

Article

Not peer-reviewed version

Protograph LDPC-Coded Superposition Bandwidth-Efficient Modulation for MIMO Channels with Mixed-ADC Architectures for Future Wireless Networks

[Duc A. Hoang](#), [Thang N. Le](#), [Hieu T. Nguyen](#)*

Posted Date: 31 July 2025

doi: 10.20944/preprints202507.2640.v1

Keywords: Large-scale MIMO; superposition modulation; protograph LDPC; low-resolution ADC; message passing; PEXIT; iterative detection and decoding; 5G/6G; IoT



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Article

Protograph LDPC-Coded Superposition Bandwidth-Efficient Modulation for MIMO Channels with Mixed-ADC Architectures for Future Wireless Networks

Duc A. Hoang ¹ , Thang N. Le ² and Hieu T. Nguyen ^{3,*}

¹ Faculty of Telecommunications, Posts and Telecommunications Institute of Technology, Hanoi 100000, Vietnam

² Postgraduate Studies Faculty, Posts and Telecommunications Institute of Technology (PTIT), Hanoi 100000, Vietnam

³ Dept. of Science and Industry Systems, Faculty of Technology, Natural Sciences and Maritime Sciences, University of Southeast Norway, NO 3603 Kongsberg, Norway

* Correspondence: hieu.nguyen@usn.no; Tel.: +47-942-90-746

Abstract

Low-resolution ADCs provide a power- and cost-efficient solution for large-scale MIMO systems in next-generation wireless networks (B5G/6G, IoT). However, coarse quantization under high-order modulation increases noise and interference, degrading signal detection performance. This paper proposes a mixed-ADC architecture combined with superposition modulation to improve both performance and spectral efficiency. A modified protograph-based EXIT (PEXIT) algorithm is developed to characterize system behavior, incorporating MIMO fading, heterogeneous ADC resolutions, and LDPC decoding effects. The method accurately predicts decoding thresholds across diverse antenna configurations and power allocation strategies. Simulation results confirm that the mixed-ADC approach yields significant performance gains, especially in setups with a limited number of receive antennas.

Keywords: Large-scale MIMO; superposition modulation; protograph LDPC; low-resolution ADC; message passing; PEXIT; iterative detection and decoding; 5G/6G; IoT

1. Introduction

LS-MIMO is a foundational technology in 5G and emerging 6G systems, offering high data rates and reliable connectivity across diverse use cases [1–3]. However, scaling LS-MIMO systems to hundreds of antennas presents a critical challenge due to the substantial power consumption of numerous RF chains and high-resolution analog-to-digital converters (ADCs) [4,5]. A promising solution is to replace traditional high-resolution ADCs with energy-efficient low-resolution alternatives [5–11,13–15].

In this work, we propose an integrated detection-and-decoding framework for LS-MIMO systems that combines superposition-based high-order modulation to improve spectral efficiency with a mixed-resolution ADC architecture (supporting both dual and triple modes) to reduce power consumption. At the receiver, a double-layer factor-graph detector is employed in conjunction with protograph-based LDPC codes to enhance error-correction capability. An analytical framework is also developed to evaluate system performance as a function of (i) MIMO array size, (ii) LDPC code structure, (iii) superposition modulation coefficients, and (iv) the configuration of ADC resolutions, including extremely low-, low-, and high-resolution chains.

Extensive research has investigated the effects of coarse quantization and hardware impairments on system performance in MIMO systems [9,11,13–15]. Nguyen et al. [10] proposed learning-based techniques that exploit parity-check redundancy or payload data to enhance robustness under imperfect or missing channel state information (CSI), demonstrating significant gains in low-resolution ADC environments. Similarly, Gao et al. [9] applied deep learning for channel estimation in mixed-ADC

architectures, where a small subset of antennas is equipped with high-resolution ADCs. Their method uses high-quality observations to infer channel information for low-resolution antennas, achieving strong performance even in the extreme case of 1-bit quantization.

In 5G New Radio, protograph LDPC codes serve as the primary channel-coding scheme, offering flexible architectures across various block lengths and code rates to support eMBB and URLLC. Looking ahead to 6G, their adaptability and high spectral efficiency make them well-suited for data-intensive applications, including holographic telepresence and intelligent machine-to-machine communication [27]. Therefore, this work adopts protograph LDPC codes for further investigation.

Moreover, superposition modulation (SM) is a non-bijective signaling method where multiple binary antipodal symbols are linearly combined using predefined weighting coefficients. A serial bitstream is first split into parallel branches, each bit mapped to +1, -1, scaled by real or complex weights, and coherently summed to form a single complex-valued symbol for transmission [30]. SM offers several advantages, notably improved spectral efficiency. For instance, multi-layer superposition modulation (MLSM) based on multiple-slope-keying chirp-spread spectrum (MSK-CSS) has been shown to enhance LoRa waveform throughput in IoT applications [31].

Recent studies highlight growing interest in mixed-resolution ADC architectures as an energy- and cost-efficient solution for massive MIMO systems. Liu et al. [18] analyzed secrecy performance in cell-free massive MIMO with mixed ADC/DACs, showing enhanced physical-layer security under hybrid quantization. In radar, Wang et al. [19] proposed a PMCW MIMO radar using mixed-ADCs for efficient angle-Doppler imaging with lower hardware cost. For extra-large MIMO, Liu et al. [20] balanced resolution and performance to support massive access. In IRS-assisted mmWave systems, robust channel estimation methods were developed to counter low-resolution ADC distortion [21,22]. Gao et al. [23] advanced detection and estimation in mmWave with mixed-ADC hardware. Elgabri et al. [24] showed that combining multi-bit and one-bit ADCs achieves good performance-cost trade-offs, while Zhao et al. [25] proposed a robust detection scheme for multi-user mixed-ADC MIMO. Collectively, these works validate the versatility of mixed-ADC designs and motivate further research into coding and modulation under hybrid quantization.

Motivated by the above observations, the present study targets mixed-ADC LS-MIMO configurations, where more than two distinct resolution levels are employed to improve receiver performance while keeping power consumption and complexity at a low level. A theoretical framework based on an extrinsic information exchange algorithm is also derived for this newly proposed mixed-ADC system to quantify its performance when the channel-coding stage employs capacity-approaching yet computationally efficient protograph LDPC schemes. This paper presents a comprehensive design and performance evaluation of energy- and bandwidth-efficient LS-MIMO systems that integrate superposition modulation, protograph LDPC coding, and a triple mixed-ADC architecture. The key contributions are as follows:

i) Mixed-ADC architecture for LS-MIMO with superposition modulation: We propose a practical *dual- and triple-resolution* mixed-ADC front-end that partitions the receive antenna array into one-bit (extremely low), two-bit (low), and five-bit (high) resolution groups. This configuration balances power efficiency and performance, with high-resolution outputs compensating for the distortion from low-resolution ADCs.

ii) PEXIT analysis for mixed-ADC superposition modulation: We develop a triple *mixed-ADC LS-MIMO PEXIT* algorithm, extendable to dual or single low-resolution setups. It tracks mutual information transfer across the joint detector-decoder graph and yields accurate iterative decoding thresholds based on superposition weights, ADC resolution mix, antenna partitioning, and LDPC protograph parameters. This framework guides the design of protograph LDPC codes optimized for various resolution splits and high-order superposition modulation schemes (e.g., 64-ary, 256-ary).

The remainder of this paper is organized as follows: Section 2 introduces the protograph LDPC-coded mixed-ADC LS-MIMO system with superposition modulation. Section 3 presents the modulation model conversion and joint detection-decoding algorithm using a double-layer factor graph. The

mixed-ADC LS-MIMO PEXIT algorithm is detailed in Section 4, followed by theoretical analysis in Section 5. Section 6 provides simulation results that validate the analytical predictions. Finally, Section 7 concludes the paper.

2. System Model

Consider a MIMO system with N_t transmit and N_r receive antennas. Let $\mathbf{b}$ be an input bit sequence of length K_c , encoded by a protograph LDPC encoder into a codeword $\mathbf{c}$ of length N_c . The coded bits are then grouped into blocks of size $2G$, where $G = \log_2(M)/2$ and M is the modulation order. Each $2G$ -bit block is split into two G -bit parts: the in-phase (I) and quadrature (Q) components. In the superposition modulation scheme, each bit $U_{I,l}$ (for $l = 1, \dots, G$) is BPSK-modulated and scaled by a factor $\alpha_{I,l}$ such that $\sum_{l=1}^G \alpha_{I,l}^2 = 0.5$. The same process is applied to the Q channel using weights $\alpha_{Q,l}$, satisfying $\sum_{l=1}^G \alpha_{Q,l}^2 = 0.5$. The resulting M -ary superposition-modulated symbols are input into a MIMO encoder using the V-BLAST architecture, where each of the N_t antennas transmits a distinct symbol, forming N_t parallel data streams. These signals propagate through the MIMO channel and are captured by N_r receive antennas, as illustrated in Figure 1.

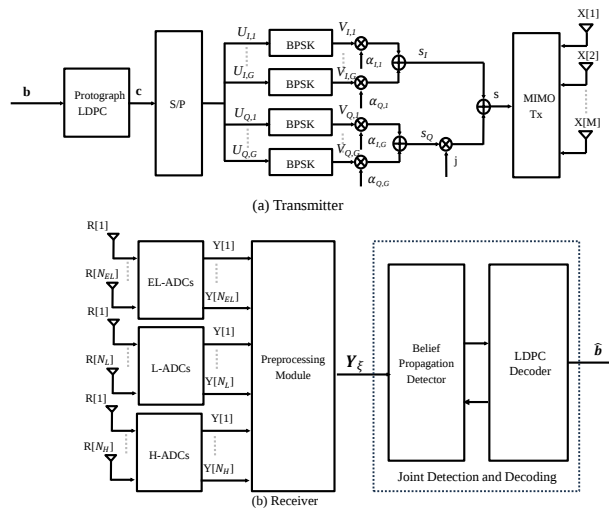


Figure 1. Block Diagram of Transceiver in Protograph LDPC Coded MIMO Communication Systems with Low-Resolution ADCs: (a) Transmitter uses superposition high-order modulation scheme for bandwidth efficiency. (b) Receiver has three sets of low-resolution ADCs to cut down the power consumption.

We represent this communication channel by the following equation:

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{W} \quad (1)$$

where

- $\mathbf{X} \in \mathbb{C}^{N_t \times 1}$ denotes the transmitted MIMO codeword,
- $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ represents the channel matrix,
- $\mathbf{W} \in \mathbb{C}^{N_r \times 1}$ is an additive white Gaussian noise (AWGN) vector with zero mean and variance N_0 .
- $\mathbf{R} \in \mathbb{C}^{N_r \times 1}$ is a received signal vector.

Assume each modulated symbol $S[m]$ (for $m = 1, \dots, N_t$) is directly mapped to the corresponding entry $X[m]$ in the MIMO codeword $\mathbf{X}$, with normalized average symbol energy $E_s = \mathbb{E}[\|\mathbf{X}\|^2] = 1$. The channel matrix $\mathbf{H}$ has i.i.d. entries $H[n, m] \sim \mathcal{CN}(0, 1)$. Perfect channel state information (CSI) is assumed at the receiver, but not at the transmitter.

By employing the superposition modulation scheme illustrated in Figure 1, the transmitted symbol $X[m]$ can be written as

$$X[m] = \sum_{l=1}^G \alpha_{I,l} V_{I,l}[m] + j \sum_{l=1}^G \alpha_{Q,l} V_{Q,l}[m], \quad (2)$$

where $\alpha_{I,l}$ and $\alpha_{Q,l}$ represent the superposition (power) allocation factors for the in-phase and quadrature components, respectively, and $V_{I,l}[m]$ and $V_{Q,l}[m]$ are the BPSK-modulated symbols on each branch. Here, j denotes the imaginary unit.

In this work, we adopt three types of ADCs: one-bit (extremely low-resolution), two-bit (low-resolution), and five-bit (higher-resolution), as illustrated in Figure 1. The use of one- and two-bit ADCs is motivated by their energy efficiency, as shown in [4], while five-bit ADCs are chosen for the high-resolution tier based on [28], which demonstrates that they offer performance comparable to full-resolution converters. This forms a triple mixed-ADC architecture. The framework can be readily adapted to a dual mixed-ADC setup by merging the low and extremely low tiers, or to a uniform single-resolution design by assigning one-bit ADCs to all tiers.

The received signal vector $\mathbf{R}$ is composed of three sub-vectors: $\mathbf{R}_{EL}$ of length N_{EL} , $\mathbf{R}_L$ of length N_L , and $\mathbf{R}_H$ of length $N_H = N_r - N_L - N_{EL}$. These correspond to the signals arriving at the extremely low-resolution, low-resolution, and high-resolution antenna sets, respectively. We can therefore decompose the received signal vector $\mathbf{R}$ into these three sub-vectors as follows:

$$\mathbf{R}_\xi = \mathbf{H}_\xi \mathbf{X} + \mathbf{W}_\xi, \xi \in \{EL, L, H\}. \quad (3)$$

3. Receiver Design

3.1. Analog-to-Digital Conversion Model

Let $Q(\cdot)$ denote the quantization function. The relationship between the input and output of the Q_ξ -bit ADC is given by

$$\mathbf{Y}_\xi = Q(\mathbf{R}_{\xi, \text{re}}) + j Q(\mathbf{R}_{\xi, \text{im}}), \quad (4)$$

where $\mathbf{R}_{\xi, \text{re}}$ and $\mathbf{R}_{\xi, \text{im}}$ denote the real and imaginary parts of the received signal $\mathbf{R}_\xi$, respectively. Under the AQNM framework, the relationship between the quantizer input and output in (4), can be described mathematically as follows [4]:

$$\mathbf{Y}_\xi = \varphi_\xi \mathbf{R}_\xi + \mathbf{W}_{\xi, \Phi} \quad (5)$$

where φ_ξ is the quantizer gain factor, and $\mathbf{W}_\xi$ and Φ_ξ denote the corresponding additive quantization noise vectors. The gain factor is given by $\varphi_\xi = 1 - \rho_\xi$, where ρ_ξ represents the inverse signal-to-quantization-noise ratio for the one-bit ADCs. In this work, we employ uniform quantizers [38] where the values of φ_ξ given in Table 1, though the following analysis can also be extended to non-uniform quantizers.

Table 1. Optimal Values of $T_\xi, \xi \in \{EL, L, H\}$ for different ADC resolution levels [28].

Resolution	1	2	3	4	5
T_ξ	1.669	2.0912	2.4613	2.7909	3.0285
φ_ξ	0.6261	0.8796	0.9628	0.9885	0.9936

In the pre-processing module, the higher-order modulation is effectively mapped to an equivalent BPSK format, allowing the belief propagation detection algorithm to operate on the double Tanner graph [13],[29]. This strategy highlights a crucial benefit of superposition modulation in massive

MIMO systems with high-order constellations, as it reduces the detector complexity for the MIMO signal. To do this, a concise representation of the transmitted MIMO symbol is given by [12]:

$$\begin{bmatrix} \mathbf{X}_I \\ \mathbf{X}_Q \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_R & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_Q \end{bmatrix} \begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_Q \end{bmatrix} \quad (6)$$

Here, $\boldsymbol{\alpha}_R$ and $\boldsymbol{\alpha}_I$, both of size $N_t \times G$, contain the superposition coefficients associated with the in-phase and quadrature channels, respectively. The quantized received signal vector is written as

$$\mathbf{Y}_{\xi} = \varphi_{\xi} \mathbf{H}_{\xi} \mathbf{V}_{\xi} + \mathbf{W}_{\xi} \quad (7)$$

By definition, the vector $\mathbf{V}_{\xi}$ contains $2 \times N_t \times G$ entries, each drawn from a BPSK constellation $\{-1, +1\}$.

As shown in Figure 2, the detection graph consists of N_r observation nodes (one per receive antenna) and $N_{\xi,t} = 2 \times G \times N_t$ binary symbol nodes. In the triple mixed-ADC setting, observation nodes are divided into three groups: N_{EL} one-bit (extremely low-resolution), N_L two-bit (low-resolution), and N_H five-bit (higher-resolution) nodes. All observation nodes are fully connected to the symbol nodes due to the broadcast nature of wireless transmission. Through these connections, reliability information from higher-resolution nodes is propagated across the graph, helping to improve the estimates at lower-resolution nodes over iterations. This structure highlights how a small number of high-resolution ADCs can significantly boost performance in mixed-ADC systems.

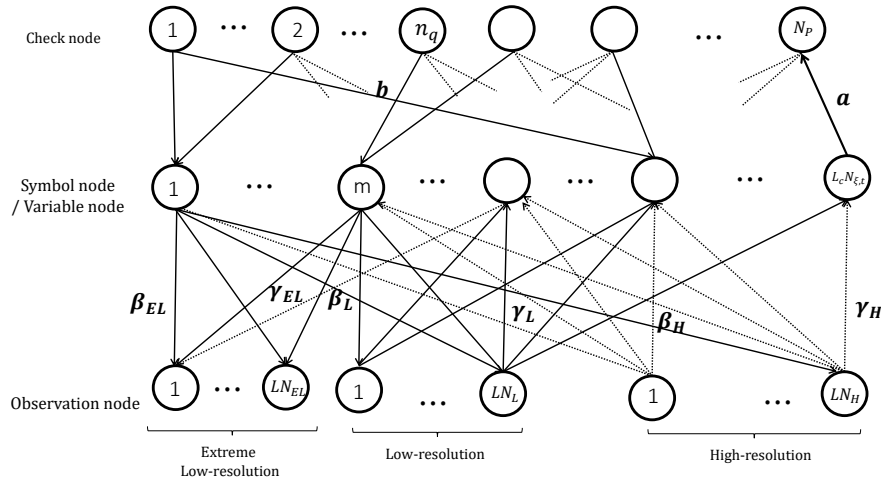


Figure 2. Double-Layer Graph for Mixed ADC Receiver: The lower layer handles MIMO signal detection using three separate sub-graphs—one for each ADC resolution level (extremely low, low, and high). The upper layer performs message passing for channel decoding.

In the iterative joint detection and decoding algorithm, five distinct message types are exchanged over the graph:

- $L_{\gamma,\xi}[n_{\xi}, m]$, $\xi \in \{EL, L, H\}$: the message passed from the n_{ξ} -th observation node to the m -th symbol node.
- $L_a[n_p, n_q]$: the message passed from the n_p -th variable node to the n_q -th check node.
- $L_b[n_q, n_p]$: the message passed from the n_q -th check node to the n_p -th variable node.
- $L_{\beta,\xi}[m, n_{\xi}]$: the message passed from the m -th symbol node to the n_{ξ} -th observation node.
- $L_{\Gamma}[m]$: the *a posteriori* log-likelihood ratio (LLR) of the symbol $V_{\xi}[m]$.

In the following, we briefly describe the workings of the message-passing joint detection and decoding receiver, which includes a soft symbol cancellation mechanism as in [12].

3.1.1. Message Passed from Observation Nodes to Symbol Nodes

$$L_\gamma[n_\xi, m] = \ln \frac{\Pr(\hat{Y}_\xi[n_\xi, m] | \mathbf{H}_\xi, V_\xi[m] = +1)}{\Pr(\hat{Y}_\xi[n_\xi, m] | \mathbf{H}_\xi, V_\xi[m] = -1)} = \frac{4\varphi_\xi}{\Psi_\xi[n_\xi, m]} \Re(H_\xi^*[n_\xi, m] \hat{Y}_\xi[n_\xi, m]). \quad (8)$$

where

$$\hat{Y}_\xi[n_\xi, m] = Y_\xi[n_\xi, m] - \varphi_\xi \sum_{t=1, t \neq m}^{N_{t,\xi}} H_\xi[n_\xi, t] \hat{V}_\xi[n_\xi, t]. \quad (9)$$

Here $\hat{Y}_\xi[n_\xi, m]$ represents the received signal for $V_\xi[m]$ at the n_ξ -th observation node after the interference cancellation step and $\hat{V}_\xi[n_\xi, m]$ is given in the equation below:

$$\hat{V}_\xi[n_\xi, m] = \tanh\left(\frac{L_\beta[m, n_\xi]}{2}\right), \quad (10)$$

By treating the residual interference as additive Gaussian noise, the power of the combined residual interference and noise term $Z_\xi[n_\xi, m]$ can be calculated as

$$\begin{aligned} \Psi_\xi[n_\xi, m] = \varphi_\xi^2 \sum_{t=1, t \neq m}^{N_{t,\xi}} |H_\xi[n_\xi, t]|^2 (1 - |\hat{V}_\xi[n_\xi, t]|^2) + \varphi_\xi^2 N_0 + \\ \varphi_\xi (1 - \varphi_\xi) \left(\sum_{m=1}^{N_{t,\xi}} |H_\xi[n_\xi, m]|^2 + N_0 \right). \end{aligned} \quad (11)$$

3.1.2. Message Passed from Variable Nodes to Check Nodes

The n_p -th variable node receives messages from two sources: the N_r observation nodes in the MIMO detection graph and the check nodes in the LDPC decoding graph. The extrinsic message it sends to the n_q -th check node is the sum of all incoming messages, excluding the one from the n_q -th check node. Hence, we obtain

$$L_a[n_p, n_q] = \sum_{\xi \in \{EL, L, H\}} \sum_{n_t \in \mathcal{S}_{o,\xi}(n_p)} L_{\gamma,\xi}[n_t, n_p] + \sum_{t \in \mathcal{S}_c(n_p) \setminus n_q} L_b[n_t, n_p]. \quad (12)$$

Here, $\mathcal{S}_c(n_p)$ is the set of check nodes that connect to the n_p -th variable node, while $\mathcal{S}_{o,\xi}(n_p), \xi \in \{EL, L, H\}$ denotes the set of observation nodes that connect to the m -th variable node.

3.1.3. Message Passed from Check Nodes to Variable Nodes

The message passed from the n_q -th check node to the n_p -th variable node follows the standard message-passing algorithm [44], and is given by

$$L_b[n_q, n_p] = \ln \frac{1 - \prod_{n_t \in \mathcal{S}_v(n_q) \setminus n_p} \frac{1 - e^{L_a[n_t, n_q]}}{1 + e^{L_a[n_t, n_q]}}}{1 + \prod_{n_t \in \mathcal{S}_v(n_q) \setminus n_p} \frac{1 - e^{L_a[n_t, n_q]}}{1 + e^{L_a[n_t, n_q]}}}. \quad (13)$$

Here, $\mathcal{S}_v(n_q)$ denotes the set of variable nodes connected to the n_q -th check node. In practice, the computation of $L_b[n_q, n_p]$ may be simplified by leveraging the $\tanh(\cdot)$ function.

3.1.4. Message Passed from Symbol Nodes to Observation Nodes

As previously described, the m -th symbol node receives messages from both observation and check nodes. The extrinsic message it sends to the n_ξ -th observation node includes all incoming

messages except the one from the n_{ξ} -th node. Thus, the message from the m -th variable node to the n_{ξ} -th observation node is given by

$$L_{\beta,\xi}[m, n_{\tau}] = \sum_{\xi \in \{EL, L, H\}} \sum_{t \in \mathcal{S}_{o,\xi}(m)} L_{\gamma}[t, m] + \sum_{n_t \in \mathcal{S}_c(n_p)} L_b[n_t, n_p] - L_{\gamma,\xi}[n_{\xi}, m], \quad (14)$$

where $\mathcal{S}_{o,\xi}(m)$ and $\mathcal{S}_c(n_p)$ denote the sets of observation nodes and check nodes, respectively, that connect to the m -th symbol node.

3.1.5. A Posteriori Messages of Codeword Bits

The posterior LLR for the m -th transmit symbol at the conclusion of each iteration is the combined sum of messages from both the observation nodes and the check nodes, expressed as

$$L_{\Gamma}[m] = \sum_{\xi \in \{EL, L, H\}} \sum_{n \in \mathcal{S}_{o,\xi}(m)} L_{\gamma,\xi}[n_{\xi}, m] + \sum_{n_q \in \mathcal{S}_c(n_p)} L_b[n_q, n_p]. \quad (15)$$

Next, the *a posteriori* LLR is passed to the hard-decision device to generate the decoded version of the codeword bit, following the rule:

$$\hat{c}[m] = \begin{cases} 0, & L_{\Gamma}[m] > 0; \\ 1, & \text{Otherwise.} \end{cases} \quad (16)$$

where $\hat{c}[m]$ represents the decoded version of $c[m]$. Consequently, the decoded information sequence $\hat{\mathbf{b}}$ is obtained.

4. Modified PEXIT Algorithm for Triple Mixed ADCs

We build upon the classic PEXIT algorithm framework [13,29,45,46] and propose an adaptation tailored for mixed-ADC LS-MIMO systems. Originally proposed for SISO AWGN channels [45], it was later used to design robust codes for AWGN [47] and extended to fading channels [46]. We present in the following section a new variant of the PEXIT algorithm designed specifically for LS-MIMO systems employing mixed-ADCs at the receiver, combined with a joint detection and decoding framework.

The mutual information flow of the joint detection-decoding receiver is illustrated in Figure 3 and Figure 4, based on a simplified version of the double-layer graph in Figure 2. For analysis, variable and symbol nodes are separated and linked by a forward combiner (forward flow) and a backward combiner (backward flow). The MIMO component includes N_{EL} one-bit, N_L two-bit, and N_H five-bit observation nodes, along with $N_{\xi,t} = 2N_t G$ symbol nodes and $N_{\xi,t} \times N_r$ edges. This structure is replicated Π times (equal to the number of channel uses) in the full double-layer graph. The LDPC decoding component consists of N_p variable nodes, N_Q check nodes, and edges defined by the proto-matrix $\mathbf{B} \in \mathbb{Z}_+^{N_Q \times N_p}$, where $\mathbf{B}[n_q, n_p]$ specifies the number of edges connecting check node n_q to variable node n_p .

We define nine main types of mutual information, corresponding to the nine extrinsic messages on the double-layer graph in Figure 2, on the joint MIMO detection and LDPC protograph decoding graph, as follows:

- $I_{\gamma_{\xi}}[n_{\xi}, m]$ is the extrinsic mutual information between the LLR value $\gamma_L[n_{\xi}, m]$ sent by the n_{ξ} -th observation node to the m -th variable node and the m -th corresponding coded bit.
- $I_a[n_p, n_q]$ is the extrinsic mutual information between the LLR value $a[n_p, n_q]$ sent by the n_p -th variable node to the n_q -th check node and the n_p -th corresponding coded bit.
- $I_b[n_q, n_p]$ is the extrinsic mutual information between the LLR value $b[n_q, n_p]$ sent by the n_q -th check node to the n_p -th variable node and the n_p -th corresponding coded bit.
- $I_{\beta_{\xi}}[m, n_{\xi}]$ is the extrinsic mutual information between the LLR value $\beta_{\xi}[m, n_{\xi}]$ sent by the m -th symbol node to the n_{ξ} -th observation node and the m -th corresponding symbol.

- $I_{\Gamma}[n_p]$ is the posteriori mutual information between the a posteriori LLR value $\Gamma[n_p]$ and the corresponding codeword bit of the n_p -th variable node.

Let P_p denote the puncturing label of the n_p -th variable node, where $P_p = 0$ if the node is punctured (i.e., its codeword bit is not transmitted), and $P_p = 1$ otherwise.

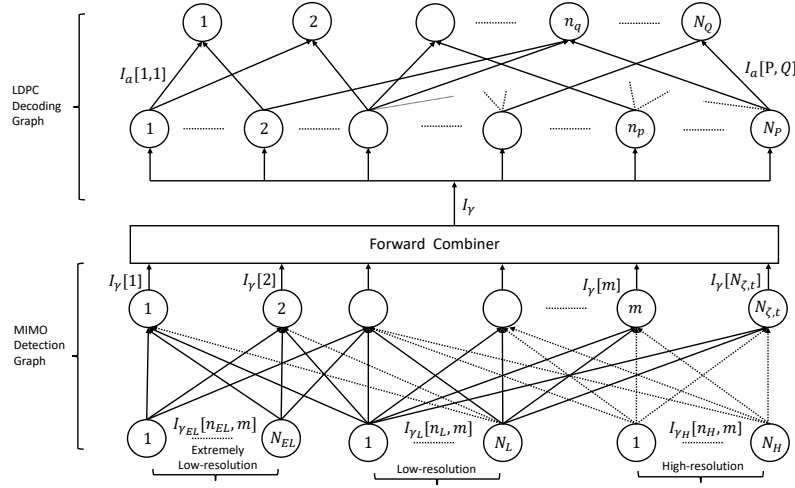


Figure 3. Forward information flow.

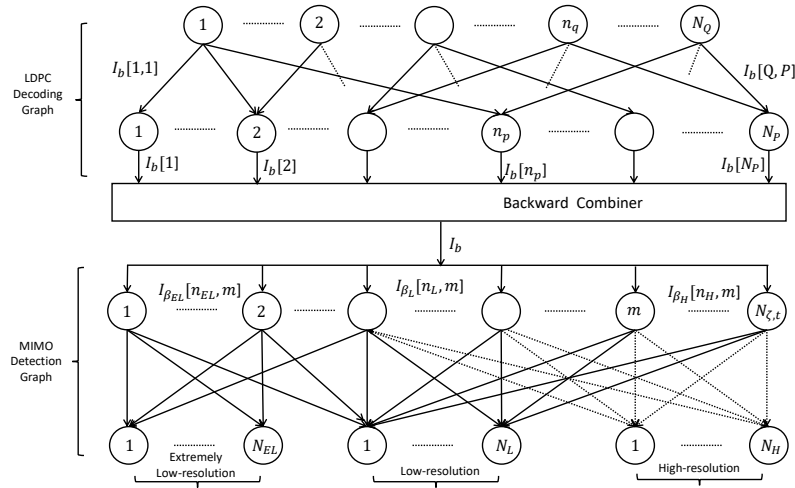


Figure 4. Backward information flow.

4.1. Forward Mutual Information Flow

The forward mutual information flow, illustrated in Figure 3, traces the propagation of extrinsic information from observation nodes through symbol and variable nodes to check nodes. We present the corresponding mutual information functions, adapted from the single-ADC scheme in [12] to the mixed-ADC setting, highlighting the interaction between low- and high-resolution ADC subgraphs.

4.1.1. Mutual Information from Observation Nodes to Symbol Nodes

The m -th symbol node receives N_{EL} , N_L , and N_H LLR messages from the respective one-bit, two-bit, and five-bit observation nodes, reflecting the broadcast nature of wireless transmission as shown by the fully connected graph in Figure 3. The mutual information from the n_{ζ} -th observation node to the m -th symbol node is given by

$$I_{\gamma_{\zeta}}[n_{\zeta}, m] = J\left(\sqrt{\frac{8\varphi_{\zeta}^2 |H_{\zeta}[n_{\zeta}, m]|^2}{\Psi_{\zeta}[n_{\zeta}, m]}}\right), \quad \zeta \in \{EL, L, H\}, \quad (17)$$

where the function $J(x)$ is defined in [44].

4.1.2. Mutual Information from Symbol Nodes to Variable Nodes

The m -th symbol node collects a total of $N_r = N_{EL} + N_L + N_H$ messages from the N_{EL} extremely low-resolution observation nodes, the N_L low-resolution observation nodes and the N_H high-resolution nodes, as depicted in Figure 3. The extrinsic mutual information $I_\gamma[m]$ is obtained by the following equation:

$$I_\gamma[m] = J\left(\sqrt{\sum_{\xi \in \{EL, L, H\}} \sum_{n_\xi=1}^{N_\xi} \frac{8 \varphi_\xi^2 |H_\xi[n_\xi, m]|^2}{\Psi_\xi[n_\xi, m]}}\right). \quad (18)$$

Assuming an infinite code length ($N_c \rightarrow \infty$), each variable node bit is equally likely to be transmitted over any of the $N_{\xi,t}$ symbol nodes. Thus, the forward combiner aggregates mutual information from all symbol nodes and passes it to the variable nodes. Let I_γ denote this average mutual information. We have

$$I_\gamma = \frac{1}{N_{\xi,t}} \sum_{m=1}^{N_{\xi,t}} I_\gamma[m], \quad (19)$$

where $I_\gamma[m]$ is given in (18).

Hence, the channel mutual information passed from the symbol nodes to the n_p -th variable node is expressed as

$$I_\gamma[n_p] = P_p I_\gamma, \quad \forall p = 1, 2, \dots, N_p, \quad (20)$$

where $P_p = 1$ if node n_p is unpunctured, and $P_p = 0$ otherwise.

4.1.3. Mutual Information Flow from Variable Nodes to Check Nodes

The expression for the mutual information transferred from the n_p -th variable node to the n_q -th check node, $I_a[n_p, n_q]$, is identical to that of the conventional PEXIT algorithm in [45] and given by

$$I_a[n_p, n_q] = J\left(\sqrt{[J^{-1}(I_\alpha[n_p])]^2 + \sigma_b^2[n_p]}\right), \quad (21)$$

where

$$\sigma_b^2[n_p] = \sum_{n_t \in \mathcal{N}_c(n_p) \setminus n_p} \mathbf{B}[n_t, n_p] [J^{-1}(I_b[n_t, n_p])]^2,$$

where $J^{-1}(x)$ is given in [44].

4.2. Backward Mutual Information Flow

The backward mutual information flow traces the path along which the extrinsic mutual information travels from the check nodes, passing through the variable and symbol nodes, and finally reaching the observation nodes, as illustrated in Figure 4. In what follows, we derive the mutual information functions that propagate in the backward direction.

4.2.1. Mutual Information Flow from Check Nodes to Variable Nodes

The process of determining the mutual information that flows from the q -th check node to the p -th variable node parallels that of the conventional PEXIT algorithm in [45]. We have $I_b[n_q, n_p]$

$$I_b[n_q, n_p] = 1 - J(\sigma_a[n_q]), \quad (22)$$

where

$$\sigma_a^2[n_q] = \sum_{n_t \in \mathcal{N}_v(n_q) \setminus n_p} \mathbf{B}[n_q, n_t] [J^{-1}(1 - I_a[n_t, n_q])]^2.$$

4.2.2. Mutual Information Flow from Variable Nodes to Symbol Nodes

Let $I_b[n_p]$ denote the total mutual information that the n_p -th variable node receives from the check nodes. We can express the total mutual information as below

$$I_b[n_p] = \sum_{n_q \in \mathcal{N}_c(n_p)} I_b[n_q, n_p]. \quad (23)$$

Under the same infinite code length assumption, the probability that any symbol node transmits a codeword bit originating from the n_p -th variable node is $1 / (\sum_{p=1}^{N_p} P_p)$. Consequently, the backward combiner aggregates the mutual information across all variable nodes before forwarding it to the symbol nodes. The average mutual information from the variable nodes to the symbol nodes is thus given by

$$I_b = \frac{\sum_{n_p=1}^{N_p} P_p I_b[n_p]}{\sum_{n_p=1}^{N_p} P_p}. \quad (24)$$

4.2.3. Mutual Information from Symbol Nodes to Observation Nodes

The mutual information conveyed from the m -th symbol node to the n_{EL} -th low-resolution observation node, $I_{\beta_{EL}}[m, n_{EL}]$, is calculated as

$$I_{\beta_{EL}}[m, n_{EL}] = J \left(\sqrt{\sigma_{\gamma_{EL}^*}^2[m] + \sigma_{\gamma_L}^2[m] + \sigma_{\gamma_H}^2[m] + \sigma_b^2} \right), \quad (25)$$

where

$$\begin{aligned} \sigma_{\gamma_{EL}^*}^2[m] &= \sum_{n_t \in \mathcal{N}_{EL,o}(m) \setminus n_{EL}} [J^{-1}(I_{\gamma_{EL}}[n_t, m])]^2 = \sum_{n_t \in \mathcal{N}_{EL,o}(m) \setminus n_{EL}} \sigma_{\gamma_{EL}}^2[n_t, m] = \\ &= \sum_{n_t \in \mathcal{N}_{EL,o}(m) \setminus n_{EL}} \frac{8\varphi_{EL}^2 |H_{EL}[n_t, m]|^2}{\Psi_{EL}[n_t, m]}. \end{aligned}$$

The mutual information transferred from the m -th symbol node to the n_L -th low-resolution observation node, $I_{\beta_L}[m, n_L]$, is calculated as

$$I_{\beta_L}[m, n_L] = J \left(\sqrt{\sigma_{\gamma_L^*}^2[m] + \sigma_{\gamma_{EL}}^2[m] + \sigma_{\gamma_H}^2[m] + \sigma_b^2} \right), \quad (26)$$

where

$$\begin{aligned} \sigma_{\gamma_L^*}^2[m] &= \sum_{n_t \in \mathcal{N}_{L,o}(m) \setminus n_L} [J^{-1}(I_{\gamma_L}[n_t, m])]^2 = \sum_{n_t \in \mathcal{N}_{L,o}(m) \setminus n_L} \sigma_{\alpha_L}^2[n_t, m] = \\ &= \sum_{t \in \mathcal{N}_{L,o}(m) \setminus n_L} \frac{8\varphi_L^2 |H_L[n_t, m]|^2}{\Psi_L[n_t, m]}. \end{aligned}$$

The mutual information transferred from the m -th symbol node to the n_H -th high-resolution observation node, $I_{\beta_H}[m, n_H]$, is calculated as

$$I_{\beta_H}[m, n_H] = J \left(\sqrt{\sigma_{\gamma_H^*}^2[m] + \sigma_{\gamma_{EL}}^2[m] + \sigma_{\gamma_L}^2[m] + \sigma_b^2} \right), \quad (27)$$

where

$$\sigma_{\gamma_H^*}^2[m] = \sum_{n_t \in \mathcal{N}_{H,0}(m) \setminus n_H} [J^{-1}(I_{\gamma_H}[n_t, m])]^2 = \sum_{n_t \in \mathcal{N}_{H,0}(m) \setminus n_H} \sigma_{\gamma_H}^2[n_t, m] = \sum_{n_t \in \mathcal{N}_{H,0}(m) \setminus n_H} \frac{8\phi_H^2 |H_H[n_t, m]|^2}{\Psi_H[n_t, m]}.$$

Equations (25)–(27) illustrate the interaction between extremely low-, low-, and high-resolution ADCs. Due to the fully connected graph, mutual information from high-resolution nodes enhances that of low-resolution nodes via symbol nodes, and *vice versa*. This mutual reinforcement is key to improving mixed-ADC LS-MIMO performance, as demonstrated in the following sections.

4.3. The APP Mutual Information

Calculate $I_\Gamma[n_p]$ for the n_p -th variable node

$$I_\Gamma[n_p] = J \left(\sqrt{[J^{-1}(I_\alpha[n_p])]^2 + \sigma_b^2[n_p]} \right), \quad (28)$$

where

$$\sigma_b^2[n_p] = \sum_{n_t \in \mathcal{N}_c(n_p)} \mathbf{B}[n_t, n_p] [J^{-1}(I_b[n_t, n_p])]^2.$$

4.4. Modified PEXIT Algorithm for Triple Mixed ADCs

The proposed PEXIT procedure is derived by integrating the mutual information functions from the earlier subsections with the parameters of a specific MIMO setting, $N_t \times N_r$, the proto-matrix size $\mathbf{B}$ of dimension $N_Q \times N_P$, the channel parameter E_b/N_0 , and the resolution levels of the triple-mixed ADCs, Q_{EL} , Q_L , and Q_H . The resulting mixed-ADC LS-MIMO-PEXIT algorithm is outlined as follows:

- **Step 0: Initialization:**
 - Select the size of proto-matrix: $\mathbf{B}$
 - Calculate the coding rate: $R_c = \frac{N_P - N_Q}{\sum_{p=1}^{N_P} P_p}$
 - Calculate $N_0 = \frac{M}{R(E_b/N_0)}$
 - Obtain the values of ϕ_{EL} , ϕ_L and ϕ_H from Table 1 accordingly their resolution levels Q_{EL} , Q_L and Q_H , respectively
 - Set $I_{\beta_{EL}} = 0$, $I_{\beta_L} = 0$ and $I_{\beta_H} = 0$
 - Generate $3 \times F$ LS-MIMO channel realization matrices $\{\mathbf{H}_{\xi,1}, \mathbf{H}_{\xi,2}, \dots, \mathbf{H}_{\xi,F}\}$, $\xi \in \{EL, L, H\}$ which follow Rayleigh distribution
 - Set the parameters for the superposition modulation to M , calculate $G = \log_2(M)/2$
 - Set values to $\alpha_{I,1}, \alpha_{I,2}, \dots, \alpha_{I,G}$
 - Set values to $\alpha_{Q,1}, \alpha_{Q,2}, \dots, \alpha_{Q,G}$
- **Step 1: Preprocessing**
 - Forming matrices $\mathbf{ff}_I$ and $\mathbf{ff}_Q$
 - For each $f = 1, 2, \dots, F$, calculate $H_{\xi,f}$
- **Step 2: Observation to variable update**
 - For $f = 1, 2, \dots, F$
 - * For $m = 1, 2, \dots, N_{\xi,t}$ and $n_\xi = 1, 2, \dots, N_\xi$, $\xi \in \{EL, L, H\}$
 - Calculate $\sigma_{\beta_\xi} = J^{-1}(I_{\beta_\xi})$

- Generate $\beta_{\xi,f}[m, n_{\xi}] \sim \mathcal{N}\left(\pm \frac{\sigma_{\beta_{\xi}}^2}{2}, \sigma_{\beta_{\xi}}^2\right)$
- Estimate soft information $\hat{V}_f[m, n_{\xi}] = \tanh\left(\frac{\beta_{\xi,f}[m, n_{\xi}]}{2}\right)$
- Calculate $\Psi_{\xi,f}[n_{\xi}, m]$ by using formula (11).
- * For $m = 1, 2, \dots, M$
 - Calculate $I_{\gamma,f}[m]$ by using formula (18)
- Calculate the average of $I_{\gamma,f}$ over all the channel realizations

$$I_{\gamma}[m] = \frac{1}{F} \sum_{f=1}^F I_{\gamma,f}[m], \forall m = 1, 2, \dots, N_{\xi,t}.$$

- For $n_p = 1, 2, \dots, N_P$, calculate $I_{\gamma}[n_p]$

$$I_{\alpha}[n_p] = P_p \left(\frac{1}{N_{\xi,t}} \sum_{m=1}^{N_{\xi,t}} I_{\gamma}[m] \right).$$

Note that if the n_p -th variable node is punctured, then $P_p = 0$. Otherwise, $P_p = 1$.

• **Step 3: Variable to check update**

- For $n_p = 1, 2, \dots, N_P$ and $n_q = 1, 2, \dots, N_Q$, calculate $I_a[n_p, n_q]$:
 - * if $\mathbf{B}[n_p, n_q] \neq 0$, $I_a[n_p, n_q]$ is then calculated by using formula (21)
 - * If $\mathbf{B}[n_p, n_q] = 0$, $I_a[n_p, n_q] = 0$.

• **Step 4: Check to variable update**

- For $n_q = 1, 2, \dots, N_Q$ and $n_p = 1, 2, \dots, N_P$
 - * if $\mathbf{B}[n_q, n_p] \neq 0$, $I_b[n_q, n_p]$ is then calculated by using formula (22)
 - * If $\mathbf{B}[n_q, n_p] = 0$, $I_b[n_q, n_p] = 0$

• **Step 5: Symbol to observation update**

- For $f = 1, 2, \dots, F$
 - * For $m = 1, 2, \dots, N_{\xi,t}$ and $n_{EL} = 1, 2, \dots, N_{EL}$, $I_{\beta_{EL},f}[m, n_{EL}]$ is then calculated by using (25)
 - * For $m = 1, 2, \dots, N_{\xi,t}$ and $n_L = 1, 2, \dots, N_L$, $I_{\beta_{L,f}}[m, n_L]$ is then calculated by using (26)
 - * For $m = 1, 2, \dots, N_{\xi,t}$ and $n_H = 1, 2, \dots, N_H$, $I_{\beta_{H,f}}[m, n_H]$ is calculated by using formula (27)
- For $m = 1, 2, \dots, N_{\xi,t}$ and $n_{EL} = 1, 2, \dots, N_{EL}$

$$I_{\beta_L}[m, n_{EL}] = \frac{1}{F} \sum_{f=1}^F I_{\beta_{EL},f}[m, n_{EL}]$$

- For $m = 1, 2, \dots, N_{\xi,t}$ and $n_L = 1, 2, \dots, N_L$

$$I_{\beta_L}[m, n_L] = \frac{1}{F} \sum_{f=1}^F I_{\beta_{L,f}}[m, n_L]$$

- For $m = 1, 2, \dots, N_{\xi,t}$ and $n_H = 1, 2, \dots, N_H$

$$I_{\beta_H}[m, n_H] = \frac{1}{F} \sum_{f=1}^F I_{\beta_{H,f}}[m, n_H]$$

- **Step 6: APP-LLR mutual information calculation**

- For $n_p = 1, 2, \dots, N_p$, $I_\Gamma[n_p]$ is then calculated by using formula (28)

- **Step 7: Repeat Step 1 - Step 6 until $I_\Gamma[n_p] = 1, \forall n_p = 1, 2, \dots, N_p$.**

The proposed PEXIT algorithm reaches convergence when the chosen E_b/N_0 surpasses the threshold. Hence, the threshold $(E_b/N_0)^*$ is the minimum E_b/N_0 value at which the mutual information between the APP-LLR messages and the associated codeword bits converges to 1.

The proposed mixed-ADC PEXIT algorithm differs from its uniform low-resolution [13] and dual mixed-ADC [50] counterparts in all steps except Step 3. It explicitly accounts for the interaction among extremely low-, low-, and high-resolution ADCs when computing mutual information along the double-layer graph. In the next section, we apply this algorithm to evaluate LS-MIMO performance with mixed-ADCs using two existing protograph LDPC codes. The resulting decoding thresholds highlight the potential advantages of the triple mixed-ADC approach.

5. Theoretical Performance Analysis

In this section, we use the proposed Mixed-ADC-LS-MIMO PEXIT in the previous section to calculate the iterative decoding threshold of a specific protograph LDPC codes: the one in (29) was designed for LS-MIMO channel [29] and the other in (30) was constructed for AWGN channel [51]. The proto-matrices of the codes are given in (29) and (30). We use the PEXIT algorithm as a powerful tool to theoretically analyze the performance gain between the equal distance and equal weight superposition modulation schemes. Importantly, we demonstrate the significant advantages of mixed-ADC architectures over single-resolution ADC systems.

$$\mathbf{B}_{6 \times 3} = \begin{bmatrix} 3 & 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 2 & 2 & 1 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (29) \quad \mathbf{B}_{8 \times 4} = \begin{bmatrix} 3 & 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 & 0 & 2 & 1 \end{bmatrix} \quad (30)$$

Figure 5. Protograph LDPC code base matrices: (a) Base matrix $\mathbf{B}_{6 \times 3}$ designed for LS-MIMO channels; (b) Base matrix $\mathbf{B}_{8 \times 4}$ designed for AWGN channels.

We evaluate a 10×40 MIMO system with a 6×3 code structure under four ADC configurations: single 1-bit, dual mixed ($N_L = 35, N_H = 5$), triple mixed ($N_{EL} = 25, N_L = 10, N_H = 5$), and single 5-bit ADCs. Table 2 compares their decoding thresholds for equal distance and equal weight superposition modulation (SM) schemes. Notably, equal weight SM consistently outperforms equal distance SM, offering 1 dB lower thresholds across all cases. This gap narrows slightly with higher ADC resolution, dropping from 1.19 dB (1-bit) to 0.97 dB (5-bit). Transitioning from a single 1-bit ADC to dual or triple mixed-ADC setups significantly reduces thresholds—by over 1.15 dB and 1.03 dB for equal distance and equal weight SM, respectively. However, the triple mixed-ADC system still lags behind the 5-bit baseline by 1.66 dB (equal distance - ED) and 1.76 dB (equal weight - EW), raising the question of optimal resolution partitioning to balance power and performance.

Table 2. Decoding thresholds for 6×3 code under 10×40 MIMO with various ADC setups.

Resolution	ED	EW	Gap
Single 1-Bit ADCs	1.34	0.15	1.19
Dual Mixed ADCs	0.77	-0.34	1.11
Triple Mixed ADCs	0.19	-0.88	1.07
Single 5-Bit ADCs	-1.57	-2.54	0.97

Table 3 presents results for a 10×60 MIMO system using a 6×3 code under four ADC configurations: single 1-bit, dual mixed ($N_L = 40, N_H = 20$), triple mixed ($N_{EL} = 40, N_L = 10, N_H = 10$), and single 5-bit ADCs. As with the 10×40 case, equal weight superposition modulation (SM) consistently

outperforms equal distance SM, with threshold gaps ranging from 0.99 to 1.12 dB across all modes. Both dual and triple mixed-ADC setups again show significant gains over the single 1-bit ADC baseline. For example, in the equal distance SM case, the triple mixed-ADC achieves a threshold of -1.72 dB, compared to -0.79 dB for the single 1-bit ADC—a 0.93 dB improvement.

Table 3. Decoding thresholds for 6×3 code under 10×60 MIMO with various ADC setups.

Resolution	ED	EW	Gap
Single 1-Bit ADCs	-0.79	-1.91	1.12
Dual Mixed ADCs	-1.39	-2.46	1.07
Triple Mixed ADCs	-1.72	-2.77	1.05
Single 5-Bit ADCs	-3.35	-4.34	0.99

Table 4 presents decoding thresholds for the 8×4 protograph code—originally designed for AWGN—under the same 10×40 MIMO configuration used for the 6×3 LS-MIMO code. The overall trends mirror those in Table 2: equal weight superposition modulation (SM) consistently outperforms equal distance SM, and both dual and triple mixed-ADC setups yield clear gains over the single 1-bit ADC. However, the 8×4 code shows consistently higher thresholds across all resolutions. For example, the equal distance SM threshold with single 1-bit ADCs rises from 1.34 dB to 1.68 dB. These results highlight that while performance trends remain consistent, codes tailored to LS-MIMO channels achieve lower thresholds, underscoring the importance of channel-specific LDPC code design.

Table 4. Decoding thresholds for 8×4 code under 10×40 MIMO with various ADC setups.

Resolution	ED	EW	Gap
Single 1-Bit ADCs	1.68	0.41	1.27
Dual Mixed ADCs	1.09	-0.09	1.18
Triple Mixed ADCs	0.49	-0.64	1.13
Single 5-Bit ADCs	-1.29	-2.33	1.04

Table 5 shows the decoding thresholds of the 8×4 protograph code under the 10×60 MIMO setup. While the threshold trends remain consistent—improving with mixed-ADC architectures and equal weight SM—the absolute values are higher than those of the LS-MIMO-optimized 6×3 code. For example, in the single 1-bit ADC case, the equal distance SM threshold is -0.47 dB, compared to -0.79 dB for the 6×3 code (Table 3), reflecting a 0.32 dB performance gap. This further confirms that codes designed for LS-MIMO channels yield better thresholds under identical conditions.

Table 5. Decoding thresholds for 8×4 code under 10×60 MIMO with various ADC setups.

Resolution	ED	EW	Gap
Single 1-Bit ADCs	-0.47	-1.66	1.19
Dual Mixed ADCs	-1.09	-2.22	1.13
Triple Mixed ADCs	-1.43	-2.54	1.11
Single 5-Bit ADCs	-3.08	-4.12	1.04

6. Simulation Results

This section presents extensive simulations to validate the theoretical decoding thresholds from the previous section. The setup mirrors earlier experiments, using two MIMO configurations (10×40 and 10×60) and two protograph LDPC codes defined by proto-matrices (29) and (30). Each code has a total length of 2400 bits, constructed via two lifting steps. First, the protograph (with 3 check nodes and 6 variable nodes) is lifted by a factor of 4 using the PEG algorithm [52] to eliminate parallel edges. The second lifting factor is 100, yielding an information block length of 1200 bits. The final LDPC code is formed using circulant permutation matrices using PEG-based selection to eliminate short cycles in the lifted graph.

To begin with, Figure 6 presents the BER performance of various ADC configurations under both equal weight (EW) and equal distance (ED) superposition modulation, using the protograph LDPC code defined by the 6×3 proto-matrix. The results were obtained for a 10×40 MIMO system operating with 16-ary superposition modulation. These simulation results are consistent with the theoretical predictions outlined in Table 2. Notably, the EW superposition modulation consistently outperforms its ED counterpart across all ADC setups, with performance gains ranging from approximately 1.0 to 1.8 dB at a BER of 10^{-5} . This performance advantage aligns closely with the decoding threshold gaps reported in Table 2, where EW modulation showed lower thresholds for all configurations.

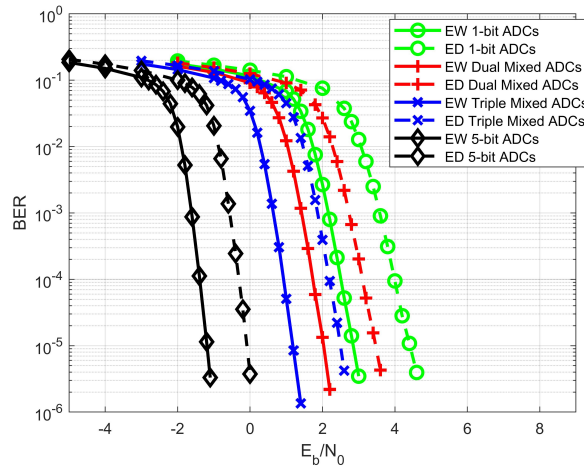


Figure 6. BER Performance: Protograph LDPC Code of Size 6×3 (29), Code Length 2400 bits, MIMO 10×40 , 16-Ary Constellation, 1–5 bit ADCs.

In terms of ADC architecture, the triple mixed-ADC configuration (comprising 25 1-bit, 10 2-bit, and 5 5-bit ADCs) achieves a 1.8 dB BER performance gain over the single 1-bit system. The dual mixed-ADC setting also demonstrates a marked improvement over the single one-bit configuration but exhibits slightly inferior performance compared to the triple mixed-ADC scheme. The simulation results confirm that the proposed mixed-ADC receiver design achieves a strong trade-off between performance and energy efficiency, as predicted by the PEXIT-based theoretical analysis. Specifically, the triple mixed-ADC system with EW modulation operates within 1.8 dB of the 5-bit baseline, while the 1-bit system has a gap of more than 3 dB to the 5-bit ADCs, thus validating the effectiveness of the mixed ADC deployment in LS-MIMO systems. It is also observed that the performance gap between equal weight and equal distance superposition modulation decreases as the ADC resolution increases. This trend is evident in both the theoretical decoding thresholds (Table 2) and the BER simulation results (Figure 6), indicating that higher-resolution ADCs reduce the relative benefit of optimal SM weighting due to improved quantization precision.

Figure 7 illustrates the BER performance for the protograph LDPC code defined by the 6×3 proto-matrix under a 10×60 MIMO system with a 16-ary superposition modulation scheme. As observed in the previous configuration (Figure 6), the equal weight (EW) superposition modulation consistently outperforms the equal distance (ED) counterpart across all ADC configurations. This trend holds true for single 1-bit ADCs, dual mixed ADCs, triple mixed ADCs, and full-resolution 5-bit ADCs, further confirming the robustness of EW modulation in various quantization environments.

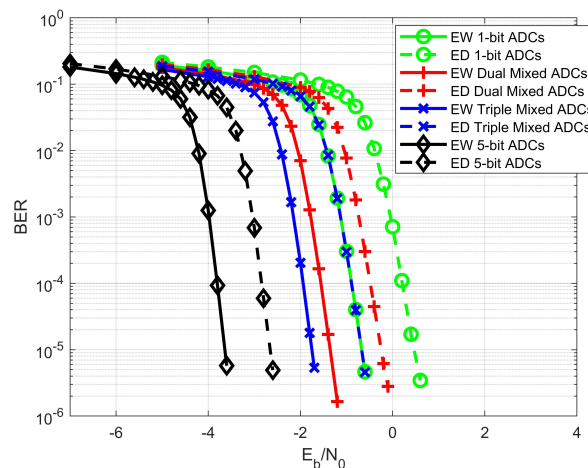


Figure 7. BER Performance: Protograph LDPC Code of Size 6×3 (29), Code Length 2400 bits, MIMO 10×60 , 16-Ary Constellation, 1–5 bit ADCs.

While the performance advantages of mixed-ADC architectures are evident, practical considerations such as hardware cost and power consumption must also be accounted for. One-bit and two-bit ADCs are significantly cheaper and more energy-efficient than five-bit ADCs, which often require complex calibration and consume higher power. Therefore, the triple mixed-ADC design offers a compelling trade-off—delivering near-optimal decoding performance while substantially reducing implementation cost and power usage compared to uniform high-resolution ADC systems.

More notably, by increasing the number of receiver antennas from $N_r = 40$ to $N_r = 60$, the system experiences a substantial improvement in BER performance across all configurations. The decoding curves for all ADC architectures shift leftward by approximately 2–4 dB, indicating that additional spatial diversity from more receive antennas enhances the detection capability and effectively compensates for quantization noise. This improvement is particularly evident in the low-resolution settings, where the 1-bit ADC system with ED modulation now operates at 0.4 dB for a BER level of 10^{-5} in the 10×60 MIMO configuration, compared to 4 dB in the 10×40 MIMO configuration.

To demonstrate the performance of the LS-MIMO code and the AWGN code, Figures 8 and 9 illustrate the BER performance of the 8×4 protograph LDPC code, originally designed for AWGN channels [51], under various MIMO setups and ADC configurations. While the code differs structurally from the 6×3 protograph used in Figures 6–7, the performance trends remain broadly consistent. Specifically, the benefits of equal weight (EW) over equal distance (ED) superposition modulation are clearly observed across all receiver configurations. Likewise, mixed-ADC architectures continue to demonstrate significant gains over the baseline single 1-bit ADC setup, and increasing the number of receive antennas from $N_r = 40$ to $N_r = 80$ results in a noticeable reduction in the required E_b/N_0 to achieve the same BER level, reaffirming the spatial diversity gain.

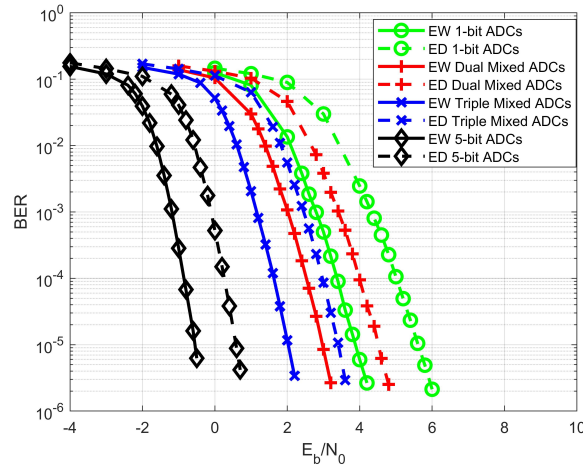


Figure 8. BER Performance: Protograph LDPC Code of Size 8×4 (30), Code Length 2400 bits, MIMO 10×40 , 16-Ary Constellation, 1–5 bit ADCs.

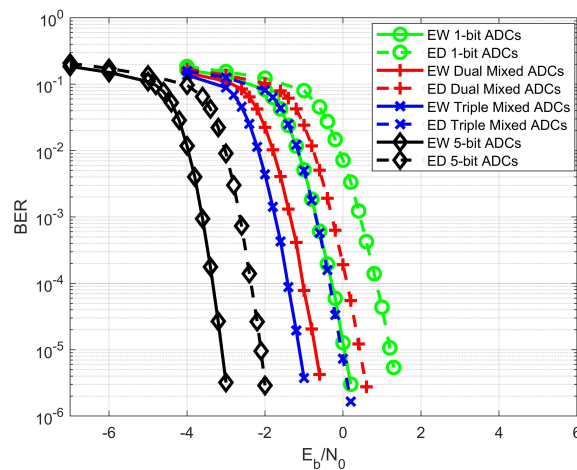


Figure 9. BER Performance: Protograph LDPC Code of Size 8×4 (30), Code Length 2400 bits, MIMO 10×60 , 16-Ary Constellation, 1–5 bit ADCs.

Moreover, the simulation results exhibit good alignment with the theoretical thresholds derived using the modified mixed-ADC PEXIT algorithm. This consistency across different code structures confirms the robustness and adaptability of the theoretical analysis tool in guiding system design and performance prediction. Interestingly, despite the fact that the 8×4 code contains more variable and check nodes—typically associated with stronger performance in AWGN settings—it exhibits inferior performance compared to the 6×3 code specifically tailored for low-resolution ADCs in massive MIMO channels. For instance, under identical system parameters, the 6×3 code consistently achieves lower BER at the same E_b/N_0 levels than its 8×4 counterpart. This observation suggests that designing LDPC codes specifically optimized for the characteristics of quantized MIMO channels (e.g., mixed-ADC distortion, spatial correlation) is a worthwhile direction for future research.

7. Conclusion

In this study, we addressed the power constraints of LS-MIMO systems by proposing a hybrid ADC receiver structure integrated with superposition modulation and protograph LDPC codes, which is crucial for future wireless networks such as B5G/6G and IoT. This work confirmed the value of triple mixed-ADC designs through both analytical and simulation-based decoding thresholds. The observed 1.8 dB improvement over 1-bit systems, approaching full-resolution ADC performance, underscores

the practical viability of triple mixed-ADC designs. Future directions include optimizing partition strategies and exploring cooperative MIMO under hybrid quantization.

References

1. D. C. Araújo, T. Maksymyuk, A. L. F. de Almeida, T. Maciel, J. C. M. Mota, and M. Jo, "Massive MIMO: Survey and future research topics," *IET Communications*, vol. 10, no. 15, pp. 1938–1946, 2016.
2. K. B. Letaief, W. Chen, Y. Shi, J. Zhang, and Y. A. Zhang, "The roadmap to 6G: AI empowered wireless networks," *IEEE Communications Magazine*, vol. 57, pp. 84–90, Aug. 2019.
3. Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, "6G wireless networks: Vision, requirements, architecture, and key technologies," *IEEE Vehicular Technology Magazine*, vol. 14, pp. 28–41, Sept. 2019.
4. L. Fan, S. Jin, C. Wen, and H. Zhang, "Uplink achievable rate for massive mimo systems with low-resolution adc," *IEEE Communications Letters*, vol. 19, pp. 2186–2189, Dec 2015.
5. J. Zhang, L. Dai, X. Li, Y. Liu, and L. Hanzo, "On low-resolution adcs in practical 5g millimeter-wave massive mimo systems," *IEEE Communications Magazine*, vol. 56, no. 7, pp. 205–211, 2018.
6. C. Zhang, Y. Jing, Y. Huang, and X. You, "Massive MIMO with ternary ADCs," *IEEE Signal Processing Letters*, p. 1, 2020.
7. T. Liu, J. Tong, Q. Guo, J. Xi, Y. Yu, and Z. Xiao, "Energy efficiency of massive MIMO systems with low-resolution adcs and successive interference cancellation," *IEEE Transactions on Wireless Communications*, vol. 18, pp. 3987–4002, Aug. 2019.
8. J. Dai, J. Liu, J. Wang, J. Zhao, C. Cheng, and J. Wang, "Achievable rates for full-duplex massive MIMO systems with low-resolution ADCs/DACs," *IEEE Access*, vol. 7, pp. 24343–24353, 2019.
9. S. Gao, P. Dong, Z. Pan, and G. Y. Li, "Deep learning based channel estimation for massive MIMO with mixed-resolution ADCs," *IEEE Communications Letters*, vol. 23, pp. 1989–1993, Nov. 2019.
10. L. V. Nguyen, D. T. Ngo, N. H. Tran, A. L. Swindlehurst, and D. H. N. Nguyen, "Supervised and semi-supervised learning for MIMO blind detection with low-resolution ADCs," *IEEE Transactions on Wireless Communications*, p. 1, 2020.
11. L. Xu, X. Lu, S. Jin, F. Gao, and Y. Zhu, "On the uplink achievable rate of massive MIMO system with low-resolution ADC and RF impairments," *IEEE Communications Letters*, vol. 23, pp. 502–505, Mar. 2019.
12. D. A. Hoang, H. T. Nguyen, and T. N. Le, "Performance Analysis of Superposition M-ary QAM Modulation in Coded Protograph LDPC MIMO Communication Systems With Low-Resolution ADCs," *IEEE Access*, vol. 13, pp. 107410–107428, 2025, doi: 10.1109/ACCESS.2025.3581775.
13. T. V. Nguyen, H. D. Vu, D. N. Nguyen, and H. T. Nguyen, "Performance analysis of protograph LDPC codes over large-scale MIMO channels with low-resolution ADCs," *IEEE Access*, vol. 7, pp. 145145–145160, 2019.
14. Y. Cho and S. Hong, "One-bit successive-cancellation soft-output (oss) detector for uplink mu-MIMO systems with one-bit ADCs," *IEEE Access*, vol. 7, pp. 27172–27182, 2019.
15. F. Mousavi and A. Tadaion, "A simple two-stage detector for massive MIMO systems with one-bit ADCs," in *Proc. 27th Iranian Conf. Electrical Engineering (ICEE)*, pp. 1674–1678, Apr. 2019.
16. Q. Bai and J. A. Nossek, "Energy efficiency maximization for 5g multi-antenna receivers," *Trans. on Emerging Telecommunications Technologies*, 2015.
17. H. N. Dang, T. V. Nguyen, and H. T. Nguyen, "Improve uplink achievable rate for massive mimo systems with low-resolution adcs," in *2020 IEEE Eighth International Conference on Communications and Electronics (ICCE)*, pp. 99–104, 2021.
18. T. Liu, A. Liu, H. Zhang, W. Xu, and H. Lei, "Secrecy performance analysis of mixed-adc/dac cell-free massive mimo in the presence of multiple eavesdroppers," *IEEE Transactions on Communications*, vol. 71, no. 3, pp. 1643–1658, 2023.
19. H. Wang, Y. D. Zhang, B. Zhang, and J. Wang, "Mixed-adc based pmcw mimo radar angle-doppler imaging," *IEEE Transactions on Aerospace and Electronic Systems*, 2023.
20. W. Liu, Y. Li, X. Wu, Y. Han, X. Chen, and H. Zhang, "Massive access in extra large-scale mimo with mixed-adc over near-field channels," *IEEE Transactions on Communications*, 2023.
21. Y. Hu, R. Xie, J. Wu, S. Jin, and H. Li, "Channel estimation for irs-assisted mmwave massive mimo systems in mixed-adc architecture," *IEEE Transactions on Wireless Communications*, 2023.
22. Y. Chen, Y. Liu, Q. Zhang, L. Li, and X. Cao, "Channel estimation for irs-aided massive mimo systems with mixed-adc architecture," *IEEE Wireless Communications Letters*, vol. 12, no. 5, pp. 851–855, 2023.

23. Y. Gao, Y. Zhang, J. Yuan, J. Zhang, and W. Zhang, "Channel estimation for mmwave massive mimo systems with mixed-adc architecture," *IEEE Wireless Communications Letters*, vol. 12, no. 6, pp. 953–957, 2023.
24. A. Elgabli and T. Y. Al-Naffouri, "Hybrid low-and full-resolution adcs in uplink massive mimo systems: Performance analysis and deep learning-based detection," *IEEE Transactions on Communications*, vol. 71, no. 5, pp. 2877–2893, 2023.
25. Y. Zhao, Y. Wei, and Y.-C. Liang, "Detection of multi-user signals in massive mimo systems with mixed-adc architecture," *IEEE Transactions on Wireless Communications*, vol. 22, no. 8, pp. 5004–5018, 2023.
26. J. Thorpe, "Low-density parity-check (ldpc) codes constructed from protographs," *IPN Progress Report*, pp. 42–154, 2003.
27. M. D. Aksun and E. Arkan, "LDPC code design for 6g wireless," *IEEE Communications Magazine*, vol. 59, pp. 76–81, Nov. 2021.
28. H. N. Dang, T. V. Nguyen, and H. T. Nguyen, "Improve uplink achievable rate for massive mimo systems with low-resolution adcs," in *2020 IEEE Eighth International Conference on Communications and Electronics (ICCE)*, pp. 99–104, 2021.
29. H. D. Vu, T. V. Nguyen, D. N. Nguyen, and H. T. Nguyen, "On design of protograph ldpc codes for large-scale mimo systems," *IEEE Access*, vol. 8, pp. 46017–46029, 2020.
30. P. A. Hoeher and T. Wo, "Superposition modulation: Myths and facts," *IEEE Communications Magazine*, vol. 49, pp. 110–116, Dec. 2011.
31. J. Luo, Y. Bai, B. Bai, C. Chen, and W. Wen, "A multi-layer superposition modulation scheme to improve the data rate for iot communications," in *Proceedings of the 2023 IEEE/CIC International Conference on Communications in China (ICCC Workshops)*, (Dalian, China), pp. 1–6, Aug. 2023.
32. D. Hao and P. A. Hoeher, "Superposition modulation with reliability-based hybrid detection," in *Proceedings of the 2010 6th International Symposium on Turbo Codes and Iterative Information Processing (ISTC)*, (Brest, France), pp. 280–284, Sept. 2010.
33. Y. H. Choi and S. W. Heo, "A novel decoding algorithm of superposition modulation for cooperative iot system," in *Proceedings of the 2020 IEEE International Conference on Consumer Electronics (ICCE)*, (Las Vegas, NV, USA), pp. 1–2, Jan. 2020.
34. C. Hasan and U. Aygolu, "An incremental relaying approach for superposition modulated cooperative transmission," in *Proceedings of the 2009 IEEE Wireless Communications and Networking Conference (WCNC)*, (Budapest, Hungary), pp. 1–6, Apr. 2009.
35. X. Lu, T. J. Li, and Y. Liu, "Multiuser cooperative transmission through superposition modulation based on braid coding," in *Proceedings of the 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, (South Brisbane, QLD, Australia), pp. 3128–3132, Apr. 2015.
36. S. Zhao, X. Ma, and B. Bai, "Decoding algorithms of ldpc coded superposition modulation," *IEEE Communications Letters*, vol. 18, pp. 487–490, Mar. 2014.
37. M. Srinivasan and S. Kalyani, "Analysis of massive MIMO with low-resolution ADC in nakagami- m fading," *IEEE Communications Letters*, vol. 23, pp. 764–767, Apr. 2019.
38. H. T. Nguyen, T. A. Ramstad, and I. Balasingham, "Wireless sensor communication system based on direct-sum source coder," *IET Wireless Sensor Systems*, vol. 1, pp. 96–104, June 2011.
39. D. Hui and D. L. Neuhoff, "Asymptotic analysis of optimal fixed-rate uniform scalar quantization," *IEEE Transactions on Information Theory*, vol. 47, pp. 957–977, March 2001.
40. W. Fukuda, T. Abiko, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Ohwatari, and Y. Kishiyama, "Low-complexity detection based on belief propagation in a massive mimo system," in *2013 IEEE 77th Vehicular Technology Conference (VTC Spring)*, pp. 1–5, June 2013.
41. T. Takahashi, S. Ibi, and S. Sampei, "On normalization of matched filter belief in gabp for large MIMO detection," in *Proc. IEEE 84th Vehicular Technology Conf. (VTC-Fall)*, pp. 1–6, Sept. 2016.
42. H. D. Vu, T. V. Nguyen, T. B. Do, and H. T. Nguyen, "Belief propagation detection for large-scale mimo systems with low-resolution adcs," in *2019 International Conference on Advanced Technologies for Communications (ATC)*, pp. 68–73, 2019.
43. T. J. Richardson, M. A. Shokrollahi, and R. L. Urbanke, "Design of capacity-approaching irregular low-density parity-check codes," *IEEE Transactions on Information Theory*, vol. 47, pp. 619–637, Feb. 2001.
44. S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Transactions on Communications*, vol. 52, pp. 670–678, Apr. 2004.
45. G. Liva and M. Chiani, "Protograph ldpc codes design based on exit analysis," in *IEEE GLOBECOM 2007 - IEEE Global Telecommunications Conference*, pp. 3250–3254, Nov 2007.

46. Y. Fang, P. Chen, L. Wang, F. C. M. Lau, and K. K. Wong, "Performance analysis of protograph-based low-density parity-check codes with spatial diversity," *IET Communications*, vol. 6, pp. 2941–2948, Nov. 2012.
47. T. V. Nguyen and A. Nosratinia, "Rate-compatible short-length protograph LDPC codes," *IEEE Communications Letters*, vol. 17, pp. 948–951, May 2013.
48. T. V. Nguyen and H. T. Nguyen, "The design of optimized fast decoding protograph LDPC codes," in *Proc. Int. Conf. Advanced Technologies for Communications (ATC)*, pp. 282–286, Oct. 2016.
49. T. Abiko, W. Fukuda, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Ohwatari, and Y. Kishiyama, "An exit chart analysis for belief-propagation based detection in a large-scale mimo system," pp. 1–5, June 2013.
50. H. N. Dang, H. T. Nguyen, and T. V. Nguyen, "Joint detection and decoding of mixed-adc large-scale mimo communication systems with protograph ldpc codes," *IEEE Access*, vol. 9, pp. 101013–101029, 2021.
51. H. Uchikawa, "Design of non-precoded protograph-based LDPC codes," in *Proc. IEEE Int. Symp. Information Theory*, pp. 2779–2783, June 2014.
52. X.-Y. Hu, E. Eleftheriou, and D. M. Arnold, "Regular and irregular progressive edge-growth tanner graphs," *IEEE Transactions on Information Theory*, vol. 51, pp. 386–398, Jan 2005.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.