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Article

A Robust Second-Order Conic Programming Model with Effective Budget of Uncertainty in the Optimal Power Flow Problem

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Abstract: Integrating large-scale wind energy in modern power systems necessitates high-efficiency mathematical models to address classical assumptions in power systems. In particular, two main assumptions for wind energy integration in power systems have not been adequately studied. First, nonlinear AC power flow equations have been linearized in most of the literature. Such simplifications can lead to inaccurate power flow calculations and result in technical issues. Second, wind power uncertainties are inevitable and have been mostly modeled using traditional uncertainty modeling techniques, which may not be suitable for large-scale wind power integration. In this study, we addressed both challenges: we developed a tight second-order conic relaxation model for the optimal power flow problem and implemented the novel effective budget of uncertainty approach for uncertainty modeling to determine the maximum wind power admissibility and address the uncertainty in the model. To the best of our knowledge, this is the first study that proposes an effective, robust second-order conic programming model that simultaneously addresses the issues of power flow linearization and wind power uncertainty with the new paradigm on the budget of uncertainty approach. The numerical results revealed the advantages of the proposed model over traditional linearized power flow equations and traditional uncertainty modeling techniques.

Keywords: Renewable energy sources; wind uncertainty; effective budget of uncertainty; second-order conic relaxation; AC power flow equations

1. Introduction

Due to recent advancements in optimization theory, optimization algorithms have been increasingly used to improve the performance of power systems and realize automatic voltage regulation [1], fault diagnosis under uncertainty [2], optimal design of battery management controller [3], and robust control methods for wind energy system [4]. Modern power systems are shifting from fossil fuels to clean, reliable, and emission-free wind energy. For example, 19.8% of electricity in North America is generated from wind energy [5]. However, integrating wind energy into large power systems is challenging and can affect the reliability of power systems due to two main reasons. First, the power system operation is usually simplified, and the effect of various aspects, such as voltage, on the system is neglected. Such simplifications can result in inaccurate estimation of the limits of the system and lead to damage to the system [6, 7]. Second, wind energy is inherently uncertain and cannot be accurately predicted. Such prediction inaccuracies can result in various operational issues [8, 9].

The optimal power flow (OPF) problem has been extensively studied and helps minimize the distribution cost while satisfying the power flow equations and operational constraints such as voltage magnitude, line flow, and generator limits [10, 11]. In OPF, the power flow equations are inherently nonlinear. However, most studies have linearized power flow equations to reduce the computational complexity; however, the effects of voltages, angles, and magnitudes are ignored [12–14]. Such simplifications lead to inaccurate power flow calculations and may lead to problems such as overloading and power mismatch[15]. To overcome this problem, relaxation techniques have been proposed in the literature [7, 16]. In recent decades, two efficient convex relaxations for nonlinear AC power flow equations have been proposed: second-order cone relaxation (SOCR) [6] and semidefinite relaxation (SDR) [17]. Such convex relaxations provide more accurate solutions for the OPF problem and consider the effects of voltages at all nodes of power systems [18]. In meshed networks, SDR is stronger than SOCR; however, SDR is more computationally challenging [19]. SOCR relaxation is accurate and highly efficient for various classes of problems in radial networks [6]. Recent surveys on these relaxations can be found in the literature [7, 20, 21].

In recent years, various prediction methods have been proposed to increase the prediction accuracy [22–25]. However, prediction errors are inevitable and can lead to severe problems in highly sensitive applications [26]. To overcome wind power uncertainties, such uncertainties must be considered in OPF so that the solution is robust under variations of wind power availability. Unlike deterministic optimization, wherein the predicated data of an optimization problem is assumed to be always perfect, robust optimization (RO) [27, 28] considers that such perfect information is not always available due to prediction errors [29]. In the context of wind power integration, an RO model assumes that wind power availability can take any value within a given uncertainty set and obtains the optimal solution under the possible worst-case scenario for wind availability. Various RO methods have been proposed in the literature. In particular, the adjustable *budget of uncertainty* method [28] adjusts the solution degree of conservatism by changing the total amount of uncertainty in the model, and the total amount of uncertainty is modeled using a parameter called the budget of uncertainty. This method has been extensively studied [9, 12, 23–31]. Alternatively, two-stage RO models [5, 26, 32–37] have been used in power systems, where the first stage decisions are made before realizing the actual wind power, and the second stage decisions are "wait-and-see" decisions that can be adjusted after the actual wind power output is known. Such two-stage RO models are computationally complex and difficult to implement in large power systems.

Recently, a new robust optimization paradigm called *effective budget of uncertainty* [38] was proposed that more effectively adjusts the solution degree of conservatism. The robust solution is not sensitive to all changes in the amount of uncertainty and, after a threshold, the solution is not affected by the level of uncertainty. This phenomenon was not considered in the traditional budget of uncertainty method; thus, such advancements in RO models are shifting the trend toward the implementation of the effective budget of uncertainty in power systems [8, 39].

In this study, we developed a new model for power distribution by considering the AC power flow equations and recent advances in robust optimization. We first identified an interval of available power under which the system can operate safely without any system limit violation. Next, given the identified interval, we implemented the robust optimization approach to handle the uncertainty in the system given the budget provided for the model. In this study, we extended the results presented in the literature [38, 39] to nonlinear AC power flow equations and developed a SOCR model by using the effective budget of uncertainty approach. To the best of our knowledge, this is the first paper that implements the recent advances of robust optimization in AC power flow equations [40]. Furthermore, we performed extensive numerical calculations to address the problems of estimating power flows as well as wind power uncertainties. The contributions of this study are presented as follows:

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We extended the recent effective budget of uncertainty approach [38] and applied it to a nonlinear model with AC power flow equations. We proposed a new modeling scheme called effective robust second-order conic programming (ERSOCP) for power systems with uncertainty. We theoretically and numerically demonstrated that the proposed model is computationally tractable and can be solved efficiently using the MOSEK solver. The proposed model effectively reduces the solution conservatism and considers the effects of voltage in wind power integration.

The proposed ERSOCP model provides a high accuracy by considering both wind power uncertainty and nonlinearity in power flow equations. The numerical results demonstrated the advantages of the proposed model in comparison with traditional methods.

The rest of the paper is organized as follows. In Section 2, the OPF problem and a reformulation using a second-order cone (SOC) are presented. In Section 3, the steps and backgrounds to implement the effective budget of uncertainty approach [38] in the proposed model are presented. In Section 4, the ERSOCP model is presented. Numerical results are provided in Section 5. Finally, Section 6 concludes the paper.

2. Optimal Power Flow Problem

The OPF problem aids in determining the best operating levels of power generators to minimize the operating cost and satisfy the power demand, transmission network constraints, ramping rates, and reserve requirements. The notations used in the proposed OPF model are listed as follows.

Notations

C_1/C_2	*Per-unit cost of power generation/wind curtailment*
$\bar{F}_l/\underline{F}_l$	*Upper/lower power flow limit at line l*
p_i^D/q_i^D	*Real/reactive power demand at bus i*
P_t^u/P_t^d	*Upward/downward spinning reserve requirement of active power at time t*
$T/L/N$	*Set of time periods/transmission lines/buses*
$\bar{U}_i^P/\underline{U}_i^P$	*Upper/lower limit of real power generation at bus i*
$\bar{U}_i^Q/\underline{U}_i^Q$	*Upper/lower limit of real power generation at bus i*
$\bar{V}_i/\underline{V}_i$	*Upper/lower limit of voltage at bus i*
$\widehat{W}_{i,t}$	*Predicted available wind power at node i at time t*
$\boldsymbol{y}_L/\boldsymbol{y}_S$	*Per-unit series/shunt admittance matrix*
$\bar{\Delta}_l/\underline{\Delta}_l$	*Upper/lower limit of voltage angle at bus i*
$\theta_{Lij}/\theta_{Sij}$	*The angle of the ij^{th} element of the series/shunt admittance matrix*
Q_t^u/Q_t^d	*Upward/downward spinning reserve requirement of reactive power at time t*

Decision variables

$p_{ij,t}/q_{ij,t}$	*Real/reactive power flow between bus i and bus j at time t*
$p_{i,t}^G/q_{i,t}^G$	*Real/reactive power produced by the generator located at bus i at time t*
$p_{i,t}^W$	*Real power produced by the wind turbine at bus i at time t*
$r_{i,t}^{p+}/r_{i,t}^{p-}$	*Upward/downward spinning reserve of the active power of generator at bus i at time t*
$r_{i,t}^{q+}/r_{i,t}^{q-}$	*Upward/downward spinning reserve of reactive power of generator at bus i at time t*
$V_{i,t}/\delta_{i,t}$	*Voltage magnitude/angle at bus i at time t*

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The OPF can be expressed as follows:

$$\min \sum_{t\in T}\sum_{g\in G} C_1 p_{g,t}^G + \sum_{t\in T}\sum_{k\in K} C_2(\widehat{W}_{k,t} - p_{k,t}^W) \tag{1}$$

$$p_{ij,t} = V_{i,t}^2 y_{Lij}\cos(\theta_{Lij}) - V_{i,t}V_{j,t}y_{Lij}\cos(\delta_{i,t} - \delta_{j,t} - \theta_{Lij}) + \frac{1}{2}V_{i,t}^2 y_{Sij}\cos(\theta_{Sij}), \quad \forall i,j \in N, t \in T \tag{2}$$

$$q_{ij,t} = -V_{i,t}^2 y_{Lij}\sin(\theta_{Lij}) - V_{i,t}V_{j,t}y_{Lij}\sin(\delta_{i,t} - \delta_{j,t} - \theta_{Lij}) - \frac{1}{2}V_{i,t}^2 y_{Sij}\sin(\theta_{Sij}), \quad \forall i,j \in N, t \in T \tag{3}$$

$$p_{i,t}^G + p_{i,t}^W - p_{i,t}^D = \sum_{j\in N} p_{ij,t}, \quad \forall i \in N, t \in T \tag{4}$$

$$q_{i,t}^G - q_{i,t}^D = \sum_{j\in N} q_{ij,t}, \quad \forall i \in N, t \in T \tag{5}$$

$$\underline{F_l} \le p_{ij,t} \le \overline{F}_l, \quad \forall (i,j): l \in L, t \in T \tag{6}$$

$$\underline{V_i} \le V_{i,t} \le \overline{V}_i, \quad \underline{\Delta}_i \le \delta_{i,t} \le \overline{\Delta}_i, \quad \forall i \in N, t \in T \tag{7}$$

$$\underline{U_l^P} \le p_{i,t}^G \le \overline{U}_l^P, \quad \forall i \in N, t \in T \tag{8}$$

$$\underline{U_l^Q} \le q_{i,t}^G \le \overline{U}_l^Q, \quad \forall i \in N, t \in T \tag{9}$$

$$\sum_{i\in N} r_{i,t}^{p+} \ge P_t^u, \quad \forall t \in T \tag{10}$$

$$\sum_{i\in N} r_{i,t}^{p-} \ge P_t^d, \quad \forall t \in T \tag{11}$$

$$\sum_{i\in N} r_{i,t}^{q+} \ge Q_t^u, \quad \forall t \in T \tag{12}$$

$$\sum_{i\in N} r_{i,t}^{q-} \ge Q_t^d, \quad \forall t \in T \tag{13}$$

$$0 \le r_{i,t}^{p+} \le \overline{U}_l^P - p_{i,t}^G, \quad \forall t \in T \tag{14}$$

$$0 \le r_{i,t}^{p-} \le p_{i,t}^G - \underline{U_l^P}, \quad \forall t \in T \tag{15}$$

$$0 \le r_{i,t}^{q+} \le \overline{U}_l^Q - q_{i,t}^G, \quad \forall t \in T \tag{16}$$

$$0 \le r_{i,t}^{q-} \le q_{i,t}^G - \underline{U_l^Q}, \quad \forall t \in T \tag{17}$$

$$0 \le p_{i,t}^W \le \widehat{W}_{i,t}, \quad \forall i \in N, t \in T \tag{18}$$

where objective function (1) minimizes the generation cost and wind curtailment cost to aid in the integration of wind power in the power system. Constraints (2) and (3) are the real and reactive power flow equations, respectively [41]. Constraints (4) and (5) are the real and reactive power balance, respectively, at each node where wind power plays a role in the active power balance constraints [42]. Constraint (6) limits the active power flow in transmission lines. Constraint (7) enforces the limits of voltage angle and magnitude. Constraints (8) and (9) enforce the upper limits for real and reactive power generations, respectively, at all generators. Constraints (10) and (11) ensure that the upward and downward spinning reserves, respectively, of active power are greater than a certain amount. The spinning reserve is the available capacity of generators to increase or decrease the power output in 10 minutes. Similarly, constraints (12) and (13) correspond to the spinning reserve constraints of reactive power. Constraints (14)–(17) enforce the limits of spinning reserves. Finally, in constraint (18), the wind power output limit is considered.

2.1. Reformulation of Power Flow Equations

Constraints (2) and (3) correspond to the real and reactive power equations, respectively [41]. Let the admittance matrix $\boldsymbol{y}$ be decomposed as $y_{ij} = G_{ij} + \mathrm{i}B_{ij}$, where G_{ij} and B_{ij} are, respectively the real and imaginary parts of the admittance matrix. Given the complex voltage, $V_i = |V_i|(\cos\theta_i + \mathrm{i}\sin\theta_i)$ can be expressed as $V_i = e_i + \mathrm{i}f_i$ in the rectangular form. To that end, the following substitutions can be made:

$$V_i^2 = e_i^2 + f_i^2$$

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$$|V_i||V_j|\cos(\delta_i - \delta_j) = e_i e_j + f_i f_j$$
$$|V_i||V_j|\sin(\delta_i - \delta_j) = e_i f_j - e_j f_i$$

Thus, power flow equations (2) and (3) can be represented in the rectangular form [43]:

$$p_{ij} = G_{ii}(e_i^2 + f_i^2) + G_{ij}(e_i e_j + f_i f_j) - B_{ij}(e_i f_j - e_j f_i) \quad (19)$$
$$q_{ij} = -B_{ii}(e_i^2 + f_i^2) - B_{ij}(e_i e_j + f_i f_j) - G_{ij}(e_i f_j - e_j f_i) \quad (20)$$

The rectangular equations (19) and (20) are nonconvex quadratic functions. However, a SOC relaxation can be obtained by defining auxiliary variables c_{ii} for each node and by defining c_{ij} and s_{ij} for transmission lines so that $c_{ii} = e_i^2 + f_i^2$, $c_{ij} = e_i e_j + f_i f_j$, and $s_{ij} = e_i f_j - e_j f_i$. Furthermore, $c_{ij}^2 + s_{ij}^2 = c_{ii} c_{jj}$. Because c_{ij} and s_{ij} correspond to each line and capture some components of the flow, it is interpreted that $c_{ij} = -c_{ji}$ and $s_{ij} = -s_{ji}$. Therefore, the power flow equations can be reformulated as follows (for all i and j):

$$p_{ij} = G_{ii} c_{ii} + G_{ij} c_{ij} - B_{ij} s_{ij} \quad (21)$$
$$q_{ij} = -B_{ii} c_{ii} - B_{ij} c_{ij} - G_{ij} s_{ij} \quad (22)$$
$$c_{ij}^2 + s_{ij}^2 = c_{ii} c_{jj} \quad (23)$$
$$c_{ij} = -c_{ji}, \qquad s_{ij} = -s_{ji} \quad (24)$$

2.2. Second-Order Conic Relaxation of Power Flow Equations

Here we present a relaxation for the power flow equations that can be represented using SOCs.

Definition 1: Q^n is an n-dimensional SOC if

$$Q = \left\{x \in \mathbb{R}^n : x_1 \geq (x_2^2 + x_3^2 + \cdots + x_n^2)^{\frac{1}{2}}\right\}.$$

SOCs are convex and can be efficiently solved using the MOSEK solver. Another variant of SOC is the rotated SOC, which can be defined as follows:

Definition 2: Q_r^n is an n-dimensional rotated SOC if

$Q = \{x \in \mathbb{R}^n : 2x_1 x_2 \geq x_3^2 + \cdots + x_n^2,\ x_1, x_2 \geq 0\}$.

Q^n and Q_r^n are equivalent [44].

In the rectangular power flow equations (21)–(24), constraint (23) is a quadratic constraint, and the rest of the constraints are linear. By converting the equality constraint (23) into an inequality constraint, we can relax (23) and rewrite it as follows

$$c_{ij}^2 + s_{ij}^2 \leq c_{ii} c_{jj}, \qquad \forall i, j \in N \quad (25)$$

The relaxed constraint (25) can be represented using a rotated SOC. If the relaxed constraint (25) becomes binding at optimality, then the proposed SOC relaxation is exact and constraint (25) is equivalent to the original power flow constraint (23) [43]. We further investigated the exactness of the proposed SOC relaxation through numerical calculations.

3. Effective Budget of Uncertainty in Power Systems

We modeled the wind power uncertainty by using the new RO paradigm of the effective budget of uncertainty.

In competitive electricity markets, wind power availability is predicted and priced in a day-ahead manner, and predictions are employed in power distribution planning [45–47]. However, in real-time operation, the actual wind power might differ from the predicted wind power; this may lead to the violation of the operational requirements and limits of the power system if such deficiencies are not considered in advance [9]. RO considers such differences in wind power availability by using an uncertainty set, which includes all possible scenarios of actual wind power. Let $W_{k,t}^{act}$ be the actual wind power of wind turbine k at time t; $\overline{W}_{k,t}$ and $\underline{W}_{k,t}$ be the upper and lower limit of the actual wind power, respectively; and $\widehat{W}_{k,t}$ be the predicted wind power. Let parameter $\widetilde{W}_{k,t}$ represent the uncertain wind power that can take any value in $[\underline{W}_{k,t}, \overline{W}_{k,t}]$. In the traditional budget of uncertainty method, the uncertainty set is represented as follows (constraints (26a), (26b), and (26d) are written for all time periods $t \in T$ and all wind turbines $k \in K$):

$$\widetilde{W}_{k,t} = \widehat{W}_{k,t} + z_{k,t}^{+}(\overline{W}_{k,t} - \widehat{W}_{k,t}) + z_{k,t}^{-}(\underline{W}_{k,t} - \widehat{W}_{k,t}) \quad (26a)$$

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$$0 \le z_{k,t}^{+}, \qquad z_{k,t}^{-} \le 1 \tag{26b}$$

$$\sum_{t \in T} (z_{k,t}^{+} + z_{k,t}^{-}) \le \Gamma_t \tag{26c}$$

$$p_{k,t}^{W} \le \widetilde{W}_{k,t} \tag{26d}$$

where constraint (26a) represents all possibilities for wind power, constraint (26b) defines the limits for the deviation variables, and constraint (26c) limits all possible deviations by a parameter called the budget of uncertainty [28, 39]. Finally, constraint (26d) considers the generated wind power that is limited by the available wind power output (i.e., $\widetilde{W}_{k,t}$) that is uncertain. Constraints (26a)–(26c) allow users to select the level of uncertainty at each time period (Γ_t) and then yields a set of acceptable values for the budget of uncertainty parameter. Γ_t also controls the degree of conservatism of the solution. For instance, $\Gamma_t = 0$ indicates that no uncertainty is considered in the system; thus, in constraints (26a)–(26c), we set $\widetilde{W}_{k,t} = \widehat{W}_{k,t}$, meaning that the uncertain parameters are equivalent to the predicted values because it is assumed that there is no uncertainty. Thus, constraint (26d) reduces to constraint (18). However, $\Gamma_t = |K|$, where $|K|$ is the number of wind turbines, means that the outputs of all wind turbines are uncertain and their inputs may deviate from the predicted values and can even take either their lower or upper bound values (i.e., $\overline{W}_{k,t}$ and $\underline{W}_{k,t}$). Further details can be found in the literature [16].

The main disadvantage of uncertainty set (26a)–(26c) is that, depending on the system worst-case scenario, it allows $\widetilde{W}_{k,t}$ to take the upper bound or lower bound of $[\underline{W}_{k,t}, \overline{W}_{k,t}]$, restricted by parameter Γ_t. However, as demonstrated in previous studies [38, 39], the uncertain wind power output of wind turbines depends on two factors: (i) the budget of uncertainty, which directly affects the uncertain wind power output ((26a)–(26c)); and (ii) the operational limits and capabilities of the system to handle wind power. For example, assume that the budget of uncertainty is $\Gamma_t = |K|$, meaning that the output of all wind turbines can have maximum deviations from the predicted values and take their upper bounds, that is, $\widetilde{W}_{k,t} = \overline{W}_{k,t}$. Thus, appropriate measures can be planned for this scenario, known as the worst-case scenario, to mitigate the risk of having issues in the system in case of unforeseen events [16]. However, planning under this scenario may conflict with the operational limits of the system. In particular, if the limits of the system are not sufficient to handle $\overline{W}_{k,t}$ amount of power, then it is obvious that there are other factors limiting the amount of wind power output rather than merely the uncertainty budget. This phenomenon [38–40] can cause various issues, such as overprotection against uncertainty, and would result in higher operational costs.

Therefore, in RO models, first, the maximum admissibility of wind power, which is the threshold beyond which the uncertainty has no effect on the system, must be determined. If the wind uncertainty is more than the threshold, the system reaches its limits, and the solution is determined by the system limits, not the budget of uncertainty [8, 39]. Therefore, the uncertainty set must be modified based on the wind power admissibility before incorporating the budget of uncertainty in the model. By doing so, the ineffective part of the uncertainty set that does not affect the solution can be removed, and the effective budget of uncertainty can be obtained [25, 38] to include in the model.

Let $\bar{s}_{k,t}$ be the maximum wind power admissibility in the system after determining the limits of the system. In other words, $\bar{s}_{k,t}$ indicates the maximum amount of power that can be handled by the system. Assuming that $\bar{s}_{k,t}$ has been obtained, the uncertainty set (26a)–(26c) can be modified as follows (constraints (27a), (27b), and (27d) are written for all time periods $t \in T$ and all wind turbines $k \in K$):

$$\widetilde{W}_{k,t}^{new} = \widehat{W}_{k,t} + z_{k,t}^{new+}(\bar{s}_{k,t} - \widehat{W}_{k,t}) + z_{k,t}^{new-}(\underline{W}_{k,t} - \widehat{W}_{k,t}) \tag{27a}$$

$$0 \le z_{k,t}^{new+}, \qquad z_{k,t}^{new-} \le 1 \tag{27b}$$

$$\sum_{t \in T} (z_{k,t}^{new+} + z_{k,t}^{new-}) \le \Gamma_t \tag{27c}$$

$$p_{k,t}^{W} \le \widetilde{W}_{k,t}^{new} \tag{27d}$$

The scaled deviation variables $z_{k,t}^{new+}$ are scaled differently so that one unit of $z_{k,t}^{new+}$ is equivalent to $(\bar{s}_{k,t} - \widehat{W}_{k,t})$ amount of deviation from the predicted wind power output $\widehat{W}_{k,t}$. In contrast, in the traditional uncertainty set (26a)–(26c), one unit of scaled deviation

$z_{k,t}^{+}$ corresponds to $(\overline{W}_{k,t} - \widehat{W}_{k,t})$ amount of deviation. As a result, in constraint (26), the budget of uncertainty Γ_t controls the total amount of deviations over all wind turbines, whereas in constraint (27), the budget of uncertainty Γ_t controls only the effective deviations and is denoted as an effective budget of uncertainty [38].

$\bar{s}_{k,t}$ is not known in the system. Thus, the maximum wind power admissibility $\bar{s}_{k,t}$ must be determined by analyzing the system requirements. $\bar{s}_{k,t}$ determines the maximum value of wind power that can be handled by the system without resulting in any operational issues. Thus, we considered the worst-case scenarios of constraints (10) and (11) because the spinning reserve of active power is a function of wind power output. Thus, by ensuring that the worst-case scenarios of these constraints are met, the maximum wind power admissibility can be determined. For each wind turbine k and time period t, problem (28) can be solved to determine $\bar{s}_{k,t}$ as follows [39]:

$$\min_{\mathbb{C}_1}\left(\overline{W}_{k,t} - \bar{s}_{k,t}\right) \tag{28a}$$

$$s.t. \quad \min_{\bar{s}_{k,t}} \sum_{i\in N} (p_{i,t}^{G} + r_{i,t}^{p+} + \bar{s}_{k,t} - p_{i,t}^{D}) \geq P_t^{u} \tag{28b}$$

$$\min_{\bar{s}_{k,t}} \sum_{i\in N} (-p_{i,t}^{G} + r_{i,t}^{p-} - \bar{s}_{k,t} + p_{i,t}^{D}) \geq P_t^{d} \tag{28c}$$

where set $\mathbb{C}_1 = \{p_{i,t}^{G}, r_{i,t}^{p+}, r_{i,t}^{p-}, \bar{s}_{k,t}\}$ is the set of decision variables that are optimized. Constraints (28b) and (28c) aim to ensure that the upward and downward active power reserve constraints are met, respectively, under the worst-case scenario of $\bar{s}_{k,t}$. In (28a), the objective function aims to increase the wind power admissibility of unit k at time t so that more wind power can be utilized as long as the worst-case scenarios of reserve constraints are met.

Problem (28) is a two-stage problem and can be solved using the strong duality theorem [28]. Thus, by introducing dual variables $\alpha_{k,t}$ and $\beta_{k,t}$, problem (28) can be transformed into an equivalent problem as follows:

$$\min_{\mathbb{C}_2}(\overline{W}_{k,t} - \bar{s}_{k,t}) \tag{29a}$$

$$s.t. \quad \sum_{i\in N} (p_{i,t}^{G} + r_{i,t}^{p+} - p_{i,t}^{D} - \alpha_{k,t}) \geq P_t^{u} \tag{29b}$$

$$\alpha_{k,t} \geq -\bar{s}_{k,t} \tag{29c}$$

$$\sum_{i\in N} (p_{i,t}^{G} - r_{i,t}^{p-} - p_{i,t}^{D} + \beta_{k,t}) \leq P_t^{d} \tag{29d}$$

$$\beta_{k,t} \geq \bar{s}_{k,t} \tag{29e}$$

where set $\mathbb{C}_2 = \{p_{i,t}^{G}, r_{i,t}^{p+}, r_{i,t}^{p-}, \bar{s}_{k,t}, \alpha_{k,t}, \beta_{k,t}\}$ is the set of decision variables.

Once $\bar{s}_{k,t}$ is obtained for all wind turbines, the effective uncertainty set (27a)–(27c) can be used. Next, a robust solution is determined under the worst-case wind power scenario to ensure a stable system under all scenarios. However, the robust solution may be overconservative because it protects against the absolute worst-case scenario, which may not happen.

Most RO models rely only on the budget of uncertainty to adjust the solution's level of conservatism and do not consider the maximum admissible wind power. However, in this study, we demonstrated that the budget of uncertainty and the wind power admissibility level should be simultaneously considered in the optimization problem to accurately control the solution's degree of conservatism. In the following subsections, we describe how the proposed RO model determines the maximum admissibility of wind power and modifies the uncertainty set based on wind power admissibility before incorporating the budget of uncertainty in the model.

4. Effective Robust Second-Order Conic Programming Model

Here we present the ERSOCP model for the OPF problem with wind power integration. The proposed model considers the effect of voltage on the system by considering a relaxation for AC power flow equations and effectively tackles wind power uncertainty by implementing the effective budget of uncertainty approach. The complete ERSOCP model is presented as follows:

ERSOCP Model

Objective function:	Eq. (1)	
s.t.		
SOCR of power flow equations:	Eqs. (21), (22), (24), (25)	(30)
Operational constraints:	Eqs. (4)–(17)	
Effective budget of uncertainty:	Eqs. (27a) –(27d)	

To obtain the ERSCOP model (30), problem (29) must be solved to obtain the maximum wind power admissibility $\bar{s}_{k,t}$.

5. Numerical Results

To demonstrate the merits of the proposed ERSCOP model, we performed numerical analysis on three test systems, namely IEEE 14, IEEE 118, and reliability test system (RTS) 96, which were modified by adding a number of wind farms to the systems. In IEEE 14, we added one wind farm (capacity: 300 MW) to bus #9. In IEEE 118, we added wind farms to bus #1 (capacity: 500 MW), bus #9 (capacity: 500 MW), and bus #26 (capacity: 800 MW). In RTS 96, we added five wind farms to the system and reconstructed the system, as explained in a previous study [39]. In the IEEE 14 and IEEE 118 test systems, the wind power forecast errors were derived from the real wind sampling data from hourly wind power measurements [48] and as described in the literature [49]. However, in the RTS 96 test system, we obtained the data from another study [39] to make a comparison. All parameters for the lower and upper bounds of the available wind power were obtained from the historical data of recent years.

We considered four models:

i. Linear model [39]: The linear model [39] is considered where power flow equation (2) is linearized and reactive power and voltage are neglected. However, this model considers the effective budget of uncertainty [38].

ii. SOCP model [49]: The SOCP OPF model employs the traditional budget of uncertainty approach.

iii. ERSOCP model: The proposed ERSOCP model combines SOCP relaxation and effective budget of uncertainty.

iv. LINDOGLOBAL: The solution of ACOPF from the LINDOGLOBAL solver is obtained as the benchmark. The LINDOGLOBAL solver employs branch-and-cut methods to obtain the global optimal solution of ACOPF for relatively simple problems.

In this study, all the models were implemented in GAMS, and the global optimal solution was obtained using MOSEK on a computer with a 2.4-GHz CPU and 8-GB RAM.

In Table 1, the objective function values (total operating cost) for various budgets of uncertainty are presented for all the models. The quality of the SOCP and ERSOCP relaxations impact the solve time. The violation of equality constraint (23) provides a measure to the relaxation quality for the AC power flow equations, and is represented in Table 1.

Table 1 Comparison of the objective function value and solution accuracy.

		Linear Model [39]	**SOCP Model** [49]		**Proposed Model**	**ERSOCP**	**LINDOGLOBAL**
System	Budget	Objective Value ($)	Objective Value ($)	Eq (23) violation	Objective Value ($)	Eq (23) violation	Objective Value ($)

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IEEE 14	$\Gamma_t = 0.5$	8,339.12	8,348.89	0.0120	**8,351.54**	0.0037	8,354.41
	$\Gamma_t = 1.0$	8,291.34	8,402.01	0.0383	**8,351.54**	0.0061	8,405.19
IEEE 118	$\Gamma_t = 0.0$	129,190.50	129,339.60	0.0447	**129541.54**	0.0193	129,660.54
	$\Gamma_t = 1.0$	**130,056.12**	129,615.73	0.0711	129,794.12	0.0507	130,189.91
	$\Gamma_t = 2.0$	129,562.02	129,894.83	0.0632	**129,983.45**	0.0596	130.221.34
	$\Gamma_t = 3.0$	129,941.88	131,713.02	0.0398	**131,972.31**	0.0164	131,962.11
RTS 96	$\Gamma_t = 0.0$	**36,717.23**	36,521.23	0.0562	36,702.23	0.0134	36,756.23
	$\Gamma_t = 1.5$	37,189.67	37,061.67	0.0491	**37,291.88**	0.0256	37,386.90
	$\Gamma_t = 3.5$	38,622.91	38.561.12	0.0560	**39,029.32**	0.0322	39,165.33
	$\Gamma_t = 5.0$	40,192.54	39,894.19	0.0598	**40,239.34**	0.0301	40,359.12

Eq (23) violation.: $\sum_{i,j} |c_{ij}^2 + s_{ij}^2 - c_{ii}c_{jj}|$

Table 2 Comparison of the average computation CPU time (in seconds).

System	Budget	Linear Model [39]	SOCP Model [49]	Proposed ERSOCP Model	LINDOGLOBAL
IEEE 14	$\Gamma_t = 0.5$	0.078	0.113	0.199	2.42
	$\Gamma_t = 1.0$	0.081	0.122	0.212	2.48
IEEE 118	$\Gamma_t = 0.0$	0.091	1.721	1.931	29.78
	$\Gamma_t = 1.0$	0.090	1.680	1.768	30.21
	$\Gamma_t = 2.0$	0.089	1.634	1.762	30.72
	$\Gamma_t = 3.0$	0.091	1.611	1.771	30.89
RTS 96	$\Gamma_t = 0.0$	0.104	1.801	1.912	42.13
	$\Gamma_t = 1.5$	0.997	1.825	1.891	45.72
	$\Gamma_t = 3.5$	0.104	1.801	1.912	42.41
	$\Gamma_t = 5.0$	0.993	1.604	1.939	41.02

The last column in Table 1 provides the global solution LINDOGLOBAL as a reference to evaluate the solution accuracy of each model. As can be observed, in most cases, the proposed ERSOCP model yielded a more accurate solution that was closer to the solution of LINDOGLOBAL compared to other alternatives. In addition, the proposed ERSOCP model outperformed the SOCP model with the traditional budget of uncertainty approach. In particular, the quality of the SCOP relaxation depends on the violations of equation (23). If the optimal solution satisfies the relaxed constraint (25) at equality, it means constraint (23) is satisfied with no violation, and thus the SOCP relaxation is exact. The proposed ERSOCP model corresponded to smaller violations in constraint (23), as

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clearly observed from the higher degree of accuracy achieved by the proposed model. Furthermore, as the budget of uncertainty increased, the objective function value increased as more budget corresponded to more uncertainty in the system, which in turn increased the operational cost. As this budget increased, the system produced a more robust solution because more budget was allowed for immunization against uncertainty. These observations are consistent with the literature [28, 38, 39].

The computation times of all four models are presented in Table 2; for each row, we solved problems three times and reported the average times. The linear model [39] exhibited the lowest computation time because it linearizes all nonlinear equations and provides a simplified version of the OPF problem. The LINDOGLOBAL approach required the maximum computational time. The proposed ERSOCP model exhibited a slightly higher computational time compared to the SOCP model [31] because, in the proposed model, a linear problem must be solved first (29), and by obtaining the values of maximum wind power admissibility, the ERSCOP model can be established. However, the additional time required by the proposed model over the SOCP model is not significant. For instance, in the IEEE 118 test system, for $\Gamma_t = 3$, the computational time of the proposed model was only 9% higher than that of the SOCP model [31].

A comparison of solution accuracy and computational time for different models can be made in Tables 1 and 2. Such comparison allows users to select the most appropriate model based on their requirements for accuracy and computational time. As can be seen from Tables 1 and 2, the proposed model is highly efficient in terms of accuracy and computational time, thus, providing a promising choice for use in real-world problems.

The proposed model is beneficial for large systems with high amounts of wind power integration. In particular, for high uncertainty and large power systems, the challenges associated with handling wind power uncertainty make commercial solvers such as LINDOGLOBAL computationally impractical (Table 2). In contrast, the proposed model is highly efficient and practical even for large power systems because it is tractable and scalable. From the managerial point of view, the proposed model aids in identifying the exact trade-off between robustness and cost to tackle wind power uncertainty within a budget.

6. Conclusions

To overcome the limitations of traditional methods, in this study, we proposed a robust OPF model considering wind power uncertainty. We considered the wind power uncertainties by adopting the recent advancements in uncertainty theory, that is, the effective budget of uncertainty method and the nonlinearities of power flow equations were considered using a tight SOCR. We demonstrated that the proposed ERSOCP model could accurately consider the effects of voltage on the power flow and effectively model uncertainties given the physical limits of the system. The numerical results demonstrated the merits of the ERSOCP model. The ERSOCP model can obtain a more accurate solution in a reasonably short computational time.

In future research, the proposed model can be employed for different problems, such as the unit commitment problem. Furthermore, the possibility of a new conic relaxation for nonlinear AC power flow equations can be explored using exponential conic programming models, power conic programming models, or a combination of both.

Author Contributions

Lie Zhang: Conceptualization; methodology, validation, supervision, project administration; Reza D. Fakhrabadi: methodology, software, validation, investigation, supervision, project administration, writing; Maryam Khoshkhoo: methodology, formal analysis, data curation and collection, investigation, writing; Husien Salama: Conceptualization; software, formal analysis, data curation, and collection, writing

Competing Interests

The authors have declared that no competing interests exist.

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