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Article

Response of a Structure Isolated by a Couple System Consisting of a QZS and FPS Under Horizontal Ground Excitation

Richie Kevin Wouako Wouako ^{1,*}, Sandra Céleste Tchato ², Euloge Felix Kayo Pokam ³,
Blaise Pascal Gounou Pokam ⁴, André Michel Pouth Nkoma ^{5,*}, Eliezer Manguelle Dicoum ¹
and Philippe Njandjock Nouck ¹

¹ Department of Physics, University of Yaoundé I, P.O. Box 812, Yaoundé, Cameroon

² National Institute of Cartography, P.O. Box 157, Yaounde, Cameroon

³ Department of Mathematical Economics and Econometrics, Omar Bongo University, Libreville P.O. Box 13113, Gabon

⁴ Institut Universitaire de Technologie de L'université de Ngaoundéré, Département de Génie Mécanique, Cameroon

⁵ National Institute of Cartography, P.O. Box 157, Yaounde, Cameroon

* Correspondence: wouakowouakor@gmail.com (R.K.W.W.); pestalazypouth12@gmail.com (A.M.P.N.)

Abstract: The study of vibration isolation devices has become an emerging area of research in view of the extensive damage to buildings caused by earthquakes. The ability to effectively isolate seismic vibrations and maintain the stability of a building is thus addressed in this paper, which evaluates the effect of horizontal ground excitation on the response of a structure isolated by a coupled isolation system consisting of a non-linear damper (QZS) and a friction pendulum system (FPS). A single-degree-of-freedom system was used to model structures whose bases are subjected to seismic excitation in order to assess the effectiveness of the QZS-FPS coupling in reducing the structural response. The results obtained revealed significant improvements in structural performance when the QZS-FPS uses a damper of optimum stiffness. A 30% reduction in displacement was recorded compared with the QZS for two signals, one harmonic and the other stochastic. In addition, the study demonstrated that the QZS-FPS combination can offer better control of building vibration in terms of horizontal displacements.

Keywords: building-structure; mechanical-system; non-linear isolation; amplitude; damper; seismic-control

1. Introduction

In building structures and mechanical systems, non-linear vibration isolation is considered as effective method of protecting a structure and is becoming increasingly popular [1-4]. The non-linear vibration isolation device has the potential to significantly reduce the fundamental frequency of the protected structure to minimise the transmission of force or displacement even for low frequency excitation cases, where traditional vibration isolations are generally not very effective [5].

Passive control devices are systems that do not require an external power source. The forces developed in these devices due to building movements are used by the devices themselves. These devices include basic isolation systems and tuned mass dampers (TMD), quazi zero stiffness QZS and friction pendulum (FPS) [6, 7]

Previous research has demonstrated the importance of isolation bearings as they play a key role in facilitating the transmission of forces from the superstructure to the substructure [3, 8, 9].

The QZS is a system generally used to damp vertical forces and its effectiveness has been demonstrated on several occasions by researchers [9-14].

Research on QZS dampers has been conducted to reduce earthquake-induced building movements. Several studies have evaluated the responses of QZS systems under both harmonic and random excitations [15, 16].

The Friction Pendulum System (FPS) is frequently used in civil engineering structures for isolation purposes. Due to its geometrical design, the FPS provides an effective friction isolation system by combining sliding motion with restoring force [17-21]. Some studies have evaluated the seismic reliability of an SPF-isolated base structure by treating isolator characteristics and principal earthquake characteristics as independent random variables [22].

This paper uses a QZS-FPS configuration to damp horizontal forces in a structure. This paper evaluates the effect of horizontal ground excitation on the response of a structure isolated by an isolation system coupling consisting of a non-linear damper (QZS) and a friction pendulum (FPS).

2. Equation of Motion

2.1. Equation of Motion for the Case of Quasi-Zero Stiffness

Figure 1 shows a QZS-isolated structure subjected to a horizontal external excitation ($\vec{u}_g$) and whose response $\vec{u}_b$ is determined from Equation (1).

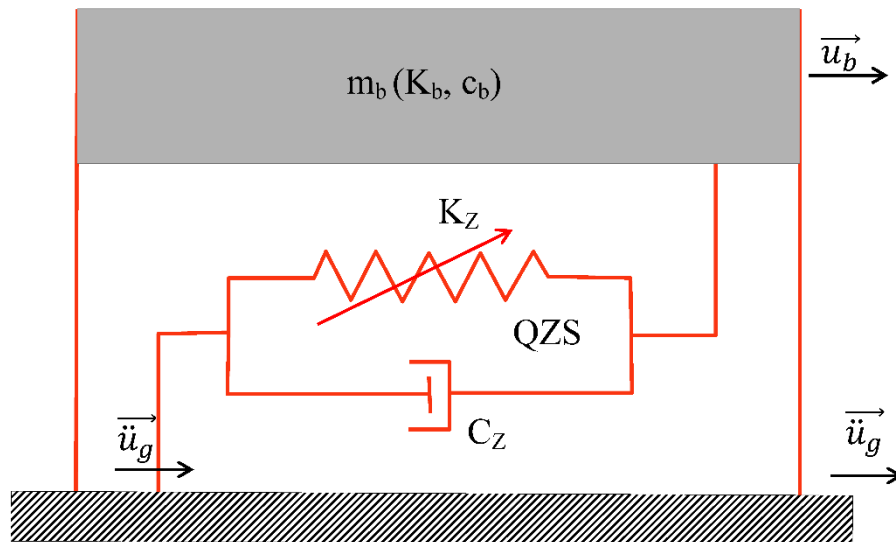


Figure 1. Structure isolated by QZS under horizontal excitation.

$$\vec{P}_{base} + \vec{R}_{sol} + \vec{F}_{armo} + \vec{T}_{elon} + \vec{F}_z = mb\vec{a}_b \quad (1)$$

With:

$\vec{P}_{base}$: the weight of the base

$\vec{R}_{sol}$: the reaction of the ground due to the load of the building

$\vec{F}_{armo}$: the inherent damping force of the base conferred on it by the various materials making it

up

$\vec{F}_z$: the force of the non-linear elastic damper

$\vec{T}_{elon}$: the elongation force of the structure imparted by the steels

$\vec{a}_b$: accelerations felt at the base (foundation)

mb : represents the mass of the base above the insulation system

γ : represents the damping ratio of the non-linear damper to that of the structure

k : represents the ratio of the stiffness of the damper structure to that of the structure

$$c_b \dot{u}_b + k_z u_b^3 + c_z \dot{u}_b + k_b u_b = -mb \cdot (\ddot{u}_b + \ddot{u}_g) \quad (2)$$

$$\ddot{u}_b + (2\varepsilon_b \omega_b \dot{u}_b + 2\varepsilon_b \omega_b \gamma \dot{u}_b) + k \omega_b^2 u_b^3 + \omega_b^2 u_b = -\ddot{u}_g \quad (3)$$

With:

$$\varepsilon_b = \frac{c_b}{2mb\omega_b}; \omega_b = \sqrt{\frac{k_b}{mb}}; \gamma = \frac{c_z}{c_b}; k = \frac{k_z}{k_b}$$

The dimensionless form of the equation of motion is obtained from the time scale $\tau = \omega_b t$ and taking the excitation signal $\ddot{u}_g = a_0 X_0(\tau)$ where a_0 is the seismic intensity scale which has the same dimension as the signal acceleration and $X_0(\tau)$ the time-dimensional function describing the time-dimensional function of the excitation signal is:

$$\ddot{\varphi}_b + 2(1 + \gamma)\varepsilon_b \dot{\varphi}_b + \varphi_b + k \frac{a_0^2}{\omega_b^4} \varphi_b^3 = X_0(\tau) \quad (4)$$

With: $\varphi_b = \frac{\omega_b^2}{a_0} u_b$

For the scaled function $X_0(\tau)$ harmonic, i.e. $X_0(\tau) = X_0 \sin(\beta\tau + \emptyset)$ and using the harmonic balance [23], the response of the structure will be expressed as; $\varphi_b = X_b \sin(\beta\tau)$, with β the scaled pulsation of the signal.

$$-X_b \beta^2 \sin(\beta\tau) + (2\varepsilon_b(1 + \gamma)X_b \beta \cos(\beta\tau)) + \left(k \frac{a_0^2}{\omega_b^4} (X_b \sin(\beta\tau))^3 + X_b \sin(\beta\tau)\right) = X_g \sin(\beta\tau + \emptyset) \quad (5)$$

However,

$$\sin(\beta\tau)^3 = \frac{3}{4} \sin(\beta\tau) - \frac{1}{4} \cos(3(\beta\tau))$$

$$\sin(\beta\tau + \emptyset) = \sin(\beta\tau) \cos(\emptyset) - \cos(\beta\tau) \sin(\emptyset)$$

Neglecting the terms in $\sin(3(\beta\tau))$ Equation (5) then becomes:

$$-X_b \beta^2 \sin(\beta\tau) + (2\varepsilon_b(1 + \gamma)X_b \beta \cos(\beta\tau)) + \left(k \frac{a_0^2}{\omega_b^4} X_b^3 \sin(\beta\tau) + X_b \sin(\beta\tau)\right) = X_g (\sin(\beta\tau) \cos(\emptyset) - \cos(\beta\tau) \sin(\emptyset)) \quad (6)$$

From Equation (6) we can deduce;

$$-X_b \beta^2 + \frac{3}{4} k \frac{a_0^2}{\omega_b^4} X_b^3 + X_b = X_g \cos(\emptyset) \quad (7)$$

$$2\varepsilon_b(1 + \gamma)X_b \beta = X_g \sin(\emptyset) \quad (8)$$

Squaring Equations (7) and (8) and adding them together gives:

$$\frac{9}{16} k^2 \frac{a_0^4}{\omega_b^8} X_b^6 + \frac{3}{2} k(1 - \beta^2) \frac{a_0^2}{\omega_b^4} X_b^4 + ((1 - \beta^2)^2 + 4\varepsilon_b^2 \beta^2 (1 + \gamma)^2) X_b^2 - X_g^2 = 0 \quad (9)$$

After posing $\varepsilon = \varepsilon_b(1 + \gamma)$, Equation (9) becomes:

$$\frac{9}{16} k^2 \frac{a_0^4}{\omega_b^8} X_b^6 + \frac{3}{2} k(1 - \beta^2) \frac{a_0^2}{\omega_b^4} X_b^4 + ((1 - \beta^2)^2 + 4\beta^2 \varepsilon^2) X_b^2 - X_g^2 = 0 \quad (10)$$

2.2. Equation of Motion for the Case of Quasi-Zero Stiffness Coupled to the FPS

Figure 2 shows the structure isolated by the coupling of the QZS and the FPS. The equation of motion governing the behaviour of this structure under external excitation is determined from Equation (11)

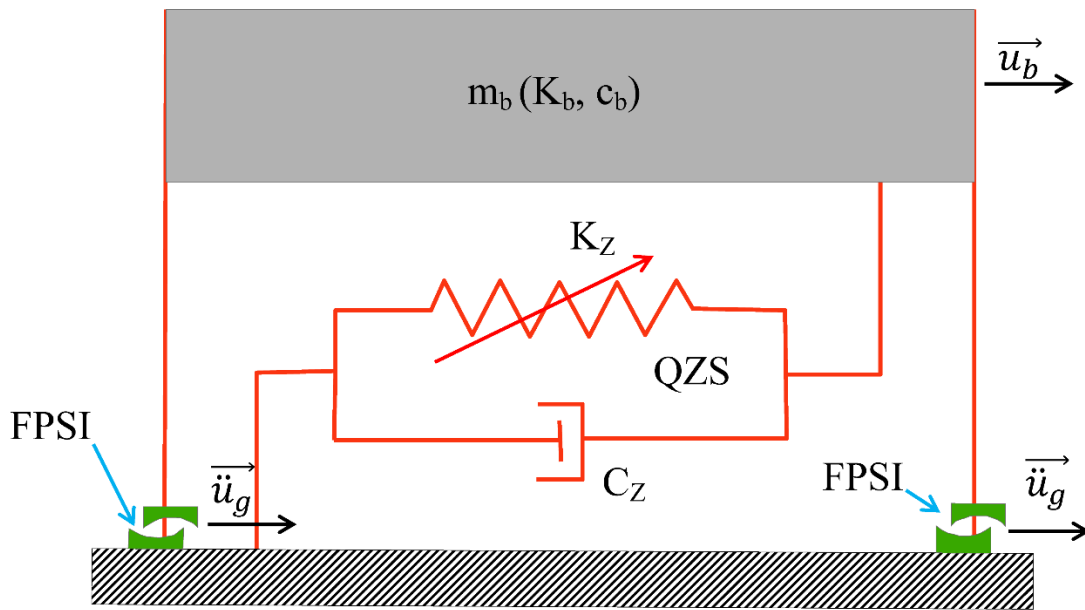


Figure 2. Structure isolated by QZS-FPS coupling under horizontal excitation.

$$\vec{P}_{base} + \vec{R}_{sol} + \vec{F}_{armo} + \vec{T}_{elon} + \vec{F}_z + \vec{F}_b = mb\vec{a}_b \quad (11)$$

With:

$\vec{P}_{base}$: the weight of the base

$\vec{R}_{sol}$: the reaction of the ground due to the load of the building

$\vec{F}_{armo}$: the inherent damping force of the base conferred on it by the various materials making it

up

$\vec{F}_z$: the force of the non-linear elastic damper

$\vec{F}_b$: the force of the double FPS

$\vec{T}_{elon}$: the elongation force of the structure conferred on it by the steels

$\vec{a}_b$: accelerations felt at the base (foundation)

mb: represents the mass of the base above the insulation system

γ : represents the ratio of the damping coefficient of the non-linear damper to that of the structure

k : represents the ratio of the stiffness of the damper to that of the structure

$\mu(\dot{u}_b)$: is the sliding friction coefficient

k_b : stiffness constant of the structure

k_z : stiffness constant of the non-linear damper

f_{max} : maximum value of the coefficient of friction

$$\ddot{u}_b + \frac{1}{mb}(c_b\dot{u}_b + k_z u_b^3 + c_z\dot{u}_b) + \frac{1}{mb} \cdot k_b u_b + \mu(\dot{u}_b)gZ(t) = -\ddot{u}_g \quad (12)$$

From Equation (12) we obtain:

$$\ddot{u}_b + (2\varepsilon_b\omega_b\dot{u}_b + 2\varepsilon_b\omega_b\gamma\dot{u}_b + k\omega_b^2 u_b^3) + \omega_b^2 u_b + \mu(\dot{u}_b)gZ(t) = -\ddot{u}_g \quad (13)$$

With:

$$\varepsilon_b = \frac{c_b}{2mb\omega_b}; \omega_b = \sqrt{\frac{k_b}{mb}}; \gamma = \frac{c_z}{c_b}; k = \frac{k_z}{k_b}; Z(t) = \text{sign}(\dot{u}_b), \text{ and } \mu(\dot{u}_b) = f_{max}$$

The principle used in Equation 3 allows us to obtain Equation (14) from Equation (13).

For $X_0(\tau)$ harmonic

$$\ddot{\phi}_b + 2(1 + \gamma)\varepsilon_b\dot{\phi}_b + \phi_b + k\frac{a_0^2}{\omega_b^4}\phi_b^3 + gf_{max}\text{sign}(\dot{\phi}_b) = X_0(\tau) \quad (14)$$

Assuming harmonic seismic excitation, i.e. $X_0(\tau) = X_g\sin(\beta\tau + \phi)$, the response of the structure will be expressed as: $\phi_b = X_b\sin(\beta\tau)$ [23]

$$-X_b\beta^2\sin(\beta\tau) + (2\varepsilon_b\omega_bX_b\beta\cos(\beta\tau) + 2\gamma\omega_bX_b\beta\cos(\beta\tau)) + (k\omega_b^2(X_b\sin(\beta\tau))^3 + \omega_b^2X_b\sin(\beta\tau)) + f_{max}\text{sign}(X_b\beta\cos(\beta\tau)) = X_g\sin(\beta\tau + \phi) \quad (15)$$

However,

$$\operatorname{sgn}(X_b \beta \cos(\beta \tau)) = \frac{4}{\pi} \beta \cos(\beta \tau)$$

Neglecting the terms in $\sin(3(\beta \tau))$ Equation (15) becomes:

$$-X_b \beta^2 \sin(\beta \tau) + (2\varepsilon_b \omega_b X_b \beta \cos(\beta \tau) + 2\varepsilon_b \omega_b \gamma X_b \beta \cos(\beta \tau)) + \left(\frac{3}{4} k \omega_b^2 X_b^3 \sin(\beta \tau) + \omega_b^2 X_b \sin(\beta \tau) \right) + \frac{4}{\pi} f_{max} \beta \cos(\beta \tau) = X_g (\sin(\beta \tau) \cos(\emptyset) + \cos(\beta \tau) \sin(\emptyset)) \quad (16)$$

From Equation (16) we get the following Equation (17);

$$(1 - \beta^2) X_b + \frac{3}{4} k \frac{a_0^2}{\omega_b^4} X_b^3 = X_g \cos(\emptyset) \quad (17)$$

$$2\varepsilon_b (1 + \gamma) X_b \beta + \frac{4}{\pi} f_{max} \beta = X_g \sin(\emptyset) \quad (18)$$

Squaring Equations (17) and (18) and adding them together gives:

$$\frac{9}{16} k^2 \frac{a_0^4}{\omega_b^8} X_b^6 + \frac{3}{2} k (1 - \beta^2) \frac{a_0^2}{\omega_b^4} X_b^4 + ((1 - \beta^2)^2 + 4\varepsilon^2 \beta^2) X_b^2 + \frac{16}{\pi} \varepsilon X_b \beta^2 f_{max} + \frac{16}{\pi^2} f_{max}^2 \beta^2 = X_g^2 \quad (19)$$

Solving this equation gives the frequency response curve for stiff, soft and linear springs for different values of ε , with $\varepsilon = \varepsilon_b (1 + \gamma)$.

2.3. Case of Stochastic Excitation

In this case, the seismic loading ($\ddot{u}_g$) is represented as random sequences of white Gaussian noise, adjusted by filtering and time modulation of varying intensity, in the context of spectral density analysis. The filter characteristics determine the frequency distribution of these random stresses and are adjusted to correspond to rigid, intermediate or loose soil conditions, as appropriate. For this purpose, the Kanai-Tajimi filter modified by Clough and Penzien (Equation (20)) is used to model the different soil types.

$$S_{\ddot{u}_g}(\beta) = \frac{\beta^4 + 4\varepsilon_g^2 \omega_g^2 \beta^2}{(\omega_g^2 - \beta^2)^2 + 4\varepsilon_g^2 \omega_g^2 \beta^2} \frac{\beta^4}{(\omega_f^2 - \beta^2)^2 + 4\varepsilon_f^2 \omega_f^2 \beta^2} S_w \quad (20)$$

$S_w = \frac{0.141 \varepsilon_g}{\omega_g \sqrt{1 + 4\varepsilon_g^2}} \ddot{u}_{g0}^2$ represents the spectral level of white noise linked to the maximum acceleration of the ground.

Where $\ddot{u}_{g0}$ is the peak ground acceleration (PGA); ε_g , ω_g , ε_f and ω_f are the filter parameters. [24, 25].

The maximum value of the recorded ground acceleration oscillates between 0.4 g and 0.6 g [26-28].

3. Results and Discussion

3.1. Amplitude-Frequency Response in the Case of a Harmonic Signal

Figure 3 shows the amplitude response of the non-isolated system isolated by the QZS according to its various characteristics as a function of the scaled frequency for a total system damping of 5%. In the case of a system isolated by a QZS with a softening stiffness ($k=-0.0249$ and $k=-0.0030$) the amplitude curves slope as the frequency increases, in other words the resonance frequency is inversely proportional to the amplitude of the oscillations. In contrast, for the system isolated by the rigid QZS ($k=0.1000$ and $k=0.6000$) the amplitude curves slope to the left, from which we deduce that the responses are weak at high resonance frequencies.

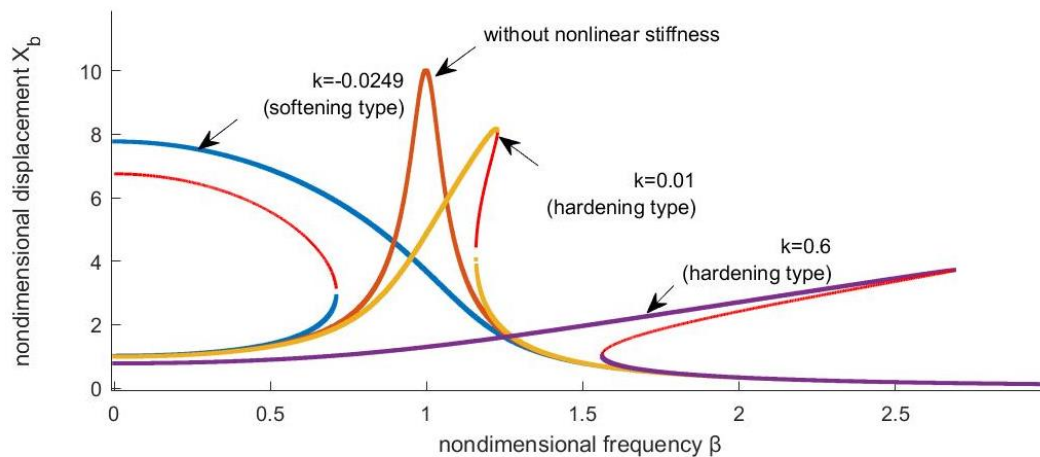


Figure 3. Amplitude-frequency response, case of QZS.

The amplitude-frequency responses of the system isolated by the FPS and by the QZS-FPS coupling are shown in Figure 4 for $\varepsilon=0.05$. The slope of the amplitude-frequency response curves in Figure 4 shows the persistence of the QZS non-linearity despite the QZS-FPS coupling. In addition, the maximum amplitudes of the responses of the structure for the case of soft-stiff QZS coupling with the FPS are greater than those of the structure isolated only with the FPS. This is quite the opposite of those where the structure is isolated by the rigid QZS coupling and the FPS, which are the lowest as the stiffness increases.

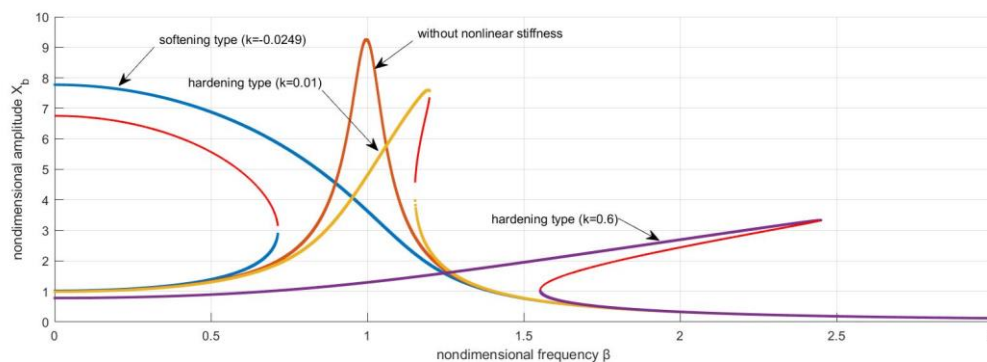


Figure 4. Amplitude-frequency response, case of QZS - friction pendulum coupling.

3.2. Amplitude Response for the Case of a Stochastic Signal

The various seismic signals were derived from Equation (17).

Case of soft ground

Figure 5 shows the time evolution of the seismic signal X_g , modelled according to the Kanai-Tajimi spectrum modified by Clough and Penzien. The signal shows amplitude variations between -4 and 3 over a period of 40 seconds. The oscillations observed are characterised by acceleration and deceleration phases, which are essential for analysing the dynamic response of structures.

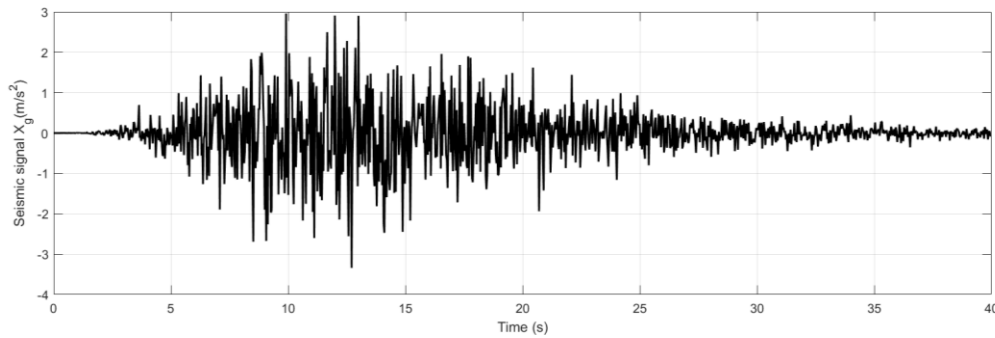


Figure 5. Seismic signal for soft ground.

The response of the structure to this signal is shown in Figure 6 for the different cases of isolation (QZS and QZS-FPS coupling). For an uninsulated structure, the amplitudes vary between -0.028 m and 0.026 m, while in the case of low stiffness insulation the amplitudes of the oscillations vary between -0.021 m and 0.019 m. The negative and positive displacements show that the building oscillates around its equilibrium position. Furthermore, the amplitudes of the oscillations in the case of rigid stiffness insulation vary between -0.021m and 0.019m. In other words, the amplitudes of the oscillations in the case of insulation with a soft stiffness are smaller than those of insulation with a rigid stiffness.

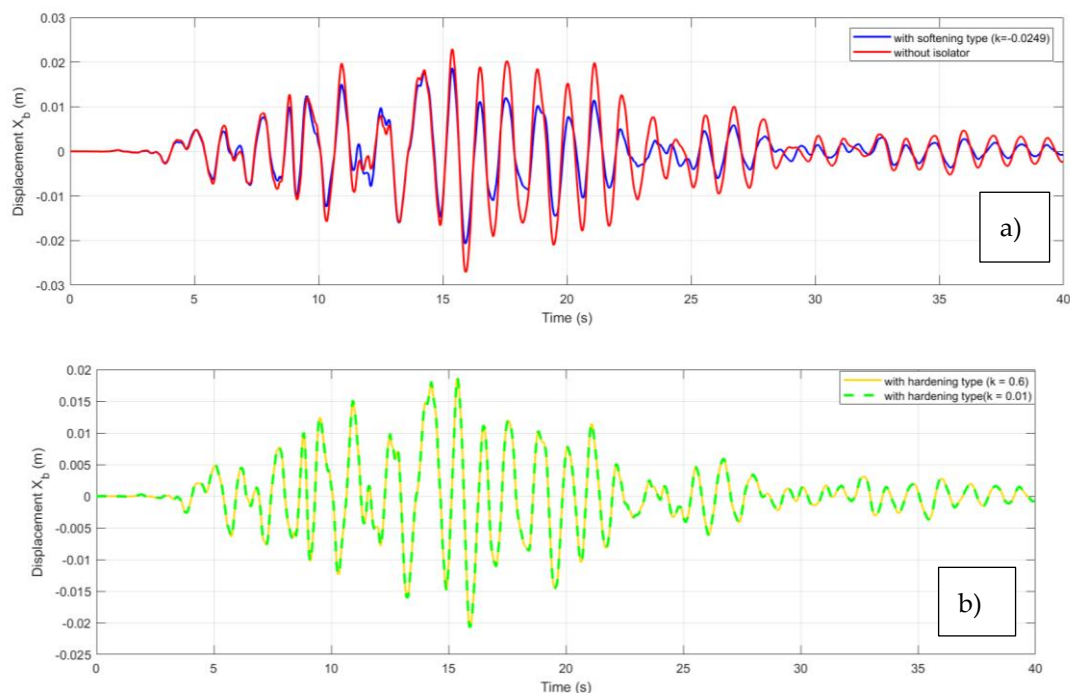


Figure 6. Response of the structure on soft soil: (a) case of a non-isolated structure and isolated by the soft stiffness QZS; (b) case of a structure isolated by the rigid stiffness QZS ($k=0.01$ and $k=0.6$).

The response of the structure to the seismic signal in a soft ground environment is shown in Figure 7. The amplitude of the oscillations for the case of a QZS coupling with soft stiffness -FPS varies between -0.0052 m and 0.0071 m while that isolated by the friction pendulum alone varies between -0.0094 m and 0.0065 m. Figure 7b on the other hand shows that for a high stiffness damper the amplitude curve varies between -0.0056m and 0.0069m.

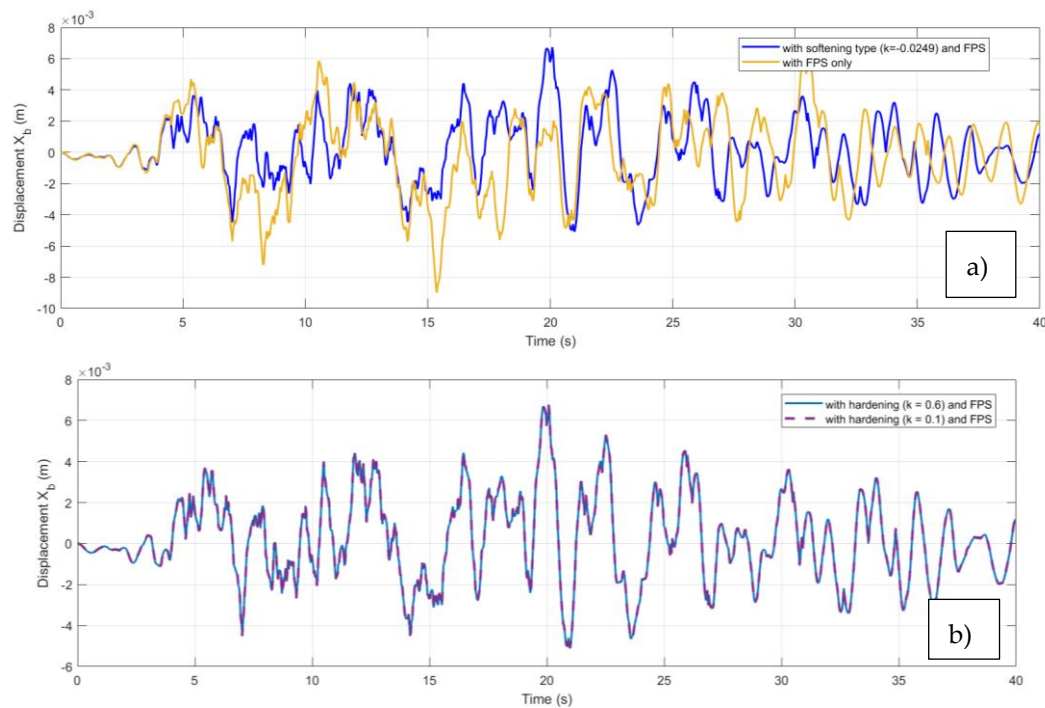


Figure 7. Response of the structure to the seismic signal on soft soil: (a) case of a structure isolated by the soft stiffness QZS-FPS coupling; (b) case of a structure isolated by the rigid stiffness QZS-FPS coupling ($k=0.01$ and $k=0.6$).

Case of hard ground

Figure 8 shows the temporal evolution of the X_g seismic signal in a rigid soil environment.

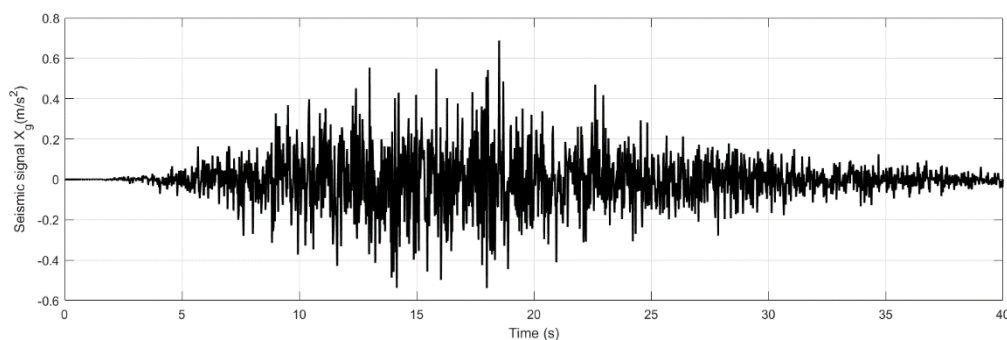


Figure 8. Seismic signal for hard ground.

Figure 9 shows the time trace of the response of the structure. For the same seismic signal, the amplitudes are constant to a few decimal places in the case of rigid QZS, whereas the amplitudes of the oscillations decrease in the case of soft QZS.

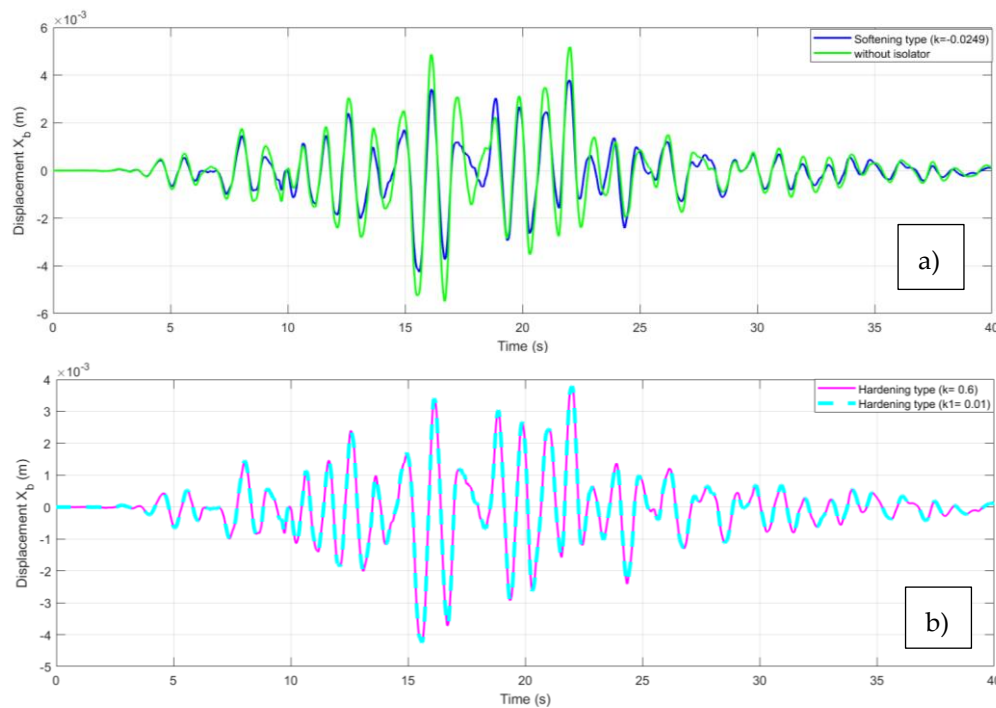


Figure 9. Response of the structure on hard ground: (a) case of a non-isolated structure isolated by the soft stiffness QZS; (b) case of a structure isolated by the rigid stiffness QZS ($k=0.01$ and $k=0.6$).

Figure 10 shows the response of the structure under different types of insulation. In fact, the blue curve in Figure 10a shows the evolution of the dependent variable (for example, performance or displacement) in a system where a softening with a coefficient $k=-0.0249$ is applied, in conjunction with SPF management, whereas the green curve shows the evolution of the same dependent variable in a system where only SPF management is applied. As for Figure 10b, the interrupted curve represents the response of the structure isolated by the FPS-QZS coupling with rigid stiffness ($k=0.6$) and the solid line represents the FPS-QZS coupling with rigid stiffness ($k=0.01$). Figure 6 shows that the structure isolated by the soft-stiff FPS-QZS coupling has a lower amplitude than that isolated by the QZS and the stiff FPS-QZS coupling.

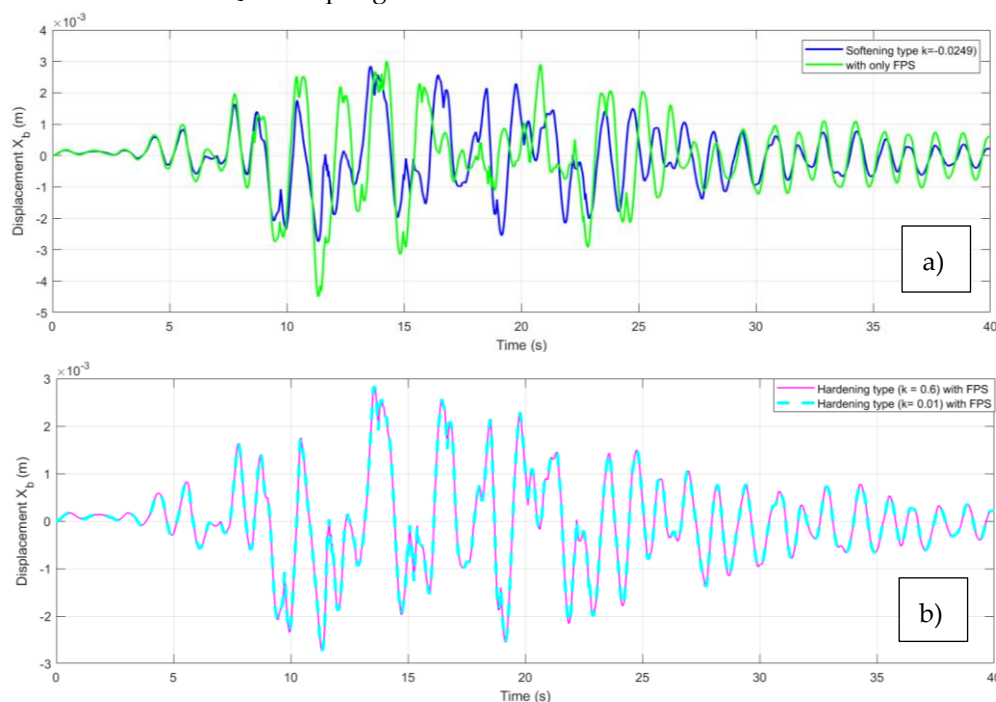


Figure 10. Response of the isolated structure: (a) by the FPS and by the coupling (FPS and FPS - QZS coupling with soft stiffness); (b) by the FPS-QZS coupling with rigid stiffnesses ($k=0.6$ and $k=0.01$).

3.3. Discussion

Figures 3 and 4 show the amplitude-frequency response of a structure that was initially uninsulated, then isolated by the QZS and by the FPS-QZS coupling at different stiffnesses. The curves observed in figure 4 show the importance of the non-linearity of the QZS compared with that of the friction pendulum [29, 30]. On the other hand, the reduction in the amplitude of oscillations and the reduction in unstable responses show that the FPS plays a major role not only in the performance but also in the stabilisation of the system. This effectiveness is illustrated by a decrease in oscillation amplitudes of around 30% on soft and hard ground in Figures 7 and 10. In addition, figures 7 and 10 show that the FPS - QZS coupling with soft stiffness is the most effective under different types of soil, probably due to the performance of the QZS with soft stiffness, as shown in figure 6a and [4].

4. Conclusions

Non-linear isolation devices are an emerging area of research and have, due to their promising potential, attracted considerable attention in the scientific community. The aim of this study was to evaluate the effect of horizontal ground excitation on the response of a structure isolated by a coupled system consisting of a non-linear damper (QZS) and a friction pendulum (FPS) with respect to the QZS.

- The FPS-QZS coupling with rigid stiffness can dampen the amplitude of the oscillations of the isolated structure by 30% with the QZS alone.
- The FPS-QZS coupling with soft stiffness can dampen the amplitude of the oscillations of the isolated structure by 40% with the QZS alone.

The soft stiffness FPS-QZS coupling dampers better than the rigid stiffness FPS-QZS coupling.

All this shows that the FPS-QZS coupling appears to be more effective than all the other isolators.

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