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Article

The Blackbody CMB Temperature, Luminosity, and Their Relation to Black Hole Cosmology, Compared to Bekenstein-Hawking Luminosity

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Abstract: We use the Stefan-Boltzmann law to derive and investigate what we will call the black hole CMB temperature luminosity. Interestingly, the CMB luminosity is always the same for every black hole at a luminosity distance equal to the Schwarzschild radius, which is fully valid in black hole cosmology, a topic that is actively discussed to this day. The CMB black hole luminosity is always $L_{cmb} = \sigma T_p^4 \frac{l_p^2}{256\pi^3}$, which can also be written in the form $L_{cmb} = \frac{c^6}{15360\pi} \frac{\hbar}{G^2 m_p^2}$, regardless of the mass of the black hole. This is in contrast to the Bekenstein-Hawking luminosity, which is linked to the Hawking temperature, that is mass dependent and equal to $L_{Hw} = \frac{c^6}{15360\pi} \frac{\hbar}{G^2 M^2}$.

Keywords: CMB temperature luminosity; black hole luminosity

1. Introduction

Black holes are considered near-perfect black bodies because they absorb all the radiation that falls on them. It has been suggested that black holes emit blackbody radiation known as Hawking radiation, which depends on the mass of the black hole. However, Hawking radiation has likely never been observed, possibly due to the difficulty in detecting it. Nevertheless, we will briefly examine it here, as the equation and derivation for Bekenstein-Hawking luminosity share many similarities with what we will coin as: "CMB luminosity" or alternatively "internal black hole luminosity". We will mathematically explore Hawking luminosity in section 2 and then CMB luminosity in section 3 and 4.

Additionally to Hawking temperature that is assumed to be a black body temperature, there is another black body temperature associated with black holes and cosmology in general including the Λ -CDM model, even if the Λ -CDM model possibly will fail short for what we will discuss in this paper, so called $R_H = ct$ cosmological models and particular growing black-hole $R_H = ct$ will much better fit what we will describe, see [1–7]. In various cosmological models, it is assumed that the Hubble sphere could be a black hole. This idea dates back to at least 1972, as suggested by Pathria [8], and was later revisited by Stuckey [9] in 1994. Although the Λ -CDM model community may overlook the possibility that the Hubble sphere is a black hole, many researchers continue to work on or discuss black hole cosmology to this day, as shown by [10–13]. Assuming, for a moment, that the Hubble sphere is a black hole, we then have a significant amount of information about the interior of a black hole. Furthermore, there is another type of black body radiation associated with the interior of black holes. The CMB temperature is often linked with a black body temperature; for example, Muller et al. in [14] state (see also [15]):

"Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057$ K."

In recent years, there has been tremendous progress in understanding the CMB (Cosmic Microwave Background) temperature from a practical perspective, particularly in terms of more

accurate measurements, as cited in [16–19]. Additionally, there has been a deeper theoretical understanding of the CMB temperature and its relationship to black hole cosmology. In section 3, we will utilize this knowledge to derive what we will refer to as the CMB luminosity. This luminosity should, in theory, apply to the interiors of all or at least most black holes, even though there are likely to be modifications for different metrics.

2. Bekenstein–Hawking luminosity of a black hole

The luminosity of a black body is given from the Stefan-Boltzman law [20,21] and is:

$$L = 4\pi R^2 \sigma T^4 \quad (1)$$

where σ is the Stefan-Boltzman constant, T is the black body temperature, and R is the luminosity distance.

For luminosity directly related to a black hole, we will set the luminosity radius equal to the black hole's event horizon radius and the temperature T to the black hole's temperature. There are actually two temperatures associated with black holes. One is the Bekenstein-Hawking temperature, as given by [22], which is described as follows:

$$T_{Hw} = \frac{\hbar g}{ck_b 2\pi} = \frac{\hbar \frac{GM}{r_s^2}}{ck_b 2\pi} = \frac{\hbar c}{k_b 4\pi R_s} \quad (2)$$

Next we input this in the Stefan-Boltzman luminosity equation:

$$\begin{aligned} L_{Hw} &= 4\pi R_s^2 \sigma T_{Hw}^4 \\ L_{Hw} &= 4\pi R_s^2 \sigma \left(\frac{\hbar c}{k_b 4\pi R_s} \right)^4 \\ L_{Hw} &= \sigma \frac{\hbar^4 c^4}{k_b^4 64\pi^3 R_s^2} \end{aligned} \quad (3)$$

and since the Stefan-Boltzman constant is $\sigma = \frac{2\pi^5 k_b^4}{15c^2 \hbar^3}$, if we insert this in above equation we get:

$$\begin{aligned} L_{Hw} &= \frac{2\pi^5 k_b^4}{15c^2 \hbar^3} \frac{\hbar^4 c^4}{k_b^4 64\pi^3 R_s^2} \\ L_{Hw} &= \frac{\hbar c^2}{3840\pi R_s^2}. \end{aligned} \quad (4)$$

Further since $R_s = \frac{2GM}{c^2}$ we can also re-write this as

$$L_{Hw} = \frac{\hbar c^6}{15360\pi G^2 M^2}. \quad (5)$$

This is the well-known Bekenstein-Hawking luminosity of a black hole. Please note that the Bekenstein-Hawking luminosity scales with the inverse square law of both the black hole's radius and its corresponding mass.

3. CMB luminosity of a black hole is the same for every black hole

Haug and Wojnow [23] has derived the CMB temperature from the Stefan-Boltzman law and got the formula:

$$T_{cmb} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_H}} \approx 2.72k. \quad (6)$$

where T_p is the Planck [24,25] temperature $T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b}$ (see also [26]). This formula holds for any Schwarzschild black hole and in more general forms is given by

$$T_{cmb} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_s}} \approx 2.72k. \quad (7)$$

This correspond also to the formula given by Tatum et al. [27], who likely is the first to obtained a formula for predicting the CMB temperature using heuristic methods that we soon also will discuss. Furthermore, Haug and Tatum [28] derived the CMB temperature from a very general geometric mean of the shortest and longest possible wavelengths in a black hole. In addition, Haug [37] has derived the same formula from quantized light bending related to black holes.

This can be generalized for any Schwarzschild [29] black hole:

$$T_{cmb} = T_{Hw} \sqrt{\frac{R_s}{2l_p}} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_s}} \approx 2.72k. \quad (8)$$

where T_{cmb} is the black body temperature associated with the black hole, the temperature on the inside of the black hole. Therefore, the term 'cosmic microwave background' might be misleading since smaller black holes will have a wavelength spectrum for their temperature that consists of much shorter wavelengths than microwaves, so alternatively we could just have called it the inside temperature of a black hole. The luminosity for any black hole, derived from the internal black body temperature, is then given by:

$$\begin{aligned} L_{cmb} &= 4\pi R_s^2 \sigma \left(\frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_s}} \right)^4 \\ L_{cmb} &= 4\pi R_s^2 \sigma \frac{T_p^4}{8^4 \pi^4} \frac{4l_p^2}{R_s^2} \\ L_{cmb} &= 4\pi \sigma T_p^4 \frac{4l_p^2}{8^4 \pi^3} \\ L_{cmb} &= \sigma T_p^4 \frac{l_p^2}{256\pi^3}. \end{aligned} \quad (9)$$

What is remarkable here is that we observe the black body luminosity as constant no matter the size of the black hole, and furthermore, it is only a function of the Planck scale. The black hole mass is not relevant. We can conclude that all black holes have the same CMB luminosity, despite the CMB temperature scaling with $\frac{1}{\sqrt{R_s}}$ for different black holes. Therefore, the temperature inside each black hole varies between black holes with different size, but the CMB luminosity at a distance of the Schwarzschild radius for the black hole of interest remains the same for all black holes.

Furthermore, since the Stefan-Boltzmann constant is given by $\sigma = \frac{2\pi^5 k_b^4}{15c^2 \hbar^3}$, we can also rewrite equation 9 as:

$$\begin{aligned} L_{cmb} &= \frac{\pi^2 k_b^4 l_p^2}{1920 c^2 \hbar^3} T_p^4 \\ L_{cmb} &= T_p^4 \frac{k_b^4 l_p^2}{15360 c^2 \pi \hbar^3}. \end{aligned} \quad (10)$$

Alternatively we can derive the black hole CMB luminosity by using the first formula presented by Tatum et. al [27] and later derived by Haug and Tatum [30] from a geometric mean approach, we get:

$$\begin{aligned} L_{cmb} &= 4\pi R_s^2 \sigma \left(\frac{\hbar c}{k_b 4\pi \sqrt{R_s 2l_p}} \right)^4 \\ L_{cmb} &= 4\pi R_s^2 \sigma \frac{\hbar^4 c^4}{k_b^4 4^4 \pi^4 R_s^2 4l_p^2} \\ L_{cmb} &= \sigma \frac{\hbar^4 c^4}{k_b^4 256 \pi^3 l_p^2} \end{aligned} \quad (11)$$

as the Stefan-Boltzman constant $\sigma = \frac{2\pi^5 k_b^4}{15c^2 h^3}$ and we can now insert this into the equation above and we get

$$\begin{aligned} L_{cmb} &= \frac{2\pi^5 k_b^4}{15c^2 h^3} \frac{\hbar^4 c^4}{k_b^4 256 \pi^3 l_p^2} \\ L_{cmb} &= \frac{\hbar c^2}{15360 \pi l_p^2} \\ L_{cmb} &= \frac{\hbar c^6}{15360 \pi G^2 m_p^2} \approx 7.50338 \times 10^{47} \text{ W} \end{aligned} \quad (12)$$

where m_p is the Planck mass. Equations 9, 10 and 12 are ultimately the same, it is just different ways to express the same formula. What is important is that the CMB luminosity of any black hole is the same. This means every black hole has a luminosity of $7.50338 \times 10^{47} \text{ W}$.

This is also fully consistent with the observed relation between the cosmological red-shift and the CMB temperature that is assume to follow of $T_t = T_0(1+z)$ and $z = \frac{T_t}{T_0} - 1$ (see for example [31]). This mean we can replace this back into the formula and we get

$$\begin{aligned} L_{cmb} &= 4\pi R_t^2 \sigma (T_0(1+z))^4 \\ L_{cmb} &= 4\pi R_t^2 \sigma T_0^4 \left(1 + \frac{T_t}{T_0} - 1 \right)^4 \\ L_{cmb} &= 4\pi R_t^2 \sigma T_0^4 \left(\frac{T_t}{T_0} \right)^4 \\ L_{cmb} &= 4\pi R_t^2 \sigma T_t^4. \end{aligned} \quad (13)$$

As $R_t = R_s$ for a Schwarzschild black hole in a $R_s = R_H = ct$ type cosmological growing black hole models then $T_t = \frac{\hbar c}{k_b 4\pi \sqrt{R_t 2l_p}}$ as pointed out by Tatum et. al [27] and Haug and Tatum [28] this leads to exactly the luminosity equations as we have derived above. That is for the first time we have demonstrated that any black hole has a the same CMB luminosity if measured from the Schwarzschild radius distance of the black body. This naturally when the luminosity distance is set equal to the Schwarzschild radius.

4. The CMB temperature in yet another perspective

We can naturally also solve the standard Stefan-Boltzman luminosity equation with respect to the temperature T , this gives the well known formula:

$$T = \sqrt[4]{\frac{L}{4\pi R^2 \sigma}}. \quad (14)$$

As we have found that the black hole luminosity is the same for every Schwarzschild black hole and is given by: $L_{cmb} = \frac{\hbar c^6}{15360\pi G^2 m_p^2}$, this means we for any Schwarzschild black hole we must have a CMB temperature (on the inside) of:

$$\begin{aligned} T_{cmb} &= \sqrt[4]{\frac{L_{cmb}}{4\pi R_s^2 \sigma}} \\ T_{cmb} &= \sqrt[4]{\frac{\frac{\hbar c^6}{15360\pi G^2 m_p^2}}{4\pi R_s^2 \sigma}} \\ T_{cmb} &= \sqrt[4]{\frac{\hbar c^6}{61440\pi^2 G^2 m_p^2 R_s^2 \sigma}} \\ T_{cmb} &= \frac{1}{\sqrt{\pi G m_p R_s}} \sqrt[4]{\frac{\hbar c^6}{61440\sigma}}. \end{aligned} \quad (15)$$

In the case we set the Schwarzschild radius to the Hubble radius we get $R_s = R_H$ which is the case in the critical Friedmann [32] universe we get:

$$T_{cmb} = \frac{1}{\sqrt{\pi G m_p R_H}} \sqrt[4]{\frac{\hbar c^6}{61440\sigma}} \approx 2.72_{-0.069}^{+0.082} K \quad (16)$$

we have here used a Hubble parameter in calculation of $R_H = \frac{c}{H_0}$ given by the recent published study by Kelly and et. al [33] of $66.6_{-3.3}^{+4.1} (km/s)/Mpc$. However different Hubble parameter studies give considerably different Hubble constant values, so the exact value will naturally vary with input, see for example [34–36].

We can naturally also re-write equation 16, by replacing R_s with $R_s = \frac{2GM}{c^2}$ this gives:

$$\begin{aligned} T_{cmb} &= \sqrt[4]{\frac{\hbar c^6}{61440\pi^2 G^2 m_p^2 \left(\frac{2GM}{c^2}\right)^2 \sigma}} \\ T_{cmb} &= \sqrt[4]{\frac{\hbar c^{10}}{245760\pi^2 G^4 m_p^2 M^2 \sigma}} \\ T_{cmb} &= \frac{1}{G\sqrt{\pi m_p M}} \sqrt[4]{\frac{\hbar c^{10}}{245760\sigma}}. \end{aligned} \quad (17)$$

If one now input the critical Friedmann mass of the universe $M = M_c = \frac{c^3}{2GH_0^2}$, we get again $T_{cmb} \approx 2.72K$. Let us next input the for the Stefan-Boltzman constant: $\sigma = \frac{2\pi^5 k_b^4}{15c^2 h^3}$ and we get:

$$\begin{aligned}
T_{cmb} &= \frac{1}{G\sqrt{\pi m_p M}} \sqrt[4]{\frac{\hbar c^{10}}{245760 \frac{2\pi^5 k_b^4}{15c^2 \hbar^3}}} \\
T_{cmb} &= \frac{1}{G\sqrt{\pi m_p M}} \sqrt[4]{\frac{\hbar c^{12}}{4096 \frac{\pi^2 k_b^4}{\hbar^3}}} \\
T_{cmb} &= \frac{\hbar c^3}{k_b 8\pi G \sqrt{M m_p}} = \frac{\hbar c}{k_b 4\pi \sqrt{R_s 2l_p}} \quad (18)
\end{aligned}$$

This is the same formula as heuristically suggested by Tatum et. al [27], which has recently been derived from the Stefan-Boltzmann law by Haug and Wojnow [23], and by the geometric mean approach by Haug and Tatum [28], and also by quantization of light bending by Haug [37]. Despite possibly, and even likely, not fitting into the Λ -CDM model, we think it can be no coincidence that the CMB formula first heuristically suggested by Tatum et. al can just be a coincidence; it predicts the right CMB temperature very accurately compared to measurements and have now been derived from a series of different complementary angles. We think cosmologies would likely benefit from taking notice of it. That said we naturally do not claim to have all the answers. There is certainly more to investigate here in relation to both theoretical CMB temperature and CMB luminosity; for example, there are multiple metrics that can be applied to black holes that should be investigated, such as the Reissner-Nordström [38,39] metric, the Kerr [40] metric, the Kerr-Newman [41,42] metric, and the recent Haug-Spavieri [43] metric, as well as other metrics used in cosmology and for black holes.

Tatum et. al [44] has clearly demonstrated that one can also use the CMB temperature formula, and when solved for H_0 , it can dramatically reduce the standard deviation in uncertainty from other methods. This is because we now, for the first time, have a much deeper theoretical understanding of the relation between CMB temperature and the Hubble parameter. The Λ -CDM model has, on the other hand, not developed any method to theoretically predict the CMB temperature now. All they have is the formula to predict the CMB temperature back in time based on the formula $T_t = T_0(1+z)$, which is also fully consistent with this work.

There are also other challenges that could be related to this and that could possibly be resolved with a deeper understanding of CMB, namely the Hubble tension [45,46]. In our model, we must have $z = \sqrt{\frac{R_H}{R_t}} - 1 = \frac{T_0}{T_t} - 1$, as recently pointed out by Haug and Tatum [30].

5. Black hole internal radiation pressure

The total energy per unit of time per unit of surface area that is radiated by a black body maintained at a temperature T is according to Stefan-Boltzmann law the radiation pressure given by:

$$P = \frac{4\sigma T^4}{3c} = \frac{L}{3\pi R^2 c} \quad (19)$$

Next we will derive the black hole internal radiation pressure that also can be called the CMB radiation pressure as it can be derived from what we called the CMB luminosity:

$$\begin{aligned}
P_{cmb} &= \frac{L_{cmb}}{3\pi R_s^2 c} \\
P_{cmb} &= \frac{\sigma T_p^4 l_p^2}{768\pi^4 R_s^2 c} \quad (20)
\end{aligned}$$

So the black hole radiation pressure follow the inverse square law rule, unlike the CMB luminosity that is the same for every black hole. Equation 20 can also be re-written as:

$$P_{cmb} = \frac{\sigma T_p^4 l_p^2 c^3}{768\pi^4 G^2 M^2} \quad (21)$$

In the special case where M is the critical mass of the Friedmann universe $M = M_c = \frac{c^2}{2GH_0}$ we get a radiation pressure of:

$$P_{cmb} = \frac{\sigma T_p^4 l_p^2 c^3}{3072\pi^4 G^2 M_c^2} \approx 1.3909 \times 10^{-14} \text{ J} \cdot \text{m}^{-3} \quad (22)$$

If we replace the Stefan-Boltzmann constant with its components: $\sigma = \frac{2\pi^5 k_b^4}{15c^2 h^3}$, we get:

$$\begin{aligned} P_{cmb} &= \frac{\frac{2\pi^5 k_b^4}{15c^2 h^3} T_p^4 l_p^2 c^3}{3072\pi^4 G^2 M^2} \\ P_{cmb} &= \frac{\frac{2\pi^5 k_b^4}{15c^2 h^3} \frac{h^4 c^4}{l_p^4 k_b^4} l_p^2 c^3}{3072\pi^4 G^2 M^2} \\ P_{cmb} &= \frac{\hbar c^5}{184320\pi^2 G^2 M^2 l_p^2} \end{aligned} \quad (23)$$

or alternatively we can write this as

$$P_{cmb} = \frac{\hbar c}{46080\pi^2 R_s^2 l_p^2}. \quad (24)$$

This is valid for any Schwarzschild black hole, including the Hubble sphere when one considers it as a Schwarzschild black hole with $R_H = R_s$, which is the case in the critical Friedmann universe. The critical Friedmann mass is given by $M_c = \frac{R_H c^2}{2G}$. Solving for R_H gives $R_H = \frac{2GM_c}{c^2}$, which is naturally mathematically identical to the Schwarzschild radius.

6. Conclusion

We have demonstrated that there is another form of black body radiation or luminosity associated with every black hole. We have named this the CMB luminosity, even though it applies to any black hole. The CMB luminosity is identical for every black hole and is always given by $L_{cmb} = \sigma T_p^4 \frac{l_p^2}{256\pi^4}$, which can also be expressed as $L_{cmb} = \frac{c^6}{15360\pi} \frac{\hbar}{G^2 m_p^2}$. It depends solely on the Planck scale and is not affected by the mass of the black hole. In addition we show that the CMB radiation pressure for any Schwarzschild black hole is given by $P_{cmb} = \frac{\hbar c}{46080\pi^2 R_s^2 l_p^2}$. This discovery can have significant implications for both cosmology and other aspects of black hole research.

Conflicts of Interest: The authors declares no conflict of interest.

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