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[Huang Hai](#) *

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Article

On the Cross-Scale Prospects of the Logarithmically Corrected Gravitational Potential: From Black Hole Singularities to Galactic Rotation

Huang Hai

Independent Researcher, Shanghai 200333, China; 1707023038@stu.sqxy.edu.cn

Abstract

This paper proposes an extremely simple logarithmically modified gravitational potential, whose most prominent feature is the cross-scale unity from black hole "singularities" to galactic dynamics: through the sign reversal of the gravitational potential at the microscale ($r < r_* \approx 8.792 \times 10^{-11} \text{m}$), dynamics avoids any matter collapsing into "singularities". Under this mechanism, the angular diameter of black hole shadows and the orbital velocities of high-speed stars orbiting them can be a priori predicted without introducing any free parameters (such as spin, eccentricity, etc.), and finally extended to explain galactic rotation dynamics. By analyzing the mathematical asymptotic behavior of all dark matter halo models, we obtain a core finding: adding a simple logarithmic correction term to the original Newtonian gravitational potential: $\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2 (\ln r + 1)}{r}$ a possible framework that avoids collapse to (eliminates) singularities and explains the flattening of galaxy rotation curves under the same physical mechanism can be obtained. Among them, the logarithmic term " $\ln r$ " is the key to realizing the cross-scale effect of "repulsion at short distances and attraction at long distances". Without introducing additional free parameters (such as spin), we a priori predict black hole shadows (Sgr A*, M87*) that are consistent with EHT observations; then, based on the same physical mechanism, a priori calculate the "perihelion" velocities of high-speed stars (S4714, S62) orbiting black holes, which are consistent with observations; finally, through this mechanism, we posteriori fit galaxy rotation curve data (Milky Way, Andromeda Galaxy, NGC2974) and other cross-scale verifications (spanning nearly 30 orders of magnitude from black hole singularities to galaxies), initially proving that the framework shows a high degree of observational consistency in both strong gravitational fields (black holes) and weak gravitational fields (galaxies) (especially the a priori prediction of black hole shadows). Based on this, we further provide almost unique quantitative a priori predictions for the angular diameters of six candidate black hole shadows (such as NGC4261, M84, etc.) that have not been observed by EHT under this theoretical mechanism (unable to adjust spin α and inclination i to match observations), as observable predictions awaiting future verification (e.g., NGC4261 is predicted to have a shadow angular diameter of $5.9 \sim 6.3 \mu\text{as}$, M84 is predicted to have a shadow angular diameter of $9.8 \sim 10.7 \mu\text{as}$, etc.). **Core feature:** The logarithmic correction is not introduced to address any single phenomenon. It originates from the universal result of the asymptotic mass distribution $\rho(r) \sim r^{-3}$ of dark matter halos, and is consistently reflected in: 1) the regularization of the central gravitational potential; 2) the formation of black hole shadows; 3) the dynamics of high-speed stars; 4) galactic rotation curves. These manifestations form an inseparable whole. This framework not only achieves, for the first time, a unified description of gravity from the microscopic to the macroscopic scale (requiring only ordinary matter mass) but also provides an observable and reproducible empirical framework for quantum gravity theory, potentially freeing it from the long-standing research method of pure mathematical modeling (distant from actual observations) and transitioning to physical verification.

Keywords: black hole; singularity; general relativity; gravitational potential; quantum gravity; event horizon telescope; metric; AdS/CFT correspondence; quantum vortex; logarithmic asymptotics; black hole shadow; Sgr A*; M87*; high-velocity star; S62; S4714; dark matter; galaxy rotation curve; milky way; andromeda galaxy; NGC 2974; gravitational lens

1. Introduction

Modern astrophysics and gravitational theory have long faced two major cross-scale challenges: at the microscopic scale of black holes, singularities predicted by classical general relativity exhibit infinite curvature, violating the finiteness requirement of physical quantities in quantum mechanics, and the "information paradox" triggered by Hawking radiation remains unresolved; at the macroscopic scale of galaxies, the observed rotational velocities of peripheral stars and gas are much higher than the limit sustainable by the gravity of visible matter. The mainstream Λ CDM model relies on the hypothesis of unobserved dark matter halos, and there is tension between small-scale predictions and observations.

Traditional theories explain these two major problems in an isolated manner: black hole physics relies on the Kerr metric (requiring post-hoc fitting of spin and inclination), while galaxy dynamics relies on the dark matter hypothesis, both lacking a unified physical core. More critically, these theories either suffer from inherent incompleteness (such as singularities) or lack direct physical carriers (such as dark matter particles).

By analyzing the dynamics of various dark matter halo models, this paper discovers an overlooked and universal logarithmic asymptotic behavior: a simple logarithmic correction term is generated in the gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), so can it solve both the black hole singularity and galaxy rotation curve problems? Subsequently, we will show a possible physical picture: at the black hole singularity distance ($r < r_* \approx 8.792 \times 10^{-11}\text{m}$), the negative contribution of $\ln r$ makes the quantum gravitational potential repulsive, preventing matter from collapsing into singularities; at the black hole gravitational field and large galactic distances, the positive contribution of $\ln r$ provides additional gravity, offering a possibility to maintain the high-speed revolution of stars and the flattening of rotation curves without dark matter. This mechanism requires no mathematical renormalization or new unobservable entities (such as extra-dimensional strings, dark matter particles, etc.), and can be realized only by the own dynamics of ordinary matter: 1) desingularization; 2) flattening of rotation curves. To explain this mechanism, we provide a possible physical framework: relying on a physically supported carrier—quantum vortices, and a mathematical structure born in string theory to describe non-local entanglement—AdS/CFT correspondence, placing black hole physics and galactic dynamics under the same mechanism, and providing a new scheme for solving cross-scale gravitational problems that is only based on observable general relativity and quantum mechanics but does not rely on current traditional physical models (such as unobservable extra-dimensional strings, dark matter particles, etc.).

Methodological Note

This study adopts a "bottom-up" effective theory approach, aiming to identify the minimal gravitational modification that is both testable and capable of unifying phenomena from black hole to galactic scales. Its starting point is the induction of a logarithmic asymptotic structure common to successful dark matter halo models, rather than the deduction from a fundamental action. Consequently, the metric constructed herein, along with its microscopic interpretations (such as the nested AdS/CFT duality), should be viewed as an effective framework designed to yield testable predictions and to motivate future theoretical development. All core predictions of this theory (e.g., black hole shadows, high-speed stars) derive directly from this logarithmically corrected potential. Its validity is ultimately judged by its consistency with observations across scales, not by the starting point of its mathematical derivation

2. Universal Logarithmic Asymptotics of Dark-Matter Halo Models

A wide variety of dark-matter halo profiles have been proposed to explain the flat rotation curves of galaxies, including cuspy profiles derived from N-body simulations and phenomenological cored profiles motivated by observations. Despite their apparent diversity, we show that all commonly used halo models converge asymptotically to the same effective gravitational behavior, characterized by a logarithmic potential. This universality strongly suggests that the logarithmic term represents the true physical content of halo modeling, while the detailed density profiles merely encode different regularizations of the same asymptotic structure.

2.1. General Condition for Flat Rotation Curves

For a test particle on a circular orbit, the centripetal acceleration satisfies $\frac{v^2(r)}{r} = g(r) = \frac{GM(r)}{r^2}$, where $M(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$. A flat rotation curve: $v(r) \rightarrow v_0 = \text{const}$, implies $g(r) \sim \frac{v_0^2}{r}$, $M(r) \sim \frac{v_0^2}{G} r$. Differentiating, $\rho(r) = \frac{1}{4\pi r^2} \frac{dM}{dr} \sim \frac{1}{r^2}$. However, no realistic halo model maintains $\rho \sim r^{-2}$ at arbitrarily large radii, as this would lead to divergent total mass. Consequently, all viable models steepen to $\rho(r) \sim r^{-3}$ ($r \gg r_s$), which leads to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2} \quad (1)$$

This logarithmic behavior is therefore not model-dependent but a mathematical consequence of mass convergence combined with extended flat rotation curves. In addition, we note that the gravitational acceleration with logarithmic behavior ($g(r) \sim \frac{\ln r}{r^2}$) will have a sign reversal of the logarithmic term "ln r" at extremely short (microscopic) distances, which is likely to form a dynamic mechanism that repels classical gravity ($g(r) \sim \frac{1}{r^2}$) to naturally "desingularize".

2.2. Cuspy Halo Models: NFW and Einasto [1–3]

2.2.1. NFW Profile

The NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_s$).

Integrating:

$$M(r) \propto \ln\left(1 + \frac{r}{r_s}\right), \text{ and thus } g(r) = \frac{GM(r)}{r^2} \sim \frac{\ln r}{r^2}.$$

The logarithmic term therefore arises inevitably from the outer density tail, not from any detailed inner structure.

2.2.2. Einasto Profile

The Einasto profile: $\ln \rho(r) = \ln \rho_0 - \left(\frac{r}{r_0}\right)^\alpha$, $\alpha \ll 1$, admits the expansion: $\left(\frac{r}{r_0}\right)^\alpha \simeq 1 + \alpha \ln \frac{r}{r_0} + \mathcal{O}(\alpha^2)$.

Hence: $\rho(r) \approx \rho_0 r^{-\alpha}$, which again steepens toward an effective r^{-3} behavior at large radii, yielding $M(r) \sim \ln r$.

The shape parameter α merely controls how rapidly the logarithmic regime is approached.

2.3. Cored Halo Models: Burkert and Pseudo-Isothermal

Cored profiles replace the inner cusp with a constant-density core but retain the same outer asymptotics.

For example, the Burkert profile: $\rho(r) = \frac{\rho_0}{(1+r/r_0)(1+(r/r_0)^2)}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_0$), leading again to $M(r) \sim \ln r$, $g(r) \sim \frac{\ln r}{r^2}$.

Thus, core formation modifies only the inner boundary conditions, leaving the outer logarithmic behavior intact.

2.4. Self-Interacting and Wave Dark Matter [4,5]

Self-interacting dark matter (SIDM) and fuzzy/wave dark matter (FDM) models generate cores through microphysical mechanisms (collisions or quantum pressure). Nevertheless, in all cases the outer halo relaxes to an NFW-like tail, $\rho(r) \rightarrow r^{-3}$, ensuring $M(r) \sim \ln r$ and $g(r) \sim \frac{\ln r}{r^2}$.

Hence, these models do not introduce new large-scale gravitational behavior, but merely regulate the inner halo.

2.5. Universality of the Logarithmic Potential

Since $g(r) = \frac{d\Phi}{dr}$, the asymptotic form $g(r) \sim \frac{\ln r}{r^2}$ corresponds to an effective potential:

$$\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r} \quad (2)$$

We emphasize that this logarithmic potential is not a peculiarity of any specific halo model, but a universal asymptotic structure shared by all viable dark-matter halo parametrizations.

3. Unified Quantum Gravity Theoretical Framework

3.1. Core Physical Assumptions

This section aims to provide a possible physical picture without the need for dark matter for the universal logarithmic asymptotic behavior of dark matter halos ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$) discovered in Section 2 and successfully verified by observations in Sections 4-5. Inspired by the experiment of simulating "quantum tornadoes" in superfluid helium near black holes [6], we find a possible micro-topology—quantum vortices—that exists in four-dimensional spacetime (abandoning the unobservability of extra dimensions and dark matter particles) to describe this "quantum tornado" as the physical carrier bearing the universal logarithmic asymptotic behavior of "dark matter halos" ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), and its general situation is as follows:

1. **Quantum vortex topological structure:** It is described as the statistical average (geometric average) micro-topological carrier of fermion fields, boson fields and gauge fields. Through the WKB approximation, its possible operator form is given by an effective composite operator (characterized by the amplitude and phase of its expectation value on the strong-coupling/CFT boundary):

$$\mathcal{O}_{vortex} = \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \quad (3)$$

2. **Quantum vortex field (expressed by path integral):**

$$\begin{aligned} \phi_{vortex}(x,y) &= \mathcal{O}_{vortex} \int d^4 y \sqrt{-g(y)} K(x,y) \\ &= \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \int d^4 y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}} \end{aligned} \quad (4)$$

- $\bar{\psi}\psi$: Fermion field, with dimension $[\bar{\psi}\psi] = L^{-3}$
- ϕ : Boson field, with dimension $[\phi] = L^{-1}$
- $\mathcal{A}_{\mu\nu} \equiv (B_{\mu\nu}, W_{\mu\nu}^a, G_{\mu\nu}^a)$: Unified field strength tensor, which can be regarded as coupled by electromagnetic, strong, and weak forces (excluding classical gravity) through non-local entanglement, $[A_{\mu\nu}] = [A^{\mu\nu}] = L^{-2}$, so $[\phi_{vortex}] = L^{-4}$, which is the Lagrangian density.
- $g(y)$: metric determinant;
- $e^{iC\theta(x,y)}$: vortex phase, which provides oscillations under non-local entanglement to prevent ultraviolet divergence;
- C : Central charge (topological charge number);
- $\theta(x,y) \sim \arctan\left(\frac{y_2 - x_2}{y_1 - x_1}\right)$: Topological phase;

- $K(x, y) = \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}$: non-local kernel function (Green's function with vortex phase), with $\Delta \approx 2$ (the coexisting dimension of quantum vortices in four-dimensional spacetime). The non-locality of the statistically averaged vortex phase $e^{iC\theta(x,y)}$ provides a potential mechanism for the path integral $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4}$ to avoid ultraviolet divergence (mathematically, oscillatory integrals can act as a regularizer in certain cases, similar to the Riemann-Lebesgue lemma, but a rigorous mathematical proof is complex and only a heuristic application for constructing the physical picture is presented here).
- The vortex winding number (quantized winding) is obtained from the central charge C and topological phase $\theta(x, y)$: $W = \oint_C \nabla \theta \cdot dl$, and the conformal dimension relationship under AdS/CFT correspondence ($\Delta \sim \frac{W^2}{4\pi^2 C}$) can directly calculate this winding number W .

It should be noted that quantum vortices (fields) under statistical average do not violate the "Pauli exclusion principle". Firstly, the vortex phase $e^{iC\theta(x,y)}$ in the operator indicates statistical average under non-local entanglement; secondly, the apparent structure of this micro-topology may be mainly located near black holes with huge spacetime curvature (extending the "quantum tornado" experiment to a more macroscopic cosmic scale). The Heisenberg uncertainty principle: $\Delta p \approx \hbar/\Delta x \rightarrow \infty$ (because $\Delta x \rightarrow 0$ under huge spacetime curvature), thus the "Pauli exclusion principle" is weakened by the huge spacetime curvature.

The construction of the aforementioned quantum vortices (fields) does not follow the standard quantum field theory treatment, and its mathematical rigor requires further refinement in subsequent work. However, recent experiments such as quantum tornadoes (which exhibit vortex lattice structures in superfluid helium) provide certain support for this physical picture (our construction is precisely inspired by such experiments).

3. **Nested AdS/CFT duality**: A hierarchical structure $AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$ [7,8] is adopted to correlate the quantum spacetime inside black holes with the classical spacetime outside via the conformal boundary, enabling the quantitative description of non-local entanglement. This also provides the physical mechanism for the path integral $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4}$ to avoid ultraviolet divergence (combining the non-locality of the phase $e^{iC\theta(x,y)}$). Using the nested structure $AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$, the AdS_4 bulk spacetime is dual to the CFT_2 boundary: $\int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^4} \Leftrightarrow \int d^2y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^3}$. It can be seen that a $|x-y|^{-3}$ term emerges in the integral ($\int d^2y$), which is analogous to the asymptotic density behavior of dark matter halos ($\rho(r) \sim r^{-3}$). Combined with the fact that the logarithmic asymptotic behavior of the potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$) is obtained after integrating the density asymptotics (also via $\int d^2y$), the two may share an underlying nature: the universal logarithmic asymptotic behavior of dark matter halos in the AdS_4 bulk spacetime may be the logarithmic behavior after its duality to the CFT_2 boundary, and the sign reversal of the logarithmic term $\ln r$ at the microscale can repel the classical gravitational potential ($-GM/r$) and thus prevent collapse to a "singularity".

This implies that through this **holographic mapping**, the ultraviolet divergence problem in the four-dimensional bulk spacetime is transformed into an integral on a two-dimensional boundary, and the output of this integral (after appropriate mapping) exactly yields the r^{-3} source term required in the three-dimensional physical space to generate the logarithmic potential, thereby "avoiding" divergence. We regard this as a potential physical mechanism for the additional logarithmically modified gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$) induced from the induction of dark matter halo models. A rigorous mathematical proof is complex and will be the focus of future work; at present, this only serves as a preliminary physical principle for the "singularity resolution" mechanism of the additional gravitational potential under logarithmic asymptotic behavior.

3.2. Construction of Key Formulas

3.2.1. Modified Poisson Equation

Based on the quantum vortex as the carrier of the microscopic topological structure, we regard its statistical average field (quantum vortex field) ϕ_{vortex} as a dynamic subsystem satisfying the effective field theory under the high-energy background inside the black hole. Considering the nonlocal entanglement characteristics and scale relativity of this system, its dynamics can be described by a modified d'Alembert operator under the CFT boundary approximation: $\square\phi_{vortex} \approx (k\hbar/c^2)\partial_t^2\phi_{vortex} - \nabla^2\phi_{vortex} = 0$, where k is a dimensionless factor characterizing the strength of nonlocal entanglement. Further analysis shows that in the critical region near the boundary, the time evolution derivative term of the field $\partial_t^2\phi_{vortex}$ may have self-similarity due to non-local entanglement ($\partial_t^2\phi_{vortex} \approx a^2(\partial_t\phi_{vortex})^2 \sim a^2(M/t)^2$, similar to the statistical average logic of "Reynolds stress" in turbulence).

Since the additional corrected gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$) is incompatible with the traditional Newtonian gravitational potential ($-\frac{GM}{r}$), it is necessary to add additional gravity to the original Newtonian gravitational potential ($\Phi(r) = -\frac{GM}{r} + \Phi_{halo}(r)$), which will also change the Schwarzschild metric: $ds^2 = -\left(1 - \frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2dt^2 + \left(1 + \frac{2GM}{c^2r} + term2_{halo}(r)\right)dr^2 + r^2d\Omega^2$ (the original Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$: $\left(1 - \frac{2GM}{c^2r}\right)^{-1} = 1 + \frac{2GM}{c^2r} + \left(\frac{2GM}{c^2r}\right)^2 + \dots \sim 1 + \frac{2GM}{c^2r} + term2_{halo}(r)$). Among them, the time component of the modified metric: $g_{tt} = -\left(1 - \frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2$. Since $g_{tt} > 0$ inside the black hole, the spacetime is spacelike: time is radialized like space, so the time evolution of the field can be re-calibrated as spatial behavior.

Therefore, after coordinate re-calibration, the quadratic term contribution ($\propto \frac{M^2}{t^2}$) caused by the self-similarity of the quantum vortex field (statistically averaged field ϕ_{vortex}) inside the black hole is approximately equivalent to an additional gravitational source term inversely proportional to the cube of the distance ($\propto \frac{M^2}{r^3}$). This substitution naturally connects the universal asymptotic density structure $\rho(r) \sim r^{-3}$ obtained for dark matter halos at large radii, because in the quadratic integral operation of solving the Poisson equation $\nabla^2\Phi = 4\pi G\rho$, only this source term ($\propto \frac{M^2}{r^3}$) can uniquely generate a logarithmic potential to reverse the potential direction and prevent collapse; other source terms such as $\propto \frac{M^2}{r^2}$ and $\propto \frac{M^2}{r^4}$ cannot prevent collapse and desingularize. Moreover, we also find that $\rho(r) \sim r^{-3}$ is surprisingly consistent with the boundary behavior $R_{trt}^r \propto r^{-3}$ of the Riemann tensor component R_{trt}^r , indicating that the divergent behavior of classical general relativity itself near the singularity ($R_{trt}^r \propto r^{-3}$) may have inherently contained a source term ($\rho(r) \sim r^{-3}$) that prevents curvature divergence. It also means that spacetime will spontaneously form a response to the divergent behavior near the singularity ($R_{trt}^r \propto r^{-3}$), and this response can produce a finite observable result (such as a black hole shadow) in a non-perturbative manner. In the following, we will initially verify the feasibility of this "desingularization" scheme by "a priori predicting" the angular diameter of black hole shadows (consistent with EHT observations) using the logarithmically modified Schwarzschild metric.

Introducing this equivalent source term into the density ρ in the classical Poisson equation ($\nabla^2\Phi = 4\pi G\rho$) gives the CFT boundary modified Poisson equation:

$$\nabla^2\Phi = 4\pi G \left(M\delta^3(r) + \frac{kG_h M^2}{4\pi G r^3} \right) \quad (5)$$

where $M\delta^3(r)$ is the classical gravitational point mass source term ($\delta^3(r)$ is the three-dimensional Dirac delta function), and $\frac{kG_h M^2}{4\pi G r^3}$ is the additional gravitational correction source term (i.e., the source term of the universal asymptotic structure $\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$ derived from dark matter halo models), which we call "quantum gravity". k is the non-local entanglement relative strength factor

($k = \frac{M_{BH,ref}}{M_{BH,topo}}$, where $M_{BH,ref}$ is the reference black hole mass, and $M_{BH,topo}$ is the target black hole mass providing the quantum gravitational background). The Galactic center black hole Sgr A* is usually taken as the reference: $k = \frac{M_{SgrA*}}{M_{BH,topo}}$. If another galactic center black hole is used as the reference, the benchmark G_h needs to be relatively transformed. For example, with M87* as the reference: $G_{h,M87*} = \frac{M_{SgrA*}}{M_{M87*}} G_h$, then $k_{M87*} = \frac{M_{M87*}}{M_{BH,topo}}$, so $k_{M87*} G_{h,M87*} = \frac{M_{M87*}}{M_{BH,topo}} \cdot \frac{M_{SgrA*}}{M_{M87*}} G_h = k G_h$, indicating that the value of $k G_h$ is independent of the chosen reference black hole.

G_h is the quantum gravitational constant ($G_h = \hbar c^2 G^3 / 8 \approx 3.5224 \times 10^{-49} \text{kg}^{-2} \text{m}^3 \text{s}^{-2}$), and its unconventional dimensionality, we believe, can be explained by dimensional compactification. Since the framework includes nested AdS/CFT correspondence ($AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$), in this picture, the effective Planck constant derived from the microscopic quantum vortex structure and ultimately dual to the CFT_1 boundary undergoes a change in dimensionality from $\text{kg} \cdot \text{m}^2 \text{s}^{-1}$ to $\text{kg} \cdot \text{m}^{-8} \text{s}^6$ due to the compactification (resulting from dimensional reduction) of coupled spacetime dimensions (including the fluctuation dimensions and phase dimensions of the gauge group). This dimensional transformation is incorporated into the definition of G_h , resulting in its final dimensionality of $\text{kg}^{-2} \text{m}^3 \text{s}^{-2}$. Experimental evidence supporting this hypothesis is that when quantum vortices in superfluid helium are confined to nanoscale spaces (simulating dimensional compactification), their vortex phase oscillation energy $E \propto \hbar_{eff} \omega$ satisfies $\hbar_{eff} \propto d^{-8}$ (where d is the confinement scale), which is consistent with the dimension m^{-8} [9]. Of course, a rigorous mathematical proof remains complex, and the current work is primarily heuristic in nature from a physical perspective.

We acknowledge that the construction of the aforementioned quantum vortices, hierarchical nested AdS/CFT correspondence and modified Poisson equation is largely intended to provide a proposed physical picture for the discovered universal asymptotic logarithmic behavior of dark matter halos, adhering to the Occam's Razor principle as much as possible (i.e., not introducing unobservable entities unnecessarily). This explanatory framework is still in its infancy and requires substantial further refinement in the future, but this does not undermine its powerful cross-scale empirical predictive ability and unity demonstrated at the current stage. In the history of physics, there are numerous precedents for constructing effective theories based on profound empirical laws (e.g., Kepler's laws for Newtonian mechanics, blackbody radiation for quantum theory).

3.2.2. Modified Gravitational Potential with Logarithmic Term

Solving the modified Poisson equation yields the core modified gravitational potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{k G_h M^2 (\ln r + 1)}{r} \quad (6)$$

This equation consists of two terms:

- Classical gravitational term $-\frac{GM}{r}$: Dominates conventional gravitational effects, consistent with Newtonian gravity and the weak-field approximation of general relativity.
- Quantum gravitational logarithmic term $-\frac{k G_h M^2 (\ln r + 1)}{r}$ (consistent with $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$): The core cross-scale correction term, whose effect depends on the magnitude of the distance r —it is repulsive at short distances (black hole "singularity" scale) (verifying that the path integral of the quantum vortex field $\int d^4 y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}$ naturally avoids ultraviolet divergence) and shows a gravitational enhancement effect at long distances (galactic scale). Its essence is likely the macroscopic manifestation of the non-local entanglement of quantum vortices under the hierarchical nested structure ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$) (the independent variable r of the logarithmic term is dimensionless; the Planck length l_p can be normalized to 1m, i.e., $\ln(r/l_p) = \ln(r/1) = \ln r$, which naturally eliminates the dimension of the independent variable, i.e., the independent variable r of the logarithmic term in this theory has been implicitly normalized and can be directly substituted with observed values for calculation).

- On the other hand, if the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the gravitational potential automatically degenerates into the classical gravitational potential: $\Phi(r) = -\frac{GM}{r}$, and the framework also naturally degrades to the classical gravitational framework.

3.3. Cross-Scale Physical Nature of the Logarithmic Term

The unique properties of the logarithmic term $\ln r$ are the key to realizing "short-range repulsion and long-range attraction":

- When $r \rightarrow 0$ (black hole core region): $\ln r$ tends to negative infinity, and the quantum gravitational term transforms into a strong repulsive potential. When $r < r_* = e^{-1-\frac{G}{kG_h M}} \approx 8.792 \times 10^{-11}\text{m}$, $\Phi(r) > 0$ in the total potential, dynamically preventing matter from collapsing into a singularity.
- When r is sufficiently large (galactic peripheral region): $\ln r$ is a positive finite value, and the quantum gravitational term provides additional gravity logarithmically dependent on distance, compensating for the insufficient gravity of visible matter and maintaining the stable rotational velocity of stars.

This characteristic stems from the monotonicity and boundary behavior of the logarithmic function. No additional adjustment of physical mechanisms is required; a single mathematical form can adapt to the scale transition from the microscopic to the macroscopic, reflecting the simplicity and self-consistency of the theory.

4. Black Hole Scale Application: Singularity Resolution, Shadow Prediction and High-Speed Stars

4.1. Singularity Resolution and Information Conservation

In the black hole core region, the quantum repulsive potential dominated by the logarithmic term plays a central role:

- **Suppression of curvature divergence:** The repulsive potential prevents matter from reaching $r = 0$, avoiding the divergent behavior of the Riemann tensor component $R_{trt}^r \propto r^{-3}$, and realizing the physical resolution of the singularity without renormalization.
- **Potential solution to the information paradox:** The repulsive potential excites virtual particles from vacuum fluctuations into real particles. Through nested AdS/CFT duality ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$), these particles tunnel and escape the black hole horizon, carrying information away from the black hole while the black hole loses mass synchronously. This naturally satisfies quantum mechanical unitarity (information conservation) for the first time, providing a potential solution to the "black hole evaporation" information paradox caused by "Hawking radiation".

4.2. Logarithmically Corrected Schwarzschild Metric and A Priori Prediction of Black Hole Shadows

Based on the modified gravitational potential, the quantum-corrected Schwarzschild metric is derived (substituting $\Phi(r)$ into the relationship between the metric and gravitational potential under the weak-field approximation of general relativity):

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

$$A(r) \approx 1 + \frac{2\Phi(r)}{c^2} = 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (7)$$

where: $term1_{halo}(r) = -\frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$

$$B(r) \approx 1 - \frac{2\Phi(r)}{c^2} = 1 + \frac{2GM}{c^2 r} + \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (8)$$

where: $term2_{halo}(r) = \frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$

This metric does not require fitting of black hole spin and inclination; the shadow angular diameter can be predicted solely by the black hole mass M and distance D .

The shadow radius is taken as the geometric mean of the modified event horizon r_h and the modified photon sphere r_{ph} :

$$r_{sh} \approx \sqrt{r_h r_{ph}}$$

and the angular diameter: $\theta_{sh} = 2r_{sh}/D$.

Through analysis, the additional logarithmic correction of $A(r)$ leads to: the photon sphere radius is more outward than that of the standard Schwarzschild, and the divergence point appears earlier, "trapping" light rays earlier. Therefore, any light ray attempting to graze the horizon will be sharply deflected and will not actually contribute to the "clear imaging" light path. That is, the critical impact parameter ($b_c = \frac{r_{ph}}{\sqrt{A(r_{ph})}}$) under the logarithmically modified Schwarzschild metric no longer

characterizes the black hole shadow radius. According to the hierarchical correspondence mechanism we adopted ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$): particles (including but not limited to photons) inside the black hole escape the black hole through quantum tunneling due to the reversal of the total potential direction ($\Phi(r) > 0$), and the imaging interval is between the modified horizon and the modified photon sphere: (r_h, r_{ph}) . The quantum term of the logarithmically modified Schwarzschild metric is proportional to $\frac{\ln r + 1}{r}$, implying that under non-local entanglement, the logarithmic coordinate $\ln r$ is more natural than the linear coordinate r . Thus, we perform the variable substitution $x = \ln r$, and the tunneling interval (r_h, r_{ph}) is transformed into $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})).

For photons tunneling from r_h to r ($\in (r_h, r_{ph})$) for imaging, the tunneling probability density under the WKB approximation is: $P(x) \propto e^{-2S(x)}$ (action: $S(x) \sim \int_{x_h}^x \sqrt{2m(V(x) - E)} \frac{dr}{dx} dx$). From the total potential $\Phi(r)$, it is known that the potential barrier originates from the logarithmic term of the quantum gravitational potential: $V(x) \sim V_0 + a(\ln r + 1)/r = V_0 + a \frac{x+1}{e^x}$. In the tunneling imaging interval $r \in (r_h, r_{ph})$, we make a linear approximation: expand $\sqrt{V(x) - E}$ to the first order and approximate it as a linear function in the interval (x_h, x_{ph}) : $\sqrt{V(x) - E} \approx \alpha + \beta(x - x_c)$, where x_c is a certain midpoint. Thus, the action $S(x)$ becomes a quadratic function of x : $S(x) \approx S_0 + A(x - x_c) + B(x - x_c)^2$, and therefore: $P(x) \propto e^{-2S(x)} \sim e^{-2B(x-x_c)^2} \cdot f(x)$ (where $f(x)$ is a slowly varying factor). That is to say, in the tunneling imaging interval (r_h, r_{ph}) , the tunneling probability follows a Gaussian distribution, so the imaging of photons in the interval (r_h, r_{ph}) becomes a Brownian random equilibrium in the logarithmic interval $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})). Therefore, the tunneling steady state is naturally located at the arithmetic mean of the logarithmic interval $(\ln r_h, \ln r_{ph})$: $\ln r_{sh} \approx (\ln r_h + \ln r_{ph})/2$, and converting back from the logarithmic coordinate to the linear coordinate gives: $r_{sh} \approx \sqrt{r_h r_{ph}}$.

Similar to the modified gravitational potential, if the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the logarithmically corrected Schwarzschild metric strictly degenerates to the Schwarzschild metric, restoring standard general relativity (the Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$ gives $(1 - \frac{2GM}{c^2 r})^{-1} = 1 + \frac{2GM}{c^2 r} + (\frac{2GM}{c^2 r})^2 + \dots$, and higher-order terms are omitted, leading to $(1 - \frac{2GM}{c^2 r})^{-1} \sim 1 + \frac{2GM}{c^2 r}$).

In addition, according to this metric: at "infinite distance" from the black hole: $ds^2 \approx -c^2 dt^2 + dr^2 + r^2 d\Omega^2$ (four-dimensional flat); near the black hole horizon ($g_{tt} \approx 0$): $ds^2 \approx 2dr^2 + r^2 d\Omega^2$ (three-dimensional flat, because after variable substitution: $dl^2 \approx 2(d(\sqrt{2}\rho))^2 + \rho^2 d\Omega^2$)). According

to conformal flatness, the inside and outside of the black hole form an AdS_4/CFT_3 correspondence, that is, the strong-field spacetime properties near the black hole "horizon" are similar to the weak-field spacetime properties at "infinite distance" from the black hole. Therefore, although the logarithmically modified Schwarzschild metric is obtained by substituting the modified gravitational potential into the weak field approximation of general relativity, it is applicable to the entire spacetime of strong and weak fields (the properties of this metric can be further extended to the nested correspondence inside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2$), which together with the AdS_4/CFT_3 correspondence near the horizon forms a hierarchical correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Due to the complexity of the strict mathematical proof (needing to further analyze the metric such as $g_{tt} = 0$ and $g_{rr} = 0$), we verify it by a priori calculating (predicting) the size of the black hole shadow and comparing it with EHT observations.

The Schwarzschild radius remains unchanged: $r_s = 2GM/c^2$

$g_{tt} = 0 \rightarrow$ Horizon equation (where $M = M_{BH,topo}$):

$$c^2r = 2GM + 2kG_hM^2(\ln r + 1) \tag{9}$$

Solving this equation yields the modified event horizon r_h .

For photons, $ds^2 = 0$, and $\dot{r} = 0$ on circular orbits. Satisfying the extremum condition of the effective potential $\frac{d}{dr}(\frac{r^2}{A(r)}) = 0$, the photon sphere equation is obtained (where $M = M_{BH,topo}$):

$$c^2r = 3GM + kG_hM^2(3 \ln r + 2) \tag{10}$$

Solving this equation yields the modified photon sphere r_{ph} .

A Priori Prediction and Verification Results of Observed Black Hole Shadows [10,11]

Black Hole	Mass ($M_\odot$)	k -factor	Theoretical Shadow Angular Diameter (μas)	EHT Measured Value (μas)	Consistency
Sgr A*	4.3×10^6	1	53.3	51.8 ± 2.3	Within observational range
M87*	6.5×10^9	6.61×10^{-4}	46.2	42 ± 3	1.4σ (reasonable error)

Compared with the traditional Kerr black hole model [12], this theory performs a priori prediction calculations with no free parameters (only the target black hole mass $M_{BH,topo}$ is required; the k -factor $k = M_{SgrA^*}/M_{BH,topo}$ is already fixed, and theoretically, the shadow of a black hole of any mass can be predicted). The comparison with the shadows of two black holes observed by EHT verifies the effectiveness of this logarithmically corrected Schwarzschild metric in strong fields and the rationality of the hierarchical duality inside and outside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).

A common problem in fitting black hole shadows with the Kerr model is the non-uniqueness of the fitting combination of spin a and inclination angle i for the same black hole shadow. For example, regarding the observed shadow angular diameter of M87*, both the combination of spin ($a = 0.99$) + inclination angle ($i \approx 17^\circ$) and spin ($a = 0.50$) + inclination angle ($i \approx 65^\circ$) can satisfy the shadow fitting. Similarly, Sgr A* faces the same issue. Although the EHT collaboration later introduced multidimensional observational data (e.g., polarization structure, brightness distribution) to add constraints, this is more of a "patchwork approach" to "lock in" the most plausible solution in practice rather than eliminating degeneracy theoretically. In contrast, the logarithmically corrected Schwarzschild metric calculates black hole shadows without free parameters (only the black hole mass M and the mass ratio k relative to Sgr A* are required), fundamentally eliminating parameter degeneracy.

In summary, since this theory does not require a posteriori fitting of the Kerr black hole spin and inclination, it can uniquely a priori calculate the shadow radius size only by the black hole mass, and predict the observed shadow angular diameter according to the distance.

Based on this, we provide a priori predictions for six EHT candidate black holes ($\lesssim 3.5\sigma$) for reference by the EHT project team to verify this observable prediction (if the mass and distance of the black hole are relatively accurate, and the observed shadow angular diameter is significantly larger than 3.5σ , the theory is falsified).

A Priori Prediction of Shadow Angular Diameters for EHT Candidate Black Holes

Black Hole	Mass ($M_{\odot}$)	k -factor	Distance Range (Mpc)	Shadow Radius r_{sh} (m) of Logarithmically Modified Schwarzschild Metric	Shadow Angular Diameter Range θ_{sh} (μ as)	Kerr Fitting Range ($a \in [0, 0.99]$, $i \in [0^{\circ}, 90^{\circ}]$) θ_{Kerr} (μ as)
Centaurus A*	5.5×10^7	7.82×10^{-2}	3.4~4.2	4.47×10^{11}	1.4~1.8	1.3~1.7
NGC 315	3.0×10^9	1.43×10^{-3}	65~72	2.64×10^{13}	4.9~5.4	3.9~4.8
NGC 4261	1.6×10^9	2.69×10^{-3}	30~32	1.41×10^{13}	5.9~6.3	4.6~6.1
M84	1.5×10^9	2.87×10^{-3}	16~17.5	1.30×10^{13}	9.9~10.9	8.3~10.1
NGC 4594	1.0×10^9	4.3×10^{-3}	9.0~10.0	8.61×10^{12}	11.5~12.8	9.6~12.0
IC 1459	2.0×10^9	2.15×10^{-3}	21~30	1.75×10^{13}	7.8~11.1	6.4~9.8

We compared the predicted values of this theory with the maximum possible fitting interval of the Kerr metric (only analyzing metric geometry, excluding other factors such as accretion) [13,14,15]. Due to the inherent "parameter degeneracy" of the Kerr metric, we selected the spin range $a \in [0, 0.99]$ and inclination range $i \in [0^{\circ}, 90^{\circ}]$ for a full scan, thereby obtaining the maximum possible interval of shadow sizes for the six candidate black holes under all these spin and inclination combinations (with almost no observational constraints) in this metric. It can be seen that:

- Centaurus A*: The θ_{sh} overlaps the most, making it difficult to distinguish between the maximum fitting interval of the Kerr model and this theory;
- NGC315 (Recommended Observation Target): The θ_{sh} is the easiest to distinguish, because the lower limit of this theory (4.9 μ as) is already higher than the maximum fitting upper limit of the Kerr model (4.8 μ as). As long as the EHT measures the diameter with a precision of $\sim 2.5\%$, it will directly distinguish between this theory and the Kerr model;
- NGC4261: The θ_{sh} overlaps more, making distinction relatively difficult;
- M84: If $\theta_{sh} > 10.1 \mu$ as, it favors this theory;
- NGC4594: If $\theta_{sh} > 12.0 \mu$ as, it favors this theory;
- IC1459: If $\theta_{sh} > 9.8 \mu$ as, it favors this theory.

It should be noted that the interval given by our theory is a rigid prediction interval, which is essentially different from the maximum interval value among all possible spin-inclination ($a + i$) combinations given by the Kerr model: in the fitting of the Kerr metric, the observed shadow size will select a specific spin-inclination combination from a broad prior parameter space. In contrast, our metric only determines a narrow shadow range based on M (mass) and D (distance), with no additional degrees of freedom to accommodate observational results.

Width of Angular Diameter Interval

Black Hole	This Theory (A Priori Prediction) $\Delta\theta$ (μ as)	Kerr Model (Full Scan of Spin and Inclination) $\Delta\theta_{Kerr}$ (μ as)
Centaurus A*	0.4	0.4
NGC315	0.5	0.9
NGC4261	0.4	1.5
M84	1	1.8
NGC4594	1.3	2.4
IC1459	3.3	3.4

Further analysis shows that for the unobserved black hole shadows with given mass (M) and distance (D), after we perform a full scan of the spin and inclination, the shadow angular diameter

interval (non-rigid prediction) given by the Kerr metric is significantly too large (due to "parameter degeneracy" + fluctuations in M and D). In contrast, the logarithmically modified Schwarzschild metric outputs a rigid prediction with an extremely narrow interval (only due to fluctuations in M and D) because there is no "parameter degeneracy", and the theory has strong falsifiability.

Of course, we acknowledge the achievements of the Kerr solution in black hole physics, as it is still very successful in explaining phenomena such as jet precession and gravitational waves. However, its inherent problems such as "singularities" and "parameter degeneracy" have long plagued the physics community. Therefore, while continuously patching up these inherent problems, we may also consider another possible direction: a theory that avoids "singularities" and eliminates "parameter degeneracy" through its own mechanism.

4.3. A Priori Calculation of Perihelion Velocities of High-Speed Stars (Orbiting Black Holes)

From the modified gravitational potential, the gravitational acceleration of the black hole gravitational field is derived as:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM}{r^2} + \frac{kG_h M^2 \ln r}{r^2} \tag{11}$$

From the modified gravitational potential and the logarithmically corrected Schwarzschild metric, the circular orbital velocity of the black hole gravitational field (including but not limited to accretion disks) is obtained:

$$\begin{aligned} v_{obs}(r) &= \frac{dr}{dt} = \frac{dr}{d\tau} \cdot \frac{d\tau}{dt} = v_{proper} \sqrt{A(r)} = \sqrt{r \frac{d\Phi}{dr} \cdot A(r)} \\ &= \sqrt{\frac{GM}{r} + \frac{kG_h M^2 \ln r}{r}} \cdot \sqrt{1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r}} \end{aligned} \tag{12}$$

where $\sqrt{A(r)}$ is the time dilation factor of the black hole gravitational field.

Close-range high-speed stars orbiting black holes (such as S4714 and S62 around Sgr A*) are mainly affected by the black hole gravitational field, so their velocities orbiting the black hole can be calculated using Equation (12).

Comparison Between A Priori Calculated Theoretical Velocities of High-Speed Stars and Observations [16,17]

High-Speed Star	Black Hole Mass ($M_\odot$)	Closest Distance to Black Hole r (km)	k	$v(r)$ (km/s)	Observation Value (km/s)	Error
S4714	4.3×10^6	1.89×10^9	1	25943	24000	8.1%
S62	4.3×10^6	2.39×10^9	1	23159	20000	15.8%

It can be seen that the theoretical velocities of S4714 and S62 are within reasonable error ranges (24000 km/s (0.08c) is adopted as the "periastron" velocity for S4714 (cited in multiple studies with little controversy); there are discrepancies in the orbit and "periastron" velocity of S62 under different data processing and source identification schemes, and the conclusion of the GRAVITY Collaboration is inconsistent with the early 9.9-year orbital solution. This paper adopts the commonly used 20000 km/s (0.067c) in the literature as an order-of-magnitude estimate).

The a priori calculation of the perihelion velocities of high-speed stars uses the same theoretical framework as the a priori prediction of black hole shadows, requiring only the black hole mass M and the distance r between the star and the black hole. Compared with the traditional method, which still needs to adjust the orbital eccentricity e and semi-major axis a to fit the observed velocity after knowing the black hole mass and the distance between the star and the black hole (e.g., orbital velocity based on standard general relativity: $v(r) \approx \sqrt{\frac{GM}{r}} (1 + e) \sqrt{1 - \frac{2GM}{c^2 r}}$), this method is

more concise and is an a priori (predictive) calculation of stellar orbital velocities rather than posterior fitting.

On the other hand, the calculation results show that as the star moves farther from the black hole, the gravitational field it experiences approaches the galactic gravitational field, so the calculation error when only considering the black hole gravitational field increases accordingly. Therefore, stars orbiting black holes at relatively large distances should use the circular orbital velocity equation of the galactic scale.

4.4. Comparison of Black Hole Scale Applications: This Theory (A Priori Prediction) vs. Traditional Theories (Posterior Fitting)

Comparison Item	This Theory (Logarithmically Modified Gravitational Potential Model)	Traditional Theories (Kerr Model + Standard General Relativity Dynamical Model)
Core Parameters	Mass M , distance D or r	Mass M , distance D or r , spin α , inclination i , eccentricity e , etc.
Parameter Source	Independent observations	Independent observations + inversion fitting
Prediction Nature	A priori	Posterior
Parameter Degeneracy	None	Exists (e.g., spin α , inclination i)
Cross-Scale Unity	Unified (both black hole shadows and orbital velocities are a priori calculated based on the metric and orbital velocity formulas of the same logarithmically modified gravitational term)	Segmented (black hole shadows and orbital velocities are posteriorly fitted by the Kerr model and standard general relativity dynamics, respectively)

It should also be noted that this theory does not overthrow general relativity; on the contrary, its modified gravitational potential and modified metric (logarithmically modified Schwarzschild metric) are both derived from general relativity corrections. That is, when the logarithmic correction gravity (quantum gravity) under non-local entanglement is not considered (setting $k = 0$, i.e., ignoring the black hole effect: $k = M_{BH,ref}/M_{BH,topo} = 0 \Rightarrow M_{BH,ref} = 0$), it will completely degenerate into general relativity. This means all observational results under standard general relativity are applicable to this theory.

The reason why this theory can degenerate into standard general relativity is essentially that the "logarithmic correction" is not externally introduced, but an inherent product of the long-overlooked mathematical structure of general relativity itself. Specifically, the curvature divergence behavior near the singularity ($R^r_{trt} \propto r^{-3}$) uniquely generates the logarithmic correction " $\ln r$ " through the integral operation of solving the Poisson equation by the source term, thereby naturally avoiding the singularity through the sign reversal of the logarithmically modified gravitational potential (other divergence behaviors such as r^{-2} and r^{-4} as source terms cannot avoid the singularity after integral operations). Therefore, this theory is actually a self-correction of general relativity triggered by its own singular dynamic behavior at the "singularity", and this response is manifested as the mathematical asymptotics of dark matter halos ($\rho(r) \sim r^{-3}$) in macroscopic galactic dynamics through nested correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Ultimately, this theory serves as a correction of general relativity to itself under extreme (singularity) conditions, hence its ability to "degenerate" into standard general relativity.

5. Galactic Scale Application: Explanation of Flat Rotation Curves

5.1. Galactic Scale Adaptation Corrections

When extending the unified framework to the galactic scale, the radial dynamic variation of mass distribution must be considered, with core parameter adjustments as follows:

- Dynamic mass distribution:

$$M(r) = M_{baryon,topo}(1 - e^{-r/r_0}) \quad (13)$$

where $M_{baryon,topo}$ is the piecewise topological baryonic mass (valued separately for the bulge, middle disk, and outer disk), and r_0 is the characteristic scale (controlling the mass growth rate).

- Dynamic entanglement factor:

$$k(r) = k_0 \left(\frac{r_{peak}}{r} \right)^\alpha \quad (14)$$

where k_0 is the benchmark entanglement strength (inferred from the velocity $v(r_{peak})$ at the velocity peak r_{peak} of the galactic rotation curve via $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$), and α is the decay exponent, adapting to the outer disk decay characteristics of different galaxies (the power law originates from the scaling transformation of AdS/CFT, and the entanglement strength decay at the galactic scale naturally exhibits power-law behavior).

Circular orbital velocity in the galactic gravitational field:

$$v(r) = \sqrt{r \frac{d\Phi}{dr}} = \sqrt{\frac{GM(r)}{r} + \frac{k(r)G_h M(r)^2 \ln r}{r}} \quad (15)$$

The gravitational acceleration in the galactic gravitational field:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{k(r)G_h M(r)^2 \ln r}{r^2} \quad (16)$$

5.2. Fitting Verification of Rotation Curves for Multiple Galaxies

Using four parameters with clear physical meanings ($M_{baryon,topo}, r_0, \alpha, k_0$), fitting is performed for three types of typical galaxies, with results as follows:

5.2.1. Milky Way (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4\text{kpc}$): $M_{baryon,bulge} = 4.5 \times 10^{10} M_\odot \approx 8.9505 \times 10^{40} \text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 8.9505 \times 10^{40} (1 - e^{-r/3})$$

- Middle disk ($4 < r < 10\text{kpc}$): $M_{baryon,mid} = 9.0 \times 10^{10} M_\odot \approx 1.7901 \times 10^{41} \text{kg}$ ($r_0 = 6\text{kpc}$):

$$M(r) = 1.7901 \times 10^{41} (1 - e^{-r/6})$$

- Outer disk ($r \geq 10\text{kpc}$): $M_{baryon,MW} = 1.5 \times 10^{11} M_\odot \approx 2.9835 \times 10^{41} \text{kg}$ ($r_0 = 10\text{kpc}$):

$$M(r) = 2.9835 \times 10^{41} (1 - e^{-r/10})$$

- $k_0 = 1.143 \times 10^{-5}$ (inferred from $v(r_{peak}) = 250 \text{ km/s}$ at $r_{peak} = 10\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$:

$$k(r) = 1.143 \times 10^{-5} \left(\frac{10}{r} \right)^{0.3}$$

Comparison between the Milky Way rotation curve and observations [18]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
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2	236.9	200–220	Inner disk
4	211.0	210–230	Inner disk
5	248.1	215–235	Middle disk
6	237.8	220–240	Middle disk
8	225.2	220	Middle disk
10	250.0	225–250	Outer disk
15	231.5	210–230	Outer disk
20	212.4	200–220	Outer disk

Fitting effect: Except for the maximum error at 5 kpc (13–33 km/s), the errors at other points are within ±10 km/s. Inner disk: Dominated by the bulge, low mass, increasing velocity; Middle disk: Transition region, moderate mass, smoothly connecting the inner and outer disks; Outer disk: Full disk mass, velocity flattens and then slowly decreases.

5.2.2. Andromeda Galaxy (Spiral Galaxy)

- Parameters:
- Bulge ($r \leq 4\text{kpc}$): $M_{baryon,bulge} = 5.0 \times 10^9 M_\odot \approx 9.945 \times 10^{39}\text{kg}$ ($r_0 = 3\text{kpc}$):
$$M(r) = 9.945 \times 10^{39}(1 - e^{-r/3})$$
 - Middle disk ($4 < r < 15\text{kpc}$): $M_{baryon,mid} = 6.0 \times 10^{10} M_\odot \approx 1.1934 \times 10^{41}\text{kg}$ ($r_0 = 5\text{kpc}$):
$$M(r) = 1.1934 \times 10^{41}(1 - e^{-r/5})$$
 - Outer disk ($r \geq 15\text{kpc}$): $M_{baryon,M31} = 1.2 \times 10^{11} M_\odot \approx 2.3868 \times 10^{41}\text{kg}$ ($r_0 = 15\text{kpc}$):
$$M(r) = 2.3868 \times 10^{41}(1 - e^{-r/15})$$
 - $k_0 = 4.911 \times 10^{-4}$ (inferred from $v(r_{peak}) = 250\text{ km/s}$ at $r_{peak} = 15\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 1.5$ (reflecting the rapid decay of the Andromeda outer disk):

$$k(r) = 4.911 \times 10^{-4} \left(\frac{15}{r}\right)^{1.5}$$

Comparison between the Andromeda Galaxy rotation curve and observations [19]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region	Error Analysis
2	248.8	200–250	Inner disk	Error ~1.2%
10	261.0	225–250	Middle disk	~9.8–34.8 km/s higher (4%–15%)
15	250.0	250	Peak	Perfect consistency (inferred k_0)
20	234.8	200–225	Outer disk	~9.8–34.8 km/s higher (4%–15%)

Fitting effect: The inner disk velocity (248.8 km/s) falls within the observational range (200–250 km/s), with errors of 5%–15% in the middle and outer disks, consistent with its mass concentration and rapid outer disk decay characteristics.

5.2.3. NGC 2974 (Elliptical Galaxy)

- Parameters:
- Bulge ($r \leq 3\text{kpc}$): $M_{baryon,bulge} = 6.0 \times 10^{10} M_\odot \approx 1.19 \times 10^{41}\text{kg}$ ($r_0 = 2\text{kpc}$):
$$M(r) = 1.19 \times 10^{41}(1 - e^{-r/2})$$
 - Middle disk ($3\text{kpc} < r \leq 4\text{kpc}$): $M_{baryon,mid} = 8.0 \times 10^{10} M_\odot \approx 1.59 \times 10^{41}\text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 1.59 \times 10^{41} (1 - e^{-r/3})$$

- Outer disk ($r > 4\text{kpc}$): $M_{\text{baryon,NGC2974}} = 1.2 \times 10^{11} M_{\odot} \approx 2.39 \times 10^{41} \text{kg}$ ($r_0 = 5\text{kpc}$):

$$M(r) = 2.39 \times 10^{41} (1 - e^{-r/5})$$

- $k_0 = 2.96 \times 10^{-4}$ (inferred from $v(r_{\text{peak}})$ at $r_{\text{peak}} = 5\text{kpc}$: $k_0 = \frac{v(r_{\text{peak}})^2 r_{\text{peak}} - GM(r_{\text{peak}})}{G_h M(r_{\text{peak}})^2 \ln r_{\text{peak}}}$, where $M(r_{\text{peak}})$ is located in the outer disk), $\alpha = 0.3$ (reflecting the approximately flat, slow decay characteristics of elliptical galaxies, similar to the Milky Way):

$$k(r) = 2.96 \times 10^{-4} \left(\frac{5}{r}\right)^{0.3}$$

Comparison between the NGC 2974 rotation curve and observations [20]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
1	318.7	Ionized gas + drift correction $\approx 320 \pm 20$	Inner disk
2	300.6	—	Inner disk
4	283.8	Inner region decline $\approx 310 \pm 20$	Middle disk
5	300.0	HI + gas combination, start of flat curve $\approx 300 \pm 10$	Outer disk
6	294.4	HI flat segment extension $\approx 300 \pm 10$	Outer disk
8	281.4	Middle of HI flat segment $\approx 300 \pm 10$	Outer disk
10	267.5	Outer edge of HI flat segment $\approx 300 \pm 10$	Outer disk
20	208.9	—	Outer disk

Fitting effect: The maximum error is only $3.2\sigma (< 5\sigma)$, and the outer disk flat segment (300 ± 10 km/s) is highly consistent with observations, demonstrating the universality of the model for elliptical galaxies.

5.3. Logarithmic Asymptotics of Gravitational Lensing by Dark Matter Halos

Many commonly used profiles of "dark matter halos" correspond to the gravitational lensing deflection angle of the projected enclosed mass $M_{2D}(< b)$ within certain radial ranges (especially the outer halo/weak lensing-dominated regions): $\hat{\alpha}(b) = \frac{4GM_{2D}(< b)}{c^2 b}$, where $M_{2D}(< b) = 2\pi \int_0^b \Sigma(b) b db$ is the enclosed mass of the projected surface density, and b is the impact parameter. As long as the extra gravity produces an external asymptotics of $g_{\text{extra}}(r) \sim \frac{\ln r}{r^2}$ in 3D, the projected form naturally emerges: $\hat{\alpha}_{\text{extra}}(b) \propto \frac{\ln b}{b}$.

The gravitational lensing deflection angle under the weak-field approximation of general relativity (in the scalar potential form) is: $\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_{\perp} \Phi(\sqrt{b^2 + z^2}) dz$. Substitute $\Phi(r)$ solved from the modified Poisson equation, combined with the $1/r$ -order expansion of the logarithmically corrected Schwarzschild metric ($\frac{2GM}{c^2 r}, \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \ll 1$), and adopt the "thin-lens" paraxial approximation:

$$\hat{\alpha}(b) \approx \frac{4GM(r)}{c^2 b} + \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \quad (17)$$

It is evident that $\hat{\alpha}_{\text{extra}}(b) \approx \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \propto \frac{\ln b}{b}$, which is consistent with the logarithmic term appearing after the projection of the aforementioned "dark matter halo". When quantum gravitational effects (dark matter halo) are not considered ($k(r) = 0$), the gravitational lensing formula naturally reduces to the general relativity form: $\hat{\alpha}(b) \approx \frac{4GM}{c^2 b}$.

5.4. Role of the Logarithmic Term at the Galactic Scale

In the peripheral regions of galaxies, the positive contribution of the logarithmic term $\ln r$ enables the quantum gravitational term to provide stable additional gravity, which is equivalent to the gravitational effect of the traditional dark matter halo but without the need to hypothesize unknown particles:

- Physical nature: The statistical average effect of non-local entanglement of quantum vortices at the galactic scale, transmitted as macroscopic gravity enhancement through AdS/CFT duality.
- Advantage: All parameters are correlatable with observations (e.g., $M_{baryon,topo}$ corresponds to stellar luminosity and gas distribution), avoiding the theoretical flaw of dark matter being "undetectable".
- Final summary: All standard halo models can be interpreted as effective parameterizations of this logarithmic term ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), and their apparent diversity is essentially a reflection of different regularizations of the same asymptotic behavior

6. Cross-Scale Consistency and Theoretical Advantages

6.1. Consistency of Dual-Scale Mechanisms

Although the effects of the logarithmic term at the black hole and galactic scales seem opposite, they originate from the same physical nature:

- Scale correlation: Both the repulsive potential at the black hole scale and the additional gravity at the galactic (black hole gravitational field) scale are macroscopic manifestations of the topological structure and non-local entanglement of quantum vortices, with only changes in the sign and magnitude of $\ln r$ caused by distance r .
- Parameter unification: Core parameters such as the k -factor and G_h have consistent definitions across dual scales; only dynamic adjustments of $M(r)$ and $k(r)$ are made to adapt to scale differences, with no additional hypotheses.

6.2. Comparative Advantages over Traditional Theories

Comparison Dimension	This Theory (Quantum Gravitational Correction with Logarithmic Term)	Traditional Theories (Kerr Black Hole + Dark Matter)
Singularity problem	Physically resolved, satisfying information conservation	Unresolved, with curvature divergence
Free parameters	None (black holes) / 4 physical parameters (galaxies)	Black holes require fitting of spin and inclination; galaxies rely on dark matter distribution hypotheses
Cross-scale unification	Covers microscopic to macroscopic scales under a single framework	Black hole and galactic dynamics are fragmented
Observational verification	Multiple verifications including black hole shadows, high-speed stars, galaxy rotation curves, and mathematical asymptotic behavior of dark matter halos	Dark matter particles not directly detected; black hole spin lacks independent verification
Physical picture	Clear image of quantum vortices + AdS/CFT duality	Dark matter nature unknown; Kerr black hole lacks microscopic physical support

7. Conclusions and Outlook

Through a unified non-perturbative quantum gravity framework, this paper reveals the cross-scale universality of the quantum gravitational correction term with a logarithmic term—its extremely simple mathematical form not only prevents collapse into singularities through repulsive potential at the black hole core and may solve the information paradox, but also maintains the high-

speed stellar velocity and the flattening of rotation curves through additional gravity in the black hole gravitational field and galactic periphery. It provides a possible scheme to solve two major cross-scale problems (desingularization + flat rotation curves) without assuming extra dimensions or dark matter particles. Starting from analyzing the mathematical asymptotic behavior of dark matter halo dynamics, this framework conducts preliminary cross-scale multiple verifications of quantum gravity with logarithmic terms through black hole shadows (EHT observations), high-speed stars, multi-galaxy rotation curves (astronomical measurements), and the mathematical asymptotics of gravitational lensing. It realizes the unified description of gravity from microscale to macroscale for the first time, and provides an observable and repeatable empirical framework for quantum gravity theory, which is different from the current mainstream paths (such as string theory, loop quantum gravity, etc.). Moreover, the rigid predictions of the angular diameters of six black hole shadows given in this paper make this theory the only theoretical framework that can provide specific and falsifiable (unable to adjust spin α and inclination i) predictions for the next-generation EHT observations.

Future research can focus on: 1) Starting from Lagrangian density and quantum field theory, combined with observations and experiments, strictly deriving the logarithmically modified gravitational potential from first principles is the core task of future work (which may require joint efforts of the academic community); 2) Attempting to apply the theoretical framework to the a priori prediction of multi-messenger astronomical research such as black hole thermodynamics, radio bursts, and astroparticle physics, and using next-generation high-precision telescopes such as EHT, JWST, CTA, Fermi-LAT, and IceCube to test the universal boundary of its application; 3) Studying the impact of this correction term on the Friedmann equation and exploring possible inspirations for cosmological problems such as dark energy dynamics and Hubble tension; 4) Directly verifying the non-local entanglement (quantum entanglement) effect corresponding to the logarithmic term through laboratory simulations (such as superfluid helium quantum vortex systems), providing a more solid micro-experimental foundation for the theory.

This study indicates that the gravitational behavior of the universe, from black holes to galaxies, may be governed by the same quantum gravitational mechanism, with the logarithmic term serving as the core carrier of this mechanism. It also strongly suggests that black holes and galaxies may share a common topological origin, which we interpret as follows: the overall dynamics of galaxy disks may be the holographic manifestation of the quantum topological structure of their central black holes on the macrocosmic scale through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). This idea resonates with multiple cutting-edge physical concepts such as quantum fluid cosmology, fractal cosmology, and recursive structures. With its simplicity and powerful cross-scale adaptability, this model may pave a brand-new path for the unified description of gravity in astrophysics.

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