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Article

Predictability of Exergy Destruction in Helically Coiled Tube Heat Exchangers: Global, Pitch-Specific and Hierarchical Correlations

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Abstract

Purpose: To quantify second-law performance in a vertically oriented helically coiled tube heat exchanger (HCTHEX) and to develop predictive correlations for the dimensionless exergy-destruction fraction $\phi_D = \dot{E}_D / \dot{E}_{in,total}$ across a matrix of operating conditions and coil pitches. **Methodology:** A steady, real fluid-to-fluid CFD model was used with water on both shell and coil sides and laminar (or weakly transitional) treatment over the stated Reynolds-number range. Exergy rates at shell/coil inlets and outlets were computed for a heat-exchanger control volume and used to evaluate $\dot{E}_D$ and ϕ_D . Predictability was assessed via (i) global (non-pitch-specific) regressions including pitch as an explicit predictor, and (ii) pitch-specific regressions trained separately at each pitch; all models were trained in log space and evaluated using five-fold cross-validation. **Findings:** The global baseline power-law regression $\phi_D = A Re_{shell}^a Re_{coil}^b p^c$ yields statistically significant dependence on pitch and Reynolds numbers (e.g., for the D_h -based case: $A = 0.2238$, $a = 0.04885$, $b = 0.04982$, $c = 0.7507$). However, cross-validation shows limited predictive fidelity for the baseline (for D_h : $R_{CV,log}^2 = 0.1687$, $RMSE_{CV,log} = 0.1402$). Among advanced surrogates, LogLog-GPR-ARDSE provides the best global performance for both characteristic-length definitions (for D_h : $R_{CV,log}^2 = 0.7171$, $RMSE_{CV,log} = 0.08181$), representing a substantial reduction in prediction error relative to the baseline. Pitch-specific analysis demonstrates that the best advanced model depends on pitch: GPR-ARDSE is selected at $p = 1.80, 1.85$, and 2.00 , while bagged trees slightly outperform GPR at $p = 1.90$ and 1.95 under the minimum $RMSE_{CV,log}$ criterion. **limitations:** The reported correlations are calibrated to the simulated geometry family and operating ranges examined (including the pitch range studied) and should not be extrapolated beyond these conditions without additional verification. **Practical implications:** The resulting correlations enable rapid estimation of ϕ_D from readily available nondimensional inputs (Re_{shell}, Re_{coil}, p), reducing the need for repeated full exergy accounting during design screening and operating-map exploration. **Originality:** This work couples a full fluid-to-fluid CFD exergy framework with systematic, cross-validated benchmarking of baseline power-law, advanced surrogate, and pitch-conditioned predictive models for ϕ_D in HCTHEX geometries, explicitly quantifying how model form and pitch conditioning affect predictive accuracy.

Keywords: HCTHEX; exergy; second law; entropy; compact heat exchangers

1. Introduction

Helically coiled tube heat exchangers (HCTHEXs) are widely used in thermal systems because their curved geometry promotes secondary flows and compact heat-transfer surface area, but the resulting thermo-hydrodynamic behavior depends strongly on operating conditions and geometric parameters such as pitch and curvature. Numerical studies have shown that pitch and Dean-number-related effects can substantially alter the local flow structures and heat-transfer/pressure-drop trends, reinforcing that “one-size-fits-all” performance descriptions can be unreliable across geometries and

Reynolds-number ranges (Ghorbani, et al., 2010) (Ghorbani, et al., 2010) (Mirgolbabaei, 2022) (Mirgolbabaei, 2018) (Mirgolbabaei, et al., 2025) (Mirgolbabaei, et al., 2011) (Mirgolbabaei, 2025) (Alabdali, et al., 2025) (Hammoodi, et al., 2025) (Elgendi, et al., 2023). At the same time, recent work in helically coiled configurations has continued to propose geometry- and Reynolds-number-based correlations for performance prediction, highlighting the practical value of reduced-order correlations for design and rapid screening (Abu-Hamdeh, et al., 2021).

From a second-law perspective, exergy analysis provides a rigorous framework to quantify where and how irreversibilities occur and to compare designs on the basis of “useful energy” rather than energy alone. The exergy method (availability analysis) is a well-established approach that uses second-law balances to relate exergy destruction directly to irreversibility and to enable component-level performance assessment beyond first-law metrics (Kotas, 1995). In this framework, exergy destruction $\dot{E}_D$ measures the lost work potential due to irreversibilities, while exergy efficiencies and destruction fractions offer interpretable, dimensionless measures suited for comparing cases with different operating conditions and geometric scales (Kotas, 1995). In this framework, exergy destruction $\dot{E}_D$ measures the lost work potential due to irreversibilities, while exergy efficiencies and destruction fractions offer interpretable, dimensionless measures suited for comparing cases with different operating conditions and geometric scales (Kumar & Sahoo, 2022) (Murugan, et al., 2022).

Exergy analysis has increasingly been applied to helical-coil heat exchangers to interpret performance tradeoffs and to derive predictive relations in terms of Reynolds numbers and geometric descriptors. For example, exergy-based performance comparisons and correlations have been reported for helically coiled heat exchangers as functions of both-side Reynolds numbers and geometric parameters including pitch, demonstrating the feasibility and usefulness of exergy-informed correlation development for coiled geometries. This motivation aligns with the broader need for compact, engineering-ready models that can estimate second-law performance without repeating full CFD/exergy accounting for every geometry–flow combination.

Accordingly, the present work develops and benchmarks predictive correlations for the dimensionless exergy-destruction fraction $\phi_D = \dot{E}_D / \dot{E}_{in, total}$ in a helically coiled tube heat exchanger over a matrix of shell- and coil-side flow conditions and coil geometries. A global baseline power-law form is first established for interpretability, consistent with traditional exergy-method practice of expressing performance in terms of nondimensional groups (Kotas, 1995), and then compared against data-driven surrogates that can capture nonlinear interactions between Reynolds-number effects and geometry. Finally, because pitch is a primary geometric control in helical coils and is known to influence the underlying transport behavior, the study also evaluates pitch-conditioned (pitch-specific) correlations and a hierarchical (varying-coefficient) power-law form to quantify the tradeoff between closed-form simplicity and predictive fidelity across geometries (Ferng, et al., 2012).

2. Computational Model

This research differs from most studies by using a real fluid-to-fluid simulation of a vertically oriented HEX in ANSYS-FLUENT 2023. Both sides are modeled with water, with no phase change assumed.

2.1. Thermophysical Properties:

While temperature-dependent properties could improve accuracy, the complex flow and uneven temperature gradients in the shell side hinder solution convergence. Therefore, the overall average temperature was estimated from prior studies, and water properties were assumed at that temperature.

2.1. Flow Regimes:

The author has previously discussed the most representative length for the shell's flow structure. (Mirgolbabaei, 2022) (Mirgolbabaei, 2018) (Mirgolbabaei, et al., 2011) (Ghorbani, et al., 2010)

(Ghorbani, et al., 2010). This study identified the shell-side flow regime based on the hydraulic diameter of the shadowed surface.

The shell-side Reynolds number in this study ranges from 8.73×10^2 to 1.31×10^4 (corresponding to $\dot{m}_{shell} = 0.01 - 0.15 \text{ kg/sec}$). For shell-side flow in helically coiled tube heat exchangers, the literature commonly applies laminar-type correlations up to $Re_{sh} \approx 10^4$, with alternative forms used above this level. Therefore, the present operating range lies predominantly in the laminar regime and only marginally extends into the onset region; accordingly, the flow is treated as laminar (or at most weakly transitional with negligible turbulence effects) within the stated range of validity, and extrapolation beyond this range is not recommended.

The regime transition of the fluid flow on the coil side is identified according to Schmidt's study (Schmidt, 1967):

$Re_{cr,coil} = 2300 \left[1 + 8.6 \left(\frac{d_{tube}}{D_{coil}} \right)^{0.45} \right]$	(1)
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The coil flow rate up to $\dot{m}_{coil} = 0.025 \text{ kg/sec}$ remains laminar. To analyze shell-side thermal variability, Re and $\dot{m}_{coil} = 0.01 - 0.025 \text{ kg/sec}$ are simulated. Exit configurations include extending the last turn or bending at 90 degrees, but these may cause turbulence. Our models ensure coils turn until fluid exits on both sides, without straight sections, to maintain laminar flow.

Dimensions of various geometries studied are listed in Table I, and the schematic for the HEX along with the exergy flows are shown in Figure 1.

The simulation operates under steady-state conditions using a pressure-based 'coupled' algorithm. It employs second-order discretization for the continuity, momentum, and energy equations, with relation factors between models. The coil wall is coupled with its shadow; the coil tube is modeled in Ansys as a 1D wall with constant copper properties. Artifacts at the coil base are modeled similarly. Protruding coil sections are treated as adiabatic with zero-thickness copper, and shell walls as adiabatic zero-thickness aluminum. The steady-state equations for continuity, momentum, and energy are solved accordingly (Versteeg & Malalasekera, 2007):

$\frac{\partial}{\partial x_i}(\rho u_i) = 0$	(2)
$\frac{\partial}{\partial x_i}(\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \left(\mu \frac{\partial u_j}{\partial x_i} \right) + \rho g_i$	(3)
$\frac{\partial}{\partial x_i}(\rho c_p u_i T) = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right)$	(4)

The computational cells and the type of meshing schemes used in the study are laid out in detail in (Mirgolbabaei, et al., 2025).

Table 1. Coils and shell specifications.

Shell	Shell	Tube	D_{coil}	P	Numb	Heigh	Coil surface	D_h
D_{oc}	D_{ic}	d_{outer}/d_{inner}	(mm)	(Pitch	er of	(mm)	area, $A_{coil,outer}$	(mm)
(mm)	(mm)	(mm/mm)		/d _{out})	turns,		(mm ²)	
N								
160	90	9.52/8.001	125	1.8	10.50	180	1.03771e+05	11.57

160	90	9.52/8.001	125	1.85	10.22	180	1.00974e+05	11.84
160	90	9.52/8.001	125	1.90	9.95	180	9.83241e+04	12.10
160	90	9.52/8.001	125	1.95	9.70	180	9.58106e+04	12.35
160	90	9.52/8.001	125	2	9.45	180	9.34230e+04	12.60
160	90	12.5/10.98	125	1.8	7.11	360	0.92330e+05	0.010.17
160	90	12.5/10.98	125	1.9	6.74	360	0.87494e+05	10.74
160	90	12.5/10.98	125	2	6.40	360	0.83142e+05	11.27

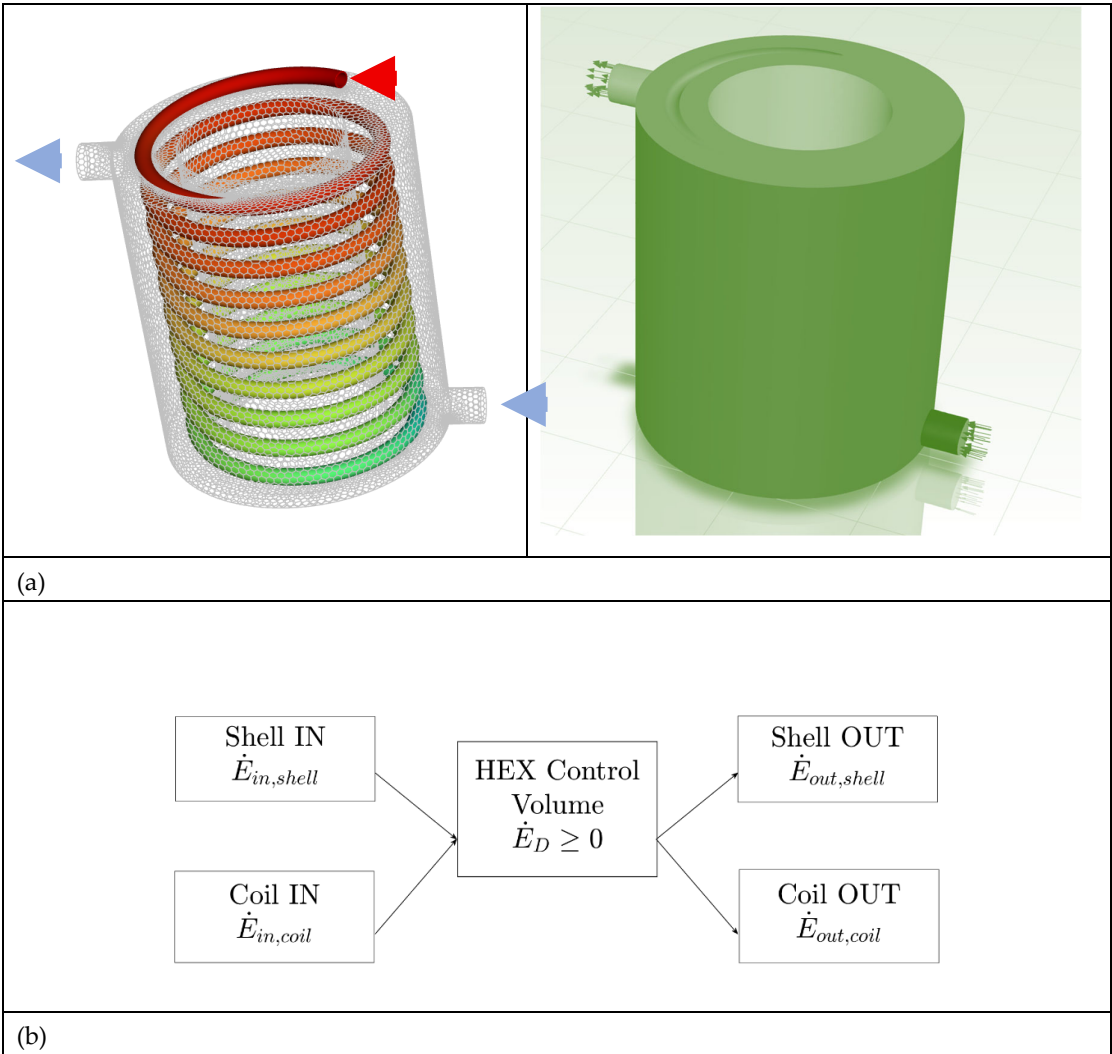


Figure 1. (a) Schematic of the helically coiled tube heat exchanger (HCTHEX) geometry and flow arrangement considered in this study. The coil-side stream enters the helical tube at the top and exits after traversing the coil, while the shell-side stream flows through the annular region between the inner and outer cylinders (arrows indicate inlet/outlet directions). (b) Exergy accounting for the heat-exchanger control volume used to compute overall exergy destruction, $\dot{E}_D \geq 0$, from the inlet and outlet exergy rates of the shell and coil streams, $\dot{E}_{in,shell}$, $\dot{E}_{out,shell}$, $\dot{E}_{in,coil}$, $\dot{E}_{out,coil}$.

2.1. Numerical Model Verification

This section confirms the accuracy of the numerical model by comparing it to a well-established formula that describes the coil's thermal and hydraulic characteristics. Benchmarking the model against experimental data was documented in earlier studies (Mirgolbabaei, 2022) and is reproduced here (Figure 2).

The Nusselt number on the shell-side is mostly determined by the curvature ratio, with the pitch having little effect, as described below (Schmidt, 1967):

$$\overline{Nu}_{coil} = 3.65 + 0.08 \left[1 + 0.8 \left(\frac{d_{tube}}{D_{coil}} \right)^{0.9} \right] Pr^{\frac{1}{3}} Re^{0.5 + 0.2903 \left(\frac{d_{inner}}{D_{coil}} \right)^{0.194}} \quad (5)$$

$\overline{Nu}_{coil}$ in our numerical models is calculated as follows (Bergman, et al., 2019):

$$q_{coil} = \dot{m}_c \bar{c}_{p,c} (\bar{T}_{c,out} - \bar{T}_{c,in}) = \bar{h}_{coil} A_{tube,inner} (\bar{T}_{b,c} - \bar{T}_{tube,inner}) \quad (6)$$

$$\overline{Nu}_{coil} = \frac{\bar{h}_{coil} d_{inner}}{k} \quad (7)$$

where $\bar{T}_{b,c}$ and $\bar{T}_{tube,inner}$ are volume-based average temperature of the coil-side medium and overall inner coil surface temperature, respectively.

These CFD-derived Nusselt numbers were compared with the widely used helical-coil correlation of Schmidt (Schmidt, 1967) (Equation (5)), which depends primarily on curvature ratio and exhibits only a weak sensitivity to pitch. Over the investigated pitch range ($P = 1.8 - 2.0$), the deviation between the simulations and the correlation remained between 0.7% and 8.7% , indicating that the model reproduces the expected magnitude of coil-side convection.

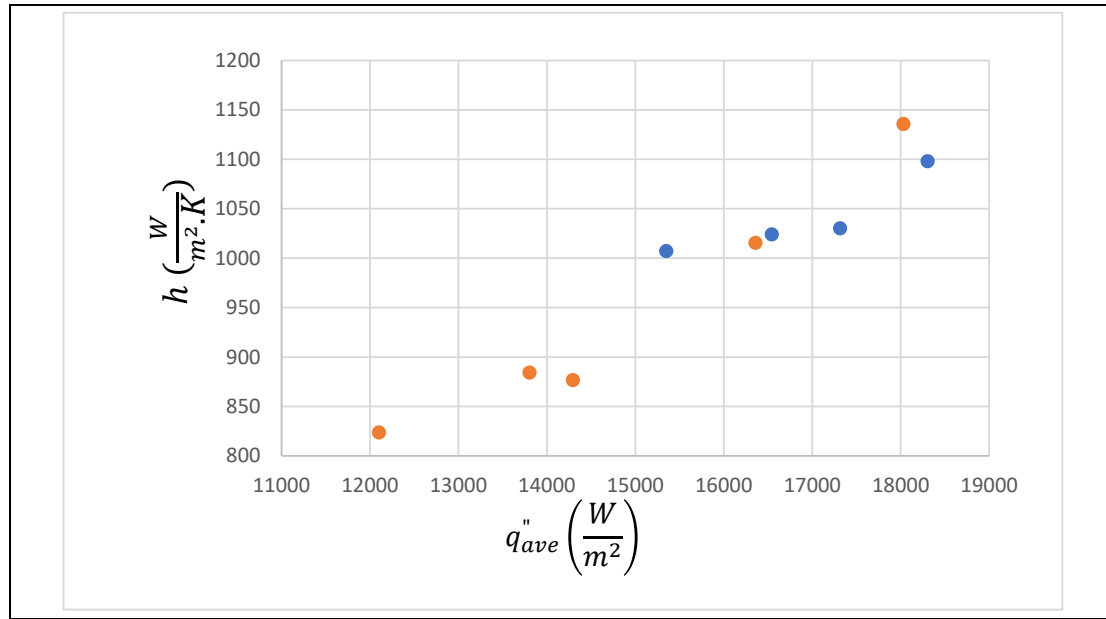


Figure 2. Comparison of the numerical simulation results with the experimental measurement reported in (Mirgolbabaei, 2022).

A second validation step was conducted to assess the hydraulic response on the coil side. The total pressure drop on the coil side derived from the CFD solution was compared with the semi-empirical correlation (Ali, 2001) commonly used for standard helical coils. Since all simulated geometries have the same curvature ratio, the same reference correlation was consistently applied across different pitch cases. Between $P = 1.8$ and 2.0 , the pressure-drop differences stayed under 10% (ranging from 3.7% to 9.1%). This consistency indicates that the numerical configuration accurately

represents the main friction loss mechanisms in the coil and offers a dependable thermo-hydraulic foundation for later shell-side analyses. While correlations with (Schmidt, 1967) and (Ali, 2001) offer useful benchmarks, they are correlation-based validations, not geometry-specific model-to-experiment validations. These semi-empirical correlations, derived from experimental data, serve as practical proxies within their applicability ranges. To enhance validation, we compare CFD predictions with direct experimental measurements from (Mirgolbabaie, 2022), conducted earlier for a similar geometry and conditions. This provides a direct model-to-experiment check, while correlation comparisons offer complementary benchmarks.

2.1. Grid Independence Analysis

A grid-refinement study was conducted for the $P = 1.8$ HCTHEX configuration to quantify discretization uncertainty in the global heat-exchanger's exergy destruction fraction $\phi_D = \dot{E}_D / \dot{E}_{in,total}$ using the Grid Convergence Index (GCI) methodology (Richardson extrapolation-based) as recommended in the ASME CFD literature (Roache, 1994) (Fluids Engineering Division of ASME, 2008).

Three systematically refined grids were considered: the original grid ($N_3 = 1,743,702$), a boundary layer adapted grid ($N_2 = 2,064,312$), and a finer grid produced by additional adaptation including global refinement ($N_1 = 26,117,622$). A representative grid spacing was defined as $h \propto N^{-\frac{1}{3}}$ for 3-D meshes, giving refinement ratios $r_{21} = h_2/h_1 = (N_1/N_2)^{1/3} = 2.33$ and $r_{32} = h_3/h_2 = (N_2/N_3)^{1/3} = 1.06$. Because r_{32} is close to unity (coarse and medium meshes are relatively close), estimating the observed order p from the full three-grid formulation becomes numerically ill-conditioned; therefore, a conservative second-order assumption ($p = 2$) was used for the fine/medium pair together with the standard safety factor F_s used in GCI practice (Roache, 1994). The GCI for a solution functional ϕ was computed from the approximate relative error $e_{a,21} = |\phi_1 - \phi_2|/|\phi_1|$ as:

$GCI_{21} = F_s \frac{e_{a,21}}{r_{21}^p - 1}$	(8)
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which provides a conservative uncertainty band about the fine-grid solution (Roache, 1994). Across the three mesh levels, the exergy-destruction fraction ϕ_D changes only slightly, increasing from 0.942584643485082 on the original grid to 0.946385161476648 on the BL-adapted grid and remaining 0.946385161476648 on the finest (BL + global refinement) grid. This corresponds to a $\sim 0.40\%$ increase from the original to BL-adapted mesh and 0.00% change between the BL-adapted and finest meshes, indicating that ϕ_D is effectively grid independent over the refinement range considered. Similarly, the exergy destruction rate $\dot{E}_D$ increases from 19400.8126261261 (original) to 19443.9804873558 (BL-adapted) and 19479.0360948075 (finest), which corresponds to $\sim 0.22\%$ (original $\rightarrow$ BL-adapted) and $\sim 0.18\%$ (BL-adapted $\rightarrow$ finest) differences. Overall, both metrics show a clear convergence trend, with ϕ_D essentially converged at the BL-adapted level and $\dot{E}_D$ exhibiting only a modest residual sensitivity; therefore, the BL-adapted mesh can be considered a suitable production grid that captures the exergy-destruction behavior close to the finest-grid solution at substantially lower computational cost.

The same GCI-based grid refinement procedure was also applied to the additional geometries with $P = 1.85, 1.9, 1.95$, and 2 to ensure consistent discretization accuracy across the full parametric set.

3. Energy Closure Metric

Figure 2 provides a first-law consistency check used to validate the heat-duty inputs required for the exergy analysis. We quantify energy closure with the normalized heat-rate imbalance, δ_q , defined from the independently calculated shell- and coil-side heat transfer rates; by normalizing with the mean of $|q_{shell}|$ and $|q_{coil}|$, δ_q becomes a dimensionless measure of mismatch that is comparable across operating conditions. The histogram (left) for the filtered "GOOD" dataset is sharply peaked around $\delta_q \approx 0$ (order 10^{-5}), indicating that the two duty estimates agree closely for the vast majority of cases and that systematic bias is negligible. The scatter plot (right) shows δ_q as a

function of shell-side Reynolds number (with point color indicating Re_{coil}), demonstrating that the closure remains centered near zero across the explored flow regime, with no clear Reynolds-number dependence; the few outliers represent operating points where measurement uncertainty, transient behavior, or model assumptions likely degrade the balance. Overall, the strong closure supports the use of these data for robust exergy accounting, since exergy rates and exergy destruction are highly sensitive to inconsistent heat duties and temperature differences.

$\delta_q = \frac{q_{shell} + q_{coil}}{\frac{1}{2}(q_{shell} + q_{coil})}$	(9)
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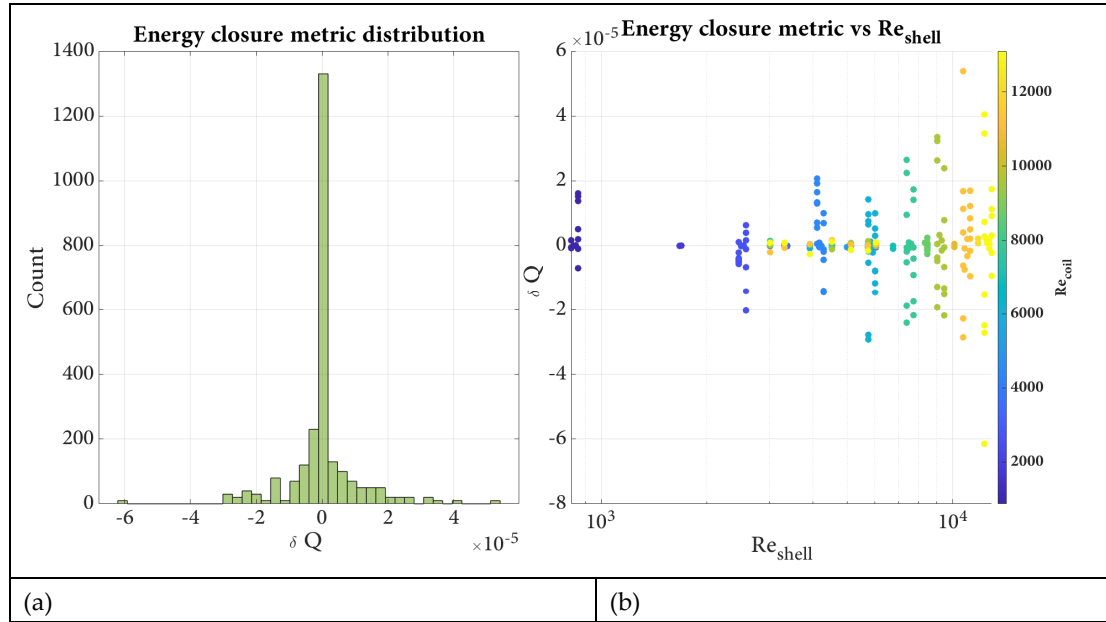


Figure 2. Left: histogram of δ_Q for all data points passing the quality filters (“GOOD” rows), showing a strong concentration near $\delta_Q \approx 0$ (note the $\times 10^{-5}$ scale), consistent with good energy balance between shell and coil calculations. Right: δ_Q versus shell-side Reynolds number (Re_{shell} , log scale), with marker color indicating Re_{coil} ; the scatter remains centered near zero with no obvious dependence on Reynolds number, while a small number of outliers highlight cases with weaker closure.

4. Results

4.1. $\dot{E}_D$ Distribution

For a steady open system, the specific flow exergy is (Cengel & Boles, 2023):

$e(T, P, z) = (h - h_0) - T_0(s - s_0) + \frac{v^2}{2} + g(z - z_0) \approx c_p \left[(T - T_0) - T_0 \ln \left(\frac{T}{T_0} \right) \right] + \frac{v^2}{2} + g(z - z_0).$	(10)
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The exergy destruction due to ... according to the global exergy balance, as follows:

$\dot{E}_D = (\dot{E}_{in,coil} + \dot{E}_{in,shell}) - (\dot{E}_{out,coil} + \dot{E}_{out,shell})$	(11)
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where

$\dot{E}_{in,coil} = \dot{m}_{coil} e(T, P, z)_{in,coil}$	(12a)
$\dot{E}_{out,coil} = \dot{m}_{coil} e(T, P, z)_{out,coil}$	(12b)
$\dot{E}_{in,shell} = \dot{m}_{shell} e(T, P, z)_{in,shell}$	(12c)
$\dot{E}_{out,shell} = \dot{m}_{shell} e(T, P, z)_{out,shell}$	(12d)

Figure 3 represents the distribution of $\dot{E}_D$ vs Re_{shell} . The overall exergy destruction rate increases with increasing shell-side Reynolds number: the highest $\dot{E}_D$ values occur at the largest Re_{shell} , while the lowest Re_{shell} cases cluster at comparatively smaller $\dot{E}_D$. At a given Re_{shell} , there is also clear spread with pitch, indicating that geometry (through p) contributes to the variability in $\dot{E}_D$ in addition to flow rate.

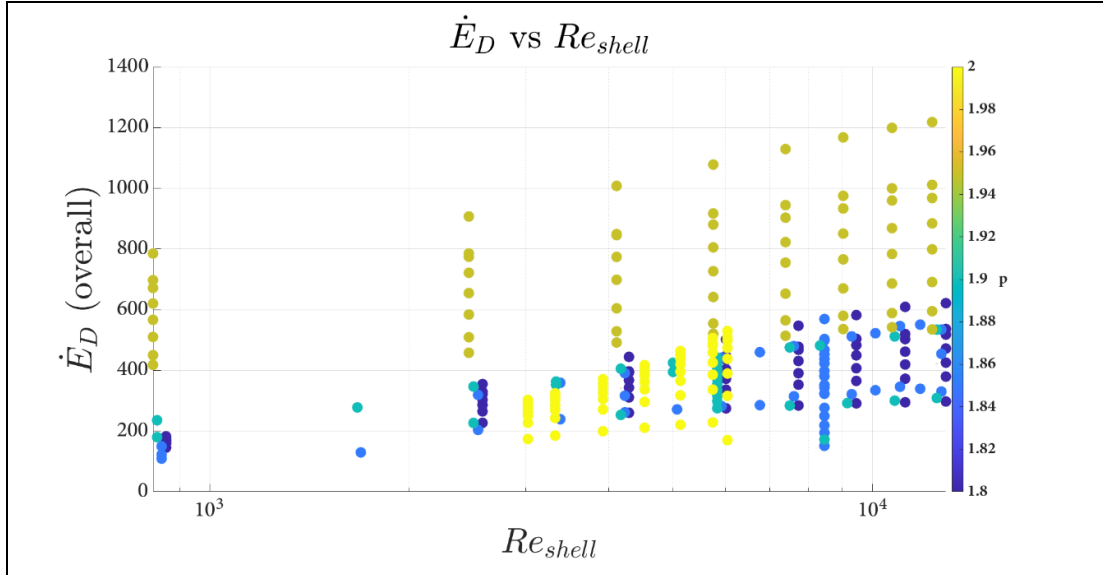


Figure 3. Overall exergy destruction rate, $\dot{E}_D$, as a function of shell-side Reynolds number, Re_{shell} (log-scale). Marker color denotes dimensionless pitch p , highlighting the combined influence of operating condition and geometry on $\dot{E}_D$.

4.2. Overall Exergy Transfer Ratio, η_D

As obvious from Figure 5 ($\eta_{overall}$ vs Re_{shell}), the overall exergy efficiency:

$$\eta_{overall} = \dot{E}_{out,total} / \dot{E}_{in,total} \quad (13)$$

is generally low for most of the dataset (many points below ~ 0.2), with the largest efficiencies appearing primarily among the lowest Re_{shell} cases. The wide scatter at a given Re_{shell} and the visible pitch-dependent grouping indicate that both operating condition and geometry affect $\eta_{overall}$.

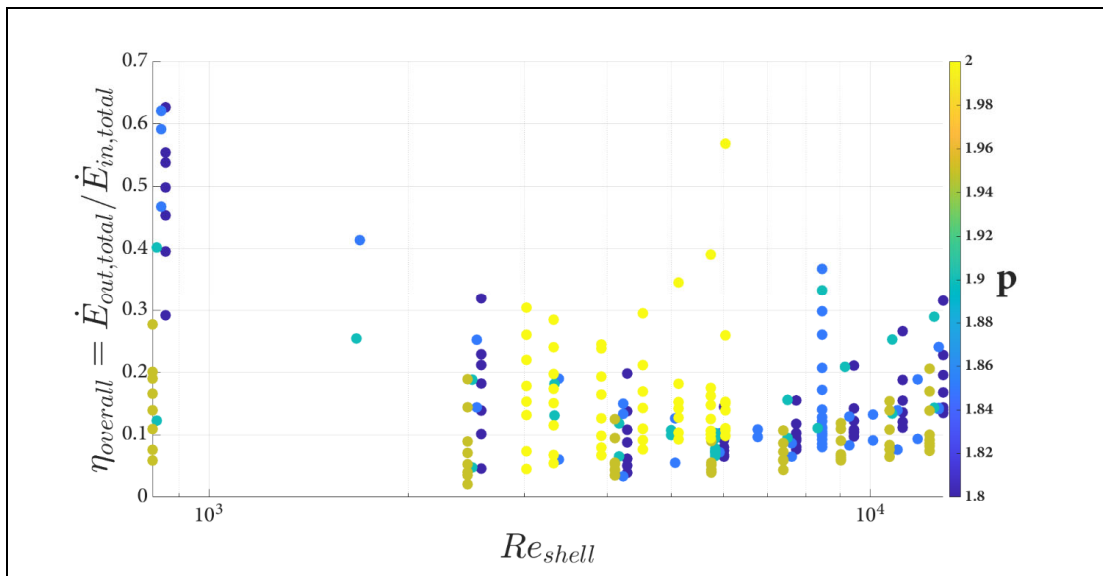


Figure 4. Overall exergy transfer ratio, $\eta_{\text{overall}} = \dot{E}_{\text{out,total}}/\dot{E}_{\text{in,total}}$, versus Re_{shell} (log-scale), with marker color indicating pitch p .

4.3. $\Delta\dot{E}_{\text{shell}}$

As demonstrated in Figure 6 ($\Delta\dot{E}_{\text{shell}}$ vs Re_{shell}), The shell-side exergy change $\Delta\dot{E}_{\text{shell}} = \dot{E}_{\text{out,shell}} - \dot{E}_{\text{in,shell}}$ is predominantly negative across the dataset, and its magnitude becomes more negative as Re_{shell} increases (points shift downward with increasing Re_{shell}). This indicates larger shell-side exergy reduction at higher shell Reynolds numbers, with additional dispersion attributable to pitch.

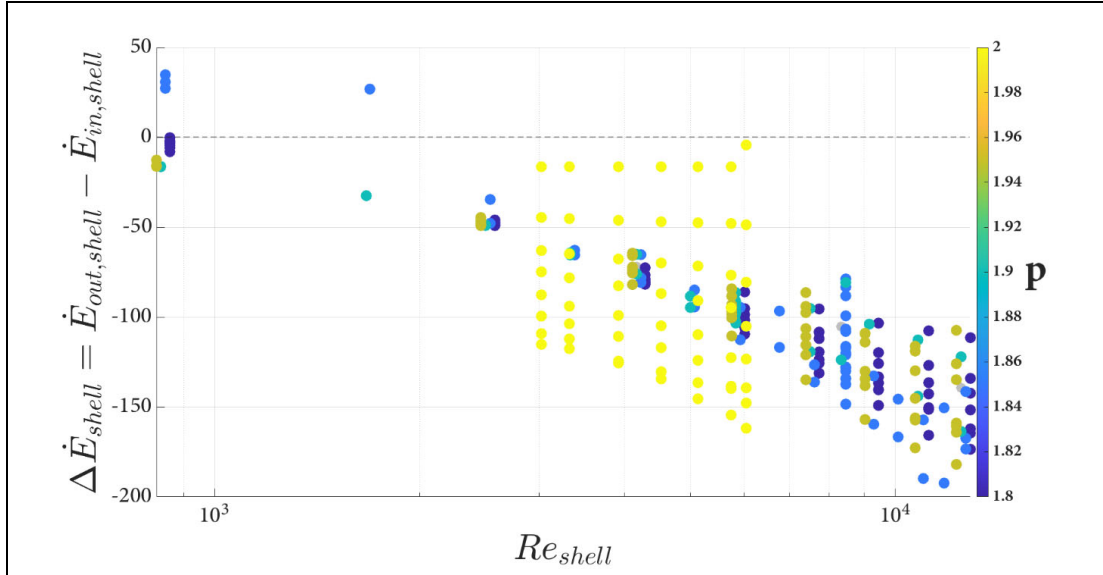


Figure 4. Shell-side exergy change, $\Delta\dot{E}_{\text{shell}} = \dot{E}_{\text{out,shell}} - \dot{E}_{\text{in,shell}}$, versus Re_{shell} (log-scale), with marker color indicating pitch p . The horizontal dashed line marks $\Delta\dot{E}_{\text{shell}} = 0$.

4.4. Exergy Destruction Fraction, ϕ_D

Figure 4 (ϕ_D vs Re_{shell}). The dimensionless exergy-destruction fraction which is defined as follows:

$$\phi_D = \frac{\dot{E}_D}{\dot{E}_{\text{in,total}}} \quad (14)$$

remains predominantly high (most points clustered roughly in the 0.8–0.95 band) across the majority of the Re_{shell} range, showing weak systematic dependence on Re_{shell} compared with its overall level. The lowest- Re_{shell} cases exhibit noticeably larger scatter (including lower ϕ_D values), and pitch contributes to that spread.

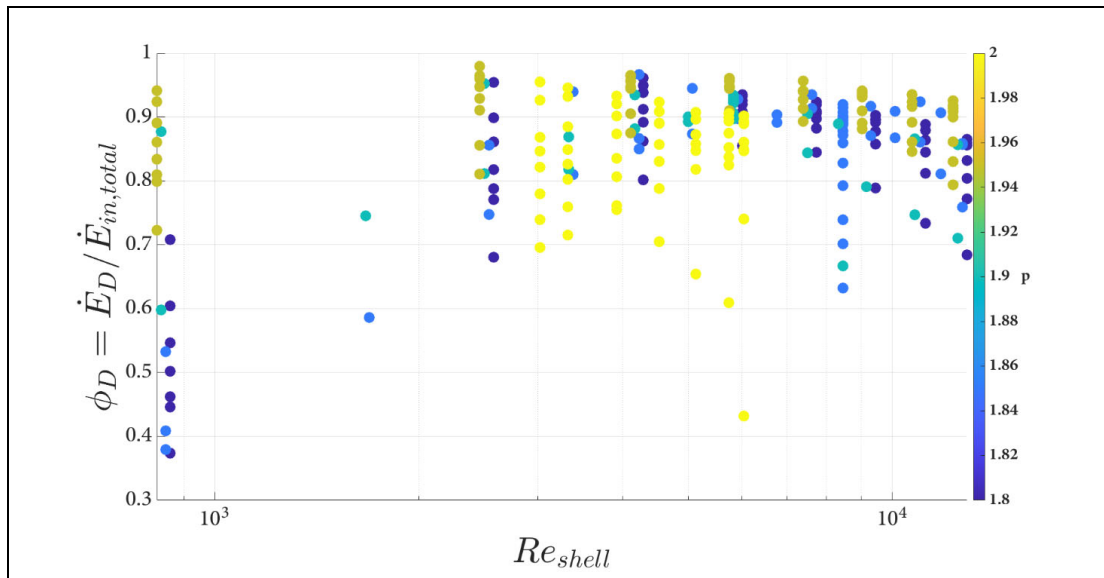


Figure 6. Dimensionless exergy-destruction fraction, $\phi_D = \dot{E}_D / \dot{E}_{in, total}$, versus Re_{shell} (log-scale), with marker color indicating pitch p .

5. Predictability

While detailed exergy balances provide the most rigorous assessment of thermodynamic performance, they are not always practical for design iteration, screening studies, or rapid operating-condition mapping because they require full field solutions (or complete exergy accounting) for every new geometry and flow-rate combination. For the helically coiled tube heat exchanger considered here, the dimensionless exergy-destruction fraction $\phi_D = \dot{E}_D / \dot{E}_{in, total}$ offers a compact, physically meaningful performance metric that normalizes irreversibility by the available exergy input, enabling direct comparison across operating conditions and geometrical variants. Developing correlations for ϕ_D therefore provides a predictive shortcut: once calibrated on the computed dataset, ϕ_D can be estimated from readily available nondimensional groups (e.g., Re_{shell} , Re_{coil} , and pitch p), allowing rapid evaluation of second-law performance without repeating the full exergy analysis. The following section quantifies this predictability by establishing baseline power-law correlations and benchmarking them against data-driven and structured hierarchical alternatives, reporting accuracy through cross-validation metrics and parity plots.

5.1. ϕ_D Correlations

The objective of the regression analysis is to construct practical correlations for the dimensionless exergy-destruction fraction (ϕ_D), which quantifies the relative magnitude of irreversibility generated by the exchanger under a given operating condition. In exergy analysis, irreversibility is fundamentally linked to entropy generation (e.g., via the Gouy–Stodola framework), making ϕ_D a compact indicator of second-law **performance** that can be correlated against flow and geometric drivers (Kotas, 1995).

Two modeling scopes were examined:

- **Global (non-pitch-specific) fits**, where all data are pooled and the pitch ratio (p) is included explicitly as an input.
- **Pitch-specific fits**, where separate models are trained at each pitch (so (p) is constant within each subset and is not included as a predictor).

All models were trained in log space (i.e., using $\ln \phi_D$ as the response), which is the standard linearization used to estimate power-law relationships and typically stabilizes multiplicative variability (Box & Cox, 1964).

Predictive performance was assessed using five-fold cross-validation, which is widely used to estimate generalization error and to guard against selecting models that only fit the training sample (Arlot, 2010).

In addition to (R^2) , the discussion emphasizes cross-validated RMSE in log-space, $RMSE_{CV}(\ln \phi_D)$, because (R^2) can become unstable (and even negative) on test folds in low-variance subsets, whereas RMSE remains directly interpretable as a prediction-error magnitude (Kvålseth, 1985).

5.1.1. ϕ_D Global Correlations

5.1.1.1. Power-Law Correlations

To establish a compact and interpretable baseline for the normalized exergy destruction fraction, $\phi_D = \dot{E}_D / \dot{E}_{in,total}$, a global power-law correlation was first fitted using a log-log linear regression of the form:

$\phi_D = A Re_{shell}^a Re_{coil}^b P^c$	(15)
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The fitted coefficients for both characteristic-length definitions are reported in Table 2, and the corresponding baseline parity behavior based on hydraulic diameter is shown in Figure 7.

For the hydraulic-diameter-based global fit (Table 2), the OLS coefficients are ($A=0.224$), ($a=0.0489$), ($b=0.0498$), and ($c=0.751$) and the pitch exponent is statistically significant and both Reynolds-number exponents are also significant at conventional levels, indicating that the dataset supports a measurable dependence of ϕ_D on both flow-state and geometry when pooled globally.

Despite its interpretability, the global power-law baseline is not the most accurate surrogate for prediction. Cross-validated performance (Table 3) for the baseline OLS model yields $RMSE_{CV}(\ln \phi_D) = 0.140$, $R^2_{CV}(\ln \phi_D) = 0.169$, and $MAPE_{CV}(\phi_D, \%)$ for D_h . This level of scatter is also visually consistent with the baseline parity dispersion in Figure 7, where systematic deviation from the 1:1 line persists in parts of the design space. These quantities are based on the following general formulas:

$RMSE_{CV} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{\phi}_{D,i}^{CV} - \phi_{D,i})^2}$	(16)
--	------

and

$MAPE_{CV} = \left \frac{y_{pred} - y_{true}}{y_{true}} \right \times 100\%,$	(17)
---	------

respectively, where $\{(\phi_{D,i}, \hat{\phi}_{D,i}^{CV})\}_{i=1}^N$ is the dataset, and $\hat{\phi}_{D,i}^{CV}$ is the cross-validated (out-of-fold) prediction for point i (i.e., predicted by a model that was trained without using point i).

5.1.1.2. Robust power-Law Variant and the Importance of Cross-Validation

Table 2 also reports a robust bisquare power-law regression. Although robust methods can reduce the influence of outliers in some settings, robust training does not automatically imply better generalization; in fact, cross-validation is needed (following section) because robust fits can still drift toward patterns that do not hold in withheld folds (Krstajic, et al., 2014). Here, the robust global model underperforms the OLS baseline in cross-validation (Table 3). The values in Table 3 confirm that the robust fit does not provide a predictive advantage for this dataset.

Table 2. Global Power-Law Fitting Specifications $\phi_D = ARe_{shell}^a Re_{coil}^b P^c$.

Characteristic Length: D_h												
Mod el	A	a	b	c	p- valu e, a	p- valu e, b	p- valu e, c	R^2 (ln	$\bar{R}^2$ (ln	RMSE(l	RMSE(	MAPE(ϕ
Pow er- law (log- log OLS)	0.22 3798	0.04 8845	0.04 9823	0.75 0697	0.03 5045	0.01 5319	0.00 0735	0.244 012	0.235 083	0.1337 4	0.097 212	9.77500 6
Pow er- law (rob ust bisq uare)	0.63 273	0.13 0155	- 012 807	0.52 501	7.00 E-26	2.59 E-30	1.12 E-06	0.597 411	0.592 656	0.1752 4	0.129 599	10.6344 8
Characteristic Length: L_n												
Mod el	A	a	b	c	p- valu e, a	p- valu e, b	p- valu e, c	R^2 (ln	$\bar{R}^2$ (ln	RMSE(l	RMSE(	MAPE(ϕ
Pow er- law (log- log OLS)	0.21 1698	0.04 8821	0.04 9842	0.79 0218	0.03 5153	0.01 5286	0.00 0485	0.243 997	0.235 067	0.1337 41	0.097 213	9.77526 2
Pow er- law (rob ust bisq uare)	0.54 5117	0.13 0227	- 012 81	0.63 0913	6.91 E-26	2.62 E-30	1.22 E-08	0.597 212	0.592 455	0.1752 3	0.129 595	10.6339 4

Table 3.

Model Class	Characteristic Length: D_h					
	R^2_{CV}	$RMSE_{CV}(\phi)$	$MAE_{CV}(\phi)$	$MAPE_{CV}(\phi_D)$	$R^2_{CV}(\ln\phi_D)$	$RMSE_{CV}(\ln\phi_D)$
LogLog_GPR_ARDSE	0.705389 889	0.0583077 96	0.039524 331	5.166476679	0.717130 032	0.08180815 2
LogLog_Ensemble_LSB oost	0.687429 86	0.0600587 85	0.042308 436	5.461087129	0.709092 452	0.08296227 6
LogLog_Ensemble_Bag gedTrees	0.595554 392	0.0683176 41	0.050094 43	6.659271866	0.599708 592	0.09731757 9
LogLog_GAM	0.565449 461	0.0708146 22	0.051430 523	6.655461266	0.596406 115	0.09771819 8
LogLog_LM_OLS (PowerLaw)	0.097491 52	0.1020536 27	0.077289 378	10.22575322	0.168722 398	0.14024138 8
LogLog_LM_RobustBis quare (Power Law)	- 0.476065 264	0.1305135 43	0.071732 376	10.78826439	- 0.308106 543	0.17592385
	Characteristic Length: L_n					
	R^2_{CV}	$RMSE_{CV}(\phi)$	$MAE_{CV}(\phi)$	$MAPE_{CV}(\%)$	$R^2_{CV}(\ln\phi_D)$	$RMSE_{CV}(\ln\phi_D)$
LogLog_GPR_ARDSE	0.705686 638	0.0582784 23	0.039509 677	5.164635358	0.717320 909	0.08178054 6
LogLog_Ensemble_LSB oost	0.682180 433	0.0605610 11	0.042698 917	5.505220257	0.705371 472	0.08349117 2
LogLog_GAM	0.579892 032	0.0696278 94	0.050546 402	6.554986429	0.607294 211	0.09639107 3
LogLog_Ensemble_Bag gedTrees	0.588766 972	0.0688885 1	0.050346 769	6.718423741	0.590102 882	0.09847831 2
LogLog_LM_OLS (Power-Law)	0.097458 559	0.1020554 9	0.077292 858	10.22617677	0.168698 947	0.14024336 6
LogLog_LM_RobustBis quare (Power-Law)	- 0.473688 958	0.1304084 44	0.071696 187	10.7814817	- 0.306434 575	0.17581138 5

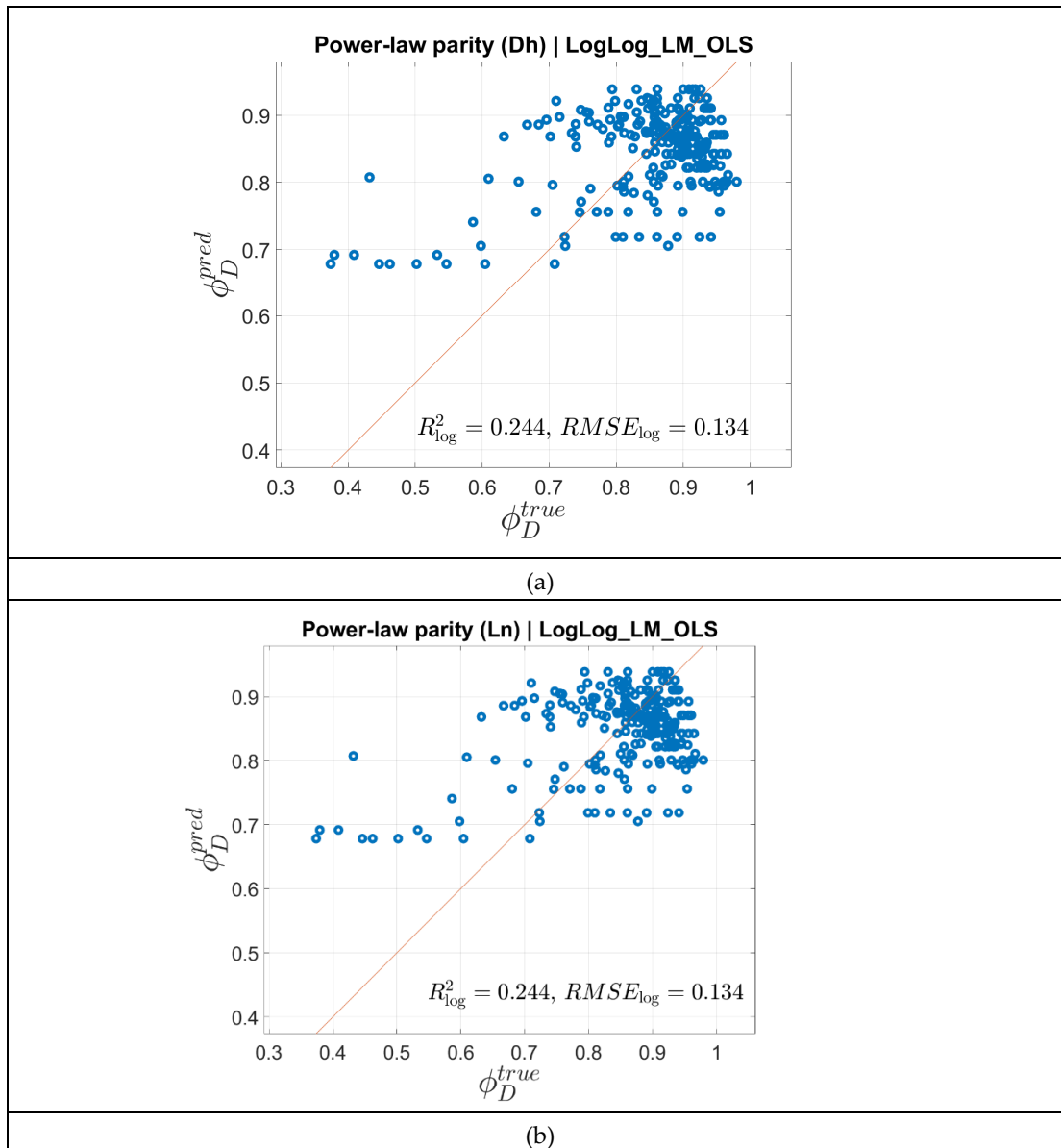


Figure 7. Baseline global power-law correlation parity plot for ϕ_D using both the D_h -based and L_n -based definition of $R_{shell}(\log\text{-}\log \text{ OLS})$. Predicted values are compared against dataset values; the 1:1 line is shown for reference.

5.1.1.2. Global advanced Models and best-Performing Surrogates

5.1.1.2.1. Comparative cross-Validated Performance

To quantify the performance ceiling beyond a fixed power-law structure, several nonlinear surrogate models were trained on the same log-transformed predictors and evaluated by five-fold cross-validation (Table 3). These models include:

Gaussian Process Regression (GPR) with an ARD squared-exponential kernel, which provides a flexible Bayesian nonparametric mapping and uses ARD to assign separate characteristic length scales to each input dimension (Rasmussen & Williams, 2005).

Tree-ensemble methods, including LSBoost (gradient boosting of regression trees) and bagged trees. Boosting is a stagewise additive optimization procedure that can reduce bias, whereas bagging is a bootstrap aggregation strategy that reduces variance by averaging many perturbed learners (Friedman, 2001).

Generalized Additive Models (GAMs), which represent the response as a sum of smooth functions of predictors and can capture nonlinearities while retaining partial interpretability (Hastie & Tibshirani, 1986).

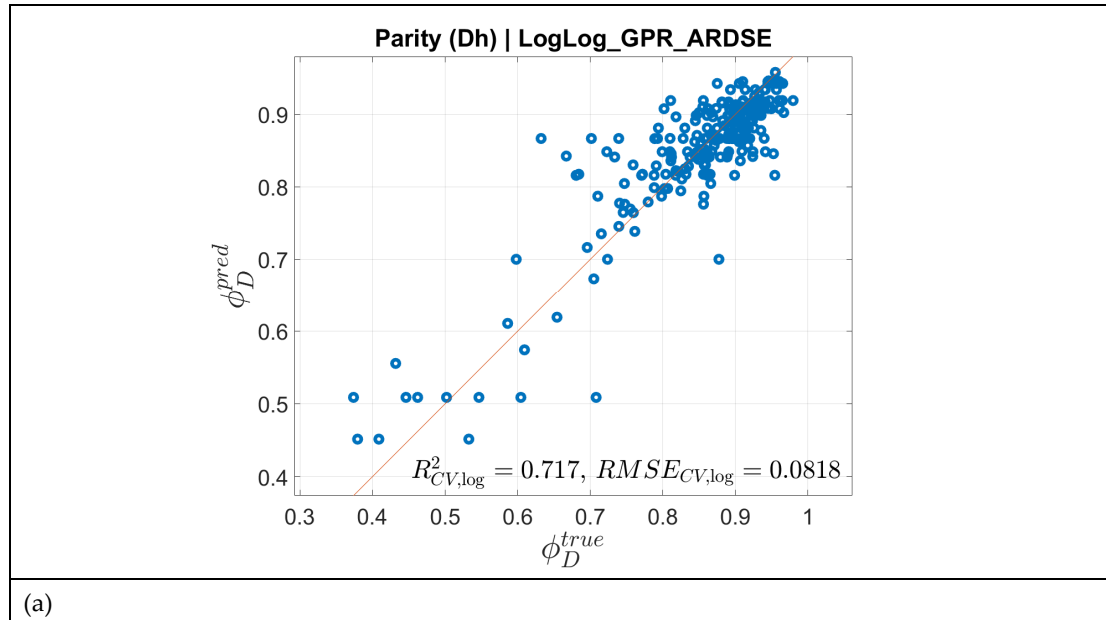
Across both Re_{shell} definitions (D_h - and L_n -based), LogLog-GPR-ARDSE produced the best overall performance (Table 3). For the D_h -based global case, GPR achieves $RMSE_{CV}(\ln\phi_D) = 0.0818$, $R^2_{CV}(\ln\phi_D) = 0.717$, and $MAPE_{CV}(\phi_D, \%) = 5.17\%$.

Relative to the global OLS power-law baseline, GPR reduces $RMSE_{CV}(\ln\phi_D)$ by approximately 41.7% and reduces $MAPE_{CV}(\phi_D, \%)$ by approximately 49.5%. This improvement indicates that nonlinear interactions and/or local curvature in the $(Re_{shell}, Re_{coil}, P)$ space are important for accurate mapping of ϕ_D , and that these effects are not well captured by a single global power-law exponent set.

LSBoost is a close second globally (Table 3), with $RMSE_{CV}(\ln\phi_D) = 0.0830$ and $R^2_{CV}(\ln\phi_D) = 0.709$, consistent with the strong performance of boosted tree ensembles on nonlinear regression tasks.

5.1.1.2.2. Characteristic Length D_h versus L_n in the Global Setting

The global results show only marginal sensitivity to whether Re_{shell} is formed using D_h or the alternative length L_n . For example, GPR yields $RMSE_{CV}(\ln\phi_D) = 0.081808$, using D_h versus 0.081781, using L_n , and nearly identical $MAPE_{CV}(\phi_D, \%)$. This close agreement is reflected directly in Figure 8, where the GPR parity plots for both characteristic lengths collapse to essentially the same predictive quality. Practically, this indicates that the dominant gains stem from the model form (nonlinear surrogate vs fixed power law) rather than from the modest rescaling introduced by the alternative shell-side length definition for this dataset.



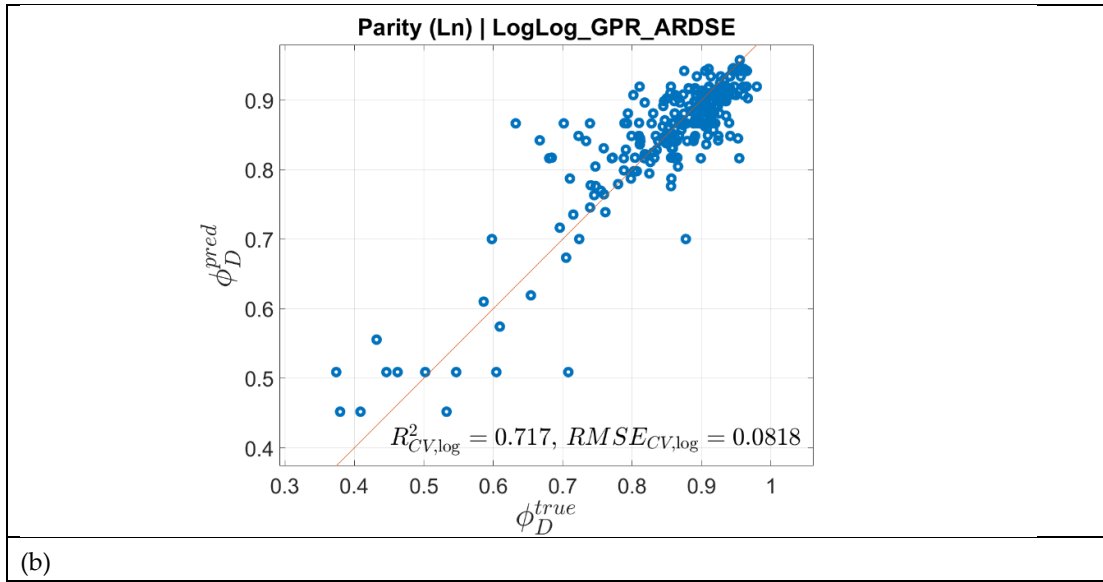


Figure 8. Global advanced-model parity plots for ϕ_D using LogLog-GPR-ARDSE: (a) Re_{shell} based on D_h , and (b) Re_{shell} based on L_n . Predicted values are compared against dataset values with the 1:1 line shown.

5.1.2. ϕ_D Pitch-Specific Correlation

5.1.2.1. Pitch-Specific Power-Law Correlations

To examine whether “one set of exponents fits all,” separate pitch-specific regressions were constructed at each pitch ratio, P , yielding:

$$\phi_D = A Re_{shell}^a Re_{coil}^b \quad (18)$$

Because (p) is constant within each subset, its influence is absorbed into the pitch-specific prefactor A . The pitch-specific baseline parity plots (power-law OLS, D_h -based) are shown in Figure 9(a–e) for all pitches ($P = 1.80, 1.85, 1.90, 1.95, 2.00$). The fitted coefficients and in-sample fit statistics are provided in Table 4. A key observation from Table 4 is that the fitted exponents a and b vary substantially with pitch and, for some pitches, take on large-magnitude values of opposite sign (e.g., $a \sim -20$ and $b \sim +20$ for $P = 1.80$ and $P = 1.90$). This pattern is a hallmark of parameter non-uniqueness that can arise when predictors are strongly correlated (multicollinearity), making the individual slope estimates ill-conditioned even if the combined two-variable mapping provides an acceptable in-sample fit (Belsley, et al., 1980). In other words, Figure 9 demonstrates that pitch-specific power laws can match the observed data within each pitch, but Table 4 indicates that the resulting exponent set may not be robustly interpretable as a stable physical scaling for every pitch.

5.1.2.2. D_h versus L_n Equivalence within a Fixed Pitch

For pitch-specific fits, results with D_h and L_n are effectively equivalent in slope because L_n/D_h depends only on geometry, and geometry is fixed within each pitch subset. In log space:

$$\ln(Re_{shell,L_n}) = \ln(Re_{shell,D_h}) + \ln\left(\frac{L_n}{D_h}\right) \quad (19)$$

At fixed P , the second term is a constant and is absorbed into $\ln(A)$, leaving a and b unchanged. This explains why D_h - and L_n -based pitch-specific models produce nearly indistinguishable parity trends once the intercept adjusts.

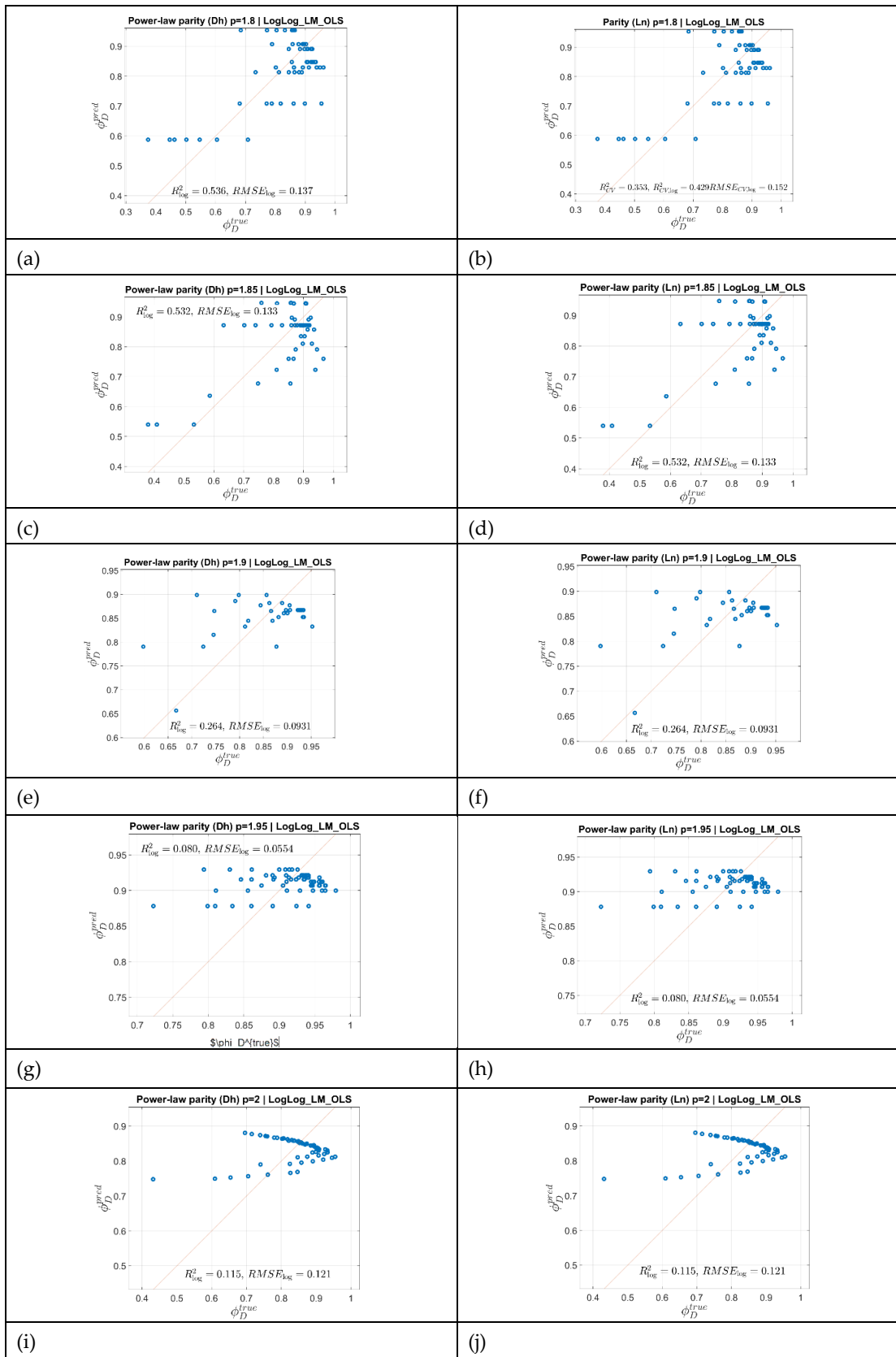


Figure 9. Pitch-specific baseline power-law (log-log OLS) parity plots for ϕ_D both the D_h -based and L_n -based definition of Re_{shell} : (a)&(b) $p = 1.80$, (c)&(d) $p = 1.85$, (e)&(f) $p = 1.90$, (g)&(h) $p = 1.95$, and (i)&(j) $p = 2.00$. Each panel shows ϕ_D^{pred} vs. ϕ_D^{true} with the 1:1 line.

5.1.2.2. P-Specific Advanced Models and Model Selection by Pitch

5.1.2.2.1. Cross-Validated Performance and Best Advanced Models

Pitch-specific predictive performance for advanced models is summarized in Table 5, and the parity plots for LogLog–GPR–ARDSE at each pitch, for both D_h and L_n , are shown in Figure 10. Several important points emerge.

First, Table 5 shows that the advanced models consistently improve accuracy over the pitch-specific OLS power-law baseline when judged by $RMSE_{CV}(\ln\phi_D)$ and $MAPE_{CV}(\phi_D, \%)$.

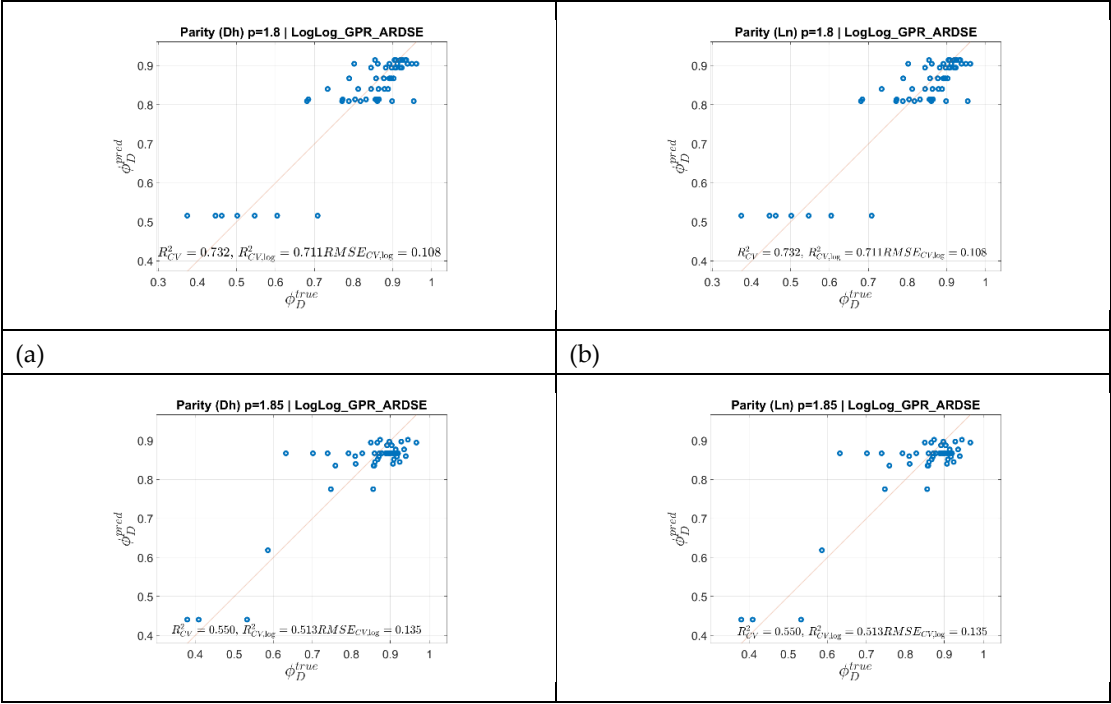
Second, the “best” advanced model is **not strictly identical** for every pitch. Using the consistent selection criterion of minimum $RMSE_{CV}(\ln\phi_D)$ (aligned with log-space training), the winning advanced models (Table 5) are:

- (p=1.80): **GPR–ARDSE**
- (p=1.85): **GPR–ARDSE**
- (p=1.90): **Bagged trees** (slightly better than GPR in $RMSE_{CV}(\ln\phi_D)$)
- (p=1.95): **Bagged trees** (difference vs GPR is minimal)
- (p=2.00): **GPR–ARDSE**

This pitch dependence is consistent with how different nonlinear learners trade bias, variance, and smoothness: bagging tends to stabilize tree-based fits through variance reduction, while GPR provides smooth function estimates with principled uncertainty control through kernel-based function priors (Breiman, 1996).

Although bagged trees are the top performer for two pitches by a narrow margin (Table 5), Figure 10 presents parity plots for **GPR–ARDSE** across all pitches for two practical reasons grounded in the observed statistics:

1. **Global best model consistency:** GPR–ARDSE is the best-performing global surrogate (Table 3), and it remains the best or near-best performer for each pitch (Table 5).
2. **Small performance gaps where GPR is not the absolute winner:** At (p=1.95), the difference between bagged trees and GPR in $RMSE_{CV}(\ln\phi_D)$ is negligible (0.051517 vs 0.051726). Even at (p=1.90), the gap is modest (bagged trees 0.0951 vs GPR 0.0995), so Figure 10 still provides a faithful visualization of near-optimal pitch-specific predictive performance while keeping the advanced-model narrative unified.



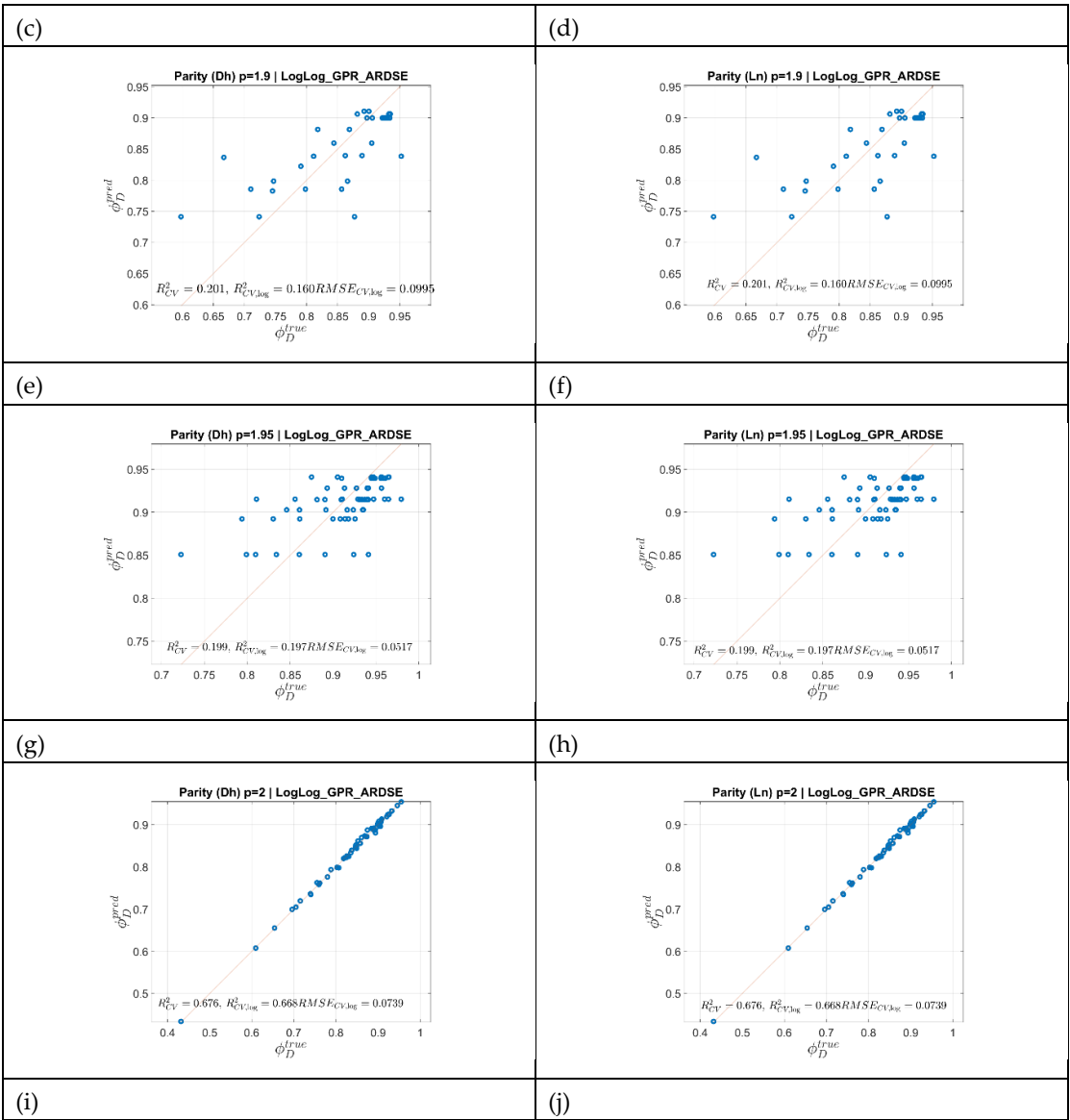


Figure 10. Pitch-specific advanced-model (LogLog-GPR-ARDSE) parity plots for ϕ_D both the D_h -based and L_n -based definition of Re_{shell} : (a)&(b) $p = 1.80$, (c)&(d) $p = 1.85$, (e)&(f) $p = 1.90$, (g)&(h) $p = 1.95$, and (i)&(j) $p = 2.00$. Each panel shows ϕ_D^{pred} vs. ϕ_D^{true} with the 1:1 line.

Table 4.

P	Model	A	a	b	c	p-value, a	p-value, b	P-value, c	$R^2(\ln\phi_i)$	$\bar{R}^2(\ln\phi_i)$	$RMSE(\ln\phi_i)$
D_h											
1.8	LogLog_LM_OLS	0.1210 46	- 20.336 8	20.521 46	0.0096 91	0.0091 58	0.5357 75	0.51825 7	0.13655 1	0.09946 8	10.5401
1.8	LogLog_LM_Robu stBisquare	0.4989 01	- 10.545 2	10.589 37	0.0387 36	0.0381 56	0.3788 65	0.35542 6	0.18644 4	0.12010 3	12.57903
1.85	LogLog_LM_OLS	0.1104 86	- 5.5996 6	5.8079 22	0.5940 79	0.5805 15	0.5320 88	0.51129 2	0.13254 5	0.09906 1	10.21113

1.85	LogLog_LM_Robu stBisquare	0.8836 2	- 1.3253 5	1.3206	0.8226 52	0.8232 95	0.4900 85	0.46742 2	0.20925 6	0.13881 5	13.08419
1.9	LogLog_LM_OLS	0.2343 81	- 19.057 5	19.105 04	0.0059 8	0.0058 97	0.2640 77	0.21659 8	0.09314 5	0.07496 2	7.468416
1.9	LogLog_LM_Robu stBisquare	0.3380 43	- 18.674 3	18.683 33	0.0022 93	0.0022 95	0.2919 08	0.24622 5	0.09900 9	0.07799 5	7.269059
1.95	LogLog_LM_OLS	0.7016 54	- 1.4314 6	1.4516 01	0.6297 67	0.6252 67	0.0800 31	0.04986 8	0.05535 6	0.04847 9	4.31082
1.95	LogLog_LM_Robu stBisquare	0.7628 04	- 1.7244 9	1.7339 43	0.5069 77	0.5050 28	0.0623 31	0.03158 8	0.05695 1	0.04947 6	4.219572
2	LogLog_LM_OLS	0.7535 42	- 0.0398 6	0.0501 34	0.5536 58	0.0135 21	0.1149 56	0.08155 8	0.12063 2	0.08897 8	9.215975
2	LogLog_LM_Robu stBisquare	0.5563 6	0.0917 12	- 0.0385 9	0.0424 64	0.0043 2	0.3255 51	0.3001	0.15068 9	0.11082 1	9.445161
<i>L_n</i>											
1.8	LogLog_LM_OLS	88655. 59	- 20.336 8	20.521 46	0.0096 91	0.0091 58	0.5357 75	0.51825 7	0.13655 1	0.09946 8	10.5401
1.8	LogLog_LM_Robu stBisquare	548.35 31	- 10.545 2	10.589 37	0.0387 36	0.0381 56	0.3788 65	0.35542 6	0.18644 4	0.12010 3	12.57903
1.85	LogLog_LM_OLS	3.9986 69	- 5.5996 6	5.8079 22	0.5940 79	0.5805 15	0.5320 88	0.51129 2	0.13254 5	0.09906 1	10.21113
1.85	LogLog_LM_Robu stBisquare	2.0661 57	- 1.3253 5	1.3206	0.8226 52	0.8232 95	0.4900 85	0.46742 2	0.20925 6	0.13881 5	13.08419
1.9	LogLog_LM_OLS	31200. 31	- 19.057 5	19.105 04	0.0059 8	0.0058 97	0.2640 77	0.21659 8	0.09314 5	0.07496 2	7.468416
1.9	LogLog_LM_Robu stBisquare	35495. 71	- 18.674 3	18.683 33	0.0022 93	0.0022 95	0.2919 08	0.24622 5	0.09900 9	0.07799 5	7.269059
1.95	LogLog_LM_OLS	1.6529 9	- 1.4314 6	1.4516 01	0.6297 67	0.6252 67	0.0800 31	0.04986 8	0.05535 6	0.04847 9	4.31082
1.95	LogLog_LM_Robu stBisquare	2.1416 08	- 1.7244 9	1.7339 43	0.5069 77	0.5050 28	0.0623 31	0.03158 8	0.05695 1	0.04947 6	4.219572
2	LogLog_LM_OLS	0.7711 22	- 0.0398 6	0.0501 34	0.5536 58	0.0135 21	0.1149 56	0.08155 8	0.12063 2	0.08897 8	9.215975
2	LogLog_LM_Robu stBisquare	0.5276 11	0.0917 12	- 0.0385 9	0.0424 64	0.0043 2	0.3255 51	0.3001	0.15068 9	0.11082 1	9.445161

Table 5.

Name	R^2_{CV}	$RMSE_{CV}(\phi)$	$MAE_{CV}(\phi)$	$MAPE_{CV}(\phi_D)$	$R^2_{CV}(\ln\phi_D)$	$RMSE_{CV}(\ln\phi_D)$
Dh						
$P = 1.8$						
LogLog_GPR_ARDSE	0.731963 309	0.0693147 01	0.049777 82	7.04497627 7	0.711317 908	0.10768113 1
LogLog_Ensemble_Bag gedTrees	0.640585 646	0.0802649 5	0.057480 06	8.60556788 9	0.590357 186	0.12827214 5
LogLog_GAM	0.609147 587	0.0837017 68	0.060722 845	8.95867011 4	0.571349 215	0.13121439 9
LogLog_Ensemble_LS Boost	0.609147 584	0.0837017 68	0.060722 837	8.95866978 1	0.571349 189	0.13121440 4
LogLog_LM_OLS	0.353489 929	0.1076504 58	0.085918 128	11.7792277 8	0.428523 539	0.15150566
LogLog_LM_RobustBi square	- 0.271784 012	0.1509853 86	0.097481 175	15.5501860 9	- 0.205620 675	0.22005691 9
$P = 1.85$						
LogLog_GPR_ARDSE	0.549880 765	0.0857890 33	0.059976 467	9.02022279 	0.512890 589	0.13523715 7
LogLog_Ensemble_Bag gedTrees	0.218977 942	0.1130055 15	0.072579 236	11.4310234 7	0.170603 023	0.17646720 1
LogLog_LM_OLS	0.118999 989	0.1200206 42	0.091032 721	13.1380441 1	0.165100 564	0.17705160 1
LogLog_Ensemble_LS Boost	0.033841 992	0.1256874 94	0.082570 22	12.6638414 9	- 0.004109 576	0.19416606 4
LogLog_GAM	- 0.006009 551	0.1282534 45	0.087207 628	13.1838265 6	- 0.024513 885	0.19612894 8
LogLog_LM_RobustBi square	- 0.082778 869	0.1330570 45	0.077416 434	12.8899353 	- 0.084258 578	0.20176656 8
$P = 1.9$						
LogLog_Ensemble_Bag gedTrees	0.266510 584	0.0735770 23	0.050823 847	6.52889849 1	0.232637 979	0.09511354 9
LogLog_Ensemble_LS Boost	0.192836 002	0.0771838 04	0.051507 869	6.55685289 2	0.170843 302	0.09886909 2
LogLog_GAM	0.179825 468	0.0778033 73	0.052063 967	6.62329712 3	0.160112 566	0.09950680 4

LogLog_GPR_ARDSE	0.201407 528	0.0767728 9	0.054359 923	6.85546638	0.159880 664	0.09952054 1
LogLog_LM_RobustBi square	- 0.412173 23	0.1020914 26	0.071894 753	9.35707918 2	- 1.155202 092	0.15939923 7
LogLog_LM_OLS	- 0.493516 911	0.1049905 85	0.075620 521	9.65298597	- 1.439734 914	0.16959520 1
P = 1.95						
LogLog_Ensemble_Bag gedTrees	0.204164 039	0.0448387 29	0.034213 91	3.89348926 3	0.203192 789	0.05151733 6
LogLog_GPR_ARDSE	0.199216 74	0.0449778 83	0.033861 992	3.84645303 7	0.196722 511	0.05172608
LogLog_GAM	0.141909 399	0.0465594 77	0.035360 079	4.01528113 7	0.143080 111	0.05342528 9
LogLog_Ensemble_LS Boost	0.141503 555	0.0465704 86	0.035390 283	4.01851238	0.142719 982	0.05343651 4
LogLog_LM_OLS	- 0.131718 122	0.0534700 58	0.041694 351	4.71972829 3	- 0.106065 122	0.06069698 5
LogLog_LM_RobustBi square	- 0.234024 225	0.0558345 94	0.041350 09	4.75318119 7	- 0.214914 523	0.06361355
P = 2						
LogLog_GPR_ARDSE	0.675606 878	0.0523990 19	0.020741 144	3.06402922 7	0.667850 996	0.07390050 9
LogLog_Ensemble_LS Boost	0.509525 398	0.0644311 07	0.036521 406	5.16525712 6	0.451988 643	0.09492393 4
LogLog_Ensemble_Bag gedTrees	0.206701 08	0.0819418 5	0.053763 724	7.48739638	0.154778 543	0.11788707 9
LogLog_GAM	0.085337 973	0.0879868 28	0.064556 259	8.72631726 8	0.065130 582	0.12398136 9
LogLog_LM_OLS	- 0.187417 279	0.1002510 7	0.076614 325	10.2108970 8	- 0.137749 499	0.13677422 9
LogLog_LM_RobustBi square	- 1.292267 252	0.1392900 07	0.079325 404	11.2709365 8	- 0.872626 982	0.1754715
Ln						
P = 1.8						
LogLog_GPR_ARDSE	0.731963 309	0.0693147 01	0.049777 82	7.04497627 7	0.711317 908	0.10768113 1

LogLog_Ensemble_Bag gedTrees	0.640585 646	0.0802649 5	0.057480 06	8.60556788 9	0.590357 186	0.12827214 5
LogLog_GAM	0.609147 587	0.0837017 68	0.060722 845	8.95867011 4	0.571349 215	0.13121439 9
LogLog_Ensemble_LS Boost	0.609147 584	0.0837017 68	0.060722 837	8.95866978 1	0.571349 189	0.13121440 4
LogLog_LM_OLS	0.353489 929	0.1076504 58	0.085918 128	11.7792277 8	0.428523 539	0.15150566
LogLog_LM_RobustBi square	- 0.271784 013	0.1509853 86	0.097481 175	15.5501861	- 0.205620 675	0.22005691 9
P = 1.85						
LogLog_GPR_ARDSE	0.549880 765	0.0857890 33	0.059976 467	9.02022279	0.512890 589	0.13523715 7
LogLog_Ensemble_Bag gedTrees	0.218977 942	0.1130055 15	0.072579 236	11.4310234 7	0.170603 023	0.17646720 1
LogLog_LM_OLS	0.118999 989	0.1200206 42	0.091032 721	13.1380441 1	0.165100 564	0.17705160 1
LogLog_Ensemble_LS Boost	0.033841 992	0.1256874 94	0.082570 22	12.6638414 9	- 0.004109 576	0.19416606 4
LogLog_GAM	- 0.006009 551	0.1282534 45	0.087207 628	13.1838265 6	- 0.024513 885	0.19612894 8
LogLog_LM_RobustBi square	- 0.082778 87	0.1330570 45	0.077416 434	12.8899353	- 0.084258 579	0.20176656 8
P = 1.9						
LogLog_Ensemble_Bag gedTrees	0.266510 584	0.0735770 23	0.050823 847	6.52889849 1	0.232637 979	0.09511354 9
LogLog_Ensemble_LS Boost	0.192836 002	0.0771838 04	0.051507 869	6.55685289 2	0.170843 302	0.09886909 2
LogLog_GAM	0.179825 468	0.0778033 73	0.052063 967	6.62329712 3	0.160112 566	0.09950680 4
LogLog_GPR_ARDSE	0.201407 528	0.0767728 9	0.054359 923	6.85546638	0.159880 664	0.09952054 1
LogLog_LM_RobustBi square	- 0.412173 163	0.1020914 24	0.071894 752	9.35707902 2	- 1.155202 028	0.15939923 4
LogLog_LM_OLS	- 0.493516 911	0.1049905 85	0.075620 521	9.65298597	- 1.439734 914	0.16959520 1

$P = 1.95$						
LogLog_Ensemble_BaggedTrees	0.204164039	0.044838729	0.03421391	3.893489263	0.203192789	0.051517336
LogLog_GPR_ARDSE	0.19921674	0.044977883	0.033861992	3.846453037	0.196722511	0.05172608
LogLog_GAM	0.141909399	0.046559477	0.035360079	4.015281137	0.143080111	0.053425289
LogLog_Ensemble_LSBoost	0.141503555	0.046570486	0.035390283	4.01851238	0.142719982	0.053436514
LogLog_LM_OLS	-0.131718122	0.053470058	0.041694351	4.719728293	-0.106065122	0.060696985
LogLog_LM_RobustBisquare	-0.234024225	0.055834594	0.04135009	4.753181197	-0.214914523	0.06361355
$P = 2$						
LogLog_GPR_ARDSE	0.675606878	0.052399019	0.020741144	3.064029227	0.667850996	0.073900509
LogLog_Ensemble_LSBoost	0.509525398	0.064431107	0.036521406	5.165257126	0.451988643	0.094923934
LogLog_Ensemble_BaggedTrees	0.20670108	0.08194185	0.053763724	7.48739638	0.154778543	0.117887079
LogLog_GAM	0.085337973	0.087986828	0.064556259	8.726317268	0.065130582	0.123981369
LogLog_LM_OLS	-0.187417279	0.10025107	0.076614325	10.21089708	-0.137749499	0.136774229
LogLog_LM_RobustBisquare	-1.292267252	0.139290007	0.079325404	11.27093658	-0.872626982	0.1754715

5.1.3. Hierarchical Varying-Coefficient Power-Law Correlation

To bridge the gap between a single global power-law correlation and fully independent pitch-specific regressions, we also evaluated a hierarchical (varying-coefficient) power-law form in which the prefactor and Reynolds exponents vary smoothly with pitch. The hierarchical (varying-coefficient) power-law correlation was formulated to retain a closed-form “engineering” model while allowing systematic pitch dependence, as follows:

$\phi_D = A(p) Re_{shell}^{a(p)} Re_{coil}^{b(p)},$	
$\ln(\phi_D) = \ln A(p) + a(p) \ln(Re_{shell}) + b(p) \ln(Re_{coil}).$	

To enforce smoothness across pitches without fitting each pitch in isolation, the pitch-dependent functions were represented by low-order polynomials using a centered pitch variable $P^* = P - 1.90$:

$\ln A(p) = c_0 + c_1 P^* + c_2 P^{*2},$	
$a(p) = a_0 + a_1 P^* + a_2 P^{*2},$	

$b(p) = b_0 + b_1P^* + b_2P^{*2}.$	
------------------------------------	--

The resulting parity trends and the recovered smooth exponent functions are shown for the D_h -based definition in Figure 11.a and for the L_n -based definition in Figure 11.b, while the corresponding cross-validated performance metrics and fitted coefficient-function parameters for both characteristic-length choices are summarized together in Table 6. Overall, this structured model improves interpretability relative to black-box regressors by retaining a closed-form “engineering correlation” while allowing systematic pitch dependence, and it yields consistent behavior between the D_h and L_n formulations (Figures 11), indicating that the identified pitch trends are robust to the characteristic-length definition; however, its predictive skill remains intermediate—better than enforcing a single exponent set globally, yet not matching the best advanced surrogate—supporting its use as a compact, physically interpretable compromise when one seeks a single correlation that still reflects pitch-driven variability (Table 6).

In Table 6, In **Table 6**, λ is the **ridge-regularization strength** used when fitting the hierarchical (varying-coefficient) model in log space. Concretely, it is for solving a **penalized least-squares** problem for the expanded linear model:

$\min_{\theta} \ y - X\theta\ ^2 + \lambda\ \theta\ ^2$	(20)
---	------

where

- $y = \ln(\phi_D)$ (vector of responses),
- $X = [B(p), \ln(Re_{shell}), B(p), \ln(Re_{coil}), B(p)]$ which is the design matrix,
- θ contains the polynomial coefficients for $\ln A(p)$, $a(p)$, and $b(p)$,

such that:

- $\lambda = 0 \rightarrow$ ordinary least squares (no regularization), can overfit/yield unstable coefficients when predictors are correlated.
- **Larger $\lambda \rightarrow$** stronger shrinkage of coefficients toward zero (smoother/more stable, but can underfit).
- Your script chooses λ by **K-fold cross-validation**, picking the value that minimizes $RMSE_{CV,log}$.

So, in Table 6, the reported λ is the **CV-selected penalty** that gave the best generalization for the hierarchical model (separately for Dh and Ln runs).

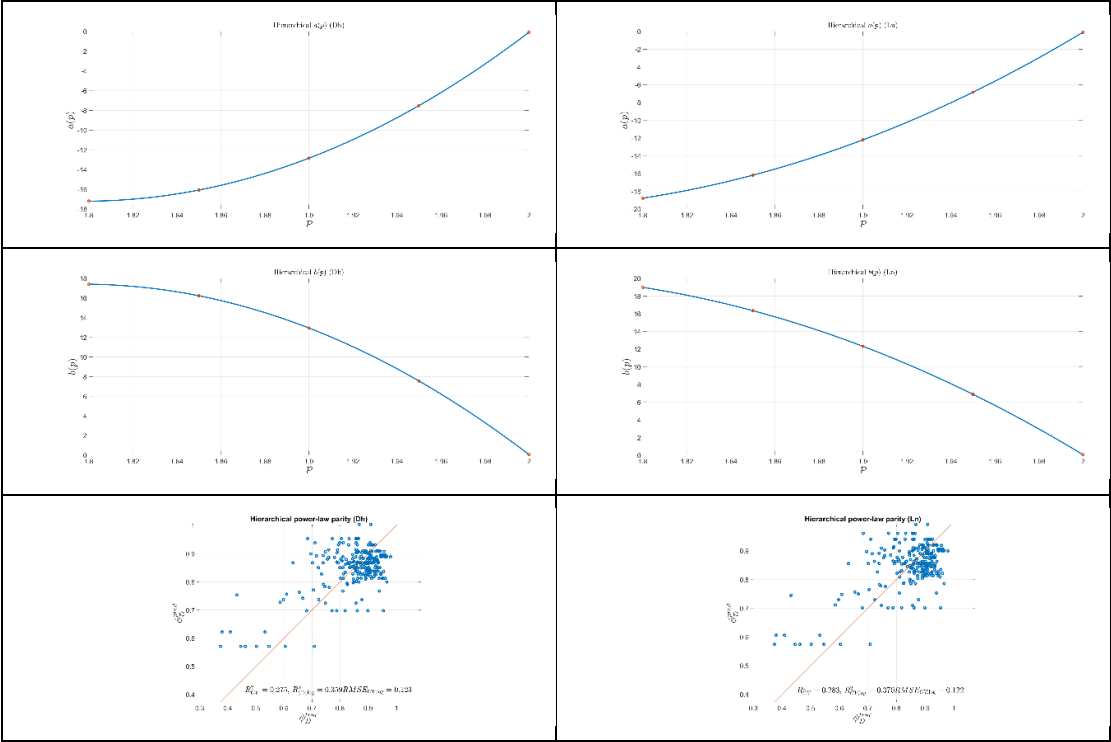


Figure 11. Hierarchical (varying-coefficient) power-law model for ϕ_D with pitch-dependent prefactor and exponents, $\phi_D = A(p) Re_{\text{shell}}^{a(p)} Re_{\text{coil}}^{b(p)}$. The top and middle panels show the fitted smooth trends of $a(p)$ and $b(p)$ versus pitch, and the bottom panels show parity between predicted and measured ϕ_D . Left: Re_{shell} based on D_h ; right: Re_{shell} based on L_n .

Table 6. Hierarchical (varying-coefficient) power-law correlation for ϕ_D , $\phi_D = A(p) Re_{\text{shell}}^{a(p)} Re_{\text{coil}}^{b(p)}$: fitted pitch-dependent coefficient functions and cross-validated predictive performance for both $Re_{\text{shell}}(D_h)$ and $Re_{\text{shell}}(L_n)$ formulations.

L_c	Degree	λ	R_{CV}^2	$R_{CV}^2(\ln \phi_D)$	$RMSE_{CV}(\ln \phi_D)$	$RMSE_{CV}(\phi_D)$	$MAPE_{CV}(\phi_D)$	$\ln A(p)$	$a(p)$	$b(p)$
	e							al	al	al
D_h	2	1e-08	0.2752	0.3594	0.1231	0.09145	9.181	$-1.514 + 11.35 \cdot p^* + 49.6 \cdot p^{*2}$	$-12.85 + 85.62 \cdot p^* + 420.8 \cdot p^{*2}$	$12.94 + -86.77 \cdot p^* + -421.8 \cdot p^{*2}$
L_n	2	1e-08	0.2825	0.3703	0.1221	0.09099	9.151	$6.039 + -50.08 \cdot p^* - 75.7 \cdot p^{*2}$	$-12.21 + 93.44 \cdot p^* + 277.7 \cdot p^{*2}$	$12.31 + -94.66 \cdot p^* + 279.7 \cdot p^{*2}$

6. Discussion

The numerical dataset used for the exergy correlation development is supported by both discretization and first-law consistency checks. The grid-refinement study shows only small changes in the key second-law metrics, with the exergy-destruction fraction ϕ_D increasing from 0.9426 (original) to 0.9464 (BL-adapted) and remaining 0.9464 on the finest grid ($\leq 0.40\%$ change), while $\dot{E}_D$ changes from 19400.8 to 19479.0 ($\leq 0.22\%$ across refinement), indicating practical grid independence over the refinement range considered.

In addition, the energy-closure analysis shows a distribution concentrated near zero mismatch and no clear Reynolds-number trend in the closure scatter, supporting the use of the filtered dataset for exergy accounting and correlation development.

For global (non-pitch-specific) correlations, the baseline power-law regression (Table 2) yields nearly identical Reynolds-number exponents for both characteristic-length definitions ($a \approx 0.0488$, $b \approx 0.0498$), while the pitch exponent is substantially larger ($c \approx 0.751$ for D_h and $c \approx 0.790$ for L_n) and statistically significant in both cases, demonstrating that pitch variation is the dominant fitted driver within the pooled multiplicative form; however, the corresponding parity plot (Figure 7) and cross-validation statistics (Table 3) show limited predictive fidelity for this baseline (for D_h : $R_{CV, \log}^2 = 0.169$, $RMSE_{CV, \log} = 0.140$, $MAPE_{CV} = 10.23\%$; nearly unchanged for L_n). In contrast, the advanced-model comparison (Table 3) shows that Gaussian process regression with an ARD squared-exponential kernel (LogLog_GPR_ARDSE) provides the best global generalization for both D_h and L_n , achieving $R_{CV, \log}^2 \approx 0.717$, $RMSE_{CV, \log} \approx 0.0818$, and $MAPE_{CV} \approx 5.17\%$, which corresponds to a $\sim 42\%$ reduction in $RMSE_{CV, \log}$ and a $\sim 49\%$ reduction in MAPE relative to the global OLS power-law baseline; these improvements are consistent with the tighter parity behavior shown in Figure 8 for both characteristic-length definitions. For pitch-specific regressions, the power-law OLS parity plots (Figure 9) and coefficients (Table 4) provide pitch-conditioned baseline correlations; the associated advanced-model evaluation (Table 5) shows that the best pitch-specific surrogate depends on pitch, with LogLog_GPR_ARDSE performing best at $p = 1.80$, 1.85 , and 2.00 , while LogLog_Ensemble_BaggedTrees is best at $p = 1.90$ and 1.95 (Figure 10), and the best advanced model reduces $RMSE_{CV, \log}$ by approximately 15–46% relative to the pitch-specific OLS baseline across the studied pitches (Table 5). Finally, the hierarchical (varying-coefficient) power-law model (Figures 11–

12) provides a compact intermediate representation in which $A(p)$, $a(p)$, and $b(p)$ vary smoothly with pitch while retaining a closed-form correlation; its cross-validated performance (Table 6) is consistent between D_h and L_n (e.g., $RMSE_{CV,log} \approx 0.123$ for D_h and ≈ 0.122 for L_n), improving upon the single global OLS power-law while remaining less accurate than the best global advanced surrogate (Table 3), thereby positioning it as a structured alternative when a single, interpretable correlation with explicit pitch dependence is desired alongside quantified predictive accuracy.

7. Concluding Remarks

This study presented a comprehensive exergy assessment of a helically coiled tube heat exchanger across a matrix of operating conditions and coil geometries, with particular emphasis on predicting the dimensionless exergy-destruction fraction ϕ_D . The exergy accounting and numerical dataset were supported by grid-refinement and energy-closure checks, providing a consistent basis for correlation development. A global power-law correlation of the form $\phi_D = A Re_{shell}^a Re_{coil}^b p^c$ was established as an interpretable baseline, and its performance was benchmarked against advanced surrogate models. Across both characteristic-length definitions of the shell-side Reynolds number (D_h and L_n), Gaussian process regression with an ARD squared-exponential kernel provided the best global predictive accuracy, as reflected by the parity plots and cross-validated error metrics. Pitch-specific analyses further showed that allowing correlations to be conditioned on pitch improves fidelity relative to a single global exponent set, with the best-performing advanced model varying with pitch level. Finally, a hierarchical (varying-coefficient) power-law model was introduced as a compact middle-ground that retains a closed-form engineering correlation while accommodating systematic pitch dependence through smoothly varying $A(p)$, $a(p)$, and $b(p)$; this model improved upon a rigid global power law while remaining less accurate than the best advanced surrogate. Collectively, the proposed correlations and benchmarking results provide a practical pathway for rapid second-law performance estimation of helically coiled tube heat exchangers, enabling geometry and operating-condition screening without repeating full exergy analyses.

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