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A Proof of Irrationality of π Based on Nested Radicals with Roots of 2

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Abstract

In this work, we consider four theorems that can be used to prove the irrationality of π. These theorems are related to nested radicals with roots of 2 of kind $c_k = \sqrt{2 + c_{k-1}}$ and $c_0 = 0$. Sample computations showing how the rational approximation tend to π with increasing the integer k are presented.

Keywords: constant π; irrationality; nested radicals; relative primes; rational approximation

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1. Introduction

In 1714, the English mathematician Roger Cotes discovered a remarkable identity [1,2]

$$ix = \ln(\cos(x) + i\sin(x)).$$

A few decades later, Swiss mathematician Leonardo Euler found a reformulated form of this identity as

$$e^{ix} = \cos(x) + i\sin(x)$$

from which it follows that

$$e^{i\pi} + 1 = 0.$$

This equation, also known as Euler's identity, is commonly considered as the most beautiful formula in mathematics as it relates the ubiquitous constants π and e to each other [2]. Sometimes these constants π and e are also regarded as Archimedes' constant and Euler's number, respectively.

A proof of irrationality of the constant e may not be difficult (see for example [3,4]). However, it was not easy to find a proof of irrationality of π; a long time passed since discovery of π by ancient Babylonians and Egyptians [5–7] to prove its irrationality.

A first proof that π is irrational was given by Swiss mathematician Johann Heinrich Lambert in 1761 [6,8] (see also [9]). In his work Lambert showed that if $x \neq 0$ in the following infinite continuous fraction

$$\tan(x) = \cfrac{x}{1 - \cfrac{x^2}{3 - \cfrac{x^2}{5 - \cfrac{x^2}{7 - \cfrac{x^2}{9 - \ldots}}}}},$$

then value of x cannot be rational when its expansion on the right side is rational. Therefore, in the equation

$$\tan\left(\frac{\pi}{4}\right) = 1$$

the constant π must be irrational.

A first proof of irrationality of π by contradiction was found in 1873 by French mathematician Charles Hermite [10]. There are several other proofs of irrationality of π [11–14,16,17]. One of them, published by Niven in 1947, is particularly interesting and attracts much attention. In his work [12], Niven proved the irrationality of π also by contradiction. In particular, with the help of the series expansion

$$F(x) = \sum_{m=0}^{n} (-1)^m \frac{d^{2m}}{dx^{2m}} f(x),$$

where

$$f(x) = \frac{x^n (a - bx)^n}{n!},$$

he showed that it is impossible to represent π as a ratio of two positive integers a and b. Despite a long history, research on the irrationality of π still remains interesting [8,15–17].

In this work, we present a proof of irrationality of π based on nested radicals of kind $c_k = \sqrt{2 + c_{k-1}}$, where $c_0 = 0$. These nested radicals have been used in our earlier publications [18,19] to generate the Machin-like formulas for π. To the best of our knowledge, this approach is new and has never been reported.

2. Preliminaries

The identity (1) below has been used in our previous publications [18,19] as a starting point to generate the Machin-like formulas for π. The following theorem shows how this identity can be derived.

Theorem 1. *The following equation [20]*

$$\frac{\pi}{4} = 2^{k-1} \arctan\left(\frac{\sqrt{2 - c_{k-1}}}{c_k}\right), \qquad k \geq 1, \tag{1}$$

where k is an integer, holds.

Proof. Using the double angle identity

$$\cos(2x) = 2\cos^2(x) - 1,$$

by induction it follows that

$$\cos\left(\frac{\pi}{2^2}\right) = \frac{1}{2}\sqrt{2} = \frac{1}{2}c_1,$$

$$\cos\left(\frac{\pi}{2^3}\right) = \frac{1}{2}\sqrt{2 + \sqrt{2}} = \frac{1}{2}c_2,$$

$$\cos\left(\frac{\pi}{2^{k+1}}\right) = \frac{1}{2}\underbrace{\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots + \sqrt{2}}}}}_{n \text{ square roots}} = \frac{1}{2}c_k. \tag{2}$$

Therefore, we get

$$\sin\left(\frac{\pi}{2^{k+1}}\right) = \sqrt{1 - \cos^2\left(\frac{\pi}{2^{k+1}}\right)}. \tag{3}$$

Thus, using equations (2) and (3) we obtain

$$\tan\left(\frac{\pi}{2^{k+1}}\right)=\tan\left(\frac{\sqrt{1-\cos^2\left(\frac{\pi}{2^{k+1}}\right)}}{\cos\left(\frac{\pi}{2^{k+1}}\right)}\right)$$

$$=\tan\left(\frac{\sqrt{2-\cos\left(\frac{\pi}{2^{k}}\right)}}{\cos\left(\frac{\pi}{2^{k+1}}\right)}\right)=\tan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)$$

or

$$\frac{\pi}{2^{k+1}}=\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)$$

and this completes the proof. □

Since the integer k can be arbitrarily large, we can also write

$$\frac{\pi}{4}=\lim_{k\to\infty}2^{k-1}\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right). \tag{4}$$

Using the limit (4) we can derive a well-known formula for π [21]

$$\pi=\lim_{k\to\infty}2^k\sqrt{2-\underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}_{k-1\text{ square roots}}}=\lim_{k\to\infty}2^k\sqrt{2-c_{k-1}}.$$

Another formula for π that can also be derived from the limit (4) is given by (see [22] and literature therein)

$$\pi=\lim_{k\to\infty}2^k\sum_{n\geq k}\frac{\sqrt{2-c_{n-1}}}{c_n}.$$

It should be noted that this limit can be further simplified as

$$\pi=\lim_{k\to\infty}2^{k-1}\sum_{n\geq k}\sqrt{2-c_{n-1}}$$

or

$$\pi=\lim_{k\to\infty}2^{k}\sum_{n\geq k}\sqrt{2-c_{n}}$$

since

$$\lim_{n\to\infty}c_n=\lim_{n\to\infty}\underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}_{n\text{ square roots}}=2.$$

3. Irrationality of π

Consider three theorems below.

Theorem 2. *The following limit*

$$\pi=\lim_{k\to\infty}\frac{2^{k+1}}{\alpha_k}, \tag{5}$$

where

$$\alpha_k = \left\lfloor \frac{2^{k+1}}{\pi} \right\rfloor \tag{6}$$

holds.

Proof. According to equations (4), (5) and (6) the constant α_k represents the integer part of the arctangent function as follows

$$\alpha_k = \left\lfloor \frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} \right\rfloor.$$

Therefore, we can express the reciprocal of the arctangent function as

$$\frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} = \alpha_k + \beta_k,$$

where β_k is the fractional part given by

$$\beta_k = \left\{ \frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} \right\} = \left\{ \frac{2^{k+1}}{\pi} \right\}.$$

Thus, equation (1) can be represented in form

$$\pi = \frac{2^{k+1}}{\alpha_k + \beta_k}, \qquad k \geq 1. \tag{7}$$

Since the fractional part β_k cannot be smaller than zero and greater than one while the integer part α_k tends to infinity with increasing k, it follows that

$$\lim_{k\to\infty} \frac{\alpha_k}{\frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)}} = \lim_{k\to\infty} \frac{\alpha_k}{\alpha_k + \beta_k} = 1.$$

Therefore, from this limit and equation (7) we have

$$\lim_{k\to\infty} \frac{2^{k+1}}{\alpha_k + \beta_k} = \lim_{k\to\infty} \frac{2^{k+1}}{\alpha_k}$$

and this completes the proof. □

Theorem 3. *The following inequality*

$$\pi < \frac{2^{k+1}}{\alpha_k}, \qquad 1 \leq k < \infty \tag{8}$$

holds.

Proof. We can show that the fractional part β_k cannot be equal to zero and, therefore, is given by the following inequality

$$0 < \beta_k < 1. \tag{9}$$

The constant β_k is always greater than zero because the reciprocal of the arctangent function in equation

$$\frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} = \frac{2^{k+1}}{\pi}$$

cannot be an integer. In particular, since π is not an even integer, the ratio on the right side of the equation above is not an integer. For example, it can be shown that π is a number located between 3.1408 and 3.1429 due to inequality [23,24]

$$\frac{223}{71} < \pi < \frac{22}{7}.$$

More explicitly, as it follows from equation (7)

$$\beta_k = \frac{2^{k+1}}{\pi} - \alpha_k, \qquad k \geq 1$$

the coefficient β_k cannot be equal to zero as π is not an even integer. Thus, according to equation (7) and inequality (9) the theorem is proved. □

Theorem 4. *The following equation*

$$\alpha_{k+1} = \begin{cases} 2\alpha_k, & 0 < \gamma_k < 1/2 \\ 2\alpha_k + 1, & 1/2 \leq \gamma_k < 1 \end{cases}$$

holds.

Proof. Since the ratio

$$\frac{2^{k+2}\arctan\left(\frac{\sqrt{2-c_k}}{c_{k+1}}\right)}{2^{k+1}\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} = \frac{\pi}{\pi} = 1,$$

we can write

$$\frac{2\arctan\left(\frac{\sqrt{2-c_k}}{c_{k+1}}\right)}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)} = \frac{\frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)}}{\frac{1}{2}\frac{1}{\arctan\left(\frac{\sqrt{2-c_k}}{c_{k+1}}\right)}} = 1.$$

This equation leads to

$$\frac{\frac{1}{\arctan\left(\frac{\sqrt{2-c_k}}{c_{k+1}}\right)}}{\frac{1}{\arctan\left(\frac{\sqrt{2-c_{k-1}}}{c_k}\right)}} = 2$$

or

$$\frac{\alpha_{k+1} + \beta_{k+1}}{\alpha_k + \beta_k} = 2. \tag{10}$$

From equation (10) it follows that

$$\alpha_{k+1} + \beta_{k+1} = 2(\alpha_k + \beta_k).$$

Taking the floor function from the both sides leads to

$$\lfloor \alpha_{k+1} + \beta_{k+1} \rfloor = \lfloor 2(\alpha_k + \beta_k) \rfloor$$

and since α_{k+1} is an integer while $0 < \beta_{k+1} < 1$, we get

$$\lfloor \alpha_{k+1} \rfloor = \lfloor 2(\alpha_k + \beta_k) \rfloor$$

or

$$\alpha_{k+1} = \lfloor 2(\alpha_k + \beta_k) \rfloor = 2\alpha_k + \lfloor 2\beta_k \rfloor.$$

As we can see from this equation, α_{k+1} is equal to $2\alpha_k$ when $0 < \beta_k < 1/2$ and equal to $2\alpha_k + 1$ when $1/2 \le \beta_k < 1$. This completes the proof. □

Finally, the lemma below shows how the limit (5) and inequality (8) lead to the irrationality of π

Lemma 1. *The constant π is irrational.*

Proof. Define the following integers

$$\gamma_k = \begin{cases} \gamma_{k-1} + 1, & \text{if } \alpha_k \text{ is odd,} \\ \gamma_{k-1}, & \text{if } \alpha_k \text{ is even,} \end{cases}$$

where $\gamma_1 = 1$. Consequently, we can construct the sequences for positive integers k, γ_k, α_k and α_{γ_k} as follows

$$\begin{aligned} \{k\}_{k=1}^{\infty} &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ \ldots\}, \\ \{\alpha_k\}_{k=1}^{\infty} &= \{1, 2, 5, 10, 20, 40, 81, 162, 325, 651, \ \ldots\}, \\ \{\gamma_k\}_{k=1}^{\infty} &= \{1, 1, 3, 3, 3, 3, 7, 7, 9, 10, \ \ldots\} \end{aligned}$$

and

$$\begin{aligned} \left\{\alpha_{\gamma_k}\right\}_{k=1}^{\infty} &= \{\alpha_1, \alpha_1, \alpha_3, \alpha_3, \alpha_3, \alpha_3, \alpha_7, \alpha_7, \alpha_9, \alpha_{10}, \ \ldots\} \\ &= \{1, 1, 5, 5, 5, 5, 81, 81, 325, 651, \ \ldots\}. \end{aligned} \tag{11}$$

The numbers α_k from the sequence $\{\alpha_k\}_{k=1}^{\infty}$ can be found in [25]. While the integers in the sequence $\{\alpha_k\}_{k=1}^{\infty}$ can be even and odd, the integers in the sequence $\left\{\alpha_{\gamma_k}\right\}_{k=1}^{\infty}$ are always odd. It means that if an integer α_k is an even number, then it has a common factor with integer 2^{k+1} as both of them are divisible by 2.

Thus, due to divisibility by 2 when α_k is an even number, we can rearrange the inequality (8) and limit (5) as

$$\pi < \frac{2^{\gamma_k + 1}}{\alpha_{\gamma_k}}, \qquad \alpha_{\gamma_k} \in 2\mathbb{N} + 1 \tag{12}$$

and

$$\pi = \lim_{k \to \infty} \frac{2^{\gamma_k + 1}}{\alpha_{\gamma_k}}, \qquad \alpha_{\gamma_k} \in 2\mathbb{N} + 1, \tag{13}$$

respectively.

Assume that π can be represented as a ratio of two positive integers p and q. Then, according to inequality (12) and equation (13), we immediately get a contradiction with our assumption that π can be represented as a ratio of two integers.

If we assume that starting from some integer k_0 the equation

$$\alpha_{k_0 + n + 1} \overset{?}{=} 2\alpha_{k_0 + n}, \qquad \forall n \ge 0$$

always holds, then the limit (13) converges in the form

$$\lim_{k \to \infty} \frac{2^{\gamma_k + 1}}{\alpha_{\gamma_k}} \overset{?}{=} \frac{2^{\gamma_{\max} + 1}}{\alpha_{\gamma_{\max}}} \tag{14}$$

where $\gamma_{\max}$ is presumably the largest integer in the sequence $\{\gamma_k\}_{k=1}^{\infty}$ and $\alpha_{\gamma_{\max}} \overset{?}{=} \alpha_{k_0}$ is presumably the largest odd integer in the sequence $\left\{\alpha_{\gamma_k}\right\}_{k=1}^{\infty}$. However, the equation (14) contradicts the inequality (12) as its right side must be greater than π. Therefore, such integers $\gamma_{\max}$ and $\alpha_{\gamma_{\max}}$ do not exist and equation (14) is incorrect.

On the other hand, if we assume that despite absence of the numbers $\gamma_{\max}$ and $\alpha_{\gamma_{\max}}$ the limit (13) still can converge as a ratio of two integers p and q such that

$$\lim_{k\to\infty} \frac{2^{\gamma_k+1}}{\alpha_{\gamma_k}} \overset{?}{=} \frac{p}{q}, \tag{15}$$

then it contradicts the fact that even numerator 2^{γ_k+1} and odd denominator α_{γ_k} are always relative primes at any value of k and, therefore, these two numbers do not have a common divisor except 1 at any value of k. As a result, it cannot converge as a ratio of two integers p and q. Thus, we can conclude that the limit (15) is also incorrect. This completes the proof that the constant π is irrational. □

4. Rational Approximation of π

The limit (5) shows that we can approximate π in form of the rational approximation as given by

$$\pi \approx \frac{2^{k+1}}{\alpha_k}, \qquad k \gg 1. \tag{16}$$

Consider the following examples (a link for the extended table showing values of α_k can be found in [25])

$$\begin{aligned}
\alpha_{70} &= \alpha_{\gamma_{70}} &&= 751,587,968,840,192,313,983 \\
\alpha_{71} &= 2\alpha_{\gamma_{70}} &&= 1,503,175,937,680,384,627,966 \\
\alpha_{72} &= 4\alpha_{\gamma_{70}} &&= 3,006,351,875,360,769,255,932 \\
\alpha_{73} &= 8\alpha_{\gamma_{70}} &&= 6,012,703,750,721,538,511,864 \\
\alpha_{74} &= 16\alpha_{\gamma_{70}} &&= 12,025,407,501,443,077,023,728
\end{aligned}$$

Although the values of the coefficient from α_{70} to α_{74} increases by a factor of 2, the corresponding ratios

$$\begin{aligned}
\frac{2^{75}}{\alpha_{74}} = \frac{2^{74}}{\alpha_{73}} = \frac{2^{73}}{\alpha_{72}} = \frac{2^{72}}{\alpha_{71}} = \frac{2^{71}}{\alpha_{70}} = \frac{2^{\gamma_{71}}}{\alpha_{\gamma_{70}}} \\
= \underbrace{3.141592653589793238462}_{\text{22 correct digits of } \pi}80398052\ldots
\end{aligned}$$

remain unchanged. This occurs because the ratio of two adjacent values is

$$\alpha_{k+1} = 2\alpha_k, \qquad 70 \leq k \leq 74.$$

However, at $k = 75$ we get

$$\alpha_{75} = (2+1)\alpha_{74}.$$

As the values

$$\begin{aligned}
\alpha_{75} &= \alpha_{\gamma_{75}} = 24,050,815,002,886,154,047,457 \\
\alpha_{76} &= 2\alpha_{75} = 48,101,630,005,772,308,094,914 \\
\alpha_{77} &= 4\alpha_{75} = 96,203,260,011,544,616,189,828
\end{aligned}$$

we get

$$\frac{2^{78}}{\alpha_{77}} = \frac{2^{77}}{\alpha_{76}} = \frac{2^{76}}{\alpha_{75}} = \frac{2^{\gamma_{76}}}{\alpha_{\gamma_{75}}} = \underbrace{3.1415926535897932384626}_{\text{23 correct digits of } \pi}7335739\ldots\ .$$

These examples showing the relations between the positive integers k, α_k, γ_k and α_{γ_k} help us to understand how the rational approximation (16) tend to π with increasing the integer k.

5. Conclusion

Four theorems that can be used to prove the irrationality of π are considered. These theorems are related to nested radicals consisting of square roots of 2 of kind $c_k = \sqrt{2 + c_{k-1}}$ and $c_0 = 0$. Examples of the rational approximation tending to π with increasing the integer k are provided.

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