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Posted Date: 15 July 2025

doi: 10.20944/preprints202507.1277.v1

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Using the Semantic Propositional Calculus (SPC) to Determine the Validity of Categorical Syllogisms: A Contribution to the Structural Teaching Approach (STA)

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Abstract: A description is provided of how to use the Semantic Propositional Calculus (SPC) for a clear, compact notation that can be applied to approach diverse topics in logic and mathematics. With two equivalent formulations (one corresponding to set theory and the other to the SPC), it is possible to determine the validity – or invalidity – of the categorical syllogisms. The *parallel* use of the SPC (preserving the syntax of the propositional calculus) and of resources from set theory facilitates learning in an area of logic and an area of mathematics. The joint learning of diverse branches of the same discipline, or of distinct disciplines which are closely related in different ways, is an essential aspect of the STA.

Keywords: semantic propositional calculus (SPC); set theory; structural teaching approach (STA)

MSC: 03B05, 97E30, 97E60

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1. Introduction

1.1. Objectives of This Article

1.1.1. First Objective

Characterize a version of the propositional calculus in which the semantic aspect of the propositions is relevant. Here this version is referred to as the Semantic Propositional Calculus (SPC). The nature of the SPC is covered in section 4.

1.1.2. Second Objective

Provide an example of the use of the SPC within the framework of the Structural Teaching Approach (STA) developed in a previous article [1]. This instructional approach is discussed in subsection 1.2 below. The example mentioned consists of determining the validity or invalidity of categorical syllogisms by applying two formalisms: a) one which uses the notion of inclusion, in a broad sense, of a set (the included set) within another set (the including set); and (b) the other which uses the SPC.

1.2. Relevant Aspects of the STA

This teaching approach is considered structural because, rather than the *way* the topic is presented (i.e., using textbooks, audiovisual aids, virtual or in-person classes, etc.), the *structure* itself of the subject matter plays a more important role.

In the most basic types of learning, note can be taken of how living beings are able to detect regularities in their environments and have certain reactions to them. Those processes are adaptive, and in the end, they are survival skills. Consider, in this regard, scientific research programs of particular interest for neuropsychology. In classical, or Pavlovian, conditioned reflexes, a stimulus (one which in isolation does not generate a response in the organism it acts on) becomes an effective

 doi:10.20944/preprints202507.1277.v1

stimulus (one which does generate a biological response) by repeatedly preceding a stimulus that generates a response. Recall the classic experiment, for example, which repeatedly uses the sound of a bell immediately before the delivery of food to a dog [2] and [3]. In operant, or Skinnerian, conditioned reflexes, behavior apparently generated randomly can become much more likely if it is repeatedly followed by a "reward" in a given setting – that is, something that favors the biological organisms used in the respective experiments [4] and [5].

Consider the following hypothetical item on a test to determine a human being's intelligence quotient (IQ): "In the numerical sequence, $2, 5, 11, 23, ?$, what number should replace the question mark?

To answer this question correctly, the test-taker should identify the regularity or pattern specified below. The second number (5) in the given numerical sequence can be calculated using the first number (2), multiplying it by 2, and adding 1 to the result: $5 = (2 \times 2) + 1$. The third number (11) in that numerical sequence can be calculated using the second (5), multiplying it by 2, and adding 1 to the result: $11 = (5 \times 2) + 1$. The fourth number (23) in that numerical sequence can be calculated using the third one (11), multiplying it by 2, and adding 1 to the result: $23 = (11 \times 2) + 1$. Therefore, it seems reasonable to find the fifth number in that numerical sequence by multiplying the fourth number (23) by 2 and adding 1 to the result. If this is done, the result is 47; $47 = (23 \times 2) + 1$. It may then be accepted that the correct response is 47.

This fictitious example of an item on an IQ test is problematical: At least some knowledge of arithmetic would be necessary to provide the correct answer. In general, it can be admitted that intelligence is a person's intellectual capacity regardless of his (or her) degree of cultural instruction. "Raven's Progressive Matrices" [6] were developed to prevent improper cultural bias. This resource is also based on the test-takers' detecting, for each item, the regularity that enables them to find the "unknown figure", that is, the figure that would be the correct answer. It should be emphasized then that the main characteristic that various tests have in common to evaluate the capacity that is conventionally known as "intelligence" is that they all are based on measuring someone's success in detecting certain regularities.

The laws of physics establish verified regularities for diverse physical entities. Thus, for example, the second law of Newton, $\vec{f} = m\vec{a}$, establishes that if the force $\vec{f}$ applied to a body of a given mass m is known, then its acceleration $\vec{a}$ can be calculated. In that law, mass serves as a proportionality constant. (Of course, given m and $\vec{a}$, $\vec{f}$ can be computed, and given $\vec{f}$ and $\vec{a}$, m can be computed.) This law is valid, according to Newtonian mechanics, for any body and at any instant. The laws of conservation in physics, such as those of linear and angular momentum and energy, establish that during the evolution through time of an isolated system, certain magnitudes remain constant; that is, they are conserved.

Likewise, laws have also been mentioned in biology. Think, for example, of Mendel's laws of inheritance. One example of a law in another factic science, economy, is the law of supply and demand. The main idea of the STA in teaching-learning processes is 1) to emphasize a careful, detailed presentation, with varied examples of the laws (i. e., of the explicit formulation of regularities) that are valid in each area of knowledge; and 2) to link different topics within the same discipline, or even from different disciplines, giving particular attention to isomorphisms or other close relationships between the corresponding laws. A seminal role was played by [7] regarding the orderly consideration of different types of systems and relationships existing between them.

The concept of law has also been found in formal sciences; one may think of the De Morgan's laws, for example. We have previously applied the STA in two formal sciences – logic and mathematics [1]. The present article contributes another example of the application of this approach.

Could the STA be useful in so-called humanistic disciplines, such as history? In these cases, it might be more difficult than in the area of exact and natural sciences to find clear examples of laws. However, this approach could facilitate the search for regularities using a systematic, comparative presentation of the topics covered. Thus, for example, if one is considering the history of diverse European countries, it is useful to present topics related to those countries the same way and in the

 doi:10.20944/preprints202507.1277.v1

same order in all cases. A tentative or preliminary example could be the proposal of an outline such as the following: 1) the inhabitants of the geographical region where each country was established, 2) the languages spoken by those inhabitants, 3) the events which led up to the founding of a given country, 4) its initial forms of government, etc. The purpose is for the learner to be able to establish simple, direct comparisons between the diverse countries discussed to find analogies and eventual differences between them. In general, the STA should have an important role in "comparative disciplines" such as comparative physiology and anatomy, just to give two examples from the field of biology. The STA makes it possible for the learner to acquire knowledge within broader than usual contexts.

2. Basic Notions on Propositional Calculus and Set Theory

Since this article is oriented mainly toward beginning level logic and mathematics teachers, it will be admitted that they are familiar with the basic notions of the disciplines addressed here. Nonetheless, to facilitate the understanding of the topic for interested students, in this section a brief review will be provided of pertinent results. They will be formulated not only with the symbols of the usual technical notation, but also in semi-technical English.

For an introduction to topics on logic, one may consult, for example, [8], [9], and [10]. For an introduction to set theory, one may use [11], [12], [13], and [14], for example.

2.1. Propositional Calculus

A propositional variable may be replaced by a proposition, and in classical bivalent logic, it is accepted that each proposition may be true or false.

When referring to more than one propositional variable, they are symbolized as $q_1, q_2, q_3, \ldots$. For concision, with a "license" common in the elemental treatment of propositional calculus, reference will be made to propositional variables as if they were propositions. Therefore, if it is stated, for example, that q_7 is true, it is supposed that q_7 has been replaced by a true proposition, and if it is stated, for example, that q_7 is false, it is admitted that the propositional variable has been replaced by a false proposition.

The *negation* operator in propositional calculus will be symbolized by a horizontal bar placed above the negated proposition. Thus, for example, given the proposition q_7, its negation – not q_7 – will be symbolized as $\overline{q}_7$.

Recall that if it is accepted that a proposition is true, its negation must be considered false, and if it is admitted that a proposition is false, its negation must be considered true. Therefore, the negation of a negation, or the "double negation" of a proposition, makes it possible to obtain that proposition once more. Thus, for instance, $\overline{\overline{q}}_{25}$ is identical to q_{25}.

The *conjunction* operator in propositional calculus will be symbolized as $\wedge$. The conjunction of any two propositions – which itself is also a proposition – is true only if those two propositions are true. Thus, for example, $q_9 \wedge q_{28}$, or q_9 and q_{28}, is true if both q_9 and q_{28} are true.

The *inclusive disjunction* operator in propositional calculus will be symbolized as $\vee$. The inclusive disjunction of any two propositions, which is also a proposition, is true only if at least one of those two propositions is true. Thus, for example, $q_{15} \vee q_{31}$ – that is, q_{15} inclusive or q_{31} – is true only if at least one of the two propositions is true. In other words, the proposition $q_{15} \vee q_{31}$ is false only if both q_{15} and q_{31} are false.

The operator for *material implication* in propositional calculus will be symbolized as $\rightarrow$. Consider any two propositions such as q_3 and q_8. The proposition $q_3 \rightarrow q_8$ states that q_3 materially implies q_8, and can be read as follows: if q_3, then q_8. For this reason, it is considered a conditional proposition in which q_3 is the antecedent and q_8 is the consequent of that proposition (which is false only if the antecedent q_3 is true and the consequent q_8 is false). The proposition $q_3 \rightarrow q_8$ is different from the proposition $q_8 \rightarrow q_3$. The latter proposition, if q_8, then q_3, is false only if its antecedent q_8 is true and its consequent q_3 is false.

The operator for *material bi-implication* or for the logical equivalence of two propositions will be symbolized as $\longleftrightarrow$. Consider any two propositions such as q_{27} and q_{54}. Both the proposition

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$q_{27} \longleftrightarrow q_{54}$ and the proposition $q_{54} \longleftrightarrow q_{27}$ (equivalent, from the logical perspective, to the former) are true only if q_{27} and q_{54} have the same truth value; that is, only in the case that both are true or in the case that both are false.

Consider any two propositions q_i and q_j, where $i = 1, 2, 3 \ldots$, and $j = 1, 2, 3 \ldots$, for $i \neq j$. Figure 1 presents the truth tables for the following propositions: $q_i \wedge q_j$, $q_i \vee q_j$, $q_i \rightarrow q_j$, $q_j \rightarrow q_i$, and $q_i \longleftrightarrow q_j$.

The presence of a 1 in a column of a truth table indicates that the proposition considered is true and the presence of a 0 in a column of a truth table indicates that the proposition considered is false.

q_i	q_j	$q_i \wedge q_j$	$q_i \vee q_j$	$q_i \rightarrow q_j$	$q_j \rightarrow q_i$	$q_i \longleftrightarrow q_j$
0	0	0	0	1	1	1
0	1	0	1	1	0	0
1	0	0	1	0	1	0
1	1	1	1	1	1	1

Figure 1. Truth table of the propositions $q_i \wedge q_j$, $q_i \vee q_j$, $q_i \rightarrow q_j$, $q_j \rightarrow q_i$, and $q_i \longleftrightarrow q_j$.

One law or tautology in propositional calculus is the following: $(q_i \rightarrow q_j) \longleftrightarrow (\overline{q}_j \rightarrow \overline{q}_i)$. In words, the proposition $q_i \rightarrow q_j$ (that is, if q_i then q_j) has the same truth value as $\overline{q}_j \rightarrow \overline{q}_i$.

Figure 2 presents the truth table for the law of propositional calculus $(q_i \rightarrow q_j) \longleftrightarrow (\overline{q}_j \rightarrow \overline{q}_i)$.

q_i	q_j	$q_i \rightarrow q_j$	$\overline{q}_i$	$\overline{q}_j$	$\overline{q}_i \rightarrow \overline{q}_j)$	$(q_i \rightarrow q_j) \longleftrightarrow (\overline{q}_j \rightarrow \overline{q}_i)$
0	0	1	1	1	1	1
0	1	1	1	0	1	1
1	0	0	0	1	0	1
1	1	1	0	0	1	1

Figure 2. Truth table of the proposition $(q_i \rightarrow q_j) \longleftrightarrow (\overline{q}_j \rightarrow \overline{q}_i)$.

Note that the proposition $(q_i \rightarrow q_j) \longleftrightarrow (\overline{q}_j \rightarrow \overline{q}_i)$ is true regardless of the truth values of q_i and q_j. That is, it is true given its logical form. Propositions which are true given their logical forms are called laws, or tautologies, in propositional calculus.

2.2. Set Theory

When using set theory within the framework of a given topic, the set to which all the elements considered within that topic belong is called the universal set $\mathbb{U}$.

The *complementation* operator for any set S will be symbolized by $+$. The set $\overset{+}{S}$ to which all the elements in the universal set $\mathbb{U}$ considered that do not belong to S belong is called the complement set of S. Note that the "double complementation" of any set S makes it possible to obtain S once more: $S = \overset{+}{\overset{+}{S}}$.

When considering more than a sole set S within the framework of the universal set $\mathbb{U}$, those different sets will be called $S_1, S_2, S_3, \ldots$.

The operator of *intersection* of any two sets will be symbolized as $\cap$. Consider any two sets S_1 and S_2. The intersection of the two sets S_1 and S_2, which also is a set, will be symbolized as $S_1 \cap S_2$. Only the members of the universal set $\mathbb{U}$ considered, which belong to both S_1 and S_2, can belong to the set $S_1 \cap S_2$.

The operator of *union* of any two sets will be symbolized by $\cup$. Consider any two sets S_1 and S_2. The union of the sets S_1 and S_2 (which also is a set) will be symbolized as $S_1 \cup S_2$. Only the members of the universal set $\mathbb{U}$ considered, which belong to both S_1 and S_2, or to at least one of those sets, belong to the set $S_1 \cup S_2$.

The operator of *inclusion* in a broad sense of one set in another set will be symbolized as $\subseteq$. The relation of inclusion of a set S_1 in another set S_2 will be symbolized as $S_1 \subseteq S_2$. Given this relation, set S_1 is called the included set, and set S_2 is called the including set. This relation can be characterized as follows: If any element whatsoever in the universal set belongs to S_1, then it also belongs to S_2. The

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relation $S_1 \subseteq S_2$ between the sets S_1 and S_2 does not exclude the possibility that the set S_1 is the same as the set S_2: $S_1 = S_2$. Of course, this equality cannot be verified if even one element that does not belong to S_1 belongs to S_2. This is the case of the "inclusion in a strict sense" of any set S_1 in a set S_2. In this article reference will not be made to that type of inclusion of one set in another set.

3. Four Types of Propositions Using Set Theory

Regarding categorical propositions one may consult [15], for example. Attention will be given below to four types of categorical propositions. Once the general logical form has been specified for each of them, each of these general logical forms is formulated in two equivalent ways using set theory symbols.

3.1. Universal Affirmative Categorical Propositions

One example of this type of categorical proposition is: *All engineers are pragmatic*.

This type of categorical proposition states that all elements belonging to a certain set that will be called S_1 belong to another set that will be called S_2. The logical form of any universal affirmative categorical proposition can be expressed in the following general way:

All S_1 are S_2.

All S_1 are S_2 ; $\quad S_1 \subseteq S_2$; $\quad \overset{+}{\overline{S_2}} \subseteq \overset{+}{\overline{S_1}}$

3.2. Universal Negative Categorical Propositions

One example of this type of categorical proposition is: *No engineer is pragmatic*.

This type of categorical proposition states that no element belonging to a certain set that will be called S_1 belongs to another set that will be called S_2. The logical form of any universal negative categorical proposition can be expressed in the following general way:

No S_1 is S_2.

No S_1 is S_2 ; $\quad S_1 \subseteq \overset{+}{\overline{S_2}}$; $\quad S_2 \subseteq \overset{+}{\overline{S_1}}$

3.3. Particular Affirmative Categorical Propositions

One example of this type of categorical proposition is: *Some engineers are pragmatic*.

This type of categorical proposition states that some of the elements belonging to a certain set that will be called S_1 belong to another set that will be called S_2. The logical form of any particular affirmative categorical proposition can be expressed in the following general way:

Some S_1 are S_2.

Some S_1 are S_2 ; $\quad (S_1 \cap S_2) \subseteq S_2$; $\quad (S_1 \cap S_2) \subseteq S_1$

3.4. Particular Negative Categorical Propositions

One example of this type of categorical proposition is: *Some engineers are not pragmatic*.

This type of categorical proposition states that some of the elements belonging to a certain set that will be called S_1 do not belong to another set that will be called S_2. The logical form of any particular negative categorical proposition can be expressed in the following general way:

Some S_1 are not S_2.

Some S_1 are not S_2 ; $\quad (S_1 \cap \overset{+}{\overline{S_2}}) \subseteq \overset{+}{\overline{S_2}}$; $\quad (S_1 \cap \overset{+}{\overline{S_2}}) \subseteq S_1$

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4. Using SPC to Formulate Different Categorical Propositions

The SPC is not different from what is known as propositional calculus but it is a less common way of using it. Often in the treatment of propositional calculus special attention is given to its syntax. The laws, or theorems, are proven based on that syntax. The SPC has precisely the same syntax as classical propositional calculus, but its semantics are essential for its use. In effect, the meaning of each proposition in the SPC determines how complex ("molecular") propositions are constructed from simple ("atomic") propositions, using the operators of propositional calculus for that purpose.

In this article, one of the applications of the SPC will be used: The expression of the diverse types of categorical propositions, with symbols like those of propositional calculus to be specified below. The notions considered here will be used in section 6.

4.1. Using SPC Symbols to State a Universal Affirmative Categorical Proposition

Recall the logical form of any universal affirmative categorical proposition and the two equivalent ways of stating it using set theory symbols:

All S_1 are S_2 ; $\quad S_1 \subseteq S_2$; $\quad \overset{+}{S}_2 \subseteq \overset{+}{S}_1$

Consider any element x belonging to the universal set $\mathbb{U}$ within the framework of which S_1 and S_2 have been characterized. The meaning of $S_1 \subseteq S_2$ is as follows: If x belongs to S_1, then it belongs to S_2. That is:

$(x \in S_1) \rightarrow (x \in S_2)$.

In the SPC it will be accepted that the expression of the antecedent of the above conditional proposition is the proposition q_1:

$q_1 : x \in S_1$.

Likewise, in the SPC it will be accepted that the expression of the consequent of that conditional proposition is the proposition q_2:

$q_2 : x \in S_2$.

Therefore, in the SPC terminology, the way of symbolizing $S_1 \subseteq S_2$ is the following:

$q_1 \rightarrow q_2 : S_1 \subseteq S_2$.

In the SPC, it will be accepted that the ways of symbolizing $\overset{+}{S}_1$ and $\overset{+}{S}_2$ are the following:

$q_{1'} : x \in \overset{+}{S}_1$

$q_{2'} : x \in \overset{+}{S}_2$

Therefore, in the SPC, the way of symbolizing $\overset{+}{S}_2$ and $\overset{+}{S}_1$ is the following:

$q_{2'} \rightarrow q_{1'} : \overset{+}{S}_2 \subseteq \overset{+}{S}_1$

Note that $q_{1'}$ has the same meaning as $\overline{q}_1$ (the negation of q_1) and $q_{2'}$ has the same meaning as $\overline{q}_2$ (the negation of q_2). In general, $q_{i'}$, where $i = 1, 2, 3, \ldots$, has the same meaning as $\overline{q}_i$. In effect, if an element x belonging to a given universal set $\mathbb{U}$ does not belong to the set S_i, where $i = 1, 2, 3, \ldots$, characterized within the framework of that $\mathbb{U}$, then it belongs to $\overset{+}{S_i}$.

4.2. Using the SPC Symbols to Express a Universal Negative Categorical Proposition

Recall the logical form of any universal negative categorical proposition and the two equivalent ways of expressing it using set theory symbols:

No S_1 is S_2; $\quad S_1 \subseteq \overset{+}{S_2}$; $\quad S_2 \subseteq \overset{+}{S_1}$

Thus, in the SPC, the ways of expressing $S_1 \subseteq \overset{+}{S_2}$ and $S_2 \subseteq \overset{+}{S_1}$ are the following:

$q_1 \rightarrow q_{2'} : S_1 \subseteq \overset{+}{S_2}$

$q_2 \rightarrow q_{1'} : S_2 \subseteq \overset{+}{S_1}$

4.3. Using the SPC Symbols to Express a Particular Affirmative Categorical Proposition

Recall the logical form of any particular affirmative categorical proposition and the two equivalent ways of expressing it using set theory symbols:

Some S_1 are S_2; $(S_1 \cap S_2) \subseteq S_2$; $(S_1 \cap S_2) \subseteq S_1$

Note that the proposition "Some S_1 are S_2" is equivalent to the expression "Some S_2 are S_1", and the way of expressing the latter proposition using set theory symbols is $(S_1 \cap S_2) \subseteq S_1$.

In the SPC the proposition that states that x belongs to the intersection set of S_1 and S_2 is symbolized as:

$q_{\cap;1,2} : x \in (S_1 \cap S_2)$

En $q_{\cap;1,2}$, the lowercase letter "q" has three subscripts. The first one is a small symbol of the operator of the intersection of sets: $\cap$. This symbol is separated by a semi-colon from the following two subscripts, 1 and 2, which are separated in turn by a comma. That is, $q_{\cap;1,2}$ states that an element x of the universal set $\mathbb{U}$ considered belongs to the set which is the intersection of the sets S_1 and S_2.

It is easy to interpret the meaning, for example, of $q_{\cap;7,9'}$:

$q_{\cap;7,9'}$: $x \in (S_7 \cap \overset{+}{S_9})$.

In words: The proposition symbolized by $q_{\cap;7,9'}$, states that an element x of the universal set $\mathbb{U}$ considered belongs to the set which is the intersection of the sets S_7 and $\overset{+}{S_9}$.

Therefore, the propositions which in the SPC express $(S_1 \cap S_2) \subseteq S_2$ and $(S_1 \cap S_2) \subseteq S_1$ are the following:

$q_{\cap;1,2} \rightarrow q_2 : (S_1 \cap S_2) \subseteq S_2$

$q_{\cap;1,2} \rightarrow q_1 : (S_1 \cap S_2) \subseteq S_1$

4.4. Using the SPC Symbols to Express a Particular Negative Categorical Proposition

Recall the logical form of any particular negative categorical proposition and the two equivalent ways of expressing it with set theory symbols.

Some S_1 are not S_2; $(S_1 \cap \overset{+}{S_2}) \subseteq \overset{+}{S_2}$; $(S_1 \cap \overset{+}{S_2}) \subseteq S_1$

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Given the notations considered, in the SPC the propositions that express $(S_1 \cap \overset{+}{S_2}) \subseteq \overset{+}{S_2}$ and $(S_1 \cap \overset{+}{S_2}) \subseteq S_1$ are the following:

$q_{\cap;1,2'} \rightarrow q_2 : (S_1 \cap \overset{+}{S_2}) \subseteq S_2$

$q_{\cap;1,2'} \rightarrow q_1 : (S_1 \cap \overset{+}{S_2}) \subseteq S_1$

4.5. A SPC Resource Not Used in This Article

The purpose of this subsection is to offer a somewhat broader view of the SPC, although the union of sets will not be used in this article.

Consider the sets S_1 and S_2 characterized within the framework of the universal set $\mathbb{U}$. The union of those two sets is a set which will be symbolized as $S_1 \cup S_2$. Given the definition of the union of two sets, any element the belongs to at least one of the sets S_1 and S_2 belongs to the set $S_1 \cup S_2$.

Consider now any element x belonging to $\mathbb{U}$ that also belongs to $S_1 \cup S_2$. The proposition which indicates its membership is as follows:

$x \in (S_1 \cup S_2)$.

In SPC terminology, this proposition will be symbolized as $q_{\cup;1,2}$. That is,
$q_{\cup;1,2}$: $x \in (S_1 \cup S_2)$.

The first subscript of q – $\cup$ – is a small symbol that represents the operator of the union of sets. This first subscript of q is separated by a semi-colon from the following two subscripts, 1 and 2, which are separated in turn by a comma.

It is clear then that the following two conditional propositions, expressed in SPC terminology, are valid:

$q_1 \rightarrow q_{\cup;1,2}$ and $q_2 \rightarrow q_{\cup;1,2}$.

It follows that in words, 1) the first of these propositions can be expressed as "If x belongs to S_1, then it belongs to the set $S_1 \cup S_2$" and 2) the second proposition can be expressed as "If x belongs to S_2, then it belongs to the set $S_1 \cup S_2$".

It is also easy to understand that given the notation that was already introduced for the SPC, propositions like the following are valid:

$q_5 \rightarrow q_{\cup;5,17'}$ and $q_{17'} \rightarrow q_{\cup;5,17'}$.

In set theory terminology, these conditional propositions can be expressed respectively as follows:

$(x \in S_5) \rightarrow (x \in (S_5 \cup \overset{+}{S}_{17})$ and $(x \in \overset{+}{S}_{17}) \rightarrow (x \in (S_5 \cup \overset{+}{S}_{17}))$.

5. Review of Notions Related to Categorical Syllogisms

Categorical syllogisms are a type of reasoning, or logical arguments, composed of three categorical propositions: the major premise, the minor promise and the conclusion.

If the syllogism is valid (that is, if its logical form is correct), then, if both premises are true, the conclusion is true.

The term of the subject of the conclusion, called "minor term", corresponds to the set which, in section 6 of this article, will be denominated S_1. The term of the predicate of the conclusion, called "major term", corresponds to the set which, in section 6 of this article, will be denominated S_3.

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"Major premise" is that in which the "major term", that corresponding to S_3, is present. "Minor premise" is that in which the "minor term", that corresponding to S_1, is present.

Both in the major premise and the minor premise, in addition to the terms specified, there is also a third term, known as the "middle term". This term corresponds to the set which, in section 6 of this article, will be denominated S_2.

The symbol $\therefore$ preceding the conclusion in each syllogism, means "therefore", "hence", or "consequently".

6. Determining the Validity or Lack of Validity of Categorical Syllogisms

6.1. Additional Results on Set Theory and the SPC

It is likely that teachers will consider these additional results on set theory and the SPC (to be used in 6.3) obvious. However, it may be useful to formulate them explicitly to make them easier to understand for students who are beginning to become familiar with the topics covered here.

Consider any 3 sets S_5, S_7 and S_9 characterized within the framework of a given universal set $\mathbb{U}$. These names have been used to prevent confusion between certain specific sets S_1, S_2 and S_3, mentioned in section 5.

Suppose that the intersection set of S_5 and S_7, or $S_5 \cap S_7$, is different from the empty set. The elements belonging to $S_5 \cap S_7$ make up a subset of both S_5 and of S_7. Therefore, the following relations of inclusion (in a broad sense) between sets are valid: $(S_5 \cap S_7) \subseteq S_5$ and $(S_5 \cap S_7) \subseteq S_7$.

If it is admitted that $S_5 \subseteq S_9$, the following result is obtained: $((S_5 \cap S_7) \subseteq S_5) \wedge (S_5 \subseteq S_9) \rightarrow (S_5 \cap S_7) \subseteq S_9$, given the law of the transitivity of inclusion, in a broad sense, between sets. Given that result, it can be seen that the elements belonging to both S_5 and S_7 also belong to S_9. From $(S_5 \cap S_7) \subseteq S_9$, it may therefore be inferred that the following relations of inclusion (in a broad sense) are valid between sets:

$$(S_5 \cap S_9) \subseteq S_5$$

$$(S_5 \cap S_9) \subseteq S_9$$

$$(S_7 \cap S_9) \subseteq S_7$$

$$(S_7 \cap S_9) \subseteq S_9$$

Using compact SPC symbols, the four relations above can be expressed, respectively, as the following four conditional propositions:

$$q_{\cap;5,9} \rightarrow q_5 : x \in (S_5 \cap S_9) \rightarrow x \in S_5$$

$$q_{\cap;5,9} \rightarrow q_9 : x \in (S_5 \cap S_9) \rightarrow x \in S_9$$

$$q_{\cap;7,9} \rightarrow q_7 : x \in (S_7 \cap S_9) \rightarrow x \in S_7$$

$$q_{\cap;7,9} \rightarrow q_9 : x \in (S_7 \cap S_9) \rightarrow x \in S_9$$

6.2. How to Analyze Categorical Syllogisms

In 6.3, each of 20 categorical syllogisms will be analyzed with the objective of determining the validity, or lack of validity, in the following way:

(a) The three categorical propositions composing the syllogism considered will be expressed in English and the corresponding sets S_1, S_2 and S_3 will be specified.

(b) The logical form of the categorical syllogism considered will be provided, as obtained when replacing the minor, middle and major terms with the symbols S_1, S_2 and S_3, respectively. The symbols $\overset{+}{S}_1$, $\overset{+}{S}_2$ and $\overset{+}{S}_3$ will also be used to refer to the complements of S_1, S_2 and S_3, respectively. Thus, for example, the universal negative categorical proposition "No S_3 are S_2" will be expressed as "All S_3 are $\overset{+}{S}_2$". Likewise, the particular negative categorical proposition "Some S_1 are not S_2" will be expressed as "Some S_1 are $\overset{+}{S}_2$". How the latter two propositions are expressed using, first, set theory symbols, and second, the SPC symbols will be reviewed below:

All S_3 are $\overset{+}{S}_2$; $S_3 \subseteq \overset{+}{S}_2$; $q_3 \rightarrow q_{2'}$

Some S_1 are $\overset{+}{S}_2$; $(S_1 \cap \overset{+}{S}_2) \subseteq \overset{+}{S}_2$; $q_{\cap;1,2'} \rightarrow q_{2'}$

The conclusion of the syllogism will be presented in two equivalent ways.

(c) Each of the categorical propositions composing the syllogism considered will be formulated with two equivalent relations of inclusion, in a broad sense, between sets. The validity, or lack of validity, of that syllogism will then be determined, using the formalism of set theory.

(d) Each of the categorical propositions composing the syllogism considered will be formulated with two equivalent conditional propositions. The validity, or lack of validity, of that syllogism will then be determined, using the formalism of the SPC.

The procedures described in (c) and in (d) have the following in common: If from the conjunction of the two premises (each expressed in one of the two ways presented), 1) a conclusion may be inferred, and 2) that conclusion is that expressed by one of the two equivalent conclusions presented in (b), the syllogism considered is valid. If 1) and 2) are not verified, the syllogism is not considered valid.

For it to be feasible to infer a conclusion as of the conjunction of the two premises of a categorical syllogism, in one of the premises, S_2 (the set corresponding to the middle term), must appear as an including set and the other premise must appear as the set included. In the SPC terminology, this may also be expressed as follows: In one of the two premises, q_2 must appear as the consequent of a conditional proposition and in the other premise, it must appear as an antecedent of the conditional proposition.

The results obtained in (c) and (d) must coincide. Of course, to achieve this objective (to determine the validity or lack of validity of the categorical syllogism considered), only one of the procedures specified in (c) and (d) would suffice. In the cases corresponding to valid syllogisms, in 6.3 both procedures have been presented for instructional purposes, so that students will have access to the two equivalent procedures specified and will be able to use them.

6.3. Determining the Validity, or Lack of Validity, of Diverse Categorical Syllogisms

Example 1

This slightly modified categorical syllogism was taken from [16], where it is attributed to Lewis Carroll.

1. a

1) Fossils cannot be madly in love.

2) Oysters can be madly in love.

∴3) Oysters are not fossils.

S_1: oysters S_2: beings that can be madly in love S_3: fossils

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1. b

1) All S_3 are $\overset{+}{S}_2$.

2) All S_1 are S_2.

$\therefore$3) All S_1 are $\overset{+}{S}_3$; All S_3 are $\overset{+}{S}_1$.

1. c

1) $S_3 \subseteq \overset{+}{S}_2$; $S_2 \subseteq \overset{+}{S}_3$

2) $S_1 \subseteq S_2$; $\overset{+}{S}_2 \subseteq \overset{+}{S}_1$

$\therefore$3) $S_1 \subseteq \overset{+}{S}_3$; $S_3 \subseteq \overset{+}{S}_1$

$((S_1 \subseteq S_2) \wedge (S_2 \subseteq \overset{+}{S}_3)) \rightarrow (S_1 \subseteq \overset{+}{S}_3)$

1. d

1) $q_3 \rightarrow q_{2'}$; $q_2 \rightarrow q_{3'}$

2) $q_1 \rightarrow q_2$; $q_{2'} \rightarrow q_{1'}$

$\therefore$3) $q_1 \rightarrow q_{3'}$; $q_3 \rightarrow q_{1'}$

$((q_1 \rightarrow q_2) \wedge (q_2 \rightarrow q_{3'})) \rightarrow (q_1 \rightarrow q_{3'})$

Both $S_1 \subseteq \overset{+}{S}_3$ and $q_1 \rightarrow q_{3'}$ can be interpreted as "All S_1 are $\overset{+}{S}_3$". Therefore, the syllogism is valid.

Example 2

2. a

1) All engineers are pragmatic.

2) Some engineers are wealthy.

$\therefore$3) Some wealthy persons are pragmatic.

S_1: wealthy persons S_2: engineers S_3: pragmatic persons

2. b

1) All S_2 are S_3.

2) Some S_2 are S_1.

$\therefore$3) Some S_1 are S_3; Some S_3 are S_1.

2. c

1) $S_2 \subseteq S_3$; $\overset{+}{S}_3 \subseteq \overset{+}{S}_2$

2) $(S_1 \cap S_2) \subseteq S_1$; $(S_1 \cap S_2) \subseteq S_2$

$\therefore$3) $(S_1 \cap S_3) \subseteq S_3$; $(S_1 \cap S_3) \subseteq S_1$

$(((S_1 \cap S_2) \subseteq S_2) \wedge (S_2 \cap S_3)) \rightarrow ((S_1 \cap S_2) \subseteq S_3)$
$((S_1 \cap S_2) \subseteq S_3) \rightarrow ((S_1 \cap S_3) \subseteq S_3)$

2. d

1) $q_2 \rightarrow q_3$; $q_{3'} \rightarrow q_{2'}$

2) $q_{\cap;1,2} \rightarrow q_1$; $q_{\cap;1,2} \rightarrow q_2$

$\therefore$3) $q_{\cap;1,3} \rightarrow q_3$; $q_{\cap;1,3} \rightarrow q_1$

$((q_{\cap;1,2} \to q_2) \wedge (q_2 \to q_3)) \to (q_{\cap;1,2} \to q_3)$
$(q_{\cap;1,2} \to q_3) \to (q_{\cap;1,3} \to q_3)$

Both $(S_1 \cap S_3) \subseteq S_3$ and $q_{\cap;1,3} \to q_3$ can be interpreted as "Some S_1 are S_3". Therefore, the syllogism is valid.

Example 3

3. a
1) No intellectual is superstitious.
2) Some French persons are intellectuals.
∴ 3) Some French persons are not superstitious.
S_1: French persons S_2: intellectuals S_3: superstitious persons

3. b
1) All S_2 are $\overset{+}{\overline{S_3}}$.
2) Some S_1 are S_2.
∴ 3) Some S_1 are $\overset{+}{\overline{S_3}}$; Some $\overset{+}{\overline{S_3}}$ are S_1.

3. c
1) $S_2 \subseteq \overset{+}{\overline{S_3}}$; $S_3 \subseteq \overset{+}{\overline{S_2}}$
2) $(S_1 \cap S_2) \subseteq S_2$; $(S_1 \cap S_2) \subseteq S_1$
∴ 3) $(S_1 \cap \overset{+}{\overline{S_3}}) \subseteq \overset{+}{\overline{S_3}}$; $(S_1 \cap \overset{+}{\overline{S_3}}) \subseteq S_1$

$(((S_1 \cap S_2) \subseteq S_2) \wedge (S_2 \subseteq \overset{+}{\overline{S_3}})) \to (S_1 \cap S_2) \subseteq \overset{+}{\overline{S_3}}$
$((S_1 \cap S_2) \subseteq \overset{+}{\overline{S_3}}) \to (S_1 \cap \overset{+}{\overline{S_3}}) \subseteq \overset{+}{\overline{S_3}}$

3. d
1) $q_2 \to q_{3'}$; $q_3 \to q_{2'}$
2) $q_{\cap;1,2} \to q_2$; $q_{\cap;1,2} \to q_1$
∴ 3) $q_{\cap;1,3'} \to q_{3'}$; $q_{\cap;1,3'} \to q_1$

$((q_{\cap;1,2} \to q_2) \wedge (q_2 \to q_{3'})) \to (q_{\cap;1,2} \to q_{3'})$
$(q_{\cap;1,2} \to q_{3'}) \to (q_{\cap;1,3'} \to q_{3'})$

Both $(S_1 \cap \overset{+}{\overline{S_3}}) \subseteq \overset{+}{\overline{S_3}}$ and $q_{\cap;1,3'} \to q_{3'}$ can be interpreted as "Some S_1 are $\overset{+}{\overline{S_3}}$". Therefore, the syllogism is valid.

Example 4

4. a
1) All men are rational.
2) All Spaniards are men.
∴ 3) All Spaniards are rational.
S_1: Spaniards S_2: men S_3: rational men

doi:10.20944/preprints202507.1277.v1

4. b

1) All S_2 are S_3.

2) All S_1 are S_2.

$\therefore$ 3) All S_1 are S_3; All $\overset{+}{\overline{S_3}}$ are $\overset{+}{\overline{S_1}}$.

4. c

1) $S_2 \subseteq S_3$; $\overset{+}{\overline{S_3}} \subseteq \overset{+}{\overline{S_2}}$

2) $S_1 \subseteq S_2$; $\overset{+}{\overline{S_2}} \subseteq \overset{+}{\overline{S_1}}$

$\therefore$ 3) $S_1 \subseteq S_3$; $\overset{+}{\overline{S_3}} \subseteq \overset{+}{\overline{S_1}}$

$$((S_1 \subseteq S_2) \wedge (S_2 \subseteq S_3)) \rightarrow (S_1 \subseteq S_3)$$

4. d

1) $q_2 \rightarrow q_3$; $q_{3'} \rightarrow q_{1'}$

2) $q_1 \rightarrow q_2$; $q_{2'} \rightarrow q_{1'}$

$\therefore$ 3) $q_1 \rightarrow q_3$; $q_{3'} \rightarrow q_{1'}$

$$((q_1 \rightarrow q_2) \wedge (q_2 \rightarrow q_3)) \rightarrow (q_1 \rightarrow q_3)$$

Both $S_1 \subseteq S_3$ and $q_1 \rightarrow q_3$ can be interpreted as "All S_1 are S_3". Therefore, the syllogism is valid.

Example 5

5. a

1) All mathematicians are clever.

2) Some Germans are mathematicians.

$\therefore$ 3) Some Germans are clever.

S_1: Germans S_2: mathematicians S_3: clever persons

5. b

1) All S_2 are S_3.

2) Some S_1 are S_2.

$\therefore$ 3) Some S_1 are S_3; Some S_3 are S_1.

This categorical syllogism has the same logical form as that of Example 2. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 5 is valid.

Example 6

6. a

1) All mammals are vertebrates.

2) Some animals are not vertebrates.

$\therefore$ 3) Some animals are not mammals.

S_1: animals S_2: vertebrates S_3: mammals

6. b

1) All S_3 are S_2.

2) Some S_1 are $\overset{+}{S}_2$.

$\therefore$3) Some S_1 are $\overset{+}{S}_3$; Some $\overset{+}{S}_3$ are S_1.

6. c

1) $S_3 \subseteq S_2$; $\overset{+}{S}_2 \subseteq \overset{+}{S}_3$

2) $(S_1 \cap \overset{+}{S}_2) \subseteq \overset{+}{S}_2$; $(S_1 \cap \overset{+}{S}_2) \subseteq S_1$

$\therefore$3) $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$; $(S_1 \cap \overset{+}{S}_3) \subseteq S_1$

$(((S_1 \cap \overset{+}{S}_2) \subseteq \overset{+}{S}_2) \land (\overset{+}{S}_2 \subseteq \overset{+}{S}_3)) \to ((S_1 \cap \overset{+}{S}_2) \subseteq \overset{+}{S}_3)$

$((S_1 \cap \overset{+}{S}_2) \subseteq \overset{+}{S}_3) \to ((S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3)$

6. d

1) $q_3 \to q_2$; $q_{2'} \to q_{3'}$

2) $q_{\cap;1,2'} \to q_{2'}$; $q_{\cap;1,2'} \to q_1$

$\therefore$3) $q_{\cap;1,3'} \to q_{3'}$; $q_{\cap;1,3'} \to q_1$

$((q_{\cap;1,2'} \to q_{2'}) \land (q_{2'} \to q_{3'})) \to (q_{\cap;1,2'} \to q_{3'})$

$(q_{\cap;1,2'} \to q_{3'}) \to (q_{\cap;1,3'} \to q_{3'})$

Both $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$ and $q_{\cap;1,3'} \to q_{3'}$ can be interpreted as "Some S_1 are $\overset{+}{S}_3$". Therefore, the syllogism is valid.

Example 7

7. a

1) Some Argentineans are not agnostics.

2) All Argentineans are South Americans.

$\therefore$3) Some South Americans are not agnostics.

S_1: South Americans S_2: Argentineans S_3: agnostics

7. b

1) Some S_2 are $\overset{+}{S}_3$.

2) All S_2 are S_1.

$\therefore$3) Some S_1 are $\overset{+}{S}_3$; Some $\overset{+}{S}_3$ are S_1.

7. c

1) $(S_2 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$; $S_2 \cap \overset{+}{S}_3 \subseteq S_2$

2) $S_2 \subseteq S_1$; $\overset{+}{S}_1 \subseteq \overset{+}{S}_2$

$\therefore$3) $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$; $(S_1 \cap \overset{+}{S}_3) \subseteq S_1$

$(((S_2 \cap \overset{+}{S}_3) \subseteq S_2) \land (S_2 \subseteq S_1)) \to ((S_2 \cap \overset{+}{S}_3) \subseteq S_1)$

7. d

1) $q_{\cap;2,3'} \to q_{3'}$; $q_{\cap;2,3'} \to q_2$))

2) $q_2 \to q_1$; $q_{1'} \to q_{2'}$

∴3) $q_{\cap;1,3'} \to q_{3'}$; $q_{\cap;1,3'} \to q_1$

$((q_{\cap;2,3'} \to q_2) \wedge (q_2 \to q_1)) \to (q_{\cap;2,3'} \to q_1)$

Both $(S_2 \cap \overset{+}{S_3}) \subseteq S_1$ and $q_{\cap;2,3'} \to q_1$ can be interpreted as "Some $\overset{+}{S_3}$ are S_1", which is equivalent to "Some S_1 are $\overset{+}{S_3}$". Therefore, the syllogism is valid.

Example 8

8. a

1) All sculptors are artists.

2) No artist is a stone.

∴3) No stone is a sculptor.

S_1: stones S_2: artists S_3: sculptors

8. b

1) All S_3 are S_2.

2) All S_2 are $\overset{+}{S_1}$.

∴3) All S_1 are $\overset{+}{S_3}$; All S_3 are $\overset{+}{S_1}$.

8. c

1) $S_3 \subseteq S_2$; $\overset{+}{S_2} \subseteq \overset{+}{S_3}$

2) $S_2 \subseteq \overset{+}{S_1}$; $S_1 \subseteq \overset{+}{S_2}$

∴3) $S_1 \subseteq \overset{+}{S_3}$; $S_3 \subseteq \overset{+}{S_1}$

$((S_3 \subseteq S_2) \wedge (S_2 \subseteq \overset{+}{S_1})) \to (S_3 \subseteq \overset{+}{S_1})$

8. d

1) $q_3 \to q_2$; $q_{2'} \to q_{3'}$

2) $q_2 \to q_{1'}$; $q_1 \to q_{2'}$

∴3) $q_1 \to q_{3'}$; $q_3 \to q_{1'}$

$((q_3 \to q_2) \wedge (q_2 \to q_{1'})) \to (q_3 \to q_{1'})$

Both $S_3 \subseteq \overset{+}{S_1}$ and $q_3 \to q_{1'}$ can be interpreted as "All S_3 are $\overset{+}{S_1}$". Therefore, the syllogism is valid.

Example 9

9. a

1) All multimillionaires are magnanimous.

2) No fanatic is magnanimous.

∴3) No fanatic is a millionaire.

S_1: fanatics S_2: magnanimous persons S_3: millionaires

doi:10.20944/preprints202507.1277.v1

9. b

1) All S_3 are S_2.

2) All S_1 are $\overset{+}{S_2}$.

$\therefore$ 3) All S_1 are $\overset{+}{S_3}$; All S_3 are $\overset{+}{S_1}$.

9. c

1) $S_3 \subseteq S_2\,;\quad \overset{+}{S_2} \subseteq \overset{+}{S_3}$

2) $S_1 \subseteq \overset{+}{S_2}\,;\quad S_2 \subseteq \overset{+}{S_1}$

$\therefore$ 3) $S_1 \subseteq \overset{+}{S_3}\,;\quad S_3 \subseteq \overset{+}{S_1}$

9. d

1) $q_3 \rightarrow q_2\,;\quad q_{2'} \rightarrow q_{3'}$

2) $q_1 \rightarrow q_{2'}\,;\quad q_2 \rightarrow q_{1'}$

$\therefore$ 3) $q_1 \rightarrow q_{3'}\,;\quad q_3 \rightarrow q_{1'}$

This categorical syllogism has the same logical form as the categorical syllogism in Example 8. Note that $(S_1 \subseteq \overset{+}{S_2}) \longleftrightarrow (S_2 \subseteq \overset{+}{S_1})$. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 9 is valid.

Example 10

10. a

1) No humanist is corrupt.

2) All despots are corrupt.

$\therefore$ 3) No despot is a humanist.

S_1: despots S_2: corrupt persons S_3: humanists

10. b

1) All S_3 are $\overset{+}{S_2}$.

2) All S_1 are S_2.

$\therefore$ 3) All S_1 are $\overset{+}{S_3}$; All S_3 are $\overset{+}{S_1}$,

10. c

1) $S_3 \subseteq \overset{+}{S_2}\,;\quad S_2 \subseteq \overset{+}{S_3}$

2) $S_1 \subseteq S_2\,;\quad \overset{+}{S_2} \subseteq \overset{+}{S_1}$

$\therefore$ 3) $S_1 \subseteq \overset{+}{S_3}\,;\quad S_3 \subseteq \overset{+}{S_1}$

10. d

1) $q_3 \rightarrow q_{2'}\,;\quad q_2 \rightarrow q_{3'}$

2) $q_1 \rightarrow q_2\,;\quad q_{2'} \rightarrow q_{1'}$

$\therefore$ 3) $q_1 \rightarrow q_{3'}\,;\quad q_3 \rightarrow q_{1'}$

This categorical syllogism has the same logical form as that of Example 1. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 10 is valid.

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Example 11

11. a
1) All logicians are wise.
2) Some logicians are Poles.
∴3) Some Poles are wise.
S_1: Poles S_2: logicians S_3: wise persons

11. b
1) All S_2 are S_3.
2) Some S_2 are S_1.
∴3) Some S_1 are S_3; Some S_3 are S_1.

11. c
1) $S_2 \subseteq S_3$; $\overset{+}{\overline{S_3}} \subseteq \overset{+}{\overline{S_2}}$
2) $(S_1 \cap S_2) \subseteq S_1$; $(S_1 \cap S_2) \subseteq S_2$
∴3) $(S_1 \cap S_3) \subseteq S_3$; $(S_1 \cap S_3) \subseteq S_1$

11. d
1) $q_2 \rightarrow q_3$; $q_{,3'} \rightarrow q_{2'}$
2) $q_{\cap;1,2} \rightarrow q_{1'}$; $q_{\cap;1,2} \rightarrow q_2$
∴3) $q_{\cap;1,3} \rightarrow q_3$; $q_{\cap;1,3} \rightarrow q_1$

This categorical syllogism has the same logical form as that of Example 2. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 11 is valid.

Example 12

12. a
1) Some mammals are dogs.
2) All mammals are vertebrates.
∴3) Some vertebrates are dogs.
S_1: vartabrates S_2: mammals S_3: dogs

12. b
1) Some S_2 are S_3.
2) All S_2 are S_1.
∴3) Some S_1 are S_3; Some S_3 are S_1.

12. c
1) $(S_2 \cap S_3) \subseteq S_3$; $(S_2 \cap S_3) \subseteq S_2$
2) $S_2 \subseteq S_1$; $\overset{+}{\overline{S_1}} \subseteq \overset{+}{\overline{S_2}}$
∴3) $(S_1 \cap S_3) \subseteq S_3$; $(S_1 \cap S_3) \subseteq S_1$

$(((S_2 \cap S_3) \subseteq S_2) \wedge (S_2 \subseteq S_1)) \rightarrow ((S_2 \cap S_3) \subseteq S_1)$
$((S_2 \cap S_3) \subseteq S_1) \rightarrow ((S_1 \cap S_3) \subseteq S_3)$

doi:10.20944/preprints202507.1277.v1

12. d
1) $q_{\cap;2,3} \to q_3$; $q_{\cap;2,3} \to q_2$
2) $q_2 \to q_1$; $q_{1'} \to q_{2'}$
$\therefore$3) $q_{\cap;1,3} \to q_3$; $q_{\cap;1,3} \to q_1$

$((q_{\cap;2,3} \to q_2) \wedge (q_2 \to q_1)) \to q_{\cap;2,3} \to q_1)$
$(q_{\cap;2,3} \to q_1) \to (q_{\cap;1,3} \to q_3)$

Both $(S_1 \cap S_3) \subseteq S_3$ and $q_{\cap;1,3} \to q_3$ can be interpreted as "Some S_1 are S_3". Therefore, the syllogism is valid.

Example 13

13. a
1) Some entrepreneurs are public accountants.
2) All public accountants are sensible.
$\therefore$3) Some sensible persons are entrepreneurs.
S_1: sensible persons S_2: public accountants S_3: entrepreneurs

13. b
1) Some S_3 are S_2.
2) All S_2 are S_1.
$\therefore$3) Some S_1 are S_3; Some S_3 are S_1.

13. c
1) $S_2 \cap S_3) \subseteq S_2$; $(S_2 \cap S_3) \subseteq S_3$
2) $S_2 \subseteq S_1$; $\overset{+}{\overline{S}}_1 \subseteq \overset{+}{\overline{S}}_2$
$\therefore$3) $(S_1 \cap S_3) \subseteq S_3$; $(S_1 \cap S_3) \subseteq S_1$

13. d
1) $q_{\cap;2,3} \to q_2$; $q_{\cap;2,3} \to q_3$
2) $q_2 \to q_1$; $q_{1'} \to q_{2'}$
$\therefore$3) $q_{\cap;1,3} \to q_3$; $q_{\cap;1,3} \to q_1$

This categorical syllogism has the same logical form as the categorical syllogism considered in Example 12. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 13 is valid.

Example 14

14. a
1) No artist is Neo-Kantian.
2) Some Germans are Neo-Kantians.
$\therefore$3) Some Germans are not artists.
S_1: Germans S_2: Neo-Kantians S_3: artists

14. b

1) All S_3 are $\overset{+}{S}_2$.

2) Some S_1 are S_2.

$\therefore$3) Some S_1 are $\overset{+}{S}_3$; Some $\overset{+}{S}_3$ are S_1

14. c

1) $S_3 \subseteq \overset{+}{S}_2$; $S_2 \subseteq \overset{+}{S}_3$

2) $(S_1 \cap S_2) \subseteq S_2$; $(S_1 \cap S_2) \subseteq S_1$

$\therefore$3) $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$; $(S_1 \cap \overset{+}{S}_3) \subseteq S_1$

$$(((S_1 \cap S_2) \subseteq S_2) \wedge (S_2 \subseteq \overset{+}{S}_3)) \rightarrow ((S_1 \cap S_2) \subseteq \overset{+}{S}_3)$$
$$((S_1 \cap S_2) \subseteq \overset{+}{S}_3) \rightarrow ((S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3)$$

14. d

1) $q_3 \rightarrow q_{2'}$; $q_2 \rightarrow q_{3'}$

2) $q_{\cap;1,2} \rightarrow q_2$; $q_{\cap;1,2} \rightarrow q_1$

$\therefore$3) $q_{\cap;1,3'} \rightarrow q_{3'}$; $q_{\cap;1,3'} \rightarrow q_1$

$$((q_{\cap;1,2} \rightarrow q_2) \wedge (q_2 \rightarrow q_{3'})) \rightarrow (q_{\cap;1,2} \rightarrow q_{3'})$$
$$(q_{\cap;1,2} \rightarrow q_{3'}) \rightarrow (q_{\cap;1,3'} \rightarrow q_{3'})$$

Both $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$ and $q_{\cap;1,3'} \rightarrow q_{3'}$ can be interpreted as "Some S_1 are S_3". Therefore, the syllogism is valid.

Example 15

15. a

1) No journalist is promiscuous.

2) Some promiscuous persons are fugitives.

$\therefore$3) Some fugitives are not journalists.

S_1: fugitives S_2: promiscuous persons S_3: journalists

15. b

1) All S_3 are $\overset{+}{S}_2$.

2) Some S_2 are S_1.

$\therefore$3) Some S_1 are $\overset{+}{S}_3$; Some $\overset{+}{S}_3$ are S_1.

15. c

1) $S_3 \subseteq \overset{+}{S}_2$; $S_2 \subseteq \overset{+}{S}_3$

2) $(S_1 \cap S_2) \subseteq S_1$; $S_1 \cap S_2 \subseteq S_2$

$\therefore$3) $(S_1 \cap \overset{+}{S}_3) \subseteq \overset{+}{S}_3$; $(S_1 \cap \overset{+}{S}_3) \subseteq S_1$

15. d

1) $q_3 \rightarrow q_{2'}$; $q_2 \rightarrow q_{3'}$

2) $q_{\cap;1,2} \rightarrow q_1$; $q_{\cap;1,2} \rightarrow q_2$

$\therefore$3) $q_{\cap;1,3'} \rightarrow q_{3'}$; $q_{\cap;1,3'} \rightarrow q_1$

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This categorical syllogism has the same logical form as the categorical syllogism considered in Example 14. Note that "Some S_2 are S_1" is equivalent to "Some S_1 are S_2". It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 15 is valid.

Example 16

16. a

1) All invertebrates are gorillas.

2) All cats are invertebrates.

∴3) All cats are gorillas.

S_1: cats S_2: invertebrates S_3: gorillas

16. b

1) All S_2 are S_3.

2) All S_1 are S_2.

∴3) All S_1 are S_3;   All $\overline{S_3}^{+}$ are $\overline{S_1}^{+}$.

16. c

1) $S_2 \subseteq S_3$; $\overline{S_3}^{+} \subseteq \overline{S_2}^{+}$

2) $S_1 \subseteq S_2$; $\overline{S_2}^{+} \subseteq \overline{S_1}^{+}$

∴3) $S_1 \subseteq S_3$; $\overline{S_3}^{+} \subseteq \overline{S_1}^{+}$

16. d

1) $q_2 \to q_3$; $q_{3'} \to q_{2'}$

2) $q_1 \to q_2$; $q_{2'} \to q_{1'}$

∴3) $q_1 \to q_3$; $q_{3'} \to q_{1'}$

This categorical syllogism has the same logical form as the categorical syllogism considered in Example 4. It was determined that the latter categorical syllogism is valid. Therefore, the syllogism considered in Example 16 is valid. Note that, according to accepted zoological knowledge, both the premises and the conclusion of this syllogism are false. That does not, in this case, affect the validity of this syllogism. Logic does not determine the truth or falsity of propositions as facts, but rather the validity or lack of validity of the diverse types of reasoning.

Example 17

17. a

1) All men are mortals.

2) Socrates is a man.

∴3) Socrates is mortal.

S_1: {Socrates} S_2: men S_3: mortals

17. b

1) All S_2 are S_3.

2) All S_1 are S_2.

∴3) All S_1 are S_3;   All $\overline{S_3}^{+}$ are $\overline{S_1}^{+}$

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In this classic syllogism, {Socrates} is considered to be a set to which one sole element belongs: Socrates.

17. c

1) $S_2 \subseteq S_3\,;\quad \overset{+}{S_3} \subseteq \overset{+}{S_2}$

2) $S_1 \subseteq S_2\,;\quad \overset{+}{S_2} \subseteq \overset{+}{S_1}$

$\therefore$3) $S_1 \subseteq S_3\,;\quad \overset{+}{S_3} \subseteq \overset{+}{S_1}$

17. d

1) $q_2 \rightarrow q_3\,;\quad q_{3'} \rightarrow q_{2'}$

2) $q_1 \rightarrow q_2\,;\quad q_{2'} \rightarrow q_{1'}$

$\therefore$3) $q_1 \rightarrow q_3\,;\quad q_{3'} \rightarrow q_{1'}$

This syllogism has the same logical form as that considered in Example 4. It was determined that the latter categorical syllogism is valid. Therefore, the classical syllogism considered in Example 17 is valid.

Example 18

18. a

1) Some professors are visionaries.

2) All poets are visionaries.

$\therefore$3) Some poets are professors.

S_1: poets S_2: visionaries S_3: professors

18. b

1) Some S_3 are S_2.

2) All S_1 are S_2.

$\therefore$3) Some S_1 are S_3; Some S_3 are S_1.

18. c

1) $(S_2 \cap S_3) \subseteq S_2\,;\quad (S_2 \cap S_3) \subseteq S_3$

2) $S_1 \subseteq S_2\quad ;\quad \overset{+}{S_2} \subseteq \overset{+}{S_1}$

$\therefore$3) $(S_1 \cap S_3) \subseteq S_3\,;\quad (S_1 \cap S_3) \subseteq S_1$

18. d

1) $q_{\cap;2,3} \rightarrow q_2\quad ;\quad q_{\cap;2,3} \rightarrow q_3$

2) $q_1 \rightarrow q_2\quad ;\quad q_{2'} \rightarrow q_{1'}$

$\therefore$3) $q_{\cap;1,3} \rightarrow q_3\quad ;\quad q_{\cap;1,3} \rightarrow q_1$

Both in the major premise $(S_2 \cap S_3) \subseteq S_2$ and in the minor premise $S_1 \subseteq S_2$, S_2 is the including set. In SPC terminology, it can be seen in 18.d that in the major premise $q_{\cap;2,3} \rightarrow q_2$ and in the minor premise $q_1 \rightarrow q_2$, q_2 is the consequent. The conclusion of the syllogism cannot be inferred from the conjunction of the premises. Therefore, this categorical syllogism considered is not valid.

Example 19

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19. a

1) All detectives are wise persons.

2) Some wise persons are Canadians.

∴3) Some Canadians are detectives.

S_1: Canadians S_2: wise persons S_3: detectives

19. b

1) All S_3 are S_2.

2) Some S_2 are S_1.

∴3) Some S_1 are S_3; Some S_3 are S_1.

19. c

1) $S_3 \subseteq S_2$; $\overset{+}{\overline{S_2}} \subseteq \overset{+}{\overline{S_3}}$

2) $(S_1 \cap S_2) \subseteq S_1$; $(S_1 \cap S_2) \subseteq S_2$

∴3) $(S_1 \cap S_3) \subseteq S_3$; $(S_1 \cap S_3) \subseteq S_1$

19. d

1) $q_3 \rightarrow q_2$; $q_{2'} \rightarrow q_{3'}$

2) $q_{\cap;1,2} \rightarrow q_1$; $q_{\cap;1,2} \rightarrow q_2$

∴3) $q_{\cap;1,3} \rightarrow q_3$; $q_{\cap;1,3} \rightarrow q_1$

As in the categorical syllogism in Example 18, in this syllogism both in the major premise $S_3 \subseteq S_2$ and in the expression of the minor premise $(S_1 \cap S_2) \subseteq S_2$, S_2 is the including set. In SPC terminology, it can be seen in 19.d that in the major premise $q_3 \rightarrow q_2$ and in the minor premise $q_{\cap;1,2} \rightarrow q_2$, q_2 is the consequent. The conclusion of the syllogism cannot be inferred from the conjunction of the premises. Therefore, this categorical syllogism is not valid.

Example 20

20. a

1) Some Germans are skiers.

2) All skiers are agile persons.

∴3) All Germans are agile persons.

S_1: Germans S_2: skiers S_3: agile persons

20. b

1) Some S_1 are S_2.

2) All S_2 are S_3.

∴3) All S_1 are S_3; All $\overset{+}{\overline{S_3}}$ are $\overset{+}{\overline{S_1}}$

20. c

1) $(S_1 \cap S_2) \subseteq S_2$; $(S_1 \cap S_2) \subseteq S_1$

2) $S_2 \subseteq S_3$; $\overset{+}{\overline{S_3}} \subseteq \overset{+}{\overline{S_2}})$

∴3) $S_1 \subseteq S_3$; $\overset{+}{\overline{S_3}} \subseteq \overset{+}{\overline{S_1}}$

$$(((S_1 \cap S_2) \subseteq S_2) \wedge (S_2 \subseteq S_3)) \rightarrow ((S_1 \cap S_2) \subseteq S_3)$$

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20. d

1) $q_{\cap;1,2} \to q_2$; $q_{\cap;1,2} \to q_1$

2) $q_2 \to q_3$; $q_{3'} \to q_{2'}$

$\therefore$ 3) $q_1 \to q_3$; $q_{3'} \to q_{1'}$

$$((q_{\cap;1,2} \to q_2) \wedge (q_2 \to q_3)) \to (q_{\cap;1,2} \to q_3)$$

In this case, also using SPC terminology, the conclusion of the syllogism can be inferred from the conjunction of the premises: $q_{\cap;1,2} \to q_3)$. This conclusion can be interpreted as: "Some Germans are agile persons". It can be seen that this conclusion is different from that specified in (20.c), $S_1 \subseteq S_3$, which can be interpreted as "All Germans are agile persons". Therefore, this categorical syllogism is not valid.

7. Discussion

In an earlier article [17], we presented inclusion diagrams to determine the validity of categorical syllogisms as an alternative to Venn diagrams For this reason, the title of this article mentions only the other technical tool used for this purpose—the SPC. Given that categorical syllogisms are composed of propositions, it is somewhat strange that, to the authors' best knowledge, no attempt has been made to use propositional calculus in some way to reach that objective.

This study shows how relations of inclusion (in a broad sense) between sets not only make this diagrammatic approach possible, but they also can be used independently from the diagrams as a formal resource for problems concerning syllogistics.

Certain isomorphisms between set theory and propositional calculus were discussed in [18]. Another relation between aspects of set theory and a particular way to use propositional calculus (the SPC) are described in the present article.

One of the essential aspects of the STA is the joint presentation of diverse approaches or formalisms of a specific discipline – or of several disciplines – emphasizing isomorphisms whenever possible; that is, the correspondences or other types of links between the different topics or approaches considered. Joint presentations like this one will be used in forthcoming articles on the STA.

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