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[Dorota Pawlus](#) *

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Article

Effect of Functionally Graded Material of Three-Layered Annular Plates on the Dynamic Stability

Dorota Pawlus

Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biala, 43-309 Bielsko-Biala, Poland; dpawlus@ubb.edu.pl

Abstract

The presented problem considers the dynamic stability of a three-layered annular plate, whose facings are made of functionally graded material in the radial direction. The plate is subjected to linearly increasing forces over time, applied along the plane of the facings at either the inner or outer edge. The effect of the heterogeneity of the plate-facing material on the dynamic response is analyzed in detail. The main parameters of the stability state, such as critical dynamic loads, critical time, maximum deflection, and buckling mode are specifically evaluated. The problem is analyzed using two approximation methods: the finite difference method and the finite element method. Numerical calculations were carried out on two tracks: using the author's program after analytical calculations and with the ABAQUS system. The final results show the importance of modeling the plate with an appropriate material function that describes the radial gradation of the material, affecting the plate's dynamic stability response and critical parameters.

Keywords: three-layered annular plate; composite plate structure; functionally graded material facings; dynamic stability; finite difference method; finite element method

1. Introduction

In the present work, a method for the calculation of composite annular plates with heterogeneous metal facings is proposed. Numerous analysis results are presented, with primary attention focused on the dynamic responses of plates under loads acting in the plane of the facings. The evaluation of the loss of plate stability, which is accompanied by a sudden change in the shape of the plate at a specific critical load, shows the sensitivity of the examined structure to its geometrical and material parameters. The application of a heterogeneous structure with smoothly functionally graded material of facings allows the construction of new plate structures with a classical three-layer layout, whose mechanical properties can fulfil the required specifications. By evaluating the buckling phenomenon for two material models, resulting from the radial distribution of the extreme material properties of steel and aluminium, a thorough numerical analysis and discussion of the obtained results have been conducted. The calculation results were obtained for two computational models: an analytical and numerical model based on the finite difference method (FDM), and a numerical model using finite elements. Special attention was given to the computational efficiency of combining analytical and numerical calculations through approximation methods, including orthogonalization and finite differences. The presented calculation techniques and evaluation of buckling behavior complement research in composite plates, creating opportunities for further exploration of structures tailored to specific requirements.

The search for new composite structures that meet technical requirements continues. Plates, including the annular plates examined in this work, are widely used construction elements in various industries, such as mechanical engineering, civil engineering, nuclear, and aerospace applications.

Composite three-layered plates with a classic structure, composed of thin metal facings and a thicker foam core, have been extensively analyzed and applied in practice.

A growing expectation in many industrial sectors is the design of structures dedicated to precise, specific technical solutions. Predicted reactions of elements demonstrating enhanced capabilities of specially composed structures, compared to classical designs, directly fulfill this expectation.

The technical potential of such structures for three-layered annular plates can be realized through the use of heterogeneous elements. Radially variable material properties of the metal facings enable the construction of new layered plates. Recognition of their static and dynamic behaviors, to assess stability loss, is the focus of the research undertaken in this work. It should be emphasized that buckling studies of layered annular plates, including three-layered plates, are presented in numerous scientific works. Thematic groups can be distinguished based on the modeling and load conditions of the examined annular plates. Thermo-mechanical environments are dominant, and most studies focus on plates with transverse material gradation.

The nonlinear bending and post-buckling behavior of a functionally graded material (FGM) annular sector plate is presented in work [1]. The plate is composed of two materials: ceramic and metal, with material parameters continuously varying across the plate thickness. The effects of material and geometrical parameters on plate responses have been examined. Two kinds of boundary conditions have been applied. Numerical calculations were conducted using graded finite elements. The free vibration analysis of the annular sector plate is presented in work [2]. The plate is also composed of metal and ceramic. Two theories were used in modeling: first-order shear deformation and Love's theory. The influence of different geometrical sector ratios and material parameters on the frequencies of the plate sector with various boundary conditions is presented. The dynamic analysis of bidirectional FGMs in a rotating annular plate with variable thickness, including the effects of geometric imperfections and different boundary conditions, is presented in work [3]. The FGM plate material parameters vary in both the radial and axial directions. The effect of specific geometrical ratios of the annular plate on low-velocity impact behavior is demonstrated through numerous results. Effects of thermo-mechanical loading, geometrical parameters, and boundary conditions on the large deflection behavior of annular and circular FGM plates are presented in paper [4]. The material grading through the plate thickness is expressed using a power law. Large deflections are derived from Karman's equations, which include thermal relations. The frequency analysis of an FGM annular plate under hygrothermal effects, with graded material across the plate thickness, is presented in paper [5]. Finite element analysis was applied to clamped-clamped and clamped-free boundary conditions for the examined plate model. Free vibration analysis of FGM annular and circular plates is presented in paper [6]. Material parameters are distributed across the plate thickness according to the power law. The effects of graded material parameters and different geometrical measurements on the natural frequency values are examined. The static temperature analysis of a three-layered annular plate with heterogeneous facings made of material with radially variable parameters is considered in work [7]. Material properties are defined by specified exponent functions. The effects of material and geometrical parameters in two plate models, built using FDM and finite element methods (FEM), on the plate response in a thermal environment are shown. The influence of a time-dependent temperature field and the effect of a stationary temperature field on the stability of a three-layered annular plate are presented in works [8] and [9]. Temperature changes, expressed as gradients in the plate's radial direction, are considered.

Complex loading fields, such as magnetic, electric, and piezo-magnetic, in addition to thermo-mechanical loads, provide additional effects that alter plate responses. The free vibration analysis of FGM annular plates integrated with piezo-magneto-electro-elastic layers is shown in paper [10]. The effect of temperature environments, corresponding to various plate parameters, has been examined. Stress and displacement analysis of annular FGP plates subjected to complex loads, including transverse mechanical and magnetic forces, is presented in work [11]. Both the elastic modulus and magnetic permeability coefficient vary across the plate thickness according to a power function.

Functionally graded piezoelectric annular plates on Winkler foundations are investigated in work [12]. The free vibration problem has been solved semi-analytically.

The behavior of plate elements is also controlled by structural properties, such as porosity. Classical functionally graded annular and circular plates with porosity are examined in paper [13]. Three porosity models were assumed: uniform, O-shaped, and X-shaped. The analyses focus on the basic natural frequencies. The effects of material and geometrical parameters, as well as porosity distribution, on axisymmetric bending of FGM annular or circular microplates under thermal and mechanical loads are presented in paper [14]. Nonlinear finite element models with various porosity distributions have been examined.

Geometrical or material heterogeneity creates complex models that require specialized solution techniques. To understand not only the macro-scale impact of structures but also the influence of microstructure on plate responses, tolerance-based modeling techniques are used. These approaches are applied to various problems, including vibrations, stability, thermo-elasticity, heat conduction, and temperature-dependent behavior [15–21].

The dynamic stability of annular plates subjected to mechanical and thermal loads is presented in works [22–26]. The studies by Chen et al. [22] and Wang et al. [23] are exemplary, addressing the dynamic stability analysis of mechanically loaded sandwich annular plates. The dynamic stability of three-layered annular plates subjected to loads acting on the facings is presented in works [24] and [25]. Plate responses to forces increasing over time are analyzed depending on material, geometrical, and loading parameters. Two plate models, built using FDM and FEM, were examined. A buckling analysis of an FGM sandwich square plate with a metal core is presented in work [26], examining the effects of geometry, material parameters, and boundary conditions.

2. Problem Analysis

The object of this analysis is a three-layered annular plate, whose outer layers are made of FGM facings. The facings are composed of two metals: steel and aluminium. The material parameters change in the plate’s radial direction according to the functional relation given by Eq. 2 for two material models: St-Al, where steel is on the inner plate edge and aluminium on the outer edge, and Al-St, where aluminium is on the inner plate edge and steel on the outer edge (see Figure 1).

The analyzed examples include plates with FGM facings and homogeneous facings. The core material is homogeneous, made of polyurethane foam.

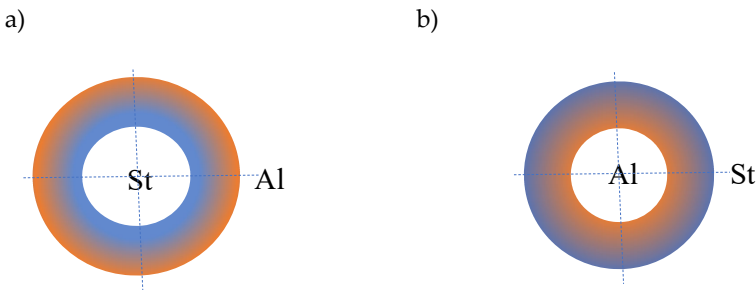


Figure 1. Plate material models: a) St-Al, b) Al-St

The plate is subjected to in-plane forces applied on the inner or outer edge of the facings. In the dynamic analysis, forces increase over time at a specified loading rate. The inner or outer edge is loaded with a uniformly distributed load, linearly increasing in time according to the following formula:

p=st , (1)

where $p(t)$ is the compressive stress, s is the rate of plate loading, and t is time.

The main focus of this study is the plate’s buckling behavior. The plate is initially imperfect and can lose its stability in complex forms, with radial and circumferential buckling waves. The main

observed examples are buckling with a half-wave in the radial direction and several waves in the circumferential direction, or the axisymmetric form without circumferential waves. Critical dynamic parameters, including the load p_{crdym} , time t_{cr} , deflection w_{cr} , and mode m (defined by the number of circumferential waves), characterize the dynamic stability of the examined FGM plate. The moment of the loss of plate stability occurs when the speed of the point of maximum deflection reaches its first peak. This criterion for dynamic stability loss is presented in work [27].

The plate scheme is shown in Figure 2. The plate consists of three layers: FGM facings and a homogeneous core, forming a classical sandwich cross-section with thinner facings and a thicker core. The analyzed plate is slideably clamped at both the inner and outer edges.

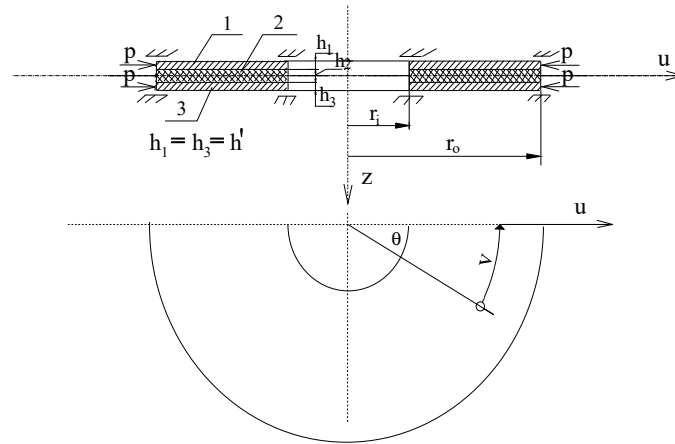


Figure 2. Plate scheme compressed on the outer edge (1, 3 – thin facings with thickness h' , 2 – foam core with thickness h_2).

3. Solution Procedure

The problem is solved using a combination of analytical and numerical calculations. It is based on the orthogonalization method for eliminating the angular variable θ (see Figure 2) and the FDM. The classical theory [28] of three-layered plate structures has been adopted. It is based on the assumptions of the broken-line hypothesis and the interaction of stresses between plate layers: normal stresses are carried by the facings, while shear stresses are carried by the core.

Additionally, an FEM plate model has been developed using the finite elements. The FEM plate structure corresponds to the classical formulation used for the FDM plate model.

3.1. Material Model

The radial changes of the FGM facing material are smooth and follow a power function:

$$VV = \left(\frac{r_i}{r_i - r_o} + \frac{r}{r_o - r_i} \right)^n, \quad (2)$$

where r_i and r_o are the inner and outer plate radii, r is the plate radius, and n is the power-law exponent.

The material parameters of the facings, such as Young's modulus E_r , Kirchhoff's modulus G_r , Poisson's ratio ν_r , and mass density μ_r depend on the plate radius r and the power-law exponent n . They are calculated according to the following equation:

$$W = W1 + VV(W2 - W1), \quad (3)$$

where $W1$ and $W2$ are the values of the material parameters (E_r , G_r , ν_r , and μ_r), VV is the rate of material variation in the radial direction, and W is the value of the facing material parameters at the plate point determined by the radius r . The distribution of material parameters in the facing plane is axisymmetric.

The facings are composed of two materials: steel, with Young's modulus $E_{St} = 210,000$ MPa, and aluminium, with Young's modulus $E_{Al} = 70,000$ MPa, located at the selected edges of the plate. Between the edges, the facings' material parameters vary according to the selected power-law exponent n (see Eq. 2), with $n = 0.2, 0.5, 1, 2, 5$. Figure 16 in Chapter 4.6 shows the distribution of Young's modulus along the plate radius for three values of n (0.2, 1, 5) for the two plate material models St-Al and Al-St (see Figure 1). The effect of the material model on the stability results is discussed in this Chapter 4.6.

3.2. Technique of the Problem Solution Using the FDM

The solution process is based on the method presented in previous works [24,25,29,30]. The solution technique uses both analytical and numerical calculations. The analytical calculations lead to the main differential equation for the deflections of the analyzed composite plate in the dynamic problem:

$$\begin{aligned} & N_1 w_{d,rrrr} + \frac{2N_1}{r} w_{d,rrr} - \frac{N_1}{r^2} w_{d,rr} + \frac{N_1}{r^3} w_{d,r} + \frac{N_1}{r^4} w_{d,\theta\theta\theta\theta} + \frac{2(N_1 + N_2)}{r^4} w_{d,\theta\theta} + \\ & \frac{2N_2}{r^2} w_{d,rr\theta\theta} - \frac{2N_2}{r^3} w_{d,r\theta\theta} - G_2 \frac{H'}{h_2} \frac{1}{r} \left(\gamma_{,\theta} + \delta + r\delta_{,r} + H' \frac{1}{r} w_{d,\theta\theta} + H' w_{d,r} + H' r w_{d,rr} \right) \\ & = \frac{2h'}{r} \left(\frac{2}{r^2} \Phi_{,\theta} w_{,r\theta} - \frac{2}{r} \Phi_{,r\theta} w_{,r\theta} + \frac{2}{r^2} w_{,\theta} \Phi_{,r\theta} - \frac{2}{r^3} \Phi_{,\theta} w_{,\theta} + w_{,r} \Phi_{,rr} + \Phi_{,r} w_{,rr} + \right. \\ & \quad \left. \frac{1}{r} \Phi_{,\theta\theta} w_{,rr} + \frac{1}{r} \Phi_{,rr} w_{,\theta\theta} \right) - M w_{d,tt} \quad (4) \end{aligned}$$

where w is the total deflection, w_d is the additional deflection, and δ , γ are the differences of radial u_1 , u_3 and circumferential v_1 , v_3 displacements of points on the middle surfaces of the facings, expressed by $\delta = u_3 - u_1$ and $\gamma = v_3 - v_1$, respectively. $N_1 = 2D_r$, $N_2 = 4D_{r\theta_r} + \nu_r N_1$, $D_r = \frac{E_r h^3}{12(1-\nu_r^2)}$, $D_{r\theta_r} = \frac{G_r h^3}{12}$ are the rigidities of the plate facings; h is the total plate thickness, $H' = h' + h_2$, where h' is the facings' thickness and h_2 is the core thickness. Φ is the stress function, $M = 2h\mu_r + h_2$, and μ_r is the core mass density.

Equation 4 was derived after formulating the dynamic equilibrium equations for each plate layer, assuming the deformation of the middle surface of the facings follows the nonlinear von Kármán theory, and describing the transversal geometry of the core deformation based on the linear broken line hypothesis. The equations of Hooke's law were applied:

$$\sigma_{r_{1(3)}} = \frac{E_r}{1-\nu_r^2} (\epsilon_{r_{1(3)}} + \nu_r \epsilon_{\theta_{1(3)}}), \quad \sigma_{\theta_{1(3)}} = \frac{E_r}{1-\nu_r^2} (\epsilon_{\theta_{1(3)}} + \nu_r \epsilon_{r_{1(3)}}), \quad \tau_{r\theta} = G_r \gamma_{r\theta} \quad (5)$$

where σ_r is the radial stress, σ_θ is the circumferential stress, and $\tau_{r\theta}$ is the shear stress.

Then, the resultant transverse forces in radial and circumferential directions and the resultant membrane forces expressed by the stress function were formulated, assuming the initial conditions that both the additional deflection w_d and the speed of the additional deflection $w_{d,t}$ for time $t = 0$ are equal to zero. Boundary conditions are expressed for the total deflection w and the differences of displacements in radial δ and circumferential γ plate directions:

$$w|_{r=r_i(r_o)} = 0, \quad w_r|_{r=r_i(r_o)} = 0, \quad \delta = \gamma|_{r=r_i(r_o)} = 0, \quad \delta_r|_{r=r_i(r_o)} = 0 \quad (6)$$

Loading conditions are defined for the plate edges: inner for radius r_i or outer for radius r_o , respectively:

$$\sigma_r|_{r=r_i} = -p(t)d_1, \quad \sigma_r|_{r=r_o} = -p(t)d_2 \quad (7)$$

The quantities d_1 and d_2 are equal to 0 or 1.

Plate geometry is modeled with the preliminary deflections. The shape is determined by the combination of the axisymmetric component and depends on the number m of circumferential waves [31,32]:

$$\zeta_o(\rho, \theta) = \xi_1(\rho)\eta(\rho) + \xi_2(\rho)\eta(\rho)\cos(m\theta), \quad (8)$$

where the dimensionless predeflection is $\zeta_o = \frac{w_o}{h}$, the dimensionless plate radius is $\rho = \frac{r}{r_o}$, ξ_1 and ξ_2 are numbers that calibrate Eq. 8, and $\eta(\rho) = \rho^4 + A_1\rho^2 + A_2\rho^2\ln\rho + A_3\ln\rho + A_4$, with A_i satisfying the conditions of clamped edges.

The shape functions used in the solution of the problem include the additional deflection $\zeta_1 = \frac{w_d}{h}$ [31,32], the functions of the radial and circumferential facings displacements $\bar{\delta}, \bar{\gamma}$ [24,25], and the stress function $F = \frac{\Phi}{Eh^2}$ [31,32]:

$$\zeta_1(\rho, \theta, t) = X_1(\rho, t)\cos(m\theta), \quad (9)$$

$$\bar{\delta}(\rho, \theta, t) = \bar{\delta}(\rho, t)\cos(m\theta), \quad \bar{\gamma}(\rho, \theta, t) = \bar{\gamma}(\rho, t)\sin(m\theta), \quad (10)$$

$$F(\rho, \theta, t) = F_a(\rho, t) + F_b(\rho, t)\cos(m\theta) + F_c(\rho, t)\cos(2m\theta), \quad (11)$$

where E is the established value of the Young's modulus (E_{St} for the St-Al plate model or E_{Al} for the Al-St model), $\bar{\delta} = \frac{\delta}{h}$, $\bar{\gamma} = \frac{\gamma}{h}$, and m is the number of circumferential waves corresponding to the number of waves shaping the plate imperfection.

The solution technique uses the orthogonalization method to eliminate the angular variable θ , along with the FDM, which approximates the derivatives with respect to ρ using central differences at discrete points. The main equation of the system of differential equations has the following form [24,25,30]:

$$\mathbf{PU} + \mathbf{Q} = \ddot{\mathbf{U}}_K, \quad (12)$$

where the vector $\ddot{\mathbf{U}}_K$ contains elements expressed by the derivative of the additional deflection with respect to time t , and the number K is given by $K = K7^2 \cdot \frac{h'}{h} \cdot r_o h_2 M$. Here, $K7$ represents the rate of mechanical loading growth calculated as the quotient of s (see Eq. 1) to the assumed value of the critical static load p_{cr} , with $K7 = \frac{s}{p_{cr}}$. The vector $\mathbf{U}$ contains the additional deflections. The vector $\mathbf{Q}$ contains the plate's material and geometrical parameters, initial and additional deflections, number m , and dimensionless radius ρ . The matrix $\mathbf{P}$ contains the plate's material and geometrical parameters, number m , and parameter b , which represent the interval in the FDM.

The Runge-Kutta integration method for the initial state of the plate was used in the numerical solution. Results of the time history of deflection parameters are presented for the dimensionless time $t^* = t \cdot K7$, which was used in the numerical calculation procedure.

3.3. Technique of the Problem Solution Using the FEM

The plate model was built using shell and solid elements. Shell elements were used to build the facing mesh, and solid elements were used to build the core mesh. The outer surfaces of the facing mesh elements were connected to the outer surfaces of the core elements using surface contact interaction. Calculations were carried out using the ABAQUS system at the Academic Computer Center CYFRONET-CRACOW via the PLGrid Portal under the awarded grant named plgplate1. The Dynamic option of the ABAQUS system was used in the dynamic stability analysis. Static analysis to calculate the critical static load was carried out using the eigenvalue buckling procedure.

4. Exemplary Numerical Results

The numerical results are presented for the selected plate models St-Al and Al-St, with various material parameters expressed by the power-law exponent n . Results were obtained using two numerical methods: the FDM and the FEM.

4.1. Parameters of the FGM Plate Model

Exemplary calculations were carried out for selected geometrical and material plate parameters. Both the facings of the FGM plate and the plate foam core are treated as elastic and isotropic. The accepted data are presented in Table 1.

Table 1. Geometrical and material plate parameters.

Description	Parameter	Value
Geometrical parameters		
plate inner radius	r_i	0.2 m
plate outer radius	r_o	0.5 m
facing thickness	h'	1 mm
core thickness	h_2	5 mm
Material parameters		
Young's modulus of steel	E_{st}	210 GPa
Poisson's ratio of steel	ν_{st}	0.3
mass density of steel	μ_{st}	7850 kg/m ³
Young's modulus of aluminium	E_{al}	70 GPa
Poisson's ratio of aluminium	ν_{al}	0.33
mass density of aluminium	μ_{al}	2700 kg/m ³
Kirchhoff's modulus of polyurethan foam of core material	G_c	5 MPa
mass density of polyurethan foam of core material	μ_c	64 kg/m ³
power-law exponent of the eq. (2)	n	0.2, 0.5, 1, 2, 5
Load parameters		
rate of dynamic loading growth	s	1000 MPa/s for plate loaded on the outer edge
		5000 MPa/s for plate loaded on the inner edge

4.2. Accuracy Analysis

Figure 3 and Table 2 present the results of the critical dynamic loads, p_{crdyn} , for various modes m for the St-Al plate material model and for three different numbers N of discrete points in the FDM: $N = 14, 20, 26$. Three values of the power-law exponent n from Eq. 2 were considered: $n = 0.2, 1, 5$. The analyzed plates are loaded on the outer edge. The values of p_{crdyn} show good compatibility for FDM plate models with the asymmetric form of buckling ($m \neq 0$). Larger differences are observed for the axisymmetric buckling form ($m = 0$). The accuracy between the p_{crdyn} values for different N does not exceed the technical error of approximately 5%. Therefore, $N = 26$ was adopted in the FDM numerical calculations.

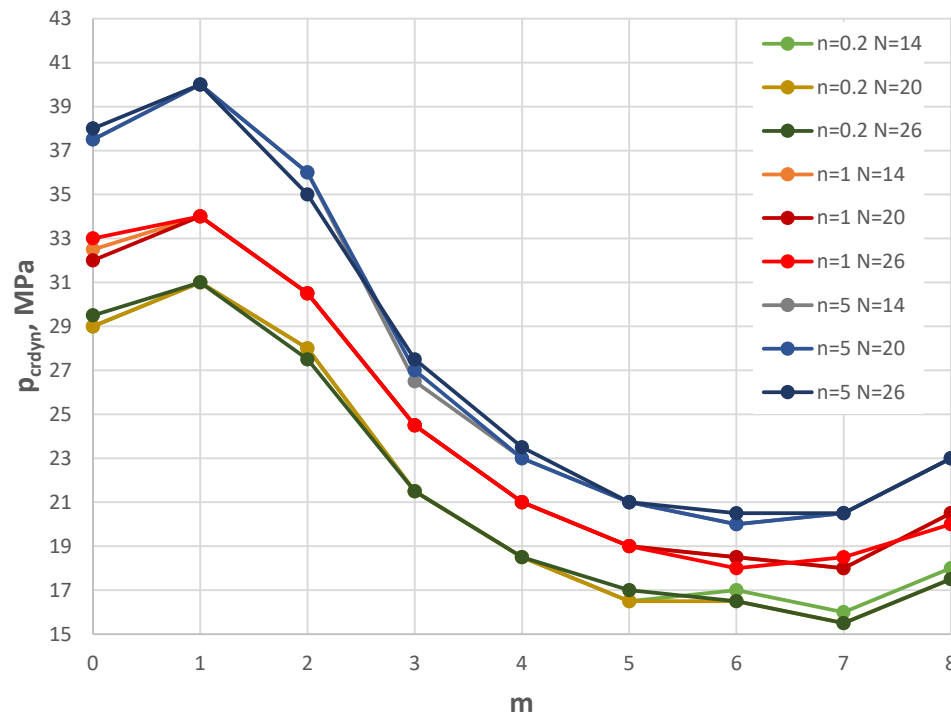


Figure 3. Distribution of the critical dynamic load p_{crdyn} versus mode number m for the St-Al plate model loaded on the outer edge with various values of n and number N of discrete points.

Table 2. Detailed values of the critical dynamic loads p_{crdyn} versus mode number m for the St-Al plate model loaded on the outer edge for different values of n and number N of discrete points.

m	p_{crdyn} , MPa					
	St-Al model $n=1$			St-Al model $n=5$		
	$N=14$	$N=20$	$N=26$	$N=14$	$N=20$	$N=26$
0	32.5	32	33	37.5	37.5	38
1	34	34	34	40	40	40
2	30.5	30.5	30.5	36	36	35
3	24.5	24.5	24.5	26.5	27	27,5
4	21	21	21	23	23	23.5
5	19	19	19	21	21	21
6	18.5	18.5	18	20	20	20.5
7	18	18	18.5	20.5	20.5	20.5
8	20.5	20.5	20	23	23	23

Additionally, the results show the following observations:

- The critical dynamic loads p_{crdyn} are much smaller for the FGM material model with power-law exponent $n = 0.2$. Greater participation of steel increases the critical load.
- The minimal values of p_{crdyn} occur for plates with several circumferential buckling waves, $m = 6$ or 7.
- There are significant differences between the p_{crdyn} values for plates without circumferential buckling waves ($m = 0$) and those with m corresponding to the minimal value of p_{crdyn} . This confirms the importance of the complex solution procedure, which includes asymmetric plate cases.

4.3. Plate loaded on the outer edge

Figure 4 shows the distribution of the critical static load p_{cr} versus the plate mode m for different facing materials: homogeneous steel, homogeneous aluminium, and FGM with different values of power-law exponent n . Numerical results were obtained using the FDM for St-Al facing models. The plate is loaded on the outer edge. The distribution of results shows a regular pattern. As the steel participation in the FGM facings increases, the critical static loads p_{cr} also increase. The results for the FGM material with $n = 5$ are close to those for homogeneous steel facings. This FGM-foam-FGM structure is lighter than a steel-foam-steel structure, without significantly reducing the stability parameters. Static analysis complements the understanding of the dynamic responses of the examined composite plate.

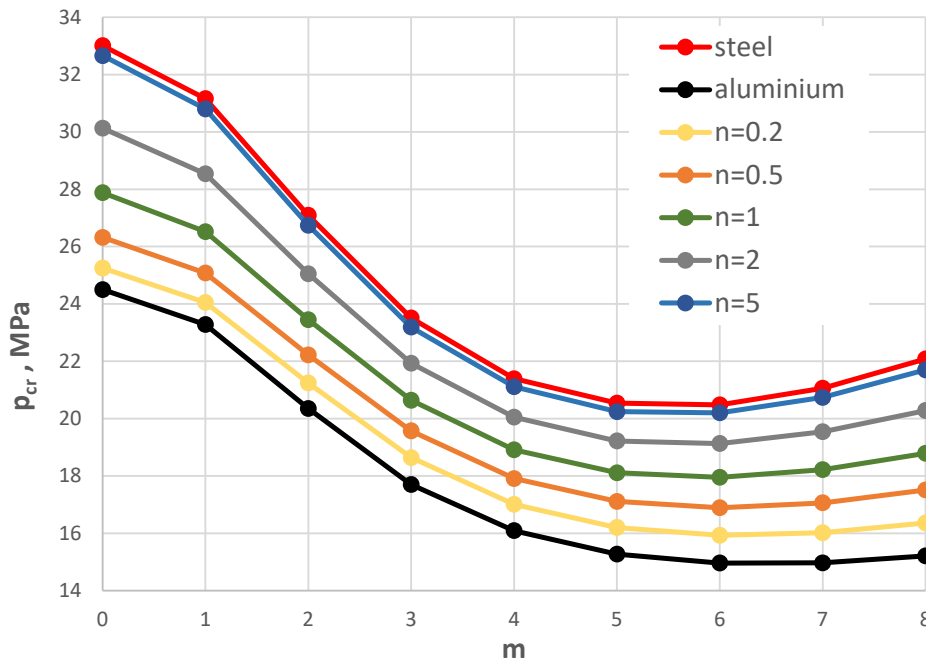


Figure 4. Distribution of the critical static load p_{cr} versus mode number m for St-Al, steel, and aluminium plate models.

Similar observations for dynamically loaded plates are shown in Figure 5. The critical dynamic loads, p_{crdyn} , for corresponding modes m are presented for St-Al facing models with different material distributions expressed by the power-law exponent n . For plates with steel facings (St-Al material, $n = 5$) and plates with aluminium facings (St-Al material, $n = 0.2$), the critical dynamic load values are particularly close for plates with larger modes m . For these examples, the dynamic stability sensitivity appears to be smaller. There are no significant differences between results for plates composed of homogeneous layers and the examined heterogeneous ones, especially for plates with minimal values of p_{crdyn} .

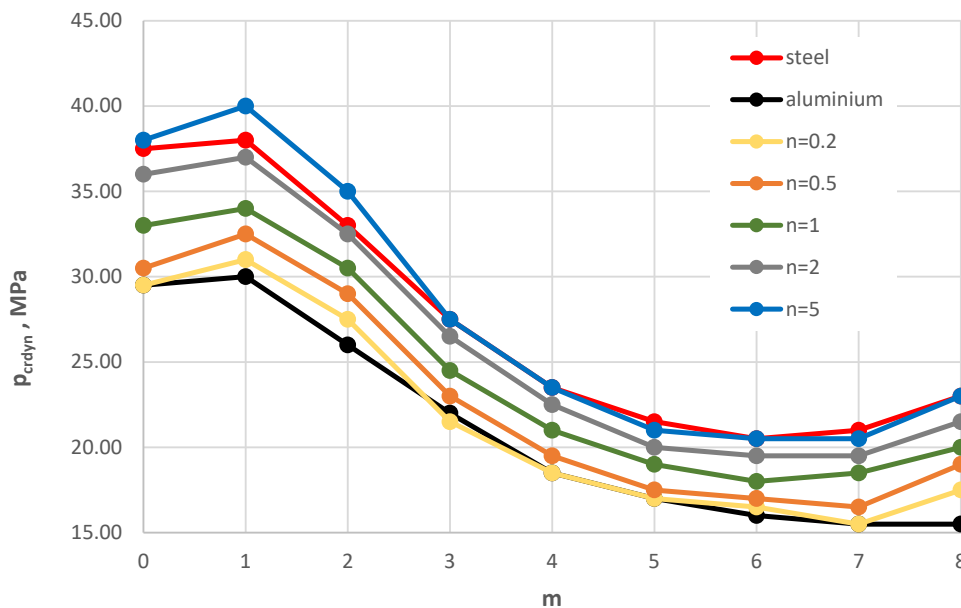


Figure 5. Distribution of the critical dynamic load $p_{cr,dyn}$ versus mode number m for St-Al, steel, and aluminium plate models.

The comparison between the critical dynamic loads, $p_{cr,dyn}$, and static loads, p_{cr} , is presented in Table 3. Results are shown for homogeneous plate structures and FGM facings with three power-law exponents, $n = 0.2, 1, 5$. Greater differences are observed for the axisymmetric buckling form ($m = 0$) and for plate structures with aluminium facings or heterogeneous facings with lower participation of steel material ($n = 0.2$). Close values are observed for asymmetric modes with multiple circumferential waves, particularly for the mode corresponding to minimal values of critical loads ($m = 6$). Slightly higher values are observed for dynamically loaded plates. This allows the conclusion that static analysis is sufficient for evaluating the stability of FGM-foam-FGM plates loaded on the outer edge.

Table 3. Critical dynamic $p_{cr,dyn}$ and static p_{cr} loads for the FDM plate model

m	FDM plate model									
	steel		aluminium		St-Al $n=0.2$		St-Al $n=1$		St-Al $n=5$	
	$p_{cr,dyn}$, MPa	p_{cr} , MPa	$p_{cr,dyn}$, MPa	p_{cr} , MPa	$p_{cr,dyn}$, MPa	p_{cr} , MPa	$p_{cr,dyn}$, MPa	p_{cr} , MPa	$p_{cr,dyn}$, MPa	p_{cr} , MPa
0	37.5	33.01	29.5	24.5	29.5	25.25	33	27.88	38	32.66
4	23.5	21.39	18.5	16.09	18.5	17.01	21	18.91	23.5	21.11
5	21.5	20.54	17	15.27	17	16.20	19	18.11	21	20.24
6	20.5	20.48	16	14.96	16.5	15.93	18	17.95	20.5	20.20
7	21	21.06	15.5	14.97	15.5	16.02	18.5	18.22	20.5	20.74

Time histories of plate deflections for various modes are shown in Figures 6, 7, and 8. The curves were obtained for three values of the power-law exponent, $n = 0.2, 1, 5$. The results show two groups of curves: one for $m < 4$, corresponding to fewer than four circumferential waves, and another for $m > 3$. Differences are observed in the character of the curves, the distribution of points representing the moment of the loss of plate stability, and in the overcritical area, where vibrations can be initiated.

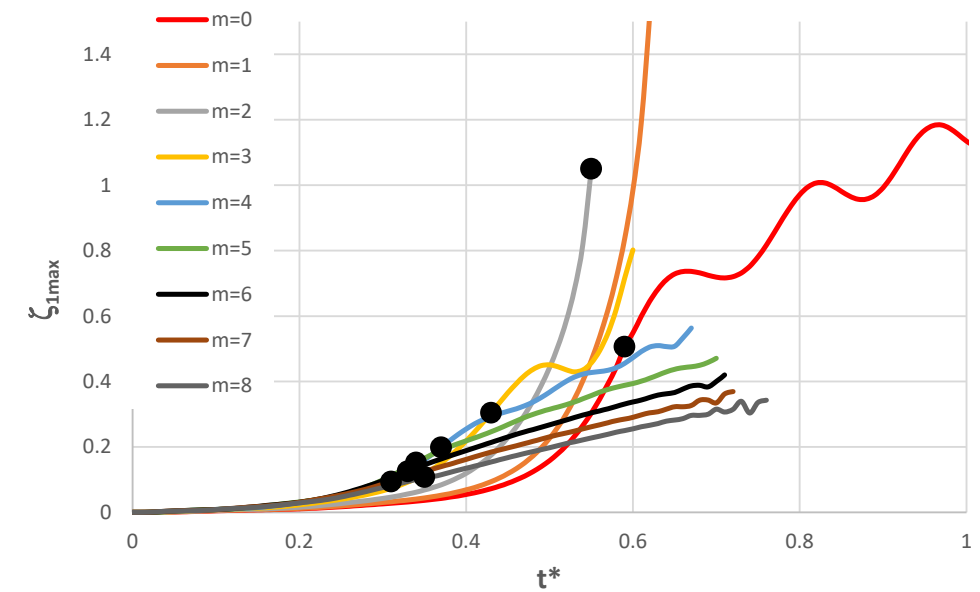


Figure 6. Time history of dimensionless deflections ζ_{1max} depending on St-Al plate mode m for $n = 0.2$.

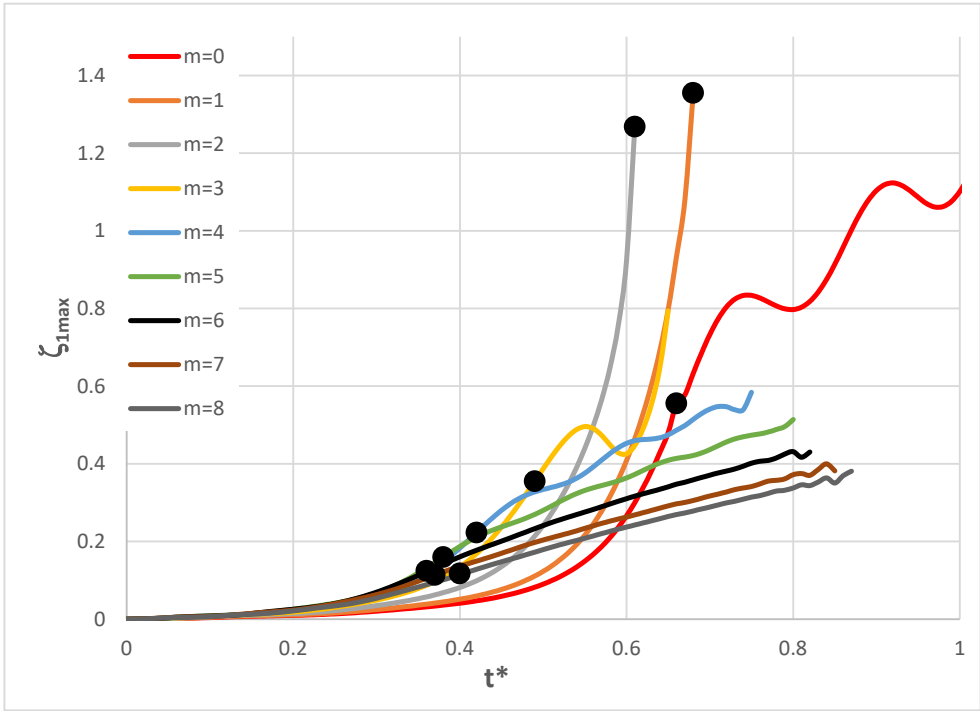


Figure 7. Time history of dimensionless deflections ζ_{1max} depending on St-Al plate mode m for $n = 1$.

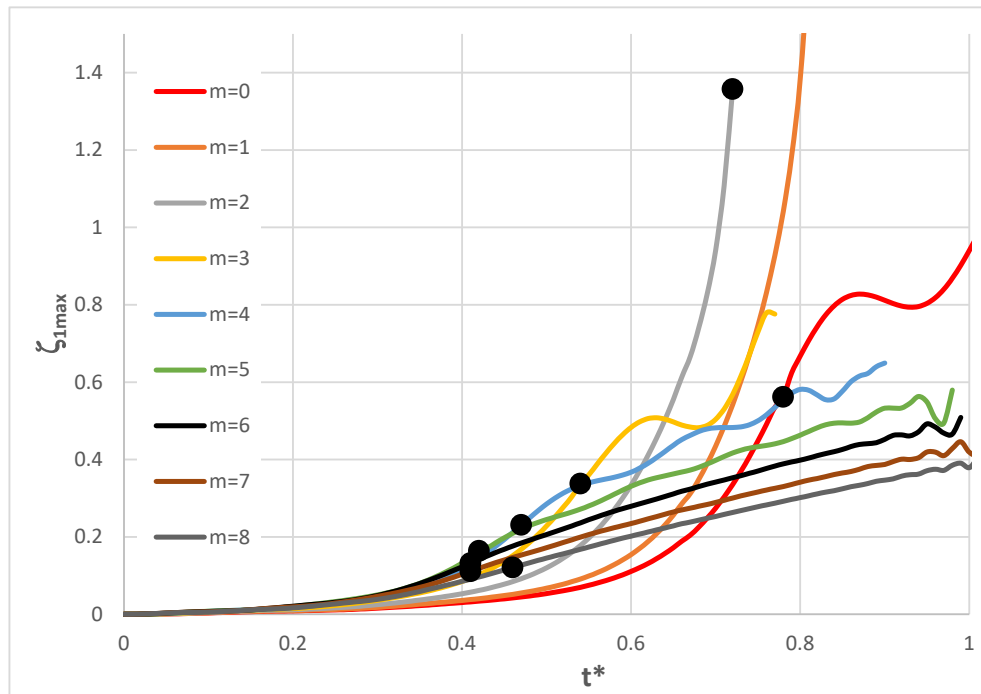


Figure 8. Time history of dimensionless deflections ζ_{1max} depending on St-Al plate mode m for $n = 5$.

4.4. Plate Loaded on the Inner Edge

Exemplary results for the St-Al plate model loaded on the inner edge are shown in Figures 9 and 10. The time histories of deflections for axisymmetric ($m = 0$) homogeneous plates and plates with FGM facings, presented in Figure 9, indicate similar dynamic responses for plates with steel-aluminium or steel facings. Increasing the steel participation in the facings prolongs the time to the loss of plate stability and increases the dynamic critical load, p_{crdyn} . The homogeneous plate with aluminium facings has less favorable stability parameters: the critical load is smaller, the critical deflection is higher, and overcritical vibrations with greater amplitude occur compared to the St-Al plate model.

Curves in Figure 10 confirm that the minimal values of p_{crdyn} for plates loaded on the inner edge correspond to the axisymmetric buckling form ($m = 0$). Results were obtained for plate modes $m = 0, 1$, and 2 . The analysis considers both homogeneous plates and plates with St-Al facings with $n = 1$. The detailed values of p_{crdyn} for axisymmetric ($m = 0$) plates are presented in Table 5. The difference between St-Al plates with material distributions expressed by $n = 0.2$ and $n = 5$ is significant. Increasing steel participation in the facings is more advantageous for the stability of the plates mechanically loaded.

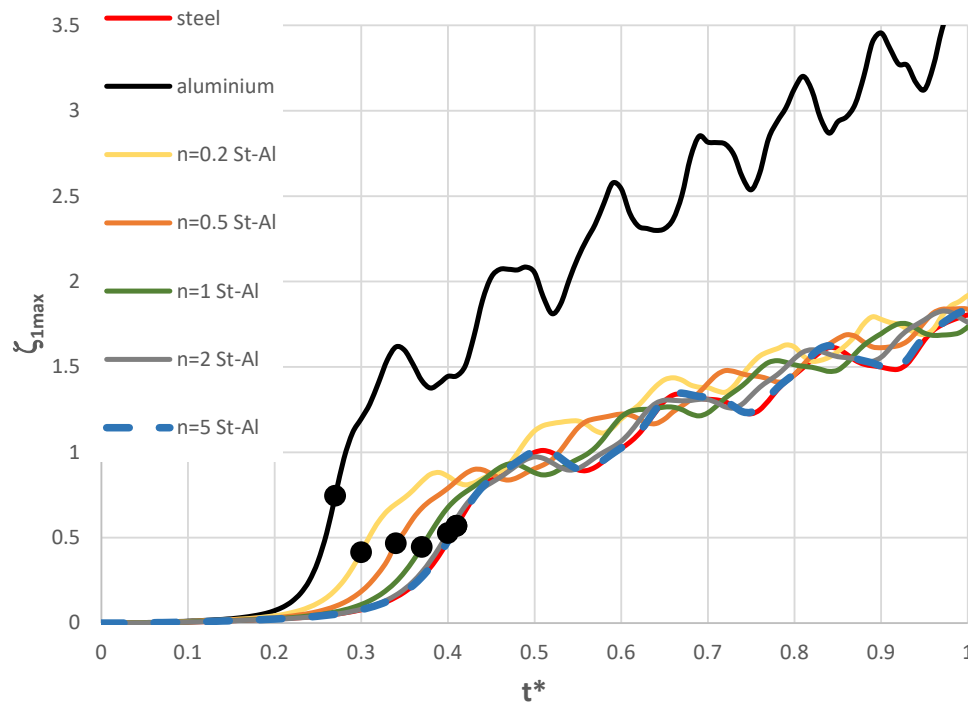


Figure 9. Time history of dimensionless deflections ζ_{1max} depending on steel, aluminium, and St-Al plate models for various power-law exponents n and plate mode $m = 0$.

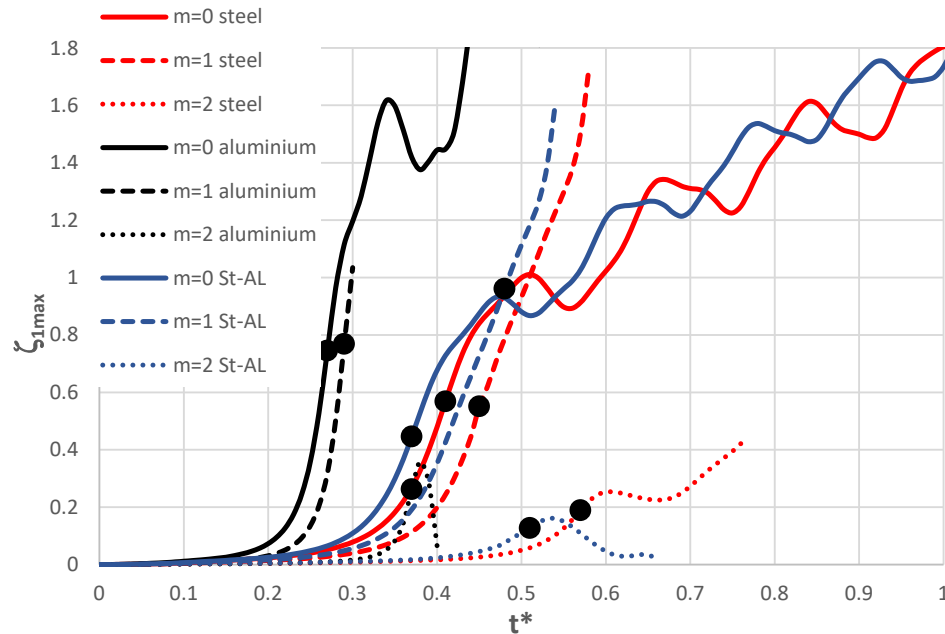


Figure 10. Time history of dimensionless deflections ζ_{1max} depending on steel, aluminium, and St-Al plate models for power-law exponent $n = 1$ and modes $m = 0, 1, 2$.

4.5. Results for Both FDM and FEM Plate Models

The comparison between the two plate models, FDM and FEM, is presented in Figure 11 and Tables 4 and 5. Figure 11 shows the critical static loads, p_{cr} , for FDM and FEM plate models. Good agreement of p_{cr} values is observed for homogeneous plates, St-Al facings with $n = 5$, and

heterogeneous plates with mode numbers $m < 5$. For higher modes, differences in p_{cr} increase for plates with facing materials characterized by power-law exponents $n = 0.2$ and $n = 1$. In these cases, lower p_{cr} values are observed for the FDM plate model.

The values of $p_{cr,dyn}$ for asymmetric plate modes are closer. Detailed values are presented in Table 4. Greater differences are observed for axisymmetric plates ($m = 0$). Table 5 shows the values of $p_{cr,dyn}$ for plates loaded on the inner edge. Good agreement is observed for plate cases with increased steel participation in the facings and for homogeneous plates. Greater differences occur between FDM and FEM plate models with St-Al facings at $n = 0.2$.

Exemplary time histories of deflections (red lines) and velocities of deflections (blue lines) for FEM plate models are shown in Figures 12 and 13. Figure 12 presents the curves for homogeneous plates and St-Al plates with $n = 0.2$, loaded on the inner edge. The curves confirm the tendency toward the initiation of overcritical vibrations. The first maximum of the velocity time history indicates the moment of loss of dynamic stability. Figure 13 shows the response of FEM plate models with different material parameters: St-Al plate facings with $n = 0.2, 1, 5$, and a plate model with steel facings. Results are presented for mode $m = 6$. No vibrations in the overcritical area are observed.

Exemplary forms of buckling—axisymmetric $m = 0$ and asymmetric with $m = 6$ and $m = 7$ circumferential waves for plates loaded on the outer edge—are shown in Figure 14.

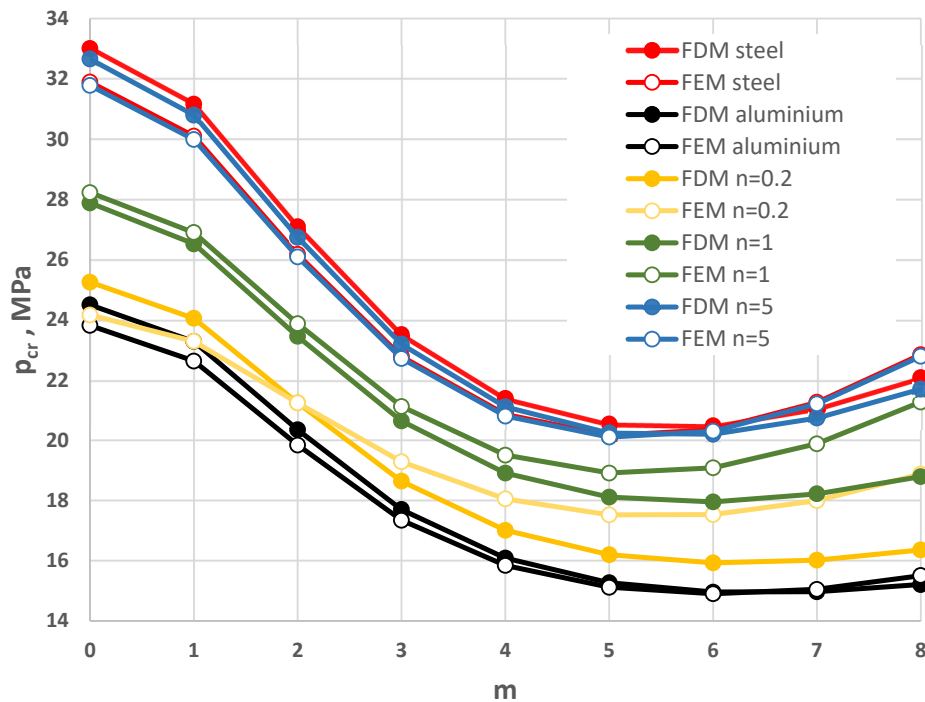


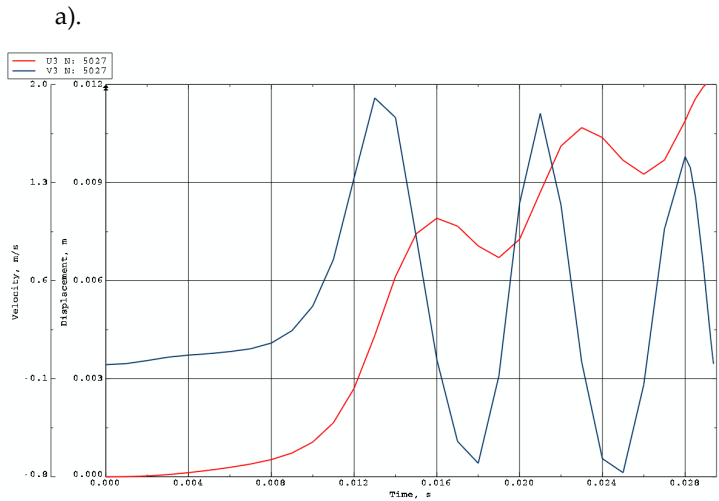
Figure 11. Distribution of the critical static load p_{cr} versus mode number m for FDM and FEM plate models with homogeneous and St-Al facings with $n = 0.2, 1, 5$.

Table 4. Results for FDM and FEM plate models loaded on the outer edge.

	p_{crdyn}, MPa											
	plate model											
	steel		aluminium		St-Al $n=0.2$		St-Al $n=1$		St-Al $n=5$			
	F	F	F	F	F	F	F	F	F	F	F	F
	DM	EM	DM	EM	DM	EM	DM	EM	DM	EM	DM	EM
	3	3	2	2	2	2	3	2	3	3		
	7.5	7	9.5	1	9.5	7	3	9	8	3		
	2	2	1	1	1	2	2	2	2	2		
	3.5	4	8.5	9	8.5	0	1	1	3.5	2		
	2	2	1	1	1	1	1	1	2	2		
	1.5	2	7	7	7	8	9	9	1	0		
	2	2	1	1	1	1	1	1	2	2		
	0.5	1	6	6	6.5	7	8	9	0.5	0		
	2	2	1	1	1	1	1	1	2	2		
	1	2	5.5	6	5.5	8	8.5	9	0.5	0		

Table 5. Results for FDM and FEM plate models loaded on the inner edge.

m	p_{crdyn}, MPa									
	plate model									
	steel		aluminium		St-Al $n=0.2$		St-Al $n=1$		St-Al $n=5$	
	FDM	FEM	FDM	FEM	FDM	FEM	FDM	FEM	FDM	FEM
0	102.5	100	67.5	65	77.5	90	92.5	100	100	100



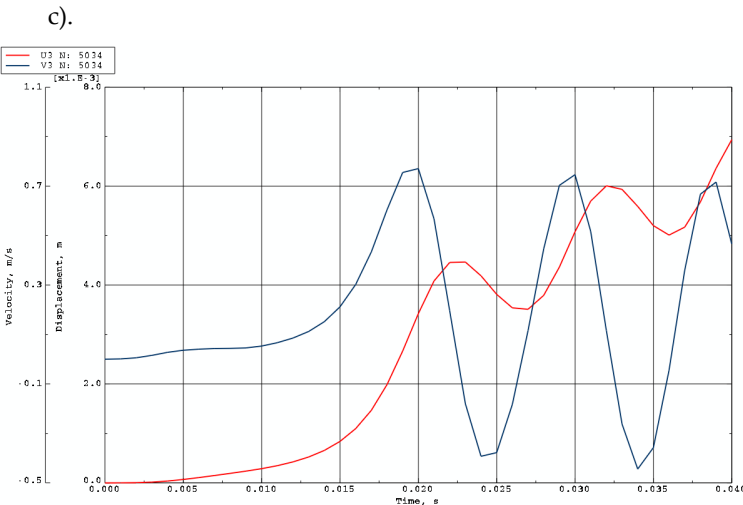
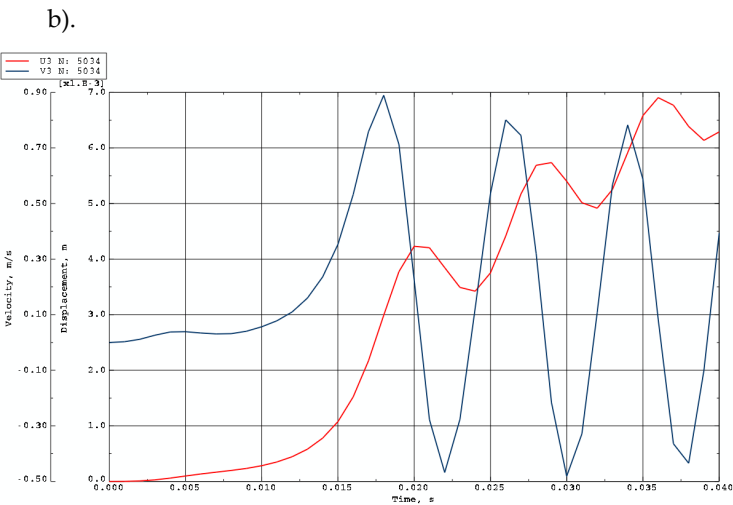
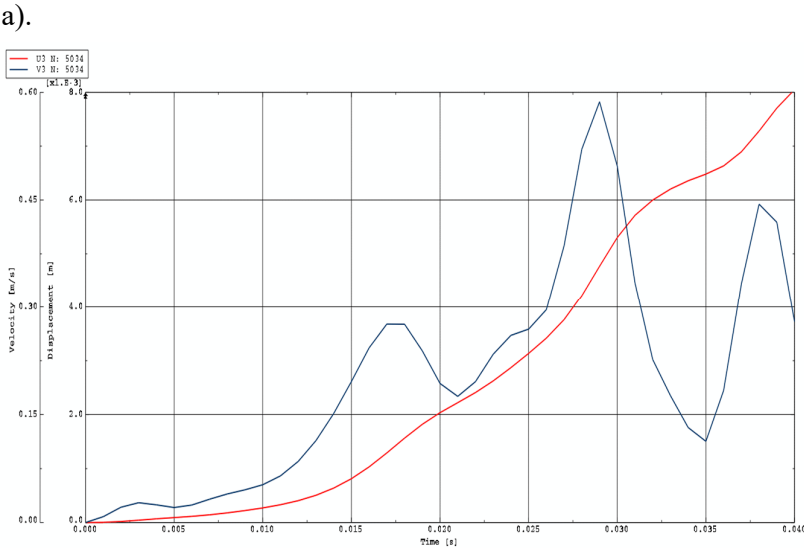
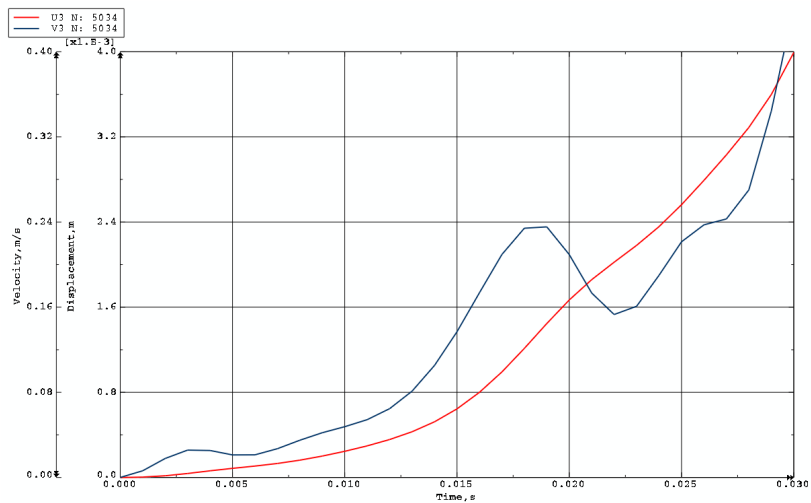


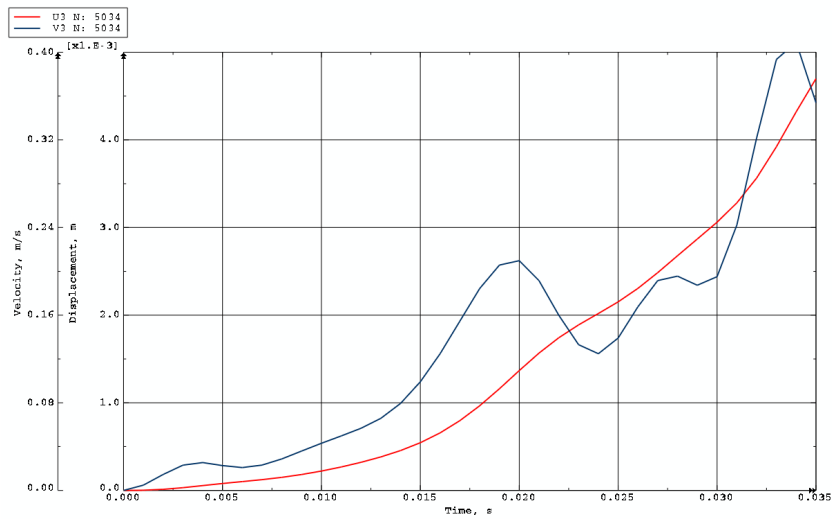
Figure 12. Time histories of deflections and velocity of deflections for plates with $m = 0$ loaded on the inner edge: a) aluminium plate model, b) St-Al plate model with $n = 0.2$, c) steel plate model.



b).



c).



d).

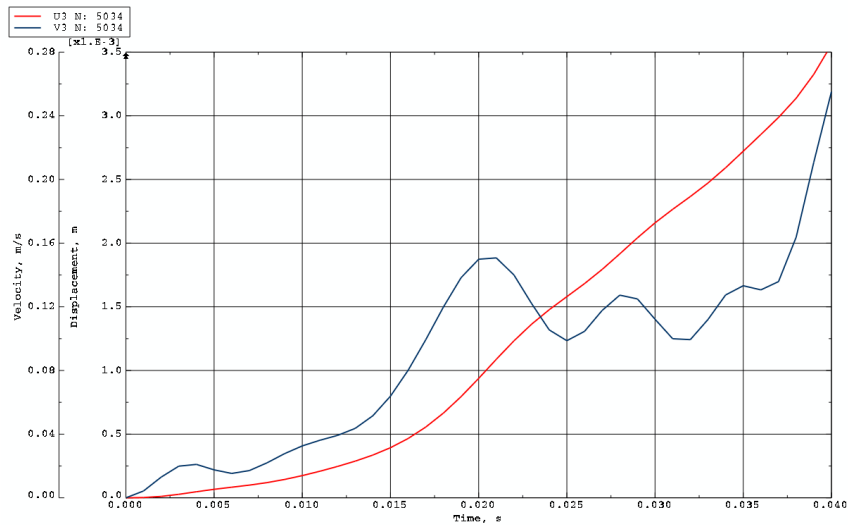


Figure 13. Time histories of deflections and velocity of deflections for plates with $m = 6$ loaded on the outer edge: a) St-Al plate model with $n = 0.2$, b) St-Al plate model with $n = 1$, c) St-Al plate model with $n = 5$, d) steel plate model.

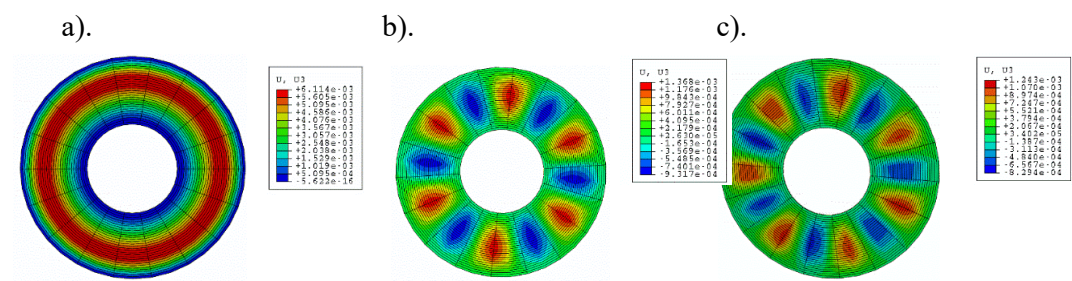


Figure 14. Exemplary forms of plate buckling loaded on the outer edge: a) axisymmetric $m = 0$ St-Al model with $n = 5$, b) asymmetric $m = 6$ St-Al model with $n = 5$, c) asymmetric $m = 7$ St-Al model with $n = 1$.

4.6. Analysis of Results for Two St-Al and Al-St FGM Plate Models

Results for the two plate models, St-Al and Al-St (see Figure 1), are presented in Figures 15–19 and Table 6. The computational analyses for the two plate material models, St-Al and Al-St, focus on demonstrating the effect of FGM material arrangement in the plate facings on the critical static loads, p_{cr} , and dynamic loads, p_{crdym} . Differences in behavior are observed. Figure 15 shows the critical static load, p_{cr} , for St-Al and Al-St FDM plate models loaded on the outer edge. The plate models exhibit opposite responses. The p_{cr} values for the Al-St plate with $n = 5$, where aluminium is located near the inner plate edge, are the smallest, whereas for $n = 0.2$, the critical loads are close to those for a homogeneous steel plate. The FGM material distribution defined by Eqs. 2 and 3 with different values of n for the Al-St plate, shown in Figure 16, explains these changes. The softer aluminium material is concentrated near the inner edge for $n = 5$, resulting in generally smaller material parameters compared to $n = 0.2$. Results obtained using FEM do not show such clear differences for the FGM plate model with $n = 0.2$ (see Figure 17). For asymmetric modes, the p_{cr} values for both St-Al and Al-St plate models converge. The opposite response trend for the two models is still observed. It can be concluded that, from a stability point of view, the lighter Al-St material distributed in the plate facings is more advantageous, providing higher critical static loads, p_{cr} , for plates loaded on the outer edge.

The forms of the axisymmetric buckling of plates loaded on the inner and outer edges are shown in Figure 18. Differences in the critical deformations of the plate structure are visible between plates loaded on the inner and outer edges. This axisymmetric form is particularly important for plates loaded on the inner edge. This axisymmetric $m = 0$ mode corresponds to the plate model with the smallest critical load. The time history of dimensionless deflections presented in Figure 19 for the FDM Al-St plate model loaded dynamically on the inner edge confirms the results obtained for the Al-St plate model loaded statically. The smallest time to loss of plate stability, identifiable by the black dots, and the smallest value of the critical dynamic load p_{crdym} occur for the plate model with $n = 5$. Larger stability parameters are observed for the plate with $n = 0.2$. Detailed values for the two material plate models, St-Al and Al-St, for both FDM and FEM computational models, are presented in Table 6. Plates are loaded statically and dynamically on the inner edge. For the FGM plate with $n = 5$, whose critical load values are close to those of a homogeneous steel or aluminium composite plate, the results depend on the material model (St-Al or Al-St). Detailed results show good agreement between the two computational models, FDM and FEM. Critical loads of the St-Al plate model are close to those of a homogeneous steel plate, while values obtained for the Al-St plate model are close to the critical loads for plates with aluminium facings. Considering the detailed values of p_{cr} or p_{crdym} calculated for the plate model with $n = 5$, the St-Al material plate model, for this specific material distribution, is more advantageous from a stability point of view for both inner and outer edge loads. The values of p_{cr} and p_{crdym} are close to those of a composite plate with steel facings. Exemplary time histories of deflections and deflection velocities for the two computational models, FDM and FEM,

are presented in Figure 20. The Al-St plate model with $n = 5$ is loaded on the inner edge. For the FDM plate model, results are presented in dimensionless form, see Figure 20a. For example, dimensionless time t^* is related to real time t , presented in Figure 20b, by the formula $t^* = tK7$, where parameter $K7$ is equal to 20. The character of the presented curves for both FDM and FEM plate models is comparable. Characteristic overcritical vibrations, initiated by the increasing applied load, are observed.

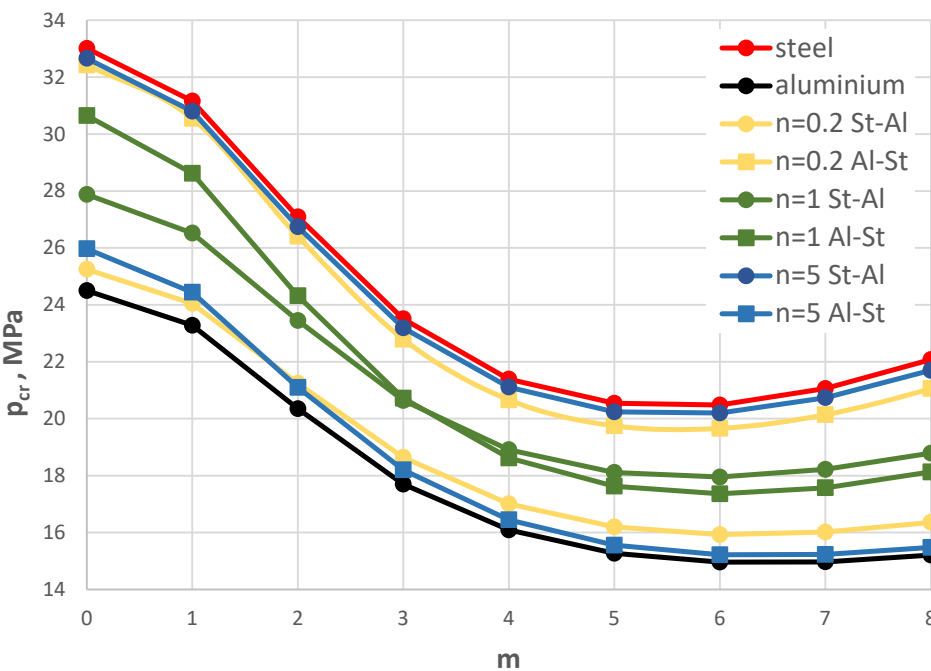


Figure 15. Distribution of the critical static load p_{cr} versus mode number m for St-Al and Al-St FDM plate models, both homogeneous and with FGM facings, for power-law exponents $n = 0.2, 1, 5$.

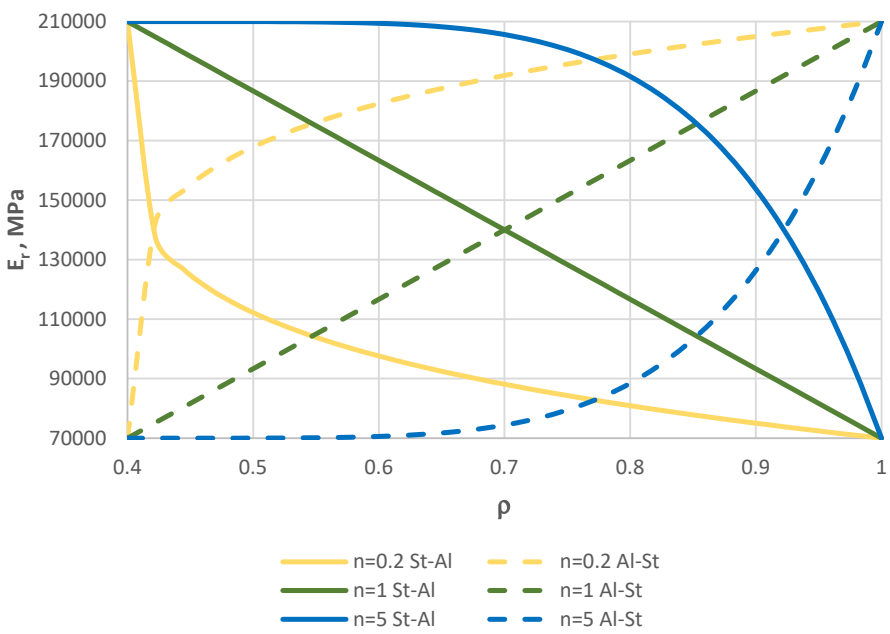


Figure 16. Distribution of Young's modulus E_r of St-Al and Al-St FGM facings versus plate radius ρ .

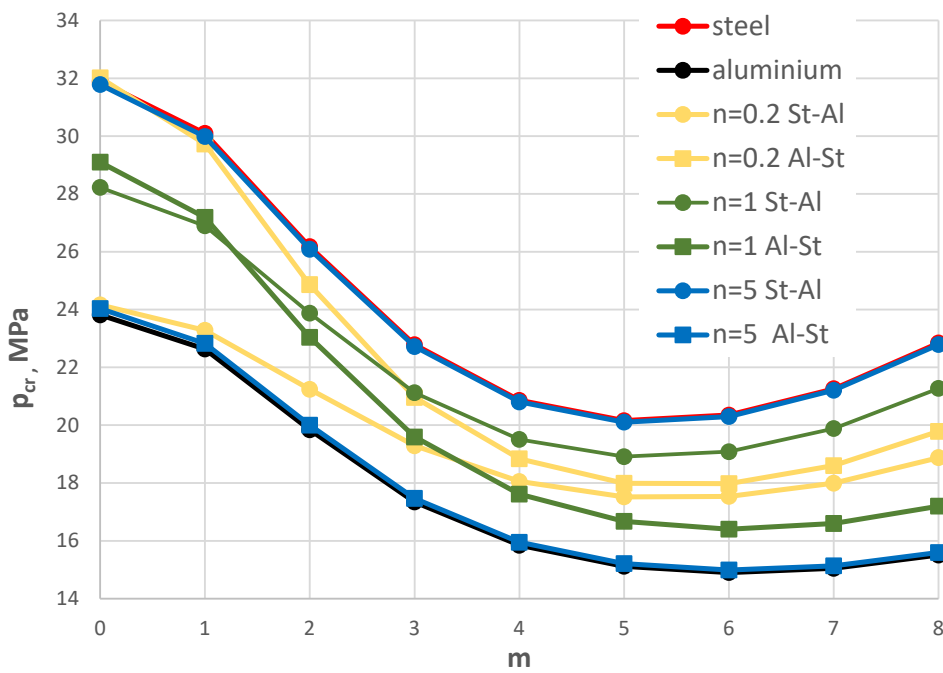


Figure 17. Distribution of the critical static load p_{cr} versus mode number m for homogeneous, St-Al, and Al-St FEM plate models, with power-law exponents $n = 0.2, 1, 5$.

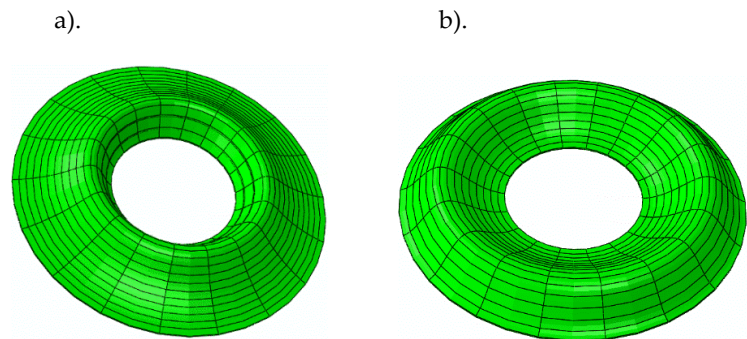


Figure 18. Axisymmetric $m = 0$ buckling modes for the FEM plate model loaded on a) inner edge, b) outer edge.

Table 6. Critical static p_{cr} and dynamic p_{crdym} loads for St-Al and Al-St FDM and FEM plate models loaded on the inner edge.

$m=0$	homogeneous plate model				FGM plate model with $n=5$			
	steel		aluminium		St-Al		Al-St	
	FDM	FEM	FDM	FEM	FDM	FEM	FDM	FEM
p_{cr} , MPa	76.57	71.78	49.23	46.75	76.56	71.83	49.36	46.67
p_{crdym} , MPa	102.5	100	67.5	65	100	95	67.5	60

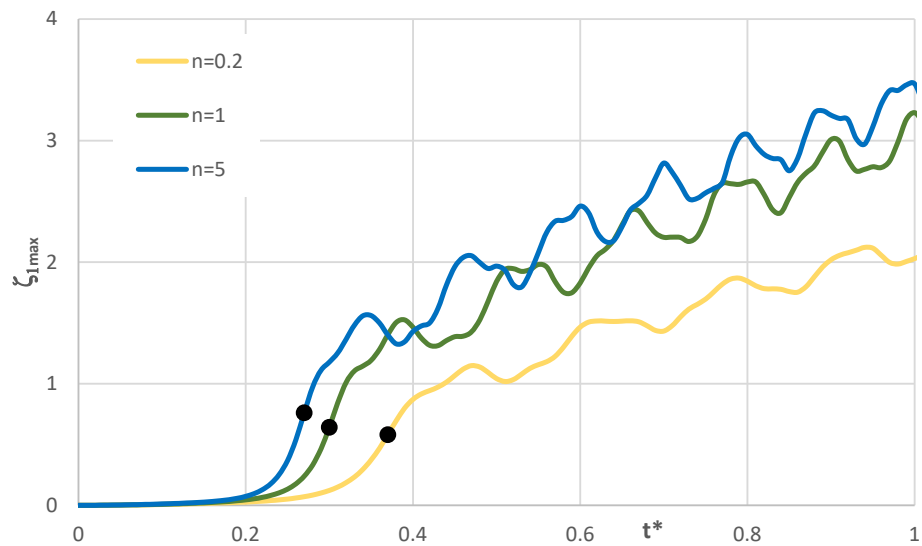
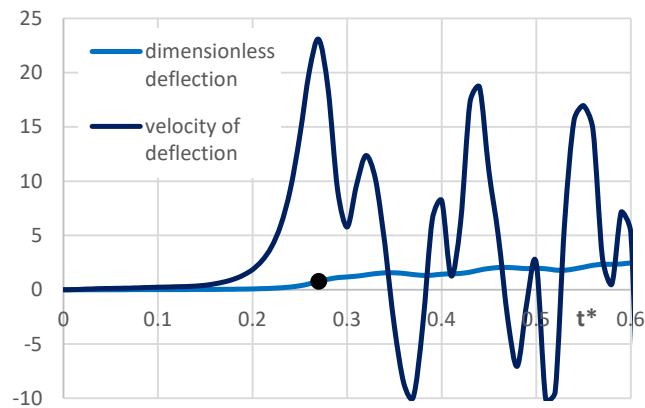


Figure 19. Time history of dimensionless deflections ζ_{1max} versus power-law exponent n for FDM Al-St plate models loaded on the inner edge.

a).



b).

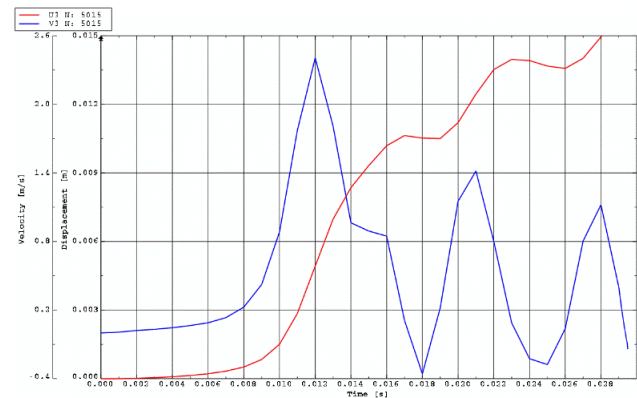


Figure 20. Dynamic results for the Al-St plate ($m = 0$) with $n = 5$ loaded on the inner edge: a) dimensionless deflections and velocities for the FDM plate model; b) deflections and velocities for the FEM plate model.

5. Conclusions

This paper presents the dynamic responses of composite annular plates with facings made of FGM. Alternating changes in material properties create two types of plate models with facings of steel-aluminium or aluminium-steel. In these models, the material with steel or aluminium parameters adheres to the inner edge, while aluminium or steel adheres to the outer edge, respectively. Radial changes of the material parameters are determined by a power function, in which the power-law exponent n takes the values $n = 0.2, 0.5, 1, 2, 5$. Numerical calculations have been conducted for two models: one using the FDM and the other using the FEM.

The results of the numerical calculations are presented with consideration of the evaluation of the plate critical state parameters. The main observations concern the critical dynamic loads, which are complemented by the evaluation of the changes in values determined for the static state of the plate, namely the critical static loads. The discussion of the calculation results highlights the importance of the radial distribution of the facing material in relation to the type of plate edge loading; inner or outer. In the pursuit of building composite plate structures with greater resistance to buckling, attention is focused on cases of plates with higher critical parameter values. Specifically, these are the cases in which the values of p_{crdyn} or p_{cr} approach their maximum values, which are associated with steel facings. The most appropriate model is the St-Al plate model with an exponent of $n = 5$ for the material power function under outer edge loading, or a plate with $n = 0.2$ under inner edge loading. The benefit of using FGM material with radially variable parameters is a significant reduction in plate weight with only a slight decrease in stability-related properties.

The computational models, built according to the assumptions and limitations of the applied approximation methods, were tested. The static and dynamic responses of both FDM and FEM plate models are consistent. Observed differences in critical load values mainly concern the plate with $n = 0.2$, whose facings are more flexible due to the increased contribution of aluminium parameters. The proposed analytical and numerical FDM solution procedure demonstrates techniques for modeling complex plate structures and enables rapid, effective calculations for variable plate parameters. From a practical point of view, the effectiveness of the proposed procedure is high.

It is possible to extend the analyses to cases of plates whose facing material functions have various forms. The presented numerical research should be complemented by experimental analyses. Additionally, considering a heterogeneous material for the plate core could significantly enrich the evaluation of the examined composite plates. This topic will be addressed in future work.

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List of Abbreviations and Main Parameters

Abbreviations
FGM – functionally graded material
FDM – finite difference method
FEM – finite element method
St-Al – plate material model with steel on the inner edge and aluminium on the outer edge
Al-St – plate material model with aluminium on the inner edge and steel on the outer edge
Main parameters
r_i – plate inner radius
r_o – plate outer radius
r – plate radius
ρ_i – dimensionless inner plate radius

- h' – facing thickness
- h_2 – core thickness
- E_{St} – Young’s modulus of steel
- E_{Al} – Young’s modulus of aluminium
- μ_{St} – mass density of steel
- μ_{Al} – mass density of aluminium
- n – FGM-facing material power-law exponent (see Eq. 2)
- G_2 – Kirchhoff’s modulus of polyurethane foam as core material
- N – number of discretization points
- p_{crdyn} – critical dynamic load
- t_{cr} – critical time
- t^* – dimensionless time
- ζ_{1max} – dimensionless additional deflection
- p_{cr} – critical static load
- m – plate buckling mode

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