varepsilon = 1$), not the extensive channel count N_{eff} . The factor 4π reflects the flux normalization convention; the remaining microscopic data are encoded entirely in the integer level k .

2.6. Scalar Sector: Mass as Occupancy Stiffness

Applying the Hessian mapping to $\delta\varphi$, the scalar block G_{MM} measures the occupancy stiffness of the localized matter content. While gauge dynamics probe the operational history on $S^3 \times S^1$, fermion masses are controlled by internal accessibility under finite resolution. We couple a scalar occupancy deformation to the gauge-invariant density operator $M = \bar{\psi}\psi$ via $\delta K_M \sim \int d^4x \sqrt{-g} \delta\varphi(x) \bar{\psi}(x)\psi(x)$.

In the symmetry decomposition of the quasi-local response, $\bar{\psi}\psi$ is a Lorentz scalar, so mass resides in the scalar block. Define χ_M as the coefficient of the leading local term $\frac{1}{2} \int d^4x \sqrt{-g} \chi_M \delta\varphi^2$ in the quasi-local expansion of the G_{MM} block; then $m \sim \chi_M^{-1}$ at the matching-scale (up to sector convention). The Hessian mapping identifies this leading scalar contribution as a mass term:

$$S_{\text{mass}} \approx - \int d^4x \sqrt{-g} m \bar{\psi}\psi.$$

The required $\mathbb{Z}_2$ grading (spinor lift) means the minimal matter content admits spinor representations [57]. Locality and this grading select Dirac-type transport as the minimal covariant transport rule [22], utilizing the operator $D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - iA_\mu^a T^a$. On a bounded region, charged operators require edge completion. The minimal localized charged excitation $D_\mu\psi$ on $\mathfrak{A}_O$ is therefore a dressed composite. We absorb the algebraic and entropic cost of attaching this excitation to the nontrivial center $\mathcal{Z}(\mathfrak{A}_O)$ into a factor $c_{\text{dress}} \geq 1$, taken as unity unless otherwise stated [U#]. Finite capacity bounds localized defect modes, yielding a graded minimal matter content supporting fermionic two-state occupancy $n \in \{0, 1\}$.

In discrete modular time, a local mass gap acts as the on-site phase accumulated per resolved update. Using the per-pixel energy budget E_{pix} (Chapter 1), we parameterize this phase as $\theta_f \simeq m_f/E_{\text{pix}}$ subject to $|\theta_f| \leq \pi$.

Finite resolution organizes the spectrum into modes with distinct effective accessible volumes Ω . If a mode explores a larger volume, its overlap with a localized anchor is diluted ($|\psi|^2 \sim 1/\Omega$), yielding the scaling $m \propto m_{\text{anchor}}/\Omega$. Here m_{anchor} is the bare, undiluted energy scale of the localized defect excitation before it explores the accessible internal volume. In practice, Ω is estimated by spectral mode counting on the internal Laplacian at the fixed cutoff M_s [31,53].

The coarse-graining regimes and the $G \rightarrow \mathfrak{g} \rightarrow T$ filtration used to derive the specific lepton masses are developed in Appendix D.

2.7. Unified Matching-Scale Action at M_s

The operators derived above for tensor, vector and scalar sectors govern the low-curvature regime. The Einstein term ($\frac{M_P^2}{2}R$) contains no saturation scale and extrapolation to $|R| \gtrsim M_s^2$ lies outside the finite-resolution regime. This means the quasi-local derivative expansion must be completed.

In the spectral/heat-kernel expansion derived earlier [31], the geometric invariants after R appear at four derivatives, so the completion begins with curvature-squared terms. Adopting a minimal isotropic saturation truncation, the scalar-curvature channel R^2 serves as the leading completion of the trace stiffness mode (additional curvature-squared structures represent higher-detail corrections).

Writing the saturated gravity sector as $\frac{M_P^2}{2}R + \lambda R^2$, the coefficient λ measures the vacuum stiffness against scalar curvature [27]. Standard GR lacks an intrinsic geometric cutoff and bottom-up EFT must introduce λ as a phenomenological parameter tuned to fit observational data. Here, because the saturation completion is extracted from the identical quasi-local spectral moments that generate the Einstein term [36], the two coefficients are structurally locked. The ratio λ/M_P^2 is fixed entirely by the same window-function moments that define the c_k coefficients:

$$\lambda = \frac{M_P^2}{12M_s^2}.$$

Cosmological implications of this parameter lock, including the relation to the Starobinsky plateau [38] and early-universe stiffness saturation, are discussed in Appendix A.

Assembling the continuum action requires mapping the extracted stiffness coefficients to their canonical local operators. We now write the three block contributions in canonical normalization and sum them. The tensor block combines the Einstein term $\frac{M_P^2}{2}R$ with the derived R^2 saturation bound. The vector block utilizes the gauge-invariant kinetic term. The scalar block merges the minimal covariant transport operator D_μ and the localized mass gap m to form the standard Dirac Lagrangian. Collecting tensor, vector and scalar contributions gives:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\underbrace{\frac{M_P^2}{2} \left(R + \frac{1}{6M_s^2} R^2 \right)}_{\text{Tensor}} - \underbrace{\frac{1}{4g(M_s)^2} F_{\mu\nu} F^{\mu\nu}}_{\text{Vector}} + \underbrace{\bar{\psi}(i\gamma^\mu D_\mu - m)\psi}_{\text{Scalar}} \right].$$

This matching-scale action is not the universal macroscopic Lagrangian. It sets the matching conditions for the continuum EFT, establishing the transition where discrete dynamics freeze out and continuous evolution takes over. By evaluating the system at M_s , the derived parameters (M_P , g , m) serve as initial conditions, providing matching conditions for standard RG flow rather than introducing them as independent EFT inputs. Higher-derivative and higher-dimension operators are suppressed by the same resolution scale M_s and are organized by the same quasi-local expansion used to extract the coefficients.

2.8. Time Evolution as Open Modular Dynamics

Up to this point, the Hessian expansion describes static restoring forces on a fixed container $(O, \mathfrak{A}_O, \delta)$. Time evolution requires translating this operational window across nested causal diamonds. However, perfectly unitary translation would require infinite precision; under finite resolution, sequential updating is inherently lossy. As the diamond shifts, its tips pinch below the fixed regulator scale δ , continuously leaking degrees of freedom and forcing the operational algebra to split into resolved and sub-resolution sectors [26]:

$$\mathfrak{A}_{\text{full}} \simeq \mathfrak{A}_{\text{res}} \otimes \mathfrak{A}_{\text{sub}}, \quad \rho_{\text{res}} = \text{Tr}_{\text{sub}} \rho_{\text{full}}.$$

Tracing out $\mathfrak{A}_{\text{sub}}$ destroys unitarity. To preserve probability conservation and state positivity, this discrete coarse-graining must act as a Completely Positive Trace-Preserving (CPTP) map Φ . In the adiabatic regime, we approximate these sequential updates by continuous Markovian evolution

in modular time τ via the GKSL (Lindblad) equation [58–60], defining the CPTP semigroup $\mathcal{E}_\tau$ (the continuous approximation to repeated applications of Φ):

$$\frac{d\rho}{d\tau} = -i[K_{\text{mod}}(\tau), \rho] + \sum_{a,\omega} \gamma_a(\omega; \tau) \left(L_{a,\omega} \rho L_{a,\omega}^\dagger - \frac{1}{2} \{L_{a,\omega}^\dagger L_{a,\omega}, \rho\} \right).$$

Here, K_{mod} generates intrinsic modular flow. Proper time t of an external observer is related by the cutoff redshift $t \simeq \delta\tau$ [5]. The dissipator encodes information leakage through the sub-resolution tip sector. We take the jump operators $L_{a,\omega}$ to be modular components of the resolved excitations supported at the tip interface: edge currents and gauge-invariant dressed spinorial excitations. To preserve local KMS structure, we impose modular covariance and detailed balance, while the leakage strength is set by the tip defect parameter κ [M11] (minimal choice identifying the tip geometric bottleneck as the common origin for transport overhead and information leakage):

$$e^{iK_{\text{mod}}s} L_{a,\omega} e^{-iK_{\text{mod}}s} = e^{-i\omega s} L_{a,\omega}, \quad \frac{\gamma_a(-\omega; \tau)}{\gamma_a(\omega; \tau)} = e^{-2\pi\omega}, \quad \gamma_a(\omega; \tau) = \kappa \hat{\gamma}_a(\omega/M_s; \tau).$$

Here, $\hat{\gamma}_a$ suppresses unresolved frequencies $\omega \gtrsim M_s$. The pointer algebra is the fixed-point subalgebra $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_\tau)$. In the low-coherence limit, repeated coarse-graining suppresses non-commuting sectors, so $\mathfrak{A}_{\text{eff}}$ approaches an abelian algebra of stable records, providing an operational route to classical geometry [61].

For compact non-abelian symmetries, because the open time evolution selects a pointer algebra that becomes increasingly commutative as coherence decreases, the surviving accessible data reduces naturally along the Lie-theoretic filtration $G \rightarrow \mathfrak{g} \rightarrow T$ (global group $\rightarrow$ local generators $\rightarrow$ commuting Cartan phases), dictating the generation mass hierarchy (Appendix D).

The same resolution scale inputs that define the static response also control the leakage. The fixed modular window $\varepsilon = 1$ that sets the pixel susceptibility χ_{pix} identically fixes the dissipative cutoff. Likewise, the lattice coordination mismatch parameter κ locks the Hessian response to the GKSL dissipation strength, eliminating arbitrary free parameters.

In the tip regime, local dissipation implies a local power $P_{\text{loc}} \sim \kappa M_s^2 \mathcal{C}$, where $\mathcal{C}$ is a dimensionless, species-weighted active-channel factor. Applying the redshift from the cutoff surface $\sqrt{g_{tt}} \approx \kappa_H / M_s$ yields the asymptotic power:

$$P_\infty \approx P_{\text{loc}} (\sqrt{g_{tt}})^2 \sim \kappa \mathcal{C} M_s^2 \left(\frac{\kappa_H}{M_s} \right)^2 = \kappa \mathcal{C} \kappa_H^2.$$

This scaling $P_\infty \propto \kappa_H^2$ applies to general causal horizons (e.g., Schwarzschild or cosmological). The precise overall normalizations, the recovery of semiclassical Hawking prefactors [40] and the derivation of the vacuum energy fraction [62] are detailed in Appendix A. In both regimes, the cancellation of M_s^2 in P_∞ ensures at this level of the scaling estimate that while finite resolution governs the local physics, it leaves no microscopic signature on large-scale radiation.

In this picture, what we call ‘now’ is the latest stable record produced by a finite-resolution CPTP update, and ‘time passing’ is the irreversible sequence of such updates.

2.9. Canonical Lie Filtration from Open Modular Updating

Finite resolution renders translation between nested operational windows intrinsically lossy. Because the physical Hilbert space does not factorize (P1), a local subsystem is defined by a boundary-completed algebra $\mathfrak{A}_O$ (P3), regulated at fixed resolution $L_s = M_s^{-1}$ (P5). As the diamond translates, its null boundaries shift; near the past and future tips, the transverse area shrinks below L_s , so degrees of freedom that were sharp in one diamond become blurry (sub-resolution) in the next. Maintaining a well-defined description therefore requires tracing out the sub-resolution sector, inducing a Completely Positive Trace-Preserving (CPTP) update map on the resolved state.

In the modular/KMS regime (P4), modular time τ supplies the intrinsic clock and fixes an equilibrium structure (modular covariance and detailed balance) for these updates. We approximate coarse-graining of many resolved updates by a continuous Markovian semigroup, yielding a GKSL (Lindblad) generator $\mathcal{L}$ [58–60]. The structural consequence is that the surviving observables are exactly those in the fixed-point (pointer) algebra $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_\tau)$ [61].

Repeated coarse-graining acts as a dephasing filter: it preserves diagonal (record-like) content while suppressing off-diagonal, phase-sensitive noncommuting correlations. This forces the stable data toward commutativity, selecting the maximal torus T (Cartan) (up to conjugacy); at intermediate coherence only local generator directions remain accessible ($\mathfrak{g}$), while at high coherence the full group structure is accessible (G). As a result, accessibility follows the canonical filtration $G \rightarrow \mathfrak{g} \rightarrow T$. Synthesizing these steps, the axiomatic basis implies:

$$(P1, P3, P5) \Rightarrow \rho_{\text{res}} = \text{Tr } \rho \xrightarrow[\text{Markov}]{\text{CTP}} \mathcal{E}_\tau = e^{\tau \mathcal{L}} \xrightarrow[\text{modular cov.}]{\text{GKSL, KMS, P4}} \mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E}_\tau) \xrightarrow[\text{dephase}]{\text{abelian.}} G \rightarrow \mathfrak{g} \rightarrow T$$

We have shown that, under finite-resolution open modular updating, compact non-abelian symmetry sectors exhibit a canonical three-stage reduction of accessible data $G \rightarrow \mathfrak{g} \rightarrow T$, turning the flavor problem into a symmetry-degradation problem.

2.10. Recovery of Standard Limits and Arrow of Time

Standard limits follow directly by varying the unified matching-scale action. For the gravitational sector, the R^2 completion yields:

$$M_P^2 G_{\mu\nu} + \frac{M_P^2}{6M_S^2} H_{\mu\nu}^{(R^2)} = T_{\mu\nu}^{\text{eff}}, \quad H_{\mu\nu}^{(R^2)} = 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2 - 2\nabla_\mu \nabla_\nu R + 2g_{\mu\nu}\square R.$$

In the Einstein regime $|R| \ll M_S^2$, the saturation correction is heavily suppressed. Stationarity of the variational functional constrains the background to the Einstein universality class [6,54], recovering $M_P^2 G_{\mu\nu} \approx T_{\mu\nu}^{\text{eff}}$, where the stiffness M_P^2 is bounded by the extensive channel capacity N_{eff} [28].

In the matter sector, the spinor-lift grading enforces spinorial matter, while the vector and scalar blocks yield the standard Yang–Mills and Dirac transport equations:

$$(i\gamma^\mu D_\mu - m)\psi = 0, \quad \nabla_\mu F^{\mu\nu} = J^\nu.$$

While deriving the Einstein sector from entropic logic is an established method [6,41], the framework yields Dirac/Yang–Mills kinematics as the minimal covariant structures compatible with the assumed boundary completion, discrete transport and sector decomposition. Standard field equations emerge as the low-density limit.

Finite resolution also supplies a structural basis for the arrow of time. Tracing out $\mathfrak{A}_{\text{sub}}$ induces the CPTP map Φ on ρ_{res} ; physically, this acts as an effective Feynman–Vernon noise kernel ($\text{Im}(I_{\text{eff}}) > 0$) encoding the influence of eliminated subpixel modes [58,63]. Because relative-entropy is contractive, we have:

$$S_{\text{rel}}(\Phi(\rho) \parallel \Phi(\sigma)) \leq S_{\text{rel}}(\rho \parallel \sigma) \Rightarrow \frac{dI}{d\tau} \leq 0.$$

While most standard approaches must postulate an arrow of time, here it follows naturally from the entropic contractivity of finite-capacity boundaries.

3. Numerical Predictions Across Scales

This chapter summarizes the quantitative predictions derived in the Appendices from the core ingredients of Chapters 1 and 2. Specifically, all entries arise from a fixed container $(\mathcal{O}, \mathfrak{A}_O, \delta)$ defined by the relative-entropic variational principle and the Hessian mapping, where the tensor, vector and scalar sectors decouple into a matching-scale response problem.

The objective is to demonstrate the numerical implications of these constraints in a central location. In this framework, the fundamental parameters of nature are not arbitrary coefficients to be fitted; rather, once the effective internal degeneracy $N \approx 1.23 \times 10^{11}$ is fixed by the entropic geometry and the fundamental resolution scale $M_s \approx 3.02 \times 10^{13}$ GeV is anchored to Newton’s constant G , these values emerge as the necessary operating settings of the regulated boundary itself.

While these predictions span disparate domains, we present them as a single summary to emphasize their shared origin. The same architecture constrains inflation, vacuum leakage and the Higgs energy budget. Splitting these results would hide the shared mechanism and make the correlations look coincidental.

3.1. Predictions vs. Observations

Table 1 synthesizes these results, mapping each prediction back to the foundational axioms (P1–P7) and the specific internal variables $(N, \mathcal{N}_*, m_p)$ that drive the numerical output. To ensure an auditable derivation, we distinguish between primary geometric consequences and dependent algebraic relations. Axiom P# references denote the core derivation basis; specific discrete minimality selections and unity normalizations used to close order-one ambiguities are listed at the end of each Appendix.

From one dimensional input $G(N, M_s)$ and seven axioms (Chapter 1), the appendices predict 26 benchmark quantities spanning many orders of magnitude; the intent is to show how strongly the architecture correlates sectors that are usually treated independently.

Classification [X, Y]: X specifies the comparator class (O=Observed/PDG, I=Inferred, S=SM/RGE, B=Bound, H=Heuristic); Y the derivation class (P=Primary, D=Dependent/Algebraic, E=Exploratory).

Table 1. Summary of Predictions vs. Observations.

Quantity	Prediction	Observation / Reference	Deviation	Class	Derivation Basis
Cosmology: Capacity Saturation and Entropic Response (Appendix A)					
Coherence $\mathcal{N}_*$	$18\pi \approx 56.55$	56.98 [64]	0.76%	[I,P]	[P4, P5, P7]
Scalar amplitude A_s	2.08×10^{-9}	2.10×10^{-9} [64]	1.0%	[O,P]	[P4, P5, P7]
Spectral tilt n_s	0.9646	0.9649 ± 0.0042 [64]	0.03%	[O,P]	$\mathcal{N}_*$; [P4, P5, P7]
Tensor ratio r	0.0038	< 0.036 [65]	Consistent	[B,P]	$\mathcal{N}_*$; [P4, P5, P7]
Running α_{run}	-6.3×10^{-4}	-0.0045 ± 0.0067 [64]	Consistent	[O,P]	$\mathcal{N}_*$; [P4, P5, P7]
Scalaron mass M_R	3.02×10^{13} GeV	3.00×10^{13} GeV [64]	0.7%	[I,D]	[P5]
Stiffness λ_{R^2}	5.42×10^8	5.44×10^8 [64]	0.4%	[I,D]	[P5, P7]
Vacuum Ω_Λ	0.667	0.689 ± 0.006 [66]	3.2%	[O,P]	[P3, P5, P7]
Struct. growth S_8	0.816	$0.76\text{--}0.84$ [64,67]	Consistent	[O,E]	[P4, P5, P7]
Accel. floor a_0	1.04×10^{-10}	$\sim 1.2 \times 10^{-10}$ [68]	13%	[H,E]	[P4]
Electroweak Saturation and Mass Generation (Appendix B)					
Higgs mass m_H	125.7 GeV	125.25 ± 0.17 [66]	0.36%	[O,P]	[P3, P5–P7]
VEV v	246.3 GeV	246.22 [66]	0.03%	[I,P]	[P4, P5, P7]
Top mass m_t	174.2 GeV	172.69 ± 0.30 [66]	0.87%	[O,P]	[P3, P5–P7]
Top Yukawa y_t	1.00	0.99 ± 0.01 [66]	1.0%	[I,D]	m_t, v ; [P3–P7]
Higgs quartic λ_H	0.130	0.126 ± 0.001 [66]	3.2%	[I,D]	m_H, v ; [P3–P7]
Gauge Couplings as Entropic Stiffness (Appendix C)					
Fine-structure α_0^{-1}	137.035999216	137.035999206 [69]	$< 10^{-10}$	[O,P]	[P2, P4, P5, P7]
UV EM $\alpha_{\text{em}}^{-1}(M_s)$	113.10	113.4 [70]	0.2%	[I,P]	[P5–P7]
UV Strong $\alpha_s^{-1}(M_s)$	37.7	38 ± 2 [70]	1.0%	[S,P]	[P5–P7]
Coupling Ratio $\mathcal{R}$	3.00	3.03 [70]	1.0%	[S,D]	[P6, P7]
Weak mix $\sin^2 \theta_W$	0.375	0.375 ± 0.005 [66]	Consistent	[S,P]	[P6, P7]
Lepton Mass Spectrum via Spectral Filtration (Appendix D)					
Proton mass m_p	942 MeV	938 MeV [66]	0.4%	[O,P]	[P5, P6]
Ratio m_p/m_e	1836.12	1836.15 [66]	10^{-5}	[O,P]	[P5, P7]
Electron mass m_e	0.511009 MeV	0.510999 MeV [66]	10^{-5}	[O,D]	m_p ; [P5, P7]
Muon mass m_μ	104.25 MeV	105.66 MeV [66]	1.3%	[O,P]	m_p ; [P5, P7]
Tau mass m_τ	1.788 GeV	1.777 GeV [66]	0.6%	[O,P]	[P5–P7]
Tau lifetime τ_τ	3.30×10^{-13} s	2.90×10^{-13} s [66]	14%	[O,D]	[P6]

The results demonstrate a non-trivial convergence with experimental data across vast energy scales. We emphasize that the underlying derivation basis is axiom-locked: the transition from the resolution scale M_s to the observed parameters is a direct consequence of the boundary geometry and the informational response of the causal diamond.

The tagged Minimal Discrete Selections (M#) and Unity Normalizations (U#) in the paper are not fit parameters in the classical sense; they represent discrete structural choices (minimal and natural) or a strict refusal to tune continuous variables (setting $O(1)$ factors to unity). Because these selections simultaneously constrain the tensor, vector and scalar sectors, the framework is radically constrained and rigid.

3.2. Derivation Summary and Insights

The architectural foundation is established in [Chapter 1](#). Rather than tuning independent sectors for inflation, forces and mass, we start from a single operational object: a causal diamond equipped with a boundary-completed algebra (edge completion) to make local subsystems well-defined [\[2,14,15\]](#), a fundamental resolution scale M_s and an effective internal degeneracy N . As such, we did not try to quantize the graviton; instead, we discretize the gravitational stiffness (and thus information capacity) of the boundary [\[6,27\]](#). We determined the constants (N, M_s) in [Chapter 1](#) using established entropic geometry to fix the topology and capacity of the regulated boundary [\[7,10,41\]](#). Those two constants carry most of the load: the fundamental resolution scale M_s sets saturation and activation thresholds, while the effective degeneracy N sets collective averaging and stiffness [\[28\]](#). As a result, the same architecture simultaneously controls inflationary amplitudes, coupling strengths and mass gaps.

The dynamical engine in [Chapter 2](#) relies on a specific identification: we treat the physical action not as a postulate, but as a relative-entropy mismatch functional [\[30,71\]](#). By identifying the Hessian of this relative entropy (Kubo–Mori metric) with the system’s linear response [\[17,44\]](#), we derive the matter sectors from geometry. Operationally, this Hessian naturally organizes into orthogonal tensor, vector and scalar blocks: tensor stiffness fixes M_P , vector stiffness fixes couplings ($1/g^2$) and scalar susceptibility fixes mass gaps. Finite resolution also makes the modular update an open-system coarse-graining [\[58,60\]](#), so fixed-point structure and spectral counting become physical properties rather than artifacts. The heat-kernel expansion then provides the map from these response kernels to effective parameters at the matching scale [\[31\]](#).

Cosmology: Capacity Saturation and Entropic Response (Appendix A)

In the ultraviolet, finite resolution makes high curvature a saturation problem: as $|R| \rightarrow M_s^2$, the response cannot grow indefinitely, and the effective dynamics falls into the plateau universality class. In the minimal saturation identification used in [Appendix A](#), the R^2 mode plays the role of the plateau degree of freedom as curvature approaches the finite-resolution ceiling, with M_s setting the scalaron scale. The scalar amplitude becomes a capacity diagnostic: the curvature readout averages over N parallel channels, so the variance scales as $1/N$, turning $A_s \sim 10^{-9}$ into a channel-count statistic. The resulting plateau relations then fix $(n_s, r, \alpha_{\text{run}})$ once $\mathcal{N}_*$ is set by a coherence budget tied to the discrete transport structure.

In the deep infrared, the same open-system machinery is used: leakage redshifts to an H^2 channel. This reflects the redshift-cancellation mechanism: the microscopic scale drops out, leaving only a dimensionless utilization ratio (active channels vs. spatial transport axes), so the vacuum fraction reduces (in the minimal closure) to $4/6$. Late-time growth suppression and acceleration floor are treated as coherence and noise limits of the same boundary, with closures stated transparently.

Electroweak Saturation and Mass Generation (Appendix B)

The key split is extensive versus intensive response. The Planck scale reflects the collective stiffness of many channels in parallel ($M_P^2 \propto N_{\text{eff}} M_s^2$); the electroweak scale reflects the intensive per-pixel energy budget $E_{\text{pix}} \sim M_s/N$. Here, the Higgs represents the minimal boundary structure needed to define and normalize the $SU(2)$ charge basis. In the minimal principal-branch estimate of [Appendix B](#), the top mass corresponds to the first fermionic excitation saturating a single-pixel unitary step once gauge-invariant dressing is imposed. This yields a compact set of matching-scale locks (m_H, v, m_t) and derived relations such as $y_t \approx 1$ and λ in the observed range, without introducing new mass parameters at the fundamental level. This recasts the hierarchy problem in geometric terms:

gravity is weak because it is extensive (shared by N channels), while electroweak thresholds are intensive (per pixel).

Gauge Couplings as Entropic Stiffness (Appendix C)

Gauge couplings are read as vector-sector stiffness: the quadratic relative-entropy cost of a background connection on the fixed algebra. Two ingredients make this rigid. First, the finite modular history fixes the integration window. Concretely, the universal modular kernel integrated over the resolved history $[\varepsilon, 2\pi - \varepsilon]$ produces a fixed susceptibility factor (with $\varepsilon = 1$ set by the fixed resolution), so the result is a protocol feature of the boundary rather than a regulator choice. Second, topological quantization (Axiom P6) replaces continuous normalization freedom with integer levels k . Once the continuum normalization is fixed, microscopic details collapse into integer data (levels and inventory counts), which is why coupling ratios become discrete locks rather than adjustable parameters. The same spectral heat-kernel machinery used for EFT coefficients then provides a global static estimate of $\alpha^{-1}(0)$ as a spectral sum on the fixed history manifold.

Lepton Mass Spectrum via Spectral Filtration (Appendix D)

Generations arise as stable accessibility limits under finite-coherence coarse-graining. Given Axioms P3 (boundary completion), P4 (modular/KMS flow) and P5 (finite resolution), the translation between nested diamonds is implemented as an open modular update (GKSL semigroup), so the relevant operational content is restricted to the fixed-point algebra $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E})$. As coherence decreases, this fixed-point algebra abelianizes (pointer limit). For a non-abelian symmetry sector, this irreversible loss of noncommuting structure reduces the accessible data strictly along the standard Lie-theoretic sequence: global group structure $G \rightarrow$ local generators $\mathfrak{g} \rightarrow$ maximal torus T . In the global limit (G), accessibility reduces to a volume count on the minimal internal closure, yielding the $6\pi^5$ benchmark ratio; in the tangent limit ($\mathfrak{g}$), it reduces to generator content; in the commuting limit (T), it reduces to Cartan phase volume modulo the axis-sign redundancy of the discrete transport. Where an empirical anchor is used (notably m_p), it is marked explicitly and used to precision-test a dimensionless geometric ratio rather than to tune the architecture.

Unification across Scales

The framework unifies scales by tying gravity, gauge couplings and mass gaps to a single finite-resolution boundary on a causal diamond. Relative-entropy variations decouple into tensor, vector, and scalar response channels, allowing cosmology, electroweak, gauge, and Lepton spectra to emerge as correlated susceptibilities. With N fixed by boundary topology and the matching scale M_s anchored to G , these sectors are no longer independent but represent the linked consequences of a single informational architecture.

4. Conclusions and Outlook

This paper proposes a finite-resolution boundary architecture as the minimal operational structure needed to define local physics in a diffeomorphism-invariant quantum theory. We work in a Wheeler–DeWitt setting ($\hat{H}\Psi = 0$) where the global state is constrained rather than time-evolved. In such a framework, the central operational problem is not to quantize the graviton, but to make a local subsystem well-defined when Hilbert spaces do not factorize cleanly across subregions.

4.1. Architectural Foundation: Discretizing Capacity

The architectural foundation is established in [Chapter 1](#). Rather than tuning independent sectors for inflation, forces and mass, we start from a single operational object: a causal diamond equipped with a boundary-completed algebra to make local subsystems well-defined.

The fundamental discretization in this framework is not a lattice of spacetime points. It is a count of effective internal degrees of freedom carried by the regulated boundary. By discretizing the

gravitational stiffness (and thus information capacity) rather than the coordinates, the gravitational variational principle is rendered finite and well-defined.

The key derived relation is the effective internal degeneracy:

$$N = \sqrt{2} \exp\left(8\pi + \frac{1}{6\pi}\right) \approx 1.23 \times 10^{11}$$

Concretely, N is the effective number of independent internal degrees of freedom available per geometric pixel to support the boundary data that keeps the vacuum geometry stable. The constitutive relation connects the macroscopic stiffness M_P to the fundamental resolution scale M_s via this effective count:

$$M_P^2 = N_{\text{eff}} M_s^2 = (\kappa N) M_s^2$$

This fixes the fundamental resolution scale of the boundary to:

$$M_s = \frac{M_P}{\sqrt{\kappa N}} \approx 3.02 \times 10^{13} \text{ GeV}$$

This result encodes a clear interpretation: M_s is the fundamental resolution cutoff per degree of freedom (sharp focus), while M_P is the collective stiffness enhanced by parallelization across N_{eff} degrees of freedom. Gravity is extensive in this internal degeneracy; gauge response is intensive and does not scale with N . This recasts the hierarchy problem in geometric terms: gravity is weak because it is extensive (diluted over N degrees of freedom), while electroweak thresholds are intensive (set by the single-pixel energy budget E_{pix}).

4.2. Axiomatic Basis

The framework is built from seven geometric axioms that formalize what is needed for local physics in constrained gravity: non-factorization, operational locality on causal diamonds, edge completion, modularity (KMS), finite capacity, gauge topology (quantized edge structure) and discrete isotropy (minimal transport). In this context, these are not ad hoc additions; they are the minimal operational requirements to define subsystems, clocks, and regulated observables without having to assume a classical background structure.

4.3. Dynamical Engine and Hessian Response

The dynamical engine in [Chapter 2](#) relies on a specific identification: we treat the physical action not as a postulate, but as a relative-entropy mismatch functional. By identifying the Hessian of this relative entropy (Kubo–Mori metric) with the system’s linear response, we derive the matter sectors directly from geometry.

Operationally, this Hessian naturally organizes into orthogonal blocks: tensor stiffness fixes M_P , vector stiffness fixes couplings ($1/g^2$), and scalar susceptibility fixes mass gaps. Furthermore, finite resolution makes the modular update an open-system coarse-graining described by GKSL (Lindblad) evolution. This means that fixed-point structure and spectral counting become operational features of the finite-resolution update rather than purely formal regulator bookkeeping. This dissipative mechanism provides a concrete leakage channel in the deep IR, with the tip defect parameter setting the structural scale of dissipation utilized in [Appendix A](#).

4.4. Cross-Sector Locking and Model Rigidity

Once Newton’s constant G fixes the overall stiffness scale, the remaining outputs are determined by the discrete boundary structure and regulator closures. [Chapter 3](#) summarizes 26 benchmark quantities spanning cosmology ([Appendix A](#)), electroweak ([Appendix B](#)), couplings ([Appendix C](#)) and masses ([Appendix D](#)).

For each target quantity, Table 1 presents the derived prediction alongside its empirical comparator, detailing the relative deviation, the derivation class, and the specific Axioms (P1–P7) and minimal closures used. Furthermore, each appendix concludes with a comprehensive Summary of Predictions (inputs, minimality assumptions, outputs, falsifiers) to ensure transparency and strictly separate explicit modeling choices from structural requirements.

A useful way to express this rigidity is to adopt resolution units by setting $M_s = 1$. In these units, the parametric freedom of the Standard Model at the matching scale largely collapses. Subject to the enumerated minimal closures, the couplings, mass ratios and cosmological figures reduce to analytic functions of discrete topological data, geometric capacities and Lie-theoretic invariants. Conventional units are then recovered solely through the dimensional conversion set by G . This represents a fundamental structural shift: the matching-scale parameters cease to be independent empirical inputs and become tightly constrained consequences of the finite-resolution architecture, leaving no free continuous parameters to be fitted beyond the stated minimal closures.

At low curvature, the extensive aggregate of degrees of freedom yields Einstein stiffness. At intermediate densities, the intensive pixel budget partitions into electroweak thresholds. Near the finite-resolution ceiling, stiffness saturates and produces plateau inflation.

In this interpretation, the scalar amplitude can be read as a capacity diagnostic: the curvature readout averages over N parallel degrees of freedom, so the variance scales as $1/N$, turning the observed amplitude ($A_s \sim 10^{-9}$) into a statistical measure of the degeneracy count. In the lepton model of Appendix D, generations correspond to accessibility limits under coarse-graining: the stable fixed-point algebra abelianizes as coherence shrinks, effectively guiding the accessible structure down the group-theoretical Lie filtration $G \rightarrow \mathfrak{g} \rightarrow T$.

4.5. Outlook

This work builds on entropic routes to gravity, edge-mode treatments of local algebras, and spectral methods for organizing the effective action. Our contribution is to treat the boundary architecture as fixed upstream and to use a single relative-entropy variational protocol to link tensor, vector and scalar responses across scales. The result is a unified set of correlated outputs under the stated axioms and minimal closures, rather than independent phenomenological inputs for each domain.

The resulting physics is not a piecemeal assembly of separate models, but a constrained architecture in which gravity, gauge response and mass are tied together at the fundamental resolution scale M_s . In this sense, the observed scales of nature appear less as unrelated constants and more as stable, self-consistent limits of what a finite-resolution quantum theory can represent.

Appendix A. Cosmology: Capacity Saturation and Entropic Response

This appendix derives UV and IR cosmological constraints from the finite-resolution boundary response set by (N, κ, M_s) and the fixed architecture.

In the UV, geometric stiffness saturates the information capacity (inflationary plateau); in the deep IR, the finite horizon surface enforces maintenance costs (vacuum fraction) and coherence limits (growth suppression and acceleration floor). Inputs such as H_0 and T_{CMB} are treated as boundary conditions of the local patch, while the functional forms and dimensionless locks are traced to the fixed boundary architecture (N, κ, M_s) and the dynamical engine of Chapter 2. Whenever the architecture fixes a quantity only up to an $\mathcal{O}(1)$ factor, we adopt the unity normalization (set the factor to 1) and tag it [U#]. We quote Planck 2018 (PR3) [64] values for continuity.

Appendix A.1. Geometric Stiffness and Activation (UV Saturation)

In Chapter 2, we showed that the regulated boundary remains weakly coupled at intermediate scales ($|R| \ll M_s^2$), naturally reproducing General Relativity at low curvature where discrete transport overhead is negligible. However, finite resolution (Axiom P5) imposes a hard limit on both geometry and the effective field theory. As the curvature scale approaches the pixel resolution ($|R| \rightarrow M_s^2$), finite

resolution bounds the growth of the response; the leading curvature-squared truncation is captured by an $R + R^2$ effective action, which places the dynamics in the Starobinsky plateau universality class [38]. The effective action in the saturation regime is:

$$S_{\text{eff}} \approx \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \lambda_{R^2} R^2 \right].$$

The standard plateau dictionary defines the mass scale M_R via $\lambda_{R^2} \equiv M_P^2 / (12M_R^2)$. We identify the onset of stiffness saturation with the resolution scale M_s up to an order-unity factor ξ : $M_R \equiv \xi M_s$. We adopt the no-hierarchy unity choice $\xi = 1$ [U2].

$$M_R = M_s \simeq 3.02 \times 10^{13} \text{ GeV}$$

(CMB-inferred (Starobinsky): $3.00 \times 10^{13} \text{ GeV}$ [64]; Relative Deviation: 0.7%)

Using $M_P^2 = N_{\text{eff}} M_s^2 = \kappa N M_s^2$ with $\kappa = 1/(6\pi)$ and the effective internal degeneracy N , the R^2 stiffness coefficient becomes:

$$\lambda_{R^2} = \frac{M_P^2}{12M_R^2} = \frac{\kappa N}{12\xi^2} \xrightarrow{\xi=1} \frac{\kappa N}{12} \approx 5.42 \times 10^8$$

(CMB-inferred (Starobinsky): 5.44×10^8 [64]; Relative Deviation: 0.4%)

Appendix A.2. Coherence Duration ($\mathcal{N}_*$)

We determine the duration of the inflationary phase by analyzing the stability of the resolved sector against discrete transport overhead. The minimal discrete transport structure (Axiom P7) induces an irreducible mismatch κ per resolved modular update $\Delta\tau = 1$ (Axiom P5). With two entangled tips, coherence is sustained until the accumulated mismatch reaches a distinguishability threshold c_{exit} along each of the $D = 3$ spatial transport axes:

$$m_* \approx \frac{2Dc_{\text{exit}}}{\kappa}.$$

To convert to e-folds, we use the standard plateau slow-roll relation $H \simeq M_R/2$ and $t \simeq \delta\tau$ at the cutoff surface, so one modular step increments $\Delta\mathcal{N} = H\Delta t \approx (M_R/2)(\delta\Delta\tau) \approx \xi/2$ (with $\xi = 1$ as fixed above). We set the distinguishability threshold $c_{\text{exit}} = 1$ as the minimal operational definition of decoherence [U3].

$$\mathcal{N}_* \approx m_* \Delta\mathcal{N} \approx \frac{\xi c_{\text{exit}} D}{\kappa} = \frac{D}{\kappa} = 18\pi \approx 56.55$$

(Observed: ≈ 56.98 [64] (n_s -inferred); Relative Deviation: 0.8%)

Appendix A.3. Primordial Observables (A_s, n_s, r, α_s)

The primordial amplitude is a capacity diagnostic: the coarse-grained curvature readout averages over N parallel channels, so we model the variance as scaling as $1/N$ [M12]. Substituting the boundary parameters into the standard plateau amplitude:

$$A_s = \frac{M_R^2 \mathcal{N}_*^2}{24\pi^2 M_P^2} \approx \frac{\xi^4 c_{\text{exit}}^2 D^2}{24\pi^2 \kappa^3 N} = \frac{81\pi}{N} \simeq 2.08 \times 10^{-9}$$

(CMB-inferred: $(2.10 \pm 0.03) \times 10^{-9}$ [64]; Relative Deviation: 1.0%)

With $\mathcal{N}_* \approx 56.55$, the standard plateau consistency relations yield the following spectral parameters (spectral index n_s , tensor-to-scalar ratio r , running of spectral index α_s):

$$n_s \approx 1 - \frac{2}{\mathcal{N}_*} = 1 - \frac{1}{9\pi} \approx 0.9646$$

(Observed: 0.9649 ± 0.0042 [64]; Relative Deviation: 0.03%)

$$r \approx \frac{12}{\mathcal{N}_*^2} \approx 0.0038$$

(Constraint: $r < 0.036$ [72]; Consistent)

$$\alpha_s \approx -\frac{2}{\mathcal{N}_*^2} \approx -6.3 \times 10^{-4}$$

(Constraint: $\alpha_s = -0.0045 \pm 0.0067$ [64]; Consistent)

Appendix A.4. Cosmological Constant as Entropic Horizon Leakage

In the deep IR, the boundary does not saturate; instead, the horizon surface becomes informationally sparse [2]. We derive the cosmological constant not as a bulk vacuum energy summation (which diverges), but as the steady-state horizon leakage cost required to balance the open modular update (Chapter 2) [62,73].

For a flat universe, the critical density is $\rho_{\text{crit}} = 3M_P^2 H^2$. As derived in Chapter 2, the local GKSL dissipation rate [58,60] redshifts to a macroscopic power $P_\infty \propto \kappa C H^2$, and M_s cancels at the level of the scaling estimate. Balancing this continuous leakage over a Hubble 4-volume implies an effective vacuum density $\rho_\Lambda \propto H^2$. Since $\rho_{\text{crit}} \propto H^2$, the vacuum fraction Ω_Λ reduces to a dimensionless coefficient fixed by the minimal discrete-transport topology:

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_{\text{crit}}} = L \times \left(\frac{\mathcal{C}_0}{z} \right)$$

We adopt the unity leakage coefficient $L = 1$ [U4]. The six orthogonal directions $\{\pm x, \pm y, \pm z\}$ of the minimal discrete transport structure (Axiom P7) set the number of independent leakage channels $z = 6$. In the deep IR, only massless long-range fields remain actively supported: the photon (2 polarizations) and the graviton (2 polarizations), yielding an active-channel factor $\mathcal{C}_0 = 4$ [M13].

Evaluating this at the current epoch yields the predicted vacuum fraction:

$$\Omega_{\Lambda,0}^{\text{pred}} = \frac{4}{6} \approx 0.667$$

(Observed: 0.689 ± 0.006 [66]; Relative Deviation: 3.2%)

Because additional massless long-range species would strictly increase $\mathcal{C}_0$ and shift the predicted band upward, this is strictly falsifiable.

At earlier epochs, the active-channel factor becomes horizon-scale dependent, $\mathcal{C}(T_H)$, because species with $m \lesssim T_H$ (equivalently $m \lesssim H/2\pi$) can contribute to the dissipative inventory, while heavier species are suppressed. A minimal interpolation replaces $\mathcal{C}_0$ with a thresholded sum:

$$\Omega_\Lambda = L \frac{\mathcal{C}(T_H)}{z} \quad (z = 6) \quad \mathcal{C}(T_H) = \sum_i g_i w\left(\frac{m_i}{T_H}\right),$$

where g_i counts helicity/polarization states and $w(x)$ is a smooth threshold with $w \rightarrow 1$ for $x \ll 1$ and $w \rightarrow 0$ for $x \gg 1$.

The vacuum fraction Ω_Λ departs from $4/6$ only if additional species become effectively massless on the horizon scale. Since $T_H \sim H/2\pi$ remains negligible compared to SM masses throughout the post-recombination epoch, $\mathcal{C}(T_H) \approx \mathcal{C}_0$ strictly holds in the late universe. Any detectable deviation in this regime would point to new ultra-light degrees of freedom.

Appendix A.5. IR Regime: Horizon Coherence and S_8 (Exploratory)

The following late-time dynamic effects (S_8, a_0) are classified as exploratory. We model late-time growth phenomenology as a coherence impedance effect to test if the channel multiplicity (N) remains the correct measure of degrees of freedom in the IR.

As the causal patch grows toward the deep IR, the discrete transport on the boundary becomes susceptible to entropic drift. We define the effective clustering strength via the tensor-sector Kubo–Mori stiffness $\mathcal{K}(O_a)$ of a horizon-sized diamond O_a :

$$f(a) \equiv \frac{G_{\text{eff}}(a)}{G_N} = \frac{\mathcal{K}(O_0)}{\mathcal{K}(O_a)}.$$

The system maintains GR-like stiffness ($f \approx 1$) only while the boundary transport remains coherent. We define the onset of decoherence a_* by the standard information-theoretic limit of distinguishability: the suppression activates when the accumulated relative-entropy drift over one Hubble time reaches the distinguishability threshold $\Delta S(a_*) \approx 1$ [U5].

Beyond this threshold, the update map becomes lossy ($f(a) < 1$). This suppression satisfies relative-entropic contractivity, consistent with monotonic information loss along RG flow. We adopt the minimal dimensional dilution (inverse spacetime dimension $d = 4$) $v_{\text{eff}} = 1/d = 1/4$ [M14]:

$$N_{\text{part}}(a) \propto \left(\frac{H(a)}{H(a_*)} \right)^{v_{\text{eff}}}.$$

This suppression appears as a finite-resolution effect in the S_8 parameter. Using the standard Λ CDM growth index approximation Ω_m^γ with $\gamma \approx 0.55$:

$$S_8^{\text{pred}} \approx S_8^{\text{Planck}} \times f(1)^{0.55} \approx 0.816$$

(Observed: 0.76–0.84 [64,67]; Consistent)

Appendix A.6. Galactic Regime: Acceleration Floor (a_0) (Exploratory)

Axiom P4 implies a thermal horizon background $T_H \simeq \hbar H / 2\pi k_B$ [6,74,75]. An inertial acceleration signal a becomes indistinguishable from the horizon noise when its Unruh temperature T_U [5] drops below T_H . The crossover $T_U \sim T_H$ fixes an acceleration floor: $a_0(z) \propto cH(z)$. Unlike static modifications of gravity, this floor is dynamic and evolves with redshift. Using the shared 2π KMS normalization:

$$a_0 = \frac{1}{2\pi} cH_0 \approx 1.04 \times 10^{-10} \text{ m s}^{-2}$$

(Observed: $\approx 1.2 \times 10^{-10} \text{ m s}^{-2}$ [68,76]; Consistent)

Appendix A.7. Summary of Predictions

Inputs: (N, κ, M_s) with $N \approx 1.2 \times 10^{11}$; $\kappa = 1/(6\pi)$; $M_s \approx 3 \times 10^{13} \text{ GeV}$.

Discrete Minimality Selections: [M12] variance $\propto 1/N$; [M13] $\mathcal{C}_0 = 4$ (photon+graviton only); [M14] $v_{\text{eff}} = 1/4$.

Unity Normalizations: [U2] $\xi = 1$; [U3] $c_{\text{exit}} = 1$; [U4] $L = 1$; [U5] $\Delta S \approx 1$.

Outputs: Inflationary plateau ($n_s \simeq 0.965, r \simeq 0.004, \alpha_s \simeq -6 \times 10^{-4}, A_s \simeq 2.1 \times 10^{-9}, \mathcal{N}_* \simeq 56.6$); stiffness ($\lambda_{R^2} \simeq 5.4 \times 10^8$); vacuum fraction ($\Omega_{\Lambda,0} \simeq 0.67$); growth suppression ($S_8 \simeq 0.82$); acceleration floor ($a_0 \simeq 1.0 \times 10^{-10}$).

Falsifiers: Violation of plateau consistency (e.g. r well below 10^{-3}); vacuum fraction significantly outside the 4/6 topological band; absence of late-time growth suppression ($S_8 \approx S_8^{\text{Planck}}$); redshift-independent acceleration floor ($a_0 \not\propto H(z)$).

Appendix B. Electroweak Saturation and Mass Generation

This appendix demonstrates that the mass scales of the electroweak sector (m_H, v, m_t) are constrained by finite-resolution boundary physics: structural entropic cost (Higgs), phase-noise floor (VEV), and principal-branch phase saturation (top quark), defined at the boundary matching scale E_{pix} . To compare our bare parameters with experimental data, we use standard SM scheme conversions [66].

Appendix B.1. Per-Pixel Energy Budget (E_{pix})

In the modular/KMS regime (Axiom P4), local dynamics are controlled by linear response via the Kubo–Mori metric [17]. The physical resolution $\delta = M_s^{-1}$ (Axiom P5) sets the shortest modular update $\varepsilon = 1$, fixing the resolution scale M_s of the boundary algebra (Axiom P3).

A pixel carries an effective internal degeneracy N (Chapter 1). Axiom P7 adds a $\mathbb{Z}_2$ spinor-lift overhead with quantum dimension $d_\sigma = \sqrt{2}$. This supplies spin structure but is not an independent dynamical channel; the addressable internal degrees of freedom count is $n_{\text{ch}} = N/\sqrt{2}$. In linear response, stiffness is additive. With no preferred channel, the per-pixel energy budget E_{pix} is defined by the minimal equipartition assumption [M15] as the energy scale per addressable channel per resolved modular update:

$$E_{\text{pix}} \equiv \frac{M_s}{n_{\text{ch}}} = \frac{\sqrt{2}M_s}{N} \approx 348.4 \text{ GeV}$$

Heavier localized excitations would require multi-pixel encoding. E_{pix} sets the single-pixel activation scale used below.

Appendix B.2. Higgs Boson: Structural Entropic Cost

The Higgs field sets the scale (metric) of the internal charge space. While gauge freedom removes three Goldstone modes at long distances, the regulated boundary description must represent the full $SU(2)$ doublet (four real components) to define the vacuum state and normalize gauge charges (Axioms P3, P6).

We define the Higgs mass m_H as the minimal entropic cost to sustain this 4-component vacuum structure ($N_H = 4$) against modular noise. A bit cost of $\ln 2$ represents one resolved unit of distinguishability per update; this is consistent with the overhead fixed in Chapter 1 [M9] and the $\{0, 1\}$ occupancy used for defect modes (Chapter 2). We adopt the one-bit increment per Higgs component as a unity choice [U6]. This yields a total structural entropy $\Delta S_H = 4 \ln 2$, corresponding to the minimal possible quantization of one bit per component (larger cost would only lower m_H).

Following a minimal Landauer-type estimate [M16] (where the thermal noise kT is replaced by the per-pixel energy budget E_{pix}), the mass corresponds to the energy required to maintain this boundary structure (monotone in ΔS_H):

$$m_H = \frac{E_{\text{pix}}}{\Delta S_H} = \frac{E_{\text{pix}}}{4 \ln 2} \approx 125.7 \text{ GeV}$$

(Measured: $125.25 \pm 0.17 \text{ GeV}$ [64]; Relative Deviation: 0.36%).

Appendix B.3. Vacuum Expectation Value: Noise Floor

The Vacuum Expectation Value (VEV) v represents the magnitude of the Higgs condensate (the radial order parameter). At resolution δ , the system cannot stably lock the absolute phase against an external reference (Axioms P4, P5), making the phase effectively random (maximum entropy) [43].

As a result, interference cancels the linear average, leaving the root-mean-square fluctuation as the stable observable. The natural per-update amplitude scale is E_{pix} . We assume the phase is uniformly distributed $\phi \in [0, 2\pi)$ [M17]:

$$v^2 = \langle (E_{\text{pix}} \cos \phi)^2 \rangle_\phi = \frac{1}{2} E_{\text{pix}}^2 \quad \rightarrow \quad v = \frac{E_{\text{pix}}}{\sqrt{2}} \approx 246.3 \text{ GeV}$$

(Inferred from G_F : 246.22 GeV [66]; Relative Deviation: 0.03%).

Appendix B.4. Top Quark: Phase Saturation Bound

Fermions cannot exist as isolated local observables (Axioms P1, P3) [14]; with edge completion, the minimal local excitation is a gauge-invariant dressed dipole. Finite resolution (Axiom P5) sets a minimal modular update scale $\varepsilon = 1$, so that local evolution is implemented on the pixel algebra by a discrete unitary step operator U_{pix} . The corresponding eigenvalues are pure phases $e^{i\theta}$; to avoid principal-branch ambiguity, we take the resolved single-step phase to lie in the principal domain $\theta \in (-\pi, \pi]$.

With the per-pixel energy budget E_{pix} representing the energy scale of this update, a mass gap m_f corresponds to a phase rotation $\theta \approx m_f / E_{\text{pix}}$ per step (analogous to discrete Dirac-walk angles [77]). Representability of the full dipole (pair energy $2m_f$) requires the total rotation angle to fit within the principal domain. We use a principal-branch saturation criterion for the dressed dipole [M18], with minimal dressing $c_{\text{dress}} = 1$ as fixed in Chapter 2 [U1]:

$$2m_f c_{\text{dress}} \lesssim E_{\text{pix}}$$

$$m_t \approx \frac{1}{2} E_{\text{pix}} \approx 174.2 \text{ GeV}$$

(PDG reference value: 172.69 ± 0.30 GeV [66]; Relative Deviation: 0.87%).

A sub-percent dressing overhead is expected for a gauge-invariant excitation that cannot sit exactly on the principal-branch boundary.

Appendix B.5. Structural Locks and Vacuum Stability

The predictions for m_t and m_H carry intrinsic uncertainties from scheme conversion and finite width (~ 1 GeV for top, ~ 0.2 GeV for Higgs) [78]. Within these margins, our values are consistent with experiment. Physically, the vacuum does not select these masses; it simply bounds any excitation that attempts to exceed the per-pixel capacity of the regulated boundary (E_{pix}). This saturation condition fixes the dimensionless couplings (specifically the top Yukawa coupling y_t and the Higgs self-coupling λ) as geometric ratios:

$$y_t \equiv \frac{\sqrt{2} m_t}{v} \approx 1$$

(Inferred: 0.99 ± 0.01 [66]; Relative Deviation: $\sim 1\%$)

$$\lambda \equiv \frac{m_H^2}{2v^2} = \frac{1}{16(\ln 2)^2} \approx 0.130$$

(Inferred: 0.126 ± 0.001 [66]; Relative Deviation: 3.2%)

This value places the vacuum right on the edge of stability, consistent with standard RG analyses where λ runs toward a near-critical small value in the 10^{11} – 10^{13} GeV range [79] (within current parameter uncertainties). This provides the matching-scale boundary conditions for the Standard Model inputs, which then evolve according to standard renormalization flows.

Appendix B.6. Summary of Predictions

Inputs: Resolution scale M_s ; effective internal degeneracy N ; spinor overhead $d_\sigma = \sqrt{2}$ (Axiom P7) $\rightarrow E_{\text{pix}} = \sqrt{2} M_s / N$.

Discrete Minimality Selections: [M15] equipartition of pixel energy; [M16] Landauer-type mapping with E_{pix} ; [M17] uniform phase $\phi \in [0, 2\pi)$; [M18] principal-branch saturation criterion.

Unity Normalizations: [U1] minimal dressing $c_{\text{dress}} = 1$; [U6] one-bit distinguishability increment per Higgs component (hence $\Delta S_H = 4 \ln 2$; consistent with the half-bit twist overhead [M9]).

Outputs: $m_H \approx 125.7$ GeV; $v \approx 246.3$ GeV; $m_t \approx 174.2$ GeV; derived locks $y_t \approx 1$; $\lambda \approx 0.130$ at the matching scale (vacuum metastability).

Falsifier: Persistent violation of these relations after standard scheme conversion/RG evolution; discovery of a fermion heavier than the top; measured Higgs quartic incompatible with the predicted boundary condition.

Appendix C. Gauge Couplings as Entropic Stiffness

This appendix derives SM gauge couplings as response properties of the boundary state, not as free parameters. We define a gauge coupling by the entropic stiffness of the variational functional: the Kubo–Mori (quantum Fisher) response of the vacuum reference state [17] to a background gauge deformation in the modular/KMS regime (Axiom P4) [4,46], evaluated on the fixed boundary algebra (Axiom P3) at finite resolution (Axiom P5) [14,16]. Topological quantization of the edge current algebra fixes an integer level k (Axiom P6) [21], while the conventional 4π factor enters through flux normalization on the diamond waist S^2 (bifurcation surface defined in Axiom P2).

Gauge couplings follow from the vector (current) sector of the Kubo–Mori Hessian, while masses follow from the scalar (density) sector (Appendix D).

Appendix C.1. Entropic Stiffness Quantization

We first establish the quantization rule that turns the vector-sector response into a definite gauge coupling at the matching scale. In Chapter 2, we fixed the measurement protocol: the coupling is read off from the quadratic relative-entropy cost of a weak background gauge deformation in the KMS regime matched to the canonical normalization of $-\frac{1}{4}F^2$.

As explained in Chapter 2, microscopically, the stiffness arises from integrating the universal modular noise kernel $\langle \mathcal{J}(\tau)\mathcal{J}(0) \rangle \propto \sin^{-2}(\tau/2)$ over the resolved modular history $[\varepsilon, 2\pi - \varepsilon]$, yielding the susceptibility factor $I_{\text{mod}} = 4 \cot(\varepsilon/2)$. Using this protocol with the fixed modular window $\varepsilon = 1$ (Axiom P5) and integer edge level k (Axiom P6), we obtain the master relation:

$$\alpha^{-1}(M_s) = 4\pi k.$$

This relation acts as a dictionary between the discrete edge currents and the continuous field. The factor 4π is not a prediction of the boundary geometry, but simply the standard geometric conversion factor for flux through a sphere ($\int_{S^2} d\Omega = 4\pi$). By fixing the continuum definition to the standard form $\alpha \equiv g^2/4\pi$ and the canonical normalization $-\frac{1}{4}F^2$, we absorb the remaining normalization into the integer level k of the matching protocol. Thus 4π fixes the continuum flux convention at the S^2 interface, while k encodes the matched normalization of the edge current algebra at M_s . Any additional order-one geometric factor from discretization would simply rescale the current and is absorbed into k by definition.

This holds at leading derivative order in the regime $\delta \ll \ell_O \ll R_{\text{curv}}$ and $|p| \ll M_s$. For an Abelian factor, independent charged sectors contribute additively to the quadratic cost, so the effective level k is an additive inventory over the charged sectors active at M_s .

Appendix C.2. Strong Sector Prediction ($k = 3$)

For the strong interaction with gauge group $SU(3)_c$, the problem is reduced to fixing one integer:

$$\alpha_s^{-1}(M_s) = 4\pi k_s$$

Non-Abelian gauge fields carry charge. Physically, the vacuum stiffness is dominated by self-interaction (anti-screening) [80], rather than the sum over matter charges characteristic of the Abelian sector. The edge current algebra must therefore be normalized by an intrinsic property of the $SU(3)$ current algebra itself. We identify the stiffness level k_s with the canonical integer that measures the

adjoint self-interaction normalization, the dual Coxeter number $h^\vee$ ($k_s = h^\vee$) [81] [M19]. For $SU(N)$, $h^\vee = N$. For $SU(3)_c$ we have $k_s = 3$ and the matching-scale coupling follows immediately:

$$\alpha_s^{-1}(M_s) = 4\pi \cdot 3 = 12\pi \simeq 37.7$$

(SM-inferred at M_s : $\alpha_s^{-1} \simeq 38 \pm 2$ [66,70] + running; Relative Deviation: $\sim 1\%$)

Appendix C.3. Electromagnetic Prediction ($k = 9$) and Consistency Check

We now apply the stiffness quantization $\alpha^{-1}(M_s) = 4\pi k$ to the electromagnetic sector. Stiffness is driven here by matter screening ($\sum Q^2$) of all long-range charged sectors active at M_s . Summing the boundary inventory fixed in Chapter 1 (3 quark/lepton generations + 1 Higgs channel):

Quarks: 3 generations $\times$ 3 colors (u, d) $\implies 9 \times [(2/3)^2 + (-1/3)^2] = 5$

Leptons: 3 generations (e, ν) $\implies 3 \times [(-1)^2 + 0^2] = 3$

Higgs: One complex charged component in the $SU(2)$ doublet $\implies 1 \times 1^2 = 1$

The total electromagnetic stiffness is the integer sum $k_{\text{em}} = 5 + 3 + 1 = 9$. Substituting this into the master formula:

$$\alpha_{\text{em}}^{-1}(M_s) = 4\pi \times 9 = 36\pi \simeq 113.10$$

We compare this geometric prediction against SM couplings extrapolated to the matching scale $M_s \simeq 3 \times 10^{13}$ GeV. Using the unbroken-basis identity with 1-loop SM values ($\alpha_1^{-1} \approx 42.6$, $\alpha_2^{-1} \approx 42.4$) [70]:

$$\alpha_{\text{SM}}^{-1}(M_s) = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1} \approx 113.4$$

(Prediction: 113.10; Relative Deviation: $\sim 0.2\%$)

This convergence yields an effective mixing angle $\sin^2 \theta_W(M_s) \approx \alpha_{\text{em}}(M_s)/\alpha_2(M_s) \approx 113.1^{-1}/42.4^{-1} \approx 0.375$, recovering the canonical projection factor $3/8$ (standard hypercharge normalization) defined by the ratio $\sum T_3^2 / \sum Q^2$ over a fermion generation [82].

Internal Consistency Check (Integer Lock)

Because both gauge sectors share the same 4π flux normalization, the ratio of inverse couplings depends only on the integers k ($k_s = 3$):

$$\frac{\alpha_{\text{em}}^{-1}(M_s)}{\alpha_s^{-1}(M_s)} = \frac{k_{\text{em}}}{k_s} = \frac{9}{3} = 3$$

(SM-inferred ratio at M_s : ≈ 3.03 ; Relative Deviation: 1%)

This integer lock is a clean falsifiable prediction at the matching scale. Any additional electrically charged sector active below M_s would shift k_{em} and spoil this integer relation.

Appendix C.4. Infrared Static Response: Fine-Structure Constant

We now derive the low-energy fine-structure constant $\alpha(0)$ (Thomson limit). Unlike the UV coupling at M_s (which counts local particle states via algebra), the static IR coupling measures the integrated susceptibility of the entire vacuum manifold. We define this observable rigorously using the Euclidean path integral: the static inverse coupling is the coefficient of the quadratic gauge kinetic term ($F_{\mu\nu}^2$) in the effective action [31] defined by the Laplace-type kinetic operator Δ_A on the operational history manifold ($S^3 \times S^1$), distinct from the internal fiber accessibility that governs masses (Appendix D) [31]. In this interpretation, $\alpha^{-1}(0)$ is the renormalized $\omega \rightarrow 0$ Kubo–Mori susceptibility of the boundary state, with microscopic fluctuations already packaged into the effective coefficient [17,30].

Spectral Expansion and Selection Rules

To evaluate this functional without explicitly performing RG running, we utilize the standard spectral representation. Just as hydrodynamics replaces microscopic collisions with effective constitutive laws, this expansion packages microscopic quantum loops into effective geometric invariants [35]. In the static limit, response coefficients are controlled by the same spectral data that determines the heat-kernel trace, so these invariants directly encode the resummed vacuum response.

We determine the static inverse coupling by summing the minimal scale-free contributions to the integrated F^2 response permitted by the heat-kernel structure on the history manifold [31].

Controlled inputs are the Kubo–Mori/Hessian definition of the response and the heat-kernel expansion organizing bulk/interface/defect contributions. For the static spectral estimate, we then work on the minimal operational history manifold $S^3 \times S^1$ implied by the diamond construction and modular periodicity (Axioms P2, P4) and fixed in Chapters 1–2, with the $\mathbb{Z}_2$ spin lift and $z = 6$ transport structure (Axiom P7) and the fixed regulator $\varepsilon = 1$ (Axiom P5) held fixed throughout.

Leading contributions must be extensive measures (scaling with history four-volume). Finite-resolution corrections must be monotone penalties: decreasing with both bulk history volume (better averaging) and boundary capacity (better resolution). This isolates the inverse product $(\text{Vol} \cdot \text{Cap})^{-1}$ as the leading admissible correction [M20].

In modular normalization, operator-dependent prefactors are fixed once by UV matching at M_s ; the static estimate therefore reduces to dimensionless history invariants on the fixed history manifold.

Below, the bulk term is the leading heat-kernel (Weyl) coefficient on $S^3 \times S^1$; the remaining contributions are the leading interface/tip corrections under the stated selection principle and minimal architectural inputs ($\mathbb{Z}_2, z = 6, \varepsilon = 1$):

Bulk volume ($4\pi^3$): Fundamental four-volume of $S^3 \times S^1$. Given the minimal history manifold $S^3 \times S^1$ (Axioms P2, P4), this term is structurally fixed as the geometric volume of vacuum manifold.

Twist sector ($\pi^2 + \pi$): The discrete transport structure (Axiom P7) imposes a $\mathbb{Z}_2$ lattice identification. This identification induces a twisted sector in the heat-kernel trace [31]; we include the leading twisted-sector contribution associated with the $\mathbb{Z}_2$ identification, which yields the half-measures of the spatial and temporal components, $\pi^2 + \pi$.

Gluing penalty ($-1/32\pi^4$): Under the monotone-penalty criterion, admissible subleading terms must decrease with increasing history volume and boundary capacity. On the minimal history $S^3 \times S^1$, the relevant scale-free invariants are $\text{Vol}(S^3 \times S^1) = 4\pi^3$ and $\text{Cap}(S^2) = 8\pi$ (Gauss–Bonnet [33]). Constants built only from the modular period do not scale with either and are excluded. The leading interface suppression is therefore $(\text{Vol} \cdot \text{Cap})^{-1}$, which first appears at order π^{-4} (hence no admissible $\pi^{-1}, \pi^{-2}, \pi^{-3}$ terms). Implementing algebraic gluing across the bifurcation interface (Axiom P3) then gives $-1/[\text{Vol}(S^3 \times S^1) \text{Cap}(S^2)] = -1/(4\pi^3 \cdot 8\pi) = -1/32\pi^4$, consistent with standard interface heat-kernel methods [83,84].

Defect correction ($+1/64\pi^6$): To model the leading contribution from the tip bottlenecks, we include the first localized term compatible with a Regge-type deficit description and the $z = 6$ coordination data. Under the minimal independence assumption for the six transport directions [U7], this yields a suppression factor $(2\pi)^{-6} = 1/64\pi^6$.

Each term is a dimensionless contribution to the same heat-kernel a_4 coefficient (bulk, interface and tip-defect carry compensating local invariants in M_s units), so they can be summed.

As shown in Chapter 2, evaluating the vacuum susceptibility over the modular S^1 geometry (by integrating the KMS response kernel subject to a short-distance cutoff) yields $I_{\text{mod}}(\varepsilon) = 4 \cot(\varepsilon/2)$. The parameter $\varepsilon = 1$ is fixed by Axiom P5 ($\tau_{\min} = 1$) and is not tunable. Small variations $\varepsilon \rightarrow 1 + \delta$ shift the integral smoothly ($I_{\text{mod}} \approx 7.32$ and $dI_{\text{mod}}/d\varepsilon \approx -8.70$ at $\varepsilon = 1$), an order-unity sensitivity that precludes fine-tuning. Physically, shifting ε moves the cutoff surface, representing a change of regulator scheme rather than a tunable fit.

Prediction under Minimal Spectral Selection

$$\alpha^{-1}(0) = 4\pi^3 + \pi^2 + \pi - \frac{1}{32\pi^4} + \frac{1}{64\pi^6} \approx 137.035\,999\,216$$

(Measured: 137.035 999 206(11) [69]; Relative Deviation $\approx 7 \times 10^{-11}$ ($< 1\sigma$)).

Observation and Falsifiability

The two most precise measurements of α ([69] and [85]) currently disagree by $> 5\sigma$. Our approach discriminates between them:

LKB 2020 (Rb): [69]: 137.035 999 206(11) $\rightarrow$ Agreement ($< 1\sigma$)

Berkeley 2018 (Cs): [85]: 137.035 999 046(27) $\rightarrow$ Disfavored ($> 5\sigma$)

Our approach supports the LKB 2020 result [69], also acting as a falsifier.

The accuracy of this result comes from rigidity, not tuning. The boundary architecture fixes the subsystem, the clock and the transport rule, so the coupling is read globally as a coarse-grained vacuum stiffness rather than a perturbative loop series. The estimate is highly constrained; changes to the inventory or topology would shift the result at $\mathcal{O}(1)$ in the spectral coefficients.

Appendix C.5. Summary of Predictions

Inputs: Resolution scale M_s ; fixed regulator $\varepsilon = 1$ (via $\tau_{\min} = 1$).

Discrete Minimality Assumptions: [M19] $k_s = h^\vee$; [M20] monotone-penalty spectral selection for the static IR estimate.

Unity Normalizations: [U7] unity prefactor for six-direction defect suppression.

Outputs: UV relation $\alpha^{-1}(M_s) = 4\pi k$; strong $\alpha_s^{-1} = 12\pi$ ($k_s = 3$); EM $\alpha_{\text{em}}^{-1} = 36\pi$ ($k_{\text{em}} = 9$); Integer lock $\alpha_{\text{em}}^{-1}/\alpha_s^{-1} = 3$; IR static prediction $\alpha^{-1}(0) \approx 137.035999216$ (spectral sum).

Falsifiers: Any new charged sector below M_s (breaks integer lock); confirmation of Berkeley (Cs) α [85] over LKB (Rb) [69].

Appendix D. Lepton Mass Spectrum via Spectral Filtration

This appendix derives the charged-lepton mass spectrum (m_e, m_μ, m_τ) from the finite-resolution boundary architecture (Axioms P1–P7), with finite capacity (Axiom P5) providing the central cutoff, and the linear-response framework (Hessian of relative entropy) established in Chapter 2.

The translation between nested causal diamonds acts as an open modular update (GKSL semi-group) (Axioms P3–P5). Because this open time evolution selects a fixed-point pointer algebra that becomes increasingly commutative as coherence decreases, the surviving accessible data for compact non-abelian symmetries is organized by the canonical Lie hierarchy $G \rightarrow \mathfrak{g} \rightarrow T$ (unique up to conjugacy). Imposing a finite capacity implies that accessibility is a physical resource rather than a formal limit [16,19].

Appendix D.1. Dressed Hadronic Anchor m_p

The resolution scale M_s sets the maximum frequency. The observable unit of mass for stable matter is the proton (m_p) [86]. We require the boundary dynamics to naturally generate this unit through confinement physics [80].

The confinement scale Λ_{QCD} is generated by running the strong coupling from M_s down to the pole, driven by the coupling $\alpha_s^{-1}(M_s) = 12\pi$ (Appendix C) [35]. Using a 1-loop beta function ($\langle b_0 \rangle \approx 7.25$ for effective running):

$$\Lambda_{\text{QCD}} \approx M_s \cdot \exp \left[-\frac{2\pi(12\pi)}{\langle b_0 \rangle} \right] \approx 200 \text{ MeV}$$

We model the proton as the ground-state of a confinement cavity $R \sim \Lambda_{\text{QCD}}^{-1}$, so m_p is set by the lowest semiclassical orbital mode [87] [M21]:

$$m_p^{\text{mech}} \approx \frac{3\pi}{2} \Lambda_{\text{QCD}} \approx 942 \text{ MeV}$$

(PDG value: 938 MeV [66]; Relative Deviation $\sim 0.4\%$)

This establishes m_p as a natural dressed anchor generated by UV data, up to standard running uncertainties. For the precision lepton ratios below, we use the empirical m_p as the precise reference for the geometric anchor.

Appendix D.2. Mass as Spectral Susceptibility

In Chapter 2, we defined local dynamics via the Hessian of relative entropy and showed that, in the linear-response limit (Kubo-Mori formalism [17]), the static susceptibility χ of the vacuum is proportional to the integrated spectral density [88] of the Laplacian Δ_{int} [31,53]:

$$\chi \sim \sum_n \frac{|\langle 0 | \mathcal{O} | n \rangle|^2}{E_n} \implies m \propto \chi^{-1} \propto \frac{m_{\text{ref}}}{\Omega}.$$

Here, Ω represents the number of resolvable modes on the manifold $\mathcal{M}_{\text{int}}$ (via Weyl's law [88], linking continuous volume to discrete capacity). Larger accessible volume lowers the gap; restricted accessibility raises it.

The finite-resolution boundary implies a loss of fine-grained information (coarse-graining) (Chapter 2). Let $\mathcal{E}_t$ denote the resulting symmetry-respecting coarse-graining semigroup [60] on the observables and let $\mathfrak{A}_{\text{eff}} = \text{Fix}(\mathcal{E})$ be its stable fixed-point algebra.

Let $\eta = t_{\text{coh}}/t_{\text{mix}}$ order the regime. Operationally, t_{mix} is set (up to order-unity factors) by Δ_{int}^{-1} , the inverse spectral gap of the relevant mixing generator. These regimes are distinguished by their accessibility class in the open modular setting, where mass and coherence time are co-determined properties of the eigenstructure, quantified via the Kubo–Mori susceptibility [17,48]. As η decreases, the fixed-point algebra $\mathfrak{A}_{\text{eff}}$ abelianizes under the coarse-graining. Non-commuting information is lost first, leaving a commuting pointer algebra [61].

For compact non-abelian symmetries, the irreversible loss of noncommuting structure drives continuous abelianization along the canonical Lie hierarchy $G \rightarrow \mathfrak{g} \rightarrow T$. For the $SU(2)$ spinorial sector (Axiom P7), this reduction has no nontrivial intermediate connected stage: the only maximal connected abelian subgroup is $U(1)$ (up to conjugacy) and $\mathfrak{g}$ is its unique local linearization. Therefore, under continuous coarse-graining, the accessible data strictly reduces as:

Global Regime ($\eta \gg 1$): Non-abelian observable content remains accessible (global group manifold G).

Tangent Regime ($\eta \sim 1$): Only local generator data remains coherent (local tangent algebra $\mathfrak{g}$).

Commuting Regime ($\eta \ll 1$): The algebra reduces to a Maximal Abelian Subalgebra (MASA) (maximal Cartan torus T , acting as geometric fiber).

Coarse-graining induces a nested reduction of accessible algebras as η decreases (non-abelian $\rightarrow$ local generators $\rightarrow$ commuting phases). We use the asymptotic limits $\eta \gg 1$ and $\eta \ll 1$ for counting; the intermediate regime is treated as a controlled proxy.

We write $m_{\text{ref}} = E_{\text{pix}} R_{\text{IR}}$. Here R_{IR} is determined by whether the gauge-invariant dressing channel is available on $\mathfrak{A}_{\text{eff}}$ (Axioms P3/P6). In the global/tangent regimes ($\eta \gtrsim 1$), dressing is available ($R_{\text{IR}} = m_p/E_{\text{pix}}$). In the commuting regime ($\eta \ll 1$), non-abelian dressing is obstructed ($R_{\text{IR}} = 1$).

Colored quarks (c, b) are confined and do not appear as asymptotic states; applying the same η -criterion to them would require colored-sector dressing and hadronic spectroscopy, which we do not attempt here.

Appendix D.3. Lepton Cascade

We apply the filtration to $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$ as a minimal internal manifold choice [M22] together with the discrete transport structure (Axiom P7). The geometric correspondence is:

$$3\text{D Space (P7)} \xrightarrow{\text{Rotation}} SO(3) \xrightarrow{\text{Quantum}} SU(2) \xrightarrow{\text{Topology}} S^3$$

Since $S^3 \cong SU(2)$ (the group of unit quaternions), this manifold naturally carries the Lie structure required by the isotropic transport (Axiom P7), making the filtration $G \rightarrow \mathfrak{g} \rightarrow T$ strictly applicable.

Define the spectral volume by $\Omega(\eta) = \text{Tr}(\Pi_{\text{axes}} \otimes \Pi_{\text{int}})$. The same accessibility Ω underlies all three generations; in the three limits it reduces to a volume count (global), a generator count (tangent) and a Cartan phase-volume count (commuting).

Generation 1: Electron (Global Limit)

Regime: Stable ($\eta \rightarrow \infty$). Resolves full global symmetry G .

Axiom P7 fixes $\text{Tr}(\Pi_{\text{axes}}) = 3$ (isotropic transport). When explicit spectral mode counting is required, the minimality criterion restricts the geometry to the smallest spheres admitting transitive actions of the relevant non-Abelian factors. The spinorial sector fixes an $SU(2)$ structure and hence $S^3 \simeq SU(2)$. For the color sector we use the smallest sphere with transitive $SU(3)$ action, $S^5 \simeq SU(3)/SU(2)$. We therefore take $\mathcal{M}_{\text{int}} \cong S^3 \times S^5$.

In the global regime, the leading Weyl term depends only on $\text{Vol}(\mathcal{M}_{\text{int}})$ (up to fixed multiplicity $\text{Tr}(\Pi_{\text{axes}})$) [88]. This fixes the accessibility Ω_e once this minimal choice is specified, removing freedom to reshuffle factors at leading order. With $\text{Vol}(S^3 \times S^5) = 2\pi^5$:

$$\Omega_e = 3 \cdot 2\pi^5 = 6\pi^5, \quad m_e = \frac{m_p}{6\pi^5} \approx 0.511009 \text{ MeV} \quad (m_p/m_e = 6\pi^5 \approx 1836.12)$$

(PDG value: 0.510999 MeV, $m_p/m_e \approx 1836.15$ [66]; Relative Deviation $\approx 10^{-5}$)

Generation 2: Muon (Tangent Limit)

Regime: Transient ($\eta \sim 1$). Resolves local algebra $\mathfrak{g}$.

Accessibility reduces to local generator content. Operationally, this regime resolves generator directions but not global identifications. The minimal compact algebra controlling the local frame is $\mathfrak{so}(3) \cong \mathfrak{su}(2)$ [81] [M23]. Thus:

$$\Omega_\mu = 3 \times \dim(\mathfrak{su}(2)) = 9, \quad m_\mu = \frac{m_p}{9} \approx 104.25 \text{ MeV}$$

(PDG value: 105.66 MeV [66]; Relative Deviation $\sim 1.3\%$)

A percent-level deviation can be expected here, as Ω_μ is a tangent-regime proxy.

Generation 3: Tau (Commuting / Cartan Fiber)

Regime: Prompt ($\eta \ll 1$). Fixed-point algebra abelianizes to T .

The accessible phase volume is the maximal Cartan torus T^4 (rank 4, strictly fixed by the minimal gauge inventory established in Chapter 2 and Appendix C). The transport symmetry $\Gamma \cong (\mathbb{Z}_2)^3$ (Axiom P7) acts as an axis-sign redundancy on the commuting phases, so $|\Gamma| = 8$ (axis permutations correspond to physical rotations and are not quotiented):

$$\Omega_\tau = \frac{\text{Vol}(T^4)}{|\Gamma|} = \frac{(2\pi)^4}{8} = 2\pi^4, \quad m_\tau = \frac{E_{\text{pix}}}{2\pi^4} \approx 1.788 \text{ GeV}$$

(PDG value: 1.777 GeV [66]; Relative Deviation $\sim 0.6\%$)

In this commuting regime, the excitation is pixel-bound, so E_{pix} sets the anchor. The mass estimate is a static (susceptibility/accessibility) statement; the lifetime check below is a separate dynamical consistency test.

Appendix D.4. Lifetime Check

The prompt regime implies a short weak lifetime. Using the standard leptonic scaling $t \propto m^{-5}$ with a channel factor $C_\tau \approx 5$ (counting lepton and color doublets) [M24] as a parametric check:

$$t_\tau \approx t_\mu \left(\frac{m_\mu}{m_{\text{tau}}} \right)^5 \frac{1}{C_\tau} \approx 3.3 \times 10^{-13} \text{ s}$$

(PDG value: $2.9 \times 10^{-13} \text{ s}$ [66])

This supports the assignment of the Tau to the commuting ($\eta \ll 1$) regime.

Appendix D.5. Summary of Predictions

Inputs: Resolution scale M_s ; Per-pixel energy budget E_{pix} ; m_p (derived from M_s via confinement, then used as a geometric anchor only).

Discrete Minimality Assumptions: [M21] cavity ground-state factor $3\pi/2$ for the proton anchor; [M22] $M_{\text{int}} \cong S^3 \times S^5$ as the minimal internal manifold; [M23] tangent-regime generator counting via $\mathfrak{so}(3) \cong \mathfrak{su}(2)$; [M24] channel factor $C_\tau \approx 5$ for the lifetime check.

Outputs: $m_e; m_\mu; m_\tau; t_\tau$.

Falsifier: Failure of geometric integers ($6\pi^5, 9, 2\pi^4$); discovery of a 4th charged lepton generation.

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