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Article

Modified Nonlocal Newtonian Dynamics and the Cosmological Constant Problem

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Abstract: In this paper, I derive the modified acceleration formula of a toy model of the universe by using a mean field approach. The dynamics are given by the equation of the Kuramoto-like form. The equilibrium solution of the equation is consistent with the usual Milgrom's law. As a result, the state of the universe at large scales enters local dynamics in small systems. I also show that the vacuum energy density arising from zero-point fluctuations and symmetry breakings is acceleration-dependent and vanishes as seen from the point of view of enough accelerated observers. Therefore, the usual vacuum stress-energy is no longer a tensor. It turns out that only the self-energy of elementary particles contributes to the vacuum energy density and the theoretical value of the cosmological constant gets significantly reduced.

Keywords: modified Newtonian dynamics; Kuramoto model; nonlocal dynamics; cosmological constant problem

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1. Introduction

The universe in its vast complexity harbors profound mysteries that challenge our understanding of fundamental physics. Observations of the flat rotation curves of galaxies reveal a striking discrepancy that galaxies rotate faster than can be accounted for by the visible matter alone, suggesting the presence of an invisible mass, known as dark matter. It is also hypothesized to explain a range of gravitational anomalies observed in gravitational lensing and the cosmic microwave background (CMB) [1]. Despite its success in accounting for these phenomena, the dark matter paradigm faces significant challenges. The lack of direct detection raises questions about its fundamental validity. The most widely accepted Cold Dark Matter (CDM) model struggles to account for observational discrepancies at smaller scales, such as the "missing satellites problem," where the predicted number of dwarf galaxies around massive galaxies exceeds observations [2–4], and the "cusp-core problem," where simulated dark matter halos exhibit central density cusps inconsistent with the flatter density profiles observed in some galaxies [5]. In addition, some observational data including rapid galaxy growth [6]and flat velocity curves extending beyond the expected virial radii of dark matter halos [7] also suggest inconsistencies with the CDM model. These issues imply that the CDM framework may need refinement or that our understanding of gravitational dynamics requires revision.

One such alternative theory is the Modified Newtonian Dynamics (MOND) proposed by Milgrom as a modification to Newtonian gravity at extremely low accelerations [8–10]. Although MOND struggles to account for phenomena at cosmological scales, such as the cosmic microwave background and large-scale structure formation, the simplicity and predictive power of MOND make it a valuable theoretical tool, prompting ongoing research into its foundations and potential extensions. Unlike the dark matter hypothesis, the MOND paradigm stipulates that the observed gravitational effects arise not from unseen mass but from a deviation in the law of gravity when accelerations fall below a critical threshold $a_0 \approx 1.2 \times 10^{-10}\mathrm{m/s^2}$. Milgrom's law can be written as $\mu(a/a_0)a = a_N$, where a_N is the Newtonian acceleration produced by the visible matter, a is the true gravitational acceleration and the interpolating function $\mu(x)$ satisfies $\mu(x) \approx 1$ when $x \gg 1$, and $\mu(x) \approx x$ when $x \ll 1$. In the deep-MOND regime $(a \ll a_0)$, MOND modifies the gravitational force law to scale inversely

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with distance rather than the square of the distance, effectively reproducing the flat rotation curves of galaxies and Tully-Fisher relation without invoking dark matter. MOND can be interpreted as a modification of gravity or inertia. In the modified gravity interpretation, Newton's law of gravity should be modified at low accelerations, leading to a stronger gravitational effect than predicted by the usual inverse-square law. This approach has been formalized in theories such as the Tensor-Vector-Scalar (TeVeS) gravity [11] and bimetric theories [12,13], which attempts to provide a relativistic extension of MOND [10]. Alternatively, the modified inertia interpretation suggests that the response of a body to a given force depends on the acceleration regime. Given the empirical validations and elegant mathematical structure of general relativity, there is more potential in modified inertia as the basis for MOND because it seems to be less drastic. Inertia governs how objects respond to applied forces. However, the origin of inertia remains an open question in modern physics. Understanding the origin of inertia could provide critical insights into the foundations of the MOND theory. In classical mechanics, inertia is an intrinsic property of mass, yet its microscopic basis is not well understood. Some theories propose that inertia arises from interactions with the vacuum [14]. Einstein's theory of general relativity offers a partial explanation, suggesting that mass influences and is influenced by the local geometry of spacetime. Alternative perspectives, like Mach's principle, suggest that inertia is a relational property, emerging from an object's interaction with the global distribution of matter in the universe. Clearly, the inertial mass of elementary particles primarily arises from the interaction with the Higgs field. For a charged particle, the interaction of particles with the electromagnetic field also contributes to inertia, known as electromagnetic mass. In general, we should consider all possible contributions. But for macroscopic objects, various intricate interactions may contribute to the inertial mass.

On the other hand, a very noteworthy coincidence of MOND is that the value of $a_0 \approx 1.2 \times 10^{-10}\mathrm{m/s^2}$ determined from galaxy dynamics is of the order of some acceleration constants of cosmological significance. It is of the same order as H_0 (H_0 is the Hubble constant) and $(\Lambda/3)^{1/2}$ (Λ is the cosmological constant). This mysterious "cosmic coincidence" raises questions about its fundamental significance and potential cosmic connections. In this paper, I argue that the "cosmic coincidence" can be understood as a "synchronization" phenomenon at cosmological scales similar to the Kuramoto model [15,16]. The Kuramoto model is a mathematical framework that describes synchronization in systems of coupled oscillators. It predicts a phase transition where oscillators spontaneously synchronize, transitioning from disorder to a collective rhythm when the coupling strength exceeds a critical value. However, on cosmological scales, the typical gravitational force that decays with distance is insufficient to induce collective emergence, as it would only result in local dynamics. But considering the gravity including the cosmological constant, the fact that the interaction strength increasing with distance inevitably leads to the state of the universe at large scales entering local dynamics in small systems, such as galaxies. This nonlocal behavior is common in nonlinear dynamical systems. Upon using a mean field theory approach, it is possible to study a system with a large or infinite number of degrees of freedom. Thus I define a mean field characterized by the Unruh temperature to represent the average effect of all the contents of the universe. It may suggest a connection between gravity and thermodynamics. In some studies, gravity is regarded as an entropic force [17–20]. In my personal view, this only means that gravity can be described in another framework at the macroscopic level, whereas at microscopic scales gravity may still be quantum. Furthermore, emergent phenomena at the cluster and cosmological scales arise from the interactions, primarily gravity, of the universe's components. Thus, it is gravity that leads to emergent phenomena at larger scales rather than gravity itself originating from emergence.

Another concern of this study is the cosmological constant problem, which remain a theoretical puzzle that bridges cosmology and quantum field theory [21]. The problem originates from the interpretation of the cosmological constant Λ as the vacuum energy density. There are at least two sources for the vacuum energy. In the cosmological context, spontaneous symmetry breakings in the early universe may have induced phase transitions, potentially contributing to the vacuum energy

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density associated with the cosmological constant. Although one can always adjust the vacuum energy today to zero by tuning the parameter of the potential, it is not a very satisfactory method because the vacuum energy cannot be zero before and after the phase transition. In addition, in quantum field theory the vacuum is filled with quantum fluctuations contributing to a zero-point energy (ZPE). Since all energy gravitates, it is expected that the ZPE contribute to the cosmological constant. However, the theoretical prediction of the cosmological constant from quantum field theory contrasts with its observed value, giving rise to a discrepancy that spans over 120 orders of magnitude. One promising avenue for addressing the cosmological constant problem is supersymmetry (SUSY), a theoretical framework that posits a symmetry between fermions and bosons [22]. But experimental searches at the Large Hadron Collider have yet to detect supersymmetric particles. In addition, the precise mechanism by which SUSY could resolve the cosmological constant problem remains elusive, as the required cancellations demand an extraordinary degree of fine-tuning in the SUSY-breaking sector. In this paper, I will calculate the vacuum energy for an accelerating observer. An interesting thing happens when we consider the Unruh effect which predicts that an accelerating observer in a vacuum will detect a thermal bath at a temperature proportional to its proper acceleration a [23]. In Ref. [24], it has been shown that the electroweak phase transition (EWPT) occurs and the electroweak gauge symmetry can be restored when the acceleration exceeds the critical value as seen from the point of view of an accelerating observer. I will show that the vacuum energy arising from zero-point fluctuations and symmetry breakings is acceleration-dependent. Therefore, it does not contribute to the vacuum stress-energy tensor. The analysis is based on the conservative assumption of maximal validity of quantum field theory and general relativity.

This paper is organized as follows. Sec.2 is dedicated to the modified nonlocal Newtonian dynamics. In Sec.3, I investigate the vacuum energy for accelerated observers. Finally, in Sec.4 I summarize the main results obtained. For convenience, I use natural units with $c = \hbar = k = 1$.

2. Modified Nonlocal Newtonian Dynamics

Let us consider a toy model of the universe consisting of N masses, each with mass m, coupled by gravity including a cosmological constant Λ. In the limit of small velocity and in the weak field approximation, Einstein's field equations should reproduce Newtonian gravity. In the Newtonian limit, Einstein's field equations with the cosmological constant reduce to a modified Poisson equation:

$$\nabla^2\phi = 4\pi G\rho - \Lambda, \tag{1}$$

where ϕ is the gravitational potential, G is Newton's gravitational constant and ρ is the mass density. The equivalent force due to the cosmological constant is directly proportional to the distance with a proportionality constant of $\Lambda m/3$. The cosmological constant Λ is very small, so this force is negligible on small scales (e.g., laboratory or planetary) and only significant on cosmological scales (e.g., galaxy clusters or larger). The fact that the couple strength increasing with distance due to the cosmological constant inevitably leads to the state of the entire system at large scales entering local dynamics in small systems. Similar to the Kuramoto model [15,16], we can sum over the acceleration of all contents in the universe to obtain an average acceleration in a specific direction to describe the overall behavior of the universe. However, assuming the universe is isotropic, it is more appropriate to describe the dynamics of the system using a scalar related to acceleration, such as the magnitude or the square of the acceleration. In addition, 4-dimensional de Sitter spacetime can be embedded into a 5-dimensional flat Minkowski spacetime to generate a geometrical symmetry group $SO(4,1)$ of the de Sitter spacetime. Here, I choose the Unruh temperature associated with acceleration as the degree of freedom. The dynamics are described by the equation of the following Kuramoto-like form:

$$\dot{T}_i = \frac{\alpha}{2\pi}F_i + \frac{\varepsilon}{N}\sum_{j=1}^{N}\frac{\alpha\Lambda m r_{ij}}{3}\Gamma_{ij}(T_j - T_i), \tag{2}$$

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where the dot denotes the derivative with respect to time t, $T_i = \frac{1}{2\pi}\left(a^2 + H_0^2\right)^{1/2}$ is the Unruh temperature of the i-th mass seen by a local comoving accelerating observer in de Sitter spacetime, α is a dimensionless numerical factor, F_i denotes the external force applied to the i-th mass or the local gravitational environment, ε is the positive coupling strength, r_{ij} is the distance and $\Gamma_{ij}(T_j - T_i)$ is a general coupling function for the interaction between the i-th and j-th masses. Here, I choose the coupling function $\Gamma_{ij}(T_j - T_i) = T_j - T_i$ because it naturally captures the acceleration differences between coupled masses in a simple and physically meaningful way. In addition, this linear term ensures that Equation (2) holds in the deep-MOND regime ($a \ll a_0$). I have neglected the contribution of the conventional Newtonian gravitational force that decays with distance to the second term on the right-hand side (RHS) of Equation (2) as it only leads to local dynamics. Let us define a mean field that represents the average effect of all the contents of the universe. Inertial observers in our universe with a positive cosmological constant detect a Gibbons–Hawking radiation with the temperature $T_{\rm GH} = H_0/2\pi$ [25]. Therefore, the mean temperature is $T_{\rm GH}$. Every accelerated object appears to be subjected to a force that drives its acceleration toward zero. It is a "synchronization" phenomenon at cosmological scales and the overall behavior of the universe encoded in H_0 enters local dynamics in small systems. Upon using a mean field approach, all of the originally coupled differential equations become coupled only to the mean field quantity $T_{\rm GH}$. Let us consider the simplest possible form of Equation (2), which is

$$\dot{T}_i = \frac{\alpha}{2\pi} F_i + \alpha m (T_{\rm GH} - T_i), \tag{3}$$

where αm is the effective coupling strength. External forces can be expressed in the form of Newton's second law, namely $F_i = m a_N$ with a_N being the Newtonian expression for the acceleration. Thus Equation (3) can be written in terms of the acceleration as

$$\frac{a\dot{a}}{\sqrt{a^2 + H_0^2}} = \alpha m \left[H_0 - \left(a^2 + H_0^2\right)^{1/2}\right] + \alpha m a_N. \tag{4}$$

The fixed point given by $\dot{a} = 0$ represents stable solutions of Equation (4) and one arrives at

$$a_N = \left(a^2 + H_0^2\right)^{1/2} - H_0, \tag{5}$$

which leads to $\mu(a/a_0)a = a_N$ with $a_0 = 2H_0$. It was interestingly noted that the local dynamics of small systems depend on the state of the universe at large scales and the Hubble constant varies during the evolution of the universe. Therefore, we should replace H_0 with the varying Hubble parameter $H(t)$ and Equation (5) becomes

$$\frac{H(t)\dot{H}(t)}{\sqrt{a^2 + H(t)^2}} = \alpha m \left[H(t) - \left(a^2 + H(t)^2\right)^{1/2}\right] + \alpha m a_N. \tag{6}$$

One immediately obtains

$$a = \frac{a_N}{2} + \frac{1}{2}\sqrt{a_N^2 - \frac{4H(t)\dot{H}(t)}{\alpha m}}, \qquad \text{for } a \gg 2H(t), \tag{7}$$

$$a = \sqrt{2H(t)a_N - \frac{2H(t)\dot{H}(t)}{\alpha m}}, \qquad \text{for } a \ll 2H(t). \tag{8}$$

It is expected that MOND has a different impact on the evolution of the universe during the period of decelerated expansion compared to the present universe. We have considered the simplest possible case. However, for the complex local gravitational environments in the actual universe, the mean field approximation does not hold, and we should employ N-body simulations. Furthermore, the parameters used to describe the overall behavior of the universe may not be unique. In addition to

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$H(t)$, we may introduce further parameters, and thus the dynamics are governed by a set of Kuramoto-like equations. The key point is that the current state described by certain parameters at cosmic scales influences the behavior of local systems.

3. Acceleration-Dependent Vacuum Energy

For an accelerating observer the electroweak $SU(2) \times U(1)$ gauge symmetry in the Standard Model is restored for acceleration larger than a critical value. The vacuum expectation value (VEV) is given by [24]

$$v(a) = v_0 \sqrt{1 - \frac{a^2}{a_{\rm EW}^2}}. \tag{9}$$

where v_0 is the VEV for the inertial observer, a is the proper acceleration and $a_{\rm EW}$ is the critical proper acceleration of the EWPT. The second-order phase transition of the restoration of electroweak symmetry occurs at $a_{\rm EW}$ and for $a > a_{\rm EW}$, we have $v = 0$. The elementary particles therefore acquire a acceleration-dependent mass which is

$$m(a) = m_0 \sqrt{1 - \frac{a^2}{a_{\rm EW}^2}}, \tag{10}$$

where m_0 is the mass of the elementary particle for the inertial observer. By introducing the Unruh-like temperature:

$$T_{\rm EW} = \frac{a_{\rm EW}}{2\pi} \tag{11}$$

and

$$T(a) = \frac{a}{2\pi}, \tag{12}$$

Equation (10) can be also written as

$$m(T) = m_0 \sqrt{1 - \frac{T^2}{T_{\rm EW}^2}}, \tag{13}$$

where $T_{\rm EW} \sim 10^2$ Gev is the critical temperature of the EWPT. It turns out that all massive particles of Standard Model become massless for the local accelerating observer when the acceleration exceeds the critical value.

The vacuum energy receives contributions from both zero-point fluctuations and symmetry breakings. The ZPE density of a real free scalar field is given by

$$\rho_Z = \frac{1}{(2\pi)^3} \frac{1}{2} \int d^3\boldsymbol{k}\, \omega(k) \tag{14}$$

with

$$\omega(\boldsymbol{k}) = \sqrt{|\boldsymbol{k}|^2 + m_0^2}, \tag{15}$$

where $(\omega, \boldsymbol{k})$ is the four-dimensional momentum and m_0 is the mass of the scalar field. Obviously, the integral is divergent in the ultraviolet region. The common method is to introduce an ultraviolet cut-off $\Lambda_{\rm UV}$ at the Planck scale, then one obtains $\rho_Z \sim 10^{76}$ GeV^4, which is larger than the observed value of vacuum energy density by a factor of 10^{123}. But a straightforward but lengthy calculation leads to

$$\rho_Z = \frac{\Lambda_{\rm UV}^4}{16\pi^2} + \frac{m_0^2 \Lambda_{\rm UV}^2}{16\pi^2} + \frac{m_0^4}{64\pi^2} \ln\left(\frac{m_0^2 e^{\frac{1}{2}}}{4\Lambda_{\rm UV}^2}\right) + \cdots, \tag{16}$$

$$p = \frac{\Lambda_{\rm UV}^4}{48\pi^2} - \frac{m_0^2 \Lambda_{\rm UV}^2}{48\pi^2} - \frac{m_0^4}{64\pi^2} \ln\left(\frac{m_0^2 e^{\frac{7}{6}}}{4\Lambda_{\rm UV}^2}\right) + \cdots, \tag{17}$$

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where p is the pressure. The Lorentz symmetry of the vacuum requires that the energy density and pressure satisfy the equation of state $p = -\rho_Z$. Notice that the first two terms of Equations (16) and (17) break Lorentz invariance and can be removed by local counterterms. Therefore, upon using a regularization scheme that preserves Lorentz symmetry of the vacuum, for any quantum field one arrives at the following expression for the ZPE density [26]

$$\rho_Z = \pm \frac{s m_0^4}{64\pi^2} \ln\left(\frac{m_0^2}{\mu^2}\right), \tag{18}$$

where μ is the renormalization scale, s represents the number of polarization states and the signs $\pm$ are associated with bosons and fermions respectively. The result can be generalized to any other interacting fields by simply replacing m_0 with the renormalized mass m_R. We see that the expression is proportional to the mass of the particle to the power four and the massless particles do not contribute to the ZPE. This result is very different from the result obtained by imposing a Planck cut-off.

Another contribution to the cosmological constant comes from the symmetry breakings. Let us now calculate the vacuum energy produced by the EWPT at the classical level. We should also consider the QCD symmetry breaking ($\sim 10^{-1}$ Gev) and other symmetry breakings at higher energy scales (e.g., the grand unification scale at 10^{14} Gev and the Planck scale at 10^{19} Gev). However, all these expressions take a similar form and the analysis parallels the electroweak case. The Higgs field consists of two complex scalar fields arranged into a doublet. After the EWPT, the field acquires a VEV and the corresponding vacuum energy density is $\rho_{\text{EW}} = \lambda v^4 \sim 10^8$ Gev^4 with λ being the coupling constant describing the self-interaction of Higgs fields. In addition to being inconsistent with observational data, such a large vacuum energy density corresponding to a large cosmological constant would also produce a high Gibbons–Hawking temperature, thereby triggering a phase transition. From Equation (9), we see that the vacuum energy density of the EWPT must satisfy the equation:

$$\rho_{\text{EW}} = \rho_0 \left(1 - \frac{T_h^2}{T_{\text{EW}}^2}\right)^2 \theta(T_{\text{EW}} - T_h), \tag{19}$$

where ρ_0 is the vacuum energy density in the absence of Gibbons–Hawking radiation, $\theta(x)$ is the Heaviside step function, $T_{\text{EW}} \sim 10^2$ GeV is the EWPT temperature and $T_h = \frac{1}{2\pi}(8\pi G\rho_s/3)^{1/2}$ is the Gibbons–Hawking temperature produced by the huge vacuum energy density ρ_s of symmetry breakings. When the vacuum energy density exceeds $T_{\text{EW}}^2/G \sim 10^{42}$ GeV^4, the broken electroweak symmetry is restored and ρ_{EW} vanishes. It can drive Λ back to zero, even when the local vacuum energy density experiences large disturbances up to the Planck scale because large disturbances will lead to a restoration of the symmetry.

It is worth noticing that all the vacuum energy is acceleration-dependent. Therefore, observers with different accelerations will measure different vacuum energy as quantum effects enter the stage. The vacuum energy vanishes when the acceleration is bigger than the critical value. As a result, the usual vacuum stress-energy is no longer a tensor. The key point is that the vacuum energy derived from zero-point fluctuations and symmetry breakings is not real for a potential covariant theory of quantum gravity. Although the mass arising from the interaction between the particle and the Higgs field depends on the acceleration and vanishes for enough accelerated observers, we can extract the covariant part of the mass arising from other interactions, such as electromagnetic interactions, to construct a vacuum stress-energy tensor. Thus, only the self-energy of elementary particles contributes to the vacuum energy density.

4. Conclusions

In this paper, I investigate the modified nonlocal Newtonian dynamics of a toy model of the universe. The system is drastically simpler by using a mean field theory approach. It is worth noting that Equation (3) can be generalized to the framework of general relativity, where the system is

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characterized by the curvature rather than temperature. It is a geometric version of the dynamics describing "synchronization" phenomena. The LHS of Equation (3) represents the derivative of the connection and can thus be replaced by a tensor $K_{\mu\nu}$ related to the true curvature of spacetime. The external force F_i is replaced by the usual Einstein tensor $G_{\mu\nu}$ related to the visible matter content. The second term on the RHS of Equation (3) is replaced by a tensor $C_{\mu\nu}$ representing nonlocal effects. As in bimetric theory, $C_{\mu\nu}$ can be constructed by introducing an auxiliary metric. The rank-2 tensor $C_{\mu\nu}$ appears to play the role of dark matter.

I then calculate the vacuum energy density based on quantum field theory. The vacuum energy is acceleration-dependent when quantum effects enter the stage. Here, I calculate the vacuum energy density arising from symmetry breaking at the tree level. If quantum corrections are considered, one only needs to replace it with the Coleman-Weinberg effective potential [27]. However, the result is the same because quantum corrections also depend on acceleration. It turns out that only the self-energy of elementary particles contributes to the vacuum energy density and the theoretical value of the cosmological constant gets significantly reduced.

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