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Article

Background Gravitational Waves as Signature of the Accelerated Cosmic Expansion

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Abstract: We propose a toy model of a spherical universe made up of an exotic dark gas with temperature T in thermal equilibrium with a black-body. Each particle of this exotic gas mimics a kind of particle of dark energy. So, each dark particle occupies a very small area of space so-called Planck area L_p^2, which represents the minimum area of the whole space-time given by the spherical surface with area $4\pi R_H^2$, where R_H is the Hubble radius. We realize that such spherical surface is the surface of the black-body for representing the dark universe as if it were the surface of an expanding balloon. Thus, we are able to derive the law of universal gravitation, thus leading us to understand the cosmological anti-gravity. We estimate the tiny order of magnitude of the cosmological constant and the acceleration of expansion of the dark sphere. In this toy model, as the dark universe can be thought of as a large black body, when we obtain its power and frequency of emission of radiation, we find very low values. We conclude that such radiation and frequency of the black body made up of dark energy is a background gravitational wave with very low frequency in the order of $\sim 10^{-17}$Hz due to the slight stretching of the fabric of space-time.

Keywords: gravitational waves; dark energy; cosmological constant; Planck length

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1. Introduction

In the early 2000s a series of works flourished combining General Relativity, Scalar Fields and extra dimensions[1–6]. All these works are based on Action S in D dimensions whose structure is composed of curvature, scalar fields and corresponding momentum energy tensor, i.e., $S = \int d^D x \sqrt{|g|} \left[-\frac{1}{4} + \frac{1}{2}\partial_\mu\phi\partial^\nu\phi - V(\phi) \right]$, where $g = \det(g_{\mu\nu})$ and the metric is $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Although these works had a certain importance in dark energy modeling, the dynamics of the dark energy is still a puzzle.

It is known that the dark energy acts as a tenuous uniform distribution of energy and with property of having a negative pressure, thus leading to the accelerated expansion of the universe.

Although the true nature of the dark energy is unknown, there are some models that seek to understand its nature. Simpler candidates include the cosmological term (cosmological constant) introduced by Einstein to modify the field equations of the General Theory of Relativity, or a scalar field that is extremely weak and has properties that are consistent with repulsive macroscopic behavior.

The universe is approximately 13.7 billion years old with Hubble radius $R_H \sim 10^{26}$m. It is composed of about 4% ordinary matter (such as hydrogen, helium, etc.), 23% dark matter, which gravitationally attracts, and 73% dark energy, whose nature is mysterious and acts as a constant energy density associated with the vacuum energy and with negative pressure, working like a cosmological anti-gravity.

The toy model treated in this paper allows us to make a connection between Unruh effect and gravity, which can be extended to the cosmological anti-gravity in view of the fact that we consider a black body made of dark energy. This issue will be explored in the section 2.

The thermodynamical aspect of gravitation is also another puzzle that should be deeper explored. Recent investigations have shown that gravitational field equations in a wide class of models can be interpreted with a thermodynamical origin. This idea, originally due to Sakharov[7], has different forms of implementation[7–22] and also other approaches[23–25].

In the section 3, we use the toy model of an expanding dark sphere given in the section 2 for representing the current universe with $r = R_H$. Due to the Unruh effect treated in this frontier R_H, we will be able to obtain a very low thermal energy $K_B T$ whose associated frequency is of an order of magnitude far below the Cosmic Microwave Background (CMB). Due to the small acceleration of the universe, such a very low frequency ($\approx 10^{-17}$Hz) will be interpreted as being the frequency of a gravitational wave emitted by the dark energy due to the expansion of the fabric of space-time.

In the section 4, as the present toy model deals with the dark universe as being the spherical surface in Figure 1 with an extremely low temperature, we will apply the Planck law for black body in order to obtain the power of emission of the background radiation with a frequency of approximately 10^{-17}Hz. However, such frequency is far outside the electromagnetic spectrum, and thus it represents a gravitational wave emitted by the own dark energy. Dark energy does not emit any kind of electromagnetic wave. Hence the fact that such energy is dark, but it has a gravitational wave signature.

2. Unruh effect and anti-gravity

In the early universe, space-time was confined to the Planck area $L_p^2 (\sim 10^{-70} m^2)$. This area has expanded very quickly as described by cosmic inflation. After inflation, it continued to expand much slower until reaching the current area $4\pi R_H^2$, $R_H (\sim 10^{26} m)$ being the Hubble radius.

The present toy model uses a dimensional compaction that preserves some physical properties like the universal gravitation law. This means that the expanding universe $3D$ works like a spherical surface $2D$ with area $4\pi R_H^2$. The reason for such model of compactation is that it is considering a spherical surface $2D$ for representing the fabric of space-time $4D$.

In this toy model, let us consider the universe made up of an exotic gas of N particles in a thermal equilibrium with the particles on the surface of the cavity of a spherical black-body, which represents a background frame for the fabric of space-time.

As the cavity $2D$ of the black-body represents the space-time, we should consider that each particle of the exotic gas is inside the tiny Planck area ($L_P^2 \sim 10^{-70} m^2$) of the big dark cavity with area $4\pi R_H^2$. So we conclude that the spherical surface with radius R_H can be divided in N areas of Planck with N exotic dark particles. So we obtain $N = 4\pi R_H^2 / L_P^2$, where N is the number of dark particles over this spherical surface, $R_H \sim 10^{26}$m being the Hubble radius with about 13.7 Gyears.

We estimate the large number N of exotic particles of dark energy inside each Planck area of the spherical surface that represents the tissue of the space-time, namely $N \sim 10^{122}$ (Figure 1).

We know that the Planck length L_p is given by $L_P = \sqrt{G\hbar/c^3}$.

Each particle of such an exotic dark gas over the spherical surface is expanding radially in only one direction r. So its total thermal energy is $E = Mc^2 = \frac{1}{2} N K_B T$, where $E = E_{dark}$, with $N \sim 10^{122}$. Such equation represents the total dark energy of a dark exotic gas made up of about 10^{122} dark particles that governs the accelerated expansion of the universe.

By using $N = 4\pi R_H^2 / L_P^2$, $L_P = \sqrt{G\hbar/c^3}$, we rewrite the thermal energy of the gas as follows:

$$E = Mc^2 = \left(\frac{2\pi c^3 K_B}{G\hbar} \right) T R_H^2, \tag{1}$$

where R_H is the Hubble radius and $E = Mc^2 = E_{dark} = M_{dark} c^2$.

We should stress that such an exotic gas is not a true gas with its particles moving randomly over a 2-dimensional surface. Actually, this exotic gas is a mimic of a gas, as we have considered only one degree of freedom represented by the radial direction r of motion of the N particles.
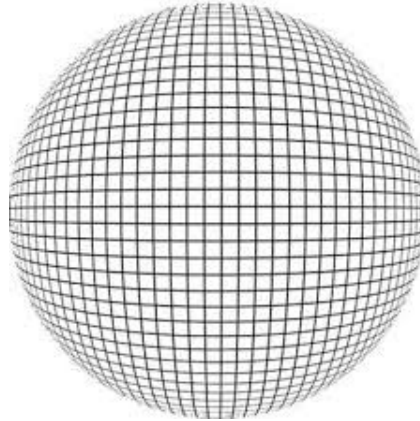
Figure 1. The surface of the dark sphere has a number $N \sim 10^{122}$ of Planck areas L_P^2. Each one of them is connected to a dark infinitesimal energy as being a dark particle of vacuum. The total mass M in Eq.(1) is the total dark mass M_{dark}. A baryonic proof particle over the surface of the sphere experiences an acceleration based on the Unruh effect. As the sphere is made up of dark energy, we have an anti-gravitational acceleration associated with the cosmological constant. Thus such a proof particle of matter is pushed out of the dark sphere due to the anti-gravity.

The proof particle of matter is over the spherical surface. Such particle is in thermal equilibrium with the sphere. So, as this particle has a radial acceleration $(a(r))$, the Unruh effect [27] states that such a particle experiences a thermal bath with the same temperature T of the sphere, i.e., $T = \hbar a / 2\pi K_B c$.

Due to the thermal equilibrium between the accelerated proof particle and the spherical surface with temperature T, we use the Unruh effect in Eq.(1), so that we obtain

$$E = M_{dark}c^2 = \left(\frac{2\pi c^3 K_B}{G\hbar} \right) \left(\frac{\hbar a}{2\pi K_B c} \right) R_H^2, \qquad (2)$$

from where we get

$$a = \frac{GM_{dark}}{R_H^2}. \qquad (3)$$

Eq.(3) is the law of universal gravitation. It was obtained within a scenario of quantum-gravity (dark energy), i.e., we have a dark mass M_{dark} made up of an exotic gas that fills the whole space-time (Figure 1). This gas is expansive, which leads to the accelerated expansion of the spherical surface (fabric of the space-time), working like an anti-gravitational acceleration.

In a recent important paper[26], as it was shown from first principles that the equation of state of vacuum (EOS) shows an anti-gravity due the negative pressure $p = -\epsilon$, ϵ being the vacumm energy density associated with a preferred reference frame of an invariant minimum speed V of a modified relativity[26], we conclude that the Eq.(3) represents the universal gravitation law applied to the accelerated expansion of the universe by avoiding its collapse. In view of this, we should interpret Eq.(3) working like a cosmological anti-gravity. So, let us just rewrite Eq.(3) as $a_0 = -\frac{GM_{dark}}{R_H^2}$, where the negative sign "$(-)$" shows that the acceleration is repulsive for a given proof particle of matter on the surface of the dark sphere. R_H is the Hubble radius and M_{dark} is the dark mass associated with the dark energy $E_{dark} = M_{dark}c^2$. This represents 73% of the whole universe.

According to recent observations[28], the density of matter in the universe is about $3 \times 10^{-30} g/cm^3$. Such density includes the contribution of dark matter plus the density of luminous

matter as stars and galaxies. This represents about 27% of the whole universe, where 23% is the dark matter and 4% is the luminous matter.

The visible universe has about 13.7 Gyears. By considering the density associated with luminous and dark matter, which represents 27% of the whole universe, we find a mass of about 3×10^{55}g. However, as the dark energy is 73% of the universe, we compute M_{dark} about 8.1×10^{55}g. This leads us to conclude that the entire energy content can be estimated as being the own dark energy M_{dark} of the order of 10^{56}g. Thus, by making $M_{dark} \sim 10^{53}$Kg, $R_H \sim 10^{26}$m and $G \sim 10^{-10}Nm^2/kg^2$ in the law of universal gravitation for representing anti-gravity, we obtain the acceleration of anti-gravity for a proof particle on the surface of the dark sphere, namely $a_0 \sim -10^{-9}m/s^2$. Such result is expected as the current acceleration of expansion of the universe (anti-gravity) is very low.

We must stress the fact that the equation for obtaining the negative acceleration a_0 of a proof particle on the surface of the sphere of radius R_H is the Newtonian law of gravitation. This is a good approximation once the dark mass in the order of $10^{56}g$ is uniformly distributed in a big sphere with volume in the order of $10^{78}m^3$, giving a too low density, i.e., we have $\rho_{dark} \approx 8.1 \times 10^{55}g/(4/3)\pi 10^{78}m^3 \approx 10^{-23}g/m^3$ or $\rho_{dark} \sim 10^{-29}g/cm^3$. Such very low density of the dark sphere is in good agreement with observations. So it justifies the use of the Newtonian approximation for a weak anti-gravity. However, if this mass in the order of $10^{53}Kg$ has a too small volume, so that the sphere presents a high density, so this result cannot be used for the approximation of classical gravitation. However, as we are only interested in the current state of the universe with an extremely low density of vacuum energy ($\sim 10^{-29}g/cm^3$), the Newtonian approach works well for obtaining the tiny value of the cosmological constant Λ_0 given by the derivative of acceleration for $r = R_H$, namely $\Lambda_0 = da/dr|r = R_H$.

The cosmological constant Λ is the acceleration rate per unit of distance r (s^{-2} in SI units). Thus we have $\Lambda = da/dr$. So, for $r = R_H$, we get its current value Λ_0, namely:

$$\Lambda_0 = \left[\frac{da}{dr}\right]_{r=R_H} = \frac{2GM_{dark}}{R_H^3} \sim 10^{-35}s^{-2}, \tag{4}$$

where $a = -GM_{dark}/r^2$ is repulsive. As $\Lambda_0 > 0$, we have an anti-gravity as expected.

We have obtained $\Lambda_0 = \Lambda(R_H) \sim 10^{-35}s^{-2}$. This is the order of magnitude of the cosmological constant Λ_0 in agreement with the current observational data[29–35].

We must be aware of the fact that the very high values obtained for the cosmological constant and the vacuum energy density by using the Quantum Field Theory (QFT) for describing the quantum vacuum energy have a high discrepancy of about 120 orders of magnitude beyond their observational values. This puzzle is well-known as the "Cosmological Constant Problem"[36][37], and it is deeply investigated in a modified relativity with an invariant minimum speed V by representing vacuum in such a modified space-time with the presence of the cosmological constant[26].

3. Gravitational waves as a signature of the dark energy due to accelerated cosmic expansion

According to the present toy model, a proof particle of matter over the dark spherical surface experiences a thermal equilibrium with the sphere with a temperature T, which is proportional to its radial acceleration ($a = a(r)$), i.e., $T \propto a$. This is the so-called Unruh effect, from where we can obtain the very low thermal energy, namely:

$$K_B T = \frac{\hbar a}{2\pi c} = \frac{G\hbar M_{dark}}{2\pi c R_H^2} \sim 10^{-51}J, \tag{5}$$

where $a \sim 10^{-9}m/s^2$ for $R_H \sim 10^{26}m$, $M_{dark} \sim 10^{53}$Kg and $T \sim 10^{-28}$K.

3.1. The very low thermal energy from the dark energy far below the electromagnetic range as a signature of gravitational waves

There is a very low thermal energy associated with the dark energy; however, as we will show that such energy is too low, in fact it is unable to emit a frequency within the well-kown electromagnetic range. Therefore, it is not a frequency of eletromagnetic origin.

From Eq.(5), we see that

$$K_B T \sim 10^{-51} J. \tag{6}$$

As $K_B \sim 10^{-23} J/K$, we find $T \sim 10^{-28}$K. Such temperature is too low, so that the predominant frequency emitted by the black body of dark energy (Figure 1) is

$$\nu = \frac{K_B T}{h} = \frac{a}{4\pi^2 c} \approx 10^{-17} Hz, \tag{7}$$

where $a \sim 10^{-9} m/s$. We have used $h\nu = K_B T$ and Eq.(5), where $\hbar = h/2\pi$.

Here it is important to stress that the visible universe is also treated as a model of black body, since it is permeated by a cosmic microwave background (CMB) radiation with a temperature of about 2.725K associated with a peak frequency of 160.4GHz, which corresponds to a wavelength of 1.9 mm. Such electromagnetic radiation is practically isotropic with very small fluctuations and comes from all directions of space.

The present model of dark sphere (dark universe) given by a black body is analogous to the first one; however it emits an isotropic background radiation of gravitational origin, since its frequency is about 10^{-17}Hz, being far below the lowest electromagnetic frequency. Thus it is not an electromagnetic wave, being a background gravitational wave emitted by the dark energy permeating the whole space, as the dark energy leads to an accelerated expansion of the fabric of space-time represented by the surface of the dark sphere (Figure 1).

We realize that the background gravitational wave emitted by the dark energy with an extremely low frequency $\nu_g \approx 10^{-17} Hz$ has a cosmological wavelength $\lambda_g = c/\nu_g \sim 10^{25} m$, which represents a super giant background gravitational wave. It has a very weak signal and it is the remnant of the cosmic inflation in the current universe.

The CMB cloaks the effects of the dark energy. What this means is that if we just assume the absence of CMB, the background temperature would be much lower in the order of 10^{-28}K. Although this is not true due to the predominance of CMB, even so the present model predicts a gravitational wave of frequency approximately 10^{-17}Hz emitted by the dark energy permeating isotropically the whole expanding universe. This lowest frequency of gravitation origin is the fossil of the inflationary phase given in the current universe.

A curved signature in the cosmic microwave background (CMB) provides proof of inflation and spacetime ripples. A CMB imprint from primordial gravitational waves would be indirect evidence, like the high-water mark left by a receding tide, but what of detecting the waves themselves? At present, the instruments most sensitive to locally produced gravitational waves are interferometers such as the dual Laser Interferometer Gravitational Wave Observatory (LIGO) installations. Each sends laser light down four-kilometer arms to look for interference effects that would be caused by passing gravitational waves stretching space in one direction and compressing it in the other. In future decades, space-based interferometers could use the same principles on much larger scales, bouncing laser light across the solar system, to directly detect the fainter primordial gravitational waves like the background gravitational waves with frequency $\nu_g \approx 10^{-17}$Hz.

Recently, a body of works has devoted interest in the dynamics of dark energy by the emission of gravitational waves[38,39]. This line of investigation is important since it can contribute to the innovation of studies involving dark energy, its cosmological role, phase of inflation and the relationship with the cosmological constant.

4. The power of emission of the black body associated to the dark sphere with Hubble radius

According to the Planck law of the black body, the power of emission of radiation of a black body is given, namely:

$$P = \frac{2\pi h A}{c^2} \int_0^\infty \frac{\nu^3}{e^{h\nu/K_B T} - 1} d\nu, \tag{8}$$

where A is the area of the surface of the black body associated with the own surface of the dark sphere, i.e., $A = 4\pi R_H^2$. So, we rewrite Eq.(8), as follows:

$$P = \frac{8\pi^2 h R_H^2}{c^2} \int_0^\infty \frac{\nu^3}{e^{h\nu/K_B T} - 1} d\nu. \tag{9}$$

The emission power P depends on R_H. In order to obtain also such a dependency in the distribution factor $(e^{h\nu/K_B T} - 1)^{-1}$, let us write Eq.(9) in its equivalent form based on the toy model of dark energy sphere, since we are applying the idea of a gravitational analogue of black body to such a dark spherical surface (2D) representing the fabric of the space-time in this model of dimensional compactation.

So, in doing this, let us first begin from Eq.(1), from where we obtain

$$T R_H^2 = \frac{G \hbar M_{dark}}{2\pi c K_B}, \tag{10}$$

from where we get

$$K_B T = \frac{G h M_{dark}}{4\pi^2 c R_H^2}, \tag{11}$$

where $\hbar = h/2\pi$ and $M = M_{dark}$.

By using Eq.(11), we can write the ratio $h\nu/K_B T$, which appears in Planck's law for black body in its equivalent form for representing the dark energy sphere, namely:

$$\frac{h\nu}{K_B T} = \frac{4\pi^2 c R_H^2 \nu}{G M_{dark}}. \tag{12}$$

Thus, according to Eq.(12), the Planck distribution factor applied to the dark sphere with radius R_H is as follows:

$$\frac{1}{e^{(h\nu/K_B T)} - 1} = \frac{1}{e^{(4\pi^2 c R_H^2 \nu/G M_{dark})} - 1}, \tag{13}$$

where we can now see that the Planck distribution factor depends on the radius R_H of the dark sphere.

Finally, substituting the equivalent distribution given by Eq.(13) in Eq.(9), we obtain

$$P(R_H) = \frac{8\pi^2 h R_H^2}{c^2} \int_0^\infty \frac{\nu^3}{e^{(4\pi^2 c R_H^2 \nu/G M_{dark})} - 1} d\nu. \tag{14}$$

Let us make the computational resolution of the integral in Eq.(14). So we write it as follows:

$$P(R_H) = \beta R_H^2 \int_0^\infty \frac{\nu^3}{e^{\alpha R_H^2 \nu} - 1} d\nu, \tag{15}$$

where $\beta = 8\pi^2 h/c^2$ and $\alpha = 4\pi^2 c/G M_{dark}$, being $\beta \sim 10^{-49} Js^3/m^2$ and $\alpha \sim 10^{-25} s/m^2$.

So, based on computational calculation, the power P obtained for $r = R_H (\sim 10^{26} m)$ in the integral given in Eq.(15) is $P(R_H) \sim 10^{-73} W$. This power of emission of the dark energy of the whole universe is extremely low, being a very weak gravitational signal.

Dark energy does not emit electromagnetic waves. However, it is important to call attention to the fact that such radiation with power in the order of $10^{-73} W$ is of gravitational nature (gravitational

wave power), since its frequency is outside of the electromagnetic spectrum, being very low in the order of $10^{-17}Hz$, which represents an isotropic background radiation of gravitational origin due to the accelerated expansion of the space-time. This means that when the fabric of space-time undergoes a stretching, background gravitational waves emerge on this dark fabric, i.e., the dark energy emits gravitational waves.

It was shown that the cosmological constant is too small according to the observations, i.e., $\Lambda_0 \sim 10^{-35}s^{-2}$, being related to a small acceleration, namely $a_0 \sim -10^{-9}m/s$. So, it is naturally expected that the frequency of the background gravitational wave is too low about 10^{-17}Hz and with a wavelength of the order of 10^{25}m. This means that such too low frequency has cosmological origin, thus being the lowest gravitational frequency within the wide range of frequencies of gravitational waves, i.e., $10^{-17}Hz < \nu < 10^3 Hz$[40]. This frequency of the order of 10^{-17}Hz obtained by the present model is in agreement with data in the literature[40].

It is well-known that gravitational wave signals are expected over a wide range of frequencies, from $10^{-17}Hz$[40] in the case of ripples in the cosmological background to 10^3Hz from the formation of neutron stars in supernova explosions. So, we notice that the present simple model was able to reproduce the order of magnitude of the lowest gravitational frequency based on the Unruh effect ($\nu \approx a_0/c \sim 10^{-17}Hz$) applied to the dark sphere (Figure 1) with a proof particle on its surface presenting a negative acceleration $a_0 \sim -10^{-9}m/s^2$.

5. Conclusions and prospects

In this paper, we were able to propose a toy model to represent the dark sector of the universe (dark energy) by means of a spherical shell of dark energy with Hubble radius R_H. Such a spherical surface behaves like a black body with a very low temperature (a gravitational analogue of black body), having a dark energy emitting a frequency of approximately 10^{-17}Hz, which is in the same order of magnitude of the frequency of background gravitational waves as being a fossil of the inflationary period. Such frequency is emitted due to the acceleration of the cosmic expansion (expansion of the spherical surface representing the tissue of space-time like a balloon that grows up) generated by the cosmological constant whose small order of magnitude was also obtained by this model acording to the observations.

As we made an analogy of the dark sphere with a black body, we were also able to obtain the emission power of the dark energy sphere, which is a very low power by indicating that the gravitational wave signal emitted by the fabric of space-time is extremely weak. So, this would make it difficult to detect.

Although the present model has used the Newtonian approximation for obtaining Eq.(3) in the current scenario of the universe, the early universe had a too large acceleration associated with a very large value of Λ, which leads to the cosmic inflation. This issue can be explored in the future, since recently it was discovered that in the exotic conditions of the early universe, gravitational waves may have shaken the space-time so hard that they spontaneously created electromagnetic radiation[41].

Here it is important to stress that the model based on an expanding two-dimensional surface is related to the bulk (3-dimensional volume) relating and conserving all the corresponding dimensional physical properties (e.g: the universal gravitational law and anti-gravity connected to the gases properties by mean of the Unruh effect). This concept is explored in Brane models that predict extra dimensions in the Universe by describing physical properties and conservation laws that are not compromised or invalidated by the dimensional reduction of the Manifold. In other words, physical properties and laws remain unchanged when the manifold is dimensionally reduced. These foundations were substantially explored in works inspired by Cosmological Branas Models [42,43], which reinforces the validity of the dimensional compaction toy model we are using.

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