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Article

Gravitational Waves and Higgs Field from Alena Tensor

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Abstract: Alena Tensor is a recently discovered class of energy-momentum tensors that proposes a general equivalence of the curved path and geodesic for analyzed spacetimes which allows the analysis of physical systems in curvilinear, classical and quantum descriptions. In this paper it is shown that Alena Tensor is related to the Killing tensor $K^{\alpha\beta}$ and describes the class of GR solutions $G^{\alpha\beta} + \Lambda g^{\alpha\beta} = 2\Lambda K^{\alpha\beta}$. In this picture, it is not matter that imposes curvature, but rather the geometric symmetries, encoded in the Killing tensor, determine the way spacetime curves and how matter can be distributed in it. It was also shown, that Alena Tensor gives decomposition of energy-momentum tensor of the electromagnetic field using two null-vectors and in natural way forces the Higgs field to appear, indicating the reason for the symmetry breaking. The obtained generalized metrics (covariant and contravariant) allow for further analysis of metrics for curved spacetimes with effective cosmological constant. The obtained solution can be also analyzed using conformal geometry tools. The calculated Riemann and Weyl tensors allow the analysis of purely geometric aspects of curvature, Petrov-type classification, and tracking of gravitational waves independently of the matter sources. A certain simplification of the analysis of gravitational waves has also been proposed, which may help both in their analysis and in the proof of the validity of the Alena Tensor. The article has been supplemented with the Alena Tensor equations with a positive value of the electromagnetic field tensor invariant (related to cosmological constant) and supplementary file containing a computational notebook used for symbolic derivations which may help in further analysis of this approach.

Keywords: alena tensor; gravitational waves; general relativity; electromagnetism; Higgs field

1. Introduction

Gravitational waves are a well-understood and researched issue [1], and it seems that the area of this research will develop dynamically both in theoretical understanding [2] and methods of waves detection [3,4]. The existence of gravitational waves is the key argument for the correctness of the General Relativity, and for this reason it is also a good tool for verifying the correctness of alternative to GR theories [5–7] and the theories of quantum gravity [8].

Alena Tensor is at the beginning of its research journey. It is a recently discovered class of energy-momentum tensors that allows for equivalent description and analysis of physical systems in flat spacetime (with fields and forces) and in curved spacetime (using Einstein Field Equations) proposing the overall equivalence of the curved path and the geodesic. In this method it is assumed that the metric tensor is not a feature of spacetime, but only a method of its mathematical description. In previous publications [9–11] it was already shown that this approach allows for a unified description of a physical system (curvilinear, classical and quantum) ensuring compliance with GR and QM results. Due to this property, the Alena Tensor seems to be a useful tool for studying unification problems, quantum gravity and many other applications in physics.

Many researchers try to reproduce the GR equations in flat spacetime or vice versa [12,13] or include electromagnetism in GR, connecting the spacetime geometry with electromagnetism [14–21]. There are known such approaches on the basis of differential geometry [22,22,23], based on field equations [24,25] as well as promising analyses of spinor fields [26] or helpful approximations for a weak field [27]. For this reason, the Alena Tensor should be viewed as another theory requiring

theoretical and experimental verification, and it seems worth checking whether the this approach ensures the existence of gravitational waves and what their interpretation is.

In this paper it will be analyzed the possibility of describing gravitational waves using the Alena Tensor. Due to the fact that research on this approach is a relatively young field, to facilitate the analysis of the article, the next section summarizes the results obtained so far and introduces the necessary notation. All computationally difficult equations and graphs can be found in the attached supplementary material as computational notebook.

In the Results section, at first it will be shown that the use of the Alena Tensor leads to the decomposition of the electromagnetic field stress-energy tensor into components dependent on two null vectors. Next, the general form of the metrics for curved spacetime described by the Alena Tensor will be obtained, the Riemann and Weyl tensors will be calculated and presented for an example field configuration. It will also be shown that the correct operation of the Alena Tensor requires the existence of the Higgs field. The article also presents tables and graphs illustrating the obtained results (Higgs field, Weyl tensor components, Newman-Penrose scalar oscillations) and it was shown, that the analysis of the equations lead to the conclusions contained in the abstract.

Although at the first moment the paradigm shift proposed by this approach may seem incomprehensible, the author hopes that the reader will resist the temptation to burn this article and trust the scientific method, which encourages us to calculate and check everything based on the correctness of the results obtained.

2. Short Introduction to Alena Tensor

The following chapter briefly explains the conclusions from the previous publications on Alena Tensor. The author uses the metric signature (+,-,-,-) which provides a positive value of the electromagnetic field tensor invariant. In previous publications it was treated as negative (reversal of the order of terms in the energy-momentum tensor of the electromagnetic field). The following equations remove this inconvenience while maintaining the correctness of the obtained results.

2.1. Transforming a Curved Path into a Geodesic

To understand the Alena Tensor, it is easiest to recreate the reasoning that led to its creation [10] using the example of the electromagnetic field. One may consider the energy-momentum tensor in flat spacetime for a physical system with an electromagnetic field in the following form

$$T^{\alpha\beta} = \rho U^\alpha U^\beta - \frac{1}{\mu_r} Y^{\alpha\beta} \quad (1)$$

where $T^{\alpha\beta}$ is energy-momentum tensor for a physical system, ρ is density of matter, U^α is four-velocity, μ_r is relative permeability, $Y^{\alpha\beta}$ is energy-momentum tensor for the electromagnetic field.

The density of four-forces acting in a physical system can be considered as a four-divergence. One may therefore denote the four-force densities occurring in the system:

- $f^\beta \equiv \partial_\alpha \rho U^\alpha U^\beta$ is the density of the total four-force acting on matter
- $\frac{1}{\mu_r} f_{em}^\beta + f_{gr}^\beta \equiv \partial_\alpha \frac{1}{\mu_r} Y^{\alpha\beta}$ are forces due to the field, where
- f_{em}^β is the density of the electromagnetic four-force
- $f_{gr}^\beta = Y^{\alpha\beta} \partial_\alpha \frac{1}{\mu_r}$ was shown in [9] as related to the presence of gravity in the system.

One may assume that the forces balance, which will provide a vanishing four-divergence of the energy-momentum tensor for the entire system

$$0 = \partial_\alpha T^{\alpha\beta} = f^\beta - \frac{1}{\mu_r} f_{em}^\beta - f_{gr}^\beta \quad (2)$$

It may be noticed, that if one wanted to use $T^{\alpha\beta}$ for a curvilinear description, which would describe the same physical system but curvilinearly, then in curved spacetime the forces due to the field can be

replaced with help of Christoffel symbols of the second kind. This means, that the entire field term can simply disappear from the equation in curved spacetime, because instead of a field and the forces associated with it, there will be corresponding curvature.

This would mean, that in curved spacetime $\frac{1}{\mu_r} Y^{\alpha\beta} = 0 \rightarrow T^{\alpha\beta} = \varrho U^\alpha U^\beta$. As shown in [10], a minor amendment to continuum mechanics provides this property. Assuming ϱ_0 as rest mass density and $\varrho U^\alpha \equiv \varrho_0 \gamma U^\alpha$ one gets mass density taking into account motion and Lorentz contraction of the volume and provides

$$\partial_\alpha \varrho U^\alpha = 0 \rightarrow U^\alpha_{,\alpha} = -\frac{d\gamma}{dt} \rightarrow U^\alpha_{;\alpha} = 0; U^\alpha U^\beta_{;\alpha} = 0 \quad (3)$$

$$\rightarrow \frac{D U^\beta}{D \tau} = 0; (\varrho U^\alpha U^\beta)_{;\alpha} = 0 \quad (4)$$

One may thus generalize $Y^{\alpha\beta}$ making the following substitution

$$Y^{\alpha\beta} \equiv \Lambda_\rho \left(\frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} - g^{\alpha\beta} \right) = \frac{1}{\mu_0} F^{\alpha\delta} g_{\delta\gamma} F^{\beta\gamma} - \Lambda_\rho g^{\alpha\beta} \quad (5)$$

where $F^{\alpha\delta}$ is electromagnetic field strength tensor, μ_0 is vacuum magnetic permeability, $g^{\alpha\beta}$ is metric tensor with the help of which the spacetime is considered, and

- $\Lambda_\rho = \frac{1}{4\mu_0} F^{\alpha\mu} g_{\mu\gamma} F^{\beta\gamma} g_{\alpha\beta}$ is invariant of the electromagnetic field tensor,
- $\mathbb{k} = g_{\mu\nu} \mathbb{k}^{\mu\nu}$ is trace of $\mathbb{k}^{\alpha\beta}$,
- $\mathbb{k}^{\alpha\beta}$ is a metric tensor of a spacetime for which $Y^{\alpha\beta}$ vanishes.

Tensor $\mathbb{k}^{\alpha\beta}$ may be calculated in flat spacetime and may be treated as fixed, since the value of $\mathbb{k}^{\alpha\beta}$ is independent of the $g^{\alpha\beta}$ adopted for analysis. In this way one obtains a generalized description of the tensor $Y^{\alpha\beta}$, which has the following properties:

- in flat spacetime $Y^{\alpha\beta}$ is the usual, classical energy-momentum tensor of the electromagnetic field
- its trace vanishes in any spacetime, regardless of the considered metric tensor $g^{\alpha\beta}$
- in spacetime for which $g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$ the entire tensor $Y^{\alpha\beta}$ vanishes
- $\mathbb{k}^{\alpha\beta} \mathbb{k}_{\alpha\beta} = 4$ which is expected property of the metric tensor (it was already shown in [10] that $\mathbb{k}^{\alpha\beta}$ indeed may be considered as metric tensor for curved spacetime)

In the above manner one obtains the Alena Tensor $T^{\alpha\beta}$ in form of

$$T^{\alpha\beta} = \varrho U^\alpha U^\beta - \frac{1}{\mu_r} \Lambda_\rho \left(\frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} - g^{\alpha\beta} \right) \quad (6)$$

with the yet unknown $\frac{1}{\mu_r}$ for which in curved spacetime ($g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$) the energy-momentum tensor of the field $Y^{\alpha\beta}$ vanishes.

The reasoning carried out above for electromagnetism is universal and allows to consider the Alena Tensor also for energy-momentum tensors associated with other fields. This leads to obtaining an energy-momentum tensor $T^{\alpha\beta}$ for the system that can be considered both in flat spacetime and in curved spacetime.

2.2. Connection with Continuum Mechanics, GR and QFT/QM

To make the Alena Tensor consistent with Continuum Mechanics in flat spacetime, it is enough to adopt the substitution $\frac{1}{\mu_r} \equiv \frac{-p}{\Lambda_\rho}$ where p is the negative pressure in the system and it is equal to $p \equiv \varrho c^2 - \Lambda_\rho$ where c is the speed of light in a vacuum. Such substitution yields

$$\varrho U^\alpha U^\beta - T^{\alpha\beta} = p \eta^{\alpha\beta} - c^2 \varrho \frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} + \Lambda_\rho \frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} \quad (7)$$

where $\eta^{\alpha\beta}$ is the metric tensor of flat Minkowski spacetime. Introducing deviatoric stress tensor $\Pi^{\alpha\beta} \equiv -c^2 \rho \frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta}$ one obtains relativistic equivalence of Cauchy momentum equation (convective form) in which only f_{em} appears as a body force

$$f^\alpha = \partial^\alpha p + \partial_\beta \Pi^{\alpha\beta} + f_{em}^\alpha \quad (8)$$

The above substitution also provides a connection to General Relativity in curved spacetime. For this purpose, one may introduce the following tensors, which can be analyzed in both flat and curved spacetime

$$R^{\alpha\beta} \equiv 2\rho U^\alpha U^\beta - p g^{\alpha\beta}; R \equiv R^{\alpha\beta} g_{\alpha\beta} = 2\Lambda_\rho - 2p; G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R \frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} \quad (9)$$

Above allows to rewrite Alena Tensor as

$$G^{\alpha\beta} + \Lambda_\rho g^{\alpha\beta} = 2T^{\alpha\beta} + \rho c^2 \left(g^{\alpha\beta} - \frac{4}{\mathbb{k}} \mathbb{k}^{\alpha\beta} \right) \quad (10)$$

Analyzing the above equation in curved spacetime ($g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$), one obtains simplifications

$$G^{\alpha\beta} + \Lambda_\rho g^{\alpha\beta} = 2T^{\alpha\beta} \quad ; \quad G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} \quad (11)$$

thus above can be interpreted as the main equation of General Relativity up to the constant $\frac{4\pi G}{c^4}$ where $G^{\alpha\beta}$ and $R^{\alpha\beta}$ can be interpreted in curved spacetime, respectively, as Einstein curvature tensor and Ricci tensor both with an accuracy of $\frac{4\pi G}{c^4}$ constant.

Analyzing the $G^{\alpha\beta}$ tensor in flat spacetime ($g^{\alpha\beta} = \eta^{\alpha\beta}$) one can also notice, that it is related to the non-body forces seen in the description of the Cauchy momentum equation

$$\partial_\beta G^{\alpha\beta} = \partial^\alpha p + \partial_\beta \Pi^{\alpha\beta} = f_{gr}^\alpha + f_{rr}^\alpha \quad (12)$$

which means that in the Alena Tensor analysis method gravity is not a body force, and as shown in [9] in above

- $f_{rr}^\alpha = \left(\frac{1}{\mu_r} - 1 \right) f_{em}^\alpha$ is the density of the radiation reaction four-force
- $f_{gr}^\alpha = \rho \left(\frac{d\phi}{d\tau} U^\alpha - c^2 \partial^\alpha \phi \right)$ is density of the four-force related to gravity, where
- $\phi = -\ln(\mu_r)$ is related to the effective potential in the system with gravity.

It can be calculated that f_{gr}^α vanishes in two cases:

- $\vec{u} = \vec{u}_{ff} \equiv -c \frac{\nabla \phi}{\partial^0 \phi}$ - which turns out to be the case of free fall
- $\partial^\alpha \phi = 0$ which occurs in the case of circular orbits

Neglecting the electromagnetic force and the radiation reaction force, using the above equation one can reproduce the motion of bodies in the effective potential obtained from the solutions of General Relativity. Such a description has already been done for the Schwarzschild metric [9] for

$$\phi + c_0 \equiv \sqrt{\frac{E^2}{m^2 c^4} - \left(\frac{1}{c} \frac{dr}{d\tau} \right)^2} = \sqrt{\left(1 - \frac{r_s}{r} \right) \left(1 + \frac{L^2}{r^2} \right)} \quad (13)$$

where c_0 is a certain constant. The solutions obtained in this way enforce the existence of gravitational waves due to time-varying ϕ (except for free fall and circular orbits).

In the above description, gravity itself is not a force, because the above description is based on an effective potential. However, one can see a similarity to Newton's classical equations for the stationary case with a stationary observer, for which f_{gr}^α can be approximated by Newton's gravitational force

with the opposite sign. Thus for stationary observer f_{gr}^α represents a force that must exist to keep a stationary observer suspended above the source of gravity in fixed place.

The description of gravity obtained in this way is surprisingly consistent with current knowledge, despite the fact that gravity itself in this description is not a force, and the force f_{gr}^α is not a body force.

The Alena Tensor constructed in presented way according to [9,11] may be simplified in flat spacetime to

$$T^{\alpha\beta} = \Lambda_\rho \eta^{\alpha\beta} - \frac{1}{\mu_0} F^{\alpha\gamma} \partial^\beta A_\gamma \quad ; \quad \mathcal{L} = T^{00} = -\Lambda_\rho = -\frac{1}{4\mu_0} F^{\alpha\beta} F_{\alpha\beta} \quad (14)$$

which allows its analysis in classical field theory and quantum theories. Obtained canonical four-momentum $H^\alpha \equiv -\frac{1}{c} \int T^{\alpha 0} d^3x$ provides $0 = H^{\alpha, \alpha} = H^\alpha H_\alpha$ and

$$\partial^\alpha H^\mu X_\mu = H^\alpha = \mu_r P^\alpha + q \mathcal{E}^\alpha \quad ; \quad -L = \frac{mc^2}{\gamma} - W_{pv} \quad (15)$$

where P^α is four-momentum, $W_{pv} = -\int p d^3x$ is pressure-volume work, and where $q\mathcal{E}^\alpha$ and $-\mu_r P^\alpha$ are in fact two gauges of electromagnetic four-potential. In above $(\mu_r - 1)P^\alpha$ is responsible for the force associated with gravity and radiation reaction force. It was also shown that canonical four-momentum H^μ may be expressed as

$$H^\mu = P^\mu + W^\mu = -\frac{\gamma L}{c^2} U^\mu + \mathbb{S}^\mu \quad (16)$$

where \mathbb{S}^μ due to its property $\mathbb{S}^\mu U_\mu = 0$, seems to be some description of rotation or spin, and where W^μ describes the transport of energy due to the field.

The quantum picture obtained from the Alena Tensor [9,11] for the system with electromagnetic field leads to the conclusion that gravity and the radiation reaction force have always been present in Quantum Mechanics and Quantum Field Theory. This conclusion follows from the fact that the quantum equations obtained from the Alena Tensor for the system with electromagnetic field [9] are actually the three main quantum equations currently used:

- simplified Dirac equation for QED:
 $\mathcal{L}_{QED} = \frac{1}{4\mu_0} F^{\alpha\beta} F_{\alpha\beta} = \frac{1}{2\mu_0} F^{0\gamma} \partial^0 A_\gamma = \frac{1}{2} \Psi (i\hbar c \mathcal{D} - mc^2) \Psi$
- Klein-Gordon equation,
- equivalent of the Schrödinger equation: $i\hbar \partial^0 \psi = -\frac{\hbar^2}{m(\gamma + \frac{1}{\gamma})} \nabla^2 \psi + cq\hat{A}^0 \psi$

where A^α and \hat{A}^α are two gauges of electromagnetic four-potential, and where the last equation in the limit of small energies (Lorentz factor $\gamma \approx 1$) turns into the classical Schrödinger equation considered for charged particles.

The above results make the Alena Tensor a useful tool for the analysis of physical systems with fields, allowing modeling phenomena in flat spacetime, curved spacetime, and in the quantum image.

3. Results

Considering a flat spacetime with an electromagnetic field, described in a way provided by Alena Tensor using notation introduced in section 2, one may reverse the reasoning presented in introduction and consider the field as a manifestation of a propagating perturbation of the curvature of spacetime which in flat spacetime is just interpreted as a field. For this purpose, one may define a certain perturbation $h^{\alpha\beta}$ of the metric tensor $\mathbb{k}^{\alpha\beta}$ that describes the deviation from flat spacetime, and also define its trace h as

$$h^{\alpha\beta} \equiv \mathbb{k}^{\alpha\beta} - \eta^{\alpha\beta} \quad ; \quad h = h^{\alpha\beta} \eta_{\alpha\beta} = \mathbb{k} - 4 \quad (17)$$

The stress-energy tensor of the electromagnetic field in flat spacetime can be thus represented as follows

$$\frac{\mathbb{k}}{4\Lambda_\rho} Y^{\alpha\beta} = h^{\alpha\beta} - \frac{h}{4} \eta^{\alpha\beta} \quad (18)$$

As one can see in the above, considering gravitational waves in the Alena Tensor is natural and does not require classical linearization. This would mean that gravitational waves in Alena Tensor approach are de facto a propagating disturbance of the energy-momentum tensor for the field (in the case analyzed, the electromagnetic field energy-momentum tensor).

Denoting the pressure amplitude \mathcal{P}_o and $\bar{h}^{\alpha\beta}$ one obtains

$$\mathcal{P}_o \equiv \frac{4\Lambda\rho}{\mathbb{k}} \quad ; \quad \bar{h}^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{h}{4}\eta^{\alpha\beta} \quad \rightarrow \quad Y^{\alpha\beta} = \mathcal{P}_o \bar{h}^{\alpha\beta} \quad (19)$$

which shows that the energy-momentum tensor of the field may be also interpreted as propagating vacuum pressure waves with tensor amplitude.

To provide an analysis of the above equation for gravitational waves and the analysis of the resulting classes of metrics, a representation using null-vectors will be useful. Therefore, in the next few steps it will be shown that Alena Tensor allows representing the energy-momentum tensor of the electromagnetic field with the use of two null-vectors.

3.1. Decomposition of the Electromagnetic Field Using Null Vectors

At first step one may recall equation (16) and define new four-vector B^μ obtaining

$$B^\mu \equiv -\frac{\gamma L}{c^2} U^\mu - \mathbb{S}^\mu \quad ; \quad H^\mu = -\frac{\gamma L}{c^2} U^\mu + \mathbb{S}^\mu \quad (20)$$

Since it is known from previous publications, that $H^\mu H_\mu = 0$ and $U^\mu \mathbb{S}_\mu = 0$, therefore above definition also yields $B^\mu B_\mu = 0$. This property allows to represent U^μ and \mathbb{S}^μ using two null-vectors H^μ and B^μ as follows

$$H^\alpha - B^\alpha = 2\mathbb{S}^\mu \quad ; \quad H^\alpha + B^\alpha = -\frac{2\gamma L}{c^2} U^\alpha \quad \rightarrow \quad H^\alpha B_\alpha = \frac{2\gamma^2 L^2}{c^2} \quad (21)$$

thus

$$H^\alpha B^\beta + B^\alpha H^\beta = \frac{2H^\mu B_\mu}{c^2} U^\alpha U^\beta - (H^\alpha H^\beta + B^\alpha B^\beta) \quad (22)$$

Next, one may define auxiliary parameter α as

$$\alpha \equiv \frac{B^0}{H^0} + \frac{2H^\mu B_\mu}{H^0 mc\gamma} \quad (23)$$

and subtract the linear combination of H^α and B^α from both sides

$$H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0} B^\alpha B^\beta = \quad (24)$$

$$= \frac{2H^\mu B_\mu}{c^2} U^\alpha U^\beta - \left([1 + \alpha] H^\alpha H^\beta + \left[1 + \frac{H^0}{B^0} \right] B^\alpha B^\beta \right) \quad (25)$$

Next, using $H \equiv cH^0$ for simplicity, one may recall from [9] coefficients related to the electromagnetic field

- relative permeability $\mu_r = \frac{\Lambda\rho}{-p} = \frac{H}{W_{pv}} = e^{-\phi}$
- volume magnetic susceptibility $\chi = \mu_r - 1 = \frac{qc^2}{-p} = \frac{mc^2\gamma}{W_{pv}}$
- relative permittivity $\epsilon_r = \frac{1}{\mu_r} = \frac{-p}{\Lambda\rho} = \frac{W_{pv}}{H}$
- electric susceptibility $\chi_e = \epsilon_r - 1 = -\frac{qc^2}{\Lambda\rho} = -\frac{mc^2\gamma}{H} = -\chi\epsilon_r$

and notice, that one obtains Alena Tensor $T^{\alpha\beta}$ as

$$\frac{\mu_r}{\Lambda_\rho} T^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu} \left(H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0} B^\alpha B^\beta \right) = \quad (26)$$

$$= \frac{\chi}{c^2} U^\alpha U^\beta - \frac{\chi}{2H^\mu B_\mu} \left([1 + \alpha] H^\alpha H^\beta + \left[1 + \frac{H^0}{B^0} \right] B^\alpha B^\beta \right) \quad (27)$$

where electromagnetic stress-energy tensor is equal to

$$\frac{1}{\Lambda_\rho} Y^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu} \left([1 + \alpha] H^\alpha H^\beta + \left[1 + \frac{H^0}{B^0} \right] B^\alpha B^\beta \right) \quad (28)$$

and where $T^{0\beta}$ actually simplifies, as shown in introduction, to

$$\frac{\mu_r}{\Lambda_\rho} T^{0\beta} = -\frac{\mu_r}{H^0} H^\beta \quad (29)$$

Completing the definition of the first invariant of the electromagnetic field tensor Λ_ρ , one may define the second invariant I_\perp by electric \vec{E} and magnetic \vec{B} fields as

$$I_\perp \equiv \frac{1}{c\mu_0} \vec{E}\vec{B} \quad (30)$$

where it is known [28], that

$$Y^{\alpha\beta} Y_{\alpha\beta} = 4(\Lambda_\rho^2 + I_\perp^2) = 4(Y^{0\beta} Y_{0\beta}) \quad (31)$$

Therefore from (28) one obtains simplifications

$$B^0 = \frac{H^\mu B_\mu}{4H^0} = \frac{\gamma^2 L^2}{2cH} \rightarrow \alpha = \frac{B^0}{H^0} \left(1 - \frac{8}{\chi e} \right) \rightarrow \frac{1}{H^0} + \frac{1}{B^0} = \frac{4c}{-L} \quad (32)$$

and by defining a useful auxiliary variable φ one gets

$$e^\varphi \equiv \frac{-\gamma L}{\sqrt{2}H} \rightarrow \gamma = \frac{1}{\sqrt{2}} \cosh(\varphi) \rightarrow e^{2\varphi} = \frac{B^0}{H^0} \quad (33)$$

Finally, defining for simplicity as below

$$e^\theta \sinh(\theta) \equiv \frac{-L}{mc^2\gamma} \quad ; \quad I_\perp^2 \equiv 1 + \frac{I_\perp^2}{\Lambda_\rho^2} \quad (34)$$

then calculating from (28) and (31)

$$I_\perp^2 = \chi^2 \gamma^2 \left(1 - \frac{2L}{mc^2\gamma} \right) = \chi^2 \gamma^2 e^{2\theta} \rightarrow \sqrt{2} \mu_r e^\varphi = \frac{-\gamma L}{W_{pv}} = I_\perp \sinh(\theta) \quad (35)$$

and expressing $\mu_r = \frac{H}{W_{pv}} = e^{-\phi}$, one gets further useful expressions

$$\frac{I_\perp}{\sqrt{2}} = \frac{e^{\varphi-\phi}}{\sinh(\theta)} \quad ; \quad \chi\gamma = I_\perp e^{-\theta} \quad (36)$$

To simplify further analysis, one may also normalize four-vectors H^μ and B^μ using (32) as follows

$$a^\mu \equiv \frac{1}{H^0} H^\mu \quad ; \quad b^\mu \equiv \frac{1}{B^0} B^\mu \rightarrow a^\mu b_\mu = 4 \quad (37)$$

where a^μ (not h^μ) was introduced to avoid confusion related to the previously defined perturbation $h^{\alpha\beta}$. After few calculations using previously derived relationships in (28)

$$\frac{\chi}{8H^0B^0}[1 + \alpha](H^0)^2 = \frac{\chi}{8} \left(-\frac{4cH^0}{L} - \frac{8}{\chi e} \right) = \mu_r \left(1 - \frac{mc^2\gamma}{2L} \right) \quad (38)$$

$$\frac{\chi}{8H^0B^0} \left[1 + \frac{H^0}{B^0} \right] (B^0)^2 = -\frac{\chi c B^0}{2L} = \chi\gamma^2 + \mu_r \frac{mc^2\gamma}{2L} \quad (39)$$

one may now rewrite the electromagnetic field tensor $Y^{\alpha\beta}$ as

$$Y^{\alpha\beta} = \mu_r \Lambda_\rho \left(1 + \frac{1}{2e^\theta \sinh(\theta)} \right) a^\alpha a^\beta + \Lambda_\rho \left(\chi\gamma^2 - \frac{\mu_r}{2e^\theta \sinh(\theta)} \right) b^\alpha b^\beta \quad (40)$$

As shown in [9] element $\mu_r \Lambda_\rho$ is responsible for electric field energy density carried by light, where $\Lambda_\rho \chi\gamma^2$ was shown as describing energy density of magnetic moment and was linked to charged matter in motion. The element $\frac{\mu_r}{2e^\theta \sinh(\theta)}$ is a new term and part of equations related to this term may be expressed as $\mathbb{S}^{\alpha\beta}$ with help of (36) as

$$\mathbb{S}^{\alpha\beta} \equiv \frac{\mu_r}{2e^\theta \sinh(\theta)} (a^\alpha a^\beta - b^\alpha b^\beta) = \frac{I_0}{2\sqrt{2}} e^{-(\theta+\varphi)} (a^\alpha a^\beta - b^\alpha b^\beta) \quad (41)$$

Since $\mathbb{S}^{\alpha\beta}$ does not actually carry energy but only momentum, it can be associated with some description of spin field effects by analogy to (21). Using (32), (33) (36) and (39), electromagnetic field tensor $Y^{\alpha\beta}$ may be, however, expressed in more useful form. Since

$$\mu_r \left(1 + \frac{1}{2e^\theta \sinh(\theta)} \right) = \frac{I_0}{2\sqrt{2}} e^{\theta-\varphi} \quad ; \quad -\frac{\chi c B^0}{2L} = \frac{I_0}{2\sqrt{2}} e^{\varphi-\theta} \quad (42)$$

thus

$$\frac{1}{\Lambda_\rho} Y^{\alpha\beta} = \frac{I_0}{2\sqrt{2}} e^{\theta-\varphi} a^\alpha a^\beta + \frac{I_0}{2\sqrt{2}} e^{\varphi-\theta} b^\alpha b^\beta \quad (43)$$

As one may notice, (37) also allows to simplify (20) by introducing $-e^\varphi s^\mu \equiv \frac{1}{H^0} \mathbb{S}^\mu$

$$a^\mu = e^\varphi \left(\frac{\sqrt{2}}{c} U^\mu - s^\mu \right) ; \quad b^\mu = e^{-\varphi} \left(\frac{\sqrt{2}}{c} U^\mu + s^\mu \right) \quad (44)$$

what using (33) yields

$$s^0 = \sinh \varphi \quad ; \quad \vec{s}\vec{s} = 1 + \cosh^2 \varphi \quad ; \quad s^\mu s_\mu = -2e^{2\varphi} \quad (45)$$

and therefore allows to analyze the system using hyperbolic (and trigonometric) functions

$$\frac{a^\mu + b^\mu}{2} = \frac{\sqrt{2}}{c} \cosh \varphi U^\mu - \sinh \varphi s^\mu \quad (46)$$

$$\frac{a^\mu - b^\mu}{2} = \frac{\sqrt{2}}{c} \sinh \varphi U^\mu - \cosh \varphi s^\mu \quad (47)$$

$$a^\alpha a^\beta = e^{2\varphi} \left(\frac{2}{c^2} U^\alpha U^\beta + s^\alpha s^\beta - \frac{\sqrt{2}}{c} [U^\alpha s^\beta + s^\alpha U^\beta] \right) \quad (48)$$

$$\frac{\sqrt{2}}{c} U^\mu = \frac{e^{-\varphi}}{2} a^\mu + \frac{e^\varphi}{2} b^\mu \quad (49)$$

One may thus denote normalized Alena Tensor in flat spacetime as $K^{\alpha\beta}$ with help of (26) and above as follows

$$K^{\mu\nu} \equiv \frac{T^{\mu\nu}}{\Lambda_\rho} = \frac{-\chi_e}{8} \left(a^\mu b^\nu + b^\mu a^\nu - \left[1 - \frac{8}{\chi_e} \right] a^\mu a^\nu - b^\mu b^\nu \right) \quad (50)$$

and notice, that it may be presented after simple calculations using (44) - (48) as

$$K^{\mu\nu} = \frac{-\chi_e}{c^2} U^\alpha U^\beta - \left(\left[1 - \frac{\chi_e}{8} (1 + e^{-2\varphi}) \right] a^\mu a^\nu + \left[-\frac{\chi_e}{8} (1 + e^{2\varphi}) \right] b^\mu b^\nu \right) \quad (51)$$

Since the expression in the brackets must be equal to $\frac{1}{\mu_r} \frac{1}{\Lambda_\rho} Y^{\alpha\beta}$, thus

$$\frac{1}{\Lambda_\rho} Y^{\alpha\beta} = \left[\mu_r + \frac{\chi}{8} (1 + e^{-2\varphi}) \right] a^\mu a^\nu + \left[\frac{\chi}{8} (1 + e^{2\varphi}) \right] b^\mu b^\nu \quad (52)$$

Therefore, according to (40), (41) there must occur $\frac{I_0}{2\sqrt{2}} e^{-(\theta+\varphi)} = \frac{\chi}{8} (1 + e^{-2\varphi})$. Indeed, with help of (33) and (36) one obtains

$$\frac{I_0}{2\sqrt{2}} e^{-\theta} = \frac{\chi}{8} (e^\varphi + e^{-\varphi}) = \frac{\chi}{4} \cosh \varphi = \frac{\chi}{2\sqrt{2}} \cdot \frac{\cosh \varphi}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \chi \gamma = \frac{I_0}{2\sqrt{2}} e^{-\theta} \quad (53)$$

and the same equality may be calculated for $\frac{\chi}{8} (1 + e^{2\varphi}) = \frac{I_0}{2\sqrt{2}} e^{\varphi-\theta}$ what confirms compliance with (43). Since θ is merely auxiliary variable (used only to highlight certain relationships) and thanks to simple algebraic transformations may be expressed in terms of φ

$$2\sqrt{2} H e^\varphi = m c^2 e^\theta \sinh(\theta) \cosh^2(\varphi) \rightarrow -\frac{8}{\chi_e} = (e^{2\theta} - 1)(e^{-2\varphi} + 1) \quad (54)$$

one may thus further simplify the description of the system. This shows that further, in-depth analysis of the system is also possible, however, modeling and simplifying the description of the electromagnetic field or searching for elementary particles that provide stable solutions requires a separate article (probably several articles). From the perspective of describing gravitational waves, other elements of the description are crucial, which will be discussed next.

Finally, one may notice, that property $a^\mu b_\mu = 4$ in (37) requires analysis in a complex basis. An example of such a basis are four-vectors

$$a^\mu(\sigma) \equiv \begin{pmatrix} 1 \\ i \sinh(i\sigma) \\ \cosh(i\sigma) \\ 0 \end{pmatrix}, \quad b^\mu(\sigma) \equiv \begin{pmatrix} 1 \\ i(2\sqrt{2} \cosh(i\sigma) - 3 \sinh(i\sigma)) \\ -3 \cosh(i\sigma) + 2\sqrt{2} \sinh(i\sigma) \\ 0 \end{pmatrix} \quad (55)$$

where the angle σ was introduced to facilitate further analysis. The null basis product is a simple consequence of equation (31) as a consequence of taking only the electromagnetic field into account in the analysis. Since reality requires other fields (e.g. electroweak), it can be assumed that changing $Y^{\alpha\beta} \rightarrow \bar{Y}^{\alpha\beta}$ into the field tensor corresponding to reality, would probably provide $\bar{Y}^{\alpha\beta} \bar{Y}_{\alpha\beta} = 2(\bar{Y}^{0\beta} \bar{Y}_{0\beta}) \rightarrow a^\mu b_\mu = 2$, which then makes it possible to assume a basis in real numbers. However, since in this paper it is considered the Alena Tensor with the electromagnetic field only, the basis (55) will be retained for further analysis as an example, especially since the transition to the generalized field in the discussed approach is a fairly simple procedure.

A cursory examination shows that this basis describes the electromagnetic field very well indeed. It has a good representation in conformal geometry (null vectors correspond to points on the equator of the Penrose sphere), where the propagation directions are perpendicular to the time axis (purely spatial), ideal for describing a circularly or elliptically polarized wave in the direction of Poyting vector $\vec{z} \equiv (0, 0, 2i\sqrt{2})$, where $a^\mu b_\mu = 4$ represents the constant phase relation between the electric and

magnetic fields. The proposed basis naturally enters the Newman–Penrose formalism, allows for a full spinor representation of the electromagnetic field, where σ is a typical massless wave, satisfies the wave equation $\square\sigma = 0$, and since it provides a spin-helicity of ± 1 , it is well suited for further analysis in the QFT framework as a photon wave representation, describing a single-particle state. However, detailed analysis of these issues is beyond the scope of this article.

3.2. Covariant Metric, Higgs Field, Riemann Tensor, Weyl Tensor and Gravitational Waves

Substituting (40) into (18) using (36) and using Alena Tensor properties one gets expression for metric $\mathbb{k}^{\alpha\beta}$ describing the system in curved spacetime

$$\mathbb{k}^{\alpha\beta} = \frac{\mathbb{k}}{4} \left(\frac{I_0}{2\sqrt{2}} e^{\theta-\varphi} a^\alpha a^\beta + \frac{I_0}{2\sqrt{2}} e^{\varphi-\theta} b^\alpha b^\beta + \eta^{\alpha\beta} \right) \quad (56)$$

It turns out that using properties of metrics, one may find a general solution for the inverted metric $\mathbb{k}_{\alpha\beta}$. Summarizing the key properties one obtains

$$\mathbb{k}^{\alpha\beta} \eta_{\alpha\beta} = \mathbb{k}_{\alpha\beta} \eta^{\alpha\beta} = \mathbb{k}; \quad \mathbb{k}^{\alpha\beta} \mathbb{k}_{\alpha\beta} = 4; \quad \det(\mathbb{k}^{\alpha\beta}) \det(\mathbb{k}_{\alpha\beta}) = 1; \quad \mathbb{k}_{\alpha\mu} \mathbb{k}^{\mu\beta} = \delta_\alpha^\beta \quad (57)$$

where in the last condition it is enough to check the index (0,0), because the null vectors are normalized ($a^0 = 1; b^0 = 1$). To simplify the calculations, it is easiest to define auxiliary variable q and start from anstaz with unknown A, B, C, D in

$$e^{-q} \equiv \frac{\mathbb{k}}{4}; \quad \mathbb{k}_{\mu\nu} \equiv \frac{A}{I_0} \frac{e^{\theta-\varphi}}{\sqrt{2}} a_\mu a_\nu + \frac{B}{I_0} \frac{e^{\varphi-\theta}}{\sqrt{2}} b_\mu b_\nu - C (a_\mu b_\nu + b_\mu a_\nu) + D \eta_{\mu\nu} \quad (58)$$

By eliminating the subsequent variables to provide equations (57), one obtains covariant metric in the form

$$\mathbb{k}_{\mu\nu} \equiv \frac{\sinh(q)}{I_0} \frac{e^{\theta-\varphi}}{\sqrt{2}} a_\mu a_\nu + \frac{\sinh(q)}{I_0} \frac{e^{\varphi-\theta}}{\sqrt{2}} b_\mu b_\nu - \sinh(q) (a_\mu b_\nu + b_\mu a_\nu) + e^q \eta_{\mu\nu} \quad (59)$$

where invariants of electromagnetic field turns out to be related to the trace

$$\frac{I_\perp^2}{\Lambda_\rho^2} = \frac{1}{e^{2q} - 2} \rightarrow I_0 = \sqrt{\frac{e^{2q} - 1}{e^{2q} - 2}}; \quad \det(\mathbb{k}_{\alpha\beta}) = e^{2q} (e^{2q} - 2) \quad (60)$$

which means that the trace \mathbb{k} is also invariant. The supplementary material contains equations confirming the correctness of the derivation.

Using arc \frown for simplicity, to emphasize the change to curvilinear description (since values in curvilinear description may be different), one may notice, that considered in curved spacetime trace $\widehat{\mathbb{k}} = 4$ yields $\widehat{q} = 0$. Therefore, the transition to curved spacetime can be understood as solutions with imaginary magnetic field $\vec{B} \rightarrow i\vec{B}$. It is also worth notice, that for the null basis example (55), the above metric seems to describe a gravitational wave in conformal geometry, where $\frac{\mathbb{k}}{4} = e^{-q}$ plays the role of a conformal factor Ω^{-2} . Further analysis in this direction should allow to isolate both the polarization and the relation of \mathbb{k} to the Ricci scalar by classical relation $R = \Omega^{-2} (\widehat{R} - 6 \square \log \Omega)$.

Expressing U^μ by null-vectors as in (48) and requesting $U^\alpha U^\beta \mathbb{k}_{\alpha\beta} = c^2$ one obtains ugly expression linking $\cosh(\varphi + \theta)$ and q (due to the complexity of the calculations and the result, it is shown in the attached supplementary material with calculations).

$$\cosh(\theta + \varphi) = \frac{-3 \sinh(q) + \cosh(q) - 1}{2\sqrt{-\frac{1}{\coth(q)-3}(\cosh(q) - 3 \sinh(q))}} \quad (61)$$

However, substituting q as the I_0 function according to (60)

$$q = \log \sqrt{2 - \frac{1}{1 - I_0^2}} \quad (62)$$

and denoting $V \equiv 1 + 2I_0^4 - 3I_0^2$, it appears that this ugly expression (61) actually expresses following dependence

$$\cosh(\theta + \varphi) = \frac{1 + \sqrt{V}}{\sqrt{2} I_0} \rightarrow V = 1 + 2I_0^2 \cosh^2(\theta + \varphi) - 2\sqrt{2} I_0 \cosh(\theta + \varphi) \quad (63)$$

where V is the classical Higgs potential and the broken symmetry in the system can be interpreted as an apparent mismatch of coefficients related to $\cosh(\varphi + \theta)$. The Figure 1 shows the classic "Mexican sombrero" potential $V(I_0)$. Since $\cosh(\varphi + \theta)$ is related in equation (41) to the angle describing the spin, this means that in this solution the spin effect of the field is described by invariant angle (depends only on I_0) and, according to (60) it is closely related to the trace of the metric and, consequently, to the Ricci scalar in the curvilinear description and thus to spacetime curvature. This also at least means that to obtain correct results in curved spacetime in the Alena Tensor model, the Higgs-like field is needed. Perhaps the above result will help explain the existence of the Higgs field and its relationship to curvature and spin.

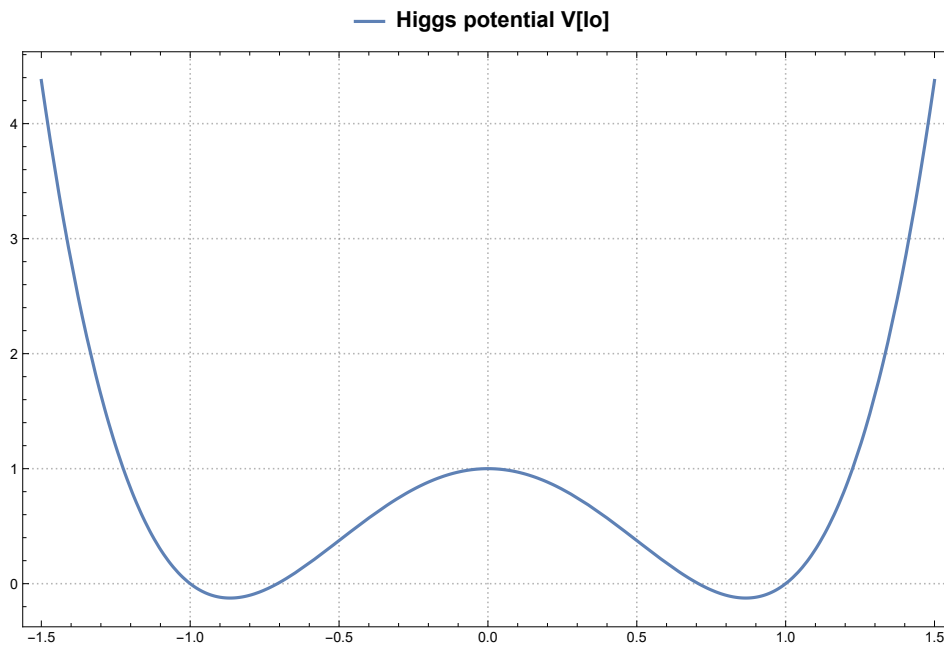


Figure 1. The existence of the Higgs field potential as a consequence of $U^\alpha U^\beta \mathbb{k}_{\alpha\beta} = c^2$.

For $I_0 = \sqrt{1 + \frac{1}{\sqrt{2}}}$ the whole metric becomes significantly simplified and symmetric

$$I_0 = \sqrt{1 + \frac{1}{\sqrt{2}}} \rightarrow V = I_0^2 \rightarrow q = \ln(\sqrt{2} I_0) \rightarrow \frac{\sinh(q)}{\sqrt{2} I_0} = \frac{I_0}{2\sqrt{2}} \quad (64)$$

which means that it may be assumed that adopting the above value leads to some significant and physically important solution of the equations for the system.

Using hyperbolic identities and expressing

$$\cosh(\theta + \varphi) = \cosh(\theta) \cosh(\varphi) + \sinh(\theta) \sinh(\varphi) = e^{-\theta} \cosh(\varphi) + e^\varphi \sinh(\theta) \quad (65)$$

then using (36) one obtains

$$\chi\gamma^2 + \mu_r e^{2\varphi} = \frac{1 + \sqrt{V}}{2} \quad (66)$$

Using (40) and (45), the relation between electromagnetic energy density Y^{00} , angle φ and spin four-vector \mathbb{S}^μ zero component is therefore

$$\frac{1}{\Lambda_\rho} Y^{00} = \mu_r + \chi\gamma^2 = \frac{1 + \sqrt{V}}{2} - 2\mu_r e^\varphi \sinh(\varphi) = \frac{1 + \sqrt{V}}{2} + \frac{2c\mathbb{S}^0}{W_{pv}} \quad (67)$$

which explains the origin of φ and the connection of spin and Lagrangian with the energy density of the electromagnetic field. Since, according to the considered Alena Tensor approach, in curved spacetime the energy density expressed by Y^{00} must vanish (the entire stress-energy tensor of the electromagnetic field vanishes), this means, that φ and thus γ becomes invariant, which allows to analyze the transition to curved spacetime.

One may now consider what value the Alena Tensor takes in curved spacetime. To do this, it is easiest to analyze the behavior of $K^{\mu\nu}$. As one may calculate, the determinant of this tensor is 0 and the matrix rank is 2, but, as described in the introduction, it degenerates to $\widehat{K}^{\mu\nu} = \frac{-\widehat{\chi}_e}{c^2} U^\alpha U^\beta$ in curved spacetime. It can be seen that in the Alena Tensor approach the metric follows from propagation. However, in curved spacetime the electromagnetic field according to this approach should vanish, remaining present only in the metric and determining geodesic. This can be achieved by degenerating vectors a^μ and b^μ to a single vector in curved spacetime $\widehat{a}^\mu = \widehat{b}^\mu$. However, in such situation the equation (48) forces this vector to be the four-velocity, divided by the Lorenz factor

$$\widehat{a}^\mu = \widehat{b}^\mu \rightarrow U^\mu = \frac{c}{\sqrt{2}} \left(\frac{e^{-\varphi}}{2} \widehat{a}^\mu + \frac{e^\varphi}{2} \widehat{a}^\mu \right) = c\gamma \widehat{a}^\mu \rightarrow \widehat{a}^\mu = \frac{1}{c\gamma} U^\mu \quad (68)$$

which degenerates $\widehat{K}^{\mu\nu}$ from (50) to the form

$$\widehat{a}^\mu = \widehat{b}^\mu = \frac{1}{c\gamma} U^\mu \rightarrow \widehat{K}^{\mu\nu} = -\widehat{a}^\mu \widehat{a}^\nu = -\frac{1}{c^2 \gamma^2} U^\mu U^\nu \rightarrow \widehat{\chi}_e = \frac{1}{\gamma^2} \quad (69)$$

As can be easily verified, the exact above relation of variables leads to the vanishing field stress-energy tensor in (52) upon transition to curved spacetime.

Since the metric is known and U^μ may be expressed by values of a^μ , b^μ in flat spacetime thanks to (48) this allows to easily obtain the normalized Einstein tensor and Ricci tensor (normalized by Λ_ρ) using the equation (11). According to the notation used in (11) one obtains

$$\frac{1}{\Lambda_\rho} R^{\mu\nu} = 2 \widehat{K}^{\mu\nu} + \left(1 + \frac{1}{\gamma^2}\right) \mathbb{K}^{\mu\nu}; \quad \frac{1}{\Lambda_\rho} G^{\mu\nu} = 2 \widehat{K}^{\mu\nu} - \mathbb{K}^{\mu\nu} \quad (70)$$

where Ricci scalar R (normalized according to notation used) may be calculated as $\frac{1}{\Lambda_\rho} R = 4 + \frac{2}{\gamma^2}$.

The described system seems to be Petrov type D or II [29], although to be sure, the Weyl tensor should be calculated. This does not seem possible for the general case (without auxiliary assumptions about symmetries), but one could simplify the obtained description considerably, based on the following observation. One may notice, that the vanishing Lorenz factor γ in $\widehat{K}^{\alpha\beta}$ can be interpreted as an important suggestion for the description of motion in curved spacetime. Such motion would correspond to a stream of particles moving without dilation, a strictly ordered flow without local perturbations, resembling a perfect, infinitely stiff fluid. This means that $\widehat{K}^{\alpha\beta}$ should be the Killing tensor and the system should have hidden symmetry (similar to the Carter constant in Kerr solutions).

As shown previously, product of the basis is in considered case $a^\mu b_\mu = 4$, which means that null vectors have global significance for spacetime geometry, thus Killing tensor should have a strong

connection with propagation along null vectors (it is not a random symmetry, but a deep feature of spacetime) and this would mean the existence of special wave surfaces, e.g. electromagnetic waves and/or gravitational radiation. This would be also a clear indication that spacetime belongs to the Petrov D or II class and is associated with wave propagation solutions.

Also, the analysis of obtained equations drive to conclusion, that energy (energy density) is not something external to geometry in Alena Tensor approach, but energy is defined by the geometry of spacetime itself. This is a result in the spirit of General Relativity, but it goes even deeper: metric $k^{\mu\nu}$ depends on e^φ , but at the same time $\frac{1}{\gamma(\varphi)}$ depends on the metric because it is trace of $\widehat{K}^{\alpha\beta}$ in this metric. The system itself defines its own energy through the structure of the field, which is similar to the idea of Self-consistent Field [30] - where the field and the source are inseparable, or Emergent Gravity [31] - where energy, gravity and geometry arise from a common structure, or Induced Geometry [32] - where energy comes from deformation of spacetime itself, as e.g. in Sakharov's theory [33]. It seems impossible to "decree" energy in Alena Tensor, but it must be calculated from geometry and the system works as a closed causal cycle.

The dependence for $\widehat{K}^{\alpha\beta}$ of its norm and trace in curved spacetime (the norm is the square of the trace) is a key property for null space and suggests that $\widehat{K}^{\alpha\beta}$ describes isometries related to null wave propagation, similar to pp-wave [34] and Robinson-Trautman solutions [35], what is actually expected in considered approach based on electromagnetic stress-energy tensor and result (56). This would also mean that the Killing tensor is directly related to the energy distribution in spacetime as expected, similar to other GR solutions (eg. Kerr solution), and lead to a rather groundbreaking but also expected result in the context of the discussed approach, that the Killing tensor directly determines the Einstein tensor in main GR equation.

Since $\widehat{K}^{\alpha\beta}$ is simply the Alena Tensor (stress-energy tensor for the system) divided by Λ_ρ , it gives correct conserved values in the Noether formalism (conserved density of energy and momentum). From the definition of the Alena Tensor as the energy-momentum tensor for the system it also follows that $\widehat{K}^{\mu\nu}$ is symmetric and $\nabla_\mu \widehat{K}^{\mu\nu}$ vanishes.

Since $\widehat{K}_{\alpha\beta} U^\alpha U^\beta = -\frac{c^2}{\gamma^2}$ this implies that along a geodesic parametrized by proper time τ , its total derivative vanishes.

$$\frac{d}{d\tau}(\widehat{K}_{\alpha\beta} U^\alpha U^\beta) = 0 = U^\lambda \nabla_\lambda \widehat{K}_{\alpha\beta} U^\alpha U^\beta \quad (71)$$

Using the symmetry of $\widehat{K}_{\mu\nu}$ one may note that

$$U^\alpha U^\mu U^\nu \nabla_\alpha \widehat{K}_{\mu\nu} = U^\alpha U^\mu U^\nu \nabla_{(\alpha} \widehat{K}_{\mu\nu)} \quad (72)$$

Therefore, the condition

$$(\nabla_{(\alpha} \widehat{K}_{\mu\nu)}) U^\alpha U^\mu U^\nu = 0 \quad \rightarrow \quad \nabla_{(\alpha} \widehat{K}_{\mu\nu)} = 0 \quad (73)$$

holds for arbitrary tangent vectors U^μ , and it indeed follows that $\nabla_{(\alpha} \widehat{K}_{\mu\nu)} = 0$. Therefore normalized Alena Tensor is Killing tensor for considered system.

In this way Alena Tensor theory becomes equivalent to some specific case of General Relativity equation expressed in the following form

$$\nabla_{(\mu} \widehat{K}_{\alpha\beta)} = 0 \quad \rightarrow \quad \frac{4\pi G}{c^4} G_{\alpha\beta} + \Lambda k_{\alpha\beta} = 2\Lambda \widehat{K}_{\alpha\beta} \quad (74)$$

where according to (11) in the above equation $\frac{4\pi G}{c^4} G_{\alpha\beta}$ is the classical Einstein tensor in commonly used notation $\frac{4\pi G}{c^4} G_{\alpha\beta} = G_{\alpha\beta}^*$ (classical Einstein tensor).

Thanks to the above, since the Weyl tensor is defined as the part of the Riemann tensor that does not depend on the Ricci tensor, this implies that since Killing tensor determines the Einstein

tensor, and the Einstein tensor determines the Ricci tensor, then one could calculate the Weyl tensor by extracting the part of the curvature that does not contain the Ricci tensor. This would also mean that the Christoffel symbols can be expressed as a function of the Killing and Einstein tensors, and the Riemann tensor can be written directly as a function of the Ricci and Killing tensors.

For the system under consideration, one may therefore construct a general Weyl tensor ansatz $C^{\mu\nu\alpha\beta}$, which must contain only components that do not become zero when the trace part is subtracted from the Riemann tensor. It should be of the form

$$C^{\mu\nu\alpha\beta} \equiv c_1 \Sigma^{\mu\nu\alpha\beta} + c_2 P^{\mu\nu\alpha\beta} + c_3 Q^{\mu\nu\alpha\beta}, \quad (75)$$

where

$$\Sigma^{\mu\nu\alpha\beta} = a^\mu b^\nu a^\alpha b^\beta - a^\mu b^\nu b^\alpha a^\beta - b^\mu a^\nu a^\alpha b^\beta + b^\mu a^\nu b^\alpha a^\beta, \quad (76)$$

$$P^{\mu\nu\alpha\beta} = a^\mu a^\nu b^\alpha b^\beta - b^\mu b^\nu a^\alpha a^\beta, \quad (77)$$

$$Q^{\mu\nu\alpha\beta} = a^\mu b^\nu \eta^{\alpha\beta} - a^\alpha b^\beta \eta^{\mu\nu}. \quad (78)$$

It is easy to check that the above tensors are linearly independent and form the basis for the representation of any Weyl tensor in the system under consideration. Analysis of their behavior indicates that

- $\Sigma_{\mu\nu\alpha\beta}$ is responsible for "pure" directional propagation - e.g. a gravitational wave propagating along null directions (purely conformal part of the Weyl tensor — described solely by null geometry),
- $P_{\mu\nu\alpha\beta}$ describes non-radiating, "axial" deformation of space - e.g. tidal sequences, consistent with mass motion without undulations,
- $Q_{\mu\nu\alpha\beta}$ describes conformal distortion of the background metric itself.

The coefficients c_1, c_2, c_3 are not known a priori, but one may determine them with help of Riemann tensor. To obtain Riemann tensor $R^{\mu\nu\alpha\beta}$ in the system under consideration, it is enough to assume the following ansatz

$$R^{\mu\nu\alpha\beta} \equiv A_1 (\mathbb{k}^{\mu\alpha} \mathbb{k}^{\nu\beta} - \mathbb{k}^{\mu\beta} \mathbb{k}^{\nu\alpha}) + \quad (79)$$

$$+ A_2 \left(\widehat{K}^{\mu\alpha} \mathbb{k}^{\nu\beta} - \widehat{K}^{\mu\beta} \mathbb{k}^{\nu\alpha} - \widehat{K}^{\nu\alpha} \mathbb{k}^{\mu\beta} + \widehat{K}^{\nu\beta} \mathbb{k}^{\mu\alpha} \right) \quad (80)$$

It has the following justification

- The Riemann tensor satisfies the known algebraic symmetries: $R_{\mu\nu\alpha\beta} = -R_{\nu\mu\alpha\beta} = -R_{\mu\nu\beta\alpha}$, $R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$, $R_{\mu[\nu\alpha\beta]} = 0$. The above ansatz satisfies them automatically.
- There are only two tensor objects available in the system: the metric $\mathbb{k}_{\mu\nu}$ and the Killing tensor $\widehat{K}_{\mu\nu} = -\frac{1}{c^2 \gamma^2} U_\mu U_\nu$. The Riemann tensor must be constructed exclusively from them.
- The first term with A_1 corresponds to the geometry of a spacetime with constant curvature, as in de Sitter spacetime: $R^{\mu\nu\alpha\beta}_{(\text{constant curvature})} \sim \mathbb{k}^{\mu\alpha} \mathbb{k}^{\nu\beta} - \mathbb{k}^{\mu\beta} \mathbb{k}^{\nu\alpha}$
- The second term with A_2 is the minimal geometrically correct extension that takes into account the presence of non-null energy (represented by $\widehat{K}^{\mu\nu}$). Its construction provides correct symmetries and enables the reproduction of a non-null Ricci tensor $R^{\nu\beta} = R_{\mu}^{\mu\nu\beta}$
- Other possible combinations (e.g. $\widehat{K} \otimes \widehat{K}$) are linearly dependent or asymmetric with respect to the required properties of the Riemann tensor — they do not provide new information in the case under consideration.
- The whole creates the most general fourth-order tensor with Riemann symmetries, which can be constructed from available geometric objects.

Computing the contraction of the above Riemann tensor anstaz with $\mathbb{k}_{\mu\alpha}$ one gets

$$\mathbb{k}_{\mu\alpha}R^{\mu\nu\alpha\beta} = 3A_1\mathbb{k}^{\nu\beta} + A_2\left(2\widehat{K}^{\nu\beta} - \frac{1}{\gamma^2}\mathbb{k}^{\nu\beta}\right) \quad (81)$$

Since normalized Ricci tensor $R^{\alpha\beta}$ is known (70), therefore considering for simplicity above Riemann tensor anstaz as normalized, one gets

$$A_1 = \frac{1}{3}\left(1 + \frac{2}{\gamma^2}\right) \quad ; \quad A_2 = 1 \quad (82)$$

Such normalized form is convenient to use because it allows one to easily obtain both: classical tensors (multiplication by the cosmological constant Λ) and tensors in the Alena Tensor notation (multiplication by field invariant Λ_ρ).

Next, one may consider normalized trace part of the Riemann tensor $S^{\mu\nu\alpha\beta}$, which provides the necessary symmetries and the vanishing trace in the Weyl tensor. For this to be satisfied, trace part must have the following form (where Ricci tensor $R^{\nu\beta}$ is considered for simplicity also as normalized):

$$S^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\mathbb{k}^{\mu\alpha}R^{\nu\beta} - \mathbb{k}^{\mu\beta}R^{\nu\alpha} - \mathbb{k}^{\nu\alpha}R^{\mu\beta} + \mathbb{k}^{\nu\beta}R^{\mu\alpha}\right) \quad (83)$$

$$- \frac{1}{6}R\left(\mathbb{k}^{\mu\alpha}\mathbb{k}^{\nu\beta} - \mathbb{k}^{\mu\beta}\mathbb{k}^{\nu\alpha}\right) = \quad (84)$$

$$\frac{1}{2}\left[\mathbb{k}^{\mu\alpha}\left(2\widehat{K}^{\beta\nu} + \left(1 + \frac{1}{\gamma^2}\right)\mathbb{k}^{\beta\nu}\right) - \mathbb{k}^{\mu\beta}\left(2\widehat{K}^{\alpha\nu} + \left(1 + \frac{1}{\gamma^2}\right)\mathbb{k}^{\alpha\nu}\right)\right. \quad (85)$$

$$\left. - \mathbb{k}^{\nu\alpha}\left(2\widehat{K}^{\beta\mu} + \left(1 + \frac{1}{\gamma^2}\right)\mathbb{k}^{\beta\mu}\right) + \mathbb{k}^{\nu\beta}\left(2\widehat{K}^{\alpha\mu} + \left(1 + \frac{1}{\gamma^2}\right)\mathbb{k}^{\alpha\mu}\right)\right] \quad (86)$$

$$- \frac{1}{3}\left(2 + \frac{1}{\gamma^2}\right)\left(\mathbb{k}^{\mu\alpha}\mathbb{k}^{\nu\beta} - \mathbb{k}^{\mu\beta}\mathbb{k}^{\nu\alpha}\right) \quad (87)$$

Since all terms related to $\widehat{K}^{\mu\nu}$ simplify after grouping the components in $C^{\mu\nu\alpha\beta} = R^{\mu\nu\alpha\beta} - S^{\mu\nu\alpha\beta}$, one obtains normalized Weyl tensor in generalized form for the system under consideration

$$C^{\mu\nu\alpha\beta} = R^{\mu\nu\alpha\beta} - S^{\mu\nu\alpha\beta} = \quad (88)$$

$$\left(1 + \frac{1}{\gamma^2}\right)\left(\mathbb{k}^{\mu\alpha}\mathbb{k}^{\nu\beta} - \mathbb{k}^{\mu\beta}\mathbb{k}^{\nu\alpha}\right) \quad (89)$$

$$- \frac{1}{2}\left(1 + \frac{1}{\gamma^2}\right)\left(\mathbb{k}^{\mu\alpha}\mathbb{k}^{\beta\nu} - \mathbb{k}^{\mu\beta}\mathbb{k}^{\alpha\nu} + \mathbb{k}^{\nu\alpha}\mathbb{k}^{\beta\mu} - \mathbb{k}^{\nu\beta}\mathbb{k}^{\alpha\mu}\right) \quad (90)$$

Trace of above Weyl tensor disappears and it has all the expected symmetries. Substituting the metric known from (56), one obtains the coefficients postulated in the anstaz (75) and as one may notice from non-zero coefficient c_1 , gravitational waves exist in the system.

$$c_1 = \left(1 + \frac{1}{\gamma^2}\right)\frac{I_0^2}{8}e^{-2q} \quad (91)$$

$$c_2 = \left(1 + \frac{1}{\gamma^2}\right)\frac{I_0^2}{4}e^{-2q}\sinh(2\theta - 2\varphi) \quad (92)$$

$$c_3 = \left(1 + \frac{1}{\gamma^2}\right)\frac{I_0}{\sqrt{2}}e^{-2q}\sinh(\theta - \varphi) \quad (93)$$

One can also demonstrate the existence of gravitational waves using classical analysis using the Weyl tensor components and Newman-Penrose scalars. Having calculated the above-mentioned tensors

and sample basis (55), one may notice, that four-vector $a^\mu(\sigma)$ is purely real. One may thus construct a Newman-Penrose tetrad around it, fitting to the geometry, as follows

$$l^\mu \equiv a^\mu(\sigma) \quad (94)$$

$$n^\mu \equiv \frac{1 - \sin \sigma}{\cos^2 \sigma} (1, 1, 0, 0) \quad (95)$$

$$m^\mu \equiv \frac{1}{\sqrt{2}(\cos \frac{\sigma}{2} + \sin \frac{\sigma}{2})} \left(-\frac{i \cos \sigma}{1 + \sin \sigma}, -\frac{i \cos \sigma}{1 + \sin \sigma}, -i, 1 \right) \quad (96)$$

$$\bar{m}^\mu \equiv \frac{\cos \frac{\sigma}{2} + \sin \frac{\sigma}{2}}{\sqrt{2}} \left(\frac{i \cos \sigma}{1 + \sin \sigma}, \frac{i \cos \sigma}{1 + \sin \sigma}, i, 1 \right) \quad (97)$$

The above four-vectors satisfy the tetrad requirement and lead to only two non-zero scalars

$$\Psi_3 = 8C_1 \left[(-1 - i\sqrt{2}) \cos\left(\frac{\sigma}{2}\right) + i(i + \sqrt{2}) \sin\left(\frac{\sigma}{2}\right) \right] \quad (98)$$

$$\Psi_4 = 4C_1 [1 - 2i\sqrt{2} \cos(\sigma) - 3 \sin(\sigma)] \quad (99)$$

what is confirmed in the supplementary materials. Since the scalar Ψ_3 can be zeroed by a rotation about n^μ without modifying Ψ_4 , this means that the system under consideration is Petrov type N, despite the non-zero Ricci tensor. It can therefore be interpreted as a certain form of the Kundt solution with null dust [36].

One may also substitute into solution an example basis (55) to Weyl tensor. Table 1 presents the values of the key components of the Weyl tensor in this basis.

Table 1. Component values of the Weyl tensor.

Component	Value
C^{0212}	$\frac{i(16C_1 - 5C_2) \cos(\sigma) - 7iC_2 \cos(3\sigma)}{\sqrt{2}} + 8C_1 \sin(\sigma) - 20C_2 \cos^2 \sigma \sin(\sigma)$
C^{1313}	0
C^{0202}	$4C_1 (1 + 3 \cos(2\sigma) - 2i\sqrt{2} \sin(2\sigma))$
C^{1010}	$4C_1 (1 - 3 \cos(2\sigma) + 2i\sqrt{2} \sin(2\sigma))$
C^{2323}	0
C^{1213}	0
C^{1323}	0

One may then solve the system of equations (36, 54, 63) presenting the results only as a function of I_o and σ . Due to the complex relationships between variables, analytical solution and obtained results are presented in supplementary file. Weyl tensor component values were calculated for the value $I_o = \sqrt{1 + \frac{1}{\sqrt{2}}}$ adopted for the analysis (64) and for chosen solution $\varphi(I_o), \theta(I_o)$. The Figure 2 below clearly shows the wave nature of the C^{0212} component of the Weyl tensor, presenting the result of the calculations performed. Obtained Weyl tensor component C^{0212} seems to encode the variation of some curvature field along the propagating direction σ .

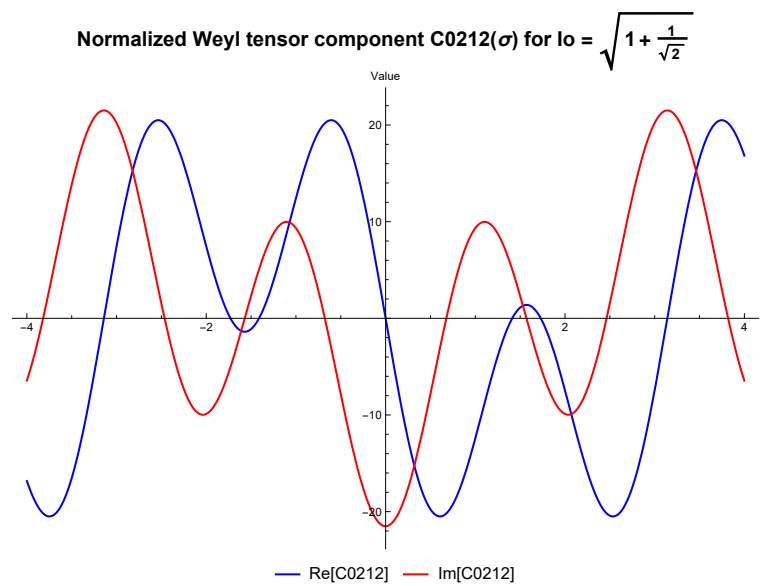


Figure 2. Weyl tensor C^{0212} component as a wave.

Gravitational waves are often modeled using Gaussian impulse as in [37], but this can also be done using the dependence of Ψ_4 on σ . One may therefore show the deformation effects for an example arrangement of test particles on a circle. In the supplementary materials there is a gif file presenting an animation of this distortion. This animation reveals the typical quadrupolar nature of spacetime distortions induced by gravitational waves (characteristic twisting and stretching).

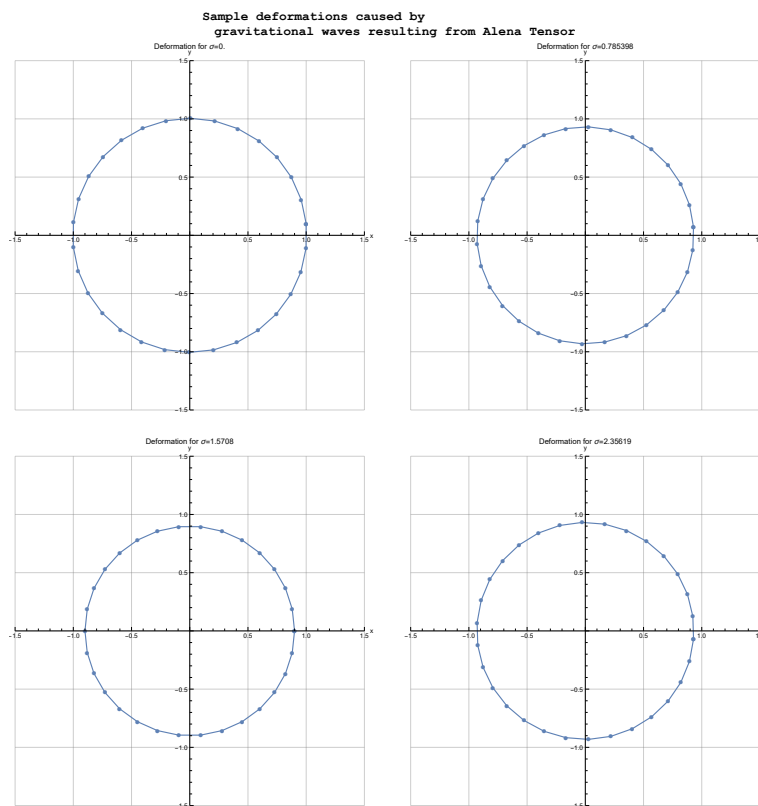


Figure 3. Deformation visualization for particles on a circle.

The above Weyl tensor components are actually normalized. One may thus present the main GR equation in the Alena Tensor notation (11) and represent it using the vacuum pressure $2p_\Lambda \equiv \frac{\Lambda c^4}{4\pi G} = \Lambda_\rho$ as follows

$$G^{\alpha\beta} + 2p_\Lambda \mathbb{k}^{\alpha\beta} = 2T^{\alpha\beta} = 2\Lambda_\rho \widehat{K}^{\alpha\beta} = 4p_\Lambda \widehat{K}^{\alpha\beta} \quad (100)$$

Since the obtained Weyl tensor is normalized (by Λ - classical notation or by Λ_ρ - Alena Tensor notation) this means that the interpretation from (19) that gravitational waves in the Alena Tensor are in fact propagating vacuum pressure disturbances also seems to be correct.

It is also worth noting that the obtained metric term $e^{-q}\eta^{\alpha\beta} = \frac{\mathbb{k}}{4}\eta^{\alpha\beta}$ can in principle be interpreted as a vacuum energy contribution (effective cosmological constant) as in [38,39] playing the role of a metric scaling factor, as e.g. described in [40].

Additionally, one may invoke the scalar field ϕ associated with the presence of matter, where $e^\phi = \frac{1}{\mu_r}$. It is known from 2.2 that e^ϕ is responsible for the presence of sources and in their absence $\mu_r = 1$. Therefore, interpreting whole $Y^{\alpha\beta}$ as the wave amplitude tensor one would get representation $\frac{1}{\mu_r}Y^{\alpha\beta} = \mathcal{P}_o \bar{h}^{\alpha\beta} e^\phi$ which would also allow to search for e^ϕ as a certain wave function.

This approach allows for two simplifications related to the analysis of gravitational waves. Considering the force f_{gr}^α responsible for effects related to gravity as shown in (13) and extracting the acceleration \mathcal{A}^α from it, one gets

$$e\mathcal{A}^\alpha \equiv f_{gr}^\alpha = e \left(\frac{d\phi}{d\tau} U^\alpha - c^2 \partial^\alpha \phi \right) \rightarrow c^2 \square \phi = \gamma^2 \frac{d^2 \phi}{dt^2} - \partial_\alpha \mathcal{A}^\alpha \quad (101)$$

since according amendment from [10] $\partial_\alpha \gamma U^\alpha = 0$.

As shown in [9], ϕ is directly related to the effective potential in gravitational systems which can be calculated from the GR equations. This would allow searching for propagating changes of the effective potential itself ($\square\phi = 0$) similarly as was postulated in [41]. It would significantly simplify both the calculations and perhaps the methods of detecting gravitational waves.

The second potential simplification results from the possibility of analyzing only the Poynting four-vector $Y^{\alpha 0}$ as $\frac{1}{\mu_r}Y^{\alpha 0} = \mathcal{P}_o \bar{h}^{\alpha 0} e^\phi$ which might also help simplify the calculations and search for experimental proof of correctness for the Alena Tensor approach.

4. Conclusion and Discussion

As shown in the above article, the Alena Tensor ensures the existence of gravitational waves and allows their physical interpretation, providing key tools for their further analysis including the ability to calculate and visualize Weyl tensor components. The obtained decomposition of the electromagnetic field stress-energy tensor (56) allows for further analysis of metrics for curved spacetimes (e.g. extending obtained solution for other fields) and also to use the proposed null basis for further Alena Tensor development in the framework of conformal geometry, the NP formalism and the description of photons in QFT. It also seems reasonable to generalize above results for non-linear electromagnetism using e.g. approach presented in [42] and search for a description of elementary particles that will provide relatively stable solutions to the obtained equations, perhaps also taking into account new approaches, such as those proposed in [43].

It remains an open question whether the Alena Tensor is a correct way to describe physical systems, but this paper shows that it exhibits many properties that are expected from such a description, including the existence of gravitational waves whose behavior and description agrees with GR. Additionally, quite surprisingly, this approach naturally forces the existence of the Higgs field and this field turns out to be necessary to even consider curved spacetime and gravitational waves. The spontaneously appearing faithful reproduction of the Higgs field may of course be a completely coincidental result, but it may also be an indication that the proposed approach is worth further investigation and development for other fields (or more general solutions), to obtain complete picture of unification.

The connection between Alena Tensor and the Killing tensor obtained in this paper reveals a deep, nonlinear connection between the matter distribution and the geometry and symmetries of spacetime. These results show that the matter distribution in this approach is not arbitrary, but precisely tuned to the geometry and hidden symmetries of spacetime, which allows for their further analysis in the language of Killing tensors and conformal Weyl curvature.

As indicated in the introduction, the Alena Tensor is a new approach and certainly requires a lot of work and further research. It seems, however, that further analysis may yield useful descriptions of the transformation between curved and flat spacetime and bring new insights that will contribute to a better understanding of issues related to the broadly understood unification of physical theories.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on [Preprints.org](https://preprints.org).

Statements: All data that support the findings of this study are included within the article (and any supplementary files). During the preparation of this work the author did not use generative AI or AI-assisted technologies. Author did not receive support from any organization for the submitted work. Author have no relevant financial or non-financial interests to disclose.

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