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Wave Function Collapse and Spontaneous Symmetry Breaking

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Abstract: In this paper, I use the spontaneous symmetry breaking mechanism as a supplement to the quantum decoherence theory to explain the wave function collapse after measurements. I show that the ergodicity breaking turns the mixed states into a certain eigenstate and thus we always measure just one particular outcome.

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1. Introduction

Quantum mechanics, the foundation of modern physics, presents revolutionary insights into the nature of reality. In addition to the mathematical framework of quantum mechanics, the "axioms of quantum mechanics" (also known as the Copenhagen Interpretation) are based on two central tenets:

1. The wave function describes the quantum state of a system, and until measurement occurs, the system exists in a superposition of all possible states.

2. The measurement of an observable leads to the collapse of the wave function and yields one of its eigenvalues as the result.

However, despite its remarkable predictive success, quantum mechanics continues to raise fundamental philosophical questions, particularly concerning the interpretation of the theory and the measurement process. The idea of wave function collapse, though widely accepted, remains mysterious because it seems to contradict the deterministic evolution of quantum states according to the Schrödinger equation, which governs the behavior of quantum systems in the absence of measurement. This rather strange aspect is known as the quantum measurement problem, where quantum systems have superpositions as predicted by the Schrödinger equation but quantum measurements always give definite outcomes.

Several potential resolutions to the measurement problem have been proposed, each of which challenges or modifies aspects of the Copenhagen interpretation. An alternative to the Collapse Postulate is the De Broglie-Bohm theory (also known as Bohmian Mechanics or Pilot-Wave theory) [1], which states that particles have definite positions at all times, and these positions are guided by a wave function. While De Broglie-Bohm theory provides an elegant interpretation, it may suffer from more severe problems than its failure to solve the quantum measurement problem. Another alternative interpretation is the Many-Worlds Interpretation (MWI) [2], which states that all possible outcomes of quantum measurements actually occur, but in separate non-interacting branches of the universe. Like De Broglie-Bohm theory, it is a reinterpretation of the mathematical framework of quantum theory, meaning that it cannot be directly tested through experiments that distinguish between different interpretations. Some other strategies to explain away the measurement problem are to argue that the collapse of the wave function is not triggered by measurements but occurs spontaneously due to some intrinsic property of quantum systems. These models include the Ghirardi–Rimini–Weber (GRW) model [3] and continuous spontaneous localization (CSL) model [4]. The GRW theory proposes that the collapse occurs at random intervals due to an objective feature of the physical world rather than something induced by measurement. In the CSL theory, the Schrödinger equation is supplemented with additional nonlinear and stochastic terms and this modification induces the collapse of the wave function. However, these theories currently lacks experimental evidence. Instead of introducing some vague concept of the unknown environmental degrees of freedom, Penrose (and Diósi, independently)

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suggested that the wave function collapse is induced by the gravity, the so-called DP model [5–8]. The wave function describing the state of a quantum system progressively loses its validity when the mass of the system becomes large enough. Although the DP model is the most influential model including gravity, it appears to have been ruled out in recent experiments [9].

It is worth mentioning that all models listed so far cannot be described within quantum mechanics. One exception is the quantum decoherence theory [10–13]. In quantum mechanics, as long as there exists a definite phase relation between the components of the superposition, the system is said to be coherent and exhibits interference effects, as seen in the famous double-slit experiment. But as soon as a system becomes entangled with the environment including the measurement apparatus, the information about the relative phases between the quantum states leaks into the environment and becomes delocalized, leading to the destruction of quantum coherence. The main result of decoherence is a reduced density matrix with the Born probabilities on the diagonal. It explains the conversion of the quantum probabilities that exhibit interference effects to the ordinary classical probability distribution of a mixed state (a classical mixture). However, quantum decoherence does not describe the actual collapse of the wave function and answer why nature always picks a specific outcome [14–16].

Thus, we can still pose these questions: What happens to the wave function during measurement? How does a system transition from a superposition of states to a definite outcome? Is there a physical mechanism that governs the collapse of the wave function? In this paper, we propose a mechanism as a supplement to the decoherence theory to provide a solution to the measurement problem. We argue that the basic origin of the wave function collapse is the same as the spontaneous symmetry breaking (SSB) [17]. Quantum phase transitions and SSB are fundamental concepts in quantum condensed matter physics, connecting microscopic quantum mechanics with macroscopic properties of materials. When the temperature approaches the quantum critical point, the states of the system might be effectively decoupled by a large energy barrier separating them. To interconvert between the different states and hence sample them in the ensemble average, we would need require to quantum mechanically tunnel through this large barrier. The wider and the higher the potential energy barrier separating two states, the longer it takes to quantum mechanically tunnel between them. The time scale for the tunneling is typically extremely long and it will take an infinitely long time to get to a different region of the phase space. Therefore, SSB occurs as the system gets stuck in a certain state. The idea is that the validity of the ensemble average of an observable is based on the premise of the Ergodic Principle. We show that the violation of the Ergodic Principle leads to the collapse of the wave function. This means that quantum mechanics is itself applicable to the description of measurements without additional assumptions.

2. Wave Function Collapse and Spontaneous Symmetry Breaking

The decoherence theory makes full use of quantum entanglement and the reduced density matrix to explain the loss of coherence of an open quantum system. Suppose a system of interest S is coupled with a measurement apparatus or an environment E. For simplicity, we assume that $|S\rangle$ is a superposition of two states: $|S\rangle = c_1 |S_1\rangle + c_2 |S_2\rangle$. Assume that before the interaction, the system is correlated with the environment in the following way: $|S_1\rangle |E_1\rangle$ and $|S_2\rangle |E_2\rangle$. Then the interaction leads to the final system-environment entangled state:

$$|\Psi\rangle = c_1 |S_1\rangle |E_1\rangle + c_2 |S_2\rangle |E_2\rangle . \tag{1}$$

In the case where the system and the environment are quantum entangled, an observer can only perform local measurements on the system S. The expectation value of an observable $\hat{O}_S$ is calculated using the reduced density matrix:

$$\hat{\rho}_S = \mathrm{Tr}_E \left(\hat{\rho}_{SE} \right) , \tag{2}$$

where $\hat{\rho}_{SE} = |\Psi\rangle\langle\Psi|$ is a density matrix describing the pure composite state. The $\hat{\rho}_S$ off-diagonal elements containing information about the relative phase of the system are determined by the overlap

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$\langle E_2|E_1\rangle$, i.e., by the distinguishability of $|E_1\rangle$ and $|E_2\rangle$. Environment-induced decoherence leads to the gradual diagonalization of $\hat{\rho}_S$, and we eventually obtain a diagonal reduced density matrix. Thus we are led to an expression for the expectation value of the observable of the two-level system:

$$\langle \hat{O}_S \rangle = \mathrm{Tr}\left(\hat{\rho}_S \hat{O}_S\right) = \sum_{i=1}^{2} p_i \langle S_i|\hat{O}_S|S_i\rangle, \tag{3}$$

where $p_1 = |c_1|^2$ and $p_2 = |c_2|^2$. In general, let $\{|S_i\rangle\}$ be a basis of the system Hilbert space $\mathcal{H}_S$ and $\{|E_i\rangle\}$ be that of the environment Hilbert space $\mathcal{H}_E$. Then $|S\rangle$ can be expressed as a linear combination of these n eigenvectors:

$$|S\rangle = \sum_{i=1}^{n} c_i\,|S_i\rangle\,. \tag{4}$$

Thus, Equation (3) becomes

$$\langle \hat{O}_S \rangle = \sum_{i=1}^{n} p_i \langle S_i|\hat{O}_S|S_i\rangle, \tag{5}$$

where $p_i = |c_i|^2$ is the Born probability. We now arrive at the central result of the quantum decoherence that open systems are effectively described by diagonal reduced density matrices, generally called the "improper mixture." An "improper mixture" refers to that we view the pure state of the total entangled system as an effective mixed state for the subsystem while the usual classical mixture is an ensemble of pure states. Although they are defined differently, no physically realizable measurements can distinguish between a classical mixture ensemble and an "improper mixture" ensemble described by the same density matrix.

Indeed, to derive the expression for the expectation value of the observable, we have made the commonly used assumption in statistical mechanics that the average of an observable over time and the average over states in phase space at a given instant in time (also known as the ensemble average) are the same. However, this assumption is reasonable based on the validity of the Ergodic Principle, which states that all states of the system are accessible and eventually explored in the dynamical evolution of the system. An ergodic system requires that different states in phase space are allowed to interconvert, and therefore we can sample all states in our ensemble average. However, in the case of the SSB, the Ergodic Principle does not hold (also known as ergodicity breaking) and the formula (5) of the ensemble average is therefore incorrect. An example of SSB is that the states of ferromagnetic materials might be effectively decoupled by a large energy barrier separating them below the Curie temperature. To interconvert between the different states and hence sample them in the ensemble average, we would need require to quantum mechanically tunnel through this large barrier. But it will take an infinitely long time to get to a different region of the phase space due to the lack of energy at low temperatures. The averages over a finite amount of time and therefore not necessarily equal to the averages over all states in phase space at an instant in time. In this case, the phase space becomes fragmented and we should compute our ensemble expectation values using only a part of the phase space.

Thus, the key point is whether different eigenstates of a quantum system can interconvert after quantum decoherence due to a measurement. Since the interference is suppressed as the system undergoes decoherence, the system becomes a classical object. As a result, it is impossible to get to a different eigenstate at the classical level. This means that the system in a mixed state gets stuck in a certain eigenstate $|S_i\rangle$ with the corresponding probability. Therefore, actual measurements always find the physical system in a definite state. In this sense, the "improper mixture" and usual classical mixture are fundamentally different.

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3. Conclusions

In this paper, I use the SSB mechanism to overcome the shortcomings of the decoherence theory in explaining the measurement problem. It is actually a new supplementation of the decoherence theory. Decoherence turns pure states into mixed states that follow classical probabilities and we always measure just one particular outcome due to the ergodicity breaking. Based on these considerations, it is possible that the evolution of a quantum system including measurements can be fully described within quantum mechanics without the Collapse Postulate.

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