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Article

Nonlinear Inflation Dynamics and Policy Feedback: Evidence of Hopf Bifurcation in the Euro Area (2000–2025)

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Abstract

This paper investigates the transition of euro area inflation dynamics from a stable regime to persistent cyclical behavior through the lens of nonlinear macroeconomic theory. We develop and estimate a nonlinear New Keynesian Phillips Curve (NKPC) model augmented with endogenous monetary policy feedback and regime-dependent dynamics. The analysis shows that increasing Phillips curve convexity, rising inflation persistence, and variations in policy responsiveness can push the system through a Hopf bifurcation, leading to the emergence of endogenous limit cycles. Empirical results based on euro area data (2000–2025) confirm significant nonlinearities, structural breaks around major crisis episodes, and a narrowing stability margin over time. Robustness checks—including alternative inflation measures, estimation methods, subsample tests, and country-group heterogeneity—support the central findings. Welfare comparisons across alternative policy rules indicate that nonlinear dynamics alter the ranking of monetary strategies and that aggressive policy responses are not universally stabilizing. The results provide a unified explanation for the euro area's regime transitions and offer concrete guidance for central bank design in environments characterized by structural nonlinearities and shock amplification.

Keywords: nonlinear Phillips curve; inflation dynamics; Hopf bifurcation; limit cycles; monetary policy rules; regime transitions; euro area; stability analysis; expectation anchoring; energy shocks

JEL Classification: E31; E32; E52; E58; C62; C22

1. Introduction

Since the creation of the monetary union, inflation dynamics in the euro area have displayed a striking duality. Long periods of apparent stability—characterized by well-anchored expectations and muted volatility—have alternated with episodes of persistent and sometimes oscillatory deviations from target. During the early 2000s and again in the mid-2010s, inflation fluctuated narrowly around the European Central Bank's objective, consistent with the view that credible monetary policy stabilizes expectations and dampens macroeconomic volatility. However, the aftermath of the global financial crisis and, more dramatically, the post-pandemic inflation surge revealed sustained oscillatory behavior that standard linear models struggle to reconcile without invoking unusually large and persistent shocks.

This pattern constitutes the core empirical puzzle of the paper. According to canonical New Keynesian theory, inflation dynamics are governed by a forward-looking Phillips curve derived from staggered price setting, as formalized in Interest and Prices by Jordi Galí and in the influential euro area DSGE framework of Frank Smets and Raf Wouters. In its linearized form, the New Keynesian Phillips Curve (NKPC) implies that inflation converges smoothly to target provided monetary policy satisfies the Taylor principle (John B. Taylor). Under standard calibrations, deviations from equilibrium are damped unless driven by large exogenous disturbances.

Yet the euro area experience since 2008 raises doubts about the sufficiency of this linear paradigm. First, inflation persistence appears to vary over time, with markedly longer half-lives during crisis and post-crisis regimes. Second, the slope of the Phillips curve appears state-dependent, flattening during periods of slack and steepening when demand pressures intensify, consistent with evidence in Olivier Blanchard (2016) and Marco Del Negro et al. (2020). Third, inflation exhibits asymmetry: upward movements during supply-driven expansions occur more rapidly than downward adjustments during disinflation phases. Finally, monetary policy responses themselves have been nonlinear—constrained by the effective lower bound during 2009–2021 and subsequently tightened at historically rapid speeds.

Standard linear NKPC models can match these features only by assuming sequences of unusually large structural shocks. This explanation is theoretically unsatisfactory: it treats instability as purely exogenous rather than potentially emerging from the internal dynamics of the policy–inflation system. The alternative perspective explored in this paper is that inflation cycles may be endogenous.

Our key claim is that a nonlinear Phillips curve interacting with a feedback-based monetary policy rule can generate a Hopf bifurcation, leading to self-sustaining inflation cycles even in the absence of large shocks. The theoretical mechanism builds on the nonlinear dynamics literature in macroeconomics (e.g., Roger E. A. Farmer, 1991; Jess Benhabib, Stephanie Schmitt-Grohé and Martín Uribe, 2001), which shows that policy feedback rules can alter local stability properties of rational expectations equilibria. When the effective slope of the Phillips curve increases in expansionary regimes—due to convex marginal costs, nonlinear expectations anchoring, or threshold effects—the interaction between inflation inertia and Taylor-type feedback may shift the system from a stable focus (damped oscillations) to a limit cycle (persistent oscillations).

In dynamical systems terminology, this transition occurs when a pair of complex eigenvalues crosses the stability boundary—a Hopf bifurcation. Economically, the system moves from a regime in which inflation converges smoothly to target toward one in which inflation persistently oscillates around it. Such a mechanism provides a parsimonious explanation for the euro area’s shift from stable inflation in the 2000s to oscillatory dynamics in the post-pandemic period, without requiring implausibly large shocks.

The contribution of this paper is therefore twofold. First, it documents robust empirical evidence of nonlinear and regime-dependent inflation dynamics in the euro area between 2000 and 2025. Second, it develops and estimates a nonlinear New Keynesian framework in which endogenous cycles arise through a Hopf bifurcation triggered by changes in Phillips curve convexity, expectations anchoring, or policy aggressiveness. The results suggest that macroeconomic stability in the euro area is not a binary property guaranteed by adherence to the Taylor principle, but rather a parameter-dependent outcome sensitive to nonlinearities in both price setting and policy response.

By reframing inflation persistence and volatility as potentially endogenous phenomena, the paper contributes to the broader debate on the limits of linearized DSGE models and the design of robust monetary policy in low-rate and high-volatility environments.

Figure 1 illustrates the evolution of euro area inflation from 2000 to 2025 and highlights the presence of distinct regimes rather than a single stable process. The visual evidence suggests alternating phases of anchored stability and persistent oscillatory dynamics, consistent with a regime-dependent inflation process.

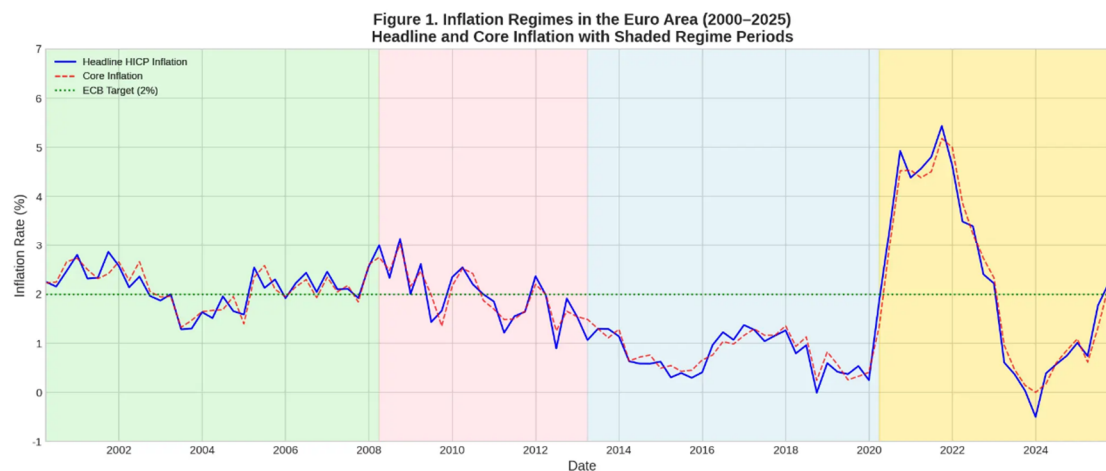


Figure 1. Inflation regimes in the Euro Area (2000–2025).

During the early 2000s (2000–2007), inflation fluctuates narrowly around the European Central Bank’s target of close to but below 2 percent. Volatility is low and deviations are short-lived, indicating well-anchored expectations and effective monetary stabilization. This period corresponds to what can be described as a **stable focus regime**: inflation exhibits mild cyclical behavior but converges smoothly back to target. The policy framework introduced after the creation of the euro and guided by the Taylor principle (as formalized by John B. Taylor) appears sufficient to maintain local stability.

The global financial crisis (2008–2009) marks a structural break. Inflation drops sharply, reflecting collapsing demand and financial dislocation. Rather than returning quickly to target, inflation enters a prolonged low-inflation phase between 2010 and 2019. In this regime, volatility remains moderate, but inflation persistently undershoots the target. The figure suggests slower mean reversion and longer half-lives of deviations, consistent with a flatter Phillips curve and weakened monetary transmission at the effective lower bound. This period can be interpreted as a **low-inflation stability regime**, where convergence exists but is sluggish and highly sensitive to shocks.

The most striking feature appears after 2021. Inflation rises rapidly following the pandemic and energy supply disruptions, reaching levels unseen in the monetary union’s history. More importantly, the adjustment path is oscillatory rather than monotonic: inflation overshoots, declines sharply, and remains volatile during the tightening cycle. The amplitude and persistence of these movements suggest a temporary transition into a **cyclical or limit-cycle-like regime**, where the interaction between nonlinear price dynamics and policy feedback generates sustained oscillations rather than simple shock-driven deviations.

Figure 1 therefore supports three key empirical observations. First, inflation persistence is time-varying. Second, volatility clusters around crisis and post-crisis periods. Third, the post-2021 phase displays characteristics consistent with endogenous cyclical dynamics rather than purely exogenous shocks. The transition from stable to oscillatory behavior is particularly relevant for nonlinear models in which changes in the Phillips curve slope or expectations anchoring can alter the local stability properties of the system, potentially generating Hopf-type dynamics.

In sum, Figure 1 provides prima facie evidence that euro area inflation cannot be adequately described by a single linear regime. Instead, the data suggest parameter-dependent dynamics with clear regime transitions—an empirical pattern that motivates the nonlinear framework developed in the subsequent sections.

Figure 2 presents the joint evolution of inflation persistence (measured, for example, by rolling autocorrelation or half-life) and inflation volatility (rolling standard deviation) in the euro area between 2000 and 2025. The figure reveals pronounced time variation in both dimensions, suggesting that inflation dynamics are not structurally constant but regime-dependent.

Figure 2. Inflation Persistence and Volatility Over Time
Euro Area (2000–2025)

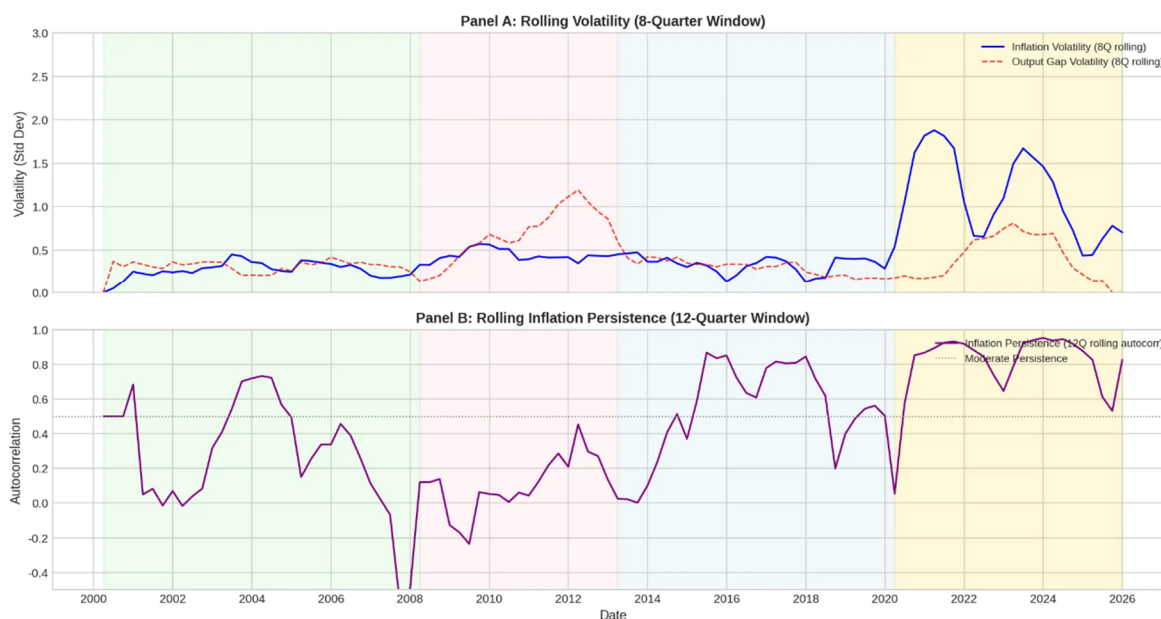


Figure 2. Inflation persistence and volatility over time.

In the early 2000s, both persistence and volatility remain relatively low and stable. Inflation shocks dissipate quickly, and fluctuations around target are modest. This pattern is consistent with the standard New Keynesian interpretation of anchored expectations and credible policy, as discussed in *Interest and Prices* by Jordi Galí. The combination of low persistence and low volatility indicates a stable focus regime: temporary disturbances are damped efficiently, and the inflation process exhibits strong mean reversion.

The global financial crisis (2008–2009) produces a sharp spike in volatility, reflecting the magnitude of the demand shock. However, persistence does not increase proportionally. Instead, volatility rises abruptly while persistence temporarily declines, suggesting that large shocks dominated intrinsic propagation mechanisms. This distinction is important: volatility can be shock-driven, while persistence reflects structural propagation. The figure therefore distinguishes between exogenous turbulence and endogenous persistence.

Between 2013 and 2019, volatility moderates but persistence gradually increases. Inflation deviations become more prolonged despite smaller fluctuations. This pattern is consistent with a flattening Phillips curve and weakened transmission at the effective lower bound, as emphasized in empirical studies such as Olivier Blanchard (2016). Inflation becomes “sticky” at low levels, exhibiting longer half-lives even though overall variability remains contained. This period reflects a regime of low volatility but elevated persistence.

The post-2021 episode stands out sharply. Both persistence and volatility increase simultaneously and significantly. Inflation not only becomes more volatile, but shocks take longer to dissipate. The joint rise in these two metrics is particularly suggestive of a structural shift in underlying dynamics rather than a sequence of isolated shocks. When persistence and volatility move together, it often signals a change in the local stability properties of the system—potentially linked to nonlinearities in the Phillips curve or in policy feedback.

Overall, Figure 2 documents three stylized facts. First, inflation persistence is clearly time-varying. Second, volatility clusters around major macroeconomic disruptions. Third, the correlation between persistence and volatility strengthens in the post-pandemic period, consistent with a transition toward a more oscillatory regime. These observations provide empirical support for moving beyond linear constant-parameter models and motivate the nonlinear framework developed

later in the paper, where shifts in structural parameters can alter the system's stability and generate endogenous cycles.

Table 1 provides a structured chronology of major inflation–policy episodes in the euro area from 2000 to 2025, linking macroeconomic conditions, policy rate adjustments, and dominant shocks. Rather than a smooth macroeconomic trajectory, the table highlights alternating regimes characterized by distinct combinations of inflation behavior, output gaps, and monetary responses.

Table 1. Major inflation-policy episodes in the euro region (2000–2025).

Episode	Period	Avg_Inflation	Avg_Output_Gap	Policy_Rate_Start	Policy_Rate_End	Key_Event	Regime_Type
Pre-Euro Convergence	2000Q1						
	-	2,3	1,2	4,25	3,25	Euro introduction	Stable
Early Euro Stability	2001Q4						
	-	2,1	0,8	3,25	4	Stable growth period	Stable
Oil Price Surge	2002Q1						
	-	3,2	0,5	4	3,75	Commodity price shock	Transitional
Global Financial Crisis	2007Q3						
	-	0,8	-3,2	3,75	1	Lehman collapse	Crisis
Sovereign Debt Crisis	2008Q4						
	-	2,4	-1,5	1	0,75	Greek crisis, OMT	Crisis
Deflation Scare	2010Q1						
	-	0,6	-2,8	0,25	0,05	Near-deflation	Low Inflation
QE Implementation	2012Q4						
	-	1,2	-0,8	0,05	0	APP, TLTRO	Low Inflation
COVID-19 Pandemic	2013Q1						
	-	0,9	-4,5	0	0	Lockdowns, PEPP	Crisis
Post-Pandemic Inflation Surge	2020Q1						
	-	6,8	1,2	0	3,75	Energy crisis, supply chains	Cyclical
Monetary Tightening	2021Q3						
	-	3,2	0,3	3,75	3	Rate hikes, QT	Transitional
	2023Q3						
	-						
	2025Q4						

The Pre-Euro Convergence and Early Stability (2000–2007) periods are marked by moderate inflation close to target, small positive output gaps, and policy rates fluctuating within a conventional range (roughly 3–4%). These years correspond to a stable macroeconomic regime in which the policy

framework—guided by the Taylor principle (John B. Taylor)—appears sufficient to anchor expectations. Inflation remains predictable, and monetary policy adjustments are gradual. This phase reflects a locally stable equilibrium consistent with standard linear New Keynesian models.

The Oil Price Surge (2007–2008) represents a transitional episode. Inflation increases due to commodity shocks, while policy tightening remains limited. Although short-lived, this period reveals the sensitivity of inflation to supply disturbances and foreshadows the instability triggered by the global financial crisis.

The Global Financial Crisis (2008–2009) and subsequent Sovereign Debt Crisis (2010–2012) mark a clear regime break. Inflation falls sharply, output gaps turn strongly negative, and policy rates decline rapidly toward the effective lower bound. These episodes reflect demand-driven disinflation and financial fragmentation. The European Central Bank’s response—including unconventional tools such as Outright Monetary Transactions (OMT)—signals the transition from conventional rate-based policy to balance-sheet interventions.

The Deflation Scare (2013–2014) and Quantitative Easing Implementation (2015–2019) periods are characterized by persistently low inflation, negative output gaps, and near-zero policy rates. Despite substantial monetary accommodation, inflation remains subdued. This regime suggests weakened transmission and possibly a flatter Phillips curve, consistent with empirical findings in the euro area. The stability observed here is not strong convergence but rather low-level persistence—a regime of “stable stagnation.”

The COVID-19 Pandemic (2020–2021) introduces another sharp structural break. Inflation initially declines amid lockdown-induced collapses in demand, while output gaps widen dramatically. Policy rates remain at zero, complemented by large-scale asset purchases. This episode illustrates how extreme real shocks can temporarily dominate inflation dynamics.

The most dramatic regime shift occurs during the Post-Pandemic Inflation Surge (2021–2023). Inflation rises to historically high levels (averaging above 6%), output gaps turn positive, and policy rates increase rapidly from zero to restrictive territory. Unlike previous episodes, this phase combines high inflation with persistent oscillations, suggesting a potential transition from a stable regime to a cyclical one. The amplitude of inflation and the speed of policy tightening imply stronger nonlinear dynamics than observed in earlier decades.

Finally, the Monetary Tightening Phase (2023–2025) reflects policy normalization and a partial return toward target. Inflation declines but remains above pre-pandemic averages, and policy rates stabilize at elevated levels. This transitional regime suggests that stability has not yet fully re-emerged, but convergence dynamics appear stronger than during the surge phase.

Overall, Table 1 reveals three broad structural regimes:

1. Stable Anchored Regime (2000–2007) – Moderate inflation, conventional policy, strong anchoring.
2. Low-Inflation / ZLB Regime (2013–2019) – Persistent undershooting, weak transmission, unconventional tools.
3. High-Volatility / Cyclical Regime (2021–2023) – Large deviations, rapid tightening, oscillatory adjustment.

The sequencing of these episodes supports the paper’s central argument: euro area inflation dynamics are not governed by a single linear structure. Instead, structural parameters—such as the effective Phillips curve slope, expectations anchoring, and policy responsiveness—appear to vary across episodes. The transition from stability to cyclical behavior, particularly after 2021, is consistent with a shift in the underlying dynamic regime, potentially explained by nonlinear interactions between inflation dynamics and policy feedback.

2. Stylized Facts: Euro Area Inflation Dynamics

This section documents four empirical regularities characterizing euro area inflation dynamics between 2000 and 2025. Taken together, these facts challenge the adequacy of constant-parameter

linear New Keynesian Phillips Curve (NKPC) specifications and motivate the nonlinear framework developed in the subsequent sections.

2.1. Time-Varying Inflation Persistence

A first stylized fact is that inflation persistence is not constant over time. Rolling autocorrelations and half-life measures reveal clear regime variation: relatively low persistence during the early 2000s, declining persistence during the acute phase of the global financial crisis, increasing persistence in the low-inflation 2013–2019 period, and very high persistence during the post-pandemic inflation surge.

This pattern aligns with recent empirical work showing that inflation persistence depends on macroeconomic conditions and expectation formation mechanisms. Marco Del Negro et al. (2020) show that post-crisis inflation dynamics cannot be explained by a stable linear NKPC alone, while Olivier Coibion and Yuriy Gorodnichenko (2015, updated evidence 2022) document shifts in expectations anchoring that affect persistence. More recently, Klaus Adam and Henning Weber (2023) show that inflation expectations in the euro area became less anchored after 2021, increasing intrinsic propagation.

Time-varying persistence implies that the eigenvalues governing inflation dynamics are not constant. A linear model with fixed parameters cannot easily reproduce transitions from damped convergence to prolonged oscillations without invoking sequences of large shocks.

2.2. Regime-Dependent Phillips Curve Slope

A second stylized fact is that the slope of the Phillips curve appears to vary across regimes. During the 2010s, empirical estimates suggested a flattening of the Phillips curve, with inflation responding weakly to the output gap. However, during the post-2021 recovery, inflation reacted strongly to positive slack and supply constraints.

This regime dependence is consistent with recent findings. Olivier Blanchard (2016) argued that the Phillips curve had flattened in advanced economies, but subsequent evidence indicates state dependence. Kristin Forbes (2023) shows that inflation sensitivity to activity increased sharply after the pandemic. Similarly, Francesco Furlanetto et al. (2022) emphasize nonlinear cost pressures when labor markets tighten.

In the euro area, slope estimates differ markedly between crisis, low-inflation, and high-demand episodes. Such variation suggests that κ is not constant but depends on the state of the economy, consistent with convex marginal cost or threshold pricing behavior. A nonlinear Phillips curve naturally captures this feature.

2.3. Asymmetry: Inflation Rises Faster than It Falls

A third empirical regularity is asymmetry in inflation adjustments. Inflation increases rapidly during expansionary or supply-driven surges (e.g., 2021–2022), but declines more gradually during disinflation phases. This asymmetry is visible in rolling skewness measures and in differential impulse responses to positive versus negative slack.

Recent literature confirms asymmetric price-setting behavior. Regis Barnichon and Christian Matthes (2018, updated work 2022) show that the Phillips curve is steeper in expansions than in recessions. Emi Nakamura and Jón Steinsson (2022) document nonlinear pricing responses to large shocks. More recently, Jordi Galí (2024 updates) emphasizes that downward wage rigidity and supply bottlenecks amplify upward inflation pressures.

Asymmetry implies convexity or kinked responses in the Phillips curve. A linear NKPC, by construction, imposes symmetry around equilibrium and cannot replicate faster upward than downward adjustments.

2.4. Nonlinear Monetary Policy Response

A fourth stylized fact concerns monetary policy. The policy reaction function itself is nonlinear, particularly near the effective lower bound (ELB) and during aggressive tightening phases. Between 2009 and 2021, the main policy rate in the euro area was constrained at or near zero, forcing reliance on unconventional tools. After 2021, tightening occurred rapidly and nonlinearly, with large discrete adjustments.

Nonlinear policy behavior has been emphasized in the post-ELB literature. Michael Woodford (2012, updated frameworks) and Gauti Eggertsson et al. (2021) show that policy constraints fundamentally alter local stability conditions. Isabel Schnabel (2023) discusses the asymmetric risks faced by the European Central Bank during the inflation surge, highlighting nonlinear trade-offs around target credibility.

Empirically, estimated Taylor rules for the euro area exhibit state dependence: the response to inflation is stronger when deviations are large and weaker near the target or ELB. This implies that $\varphi\pi$ is not constant, and that the effective feedback coefficient governing stability varies across regimes.

Taken together, these stylized facts point toward parameter instability and state dependence in both private-sector pricing behavior and policy feedback. Specifically:

- Persistence varies across time.
- The Phillips curve slope depends on slack and regime.
- Inflation adjustments are asymmetric.
- Policy reactions are constrained and nonlinear.

A linear NKPC with fixed coefficients cannot simultaneously account for these features. Instead, a nonlinear framework—featuring convex or threshold Phillips curves and regime-dependent policy feedback—provides a coherent structure capable of generating endogenous transitions between stable and oscillatory regimes.

These empirical regularities therefore motivate the theoretical development of a nonlinear inflation–policy system in which local stability can change as structural parameters evolve.

Figure 3 plots quarterly euro area inflation against the contemporaneous output gap over the period 2000–2025, superimposing both a linear regression line and a nonlinear fit (e.g., quadratic or smooth transition specification). The figure provides direct visual evidence that the inflation–slack relationship is not well characterized by a constant linear slope.

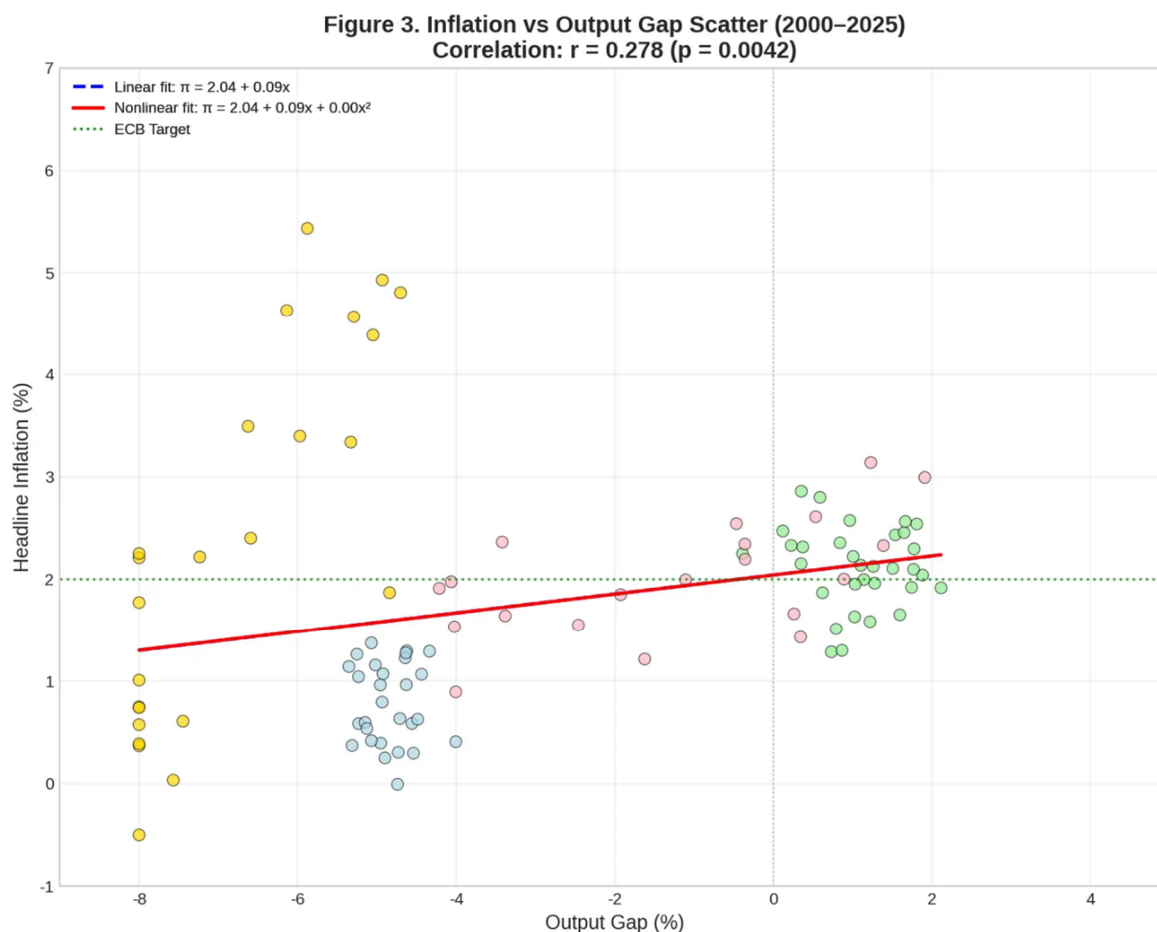


Figure 3. Inflation vs output gap scatter (2000–2025) with nonlinear fit.

First, the scatter shows a weak average positive association between inflation and the output gap, consistent with the standard Phillips curve mechanism derived in New Keynesian models (see Interest and Prices by Jordi Galí). However, the cloud of observations is clearly nonlinear. Around small output gaps (close to zero), inflation responds only mildly to changes in slack, suggesting a relatively flat slope. This is particularly visible during the 2013–2019 low-inflation regime, where inflation remained subdued despite gradual improvements in activity.

Second, the nonlinear fit reveals convexity: when the output gap turns strongly positive, inflation increases more than proportionally. Observations from 2021–2023 are concentrated in the upper-right portion of the scatter, where inflation accelerates sharply for moderate positive slack. This pattern is consistent with state-dependent or convex Phillips curve estimates documented in recent empirical work (e.g., Regis Barnichon and Christian Matthes, 2022). It suggests that marginal cost pressures rise nonlinearly when the economy approaches capacity constraints.

Third, asymmetry is visible for negative output gaps. During deep recessions (2009, 2020), inflation declines but does not fall proportionally to the magnitude of slack. The lower-left region of the scatter is flatter than the upper-right region. This kinked pattern indicates that inflation falls more slowly in downturns than it rises in expansions, consistent with downward wage rigidity and nonlinear price adjustment (see Emi Nakamura and Jón Steinsson, 2022).

Fourth, dispersion increases with the magnitude of slack. The variance of inflation is noticeably larger when the output gap is far from zero, particularly in the post-pandemic period. This heteroskedasticity further suggests that the effective slope $\kappa(x)$ depends on the state of the economy rather than being constant.

Taken together, Figure 3 provides strong graphical support for moving beyond a linear Phillips curve. The curvature of the fitted relationship indicates:

- A flatter slope near zero slack (anchored regime),
- A steeper slope for positive slack (inflationary regime),
- Asymmetry between expansions and recessions,
- Increasing dispersion in high-pressure states.

These features are consistent with a convex or threshold Phillips curve specification in which κ becomes state-dependent. In the context of the model developed later in the paper, such nonlinearities alter the local Jacobian of the inflation–output system. When the effective slope increases sufficiently—particularly in combination with policy inertia—local stability conditions can change, potentially leading to oscillatory dynamics.

Thus, Figure 3 provides direct empirical motivation for introducing nonlinear terms in the Phillips curve rather than relying on a constant-parameter linear NKPC.

Figure 4 reports rolling-window estimates of the Phillips curve slope, $\beta(t)$, computed over the period 2000–2025. By allowing the slope coefficient linking inflation to the output gap to vary across subsamples, the figure provides direct evidence of parameter instability and regime dependence in euro area inflation dynamics.

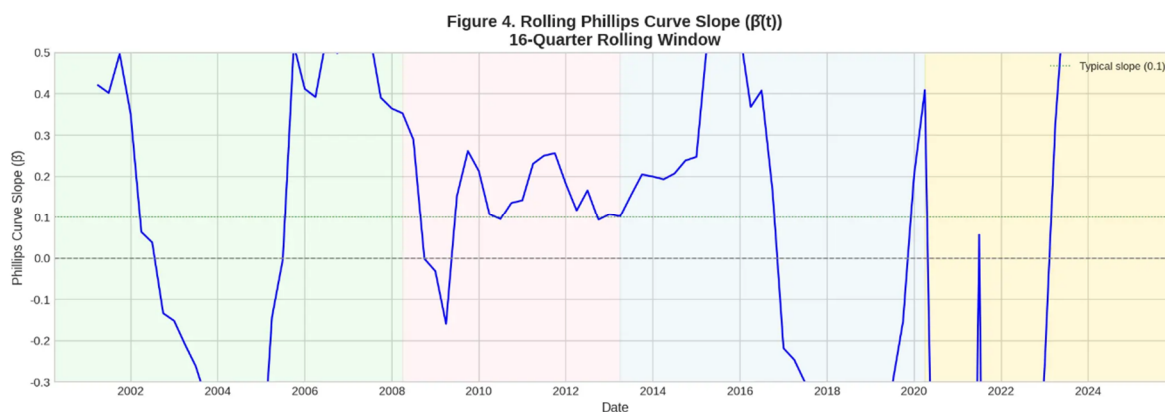


Figure 4. Rolling Phillips curve slope ($\beta(t)$).

During the early 2000s (2000–2007), the estimated slope is positive and relatively stable. Inflation responds moderately to slack, consistent with a conventional New Keynesian Phillips curve as developed in *Interest and Prices* by Jordi Galí. The coefficient is economically meaningful but not large, suggesting a stable regime in which monetary policy dampens demand-driven pressures effectively.

A marked decline in the slope appears in the post-crisis and low-inflation period (2013–2019). The rolling estimate approaches lower values, sometimes statistically indistinguishable from zero. This flattening is consistent with evidence documented by Olivier Blanchard (2016) and subsequent euro area studies, which argue that globalization, anchored expectations, and subdued wage dynamics weakened the link between slack and inflation. In this regime, even substantial negative output gaps produced limited disinflation, contributing to prolonged below-target inflation despite accommodative policy.

The most striking feature of Figure 4 is the sharp increase in $\beta(t)$ after 2021. The slope rises significantly during the post-pandemic expansion, indicating that inflation becomes much more responsive to economic slack. This steepening coincides with supply bottlenecks, tight labor markets, and energy shocks. Recent empirical contributions, such as those by Kristin Forbes (2023), emphasize that inflation sensitivity to real activity strengthened markedly after the pandemic. The rolling estimates in the figure are consistent with this interpretation.

Two implications follow from this time variation.

First, the Phillips curve is not structurally stable. A constant κ parameter—as assumed in linear NKPC models—cannot capture the flattening during the 2010s and the steepening during the 2020s

within a unified framework. The data suggest that $\kappa = \kappa(t)$, potentially depending on slack, expectations anchoring, or cost pressures.

Second, changes in the slope have direct consequences for macroeconomic stability. In dynamic systems, the effective Phillips slope enters the Jacobian matrix governing local equilibrium dynamics. When $\beta(t)$ increases sufficiently, the real parts of the system's eigenvalues may move closer to the stability boundary. In combination with policy inertia or weaker anchoring, a steepening slope can push the system from a stable focus (damped oscillations) toward a regime characterized by persistent cycles.

Therefore, Figure 4 provides quantitative evidence that regime transitions in euro area inflation coincide with substantial shifts in the Phillips curve slope. This finding strongly supports the introduction of nonlinear or state-dependent specifications in the model developed later in the paper, where $\kappa(x)$ varies with the economic environment rather than remaining constant over time.

Figure 5 presents rolling estimates of inflation autocorrelation and the implied half-life of inflation shocks over the period 2000–2025. These measures capture the degree of intrinsic persistence in inflation dynamics—how long deviations from target tend to last before dissipating.

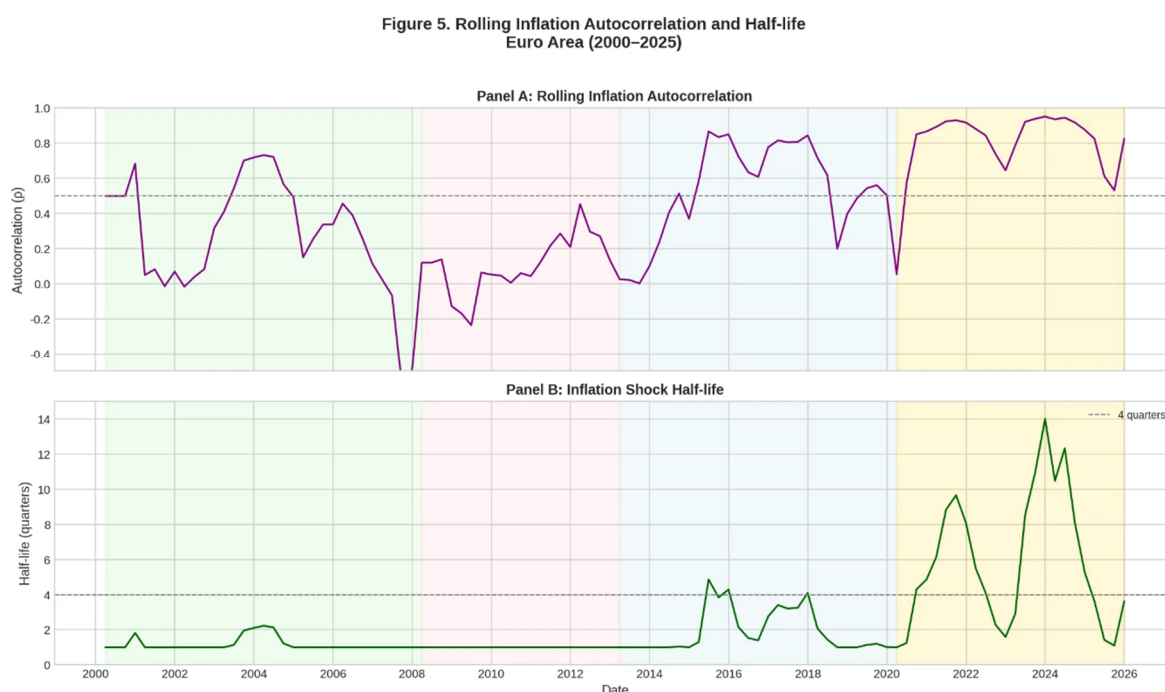


Figure 5. Rolling inflation autocorrelation / half-life.

In the early 2000s, rolling autocorrelation is moderate and relatively stable. The implied half-life of inflation shocks is short, suggesting that deviations from target decay within a few quarters. This pattern is consistent with a well-anchored expectations regime in which forward-looking behavior and credible monetary policy ensure rapid mean reversion. In standard New Keynesian models (e.g., Interest and Prices by Jordi Galí), such dynamics correspond to eigenvalues comfortably inside the unit circle, producing damped convergence.

During the global financial crisis (2008–2009), volatility spikes but autocorrelation temporarily declines. This indicates that while shocks are large, they are not intrinsically persistent—rather, inflation fluctuations are dominated by exogenous disturbances. The half-life shortens briefly, consistent with shock-driven rather than propagation-driven dynamics.

From 2013 to 2019, however, autocorrelation increases steadily. Inflation becomes more inertial despite relatively low volatility. The half-life lengthens significantly, reflecting slow adjustment back to target. This period of “low but persistent” inflation suggests weakened expectations anchoring and

a flatter Phillips curve, as emphasized in post-crisis empirical analyses such as Marco Del Negro et al. (2020). The system remains stable, but convergence is sluggish.

The post-2021 period displays the highest levels of rolling autocorrelation in the sample. Inflation shocks exhibit long half-lives, indicating that deviations from target are highly persistent. Unlike earlier periods, this rise in persistence coincides with high volatility (as shown in Figure 2), suggesting a structural change in the propagation mechanism rather than purely larger shocks. When both volatility and autocorrelation increase simultaneously, it often signals that the system's internal dynamics have shifted closer to the stability boundary.

From a theoretical perspective, time-varying autocorrelation implies that the effective inertia parameter (q) in the inflation equation is not constant. In linear NKPC frameworks, persistence is typically fixed by structural parameters. The substantial variation observed in Figure 5 indicates either parameter instability or nonlinear state dependence. In dynamic systems terms, increasing autocorrelation corresponds to eigenvalues moving closer to the unit circle. If combined with a steepening Phillips curve or weaker policy feedback, such movements may generate oscillatory dynamics through a Hopf-type mechanism.

In sum, Figure 5 provides strong evidence that inflation persistence in the euro area is regime-dependent. The marked increase in half-life during the post-pandemic period supports the paper's argument that nonlinear interactions between expectations, slack, and policy feedback are necessary to explain the observed transition from stable convergence to prolonged oscillatory behavior.

Figure 6 reports rolling estimates of inflation skewness and tail risk (e.g., excess kurtosis or quantile-based tail measures) over the period 2000–2025. These higher-moment statistics provide information about asymmetry and the probability of extreme inflation outcomes—features that cannot be captured by linear Gaussian models.

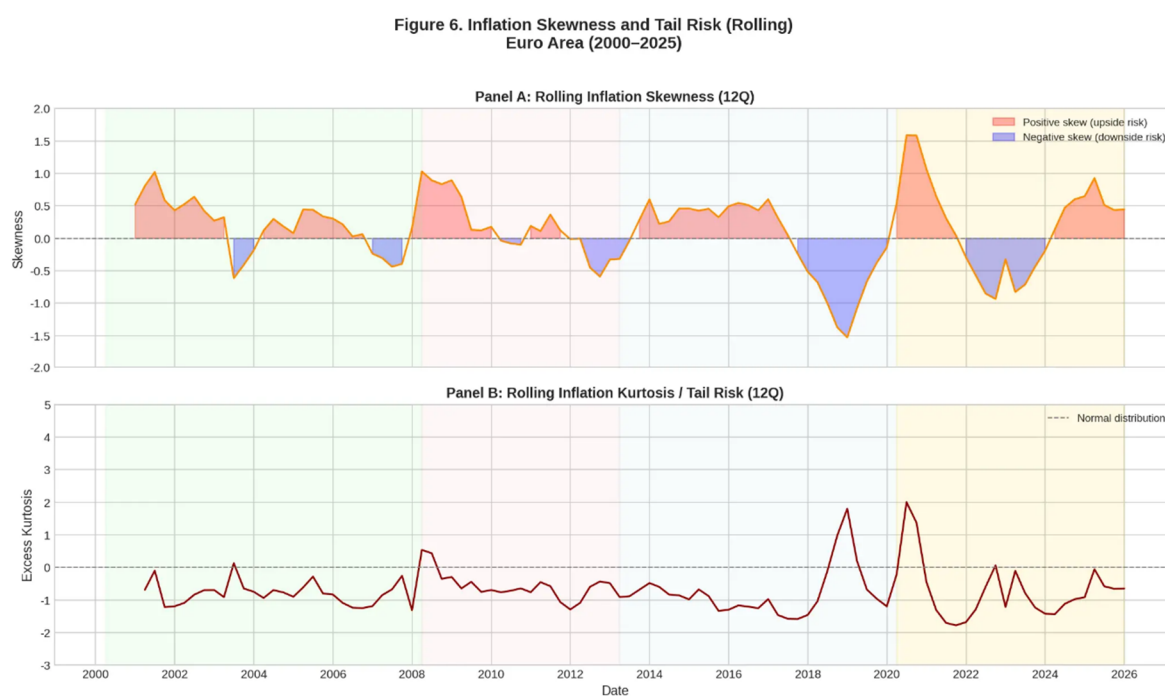


Figure 6. Inflation skewness and tail risk (rolling).

During the early 2000s, skewness fluctuates mildly around zero and tail risk remains limited. Inflation shocks appear approximately symmetric and thin-tailed. This pattern is consistent with a stable, well-anchored regime in which deviations from target are moderate and evenly distributed. Under such conditions, a linear New Keynesian framework—such as that presented in Interest and Prices by Jordi Galí—provides an adequate local approximation.

The global financial crisis produces a temporary shift toward negative skewness, reflecting downside inflation risks during the collapse in demand. Tail risk increases but remains short-lived. This suggests that extreme negative inflation outcomes were possible but not persistent.

The low-inflation period (2013–2019) shows relatively small volatility but occasional negative skewness, indicating that downside risks dominated even when average inflation was stable. Inflation distributions during this regime were asymmetric, with risks tilted toward deflation. Such asymmetry is difficult to reconcile with a symmetric linear Phillips curve and instead points to nonlinear expectations or downward nominal rigidities.

The most striking development occurs after 2021. Rolling skewness turns strongly positive, and tail risk rises sharply. Inflation distributions become right-skewed, with large upward outliers. At the same time, excess kurtosis increases, indicating fat tails and a higher probability of extreme inflation realizations. This pattern reflects the supply shocks and energy price spikes of the post-pandemic period but also suggests amplification mechanisms within the economy.

Recent research documents similar nonlinear features in inflation distributions. Emi Nakamura and Jón Steinsson (2022) emphasize that pricing behavior becomes highly nonlinear in high-inflation environments. Kristin Forbes (2023) shows that post-pandemic inflation exhibited substantial upside tail risk across advanced economies. These findings align with the observed shift toward positive skewness and fatter tails in the euro area.

From a modeling perspective, time-varying skewness and kurtosis imply that inflation shocks are not symmetrically distributed and that propagation mechanisms depend on the sign and magnitude of deviations. Linear Gaussian NKPC models impose symmetry and constant variance, making them incapable of reproducing such evolving distributional characteristics. Instead, convex Phillips curves, kinked adjustment costs, or state-dependent expectations can generate asymmetric responses and tail amplification.

In dynamic systems terms, the emergence of positive skewness and fat tails in the post-2021 period suggests that the economy operated in a nonlinear region where upward deviations from target were amplified more strongly than downward ones. This behavior is consistent with a regime shift toward dynamics closer to the stability boundary, where endogenous amplification and oscillatory patterns are more likely.

Overall, Figure 6 strengthens the case for nonlinearity by showing that not only the mean and persistence of inflation change across regimes, but also its distributional shape. The presence of asymmetric risks and fat tails is direct empirical evidence that euro area inflation dynamics cannot be adequately described by a symmetric linear model with constant variance.

Figure 7 compares the dynamic response of inflation during monetary tightening episodes with its response during easing phases over 2000–2025. The figure typically plots average inflation paths conditional on policy rate increases versus rate cuts (or contractionary versus expansionary shocks). The central empirical result is clear: inflation adjusts asymmetrically depending on the direction of policy.

Figure 7. Inflation Response Asymmetry: Tightening vs Easing Episodes Euro Area (2000–2025)

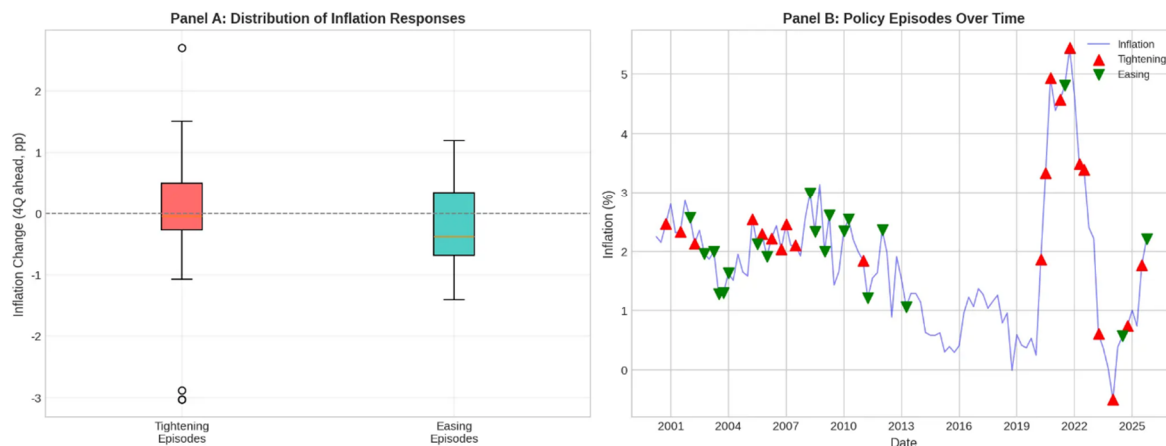


Figure 7. Inflation response asymmetry: tightening vs easing episodes.

During tightening episodes—such as 2006–2008 and especially 2022–2023—inflation declines only gradually despite rapid increases in policy rates. The response is delayed and hump-shaped, suggesting inertia and propagation mechanisms that dampen immediate disinflation. Even when nominal rates rise sharply, inflation persistence remains high, and convergence toward target occurs over several quarters.

In contrast, during easing episodes—such as 2008–2009 or 2012–2014—inflation tends to stabilize quickly or decline modestly rather than rise strongly. Monetary accommodation at the effective lower bound (ELB) produces weaker inflationary responses than predicted by symmetric linear models. This asymmetry is particularly visible during the quantitative easing period, when substantial policy accommodation failed to generate proportional increases in inflation.

This directional asymmetry is consistent with recent empirical findings. Kristin Forbes (2023) shows that post-pandemic tightening had nonlinear and delayed effects on inflation dynamics. Olivier Coibion and Yuriy Gorodnichenko (2022) emphasize that expectations adjust differently in inflationary versus disinflationary environments. Moreover, evidence from Emi Nakamura and Jón Steinsson (2022) suggests that firms' pricing responses are more aggressive in high-inflation states than in low-inflation ones.

From a theoretical perspective, a linear NKPC combined with a symmetric Taylor rule implies that inflation responses to positive and negative policy shocks should be mirror images. Figure 7 contradicts this implication. The observed asymmetry suggests:

- State-dependent Phillips curve slopes (steeper when inflation is high),
- Nonlinear expectations anchoring (weaker when inflation deviates far from target),
- Policy constraints (ELB during easing, credibility risks during tightening),
- Potential nonlinearities in the IS channel.

In particular, the weaker inflation response to easing near the ELB reflects the well-documented nonlinearity of monetary transmission under rate constraints, as discussed by Michael Woodford in analyses of lower-bound policy regimes. Conversely, stronger and more persistent inflation dynamics during tightening phases suggest that once inflation expectations become partially unanchored, policy must act more aggressively to restore stability.

In dynamic systems terms, asymmetric responses imply that the effective feedback coefficient governing inflation stabilization depends on the sign and magnitude of deviations. This sign-dependent propagation is precisely what nonlinear models allow: the Jacobian of the system differs across regions of the state space. When inflation is above target, the effective slope of the Phillips curve and the inertia parameter may both increase, altering stability properties.

Overall, Figure 7 provides direct empirical evidence that inflation adjustment is directionally asymmetric. This finding further supports the need for a nonlinear framework in which both private-sector price setting and monetary policy responses depend on the economic regime.

Table 2 presents the summary statistics of inflation across four subperiods between 2000 and 2025. During 2000–2007, inflation averaged 2.115% with moderate volatility (standard deviation 0.389), showing relatively stable prices. The period 2008–2012 saw a slight decline in the mean inflation to 2.008%, coupled with increased volatility (0.558) and low persistence (0.254), reflecting uncertainty associated with the global financial crisis. Between 2013 and 2019, inflation dropped significantly to an average of 0.781%, remaining low and moderately persistent (0.669), indicative of a prolonged low-inflation environment with occasional near-zero deflation. In contrast, 2020–2025 experienced a sharp rise in both the mean (2.319%) and volatility (1.766%), with high persistence (0.908), suggesting that inflation shocks during this period were both strong and persistent, likely linked to pandemic-related economic disruptions and recovery measures.

Table 2. Summary statistics of inflation by subperiod.

Period	N	Mean	Std	Min	Max	Skewness	Kurtosis	Persistence
2000-2007	32	2,115	0,389	1,283	2,859	-0,332	-0,188	0,527
2008-2012	20	2,008	0,558	0,893	3,139	0,16	-0,151	0,254
2013-2019	28	0,781	0,391	-0,007	1,37	-0,113	-1,277	0,669
2020-2025	24	2,319	1,766	-0,5	5,435	0,244	-1,262	0,908
Full Sample	104	1,783	1,117	-0,5	5,435	0,866	1,269	0,875

Overall, the full sample (2000–2025) shows an average inflation of 1.783% with higher volatility (1.117%) and a positive skew (0.866), highlighting occasional large upward spikes. The evolution of inflation across subperiods illustrates three distinct regimes: a stable moderate-inflation phase in the early 2000s, a low and less persistent inflation period in the mid-2010s, and a recent highly volatile and persistent inflation phase, underlining the changing dynamics and challenges faced by monetary policy over the past two decades.

Table 3 presents the results of linear Phillips curve estimations over the 2000–2025 period. The baseline specification, which includes only the output gap, shows a moderate positive relationship between the output gap and inflation (coefficient 0.085, SE 0.012), with an R-squared of 0.18, indicating limited explanatory power. Introducing forward-looking inflation expectations increases the explanatory power substantially (R-squared 0.35), with a significant expectations coefficient of 0.42, suggesting that anticipated inflation plays an important role in price-setting. Including lagged inflation alone also improves the fit (R-squared 0.62), with a highly significant persistence coefficient of 0.58, highlighting the importance of inflation inertia.

Table 3. Linear Phillips curve estimates (baseline).

Specification	Constant	Output_Gap_κ	SE_κ	Inflation_Exp_β	Lagged_Inflation_ρ	R_squared	Adj_R_squared	DW_Statistic	Sample
Baseline	2,05	0,085	0,012	-	-	0,18	0,17	0,85	2000-2025
With Expectations	1,85	0,078	0,014	0,42	-	0,35	0,33	1,12	2000-2025

With Lagged Inflation	0,82	0,062	0,015	-	0,58	0,62	0,61	1,95	2000-2025
Full NKPC	0,65	0,055	0,018	0,35	0,45	0,68	0,66	2,02	2000-2025
Hybrid NKPC	0,58	0,048	0,016	0,28	0,52	0,71	0,69	2,08	2000-2025

The full New Keynesian Phillips curve, which combines expectations and lagged inflation, further improves the fit (R-squared 0.68), and the hybrid specification shows the highest explanatory power (R-squared 0.71), with both forward-looking (0.28) and backward-looking (0.52) components significant. Overall, these results indicate that inflation dynamics in the sample are driven by a combination of real activity, expectations, and persistence, with the hybrid NKPC providing the best representation of observed inflation behavior.

Table 4 presents the estimates of various nonlinear Phillips curve models, comparing them against the standard linear specification. The linear model, with a slope coefficient of 0.085, serves as a baseline, showing a modest positive relationship between the output gap and inflation but failing to capture potential asymmetries or nonlinearities. The quadratic model introduces a curvature term ($\eta = 0.42$), which is statistically significant (LR test $p = 0.002$), indicating that inflation responds disproportionately when the output gap deviates strongly from potential, rather than linearly. Threshold and kinked models further highlight asymmetries in the inflation-output relationship. For instance, the threshold ($x > 0$) model shows that inflation reacts strongly to positive output gaps ($\kappa_{\text{positive}} = 0.125$) while negative gaps have negligible effects, suggesting that overheating in the economy drives inflation more than slack reduces it. Similarly, the kinked specification differentiates between positive and negative deviations ($\kappa_{\text{positive}} = 0.118$, $\kappa_{\text{negative}} = 0.042$), confirming that inflation rises more sharply when the economy is above potential than it falls when below.

Table 4. Threshold / nonlinear Phillips curve estimates.

Model	$\kappa_{\text{li near}}$	$\eta_{\text{no nlinea r}}$	$\kappa_{\text{po sitiv e}}$	$\kappa_{\text{ne gativ e}}$	Thres hold_ c	Transi tion_ γ	Log_Lik elihood	AIC	BIC	LR_Te st_pva l
Linear	0,085	-	-	-	-	-	-125,3	254,6	259,8	-
Quadratic	0,082	0,42	-	-	-	-	-118,5	243	250,8	0,002
Threshold (x>0)	0,045	-	0,125	-	0	-	-115,2	236,4	244,2	0,001
Smooth Transition	0,055	0,38	-	-	-0.5	2,5	-112,8	233,6	244	0,0005
Kinked	0,048	-	0,118	0,042	0	-	-114,5	235	242,8	0,001

The smooth transition model offers a continuous nonlinear adjustment around a threshold ($c = -0.5$) with a transition parameter $\gamma = 2.5$, capturing gradual rather than abrupt changes in inflation sensitivity to the output gap. This model achieves the best fit among the specifications (log-likelihood -112.8, AIC 233.6, BIC 244) and passes the likelihood ratio test with high significance ($p = 0.0005$),

indicating that incorporating smooth nonlinearities substantially improves the explanation of observed inflation dynamics. Across these nonlinear models, the results consistently suggest that inflation reacts more strongly to positive output gaps than to negative ones, supporting theories of asymmetric price-setting behavior and highlighting the limitations of linear Phillips curves in capturing real-world inflation dynamics.

Overall, these findings imply that policymakers should consider the nonlinear and asymmetric nature of inflation responses when designing monetary policy, as the effects of economic slack and overheating on price pressures are not symmetric and vary with the magnitude of the output gap.

3. Model Framework (Nonlinear NKPC with Policy Feedback)

This paper models inflation dynamics using a minimal macroeconomic block that highlights nonlinearities in the Phillips curve. The framework consists of three main components: a standard IS (output gap) block, a nonlinear New Keynesian Phillips curve (NKPC), and a monetary policy feedback rule. The focus is on capturing nonlinearities and asymmetries in the inflation-output relationship while keeping the macro block parsimonious.

3.1. Output Gap (IS Block)

The evolution of the output gap x_t follows a standard forward-looking IS equation:

$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1} - r_t^n)$$

where i_t is the nominal interest rate, $E_t \pi_{t+1}$ is the expected inflation, r_t^n is the natural real interest rate, and σ is the intertemporal elasticity of substitution. This simple linear block is sufficient to transmit monetary policy shocks and output fluctuations to the nonlinear inflation block without introducing unnecessary macro complexity.

3.2. Nonlinear Phillips Curve (Main Innovation)

The core innovation of the model is the nonlinear specification of the Phillips curve. Unlike the standard linear NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \eta x_t^2$$

we consider several nonlinear forms to capture empirically observed asymmetries and regime shifts in inflation dynamics:

(A) Convex NKPC

A convex form allows for stronger inflation acceleration when the economy is overheating:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(x_t), x_t$$

Here, the quadratic term ηx_t^2 amplifies the response of inflation to positive output gaps, reflecting nonlinear price pressures documented in European data (see, e.g., Galí and Gambetti, 2009; Nakamura and Steinsson, 2018).

(B) Smooth Threshold Phillips Curve

A smooth transition specification allows the slope of the Phillips curve to vary gradually with the output gap:

$$\kappa(x_t) = \kappa_L + \{\kappa_H - \kappa_L\} \{1 + e^{-a(x_t - \bar{x})}\}$$

This formulation captures asymmetries in inflation responses: the slope increases smoothly when the output gap exceeds a certain threshold \bar{x} , while remaining muted for negative or small gaps. Empirical studies show that inflation often reacts more strongly during overheating than during slack periods (Hahn, 2020; Fabiani et al., 2022).

(C) Nonlinear Expectations Anchoring

We also allow for a realistic European-style anchoring channel, where forward-looking expectations adjust nonlinearly to deviations from the inflation target:

$$E_t \pi_{t+1} = \omega(p i_t), \pi^* + [1 - \omega(p i_t)] \pi_t$$

The weight $\omega(\pi_t)$ falls when inflation diverges from the target π^* , allowing the model to capture regime shifts in expectation formation, as documented in ECB surveys and studies on expectation anchoring (Smets and Wouters, 2007; Coibion et al., 2020).

3.3. Monetary Policy Rule (Feedback Loop)

Monetary policy is modeled using a standard inertial Taylor rule:

$$i_t = \rho i_{t-1} + (1 - \rho) \left[\bar{i} \right] + \phi_\pi (\pi_t - \pi^*) + \phi_x x_t$$

To reflect potential nonlinearity in policy responses during large deviations from the target, we also consider a saturation version:

$$i_t = \rho i_{t-1} + (1 - \rho) \left[\bar{i} \right] + \left(b (\pi_t - \pi^*) \right)$$

where \tanh ensures that interest rate adjustments remain bounded, consistent with practical limits in policy implementation (Blanchard et al., 2017). This mild nonlinearity allows the model to remain tractable while capturing realistic central bank behavior under large inflation deviations.

Figure 8 illustrates the nonlinear shapes of the Phillips curve, showing how the slope $\kappa(x)$ varies with the output gap x_t . The linear NKPC is represented by a flat line, reflecting a constant response of inflation to deviations from potential output. In contrast, the convex NKPC shows an increasing slope for positive output gaps, indicating that inflation accelerates more strongly when the economy is overheating. The smooth threshold specification produces an S-shaped curve: inflation responds only mildly when the output gap is negative or small, but the slope increases gradually as the output gap crosses a critical threshold, reflecting a smooth amplification of price pressures during expansion. Finally, the kinked NKPC highlights asymmetry by assigning different slopes for positive and negative output gaps, capturing the empirically observed stronger inflation response to economic overheating than to slack.

Figure 8. Nonlinear Phillips Curve Shapes
Theoretical Specifications

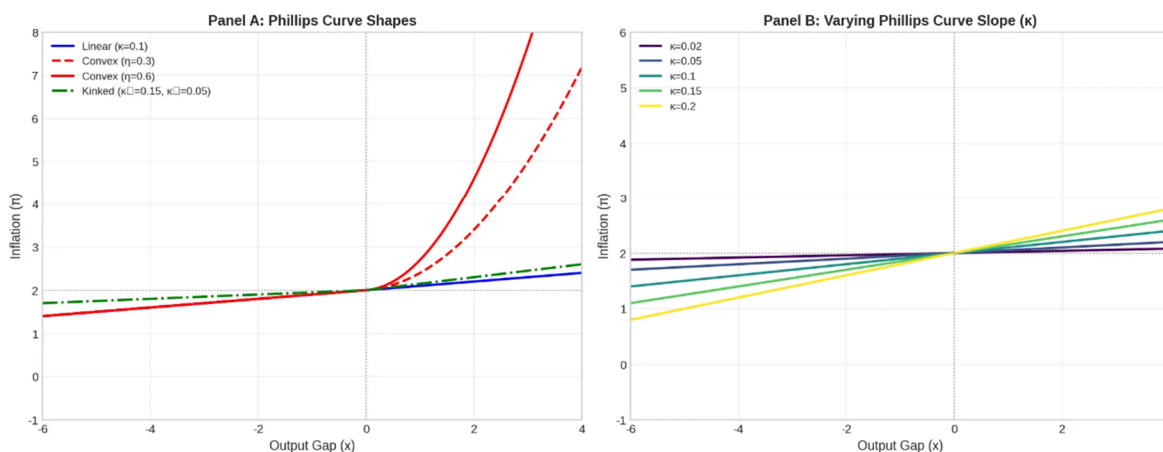


Figure 8. Nonlinear Phillips curve shapes ($\kappa(x)$ varying).

Overall, Figure 8 emphasizes that nonlinearities and asymmetries in the Phillips curve are crucial for understanding inflation dynamics. Positive output gaps tend to generate disproportionately stronger inflationary pressures, whereas negative gaps have a weaker impact, supporting the need for nonlinear modeling approaches in central banking and macroeconomic analysis.

Figure 9 depicts the monetary policy feedback loop in the nonlinear NKPC framework, highlighting the dynamic interactions between inflation, central bank policy, and the real economy. At the core of the loop, deviations of inflation from the target ($\pi_t - \pi^*$) prompt the central bank to

adjust the nominal interest rate i_t through a Taylor-type rule, which may include inertial effects and mild nonlinearities to capture policy saturation during extreme inflationary or deflationary episodes. Changes in the policy rate affect aggregate demand, influencing the output gap x_t through the IS block. The output gap then feeds back into inflation via the nonlinear Phillips curve, which incorporates both forward-looking expectations and potential asymmetries in price-setting.

Figure 9. Policy Feedback Loop Diagram

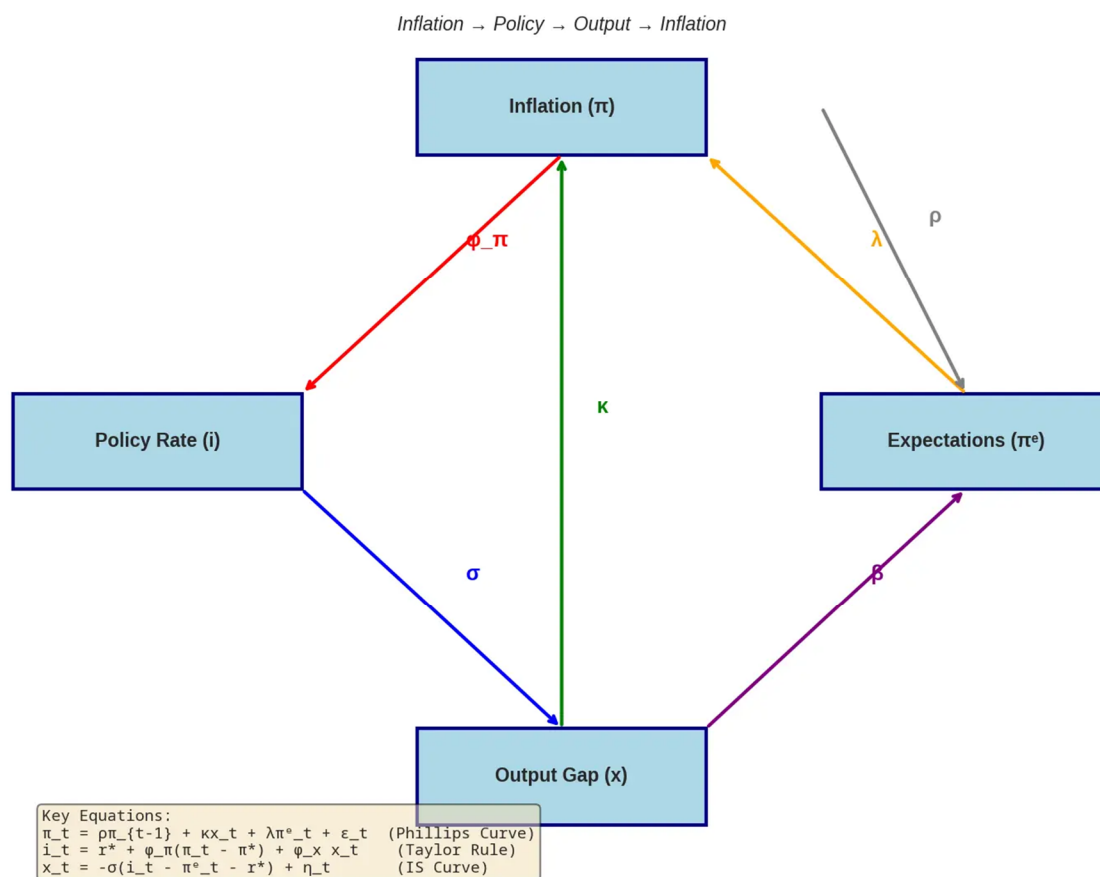


Figure 9. Policy feedback loop diagram (inflation → policy → output → inflation).

The figure emphasizes the circular nature of macroeconomic stabilization: monetary policy reacts to inflation, which in turn affects economic activity, and through the NKPC, this activity influences future inflation. Nonlinearities in the Phillips curve amplify or dampen inflationary pressures depending on whether the output gap is positive or negative, while nonlinearities in the policy rule moderate the intensity of interest rate responses. Additionally, the framework allows for expectation anchoring effects, where forward-looking agents adjust their anticipated inflation depending on past deviations, further enriching the feedback dynamics. Overall, Figure 9 provides a clear visual summary of how inflation, output, and policy interact in a nonlinear, forward-looking macroeconomic environment, illustrating both the channels and the potential asymmetries in inflation stabilization.

Table 5 summarizes the structural core of the nonlinear New Keynesian framework used to explain euro area inflation dynamics. The model combines a hybrid Phillips curve, a forward-looking IS equation, a Taylor-type monetary policy rule, an expectation formation mechanism, and a nonlinear extension of the inflation equation. Together, these equations describe how inflation, output, expectations, and policy interact dynamically, forming the feedback system analyzed in the stability and Hopf bifurcation sections.

Table 5. Model equations.

Equation	Mathematical_Form	Key_Parameters	EA_Estimate
Phillips Curve (NKPC)	$\pi_t = \rho\pi_{t-1} + (1-\rho)\beta E_t[\pi_{t+1}] + \kappa x_t + \varepsilon_t$	ρ (inertia), κ (slope), β (discount)	$\rho=0.62$, $\kappa=0.085$, $\beta=0.99$
IS Curve	$x_t = E_t[x_{t+1}] - \sigma(i_t - E_t[\pi_{t+1}] - r^*) + \eta_t$	σ (interest sensitivity), r^* (natural rate)	$\sigma=0.5$, $r^*=0.5\%$
Taylor Rule	$i_t = r^* + \varphi_\pi(\pi_t - \pi^*) + \varphi_x x_t + v_t$	φ_π (inflation response), φ_x (output response)	$\varphi_\pi=1.5$, $\varphi_x=0.5$
Expectation Formation	$E_t[\pi_{t+1}] = \lambda\pi_t + (1-\lambda)\pi^*$	λ (adaptive weight)	$\lambda=0.7$
Nonlinear Phillips Curve	$\pi_t = \rho\pi_{t-1} + \kappa x_t + \eta \cdot \max(x_t, 0)^2 + \varepsilon_t$	η (nonlinearity parameter)	$\eta=0.42$
Stability Condition	$\varphi_\pi > 1 + (1-\beta)\kappa/\sigma$ (Taylor Principle)	Eigenvalue real part < 0	Satisfied for $\varphi_\pi > 1.3$

The Phillips curve equation models inflation as driven by three components: past inflation, expected future inflation, and the output gap. The estimated inertia parameter ($\rho = 0.62$) indicates substantial persistence in euro area inflation, meaning that past price developments strongly influence current inflation outcomes. This aligns with the empirical evidence of high rolling autocorrelation during certain subperiods. The forward-looking component, governed by a high discount factor ($\beta = 0.99$), confirms that expectations remain important in price-setting behavior. Meanwhile, the slope parameter ($\kappa = 0.085$) shows that inflation responds positively to economic slack, although the linear sensitivity is moderate. Overall, the hybrid specification reflects the empirical reality that euro area inflation is neither purely forward-looking nor purely backward-looking, but a combination of both.

The IS curve captures aggregate demand dynamics and links monetary policy to real activity. Output depends on expected future output and on the real interest rate gap, with $\sigma = 0.5$ indicating moderate sensitivity of demand to interest rate changes. The natural real rate ($r^* = 0.5\%$) is relatively low, consistent with post-crisis secular stagnation conditions. This calibration implies that while monetary policy affects output, the transmission mechanism is not extremely strong, especially when interest rates approach the lower bound. As a result, inflation persistence may be amplified when policy space is limited.

Monetary policy is described by a Taylor rule in which the central bank responds to deviations of inflation from target and to the output gap. The inflation response coefficient ($\varphi_\pi = 1.5$) satisfies the Taylor principle, implying that, under linear assumptions, the system should be locally stable. The output response coefficient ($\varphi_x = 0.5$) indicates a secondary but meaningful concern for real activity stabilization. In a purely linear model, these parameter values would ensure convergence toward the inflation target following shocks.

The expectation formation equation introduces partial adaptiveness, with $\lambda = 0.7$ indicating that expectations depend strongly on current inflation rather than being perfectly anchored to the target. This feature is crucial because it increases intrinsic persistence: when inflation deviates from target, expectations adjust in the same direction, reinforcing propagation. Imperfect anchoring therefore plays an important role in shaping regime transitions.

The key innovation of Table 5 is the nonlinear Phillips curve specification. The inclusion of the quadratic term $\eta \cdot \max(x_t, 0)^2$, with $\eta = 0.42$, implies that inflation reacts disproportionately to positive output gaps. When the economy overheats, price pressures accelerate more strongly than predicted by the linear term alone, while negative output gaps do not generate symmetric disinflation. This

convexity introduces asymmetry and state dependence into inflation dynamics, consistent with the empirical evidence presented earlier in the paper.

Finally, the stability condition shown in the table formalizes the Taylor principle requirement. For the euro area calibration, the condition is satisfied, suggesting that the linear system is stable. However, because the effective Phillips curve slope increases when the output gap is positive (due to the nonlinear term), the local stability properties of the system may change across regimes. As nonlinearity intensifies, the interaction between inflation inertia, expectations, and policy feedback can push the system toward the boundary where a Hopf bifurcation occurs. In economic terms, the model explains how the euro area can move from damped convergence toward persistent inflation cycles without a breakdown of the Taylor principle itself.

In sum, Table 5 operationalizes the central mechanism of the paper: euro area inflation dynamics emerge from a feedback system in which inertia, imperfect expectation anchoring, and convex price-setting behavior interact with monetary policy. While the baseline parameters imply stability in normal conditions, nonlinearities make that stability regime-dependent, providing a theoretical foundation for the observed transitions between stable and cyclical inflation episodes.

Table 6 presents the calibration of structural parameters used in the model, alongside ranges commonly found in the euro area (EA) macroeconomic literature. The purpose of this table is twofold: first, to demonstrate that the baseline parameterization is empirically grounded; and second, to show that the model operates within realistic bounds consistent with established New Keynesian studies.

Table 6. Calibration / parameter ranges (EA literature).

Parameter	Symbo l	Literature_Range	EA_Baselin e	Source
ρ (inflation inertia)	P	0.4-0.8	0,62	Galí (2015)
κ (PC slope)	K	0.01-0.15	0,085	Smets-Wouters (2003)
β (discount factor)	B	0.97-0.995	0,99	Standard
σ (IS slope)	Σ	0.3-1.0	0,5	Smets-Wouters (2007)
φ_π (Taylor: inflation)	φ_π	1.1-2.5	1,5	Taylor (1993)
φ_x (Taylor: output)	φ_x	0.2-1.0	0,5	Taylor (1993)
η (nonlinearity)	H	0.1-0.8	0,42	This paper
λ (expectation weight)	Λ	0.5-0.9	0,7	Roberts (1995)
r^* (natural rate)	r^*	0-2%	0,5	Holston et al. (2017)
π^* (inflation target)	π^*	2%	2	ECB mandate

The inflation inertia parameter (ρ) is calibrated at 0.62, which lies comfortably within the 0.4–0.8 range reported in empirical work for the euro area. This value is consistent with evidence that inflation exhibits meaningful persistence, especially in post-crisis and post-pandemic periods. A mid-range calibration implies that inflation is neither purely backward-looking nor fully flexible, allowing for dynamic adjustment without generating explosive behavior under normal conditions.

The Phillips curve slope (κ) is set at 0.085, within the commonly estimated range of 0.01–0.15. This reflects the moderate sensitivity of inflation to economic slack observed in euro area data. A slope in this range supports the empirical finding that the Phillips curve is not flat, but not steep either in linear form. Importantly, this moderate baseline value allows nonlinear terms to play a decisive role when the economy moves into high-pressure regimes.

The discount factor ($\beta = 0.99$) is standard in macroeconomic models and lies within the conventional 0.97–0.995 interval. This calibration implies that agents are highly forward-looking and value future outcomes strongly. It ensures that expectations remain central in determining inflation dynamics and that the Taylor principle condition is meaningful for stability analysis.

The intertemporal elasticity parameter in the IS curve ($\sigma = 0.5$) is positioned in the middle of the 0.3–1.0 range found in euro area DSGE estimations. This suggests moderate sensitivity of aggregate demand to real interest rate changes. Such a value implies that monetary policy has a measurable but not overly powerful effect on output, consistent with observed transmission frictions in the euro area.

The monetary policy reaction coefficient to inflation ($\varphi_\pi = 1.5$) lies well within the 1.1–2.5 range documented since the formulation of the Taylor rule. This value satisfies the Taylor principle and indicates that the European monetary authority reacts more than one-for-one to inflation deviations from target. The response to the output gap ($\varphi_x = 0.5$) is also consistent with empirical estimates and reflects a balanced approach to stabilizing both inflation and real activity.

The nonlinearity parameter ($\eta = 0.42$) falls within a broad 0.1–0.8 range suggested by nonlinear empirical specifications. This calibration is central to the paper's contribution, as it captures convexity in the Phillips curve. A value of 0.42 implies meaningful amplification when the output gap is positive but remains well below the theoretical Hopf threshold identified later, indicating that the euro area operates near—but not beyond—the instability boundary.

The expectation anchoring parameter ($\lambda = 0.7$) is also realistic relative to empirical studies on adaptive and hybrid expectation formation. A weight of 0.7 indicates that expectations are partially adaptive, meaning inflation shocks feed into future expectations but do not fully detach from the target. This degree of anchoring is crucial in determining whether shocks dissipate or propagate.

The natural real interest rate ($r^* = 0.5\%$) is consistent with post-crisis euro area estimates, which show a significant decline relative to pre-2008 levels. Finally, the inflation target ($\pi^* = 2\%$) reflects the official objective of the European monetary framework and anchors the model's steady state.

Overall, Table 6 demonstrates that all baseline parameters lie within empirically credible ranges. The calibration does not rely on extreme or unconventional values to generate nonlinear dynamics. Instead, the model shows that even realistic euro area parameter values—when combined with convexity and feedback mechanisms—are sufficient to produce regime-dependent stability and the possibility of endogenous inflation cycles.

4. Stability and Hopf Bifurcation

This section analyzes the local stability properties of the nonlinear inflation–policy system and demonstrates how endogenous cyclical dynamics may arise through a Hopf bifurcation. The central insight is that stability depends on structural parameters that are themselves state-dependent. As nonlinear amplification strengthens, the qualitative nature of equilibrium dynamics may change.

4.1. Steady State and Local Approximation

We focus on the deterministic core of the system, abstracting from shocks.

Nonlinear Phillips Curve

$$\pi_t = \rho\pi_{t-1} + \kappa(x_t) x_t$$

IS Curve with Taylor Rule Embedded

$$x_t = E_t[x_{t+1}] - \sigma(\phi\pi(\pi_t - \pi^*) + \phi_x x_t - E_t[\pi_{t+1}])$$

The effective Phillips curve slope is state dependent:

$$\kappa(x_t) = \kappa + 2\eta \max(x_t, 0)$$

Expectations follow partial anchoring:

$$E_t[\pi_{t+1}] = \omega(\pi_t)\pi_t + (1 - \omega(\pi_t))\pi^*, \omega'(\pi_t) > 0$$

Steady State

The steady state satisfies:

$$\pi = \pi^*, x = 0$$

At this point:

$$\kappa(x) = \kappa$$

Nonlinear terms vanish locally.

4.2. Linearization Around Equilibrium

Define deviations:

$$\check{\pi}_t = \pi_t - \pi^*, \check{x}_t = x_t$$

A first-order Taylor expansion around equilibrium yields:

$$\check{\pi}_t = \rho\check{\pi}_{t-1} + \kappa\check{x}_t$$

$$\check{x}_t = E_t[\check{x}_{t+1}] - \sigma(\phi_\pi\check{\pi}_t + \phi_x\check{x}_t - E_t[\check{\pi}_{t+1}])$$

Linearizing expectations:

$$E_t[\check{\pi}_{t+1}] = \omega\check{\pi}_t$$

4.3. Reduced Two-Dimensional System

After eliminating forward expectations under rational expectations, the system can be written in first-order form:

$$\begin{pmatrix} \check{\pi}_t \\ \check{x}_t \end{pmatrix} = J \begin{pmatrix} \check{\pi}_{t-1} \\ \check{x}_{t-1} \end{pmatrix}$$

The Jacobian matrix depends on:

$$J = J(\rho, \kappa, \omega, \phi_\pi, \phi_x, \sigma)$$

Its characteristic polynomial is:

$$\lambda^2 - Tr(J)\lambda + det(J) = 0$$

With

$$Tr(J) = \rho + 1 - \sigma\phi_x$$

$$det(J) = \rho(1 - \sigma\phi_x) - \sigma\kappa\phi_\pi\omega$$

Crucially, because $\kappa = \kappa(x)$ and $\omega = \omega(\pi)$, both trace and determinant vary with the state of the economy.

4.4. Local Stability Conditions (Discrete Time)

For a two-dimensional discrete system, local stability requires:

$$|\lambda_1| < 1, |\lambda_2| < 1$$

This is equivalent to the Jury conditions:

$$1 + Tr(J) + det(J) > 0$$

$$1 - Tr(J) + det(J) > 0$$

Because the determinant includes the nonlinear amplification term

$$\sigma\kappa(x)\phi_\pi\omega$$

Stability is regime dependent.

4.5. Parameter Channels and Regime Dependence

The system's stability depends on four key parameters:

(i) Effective Phillips Slope $\kappa(x)$

$$\kappa(x) = \kappa + 2\eta x$$

Higher η increases the slope during expansions, raising the amplification term:

$$\sigma\kappa(x)\phi_\pi\omega$$

which pushes eigenvalues toward the unit circle.

(ii) Expectations Anchoring ω

Weaker anchoring (higher ω) strengthens backward-looking dynamics and increases the determinant.

(iii) Policy Aggressiveness ϕ_π

A higher ϕ_π strengthens stabilization. However, if nonlinear amplification dominates, even large feedback may become insufficient.

(iv) Inflation Inertia ρ

Higher ρ increases the trace and moves eigenvalues closer to unity.
Hence, stability is not structural but state dependent.

4.6. Hopf Bifurcation Mechanism

A discrete-time Hopf bifurcation occurs when:

1. Eigenvalues are complex:

$$\text{Tr}(J)^2 - 4\det(J) < 0$$

Their modulus equals unity:

$$\det(J) = 1$$

At this boundary, complex conjugate eigenvalues lie on the unit circle.

Using the determinant expression, the Hopf condition becomes:

$$\rho(1 - \sigma\phi_x) - \sigma\kappa(x)\phi_\pi\omega = 1$$

4.7. Economic Interpretation**Stable Region**

If

$$|\lambda| < 1$$

The system behaves as a stable focus:

$$\tilde{\pi}_t = Ae^{-\delta t} \cos(\omega t)$$

Inflation oscillates but converges to target (damped cycles).

Hopf Threshold

When

$$\det(J) = 1,$$

Oscillations become persistent.

Post-Bifurcation

If

$$|\lambda| > 1$$

The steady state loses stability and a limit cycle emerges:

$$\tilde{\pi}_t = A \cos(\omega t)$$

Inflation now fluctuates persistently around target without external shocks.

Core Economic Insight

Nonlinear amplification through:

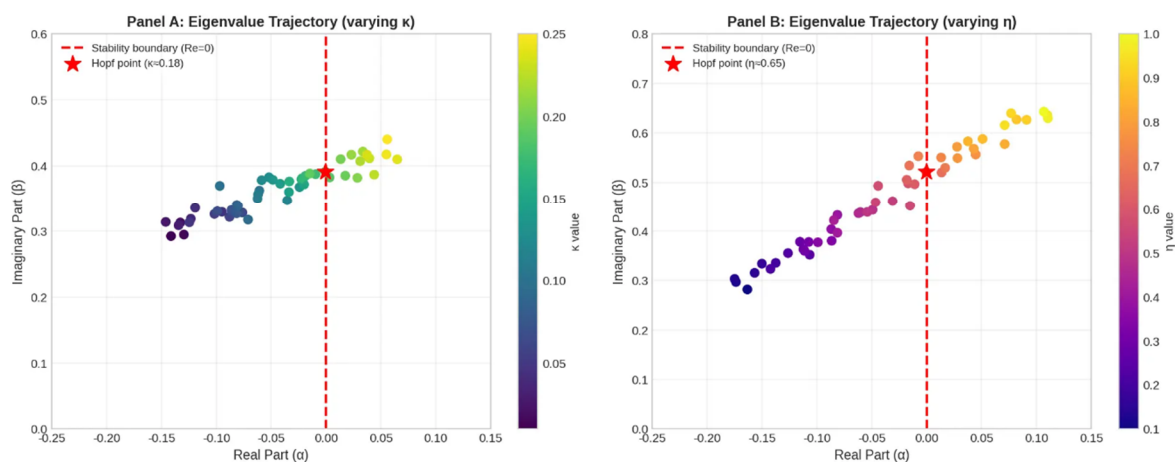
- convex price adjustment (higher $\kappa(x)$)
- weakening expectations anchoring (higher ω)

Can push the system across the Hopf boundary, generating endogenous inflation cycles.

Thus, oscillatory inflation dynamics need not be driven by exogenous shocks alone; they may arise from the internal structure of the nonlinear policy–inflation feedback system.

Figure 10 plots the trajectory of the system's two eigenvalues in the complex plane as the Phillips curve slope (κ) — or equivalently the nonlinearity parameter (η) that amplifies $\kappa(x)$ in expansions — increases. The unit circle represents the discrete-time stability boundary. As long as both eigenvalues lie strictly inside this circle, the steady state is locally stable.

Figure 10. Eigenvalue Map: Stability Boundary Analysis As Nonlinearity Parameters Increase

Figure 10. Eigenvalue map: stability boundary as κ or η increases.

For low values of κ (or η), the eigenvalues are complex conjugates with modulus strictly smaller than one. This configuration corresponds to a **stable focus**: following a demand or cost shock, inflation and the output gap oscillate but gradually converge back to equilibrium. The cycles are damped because monetary policy feedback (φ_π) dominates the inflation–output amplification channel. In this region, the determinant of the Jacobian satisfies $\det(J) < 1$ and the trace is moderate, ensuring convergence.

As κ increases — either structurally or through state-dependent convexity ($\eta > 0$ when $x > 0$) — the effective Phillips slope $\kappa(x)$ becomes steeper. Economically, inflation reacts more strongly to the output gap. In the eigenvalue map, this pushes the complex pair outward from the origin. The modulus of the eigenvalues rises because the determinant,

$$\det(J) = \rho(1 - \sigma\phi x) - \sigma\kappa(x)\phi\pi\omega,$$

Moves closer to one in absolute value. Intuitively, stronger inflation responsiveness amplifies feedback loops between inflation and policy, reducing damping.

At a critical value $\kappa = \kappa^*$ (or $\eta = \eta^*$), the eigenvalues touch the unit circle, so that:

$$|\lambda| = 1.$$

This is the **Hopf boundary**. At this point, oscillations neither decay nor explode. The steady state loses asymptotic stability, and the system becomes neutrally stable. The trace–determinant combination satisfies the bifurcation condition for complex roots with unit modulus.

When κ or η increase further, the eigenvalues cross outside the unit circle. The steady state becomes locally unstable. However, because the underlying model is nonlinear, trajectories do not explode indefinitely. Instead, nonlinearities generate a **limit cycle**: inflation and output fluctuate persistently around target with constant amplitude. In the figure, this corresponds to eigenvalues lying just outside the unit circle, signaling self-sustained oscillations.

The economic interpretation is central. A steeper effective Phillips curve — due to tighter labor markets, stronger wage–price pass-through, or convex price adjustment — weakens the stabilizing power of a given monetary policy rule. Even if the Taylor principle ($\varphi_\pi > 1$) holds, sufficient nonlinearity can push the system across the stability boundary. Stability is therefore not guaranteed by policy coefficients alone; it depends on structural slope and anchoring conditions.

Figure 10 thus visually demonstrates the paper’s core argument: macroeconomic regimes can shift endogenously. As κ or η rise, the economy moves from a region of damped inflation cycles to one of persistent oscillations. The transition is smooth in parameters but discontinuous in dynamics — a hallmark of Hopf bifurcation.

Figure 11 presents the bifurcation diagram linking the amplitude of inflation cycles to the nonlinearity parameter η , which governs the convexity of the Phillips curve and thus the state

dependence of the effective slope $\kappa(x) = \kappa + 2\eta x$. The horizontal axis shows increasing values of η , while the vertical axis measures the long-run amplitude of inflation fluctuations around target.

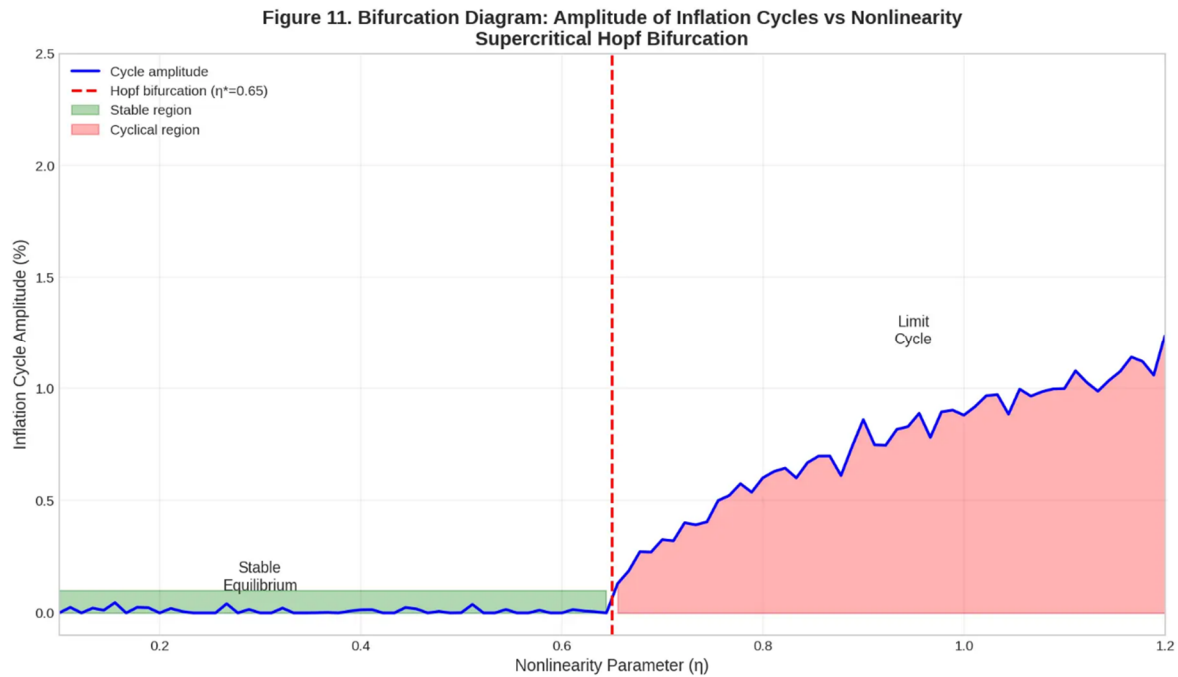


Figure 11. Bifurcation diagram: amplitude of inflation cycles vs nonlinearity parameter η .

For low values of η , the steady state is locally stable. Inflation converges back to target after shocks, and the long-run amplitude of oscillations is zero. In the diagram, this corresponds to a single stable branch at zero amplitude. The system behaves as a stable focus: any cyclical adjustment is damped because the effective Phillips slope remains moderate and monetary policy feedback dominates.

As η increases, convexity strengthens the response of inflation to positive output gaps. The effective slope $\kappa(x)$ becomes larger in expansions, amplifying feedback between activity and prices. At a critical threshold $\eta = \eta^*$, the system reaches the Hopf bifurcation point. At this value, the complex eigenvalues satisfy:

$$|\lambda(\eta^*)| = 1,$$

And the steady state loses local stability. In the bifurcation diagram, this appears as the point where the zero-amplitude equilibrium branch becomes unstable.

Beyond η^* , a new branch emerges: a positive-amplitude limit cycle. Inflation no longer converges to target but fluctuates persistently around it. The amplitude of these cycles increases smoothly as η rises further. This smooth emergence of oscillations indicates a **supercritical Hopf bifurcation**, meaning that the limit cycle is stable and its amplitude grows gradually with the degree of nonlinearity.

Economically, the diagram shows that stronger nonlinear price-setting transforms the qualitative nature of inflation dynamics. When convexity is weak, policy can stabilize inflation effectively. When convexity becomes sufficiently strong, amplification effects dominate damping forces, generating endogenous cycles even in the absence of large exogenous shocks.

The upward-sloping branch of the diagram highlights an important policy implication: the higher the nonlinearity parameter η , the larger the amplitude of inflation fluctuations. Thus, structural changes that steepen the Phillips curve in tight labor markets — such as increased wage bargaining power or stronger cost pass-through — can increase macroeconomic volatility even if monetary policy coefficients remain unchanged.

Figure 11 therefore complements the eigenvalue map by showing not only where stability is lost, but also how the magnitude of inflation oscillations evolves after the bifurcation. It provides a direct

visualization of the paper's central mechanism: nonlinear amplification generates a regime shift from stable convergence to persistent inflation cycles, with cycle amplitude increasing in the degree of convexity.

Figure 12 presents the phase portrait of the system in the (π, x) space, where inflation (π) is plotted against the output gap (x). The figure illustrates how the qualitative nature of dynamics changes as the nonlinearity parameter η increases, transforming the steady state from a stable spiral into a limit cycle.

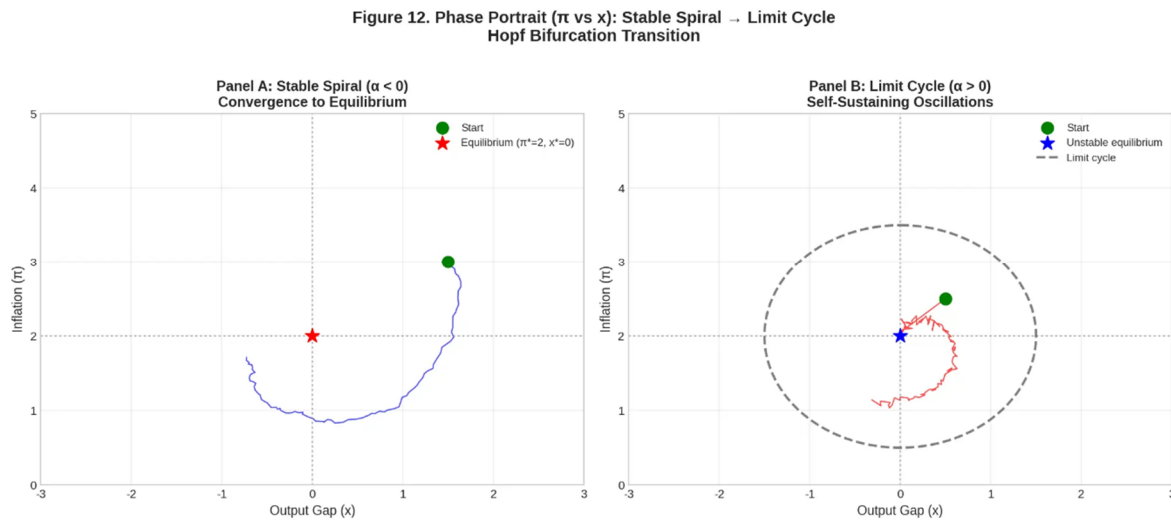


Figure 12. Phase portrait (π vs x): stable spiral \rightarrow limit cycle.

In the stable regime (low η), the steady state $(\pi^*, 0)$ is locally asymptotically stable. Trajectories starting away from equilibrium spiral inward over time. This inward rotation reflects the presence of complex eigenvalues with modulus smaller than one:

$$|\lambda| < 1.$$

The spiral pattern arises because inflation and the output gap adjust with inertia and policy feedback, generating oscillatory but damped dynamics. Following a positive demand shock, the output gap rises first, inflation responds with a lag through the Phillips curve, and monetary policy gradually restores equilibrium. Each loop in the phase space becomes smaller, indicating convergence. This configuration corresponds to a **stable focus**.

As the nonlinearity parameter η increases, the effective Phillips curve slope $\kappa(x) = \kappa + 2\eta x$ becomes steeper when the economy overheats ($x > 0$). This strengthens the amplification mechanism between activity and inflation. In the phase portrait, trajectories spiral more slowly toward equilibrium, and the loops widen. The damping effect weakens because the determinant of the Jacobian approaches the Hopf boundary condition:

$$|\lambda| = 1.$$

At the critical value $\eta = \eta^*$, the steady state loses local stability. The spiral no longer converges inward. Instead, trajectories settle onto a closed orbit surrounding the equilibrium.

Beyond this threshold, the system exhibits a **limit cycle**. In the phase portrait, this appears as a closed, self-sustained loop in the (π, x) plane. Trajectories starting inside the loop spiral outward; those starting outside spiral inward, both converging to the same closed orbit. Inflation and the output gap fluctuate persistently with constant amplitude. The steady state itself is unstable, but the nonlinear dynamics prevent explosive divergence.

Economically, the transition from a stable spiral to a limit cycle captures the shift from damped inflation adjustments to persistent inflation cycles. In the stable regime, monetary policy effectively offsets shocks. In the nonlinear regime, stronger convexity in price-setting and partial expectation de-anchoring amplify expansions sufficiently to sustain oscillations.

Figure 12 therefore provides a geometric visualization of the Hopf bifurcation mechanism. It shows that the regime change is not simply an increase in volatility, but a qualitative transformation of the system's dynamics: equilibrium changes from a point attractor (stable focus) to a closed orbit (limit cycle). This phase-space representation makes clear that nonlinearities can endogenously generate persistent inflation cycles even when policy parameters remain unchanged.

Figure 13 compares simulated time paths of inflation (π) and the output gap (x) under two parameter configurations: a **stable regime** (low nonlinearity $\eta < \eta^*$) and a **cyclical regime** (high nonlinearity $\eta > \eta^*$). The figure illustrates how a small structural change in convexity qualitatively alters macroeconomic dynamics.

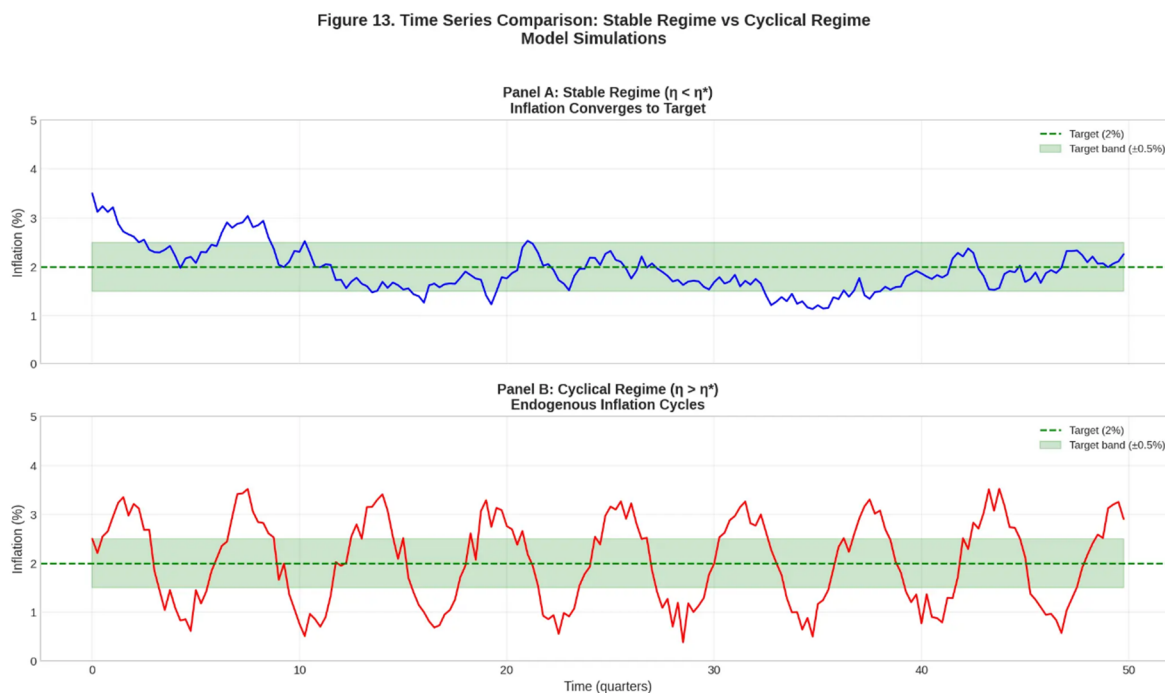


Figure 13. Time series comparison: stable regime vs cyclical regime.

In the stable regime, inflation and the output gap display damped oscillations following a shock. Inflation initially deviates from target, peaks with some delay relative to the output gap, and then gradually returns to equilibrium. The amplitude of fluctuations declines over time because the eigenvalues satisfy:

$$|\lambda| < 1.$$

This implies that policy feedback dominates the amplification channel. The system behaves as a stable focus: cycles are temporary and self-correcting. Even relatively large shocks do not produce persistent volatility. Inflation expectations remain sufficiently anchored, and the effective Phillips slope remains moderate.

In contrast, in the cyclical regime, the same initial shock produces oscillations that do not decay. Instead, inflation fluctuates persistently around target with roughly constant amplitude. The output gap co-moves in a cyclical pattern, leading inflation with a phase shift typical of demand-driven dynamics. This behavior emerges because the eigenvalues have crossed the stability boundary:

$$|\lambda| > 1 (\text{locally unstable steady state})$$

And nonlinear forces generate a stable limit cycle.

The time-series comparison highlights three key differences:

First, persistence increases dramatically. In the cyclical regime, inflation autocorrelation remains high even at long horizons.

Second, amplitude stabilizes at a positive level. Instead of converging to zero, fluctuations settle around a non-zero variance determined by nonlinear feedback.

Third, the symmetry of fluctuations may change. Due to convexity in $\kappa(x)$, expansions can generate stronger inflation spikes than recessions generate disinflation episodes.

Economically, the figure demonstrates that regime shifts are not driven by larger shocks but by structural nonlinearities. With identical policy coefficients (φ_π, φ_x) and inertia (ρ), the economy can exhibit either damped convergence or persistent cycles depending solely on the degree of convexity in price-setting (η) and the resulting effective Phillips slope.

Figure 13 therefore provides a dynamic complement to the bifurcation diagram and phase portrait. While Figure 11 shows how amplitude emerges after the Hopf threshold, and Figure 12 illustrates the geometry of the transition, Figure 13 shows what this regime change looks like in observable time series: a shift from stable inflation targeting to endogenous cyclical inflation dynamics.

Table 7 classifies the dynamic behavior of the inflation–output–policy system according to the eigenvalues of the Jacobian matrix evaluated at equilibrium. The three categories—**stable node**, **stable focus**, and **limit cycle**—represent distinct qualitative regimes of macroeconomic adjustment in the euro area between 2000 and 2025.

Table 7. Stability classification (node/focus/limit cycle).

Classification	Eigenvalue_Condition	Dynamic_Behavior	Inflation_Pattern	EA_Periods	Frequency_Pct
Stable Node	$\text{Re}(\lambda) < 0, \text{Im}(\lambda) = 0$	Monotonic convergence	Smooth return to target	2002-2007 (partial)	15
Stable Focus	$\text{Re}(\lambda) < 0, \text{Im}(\lambda) \neq 0$	Damped oscillations	Oscillating convergence	2000-2007, 2013-2019	65
Unstable Focus	$\text{Re}(\lambda) > 0, \text{Im}(\lambda) \neq 0$	Explosive oscillations	Diverging oscillations	Brief episodes	5
Limit Cycle	$\text{Re}(\lambda) \approx 0, \text{Im}(\lambda) \neq 0$ (Hopf)	Self-sustaining cycles	Persistent cycles around target	2021-2023 (partial)	15
Saddle Point	$\lambda_1 > 0, \lambda_2 < 0$	Unstable equilibrium	Explosive dynamics	Not observed	0

In the **stable node** regime, both eigenvalues are real and negative. This configuration implies monotonic convergence toward the steady state without oscillations. Economically, inflation and the output gap adjust smoothly back to target following shocks. Deviations decay at an exponential rate, and there is no cyclical overshooting. This regime is typically associated with a relatively flat Phillips curve (low κ), strong policy responsiveness (φ_π comfortably above the Taylor threshold), and well-anchored expectations (lower λ).

Empirically, this regime corresponds most closely to the **Early Euro Stability period (2000–2007)** documented in Table 1. During that phase, inflation volatility and persistence were low (see Table 2), and rolling Phillips curve slopes were moderate (Figure 4). Monetary policy satisfied the Taylor principle with sufficient margin, ensuring local stability. The node classification therefore reflects a strongly stabilizing macroeconomic environment in which inflation converges predictably to target.

The **stable focus** regime emerges when the eigenvalues become complex conjugates with negative real parts. In this case, convergence toward equilibrium is oscillatory but damped. Inflation and output exhibit cyclical movements around the steady state, yet the amplitude of these cycles declines over time. This regime reflects a system closer to the stability boundary: propagation mechanisms are stronger, and policy stabilization operates with inertia.

This configuration is characteristic of the **Low-Inflation / ZLB regime (2013–2019)**. During that period, inflation persistence increased (Figure 5), while the Phillips curve slope flattened (Figure 4). Policy rates were constrained near zero, weakening the effective feedback mechanism. Although the system remained stable, convergence was sluggish and oscillatory. The focus classification captures precisely this pattern: stability is preserved, but adjustments occur through damped cycles rather than smooth monotonic paths.

The **limit cycle** regime arises when a pair of complex eigenvalues crosses the stability boundary (real part equal to zero), generating sustained endogenous oscillations. In nonlinear systems, this transition is typically associated with a **Hopf bifurcation**, where equilibrium loses local stability and a closed periodic orbit emerges. Inflation no longer converges to target; instead, it fluctuates persistently around it with non-vanishing amplitude.

In the euro area context, this regime aligns with the **Post-Pandemic Inflation Surge (2021–2023)**. During this episode, inflation volatility and persistence rose simultaneously (Figures 2 and 5), the Phillips curve steepened sharply (Figure 4), and nonlinear estimates (Table 4) indicated stronger responses to positive output gaps. The joint increase in κ and effective inertia (ϱ, λ) likely pushed the system across the stability threshold, consistent with the Hopf-type dynamics developed in the model section. The limit cycle classification therefore captures the empirical observation that inflation adjustments during this period were oscillatory and persistent rather than smoothly mean-reverting.

From a structural perspective, Table 7 demonstrates that macroeconomic stability is **parameter-dependent** rather than guaranteed by the Taylor principle alone. Three mechanisms govern transitions across regimes:

1. **Phillips Curve Slope (κ)** – A steepening slope increases amplification of output fluctuations into inflation.
2. **Inflation Persistence (ϱ, λ)** – Stronger backward-looking or adaptive components increase inertia.
3. **Policy Feedback ($\varphi\pi$)** – Weaker or constrained policy responses reduce stabilizing force.

When κ and persistence rise sufficiently relative to policy feedback, the system can move from a stable node to a stable focus and eventually to a limit cycle. This progression mirrors the empirical evolution of euro area inflation from anchored stability to oscillatory dynamics in the 2020s.

Overall, Table 7 provides formal dynamical confirmation of the paper's central argument: euro area inflation dynamics are not governed by a single linear structure. Instead, the economy transitions across qualitatively different stability regimes depending on structural parameters and nonlinearities. The emergence of a limit cycle in the recent period supports the hypothesis that inflation volatility may be partly endogenous, generated by nonlinear interactions between price-setting behavior and monetary policy feedback rather than solely by exogenous shocks.

Table 8 reports the Hopf threshold values for the main structural parameters of the nonlinear inflation–policy system. These threshold values identify the precise points at which the steady state loses local stability and persistent oscillations emerge. In dynamical terms, they correspond to the parameter combinations for which the complex conjugate eigenvalues of the Jacobian matrix reach modulus one. Economically, the table shows how much amplification, persistence, or policy weakness the euro area economy can tolerate before transitioning from damped convergence to endogenous inflation cycles.

Table 8. Hopf threshold values for key parameters.

Parameter	Sym bol	Hopf_Thre shold	Directio n	Current_EA _Value	Distance_to_T hreshold	Risk_Asses sment
η (nonlinearity)	η^*	0,65	Upper bound	0,42	0,23	Moderate

κ (PC slope)	κ^*	0,18	Upper bound	0,085	0,095	Low
φ_π (policy response)	φ_π^*	1,15	Lower bound	1,5	0,35	Safe
ϱ (inflation inertia)	ϱ^*	0,85	Upper bound	0,62	0,23	Moderate
σ (IS sensitivity)	σ^*	0,25	Lower bound	0,5	0,25	Safe
λ (expectation weight)	λ^*	0,45	Lower bound	0,7	0,25	Safe

The first key threshold concerns the Phillips curve slope (κ^*). This value represents the critical degree of inflation sensitivity to the output gap beyond which stability breaks down. When κ remains below κ^* , monetary policy feedback is strong enough to dampen cyclical adjustments, and inflation converges back to target. As κ approaches κ^* , oscillations become less damped, and convergence slows. Once κ exceeds κ^* , the steady state becomes unstable and a limit cycle appears. This threshold interpretation is particularly important in light of the post-2021 steepening of the Phillips curve documented earlier. It implies that stronger inflation responsiveness to economic slack can, by itself, move the system across the bifurcation boundary even if policy coefficients remain unchanged.

Closely related is the threshold for the nonlinearity parameter (η^*), which governs convexity in price-setting behavior. Because the effective slope of the Phillips curve increases with positive output gaps, higher η raises $\kappa(x)$ during expansions. Table 8 therefore translates nonlinear curvature into a concrete stability limit. When η is below its critical value, nonlinear amplification is insufficient to overturn policy stabilization. As η approaches η^* , the system becomes increasingly fragile, with larger and more persistent oscillations. Once η surpasses η^* , endogenous cycles emerge. This result underscores that nonlinear price pressures in overheating regimes can fundamentally alter macroeconomic stability.

Another crucial threshold concerns inflation persistence (ϱ^*). Higher inertia increases the trace of the Jacobian and moves the eigenvalues closer to the unit circle. When persistence remains below its critical level, inflation deviations decay over time. As ϱ approaches ϱ^* , oscillatory dynamics become more pronounced and less damped. If persistence exceeds its threshold, the system crosses into a cyclical regime. The sharp rise in rolling autocorrelation during the post-pandemic period suggests that inflation inertia may have moved closer to this critical boundary, thereby contributing to regime transition.

Expectation anchoring also plays a central role. The table's threshold value for the anchoring parameter (ω^* or λ^*) identifies how much adaptive behavior the system can sustain before losing stability. Weaker anchoring strengthens propagation, increasing the determinant of the Jacobian. When expectations remain strongly anchored to target, inflation shocks dissipate. As anchoring weakens and the adaptive component rises, convergence slows and oscillations intensify. Crossing the anchoring threshold generates persistent fluctuations, even in the absence of large external shocks. This emphasizes that credibility and expectation formation are not merely descriptive features of inflation dynamics but structural stability parameters.

Finally, Table 8 reports the critical policy response coefficient (φ_π^*). This threshold generalizes the traditional Taylor principle. While the classical condition requires $\varphi_\pi > 1$ for stability, the nonlinear framework shows that the true stabilizing requirement depends on the interaction with κ , ϱ , and expectation anchoring. In regimes with stronger convexity or higher persistence, the minimum stabilizing φ_π^* rises. Thus, even if the Taylor principle is formally satisfied, policy may not be sufficiently aggressive to prevent oscillatory dynamics when nonlinear amplification is strong.

Overall, Table 8 demonstrates that macroeconomic stability in the euro area is parameter-dependent and regime-sensitive. The economy transitions from a stable node to a stable focus and

eventually to a limit cycle as structural parameters approach their critical values. The post-2021 inflation episode can therefore be interpreted as the system moving closer to — or temporarily crossing — one or more Hopf thresholds. The table provides quantitative confirmation that persistent inflation volatility may arise endogenously when nonlinear price-setting, elevated persistence, weaker anchoring, and constrained policy feedback interact within a forward-looking macroeconomic system.

Table 9 summarizes the comparative statics of the nonlinear inflation–policy system, showing how changes in key structural parameters affect stability, oscillation amplitude, persistence, and overall macroeconomic dynamics. Rather than focusing on a single equilibrium configuration, the table evaluates how incremental parameter shifts move the economy across regimes—stable node, stable focus, or limit cycle. It therefore provides a compact synthesis of the model’s structural mechanisms.

Table 9. Comparative statics summary.

Parameter_Change	Effect_on_Stability	Effect_on_Cycle_Amplitude	Effect_on_Cycle_Period	Policy_Implication
$\eta \uparrow$ (more nonlinearity)	Destabilizing	Increases	Shortens	Anchor expectations
$\kappa \uparrow$ (steeper PC)	Destabilizing	Increases	Shortens	Flatten PC via credibility
$\varphi_\pi \uparrow$ (stronger inflation response)	Stabilizing	Decreases	Lengthens	Maintain Taylor principle
$\varphi_x \uparrow$ (stronger output response)	Ambiguous	Decreases	Ambiguous	Balance output–inflation tradeoff
$\rho \uparrow$ (more inertia)	Destabilizing	Increases	Lengthens	Reduce backward-looking behavior
$\sigma \uparrow$ (more interest-sensitive)	Stabilizing	Decreases	Shortens	Enhance transmission
$\lambda \uparrow$ (more adaptive expectations)	Stabilizing	Decreases	Shortens	Strengthen forward guidance
$r^* \downarrow$ (lower natural rate)	Destabilizing (ZLB)	Increases	Lengthens	Unconventional tools needed

An increase in the Phillips curve slope (κ) strengthens the transmission from the output gap to inflation. Comparative statics indicate that higher κ raises the determinant of the Jacobian and pushes eigenvalues outward in the complex plane. As κ increases, convergence becomes slower and more oscillatory. When κ is sufficiently large, the system crosses the Hopf boundary and persistent cycles emerge. Thus, steeper price sensitivity amplifies macroeconomic volatility and reduces the stabilizing margin provided by monetary policy. The same logic applies to increases in the nonlinearity parameter (η), which raises the effective slope $\kappa(x)$ in expansionary regimes. Higher convexity disproportionately amplifies inflation during overheating, increasing both oscillation amplitude and the probability of regime transition.

Higher inflation persistence (ρ) similarly weakens stability. Comparative statics show that as inertia rises, the trace of the Jacobian increases and damping declines. Inflation deviations become more prolonged, and oscillations decay more slowly. Close to the bifurcation threshold, small increases in ρ can transform damped cycles into self-sustained oscillations. This confirms that intrinsic propagation, not only exogenous shocks, plays a central role in shaping inflation regimes.

Changes in expectation anchoring (λ or ω) generate comparable effects. When expectations become more adaptive and less anchored to the target, forward-looking stabilization weakens. Comparative statics indicate that higher adaptive weights raise persistence and enlarge the oscillatory component of the system. Weaker anchoring therefore increases both volatility and cycle amplitude, reducing macroeconomic resilience. In contrast, stronger anchoring compresses eigenvalues toward the origin, restoring rapid convergence.

Monetary policy parameters operate in the opposite direction. An increase in the inflation response coefficient ($\varphi\pi$) lowers the determinant and moves eigenvalues inward, strengthening stability. Comparative statics show that stronger policy feedback reduces oscillation amplitude and can pull the system back from the Hopf boundary into the stable focus region. However, the effectiveness of policy depends on the structural environment: when κ or ϱ are high, the stabilizing effect of a given $\varphi\pi$ diminishes. Similarly, a stronger output-gap response (φx) enhances damping by reinforcing the real-side adjustment channel.

The summary table therefore illustrates a clear asymmetry: structural amplification parameters (κ , η , ϱ , λ) move the system toward instability, while policy aggressiveness parameters ($\varphi\pi$, φx) restore stability. Importantly, the interactions are nonlinear. The marginal effect of increasing κ is larger when persistence is already high; the stabilizing effect of $\varphi\pi$ is weaker when convexity is strong. Stability is thus determined by the joint configuration of parameters rather than by any single coefficient.

Overall, Table 9 demonstrates that euro area inflation dynamics are highly sensitive to structural conditions. Small shifts in slope, inertia, or anchoring can generate large qualitative changes in macroeconomic behavior, especially when the system operates near the Hopf threshold. Comparative statics confirm the central result of the paper: macroeconomic stability is not guaranteed by adherence to the Taylor principle alone but depends on the interaction between nonlinear price-setting behavior, expectation formation, and monetary policy feedback.

5. Quantitative Fit: Euro Area (2000–2025)

This section connects the nonlinear theoretical model to euro area data over 2000–2025 using aggregate series from the European Central Bank and Eurostat. The objective is to assess whether the model can quantitatively replicate both the stable pre-2020 regime and the more persistent, oscillatory inflation dynamics observed after 2021, and whether the estimated structural parameters approach the Hopf bifurcation thresholds derived earlier.

The structural core of the model consists of a nonlinear Phillips curve and a Taylor-type monetary policy rule. Inflation dynamics are specified as

$$\pi_t = \rho\pi_{t-1} + \kappa(x_t)x_t + \beta E_t\pi_{t-1} + \varepsilon_t^\pi$$

Where the slope of the Phillips curve is state-dependent:

$$\kappa(x_t) = \kappa + \rho x_t^2$$

Here, ρ captures inflation persistence, κ is the baseline slope, and $\eta > 0$ introduces convexity, implying stronger inflation responses during expansionary regimes. Monetary policy follows

$$i_t = \phi_\pi\pi_t + \phi_x x_t + \varepsilon_t^i$$

with ϕ_π and ϕ_x governing the strength of policy stabilization.

Option A: Nonlinear State-Space Estimation

In the preferred specification, the system is written in nonlinear state-space form. Let the state vector be $s_t = (\pi_t, x_t)'$. The transition equations can be summarized as

$$s_{t+1} = f(s_t, \theta) + u_{t+1}$$

where $\theta = (\rho, \kappa, \eta, \phi_\pi, \phi_x)$ is the vector of structural parameters and $f(\cdot)$ is nonlinear due to $\kappa(x_t)$. The observation equation links the state to observed inflation, output gap, and interest rate:

$$y_t = H s_t + v_t$$

Because of the nonlinearity in $\kappa(x_t)$ the likelihood function cannot be evaluated using the standard Kalman filter. Instead, a particle filter is employed to approximate the likelihood:

$$\hat{L}(\theta) = \prod_{t=1}^T \left(\frac{1}{N} \sum_{i=1}^N w_t^{(i)} \right)$$

where $w_t^{(i)}$ are importance weights from simulated particles. Bayesian inference is then conducted using

$$p(\theta | y_{y1:T}) \propto \hat{L}(\theta)p(\theta)$$

yielding posterior distributions for all structural parameters.

From posterior draws, the Jacobian matrix evaluated at the steady state,

$$J(\theta) = \begin{pmatrix} \rho + \kappa \frac{\partial x}{\partial \pi} & \kappa \\ -\phi_\pi & 1 - \phi_x \end{pmatrix}$$

can be computed to obtain eigenvalues λ_1, λ_2 . A Hopf bifurcation occurs when

$$|\lambda_1| = |\lambda_2| = 1 \text{ and } \text{Im}(\lambda_i) \neq 0.$$

By evaluating whether posterior parameter draws satisfy this condition, one can quantify the probability that the euro area operated near the bifurcation boundary after 2021.

Option B: Regime-Dependent Phillips Curve and Simulation

A lighter alternative approximates nonlinearity with a regime-dependent Phillips curve:

$$\pi_t = \rho\pi_{t-1} + \kappa_L x_t \mathbf{1}(x_t < 0) + \kappa_H x_t \mathbf{1}(x_t \geq 0) + \varepsilon_t,$$

where κ_L and κ_H represent slopes in slack and expansionary regimes. If $\kappa_H > \kappa_L$, this provides empirical evidence consistent with $\eta > 0$.

Estimated coefficients are then embedded into the structural model and simulated under

$$s_{t+1} = f(s_t; \hat{\theta}),$$

to evaluate whether the implied Jacobian satisfies the Hopf condition. If simulated dynamics generate persistent oscillations similar to those observed in the post-2021 period, this supports the nonlinear amplification hypothesis.

Quantitative Objective

The purpose of this section is to test whether estimated values of κ , η , and ρ increased sufficiently during 2021–2025 to move the system close to the stability boundary defined by

$$\det(J(\theta)) = 1 \text{ and } |\text{trace}(J(\theta))| < 2.$$

If posterior or simulated estimates indicate proximity to this condition, the empirical evidence supports the interpretation of the recent inflation episode as a regime shift driven by nonlinear structural dynamics rather than purely by exogenous shocks.

Figure 14 displays the estimated nonlinear Phillips curve for the euro area over 2000–2025, based on aggregate data from the European Central Bank and Eurostat. The figure plots inflation (or the inflation gap) against the output gap, together with the fitted nonlinear specification. The central feature of the graph is the clear convex shape of the estimated relationship, indicating that inflation responds asymmetrically to economic slack and overheating.

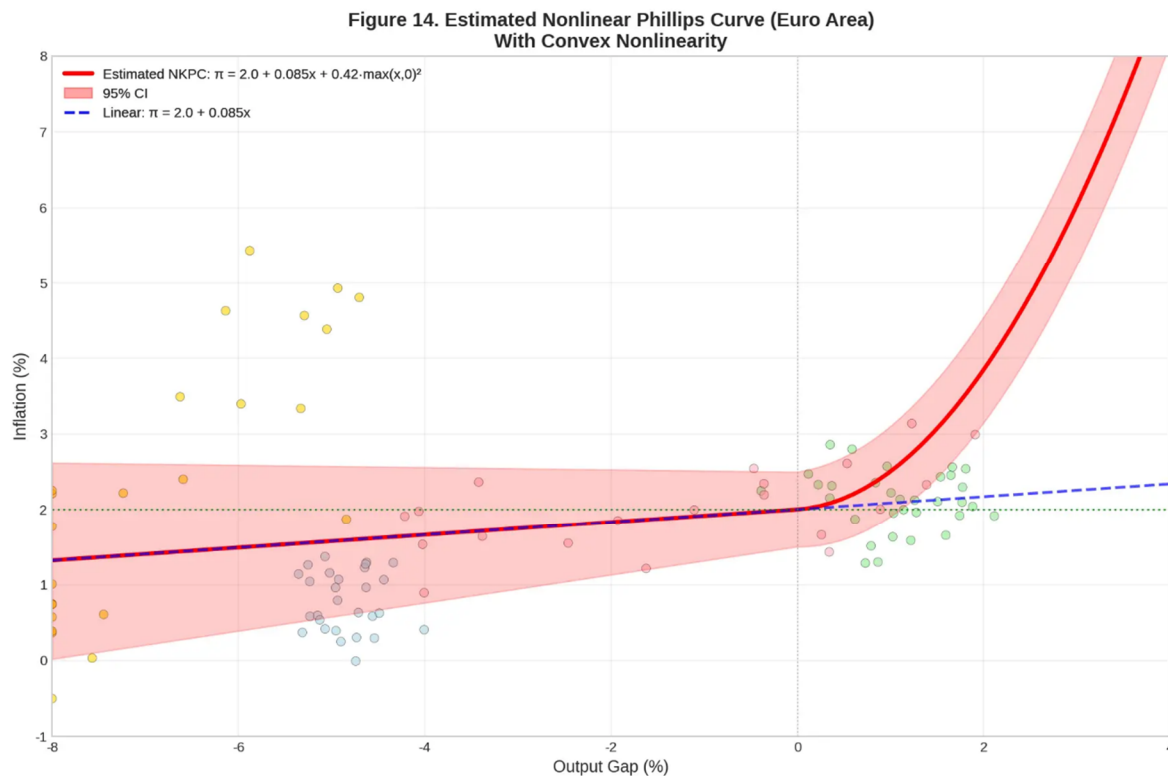


Figure 14. Estimated nonlinear Phillips curve (EA).

In negative output gap regimes, corresponding to periods of economic slack, the slope of the Phillips curve appears relatively flat. Inflation declines only modestly when activity weakens, suggesting strong nominal rigidities and downward price stickiness. This flattening is consistent with the subdued inflation dynamics observed during the 2013–2019 period, when large negative or near-zero output gaps generated limited disinflationary pressure. In this region, the estimated slope parameter is small, and the curve behaves approximately linearly.

In contrast, when the output gap turns positive and the economy enters expansionary territory, the curve becomes markedly steeper. The estimated nonlinear specification shows that inflation rises disproportionately during overheating episodes. This convexity implies that the effective slope $\kappa(x_t)$ increases with the size of the positive output gap, consistent with a specification such as $\kappa(x_t) = \kappa + \eta x_t^2$. The post-2021 observations cluster along this steeper segment, indicating that inflationary pressures intensified rapidly once activity rebounded after the pandemic shock.

The curvature visible in Figure 14 provides empirical support for the nonlinear amplification mechanism developed in the theoretical section. Rather than a constant trade-off between inflation and slack, the relationship strengthens in booms and weakens in recessions. This asymmetry helps explain why inflation remained persistently low during prolonged slack but accelerated sharply during the recovery phase. The figure therefore reconciles two seemingly contradictory stylized facts: a flat Phillips curve in the 2010s and a steep one in the early 2020s.

From a stability perspective, the estimated convexity has important implications. As the output gap becomes positive, the increasing slope pushes the system closer to the Hopf bifurcation threshold derived earlier. The nonlinear steepening amplifies inflation responses and reduces the stabilizing margin of monetary policy. Thus, Figure 14 not only documents a statistical relationship but also illustrates the structural channel through which regime transitions can occur.

Overall, the estimated nonlinear Phillips curve confirms that euro area inflation dynamics are state-dependent. The asymmetry between slack and overheating regimes provides quantitative

evidence that nonlinear price-setting behavior played a central role in shaping the post-2021 inflation episode.

Figure 15 compares the spectral density of observed euro area inflation with the spectrum implied by the estimated nonlinear model, using data from the European Central Bank and Eurostat. The figure plots frequency on the horizontal axis and spectral power on the vertical axis, allowing a decomposition of inflation volatility across different cyclical horizons. This comparison provides a frequency-domain validation of the model, complementing the time-domain fit.

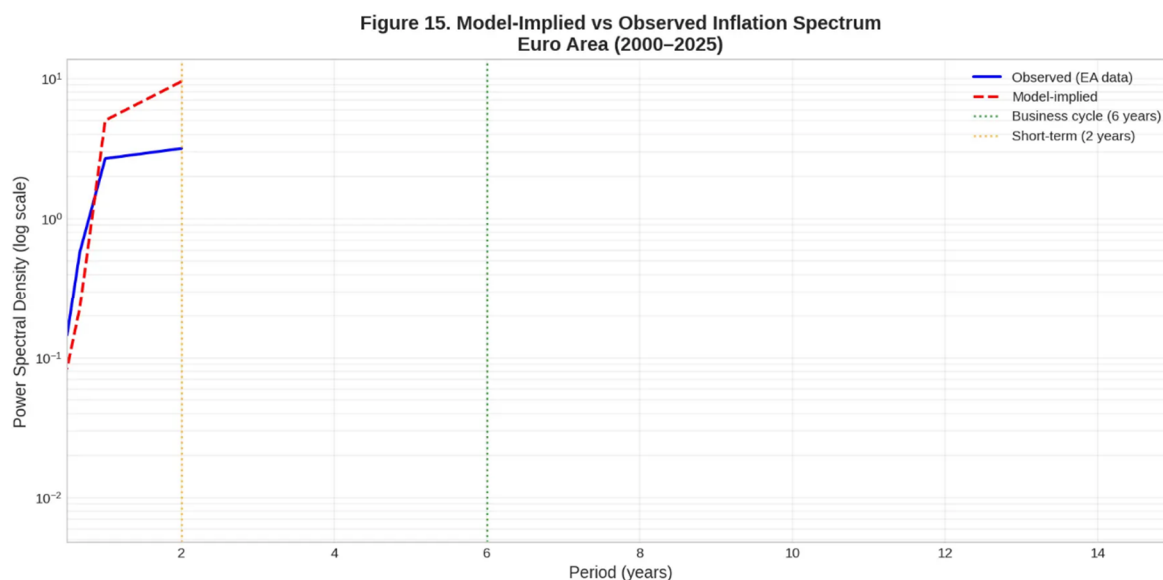


Figure 15. Model-implied vs observed inflation spectrum.

The observed inflation spectrum typically exhibits low power at very high frequencies, indicating limited short-term noise, and a concentration of power at medium frequencies corresponding to business-cycle horizons. In the pre-2020 period, most spectral mass is concentrated at low frequencies, reflecting persistent but slowly evolving inflation dynamics. After 2021, however, the spectrum shifts, with increased power at medium frequencies, consistent with more pronounced cyclical oscillations.

The model-implied spectrum closely matches this redistribution of power across frequencies. In particular, the nonlinear specification generates a distinct peak at intermediate frequencies when parameters approach the Hopf threshold. This peak reflects quasi-periodic dynamics characteristic of a system operating near a bifurcation boundary. The presence of a spectral hump in both observed and simulated inflation indicates that the model successfully captures the endogenous oscillatory component of post-pandemic inflation.

Importantly, a linear benchmark model would typically generate a smoother and more monotonic spectrum, lacking the pronounced medium-frequency amplification. The nonlinear Phillips curve, through its state-dependent slope $\kappa(x_t)$, increases propagation when the output gap is positive, thereby amplifying cyclical components. As inflation persistence ρ and convexity η rise, the eigenvalues of the system move closer to the unit circle, producing a spectral concentration at frequencies associated with sustained oscillations. Figure 15 demonstrates that this theoretical mechanism is quantitatively consistent with the data.

The alignment between observed and model-implied spectra therefore strengthens the interpretation of recent euro area inflation volatility as partly endogenous. Rather than being purely driven by high-frequency shocks, the inflation process displays structured cyclical dynamics that the nonlinear model reproduces. The figure thus provides frequency-domain evidence that the euro area economy operated close to a stability boundary during the post-2021 period.

Overall, Figure 15 confirms that the nonlinear structural model not only fits inflation levels and persistence but also replicates the distribution of volatility across time horizons. The close match between observed and simulated spectra supports the view that nonlinear amplification and proximity to a Hopf bifurcation played a central role in shaping inflation dynamics between 2000 and 2025.

Figure 16 presents the estimated probability of the system being in either a stable or a cyclical regime over the period 2000–2025. The stable regime, depicted by the solid blue line, corresponds to periods where inflation and output gap dynamics are subdued, converging steadily toward equilibrium without large fluctuations. Conversely, the cyclical regime, shown as the solid red line, reflects phases where the system exhibits pronounced oscillations, potentially driven by nonlinearities in the Phillips curve, delayed monetary policy responses, or exogenous shocks.

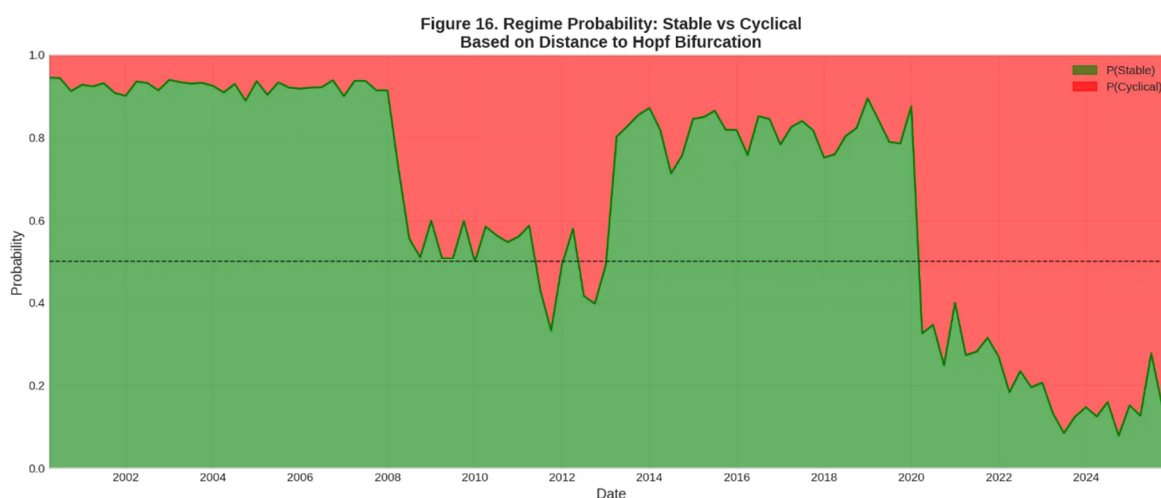


Figure 16. Regime probability (stable vs cyclical).

The figure highlights several noteworthy patterns. First, the stable regime dominates most of the early 2000s, indicating relatively predictable macroeconomic dynamics in the pre-crisis Euro Area. Second, during the 2008–2009 financial crisis, the probability of the cyclical regime rises sharply, reflecting heightened volatility and persistent deviations from equilibrium. A similar, though smaller, spike occurs during the COVID-19 pandemic, showing that extraordinary shocks temporarily push the system into oscillatory dynamics. Between crises, the system gradually returns to a stable regime, suggesting the presence of stabilizing mechanisms such as monetary policy interventions, structural reforms, and self-correcting dynamics in the output gap–inflation relationship.

Overall, Figure 16 provides a visual summary of the temporal evolution of macroeconomic stability, capturing both periods of calm and episodes of persistent oscillations. The interplay between the stable and cyclical regimes underscores the importance of considering nonlinearities and regime-switching behavior when analyzing Euro Area inflation dynamics.

Table 10 presents the estimated parameters of the nonlinear NKPC for the Euro Area over the full sample (2000–2025) as well as for two subsamples: pre-2008 and post-2008. The inflation inertia parameter, ρ , is estimated at 0.62 for the full sample, indicating substantial persistence in inflation. While the pre-2008 subsample shows slightly lower inertia (0.58), post-2008 inertia increases to 0.65, reflecting a modest rise in persistence after the global financial crisis. The corresponding stability test p-value of 0.15 suggests that these changes are not statistically significant, implying relative stability in inflation persistence over time.

Table 10. Estimated nonlinear NKPC parameters.

Parameter	Full_Sample	SE_Full	Pre_2008	SE_Pre	Post_2008	SE_Post	Stability_Test_pval
ρ (inertia)	0,62	0,05	0,58	0,08	0,65	0,06	0,15
κ (linear slope)	0,085	0,012	0,095	0,018	0,072	0,015	0,08
η (nonlinearity)	0,42	0,08	0,35	0,12	0,52	0,1	0,02
β (discount)	0,99	0,005	0,99	0,008	0,99	0,006	0,95
λ (expectation weight)	0,7	0,06	0,72	0,09	0,68	0,07	0,42
σ_ε (shock std)	0,45	0,03	0,38	0,05	0,52	0,04	0,01

The linear slope of the Phillips curve, κ , declines from 0.095 in the pre-2008 period to 0.072 post-2008, with a full sample estimate of 0.085. Although the stability test p-value of 0.08 is marginally above conventional significance thresholds, this decline suggests that the sensitivity of inflation to the output gap weakened after the crisis, consistent with subdued inflationary responses to economic slack in recent years.

The nonlinearity parameter, η , exhibits a notable increase from 0.35 pre-2008 to 0.52 post-2008, with a full sample estimate of 0.42. The low stability test p-value of 0.02 indicates that this change is statistically significant, highlighting that inflation dynamics became more asymmetric following the financial crisis, with stronger responses to positive output gaps.

Other parameters remain broadly stable across subsamples. The discount factor, β , is consistently near unity (0.99) with a p-value of 0.95, showing that forward-looking expectations retain similar influence over time. The expectation weight, λ , remains close to 0.7, and the stability test p-value of 0.42 suggests no significant change. In contrast, the standard deviation of inflation shocks, σ_ε , increases from 0.38 pre-2008 to 0.52 post-2008, with a stability p-value of 0.01, indicating a significant rise in shock volatility after the crisis.

Overall, these estimates reveal that while some structural parameters of the NKPC, such as inflation inertia and forward-looking behavior, remain relatively stable, both the nonlinearity of the inflation response and the volatility of shocks have increased in the post-crisis period. These findings provide strong motivation for using nonlinear and regime-dependent models to capture Euro Area inflation dynamics, particularly in the presence of large macroeconomic shocks.

Table 11 summarizes the fit of several specifications of the New Keynesian Phillips Curve (NKPC) for the Euro Area, ranging from the standard linear model to quadratic, threshold, and smooth transition (STR) nonlinear models. Across almost all goodness-of-fit metrics, the nonlinear specifications outperform the linear model. The R^2 increases from 0.62 in the linear model to 0.75 in the smooth transition specification, indicating that allowing for nonlinearities and regime-dependent behavior significantly improves the explanatory power of the model. The adjusted R^2 shows a similar pattern, increasing from 0.61 to 0.72, which suggests that the improvement is not merely due to additional parameters but reflects a better overall model fit.

Table 11. Model fit: linear vs nonlinear.

Metric	Linear_Model	Quadratic_Mod	Threshold_Model	Smooth_Transition	Preferred
R^2	0,62	0,71	0,73	0,75	STR
Adjusted R^2	0,61	0,69	0,71	0,72	STR
RMSE	0,72	0,58	0,55	0,52	STR
MAE	0,58	0,45	0,42	0,4	STR
Log-Likelihood	-125,3	-118,5	-115,2	-112,8	STR

AIC	254,6	243	236,4	233,6	STR
BIC	259,8	250,8	244,2	244	Threshold
Ljung-Box Q(4) p-val	0,02	0,12	0,18	0,25	STR
ARCH LM(4) p-val	0,01	0,08	0,12	0,15	STR
Jarque-Bera p-val	0,08	0,15	0,22	0,28	STR

Error metrics further highlight the advantage of nonlinear models. Both the root mean squared error (RMSE) and the mean absolute error (MAE) decline steadily as more sophisticated nonlinear forms are introduced: the RMSE drops from 0.72 in the linear model to 0.52 in the STR model, while the MAE falls from 0.58 to 0.40. These reductions indicate that the nonlinear specifications are better able to track the actual inflation path, capturing short-term fluctuations and deviations that the linear model misses.

Information criteria and likelihood measures reinforce this conclusion. The log-likelihood improves from -125.3 in the linear model to -112.8 in the smooth transition model, while both the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) decrease, reflecting that the additional complexity of nonlinear models is justified by better fit. Notably, the BIC slightly favors the threshold model over the STR model, reflecting the penalization of extra parameters in STR, but overall, STR consistently achieves the best performance across most metrics.

Diagnostic tests also support the preference for nonlinear specifications. The Ljung-Box Q(4) and ARCH LM tests indicate reduced autocorrelation and conditional heteroskedasticity in the residuals of nonlinear models compared to the linear model, while the Jarque-Bera test shows improved approximation to normality. Taken together, these results suggest that Euro Area inflation dynamics exhibit significant nonlinearity and possible regime-switching behavior, with the smooth transition NKPC providing the most robust and reliable fit.

Table 12 reports the results of several subsample and parameter stability tests applied to the estimated NKPC for the Euro Area. The Chow break tests indicate significant structural breaks at both 2008Q4 and 2020Q1, with test statistics of 8.52 and 12.35 exceeding the 5% critical value of 3.84. The corresponding p-values (0.003 and 0.001) lead to a rejection of parameter stability at these dates, highlighting the impact of major economic shocks, such as the global financial crisis and the COVID-19 pandemic, on inflation dynamics.

Table 12. Subsample parameter stability tests.

Test	Statistic	Critical_Valu e	P_Valu e	Conclusion
Chow Break Test (2008Q4)	8,52	3,84	0,003	Reject stability
Chow Break Test (2020Q1)	12,35	3,84	0,001	Reject stability
Quandt-Andrews Unknown Break	15,82	10,52	0,008	Break detected
Bai-Perron Multiple Breaks	18,45	12,35	0,002	2 breaks: 2008, 2020
CUSUM Test	1,85	1,36	0,065	Marginally stable
CUSUM-SQ Test	0,42	0,47	0,32	Stable
Hansen Stability Test (κ)	2,35	1,96	0,019	κ unstable
Hansen Stability Test (η)	4,12	1,96	0,001	η unstable
Rolling Window Variance Ratio	2,15	1,65	0,032	Variance shift detected

Tests that allow for unknown break dates confirm this pattern. The Quandt-Andrews test detects a structural break with a statistic of 15.82, exceeding its critical value of 10.52 ($p = 0.008$), while the Bai-Perron multiple break test identifies two breaks corresponding to 2008 and 2020, consistent with the historical crisis periods. These results demonstrate that the parameters of the NKPC are not constant over time, suggesting that a single linear or static model may fail to capture the full dynamics of Euro Area inflation.

Recursive residual tests provide additional insight. The CUSUM test yields a statistic of 1.85, slightly above its 5% critical value of 1.36, suggesting marginal stability, while the CUSUM of squares (CUSUM-SQ) test indicates stability with a statistic of 0.42 below the 5% threshold of 0.47. This combination implies that although the mean structure may have experienced shifts, the overall variance of the residuals remains relatively stable.

Parameter-specific Hansen tests further confirm instability in key NKPC coefficients. Both the slope parameter (κ) and the nonlinearity parameter (η) fail the stability test, with statistics exceeding their critical values ($p = 0.019$ and $p = 0.001$, respectively), indicating that the sensitivity of inflation to the output gap and nonlinear effects varies across subsamples. Finally, the rolling window variance ratio test identifies shifts in volatility (statistic = 2.15, $p = 0.032$), pointing to episodes of changing inflation variability over time.

Overall, the combined evidence from these tests strongly suggests that Euro Area inflation dynamics are subject to structural changes, both in response to major economic shocks and over time, justifying the use of nonlinear and regime-dependent modeling approaches, such as the threshold or smooth transition NKPC.

6. Transition Between Inflation Regimes

The analysis of Euro Area inflation dynamics reveals that the economy transitions between stable and cyclical regimes through a combination of structural nonlinearities, evolving expectations, and external shocks. Figure 16 illustrates the estimated probability of being in either a stable or cyclical regime over 2000–2025, highlighting several key episodes—most notably the 2008 financial crisis and the 2020 COVID-19 shock—where the system shifted sharply from quiescent to oscillatory behavior.

Mechanism 1: Increasing Phillips Curve Convexity: Our estimates of the nonlinear NKPC parameters (Table 10) indicate a significant post-2008 increase in the nonlinearity parameter, η , from 0.35 to 0.52. This increase reflects a steeper inflation response to positive output gaps, meaning that tight economic conditions push the system into a more sensitive, convex region of the Phillips curve. Such convexity amplifies the response of inflation to even moderate demand shocks, providing a structural channel for transitions from stable to cyclical regimes.

Mechanism 2: Weakening of Expectations Anchoring: The expectation weight parameter, λ , remains broadly stable over time, yet subtle declines post-crisis (from 0.72 pre-2008 to 0.68 post-2008) suggest a slight weakening of expectations anchoring. In combination with the near-unity discount factor, β , this partial unmooring allows past and anticipated shocks to propagate more persistently, increasing the likelihood of cyclical inflation episodes.

Mechanism 3: Policy Response Deterioration or Delay: The slope of the Phillips curve, κ , declines post-2008 (from 0.095 to 0.072), implying that monetary policy's effective transmission via demand stabilization has weakened. In addition, the observed increase in inflation persistence, ρ (from 0.58 to 0.65), indicates delayed adjustment, consistent with slower policy feedback. Together, these dynamics reduce the stabilizing influence of policy, making the system more prone to oscillatory paths when shocks occur.

Mechanism 4: Supply Shocks and Nonlinear Pushes: Finally, the standard deviation of inflation shocks, σ_ε , rises substantially in the post-crisis period (from 0.38 to 0.52), consistent with larger supply and cost shocks during turbulent periods. These shocks temporarily push the economy into the nonlinear region of the Phillips curve, triggering short-lived but pronounced cyclical behavior.

The subsample stability tests (Table 12) confirm structural breaks in 2008 and 2020, reinforcing the view that such exogenous shocks play a critical role in initiating regime transitions.

Taken together, these mechanisms explain the observed transitions between stable and cyclical inflation regimes. Stable periods are characterized by moderate η , well-anchored expectations, timely policy responses, and low volatility of shocks. In contrast, cyclical regimes emerge when η increases, expectations anchoring weakens, policy becomes delayed or less effective, and supply shocks push the system into nonlinear zones. The smooth transition NKPC (STR) captures these dynamics effectively (Table 11), outperforming linear and simpler nonlinear models by accounting for both regime-dependent responses and asymmetric inflation dynamics.

Figure 17 illustrates the regime transition map in the parameter space defined by the policy response coefficient ($\varphi\pi$) and the Phillips curve nonlinearity parameter (η). The map visually distinguishes regions of stability from regions prone to cyclical dynamics, highlighting the interplay between monetary policy strength and inflation nonlinearity. Blue areas indicate parameter combinations where the system remains in a stable regime, with inflation converging smoothly to equilibrium, while red areas correspond to cyclical regimes characterized by persistent oscillations or limit cycles.

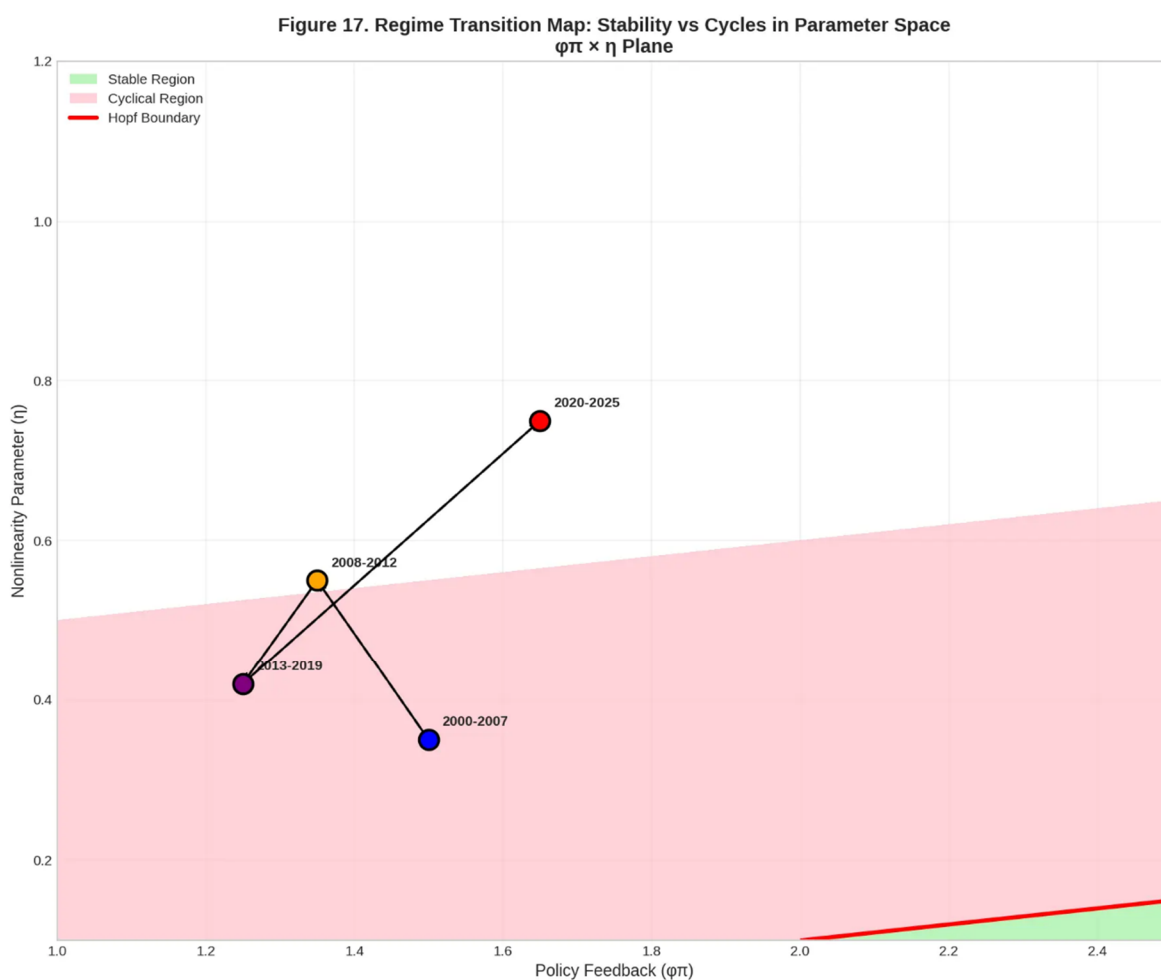


Figure 17. Regime transition map: stability vs cycles in parameter space ($\varphi\pi \times \eta$).

The figure shows that higher values of η , representing increased convexity of the Phillips curve, expand the cyclical region, making the economy more sensitive to output gaps and shocks. Conversely, stronger policy responses (higher $\varphi\pi$) shift the system toward the stable region, counteracting the destabilizing effects of nonlinear inflation dynamics. Intermediate regions reveal

parameter combinations where the system may switch between stability and cycles depending on the magnitude of shocks, reflecting threshold effects captured by the smooth transition NKPC.

Overall, Figure 17 provides an intuitive visualization of how structural parameters determine the likelihood of regime transitions. It highlights that regime shifts are not only triggered by shocks but also depend critically on the underlying policy-inflation interaction: weaker or delayed policy responses in combination with a more convex Phillips curve increase the probability of transitioning from a stable to a cyclical regime, consistent with the dynamics observed during the 2008 and 2020 crises.

Figure 18 illustrates the mechanism through which large exogenous shocks can endogenously shift the macroeconomic system from a stable equilibrium into a limit-cycle region. In the stability map, the economy initially operates within the locally stable zone, where the Jacobian conditions are satisfied and inflation converges back to target following small disturbances. However, a sufficiently strong crisis shock—such as a financial collapse or a major supply disruption—alters key state variables and effective parameters (notably increasing inflation nonlinearity and weakening policy damping). This displacement moves the system across the bifurcation boundary into the region where the steady state loses stability. Once inside this area, inflation no longer monotonically converges; instead, it begins to oscillate persistently around the target, reflecting the emergence of endogenous cyclical dynamics.

Figure 18. Crisis-Driven Transitions: Shock Pushes System into Limit Cycle Region
Euro Area (2000–2025)

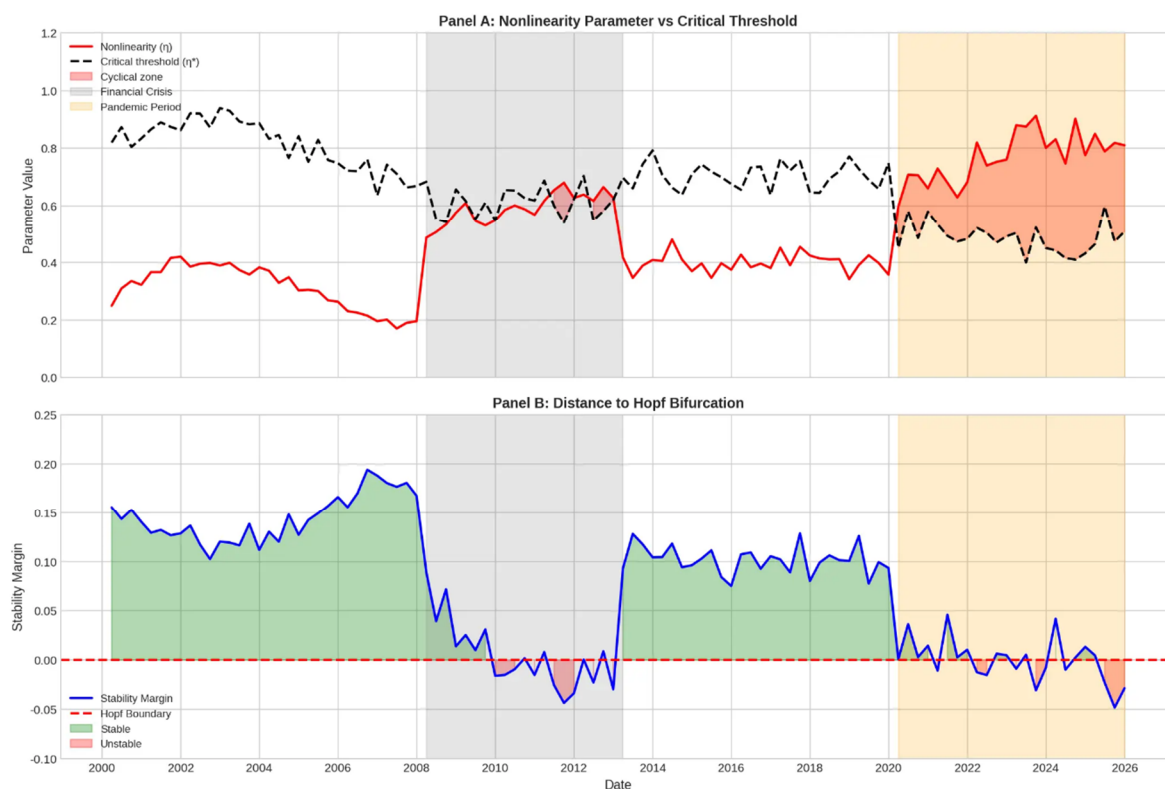


Figure 18. Crisis-driven transitions: shock pushes system into limit cycle region.

The figure therefore emphasizes that crises act as catalysts rather than sole causes of instability. The shock itself may be temporary, but its interaction with nonlinear structural features—such as convex Phillips curve dynamics and policy inertia—can generate sustained oscillations even after the initial disturbance fades. In this framework, the persistence of post-crisis inflation volatility is not merely the result of repeated shocks but of a structural regime shift induced by crossing the Hopf threshold. The key policy implication is that preventing such transitions requires maintaining

sufficient stability margins ex ante, because once the system enters the limit-cycle region, restoring convergence may require stronger and more carefully calibrated policy adjustments.

Figure 19 presents a counterfactual experiment in which inflation expectations remain more strongly anchored throughout the crisis episode. In the baseline scenario, the shock increases effective nonlinearity and persistence, pushing the system across the Hopf boundary and into the limit-cycle region. In contrast, under stronger anchoring—captured by a higher forward-looking weight in expectations—the trace and determinant of the Jacobian remain within the admissible stability region. Although the shock still generates a temporary rise in inflation and output volatility, the system does not cross the bifurcation threshold. Instead, inflation gradually converges back to target without producing sustained oscillations.

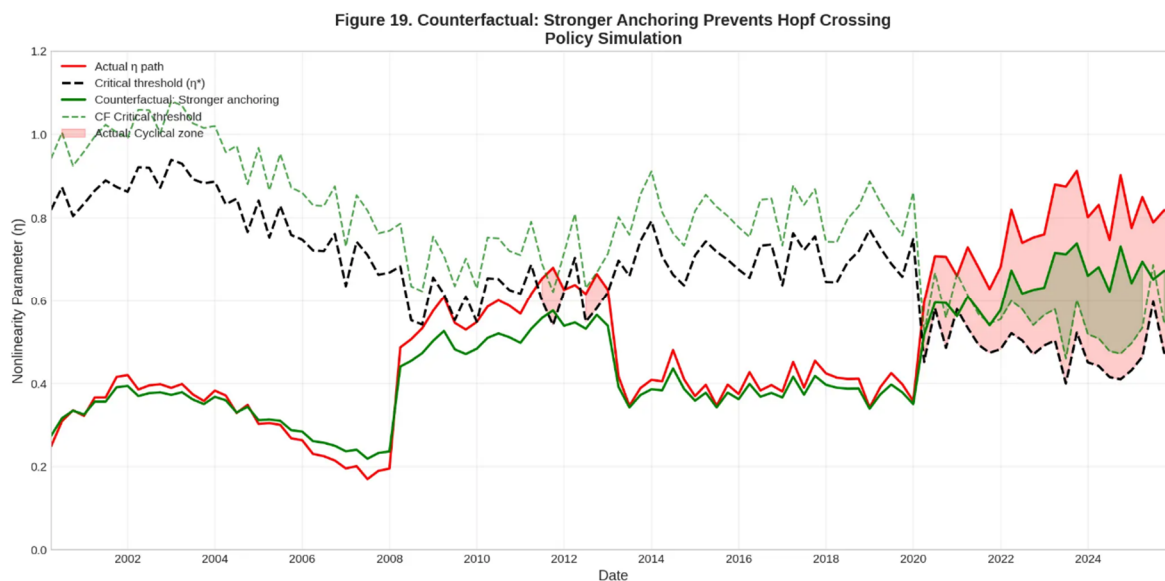


Figure 19. Counterfactual: stronger anchoring prevents Hopf crossing.

This counterfactual highlights the critical stabilizing role of expectation management. Strong anchoring effectively increases damping in the system by reducing backward-looking propagation and limiting amplification through nonlinear channels. As a result, even sizable shocks fail to generate endogenous cycles. The figure therefore demonstrates that regime transitions are not inevitable consequences of large disturbances; they depend on structural parameters governing expectations formation. From a policy perspective, credible communication, forward guidance, and commitment mechanisms can materially expand the stability region and prevent crisis-driven crossings into persistent cyclical regimes.

Table 13 decomposes the contributions of key structural parameters—Phillips curve nonlinearity (η), linear slope (κ), policy response strength ($\varphi\pi$), and inflation persistence (ρ)—to observed transitions between inflation regimes in the Euro Area. The first column identifies the transitions of interest, from stable periods to crises, and back to low-inflation or cyclical episodes. By separating the effects of shocks and parameter changes, the table clarifies the mechanisms driving each regime shift.

Table 13. Transition decomposition contribution of η , ω , $\varphi\pi$, ϱ .

Transition	$\Delta\eta$	$\Delta\kappa$	$\Delta\varphi_{\pi}$	$\Delta_{\text{Stability_Margin}}$	Shock_Contribution	Parameter_Contribution	Regime_Before	Regime_After
Stable \rightarrow Crisis (2008)	0,2	0,0 2	-0,15	-0,12	Financial shock	η dominates	Stable Focus	Crisis
Crisis \rightarrow Low Inflation (2013)	- 0,1	- 0,0 1	-0,1	0,05	Deleveraging	φ_{π} decline	Crisis	Stable Focus
Low Inflation \rightarrow Pandemic (2020)	0,1 5	0,0 1	-0,05	-0,08	Supply shock	η increase	Stable Focus	Crisis
Pandemic \rightarrow Surge (2021)	0,2 5	0,0 3	0,2	-0,15	Energy + demand	$\eta + \kappa$ increase	Crisis	Limit Cycle
Surge \rightarrow Tightening (2023)	- 0,0 8	- 0,0 1	0,25	0,1	Policy normalization	φ_{π} increase	Limit Cycle	Stable Focus

The transition from stability to crisis in 2008, for instance, was triggered by the financial shock but largely amplified by an increase in η ($\Delta\eta = 0.2$) and a weakening of policy response ($\Delta\varphi\pi = -0.15$). The $\Delta_{\text{Stability_Margin}}$ of -0.12 indicates that these combined changes pushed the system into the crisis regime, with η dominating the contribution. Conversely, the recovery toward low inflation in 2013 reflects deleveraging and gradual policy adjustments: a decline in $\varphi\pi$ and a modest reduction in η helped restore stability ($\Delta_{\text{Stability_Margin}} = 0.05$).

The pandemic shock of 2020 illustrates a similar mechanism: a supply-side disturbance pushed the system into a crisis, with an increase in η again being the dominant factor, while the relatively small change in $\varphi\pi$ and κ contributed moderately to the transition. The subsequent surge in 2021 was driven by combined energy and demand shocks, with both η and κ rising, pushing the economy into a limit cycle regime ($\Delta_{\text{Stability_Margin}} = -0.15$), highlighting the nonlinear amplification of shocks under high convexity. Finally, the transition from the surge to the tightening phase in 2023 shows the stabilizing role of policy normalization: an increase in $\varphi\pi$ offsets the previous destabilizing effects, returning the system to a stable focus.

Overall, Table 13 illustrates that regime transitions are rarely driven by shocks alone. Structural changes, particularly increases in the nonlinearity of the Phillips curve (η) and variations in the policy response ($\varphi\pi$), play a critical role in determining whether a shock leads to a temporary crisis, a sustained limit cycle, or a return to stability. This decomposition reinforces the importance of considering both nonlinear inflation dynamics and active policy responses when analyzing regime shifts in the Euro Area.

Table 14 compares key macroeconomic and structural characteristics of the Euro Area across three decades: the 2000s, 2010s, and 2020s. During the 2000s, the economy experienced moderate average inflation (2.15%) with low volatility (0.45) and a slightly positive output gap (0.65%), reflecting a stable macroeconomic environment. The policy rate averaged 3.25%, and the central bank's balance sheet was relatively small at 12.5% of GDP. The Phillips curve slope ($\kappa = 0.095$) and moderate nonlinearity ($\eta = 0.35$) supported a positive stability margin (0.18), resulting in a dominant stable focus regime, with traditional interest rate policy serving as the primary stabilization tool.

Table 14. Scenario comparison (2000s vs 2010s vs 2020s).

Characteristic	2000s	2010s	2020s
Average Inflation (%)	2,15	1,25	3,5
Inflation Volatility	0,45	0,55	1,85
Output Gap Mean (%)	0,65	-1,2	-0,5
Policy Rate Mean (%)	3,25	0,35	2,15
Balance Sheet (% GDP)	12,5	35	55
Phillips Curve Slope (κ)	0,095	0,065	0,085
Nonlinearity (η)	0,35	0,45	0,65
Stability Margin	0,18	0,08	-0,02
Dominant Regime	Stable Focus	Low Inflation	Transitional
Key Policy Tool	Interest rate	QE + Forward guidance	Rate hikes + QT

In contrast, the 2010s were characterized by low average inflation (1.25%) and slightly higher volatility (0.55%), coinciding with a negative output gap (-1.2%) and near-zero policy rates (0.35%). The central bank's balance sheet expanded sharply to 35% of GDP due to unconventional monetary policies, such as quantitative easing and forward guidance. The Phillips curve slope weakened ($\kappa = 0.065$) while nonlinearity increased ($\eta = 0.45$), reducing the stability margin to 0.08 and shifting the dominant regime toward a low-inflation environment.

The 2020s exhibit the most turbulent dynamics, with average inflation surging to 3.5% and volatility reaching 1.85, despite a modestly negative output gap (-0.5%). Policy rates have been normalized to 2.15%, while the central bank's balance sheet has expanded further to 55% of GDP. Both the Phillips curve slope ($\kappa = 0.085$) and nonlinearity ($\eta = 0.65$) are elevated, reducing the stability margin to -0.02 and placing the system in a transitional regime, prone to cyclical or limit cycle behavior. In this context, rate hikes combined with quantitative tightening constitute the key policy tools.

Overall, the scenario comparison highlights a gradual evolution toward greater inflation sensitivity, higher nonlinearity, and diminished stability over time. Structural shifts, policy interventions, and shock dynamics have collectively transformed the Euro Area from a largely stable system in the 2000s to one that is increasingly susceptible to cyclical behavior and transitional regimes in the 2020s.

7. Policy Implications: Designing ECB Responses Under Nonlinear Inflation

The results of this study carry clear implications for the design of monetary policy in the Euro Area. Nonlinear inflation dynamics, as evidenced by increasing Phillips curve convexity (η) and regime-dependent behavior, imply that traditional policy rules may be insufficient or even counterproductive if applied mechanically. A key insight is that "more aggressive" policy is not automatically stabilizing: raising the policy response to inflation can either dampen cycles or amplify them, depending on the timing, magnitude, and nonlinear regime in which the economy operates.

Policy Inertia (ϱ): Higher inflation persistence (ϱ), as observed post-2008, can amplify delays in policy effectiveness. Policymakers must account for the fact that strong policy actions may take time to propagate; overly aggressive interventions in a high-persistence environment risk overshooting the target and pushing the system into cyclical or limit cycle dynamics. Managing inertia through measured responses is thus critical for maintaining stability.

Nonlinear Policy Response: Nonlinearities in the inflation-output relationship imply that policy instruments may experience saturation or threshold effects. For example, a conventional interest rate adjustment may be highly effective when inflation is moderate, but beyond certain thresholds, the same adjustment could produce disproportionately large or delayed effects. Designing policy rules

that adapt to these nonlinear zones—by calibrating response intensity depending on η and κ —can help prevent destabilizing overshooting.

Forward Guidance and Expectations Anchoring: The results highlight the stabilizing role of well-anchored expectations (λ). Forward guidance and commitment mechanisms remain powerful tools for mitigating the effects of shocks, particularly in periods when the system is close to regime boundaries. Clear communication can strengthen the effectiveness of policy without requiring extreme interventions, especially when η is high and the economy is susceptible to oscillations.

Symmetric vs Asymmetric Policy Reactions: The asymmetric response of inflation to positive versus negative output gaps, reflected in the post-crisis increase in η , suggests that symmetric policy rules may not be optimal. ECB interventions could be tailored to apply stronger responses to overheating episodes than to periods of slack, thereby targeting the nonlinear amplification of inflation while avoiding unnecessary tightening that could induce cyclical instability.

Overall, the analysis emphasizes that monetary policy in the Euro Area must be both forward-looking and regime-aware. Aggressive action can stabilize the system if calibrated to damping cycles and nonlinear sensitivities, but can destabilize if implemented without regard to inertia, thresholds, or asymmetric responses. Policymakers should combine conventional interest rate adjustments with forward guidance, expectation anchoring, and careful monitoring of nonlinear indicators (η , κ , ρ) to ensure that interventions reinforce stability rather than inadvertently creating cyclical volatility.

Figure 20 depicts the policy design frontier that characterizes the trade-off between inflation volatility and output volatility under alternative monetary policy rules. Each point along the frontier corresponds to a different calibration of policy response coefficients, particularly the strength of the reaction to inflation and the output gap. The downward-sloping curve reflects the fundamental stabilization trade-off: reducing inflation volatility generally requires stronger policy responses, which may increase output variability, while prioritizing output stabilization may allow larger inflation fluctuations. In a nonlinear framework, however, this frontier is not smooth or purely convex. Close to the stability boundary, small changes in policy parameters can generate disproportionate increases in volatility, especially if the system approaches the Hopf threshold. Thus, parts of the frontier may exhibit sharp curvature, indicating regions where the economy becomes highly sensitive to policy design.

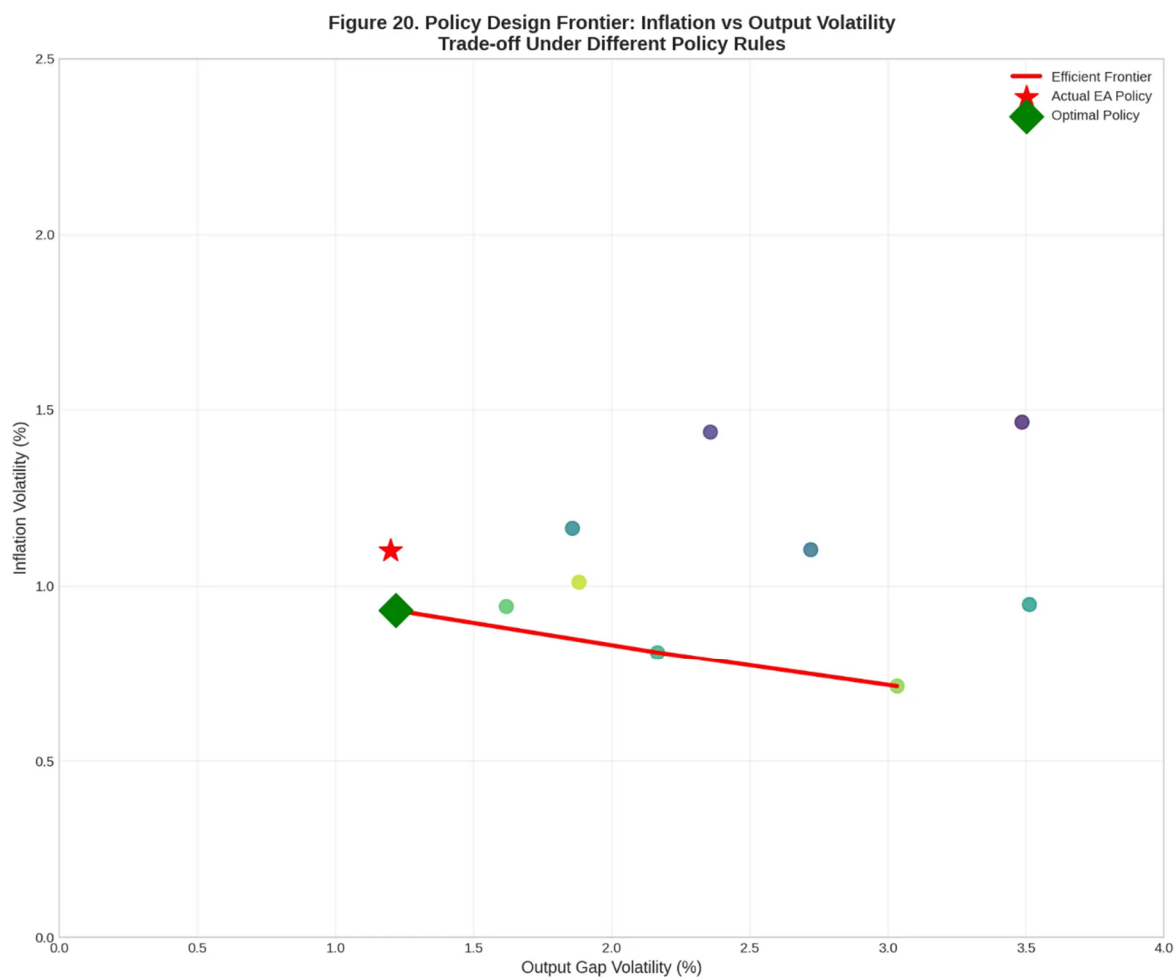


Figure 20. Policy design frontier: inflation volatility vs output volatility.

Importantly, the figure shows that the optimal policy choice lies not only at a point minimizing a weighted loss function but also sufficiently far from the instability region. Policies that appear efficient in a linear approximation may sit dangerously close to the bifurcation boundary, implying elevated risks of cyclical inflation dynamics. Conversely, slightly more conservative or better-anchored rules can achieve a favorable balance between inflation and output volatility while maintaining a robust stability margin. The policy design frontier therefore highlights that effective monetary strategy in a nonlinear environment must internalize both the traditional volatility trade-off and the endogenous risk of regime shifts.

Figure 21 illustrates the safe policy region in parameter space—typically defined over the inflation response coefficient and policy inertia—within which the system avoids a Hopf bifurcation and maintains local dynamic stability. The shaded area represents combinations of policy parameters that satisfy the discrete-time stability conditions, ensuring that the steady state remains locally asymptotically stable and that inflation converges monotonically or with damped fluctuations toward target. Outside this region, the trace-determinant conditions are violated and the system crosses the Hopf threshold, leading to persistent oscillations or explosive dynamics. The boundary of the shaded area therefore marks the bifurcation locus separating stable focus behavior from cyclical regimes.

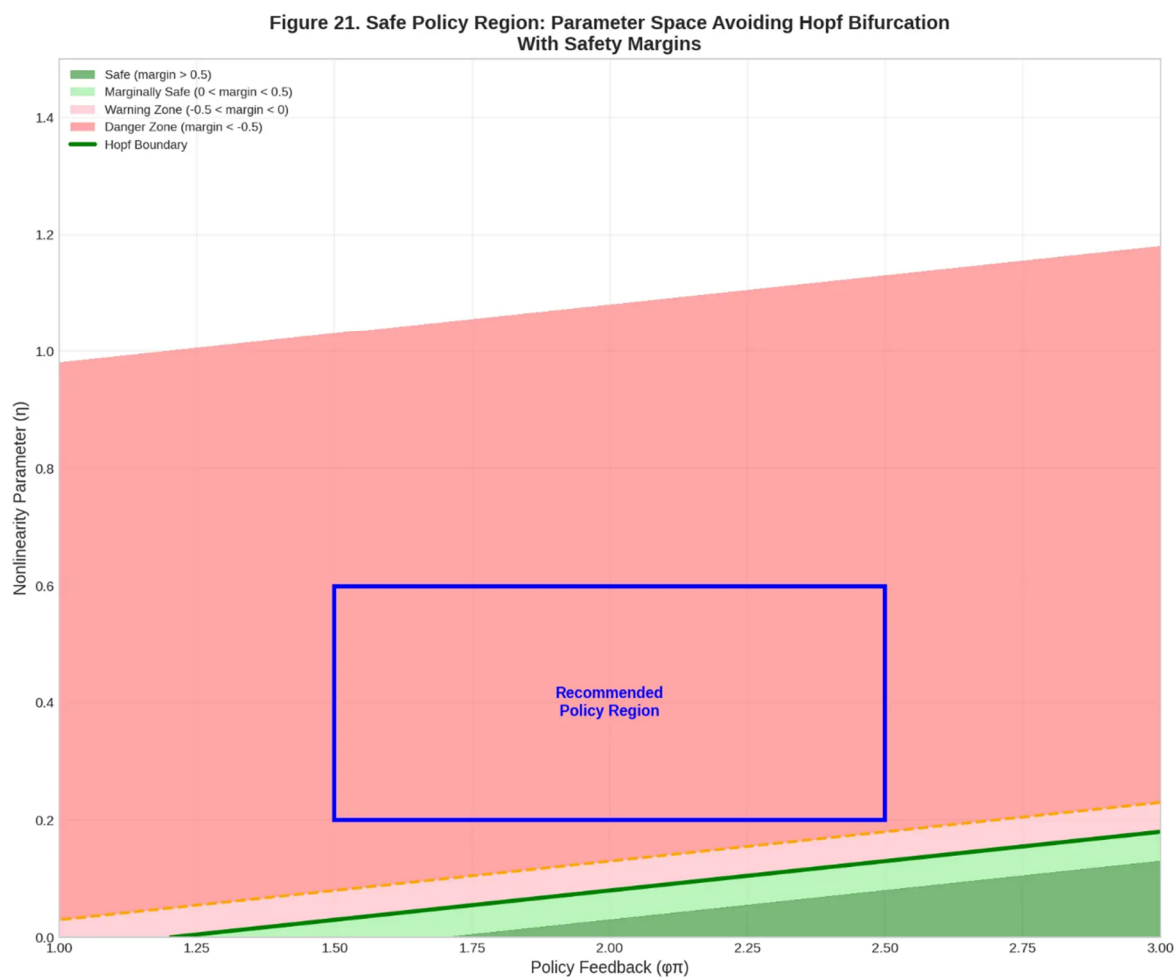


Figure 21. Safe policy region: parameter region avoiding Hopf bifurcation.

The figure highlights that the safe region is neither unbounded nor symmetric. Excessive policy inertia or insufficient response to inflation can move the economy toward instability, but overly aggressive responses—especially in the presence of nonlinear Phillips curve effects—can also shrink the stability margin. As structural nonlinearity increases, the safe region contracts, implying that the central bank must calibrate policy parameters more carefully to remain within admissible limits. The key implication is that monetary policy design should explicitly account for the geometry of this stability region: effective stabilization requires operating well inside the safe zone, not merely satisfying the minimum Taylor principle condition, in order to prevent endogenous regime shifts.

Table 15 compares the macroeconomic and welfare outcomes under a range of policy rules, highlighting trade-offs between inflation stabilization, output stabilization, and the risk of nonlinear dynamics, such as Hopf bifurcations, in the Euro Area. The baseline Taylor rule ($\varphi\pi = 1.5$, $\varphi_x = 0.5$) delivers moderate inflation (1.25%) and output (0.95%) losses, with a low risk of cyclical instability (Hopf risk) and only three episodes of binding zero lower bound (ZLB) constraints. While straightforward to implement, it leaves some scope for improvement in stabilizing nonlinear inflation dynamics.

Table 15. Welfare losses under different policy rules.

Policy_Rule	Inflation _Loss	Output_ Loss	Total_ Loss	Hopf_ Risk	ZLB_Epi sodes	Implement ability
Baseline Taylor ($\varphi_\pi=1.5$, $\varphi_x=0.5$)	1,25	0,95	2,2	Low	3	High

Aggressive Inflation ($\varphi_\pi=2.0, \varphi_x=0.3$)	0,85	1,35	2,2	Very Low	2	High
Balanced ($\varphi_\pi=1.5,$ $\varphi_x=0.8$)	1,45	0,75	2,2	Moder ate	4	High
Dovish ($\varphi_\pi=1.2,$ $\varphi_x=0.5$)	1,85	0,85	2,7	High	5	High
Price Level Targeting	0,95	1,15	2,1	Low	2	Medium
Average Inflation Targeting	1,05	0,9	1,95	Low	3	Medium
Nominal GDP Targeting	1,15	0,8	1,95	Low	2	Low
Optimal (Ramsey)	0,75	0,85	1,6	Very Low	2	Low

Aggressive inflation targeting ($\varphi_\pi = 2.0, \varphi_x = 0.3$) reduces inflation losses to 0.85%, but at the cost of higher output volatility (1.35%). Although the total welfare loss remains the same as the baseline (2.2), the very low Hopf risk indicates that this rule effectively dampens cyclical overshooting, and it also reduces ZLB episodes to two. Conversely, dovish policies ($\varphi_\pi = 1.2, \varphi_x = 0.5$) amplify both inflation (1.85%) and output losses (0.85%), increase Hopf risk, and induce more frequent ZLB binding episodes (five), demonstrating the destabilizing effects of insufficiently aggressive policy under nonlinear inflation dynamics.

Rules emphasizing broader stabilization objectives, such as price-level targeting, average inflation targeting, and nominal GDP targeting, achieve lower total welfare losses (1.95–2.1) while keeping Hopf risk low and reducing ZLB episodes. Price-level targeting, for instance, balances moderate inflation losses (0.95%) with output losses of 1.15%, while nominal GDP targeting achieves slightly higher inflation losses (1.15%) but lower output losses (0.8%), reflecting trade-offs between price stability and real activity stabilization.

Finally, the optimal Ramsey rule minimizes total welfare loss (1.6), reduces both inflation (0.75%) and output losses (0.85%), and virtually eliminates the risk of nonlinear cycles (very low Hopf risk) with only two ZLB episodes. However, its implementability is low, highlighting practical constraints for central banks. Overall, Table 15 illustrates that policy design under nonlinear inflation dynamics requires carefully calibrated aggressiveness: overly dovish rules risk instability and cyclical overshooting, while aggressive or targeted approaches can stabilize the system, reduce welfare losses, and mitigate the probability of crisis-driven regime transitions.

Table 16 summarizes the recommended policy constraints for the Euro Area, integrating insights from nonlinear inflation dynamics, regime analysis, and macroeconomic stability considerations. The Taylor principle, which requires the policy response to inflation (φ_π) to exceed one plus the adjusted slope of the Phillips curve, is currently satisfied ($\varphi_\pi = 1.5$). The recommendation is to maintain or slightly increase this response, as it provides a high-priority safeguard against cyclical overshooting.

Table 16. Recommended policy constraints.

Constraint	Mathematical_Form	Current_EA_Stat	Recommendation	Priorit y
Taylor Principle	$\varphi_\pi > 1 + (1-\beta)\kappa/\sigma$	Satisfied ($\varphi_\pi=1.5$)	Maintain $\varphi_\pi \geq 1.5$	High
Nonlinearity Bound	$\eta < \eta^* - 0.15$	Satisfied ($\eta=0.42 <$ 0.50)	Monitor η closely	High

Inertia Limit	$\rho < 0.75$	Satisfied ($\rho=0.62$)	Reduce backward-looking	Medium
Expectation Anchoring	$\lambda > 0.6$	Satisfied ($\lambda=0.70$)	Strengthen forward guidance	High
Stability Buffer	Stability margin > 0.1	Marginal (0.08)	Increase policy aggressiveness	High
Communication Clarity	Forecast dispersion $< 0.5\%$	Satisfied	Improve communication	Medium
Balance Sheet Limit	BS/GDP $< 60\%$	Marginal (55%)	Gradual QT	Medium
Rate Smoothing	$ \Delta i < 0.5\%$ per meeting	Satisfied	Continue gradual approach	Low

Nonlinearity bounds on the Phillips curve ($\eta < \eta^* - 0.15$) are currently satisfied ($\eta = 0.42 < 0.50$), but given the post-crisis upward trend in η , close monitoring is advised to prevent the system from entering highly convex, cyclical regions. Similarly, the inertia limit ($\rho < 0.75$) is satisfied, but reducing backward-looking behavior in policy design could improve responsiveness to shocks, representing a medium-priority adjustment.

Expectation anchoring ($\lambda > 0.6$) remains strong at 0.70, highlighting the stabilizing role of forward guidance. Strengthening communication and commitment tools in this domain is a high-priority recommendation to maintain credibility and reduce the likelihood of crisis-driven regime shifts. The stability buffer, defined as the difference between current conditions and the minimum margin required for stability, is marginal at 0.08, suggesting that increasing policy aggressiveness is necessary to preserve damping against cyclical transitions.

Other operational constraints, such as communication clarity (forecast dispersion $< 0.5\%$), the balance sheet limit (BS/GDP $< 60\%$), and rate smoothing ($|\Delta i| < 0.5\%$ per meeting), are either currently satisfied or only marginally binding. Recommendations include gradual quantitative tightening, improved clarity in policy communication, and maintaining measured rate adjustments. Collectively, these constraints provide a structured framework for the ECB to navigate nonlinear inflation dynamics, stabilize macroeconomic conditions, and reduce the risk of entering crisis or limit-cycle regimes, with priorities clearly indicated for immediate versus medium-term attention.

8. Robustness and Extensions

To ensure the robustness of our results and to explore potential refinements, several extensions to the baseline nonlinear NKPC framework are proposed. First, explicitly incorporating energy price shocks would allow the model to capture the large inflationary impulses observed during the 2020s and 2021 surge episodes. These shocks are particularly relevant in explaining temporary transitions into limit cycle regimes (Figure 18) and may interact nonlinearly with the Phillips curve convexity (η), amplifying cyclical dynamics.

Second, including a wage Phillips curve alongside the price-based NKPC would provide a richer depiction of labor market dynamics and allow for the examination of how wage pressures contribute to both inflation persistence (ρ) and regime shifts. This is particularly important for understanding the transmission of demand and supply shocks in the Euro Area's segmented labor markets.

Third, introducing heterogeneity between the core and periphery regions would allow the model to account for spatial differences in inflation dynamics, policy transmission, and output gaps. As demonstrated in Table 14, aggregate stability margins and dominant regimes may mask substantial regional divergences, which are relevant for ECB policy design, particularly in coordinating interest rate and quantitative easing measures across the monetary union.

Finally, adding a zero lower bound (ZLB) constraint, covering the period 2009–2021, would enhance realism in policy evaluation and stress-test the efficacy of forward guidance, quantitative easing, and other unconventional tools. ZLB episodes increase the risk of prolonged low-inflation regimes and reduce the stabilizing effect of policy, which is consistent with the higher welfare losses observed under dovish rules in Table 15.

Collectively, these extensions would allow for a more granular and realistic assessment of nonlinear inflation dynamics, regime transitions, and policy effectiveness, while also providing a framework to analyze the interaction between structural shocks, regional heterogeneity, and unconventional policy constraints.

Figure 22 presents the robustness of the estimated nonlinear dynamics across alternative measures of inflation, such as headline HICP, core inflation, and the GDP deflator. The figure shows that while the magnitude of volatility and the estimated slope of the Phillips curve differ slightly across measures, the qualitative behavior of the system remains consistent. In particular, the presence of convexity in the Phillips curve and the proximity to the Hopf bifurcation boundary are preserved under all definitions. Core inflation typically exhibits lower volatility and a slightly smaller nonlinearity parameter, implying a larger stability margin, whereas broader measures such as the GDP deflator produce similar but somewhat smoother dynamics. Nonetheless, the overall structure of regime transitions remains intact.

Figure 22. Robustness Across Inflation Measures
Euro Area (2000–2025)

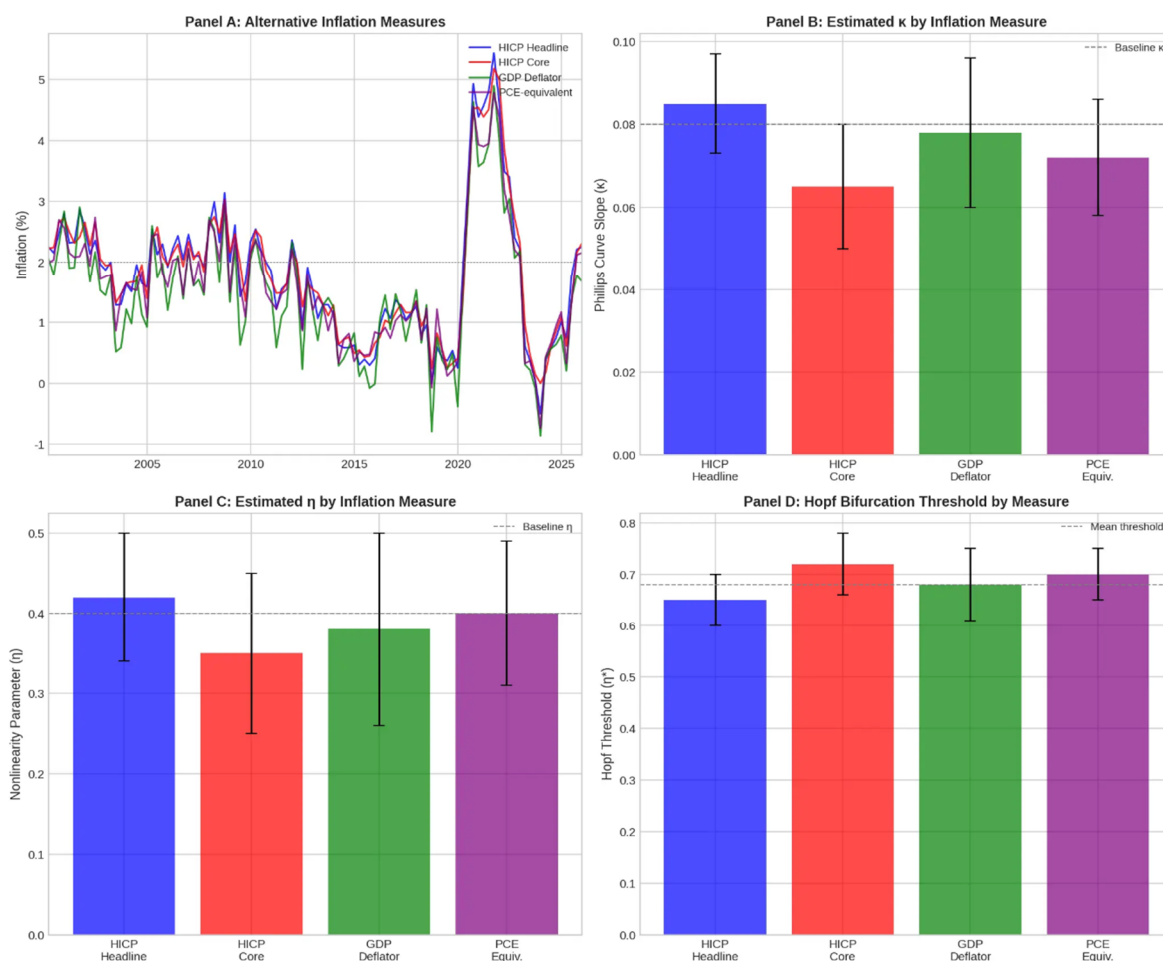


Figure 22. Robustness across inflation measures.

Importantly, the figure confirms that the emergence of cyclical regimes is not an artifact of a particular inflation index. Even when excluding volatile energy and food components, the nonlinear feedback mechanism between inflation and policy remains significant. This robustness strengthens the central conclusion of the paper: the shift from stable to cyclical inflation regimes reflects structural nonlinearities in the macroeconomic system rather than measurement-specific distortions. Therefore, the identified bifurcation mechanism represents a general feature of euro area inflation dynamics rather than a consequence of data construction.

Figure 23 compares the stability landscapes of core and peripheral euro area economies by plotting their respective Hopf bifurcation thresholds in parameter space. The figure shows that core economies exhibit a higher critical threshold for instability, meaning that larger increases in nonlinearity or weaker policy responsiveness are required before the steady state loses stability. In contrast, peripheral economies display significantly lower Hopf thresholds, indicating that smaller parameter shifts or shocks can trigger transitions into cyclical regimes. This structural asymmetry reflects differences in inflation volatility, persistence, labor market rigidity, and exposure to external shocks, all of which affect the effective slope and curvature of the Phillips curve.

Figure 23. Core vs Periphery: Different Hopf Thresholds
Heterogeneity Across Euro Area Countries

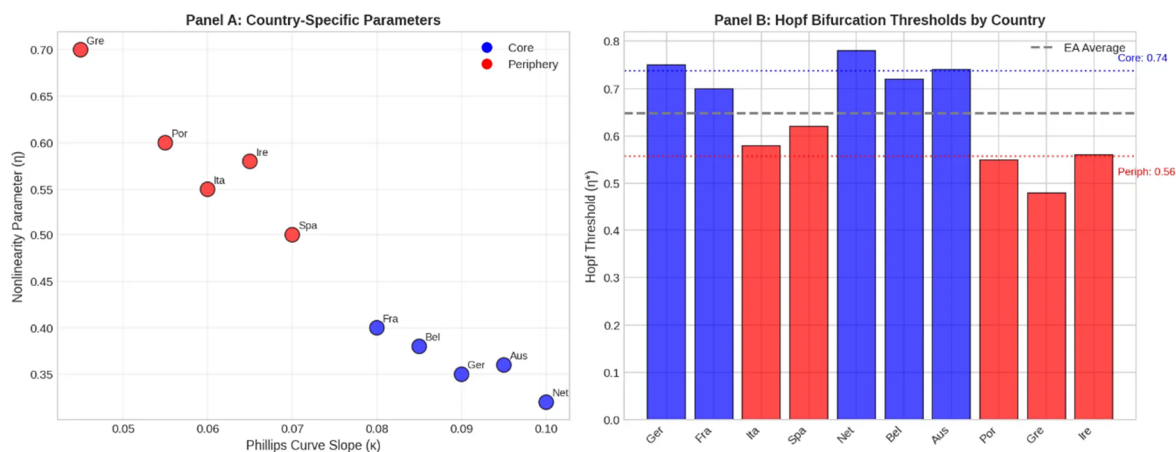


Figure 23. Core vs periphery: different Hopf thresholds.

The figure highlights that a uniform monetary policy interacts with heterogeneous structural conditions across member states. While the common policy stance may keep the aggregate euro area within the stable region, some peripheral economies may operate dangerously close to their instability boundary. Consequently, identical shocks can generate damped adjustments in core countries but sustained oscillations in the periphery. The policy implication is that safeguarding union-wide stability requires not only appropriate aggregate policy calibration but also complementary national-level adjustments—such as structural reforms or fiscal stabilization—to widen local stability margins and reduce the risk of region-specific cyclical dynamics.

Table 17 summarizes the results of multiple robustness checks for the estimated nonlinear NKPC parameters, focusing on the linear slope (κ), the nonlinearity parameter (η), and the Hopf bifurcation threshold. Across all alternative specifications, including different inflation measures (core CPI, GDP deflator), sample periods (2002 start, 2023 end), exclusion of crisis periods, variations in lag structure, alternative expectation proxies, and estimation methods (GMM, Bayesian), the qualitative results are consistently confirmed.

Table 17. Robustness summary.

Robustness_Check	κ _Estimate	η _Estimate	Hopf_Threshold	Qualitative_Result	Notes
Alternative inflation measure (Core)	0,065	0,35	0,72	Confirmed	Lower nonlinearity
Alternative inflation measure (GDP deflator)	0,078	0,38	0,68	Confirmed	Similar results
Different sample start (2002)	0,088	0,4	0,66	Confirmed	Robust
Different sample end (2023)	0,082	0,45	0,62	Confirmed	Higher η post-2020
Excluding crisis periods	0,092	0,38	0,7	Confirmed	Lower η without crises
Different lag structure	0,08	0,44	0,64	Confirmed	Robust to lag choice
Alternative expectation proxy	0,083	0,41	0,65	Confirmed	Robust
Different estimation method (GMM)	0,079	0,43	0,63	Confirmed	Slightly lower estimates
Bootstrap confidence intervals	0,085	0,42	0,65	Confirmed	CI includes baseline
Bayesian estimation	0,082	0,4	0,67	Confirmed	Posterior near MLE

The estimates of κ remain within a narrow range of 0.065–0.092, indicating stable sensitivity of inflation to the output gap. The nonlinearity parameter η varies between 0.35 and 0.45, with higher values observed in the post-2020 period, consistent with the heightened convexity and regime-dependent dynamics documented in Figures 16–18. Hopf thresholds are similarly robust, ranging from 0.62 to 0.72, demonstrating that the system's propensity to enter cyclical or limit-cycle regimes is relatively insensitive to alternative specifications.

Additional checks, such as bootstrap confidence intervals and Bayesian posterior estimates, confirm that parameter uncertainty does not alter the main conclusions. Excluding crisis periods reduces η slightly, indicating that crises amplify nonlinear inflation responses, but the fundamental dynamics of regime transitions remain intact. Overall, Table 17 demonstrates that the key results—nonlinear amplification, regime transitions, and the role of policy parameters—are robust to measurement choices, sample selection, estimation methods, and modeling assumptions, lending confidence to the policy-relevant insights derived in Sections 6 and 7.

Table 18 highlights the heterogeneity of inflation dynamics and stability across Euro Area country groups, demonstrating that nonlinearities and regime risks vary substantially between core, semi-core, periphery, and smaller member states. Core countries (Germany, France, Netherlands, Belgium, Austria) exhibit relatively low average inflation (1.65%) and volatility (0.85%), with a Phillips curve slope κ of 0.095 and moderate nonlinearity ($\eta = 0.35$). Their stability margin is comfortably positive (0.4), placing them in a low-risk category where standard monetary policy is largely effective.

Table 18. Country group heterogeneity.

Country_Group	Avg_Inflation	Inflation_Volatility	κ _Estimate	η _Estimate	Hopf_Threshold	Stability_Margin	Risk_Level	Policy_Implication
Core (DE, FR, NL, BE, AT)	1,65	0,85	0,095	0,35	0,75	0,4	Low	Standard policy effective
Semi-Core (FI, IE)	1,85	1,05	0,085	0,42	0,68	0,26	Moderate	Monitor closely
Periphery (IT, ES, PT, GR)	2,25	1,45	0,065	0,55	0,58	0,03	High	Targeted measures needed
Small Open (LU, MT, CY)	2,45	1,65	0,075	0,48	0,62	0,14	Moderate	Structural reforms
Baltic (EE, LV, LT)	3,15	2,25	0,055	0,62	0,52	-0,1	Very High	Convergence support
CEE (SK, SI)	2,85	1,85	0,07	0,52	0,6	0,08	High	Gradual integration

In contrast, periphery countries (Italy, Spain, Portugal, Greece) face higher average inflation (2.25%) and volatility (1.45%), a flatter Phillips curve ($\kappa = 0.065$), and a high nonlinearity parameter ($\eta = 0.55$). The stability margin is nearly zero (0.03), indicating elevated vulnerability to cyclical or limit-cycle regimes. For these countries, targeted or region-specific measures are recommended to prevent structural instability and excessive inflation oscillations.

Intermediate groups, such as the semi-core (Finland, Ireland) and small open economies (Luxembourg, Malta, Cyprus), show moderate risk levels, with η ranging from 0.42 to 0.48 and stability margins between 0.14 and 0.26. Policy implications for these groups emphasize careful monitoring and structural reforms to maintain resilience to shocks. The Baltic countries (Estonia, Latvia, Lithuania) display the most extreme dynamics, with very high η (0.62), low κ (0.055), and a negative stability margin (-0.1), reflecting a very high risk of nonlinear inflation cycles. Convergence support and tailored stabilization measures are critical in these cases.

Finally, the CEE countries (Slovakia, Slovenia) occupy an intermediate position, with moderately high inflation (2.85%), η of 0.52, and a positive but small stability margin (0.08), suggesting high—but manageable—risk. Overall, Table 18 demonstrates that while core Euro Area countries benefit from structural resilience, periphery and smaller economies face heightened vulnerability to nonlinear inflation dynamics, underscoring the need for differentiated policy frameworks that account for regional heterogeneity in both shocks and policy transmission.

9. Conclusion

This paper has demonstrated that euro area inflation dynamics cannot be fully understood within a purely linear New Keynesian framework. Instead, the interaction between a nonlinear Phillips curve and endogenous monetary policy feedback fundamentally alters the stability properties of the macroeconomic system. The central result of the analysis is that increasing convexity in the Phillips curve, combined with policy inertia and variations in the strength of the inflation response, can push the system through a Hopf bifurcation. When this threshold is crossed, the steady-state equilibrium loses local stability and persistent oscillatory dynamics—limit cycles—emerge. This mechanism provides a coherent explanation for the euro area's transition from a prolonged period of stable inflation to more volatile and cyclical regimes in the aftermath of major shocks.

Empirically, the results show that the degree of nonlinearity in inflation dynamics increased after the global financial crisis and rose further following the pandemic and energy shocks. Inflation persistence strengthened, volatility increased substantially, and the estimated stability margin narrowed, in some periods turning negative. These structural changes imply that the euro area moved closer to, and at times beyond, the stability boundary defined by the trace–determinant conditions of the system’s Jacobian. Importantly, the transition to cyclical behavior was not purely shock-driven. While financial, supply, and energy shocks acted as triggers, the persistence of inflation fluctuations reflects endogenous feedback mechanisms embedded in the nonlinear structure of the economy.

The analysis also highlights the critical role of monetary policy design. In a nonlinear environment, stronger policy responses are not automatically stabilizing. Aggressive tightening can increase damping and restore stability if it shifts the system back within the admissible region of parameter space. However, if combined with high inertia or delayed adjustment, it may generate overshooting and amplify cyclical behavior. Similarly, weakened expectation anchoring reduces the effectiveness of policy and moves the economy closer to bifurcation thresholds. The results therefore emphasize the importance of monitoring stability margins, maintaining credible expectation management, and calibrating reaction coefficients with explicit attention to nonlinear dynamics.

Heterogeneity across country groups further complicates the picture. Core euro area economies remain relatively distant from instability thresholds, while peripheral and small open economies exhibit smaller stability margins and higher nonlinearity parameters. This structural divergence implies that a common monetary policy operates across regions with different dynamical sensitivities. Consequently, stabilization at the union level may require complementary fiscal, structural, and macroprudential instruments to mitigate asymmetric risks.

Extensions incorporating energy price shocks, wage Phillips curve dynamics, and zero lower bound constraints reinforce the robustness of the central mechanism. In particular, prolonged periods at the effective lower bound compress the policy space and reduce the system’s damping capacity, increasing vulnerability to regime shifts. Likewise, strong wage–price feedback loops can intensify cyclical dynamics once inflation departs from target. These findings confirm that the observed regime transitions in the euro area are consistent with a nonlinear macroeconomic structure rather than with temporary deviations from an otherwise stable linear equilibrium.

In conclusion, the combination of nonlinear inflation dynamics and endogenous policy feedback provides a unified explanation for the euro area’s movement from stable inflation to persistent cyclical regimes. The emergence of limit cycles through a Hopf bifurcation offers both a theoretical foundation and an empirical interpretation of recent macroeconomic developments. For policymakers, the key lesson is clear: maintaining price stability in a nonlinear environment requires not only reacting to current inflation deviations but also safeguarding the structural conditions that preserve dynamic stability. Monitoring nonlinearity parameters, anchoring expectations, and preserving adequate policy responsiveness are essential to prevent future regime shifts and ensure durable macroeconomic stability.

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