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Compact Non-Linear Gravitomagnetic Fields as Dark Matter

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Abstract: We introduce a model of dark matter inspired by the non-linear gravito-electromagnetism formalism of Roy Maartens and Bruce Bessett. The Maartens and Bessett model is a fully covariant 1+3 electromagnetic analogue of gravity characterized by Weyl gravito-electric and gravito-magnetic spatial tensor fields in which the Bianchi identities are the dynamical equations. The nonlinear rotationally invariant invariant vacuum Bianchi equations describe the properties of spacetime within a given locality. We identify compact gravito-magnetic fields based on this model as a Ricci soliton which when included into General Relativity describes dark matter. This component provides the extra gravity which is currently attributed to a variety of theorized forms of non-baryonic matter collectively known as dark matter.

Keywords: dark matter; non-linear gravito-electromagnetism; Ricci Soliton; baryonic Tully-Fisher relation; Milgrom's constant

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I. Introduction

Currently, one of the most intriguing questions in fundamental physics is, what is the nature and origins of dark matter (DM) and dark energy (DE)? In astrophysics and cosmology, DM is attributed to either some exotic non-baryonic matter or to a modification of the law of gravity at large scales. Particles beyond the standard model of particle physics are actively being searched either through direct detection experiments or through collider experiments. So far, these particles have proved elusive with some models falling out of favor as a series of direct detection experiments have ruled out a gamut of predicted energy scales and cross-sectional areas [1–6]. A broad and in-depth review of the variety of proposed ideas on particulate DM can be found in [7–11] and the references therein. Within the formalism of modified gravity $F(R)$, $F(G)$ and $F(T)$ gravity theories are actively being explored with the hope that a specific model can be found that reduces to the phenomenological model referred to as MOND [12,13]. Further reviews of such approaches can be found in [14–17].

In the present work, we attempt to shed some light on the enigmas of DM from the perspective of gravito-electromagnetism (GEM), specifically through the Maartens-Bassett formalism of non-linear GEM [18–20]. GEM arises as the name suggests from a rich and detailed correspondence between General Relativity (GR) and electromagnetism [21–23]. This strong correspondence is reflected in the Weyl tensor which has a Maxwell like form, the Bel-Robinson tensor with similarities to the electromagnetic energy-momentum tensor and the Bianchi identities' similarities to the dynamical equations of electromagnetism.

The paper is structured as follows: first, in the preliminaries section, we recall the fundamental equations of the Maartens-Basset formalism of non-linear GEM. We then implement this formalism in the linearized form to obtain the modified field equations of GR that include effects attributed to DM.

2. Preliminaries

In Ref. [23], Ellis provides Maxwell's equations in the 1+3 streamlined form as follows:

$$D^a E_a = -2\omega^a H_a + \varrho \,, \tag{1}$$

$$D^a H_a = 2\omega^a E_a \,, \tag{2}$$

$$\dot{E}_{\langle a\rangle} - \nabla\times H_a = \tfrac{2}{3}\Theta E_a + \sigma_{ab}E^b - [\omega, E]_a + [\dot{u}, H]_a - j_a \,, \tag{3}$$

$$\dot{H}_{\langle a\rangle} + \nabla\times E_a = \tfrac{2}{3}\Theta H_a + \sigma_{ab}H^b - [\omega, H]_a - [\dot{u}, E]_a \,, \tag{4}$$

where $\varrho = -J_a u^a$ is the electric charge density, u^a the four velocity, $\Theta = D^a u_a$, the shear $\sigma_{ab} = D_{\langle a}u_{b\rangle}$, $\omega_a = -\frac{1}{2}\nabla\times u_a$ is the vorticity and $\dot{u}_a = \dot{u}_{\langle a\rangle}$ is the four acceleration.

The above operators obey the following covariant identities:

$$\dot{(D_a f)} = D_a\dot{f} - \tfrac{1}{3}\Theta D_a f + \dot{u}_a\dot{f} - \sigma_a{}^b D_b f - [\omega, Df]_a + u_a\dot{u}^b D_b f \,, \tag{5}$$

$$\nabla\times D_a f = 2\dot{f}\omega_a \,, \tag{6}$$

$$D^a[V, W]_a = W^a\nabla\times V_a - V^a\nabla\times W_a \,, \tag{7}$$

And

$$D^a[A, B]_a = B^{ab}\nabla\times A_{ab} - A^{ab}\nabla\times B_{ab} \,, \tag{8}$$

The Maxwellean equivalent of GR is based on the correspondence between the Weyl tensor C_{abcd} and the electromagnetic tensor F_{ab} for any u_a. Thus

$$E_{ab} = C_{abcd}u^c u^d = E_{\langle ab\rangle}, H_{ab} = * \, C_{abcd}u^c u^d = H_{\langle ab\rangle} \tag{9}$$

These gravito-electric/magnetic spatial tensors are in principle physically measurable in the frames of comoving observers, and together they are equivalent to the space-time Weyl tensor.

The gravito-electromagnetic version of the electromagnetic tensor is

$$C_{ab}{}^{cd} = 4\{u_{[a}u^{[c} + h_{[a}{}^{[c}\}E_{b]}{}^{d]} + 2\varepsilon_{abe}u^{[c}H^{d]e} + 2u_{[a}H_{b]e}\varepsilon^{cde} \,. \tag{10}$$

In the 1 + 3 covariant approach to GR, the source of gravity is a fluid in which the energy density , pressure and the gravito-electromagnetic tensors are the fundamental quantities and not the metric. These quantities are governed by the Bianchi identities and the Ricci identities for u^a. Einstein's equations incorporated through the algebraic definition of the Ricci tensor R_{ab} in terms of the energy-momentum tensor T_{ab}. The Bianchi identities are

$$\nabla^a C_{abcd} = \nabla_{[a}\left(-R_{b]c} + \tfrac{1}{6}R g_{b]c}\right) . \tag{11}$$

Here $R = R_a{}^a$ and $R_{ab} = \kappa\left(T_{ab} - \frac{1}{2}T_c{}^c g_{ab}\right)$ where κ is the Einstein constant. The gravitational equivalents of Maxwell's equations are therefore

$$D^a E_{ab} = -3\omega^a H_{ab} + \tfrac{1}{3}D_a\rho + [\sigma, H]_a \,, \tag{12}$$

$$D^a H_{ab} = 3\omega^a E_{ab} - \tfrac{1}{3}(\rho + p)\omega_a\rho - [\sigma, E]_a \,, \tag{13}$$

$$\dot{E}_{\langle ab\rangle} - \nabla\times H_{ab} = \Theta E_{ab} + \sigma_{c\langle a}E_{b\rangle}{}^c - \omega^c\varepsilon_{cd(a}E_{b)}{}^d + 2\dot{u}^c\omega^c\varepsilon_{cd(a}H_{b)}{}^d - \tfrac{1}{2}(\rho + p)\sigma_{ab}, \tag{14}$$

$$\dot{H}_{\langle ab\rangle} - \nabla\times E_{ab} = -\Theta H_{ab} + \sigma_{c\langle a}H_{b\rangle}{}^c - \omega^c\varepsilon_{cd(a}H_{b)}{}^d - 2\dot{u}^c\omega^c\varepsilon_{cd(a}E_{b)}{}^d \,. \tag{15}$$

These are the full nonlinear gravito-electromagnetic equations in covariant form. In both electromagnetism and gravito-electromagnetism vorticity produces source terms however, gravity has additional sources from a tensor coupling of the shear to the field. Furthermore, in electromagnetism, there is no magnetic charge sources, but the gravito- magnetic field H_{ab} has the

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source $(\rho+p)\omega_a$. Since $\rho+p$ is the relativistic inertial mass-energy density, $(\rho+p)\omega_a$ is the 'angular momentum density', which we identify as a gravito- magnetic 'charge' density.

3. Modification of General Relativity

The electromagnetic analogy suggests a further interesting interpretation of the vorticity. In flat spacetime, relative to inertial observers, the electric and magnetic vectors may be written as

$$\vec{E} = \vec{\nabla}V - \partial_t\vec{\alpha} \ , \ \vec{H} = \vec{\nabla}\times\vec{\alpha}, \tag{16}$$

where, V is the electric scalar potential and $\vec{\alpha}$ is the gravitomagnetic vector potential. We express the gravitomagnetic vector potential as $\vec{\alpha} = \vec{v} = \mu(t)\vec{H}_0 \times \vec{r}$. Here, the magnitude of $\vec{H}_0$ is the Hubble constant, $\vec{r}$ the radius vector, $\vec{v}$ the tangential velocity and $\mu(t)$ the time dependent relative gravito-magnetic permeability. The curl of the tangential velocity for a given r results in an angular velocity about the z direction of

$$\omega = \mu(t)H_0\,. \tag{17}$$

Einstein's field equations (EFE) need to be modified such that they include the contribution by the magnetic charge. We now proceed with a similar approach as in [24]. As stated earlier, in the 1 + 3 covariant approaches to GR, the source of gravity is a fluid in which the energy density, pressure and the gravito-electromagnetic tensors are the fundamental quantities and not the metric. This allows us to intuitively make the assumption that the gravito-magnetic field is a compact and expanding region of spacetime that can be described as a Ricci soliton of the form $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = kg_{\mu\nu}$ with the Ricci tensor of the compact manifold expressed as a compact energy-momentum tensor field $R_{ab} = \kappa\left(T_{ab} - \frac{1}{2}Tg_{ab}\right) = kg_{ab}$. Here, κ is the Einstein gravitational constant k is a constant proportional to the cosmological constant. These expressions can only be self-consistent if and only if the traces are $T = 0$ and $R = 0$. Here, the compact stress momentum tensor field $\left(T_{ab} - \frac{1}{2}Tg_{ab}\right)$ is considered as a barotropic fluid. A system consisting of gravitationally bound baryonic matter can assume this barotropic fluid like morphology when modeled at a sufficiently large scale. For example, the solar system, star and galactic clusters can be considered as fluid at sufficiently large scales relative to the intrinsic scale of the gravitationally bound system. This approach has been successfully applied at cosmic scales to model the universe using the FLRW metric. Here the approach is applied at much smaller clustering of gravitationally bound astrophysical systems. By modelling the system as barotropic, it satisfies the condition of being homogeneous and isotropic in the volume it occupies. Einstein's field equations (EFE) are then modified such that they include the GEM field modelled as 4-D compact Einstein manifold and thus expressing the EFE as follows:

$$G_{ab} - kg_{ab} + \Lambda g_{ab} = \kappa T_{ab}\,. \tag{18}$$

Here, $k = \mu^2\Lambda = 3\left(\frac{\mu H_0}{c}\right)^2$ with a trace $R = 0$. Here, μH_0 is the angular velocity of a Ricci soliton of reduced radius $r = \frac{c}{2\pi\mu H_0}$. This implies a rotational velocity on the surface of the soliton of $v = H_0 r = \frac{c}{2\pi\mu}$. This circular motion on the surface of the Ricci soliton arises from the traceless condition $T = 0$. This condition demands marginally stable or zero energy orbits on the surface in which the kinetic energy is equal to the gravitational potential energy.

Equation(18) must therefore satisfy a metric solution of the form

$$-c^2\tau^2 = -c^2t^2 + \mu(t)^2 d\Sigma^2 \tag{19}$$

Where $d\Sigma^2 = \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)$ in reduced-circumference polar coordinates.

The analytic solutions to Equation (19) in which the energy momentum tensor is isotropic and homogeneous are the following equations:

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$$\left(\frac{\dot{\mu}}{\mu}\right)^2+\frac{\mu^2\Lambda c^2}{3\mu^2}-\frac{\Lambda c^2}{3}=\frac{\kappa c^4}{3}\rho\ \rightarrow\ \left(\frac{\dot{\mu}}{\mu}\right)^2+\frac{2\Lambda c^2}{3}=\frac{\kappa c^4}{3}\rho\ , \tag{20}$$

$$2\frac{\ddot{\mu}}{\mu}+\left(\frac{\dot{\mu}}{\mu}\right)^2+\frac{\mu^2\Lambda c^2}{\mu^2}-\Lambda c^2=-\kappa c^2 p\ \rightarrow 2\frac{\ddot{\mu}}{\mu}+\left(\frac{\dot{\mu}}{\mu}\right)^2=-\kappa c^2 p\ , \tag{21}$$

$$R=-6\left[\frac{\ddot{\mu}}{\mu}+\left(\frac{\dot{\mu}}{\mu}\right)^2+\Lambda c^2\right]=0\ . \tag{22}$$

From Equation (22) we obtain

$$\frac{\ddot{\mu}}{\mu}=-\left(\frac{\dot{\mu}}{\mu}\right)^2-\ \Lambda c^2. \tag{23}$$

Substituting Equation (23) into Equation (21) yields

$$\left(\frac{\dot{\mu}}{\mu}\right)^2=\kappa c^2 p-2\Lambda c^2\ . \tag{24}$$

Substituting Equation (24) into Equation (20) and taking $p=-\rho c^2$ yields

$$\Lambda=-\kappa c^2\rho\ . \tag{25}$$

Given a measured value of the cosmological constant of $1.1\times 10^{-52}m^{-2}$ we compute a critical baryonic matter density for the Ricci soliton of $\rho_{cb}=\rho_{DE}=5.6\times\ 10^{-27}kgm^{-3}$.

The Ricci soliton therefore expands exponentially when $p=-\rho c^2$ with a scale factor $\mu(t)=\mu_o e^{H_0 t}$. Here $H_o=\sqrt{\kappa c^2 p-2\Lambda c^2}=\sqrt{\kappa c^4\rho}$. This expansion occurs when the covariant flatness condition of Equation (19) is satisfied. Under such conditions the centripetal acceleration on the surface of the soliton is given by the expression

$$\frac{v^2}{r}=\frac{(H_0 r)^2}{r}=H_0{}^2 r=\frac{H_0{}^2 c}{2\pi\mu H_0}=\frac{H_0 c}{2\pi\mu}\ . \tag{26}$$

In the comoving reference frame $\mu=1$ which implies a scale invariant acceleration

$$\mu_0=\frac{H_0 c}{2\pi}\ . \tag{27}$$

Under such conditions the radius is computed from Equation (26) as

$$r=\frac{2\pi v^2}{H_0 c} \tag{28}$$

Substituting r in the expression $\frac{GM(r)}{r}=v^2$ we obtain

$$v^4=\frac{GM(r)H_0 c}{2\pi}\ . \tag{29}$$

This is the baryonic Tully Fisher relation and $a_0=\frac{H_0 c}{2\pi}\cong 1.1\times 10^{-10}m/s^2$ is the empirically observed Milgrom's acceleration constant [60,61]. The theoretically computed value is in agreement with observations.

Since $r=r_0 e^{H_0 t}$ then the velocity evolves with the scale factor as follows:

$$v^4=\frac{GM(r)H_0 c}{2\pi}e^{4H_0 t} \tag{30}$$

From the above equations, we also obtain the following equations of galactic and galactic cluster evolution

$$r=\frac{1}{H_0}e^{(H_0 t)}(GM_B(r)\frac{H_0}{2\pi}c)^{\frac{1}{4}}=\frac{v_n}{H_0}\ , \tag{31}$$

$$v=\frac{dr}{dt}=e^{(H_0 t)}(GM_B(r)\frac{H_0}{2\pi}c)^{\frac{1}{4}}=H_0 r\ , \tag{32}$$

$$a=\frac{dv}{dt}=H_0 e^{(H_0 t)}(GM_B\frac{H_0}{2\pi}c)^{\frac{1}{4}}=H_0 v\ . \tag{33}$$

4. Discussion and Concluding Remarks

The gravito-electromagnetism formulation of Roy Maartens and Bruce Bassett draws parallels between the equations governing electromagnetism and gravity. This analogy helps in developing a unified theoretical framework that simplifies the understanding of gravitational phenomena using well-established electromagnetic principles. The concept of super-energy and the super-Poynting vector offers deeper insights into the dynamics of spacetime. In this paper we have taken insights from the Roy Maartens and Bruce Bassett formulation to model dark matter as a compact gravito-magneto field with the same properties as a Ricci soliton which was first discussed in Ref. [24]. This model of dark matter offers a unique perspective compared to traditional dark matter models. It provides a classical, geometric approach to dark matter, contrasting with the particle-based explanations of traditional models. This could lead to new insights and potentially broaden our understanding of dark matter and dark energy.

Data Availability: Empirical data used in this research can be found in the cited articles.

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Conflict of Interest: We declare no conflict of interest.

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