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Abstract

A Soft Set is a parameterized family of subsets of a universe, where each parameter selects elements relevant under that condition. A ContraSoft Set is a parameterized soft set in which each parameter's values are linked with a contradiction degree, and a threshold mechanism is applied to retain only those values that do not exceed a specified level of contradiction with respect to a chosen reference. In this paper, we explore two new concepts, namely the *Contra-HyperSoft Set* and the *Contra-SuperHyperSoft Set*, extending the framework of contradiction-aware modeling.

Keywords: soft set; contradiction; ContraSoft set; Contra-HyperSoft set; Contra-SuperHyperSoft set

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1. Preliminaries

We collect the basic terminology and notation used in what follows. The definitions in this paper are assumed to be finite.

1.1. Soft Set

A Soft Set is a parameterized family of subsets selecting universe elements relevant to each parameter, supporting flexible decision modeling [1–3]. The definitions of the Soft Set are provided below.

Definition 1 (Soft Set)**.** *[1] Let U be a universal set and E a set of parameters. A soft set over U is defined as an ordered pair (F, E), where F is a mapping from E to the power set $\mathcal{P}(U)$:*

$$F : E \to \mathcal{P}(U).$$

For each parameter $e \in E$, $F(e) \subseteq U$ represents the set of e-approximate elements in U, with (F, E) forming a parameterized family of subsets of U.

Example 1 (Soft Set — Hotel Filtering with Parameterized Conditions)**.** ***Universe and parameters.*** *Let the universe of candidate hotels be*

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}.$$

Let the parameter set be

$$E = \{\text{near\_station},\ \text{free\_breakfast},\ \text{onsen},\ \text{under¥12000},\ \text{twin\_room}\}.$$

Define a soft set (F, E) *with* $F : E \to \mathcal{P}(U)$ *by*

$$\begin{aligned} F(\text{near\_station}) &= \{h_1, h_2, h_5, h_7\}, \\ F(\text{free\_breakfast}) &= \{h_2, h_4, h_5, h_6\}, \\ F(\text{onsen}) &= \{h_3, h_5, h_8\}, \\ F(\text{under¥12000}) &= \{h_1, h_4, h_5, h_7, h_8\}, \\ F(\text{twin\_room}) &= \{h_2, h_4, h_6, h_7\}. \end{aligned}$$

Concrete queries and explicit computations.

$$\begin{aligned} \textit{(i) near\_station \& under¥12000:} \quad & F(\text{near\_station}) \cap F(\text{under¥12000}) = \{h_1, h_2, h_5, h_7\} \cap \{h_1, h_4, h_5, h_7, h_8\} \\ & = \{h_1, h_5, h_7\}, \quad |\cdot| = 3. \\ \textit{(ii) (free\_breakfast} \cup \textit{onsen) \& twin\_room:} \quad & (F(\text{free\_breakfast}) \cup F(\text{onsen})) \cap F(\text{twin\_room}) \\ & = (\{h_2, h_4, h_5, h_6\} \cup \{h_3, h_5, h_8\}) \cap \{h_2, h_4, h_6, h_7\} \\ & = \{h_2, h_4, h_6\}, \quad |\cdot| = 3. \\ \textit{(iii) onsen \& twin\_room:} \quad & F(\text{onsen}) \cap F(\text{twin\_room}) = \{h_3, h_5, h_8\} \cap \{h_2, h_4, h_6, h_7\} = \varnothing. \end{aligned}$$

These results illustrate how a soft set supports multi-criterion filtering by standard set operations with exact outputs.

1.2. ContraSoft Set

A ContraSoft Set is a parameterized soft set where each parameter's values are associated with a contradiction degree, and thresholding is used to aggregate only those values that are not too contradictory with respect to a chosen reference. This allows soft-set modeling to filter or weight information based on contradiction, rather than uncertainty.

Definition 2 (Contradiction on attribute values)**.** *Let* V *be a nonempty finite set of attribute values. A* contradiction function *on* V *is a map*

$$c : V \times V \longrightarrow [0, 1]$$

such that

$$c(v, v) = 0 \quad (\textit{reflexivity}), \qquad c(v, w) = c(w, v) \quad (\textit{symmetry}).$$

The quantity $c(v, w)$ *measures the degree of* contradiction *between* v *and* w *(larger means more contradictory).*

Example 2 (Contradiction on attribute values — temperature preference)**.** *Let* $V = \{\text{cold}, \text{mild}, \text{hot}\}$*. Define the symmetric contradiction* $c : V \times V \to [0, 1]$ *(with* $c(v, v) = 0$*) by*

$c =$

	cold	mild	hot
cold	0	0.4	0.9
mild	0.4	0	0.5
hot	0.9	0.5	0

so, e.g., $c(\text{cold}, \text{hot}) = 0.9$ *expresses a strong contradiction, while* $c(\text{cold}, \text{mild}) = 0.4$ *is moderate.*

Definition 3 (ContraSoft structure)**.** *Let* U *be a nonempty universe and* E *a nonempty set of parameters. For each* $e \in E$ *fix:*

- *a nonempty finite value set* V_e*;*
- *a contradiction function* $c_e : V_e \times V_e \to [0, 1]$ *(Definition 2);*
- *a designated* reference value $v_e^\star \in V_e$.

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Write $V := \bigsqcup_{e\in E}(\{e\} \times V_e)$ for the disjoint union of all parameter–value pairs.

Example 3 (ContraSoft structure — hotels by noise and price). *Let the universe be $U = \{h_1, h_2, h_3, h_4\}$ and parameters $E = \{\text{noise}, \text{price}\}$. For each $e \in E$ fix a finite value-set V_e, a contradiction $c_e : V_e \times V_e \to [0,1]$, and a reference value $v_e^\star \in V_e$:*

$$V_{\text{noise}} = \{\text{quiet}, \text{moderate}, \text{loud}\}, \quad c_{\text{noise}} = \begin{array}{c|ccc} & \text{quiet} & \text{moderate} & \text{loud} \\ \hline \text{quiet} & 0 & 0.3 & 0.8 \\ \text{moderate} & 0.3 & 0 & 0.4 \\ \text{loud} & 0.8 & 0.4 & 0 \end{array}, \quad v^\star_{\text{noise}} = \text{quiet}.$$

$$V_{\text{price}} = \{\text{cheap}, \text{mid}, \text{expensive}\}, \quad c_{\text{price}} = \begin{array}{c|ccc} & \text{cheap} & \text{mid} & \text{expensive} \\ \hline \text{cheap} & 0 & 0.2 & 0.7 \\ \text{mid} & 0.2 & 0 & 0.3 \\ \text{expensive} & 0.7 & 0.3 & 0 \end{array}, \quad v^\star_{\text{price}} = \text{mid}.$$

These choices realize Definition (ContraSoft structure) *by specifying value domains, their contradiction degrees, and per-parameter references.*

Definition 4 (ContraSoft Set). *Let U be a finite universe of objects and E a finite set of parameters. A* ContraSoft Set *is a quadruple*

$$\mathsf{CS} := (U, E, F, c),$$

where

- *$F : E \to \mathcal{P}(U)$ is the (crisp) soft mapping; $F(e) \subseteq U$ is the set of objects* accepted *(or classified as positive) under parameter e;*
- *$c : E \times E \to [0,1]$ is a* contradiction degree *on parameters, symmetric and reflexive on the diagonal:*

$$c(e,e) = 0, \qquad c(e,f) = c(f,e) \quad (\forall\, e, f \in E).$$

For $x \in U$ and $e \in E$, the atomic lemma "x is accepted by e" is represented by

$$A(x,e): \quad x \in F(e),$$

with truth value **T** *if $x \in F(e)$ and* **F** *otherwise.*

Example 4 (ContraSoft Set — Noise-Aware Hotel Selection with Contradiction Thresholding). ***Universe, parameters, and soft mapping.*** *Let the same universe U be as above. Consider parameters*

$$E = \{\text{quiet}, \text{nightlife}, \text{coworking}, \text{scenic}\}.$$

Define $F : E \to \mathcal{P}(U)$ by

$$F(\text{quiet}) = \{h_1, h_3, h_5, h_8\}, \quad F(\text{nightlife}) = \{h_2, h_4, h_6\},$$
$$F(\text{coworking}) = \{h_2, h_5, h_6, h_7\}, \quad F(\text{scenic}) = \{h_3, h_5, h_7, h_8\}.$$

Contradiction degrees on parameters. *Let $c : E \times E \to [0,1]$ be symmetric with*

$c(\cdot,\cdot)$	quiet	nightlife	coworking	scenic
quiet	0	0.9	0.4	0.2
nightlife	0.9	0	0.3	0.5
coworking	0.4	0.3	0	0.4
scenic	0.2	0.5	0.4	0

(diagonal 0, larger values mean more contradictory).

Reference and thresholded aggregation. *Fix the* reference parameter $e^\star$ = quiet *and threshold* $\tau = 0.4$. *Define the* accepted envelope

$$S^{(\tau)}(e^\star) := \bigcup_{e\in E:\ c(e,e^\star)\le\tau} F(e).$$

Eligible parameters are those within the contradiction radius:

$$c(\text{quiet},\text{quiet}) = 0, \quad c(\text{coworking},\text{quiet}) = 0.4, \quad c(\text{scenic},\text{quiet}) = 0.2, \quad c(\text{nightlife},\text{quiet}) = 0.9 > \tau.$$

Hence

$$\begin{aligned} S^{(\tau)}(\text{quiet}) &= F(\text{quiet}) \cup F(\text{coworking}) \cup F(\text{scenic}) \\ &= \{h_1,h_3,h_5,h_8\} \cup \{h_2,h_5,h_6,h_7\} \cup \{h_3,h_5,h_7,h_8\} \\ &= \{h_1,h_2,h_3,h_5,h_6,h_7,h_8\}, \quad |S^{(\tau)}| = 7. \end{aligned}$$

Tighter threshold for comparison. *With* $\tau' = 0.2$, *only* quiet *and* scenic *are admitted:*

$$S^{(\tau')}(\text{quiet}) = F(\text{quiet}) \cup F(\text{scenic}) = \{h_1,h_3,h_5,h_7,h_8\}, \quad |S^{(\tau')}| = 5.$$

Thus $S^{(\tau)}$ *is monotone in* τ, *and the contradiction metric controls how widely we aggregate across potentially conflicting parameters.*

1.3. HyperSoft Set and SuperHyperSoft Set

HyperSoft Set maps each multi-attribute tuple from a Cartesian product to a subset of the universe consistent with those values [4–8]. SuperHyperSoft Set maps tuples of subsets from power-set domains to universe subsets, generalizing HyperSoft; singletons in each coordinate recover HyperSoft [9,10].

Definition 5 (HyperSoft Set)**.** *[4] Let* U *be a finite universe and let* $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ *be* m *attribute value domains. Consider the Cartesian product*

$$\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m,$$

so that each parameter $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m) \in \mathcal{C}$ *chooses a single value* $\gamma_i \in \mathcal{A}_i$ *for every attribute. A* HyperSoft Set *over* U *is a pair* $(G, \mathcal{C})$ *where*

$$G : \mathcal{C} \longrightarrow \mathcal{P}(U)$$

assigns to each multi-attribute parameter γ *a subset* $G(\gamma) \subseteq U$. *Equivalently,*

$$(G, \mathcal{C}) = \{\, (\gamma,\ G(\gamma)) \ :\ \gamma \in \mathcal{C} \,\}.$$

Example 5 (HyperSoft Set — Multi-Attribute Restaurant Finder)**.** ***Universe and attributes.*** *Let the universe of candidate restaurants be*

$$U = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9\}.$$

Let the attribute domains be

$$\mathcal{A}_1 = \{\text{jpn}, \text{ita}, \text{ind}\}, \quad \mathcal{A}_2 = \{\text{low}, \text{mid}, \text{high}\}, \quad \mathcal{A}_3 = \{\text{omn}, \text{veg}, \text{vgn}\}.$$

The parameter space is the Cartesian product $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$. *A HyperSoft Set is a mapping* $G : \mathcal{C} \to \mathcal{P}(U)$ *that assigns a subset of restaurants to each single-valued tuple* (cuisine, price, diet).

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Specification (nonempty images).

$$G(\text{jpn}, \text{mid}, \text{veg}) = \{r_2, r_5\}, \quad G(\text{jpn}, \text{low}, \text{omn}) = \{r_1, r_3\},$$
$$G(\text{ita}, \text{mid}, \text{veg}) = \{r_4\}, \qquad G(\text{ind}, \text{low}, \text{vgn}) = \{r_6, r_8\},$$
$$G(\text{ita}, \text{high}, \text{omn}) = \{r_7, r_9\},$$

and $G(\gamma) = \varnothing$ *for all other* $\gamma \in \mathcal{C}$.

Concrete queries with exact set calculations.

$$\text{(i) Exactly } (\text{jpn}, \text{mid}, \text{veg}): \quad G(\text{jpn}, \text{mid}, \text{veg}) = \{r_2, r_5\}, \ |\cdot| = 2.$$
$$\text{(ii) Union of two precise asks:} \quad G(\text{jpn}, \text{low}, \text{omn}) \cup G(\text{ita}, \text{high}, \text{omn}) = \{r_1, r_3\} \cup \{r_7, r_9\} = \{r_1, r_3, r_7, r_9\}, \ |\cdot| = 4.$$
$$\text{(iii) Disjointness of incompatible tuples:} \quad G(\text{jpn}, \text{low}, \text{omn}) \cap G(\text{ind}, \text{low}, \text{vgn}) = \{r_1, r_3\} \cap \{r_6, r_8\} = \varnothing.$$

The HyperSoft Set captures single-value *choices per attribute; each tuple pinpoints a crisp slice of* U.

Definition 6 (SuperHyperSoft Set)**.** *[9,11] Let* U *be a finite universe. Let* $a_1, a_2, \ldots, a_n$ *be distinct attributes with finite, pairwise disjoint value-sets* $A_1, A_2, \ldots, A_n$ *(i.e.,* $A_i \cap A_j = \varnothing$ *for* $i \neq j$*). Write* $\mathcal{P}(A_i)$ *for the power set of* A_i *and form*

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n).$$

A SuperHyperSoft Set *over* U *is a pair* $(F, \mathcal{C})$ *with*

$$F : \mathcal{C} \longrightarrow \mathcal{P}(U),$$

so that for each $\gamma = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathcal{C}$ *(where* $\alpha_i \subseteq A_i$*) we have a subset* $F(\gamma) \subseteq U$*. Formally,*

$$(F, \mathcal{C}) = \{ (\gamma, F(\gamma)) : \gamma \in \mathcal{C}, \ F(\gamma) \subseteq U \}.$$

Example 6 (SuperHyperSoft Set — Flexible Restaurant Finder with Set-Valued Coordinates)**.** ***Universe and attributes.*** *Use the same* U *and attribute value-sets* $A_1 = \{\text{jpn}, \text{ita}, \text{ind}\}$, $A_2 = \{\text{low}, \text{mid}, \text{high}\}$, $A_3 = \{\text{omn}, \text{veg}, \text{vgn}\}$. *In the SuperHyperSoft setting, the parameter space is*

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3),$$

so each coordinate is a subset *of admissible values (a flexible filter).*

Mapping (nonempty images). *Define* $F : \mathcal{C} \to \mathcal{P}(U)$ *by*

$$F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\}) = \{r_2, r_4, r_5\},$$
$$F(\{\text{ind}\}, \{\text{low}, \text{mid}\}, \{\text{vgn}\}) = \{r_6, r_8\},$$
$$F(\{\text{jpn}, \text{ind}\}, \{\text{low}\}, \{\text{omn}, \text{veg}\}) = \{r_1, r_3, r_6\},$$
$$F(\{\text{ita}\}, \{\text{high}\}, \{\text{omn}\}) = \{r_7, r_9\},$$

and $F(\alpha) = \varnothing$ *otherwise.*

Reading the parameters. *For example,* $\alpha = (\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\})$ *means: cuisine is Japanese* or *Italian, price is low* or *mid, diet is vegetarian. Then* $F(\alpha) = \{r_2, r_4, r_5\}$ *is the recommended subset.*

Coherence with HyperSoft via singletons. *If we restrict to singletons in each coordinate, SuperHyperSoft reduces to HyperSoft. Concretely,*

$$F(\{\text{jpn}\}, \{\text{mid}\}, \{\text{veg}\}) = \{r_2, r_5\} = G(\text{jpn}, \text{mid}, \text{veg}),$$

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so the singleton tuple reproduces the HyperSoft slice exactly. Moreover,

$$F(\{\text{jpn}\}, \{\text{mid}\}, \{\text{veg}\}) \subseteq F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\}) = \{r_2, r_4, r_5\},$$

exhibiting the intended flexible expansion *when coordinates are broadened from single values to sets of values.*

Cardinality checks.

$$|F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\})| = 3, \quad |F(\{\text{ind}\}, \{\text{low}, \text{mid}\}, \{\text{vgn}\})| = 2.$$

Thus SuperHyperSoft enables compact specification of multi-value preferences per attribute and directly returns the filtered subset of U.

2. Main Results

In this section, we present and analyze the principal outcomes of our study.

2.1. Contra-HyperSoft Set

Contra-HyperSoft Set augments HyperSoft with a tuple-wise contradiction metric, reference selector, and threshold, uniting parameter slices within the admissible radius.

Definition 7 (Coordinatewise contradiction). *Let $\mathcal{A}_1, \dots, \mathcal{A}_m$ be nonempty finite sets. For each $i \in \{1, \dots, m\}$ a* contradiction function *is a map*

$$c_i : \mathcal{A}_i \times \mathcal{A}_i \longrightarrow [0,1] \quad \textit{with} \quad c_i(a,a) = 0, \; c_i(a,b) = c_i(b,a).$$

When needed for exact reductions, we assume the zero-separation *property $c_i(a,b) = 0 \Rightarrow a = b$.*

Definition 8 (Tuple-level contradiction). *Let $\mathcal{C} := \mathcal{A}_1 \times \cdots \times \mathcal{A}_m$ and write $\gamma = (\gamma_1, \dots, \gamma_m)$, $\delta = (\delta_1, \dots, \delta_m) \in \mathcal{C}$. Define the aggregated contradiction by*

$$\Delta(\gamma, \delta) := \max_{1 \le i \le m} c_i(\gamma_i, \delta_i) \in [0,1].$$

Then $\Delta(\gamma, \delta) = \Delta(\delta, \gamma)$ and $\Delta(\gamma, \gamma) = 0$. If each c_i is zero-separating, then $\Delta(\gamma, \delta) = 0 \iff \gamma = \delta$.

Definition 9 (Reference selector). *A* reference selector *is a map $\rho : \mathcal{C} \to \mathcal{C}$. Two canonical choices are*

$$\textit{(self-centered)} \; \rho(\gamma) = \gamma, \qquad \textit{(fixed-reference)} \; \rho(\gamma) \equiv r \; \textit{for a fixed } r \in \mathcal{C}.$$

Definition 10 (Contra-HyperSoft Set (CHS)). *Let U be a finite universe and let $G : \mathcal{C} \to \mathcal{P}(U)$ be a HyperSoft mapping. Fix contradiction kernels $\{c_i\}_{i=1}^m$, a reference selector ρ, and a threshold $\tau \in [0,1]$. The associated* Contra-HyperSoft Set *is the tuple*

$$\mathsf{CHS} := (U, \{\mathcal{A}_i, c_i\}_{i=1}^m, G, \rho, \tau),$$

together with the filtered mapping

$$G_\rho^{(\tau)} : \mathcal{C} \longrightarrow \mathcal{P}(U), \qquad G_\rho^{(\tau)}(\gamma) := \bigcup_{\delta \in \mathcal{C} \,:\, \Delta(\delta, \rho(\gamma)) \le \tau} G(\delta).$$

Example 7 (Contra-HyperSoft Set — Candidate Shortlisting under Conflicting Signals (self-centered selector)). ***Universe and attributes.*** *Let the candidate pool be $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$. Consider three single-valued attribute domains:*

$$\mathcal{A}_1 = \{\text{junior}, \text{mid}, \text{senior}\}, \quad \mathcal{A}_2 = \{\text{onsite}, \text{hybrid}, \text{remote}\}, \quad \mathcal{A}_3 = \{\text{backend}, \text{frontend}, \text{data}\}.$$

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The parameter space is $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$.

Coordinatewise contradictions. *All* c_i *are symmetric with* 0 *on the diagonal.*

$c_1 =$

	junior	mid	senior
junior	0	0.4	0.9
mid	0.4	0	0.4
senior	0.9	0.4	0

$c_2 =$

	onsite	hybrid	remote
onsite	0	0.3	0.8
hybrid	0.3	0	0.3
remote	0.8	0.3	0

$c_3 =$

	backend	frontend	data
backend	0	0.5	0.4
frontend	0.5	0	0.6
data	0.4	0.6	0

Aggregate tuple-contradiction: $\Delta(\gamma, \delta) := \max\{c_1(\gamma_1, \delta_1), c_2(\gamma_2, \delta_2), c_3(\gamma_3, \delta_3)\}$.

HyperSoft mapping $G : \mathcal{C} \to \mathcal{P}(U)$ ***(nonempty images).***

$$G(\text{junior}, \text{remote}, \text{frontend}) = \{c_1, c_3\}, \quad G(\text{mid}, \text{hybrid}, \text{backend}) = \{c_2, c_5\},$$
$$G(\text{senior}, \text{onsite}, \text{data}) = \{c_4\}, \quad G(\text{mid}, \text{remote}, \text{data}) = \{c_6\},$$
$$G(\text{senior}, \text{hybrid}, \text{backend}) = \{c_7, c_8\}.$$

CHS filter. *Choose the* self-centered *selector* $\rho(\gamma) = \gamma$ *and threshold* $\tau = 0.4$. *Let* $\gamma^\star = (\text{mid}, \text{hybrid}, \text{backend})$. *Compute* $\Delta(\cdot, \gamma^\star)$ *on the above tuples:*

δ	$\Delta(\delta, \gamma^\star)$
(mid, hybrid, backend)	$\max(0, 0, 0) = 0 \ (\le \tau)$
(senior, hybrid, backend)	$\max(0.4, 0, 0) = 0.4 \ (\le \tau)$
(mid, remote, data)	$\max(0, 0.3, 0.4) = 0.4 \ (\le \tau)$
(senior, onsite, data)	$\max(0.4, 0.3, 0.4) = 0.4 \ (\le \tau)$
(junior, remote, frontend)	$\max(0.4, 0.3, 0.5) = 0.5 \ (> \tau)$

Hence

$$G_\rho^{(\tau)}(\gamma^\star) = \{c_2, c_5\} \cup \{c_7, c_8\} \cup \{c_6\} \cup \{c_4\} = \{c_2, c_4, c_5, c_6, c_7, c_8\}, \quad |\cdot| = 6.$$

With a tighter threshold $\tau' = 0.3$, *only the base slice survives:* $G_\rho^{(\tau')}(\gamma^\star) = \{c_2, c_5\}$, *illustrating monotonicity in* τ.

Example 8 (Contra-HyperSoft Set — Travel Package Selection (fixed reference))**.** ***Universe and attributes.*** *Let* $U = \{\text{pkg}_1, \dots, \text{pkg}_{10}\}$ *be travel packages. Attributes:*

$$\mathcal{A}_1 = \{\text{winter}, \text{spring}, \text{summer}, \text{autumn}\}, \quad \mathcal{A}_2 = \{\text{ski}, \text{beach}, \text{culture}\}, \quad \mathcal{A}_3 = \{\text{solo}, \text{couple}, \text{family}\}.$$

Contradiction matrices (symmetric, 0 ***on diagonal).***

$c_1 =$

	winter	spring	summer	autumn
winter	0	0.3	0.8	0.5
spring	0.3	0	0.3	0.5
summer	0.8	0.3	0	0.3
autumn	0.5	0.5	0.3	0

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$c_2 =$

	ski	beach	culture
ski	0	0.9	0.4
beach	0.9	0	0.4
culture	0.4	0.4	0

,

$c_3 =$

	solo	couple	family
solo	0	0.2	0.6
couple	0.2	0	0.3
family	0.6	0.3	0

.

Aggregate $\Delta(\gamma,\delta) := \max\{c_1(\gamma_1,\delta_1), c_2(\gamma_2,\delta_2), c_3(\gamma_3,\delta_3)\}$.

HyperSoft mapping (nonempty images).

$$G(\text{winter},\text{ski},\text{family}) = \{\text{pkg}_1,\text{pkg}_2\},\quad G(\text{summer},\text{beach},\text{couple}) = \{\text{pkg}_3,\text{pkg}_4\},$$
$$G(\text{spring},\text{culture},\text{solo}) = \{\text{pkg}_5\},\quad G(\text{autumn},\text{culture},\text{family}) = \{\text{pkg}_6,\text{pkg}_7\},$$
$$G(\text{summer},\text{culture},\text{family}) = \{\text{pkg}_8\},\quad G(\text{winter},\text{beach},\text{solo}) = \{\text{pkg}_9\}.$$

CHS filter (fixed reference). *Choose the fixed reference* $r = (\text{summer},\text{beach},\text{family})$ *and threshold* $\tau = 0.5$. *Evaluate* $\Delta(\cdot, r)$*:*

γ	$\Delta(\gamma, r)$
(winter, ski, family)	$\max(0.8, 0.9, 0) = 0.9\ (> \tau)$
(summer, beach, couple)	$\max(0, 0, 0.3) = 0.3\ (\le \tau)$
(spring, culture, solo)	$\max(0.3, 0.4, 0.6) = 0.6\ (> \tau)$
(autumn, culture, family)	$\max(0.3, 0.4, 0) = 0.4\ (\le \tau)$
(summer, culture, family)	$\max(0, 0.4, 0) = 0.4\ (\le \tau)$
(winter, beach, solo)	$\max(0.8, 0, 0.6) = 0.8\ (> \tau)$

Thus the accepted tuples are the 2nd, 4th, and 5th. The CHS envelope at r *is*

$$G^{(\tau)}_{\rho\equiv r}(r) = \{\text{pkg}_3,\text{pkg}_4\} \cup \{\text{pkg}_6,\text{pkg}_7\} \cup \{\text{pkg}_8\} = \{\text{pkg}_3,\text{pkg}_4,\text{pkg}_6,\text{pkg}_7,\text{pkg}_8\},$$

with cardinality 5. *If we tighten to* $\tau' = 0.3$, *only the 2nd tuple remains, so* $G^{(\tau')}_{\rho\equiv r}(r) = \{\text{pkg}_3,\text{pkg}_4\}$, *demonstrating the control afforded by the contradiction threshold.*

Proposition 1 (Basic properties)**.** *For fixed* $(U, \{\mathcal{A}_i, c_i\}, G, \rho)$ *the family* $\{G^{(\tau)}_\rho\}_{\tau\in[0,1]}$ *is monotone in* τ*: if* $0 \le \tau_1 \le \tau_2 \le 1$ *then* $G^{(\tau_1)}_\rho(\gamma) \subseteq G^{(\tau_2)}_\rho(\gamma)$ *for all* $\gamma \in \mathcal{C}$*. Moreover* $G^{(1)}_\rho(\gamma) = \bigcup_{\delta\in\mathcal{C}} G(\delta)$ *for all* γ*.*

Proof. If $\tau_1 \le \tau_2$ then $\{\delta : \Delta(\delta, \rho(\gamma)) \le \tau_1\} \subseteq \{\delta : \Delta(\delta, \rho(\gamma)) \le \tau_2\}$, hence the unions are nested. For $\tau = 1$ the constraint is vacuous since $\Delta \in [0,1]$. □

Theorem 1 (CHS generalizes the HyperSoft Set)**.** *Assume each* c_i *is zero-separating and take the self-centered selector* $\rho(\gamma) = \gamma$*. Then for* $\tau = 0$ *one has*

$$G^{(0)}_\rho(\gamma) \;=\; G(\gamma) \qquad (\forall\, \gamma \in \mathcal{C}).$$

Proof. By definition, $G^{(0)}_\rho(\gamma) = \bigcup_{\delta:\,\Delta(\delta,\gamma)\le 0} G(\delta)$. Since $\Delta(\delta,\gamma) \ge 0$ always, the inequality forces $\Delta(\delta,\gamma) = 0$. Zero-separation gives $\delta = \gamma$, thus the union is $G(\gamma)$. □

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Definition 11 (Neighborhood-based ContraSoft on a single attribute). *Let V be a finite set with contradiction $c : V \times V \to [0,1]$, and let $F : V \to \mathcal{P}(U)$. For $\tau \in [0,1]$, the* neighborhood-based *ContraSoft transform is*

$$F^{(\tau)}(v) \;:=\; \bigcup_{w \in V:\ c(w,v) \le \tau} F(w) \qquad (v \in V).$$

Fixing $v^\star \in V$ yields the fixed-reference *variant $F^{(\tau;v^\star)}(v) := \bigcup_{w:\ c(w,v^\star) \le \tau} F(w)$.*

Example 9 (Neighborhood-based ContraSoft — Destination Selection by Climate Preference). ***Universe and attribute.*** *Let the universe of candidate destinations be*

$$U = \{u_1 = \text{Reykjavik},\ u_2 = \text{Zurich},\ u_3 = \text{Lisbon},\ u_4 = \text{Dubai},\ u_5 = \text{Helsinki},\ u_6 = \text{Vancouver}\}.$$

Consider a single attribute "preferred climate" with value set

$$V = \{\text{cold},\ \text{mild},\ \text{warm},\ \text{hot}\}.$$

Contradiction on V. *Let $c : V \times V \to [0,1]$ be symmetric with $c(v,v) = 0$:*

$c =$

	cold	mild	warm	hot
cold	0	0.3	0.6	0.9
mild	0.3	0	0.3	0.7
warm	0.6	0.3	0	0.3
hot	0.9	0.7	0.3	0

Baseline soft mapping. *Define $F : V \to \mathcal{P}(U)$ by*

$$F(\text{cold}) = \{u_1, u_5\}, \quad F(\text{mild}) = \{u_2, u_6\}, \quad F(\text{warm}) = \{u_3\}, \quad F(\text{hot}) = \{u_4\}.$$

Neighborhood-based ContraSoft transform. *For threshold $\tau \in [0,1]$ and center $v \in V$,*

$$F^{(\tau)}(v) = \bigcup_{w \in V:\ c(w,v) \le \tau} F(w).$$

Case 1 (moderate neighborhood). Let $v = \text{mild}$ and $\tau = 0.35$. Eligible neighbors satisfy $c(w, \text{mild}) \le 0.35$:

$$c(\text{cold},\text{mild}) = 0.3\,(\checkmark), \quad c(\text{mild},\text{mild}) = 0\,(\checkmark), \quad c(\text{warm},\text{mild}) = 0.3\,(\checkmark), \quad c(\text{hot},\text{mild}) = 0.7\,(\times).$$

Therefore

$$\begin{aligned} F^{(0.35)}(\text{mild}) &= F(\text{cold}) \cup F(\text{mild}) \cup F(\text{warm}) \\ &= \{u_1,u_5\} \cup \{u_2,u_6\} \cup \{u_3\} = \{u_1,u_2,u_3,u_5,u_6\}, \quad |\cdot| = 5. \end{aligned}$$

Case 2 (tight neighborhood). Let $\tau = 0.20$. Only $w = \text{mild}$ satisfies $c(w,\text{mild}) \le 0.20$, hence

$$F^{(0.20)}(\text{mild}) = F(\text{mild}) = \{u_2, u_6\}, \quad |\cdot| = 2.$$

These computations show how increasing τ expands the accepted neighborhood in V and unions the corresponding destination sets in U.

Theorem 2 (CHS generalizes ContraSoft). *Suppose $m = 1$, so $\mathcal{C} = \mathcal{A}_1 =: V$, and let $c_1 = c$. Identify $G : V \to \mathcal{P}(U)$ with F. Then:*

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(a) *With the self-centered selector* $\rho(v) = v$*, one has*

$$G_\rho^{(\tau)}(v) \;=\; \bigcup_{w:\, c(w,v)\le\tau} G(w) \;=\; F^{(\tau)}(v), \qquad \forall v \in V.$$

(b) *With the fixed-reference selector* $\rho(v) \equiv v^\star$*, one has*

$$G_\rho^{(\tau)}(v) \;=\; \bigcup_{w:\, c(w,v^\star)\le\tau} G(w) \;=\; F^{(\tau;v^\star)}(v), \qquad \forall v \in V.$$

Hence, for $m = 1$ *the CHS construction recovers both standard ContraSoft variants.*

Proof. When $m = 1$, $\Delta(w, \rho(v)) = c(w, \rho(v))$. Substituting $\rho(v) = v$ gives (a); substituting $\rho(v) \equiv v^\star$ gives (b). The set-theoretic unions agree by definition in both cases. □

2.2. Contra-SuperHyperSoft Set

Contra-SuperHyperSoft Set extends to set-valued coordinates, using lifted subset contradictions and aggregate radius; selector-threshold filtering unions nearby SuperHyperSoft slices effectively.

Definition 12 (Base and lifted contradictions). *Let* $A_1, \ldots, A_m$ *be nonempty finite sets of attribute values and let* $c_i : A_i \times A_i \to [0, 1]$ *be* contradiction functions *(symmetric and reflexive:* $c_i(a, a) = 0 = c_i(a, a)$*,* $c_i(a, b) = c_i(b, a)$*). Assume the* zero-separation *property* $c_i(a, b) = 0 \Rightarrow a = b$*. For subsets* $S, T \subseteq A_i$ *define the* lifted *contradiction*

$$\widehat{c}_i(S,T) \;:=\; \begin{cases} \max\Big\{ \max_{a\in S}\min_{b\in T} c_i(a,b),\ \max_{b\in T}\min_{a\in S} c_i(a,b) \Big\}, & S,T \neq \varnothing,\\ 0, & S = T = \varnothing,\\ 1, & \text{otherwise.} \end{cases}$$

(With finiteness, the max / min *are attained.)*

Lemma 1 (Symmetry, reflexivity, and zero-separation on subsets). *For each* i *and* $S, T \subseteq A_i$*:*

(a) $\widehat{c}_i(S, T) = \widehat{c}_i(T, S)$ *and* $\widehat{c}_i(S, S) = 0$.
(b) *If* c_i *is zero-separating, then* $\widehat{c}_i(S, T) = 0$ *implies* $S = T$.

Proof. (a) Symmetry follows by exchanging the two max terms; reflexivity is immediate. (b) If $\widehat{c}_i(S, T) = 0$ with $S, T \neq \varnothing$, then $\max_{a\in S} \min_{b\in T} c_i(a, b) = 0$ and $\max_{b\in T} \min_{a\in S} c_i(a, b) = 0$. Thus, for each $a \in S$ there is $b \in T$ with $c_i(a, b) = 0$, hence $a = b$ by zero-separation, so $S \subseteq T$. The second equality gives $T \subseteq S$. The cases with empties are by definition. □

Definition 13 (Product parameter space and aggregate contradiction). *Let* $\mathcal{C} := \mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_m)$ *and write* $\alpha = (\alpha_1, \ldots, \alpha_m)$*,* $\beta = (\beta_1, \ldots, \beta_m) \in \mathcal{C}$*. Define the tuple-level contradiction by*

$$\Delta(\alpha, \beta) \;:=\; \max_{1\le i\le m} \;\widehat{c}_i(\alpha_i, \beta_i) \;\in [0, 1].$$

Then $\Delta(\alpha, \beta) = \Delta(\beta, \alpha)$ *and* $\Delta(\alpha, \alpha) = 0$*; if each* c_i *is zero-separating, then by Lemma 1 we have* $\Delta(\alpha, \beta) = 0 \iff \alpha = \beta$*.*

Definition 14 (Reference selector). *A* reference selector *is any map* $\rho : \mathcal{C} \to \mathcal{C}$*. Two common choices are the* self-centered *selector* $\rho(\alpha) = \alpha$ *and a* fixed-reference *selector* $\rho(\alpha) \equiv r$ *for a fixed* $r \in \mathcal{C}$*.*

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Definition 15 (Contra-SuperHyperSoft Set (CSHS))**.** *Let U be a finite universe and let $F : \mathcal{C} \to \mathcal{P}(U)$ be a SuperHyperSoft mapping. Fix contradiction kernels $\{c_i\}$, their lifts $\{\widehat{c}_i\}$, an aggregate Δ, a selector ρ, and a threshold $\tau \in [0,1]$. The associated* Contra-SuperHyperSoft Set *is the tuple*

$$\mathsf{CSHS} := \big(U,\ \{A_i, c_i\}_{i=1}^{m},\ F,\ \rho,\ \tau\big),$$

together with the filtered *mapping*

$$F_\rho^{(\tau)} : \mathcal{C} \longrightarrow \mathcal{P}(U), \qquad F_\rho^{(\tau)}(\alpha) := \bigcup_{\beta \in \mathcal{C}\,:\,\Delta(\beta,\rho(\alpha)) \le \tau} F(\beta).$$

Example 10 (CSHS in E-commerce Fraud Review (self-centered selector))**.** ***Setup.*** *Let the universe of orders be $U = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8\}$. Take two attribute domains:*

$$A_1 = \{\text{card}, \text{crypto}\}, \qquad A_2 = \{\text{verified}, \text{partial}, \text{missing}\}.$$

Base contradictions $c_1, c_2 : [\cdot] \to [0,1]$ (symmetric, 0 on the diagonal):

$c_1 =$

	card	crypto
card	0	0.8
crypto	0.8	0

, $c_2 =$

	verified	partial	missing
verified	0	0.3	0.9
partial	0.3	0	0.6
missing	0.9	0.6	0

.

Lift $\widehat{c}_i$ to subsets by Definition (lifted contradiction) and aggregate

$$\Delta\big((S_1, S_2), (T_1, T_2)\big) := \max\{\widehat{c}_1(S_1, T_1), \widehat{c}_2(S_2, T_2)\}.$$

The SuperHyperSoft mapping $F : \mathcal{P}(A_1) \times \mathcal{P}(A_2) \to \mathcal{P}(U)$ is specified by

$$\begin{aligned} &F(\{\text{crypto}\}, \{\text{missing}\}) = \{o_1, o_2\}, \quad F(\{\text{crypto}\}, \{\text{partial}\}) = \{o_3\}, \\ &F(\{\text{card}\}, \{\text{missing}\}) = \{o_4\}, \quad F(\{\text{card}\}, \{\text{partial}\}) = \{o_5, o_6\}, \\ &F(\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\}) = \{o_7\}, \quad \textit{all other pairs map to } \varnothing. \end{aligned}$$

Choose the self-centered *selector $\rho(\alpha) = \alpha$ and threshold $\tau = 0.6$.*

Filtering at $\alpha_0 = (\{\text{crypto}\}, \{\text{missing}\})$.

To avoid overfull lines, we list the computations in an aligned display:

$$\begin{aligned} \Delta\big((\{\text{crypto}\}, \{\text{missing}\}),\, \alpha_0\big) &= \max(0,\, 0) = 0 \ \le\ \tau, \\ \Delta\big((\{\text{crypto}\}, \{\text{partial}\}),\, \alpha_0\big) &= \max\Big(0,\, c_2(\text{partial}, \text{missing})\Big) = \max(0,\, 0.6) = 0.6 \ \le\ \tau, \\ \Delta\big((\{\text{card}\}, \{\text{missing}\}),\, \alpha_0\big) &= \max(0.8,\, 0) = 0.8 \ >\ \tau, \\ \Delta\big((\{\text{card}\}, \{\text{partial}\}),\, \alpha_0\big) &= \max(0.8,\, 0.6) = 0.8 \ >\ \tau, \\ \Delta\big((\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\}),\, \alpha_0\big) &= \max\Big(\widehat{c}_1(\{\text{card}, \text{crypto}\}, \{\text{crypto}\}),\ \widehat{c}_2(\{\text{partial}, \text{missing}\}, \{\text{missing}\})\Big) \\ &= \max(0.8,\, 0.6) = 0.8 \ >\ \tau. \end{aligned}$$

Hence

$$F_\rho^{(\tau)}(\alpha_0) = \{o_1, o_2\} \cup \{o_3\} = \{o_1, o_2, o_3\}, \quad |F_\rho^{(\tau)}(\alpha_0)| = 3.$$

Filtering at $\alpha_1 = (\{\text{card}\}, \{\text{partial}\})$.

β	$\Delta(\beta, \alpha_1)$
$(\{\text{card}\}, \{\text{partial}\})$	$0 \le \tau$
$(\{\text{card}\}, \{\text{missing}\})$	$\max(0, 0.6) = 0.6 \le \tau$
$(\{\text{crypto}\}, \{\text{partial}\})$	$\max(0.8, 0) = 0.8 > \tau$
$(\{\text{crypto}\}, \{\text{missing}\})$	$\max(0.8, 0.6) = 0.8 > \tau$
$(\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\})$	$\max(0.8, 0.6) = 0.8 > \tau$

Therefore

$$F_\rho^{(\tau)}(\alpha_1) = \{o_5, o_6\} \cup \{o_4\} = \{o_4, o_5, o_6\}, \quad |F_\rho^{(\tau)}(\alpha_1)| = 3.$$

This illustrates how the CSHS envelope aggregates nearby subset-parameters under the contradiction metric.

Example 11 (CSHS in Cloud Deployment Recommendation (self-centered selector))**.** ***Setup.*** *Let the universe of candidate nodes be* $U = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7\}$. *Attributes:*

$$A_1 = \{\text{us-east}, \text{us-west}, \text{eu}\}, \qquad A_2 = \{\text{cpu}, \text{gpu}, \text{memory}\}.$$

*Base contradictions (*0 *on the diagonal, symmetric):*

$$c_1 = \begin{array}{c|ccc} & \text{us-east} & \text{us-west} & \text{eu} \\ \hline \text{us-east} & 0 & 0.4 & 0.7 \\ \text{us-west} & 0.4 & 0 & 0.8 \\ \text{eu} & 0.7 & 0.8 & 0 \end{array}, \qquad c_2 = \begin{array}{c|ccc} & \text{cpu} & \text{gpu} & \text{memory} \\ \hline \text{cpu} & 0 & 0.5 & 0.3 \\ \text{gpu} & 0.5 & 0 & 0.7 \\ \text{memory} & 0.3 & 0.7 & 0 \end{array}.$$

Lift to subsets by $\widehat{c}_i$ *and aggregate by* $\Delta\big((S_1, S_2), (T_1, T_2)\big) = \max\{\widehat{c}_1(S_1, T_1), \widehat{c}_2(S_2, T_2)\}$.

SuperHyperSoft mapping F (nonempty images shown):

$$F(\{\text{us-east}\}, \{\text{cpu}\}) = \{n_1, n_2\}, \quad F(\{\text{us-east}\}, \{\text{gpu}\}) = \{n_3\},$$
$$F(\{\text{us-west}\}, \{\text{cpu}\}) = \{n_4\}, \quad F(\{\text{eu}\}, \{\text{cpu}\}) = \{n_5\},$$
$$F(\{\text{us-east}, \text{us-west}\}, \{\text{cpu}\}) = \{n_6\},$$
$$F(\{\text{us-east}\}, \{\text{cpu}, \text{gpu}\}) = \{n_7\}.$$

Choose the self-centered *selector* $\rho(\alpha) = \alpha$ *and threshold* $\tau = 0.5$.

Filtering at $\alpha^\star = (\{\text{us-east}\}, \{\text{cpu}\})$. *For each* β *with* $F(\beta) \neq \varnothing$, *compute* $\Delta(\beta, \alpha^\star)$*:*

β	$\Delta(\beta, \alpha^\star)$
$(\{\text{us-east}\}, \{\text{cpu}\})$	$\max(0, 0) = 0 \le \tau$
$(\{\text{us-east}\}, \{\text{gpu}\})$	$\max(0,\ c_2(\text{gpu}, \text{cpu}) = 0.5) = 0.5 \le \tau$
$(\{\text{us-west}\}, \{\text{cpu}\})$	$\max(c_1(\text{us-west}, \text{us-east}) = 0.4,\ 0) = 0.4 \le \tau$
$(\{\text{eu}\}, \{\text{cpu}\})$	$\max(0.7, 0) = 0.7 > \tau$
$(\{\text{us-east}, \text{us-west}\}, \{\text{cpu}\})$	$\max(\widehat{c}_1(\{\text{us-east}, \text{us-west}\}, \{\text{us-east}\}) = 0.4,\ 0) = 0.4 \le \tau$
$(\{\text{us-east}\}, \{\text{cpu}, \text{gpu}\})$	$\max(0,\ \widehat{c}_2(\{\text{cpu}, \text{gpu}\}, \{\text{cpu}\}) = 0.5) = 0.5 \le \tau$

Hence

$$F_\rho^{(\tau)}(\alpha^\star) = \{n_1, n_2\} \cup \{n_3\} \cup \{n_4\} \cup \{n_6\} \cup \{n_7\} = \{n_1, n_2, n_3, n_4, n_6, n_7\},$$

with $|F_\rho^{(\tau)}(\alpha^\star)| = 6$. *Nodes requiring a European region are excluded by the contradiction bound.*

Proposition 2 (Monotonicity in the threshold)**.** *If* $0 \le \tau_1 \le \tau_2 \le 1$, *then* $F_\rho^{(\tau_1)}(\alpha) \subseteq F_\rho^{(\tau_2)}(\alpha)$ *for all* $\alpha \in \mathcal{C}$. *Moreover,* $F_\rho^{(1)}(\alpha) = \bigcup_{\beta \in \mathcal{C}} F(\beta)$ *for all* α.

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Proof. The index sets $\{\beta : \Delta(\beta, \rho(\alpha)) \le \tau\}$ are nested as τ grows, hence so are the unions. For $\tau = 1$ the constraint is vacuous since $\Delta \in [0,1]$. □

Theorem 3 (CSHS generalizes SuperHyperSoft). *Assume zero-separation for each c_i and take the self-centered selector $\rho(\alpha) = \alpha$. Then for $\tau = 0$ we have*

$$F_\rho^{(0)}(\alpha) = F(\alpha) \qquad (\forall\, \alpha \in \mathcal{C}).$$

Proof. By definition, $F_\rho^{(0)}(\alpha) = \bigcup_{\beta:\, \Delta(\beta,\alpha)\le 0} F(\beta)$. Since $\Delta \ge 0$, the inequality forces $\Delta(\beta, \alpha) = 0$. By Definition 13 and Lemma 1, this occurs iff $\beta = \alpha$. Hence the union collapses to $F(\alpha)$. □

Definition 16 (Singleton embedding of HyperSoft into SuperHyperSoft). *Let $\iota : A_1 \times \cdots \times A_m \longrightarrow \mathcal{C}$ be*

$$\iota(\gamma_1, \ldots, \gamma_m) := (\{\gamma_1\}, \ldots, \{\gamma_m\}).$$

We say $F : \mathcal{C} \to \mathcal{P}(U)$ is singleton-supported *for $G : A_1 \times \cdots \times A_m \to \mathcal{P}(U)$ if $F(\iota(\gamma)) = G(\gamma)$ for all γ and $F(\alpha) = \varnothing$ whenever some α_i is not a singleton.*

Lemma 2 (Compatibility of contradictions on singletons). *For any $a, b \in A_i$ we have $\widehat{c}_i(\{a\}, \{b\}) = c_i(a, b)$. Consequently, for $\gamma, \delta \in A_1 \times \cdots \times A_m$,*

$$\Delta\big(\iota(\gamma), \iota(\delta)\big) = \max_{1\le i\le m} c_i(\gamma_i, \delta_i).$$

Proof. From Definition 12 with singletons, $\max\{\min c_i(a,b), \min c_i(b,a)\} = c_i(a,b)$ by symmetry, and the product case follows. □

Theorem 4 (CSHS generalizes Contra-HyperSoft). *Let $G : A_1 \times \cdots \times A_m \to \mathcal{P}(U)$ be a HyperSoft set and F be a singleton-supported extension (Definition 16). Let ρ_H be a selector on $A_1 \times \cdots \times A_m$ and define ρ_S on $\mathcal{C}$ by $\rho_S(\iota(\gamma)) := \iota(\rho_H(\gamma))$, with arbitrary values elsewhere. Then for every γ and $\tau \in [0,1]$,*

$$F_{\rho_S}^{(\tau)}(\iota(\gamma)) \;=\; \bigcup_{\delta:\, \max_i c_i(\delta_i, \rho_H(\gamma)_i) \le \tau} G(\delta) \;=:\; G_{\rho_H}^{(\tau)}(\gamma),$$

i.e. the CSHS filtered mapping restricted to singleton parameters coincides with the Contra-HyperSoft mapping for G built from $\{c_i\}$ and the same threshold.

Proof. By Definition 15,

$$F_{\rho_S}^{(\tau)}(\iota(\gamma)) = \bigcup_{\beta:\, \Delta(\beta, \rho_S(\iota(\gamma)))\le\tau} F(\beta).$$

Since F is singleton-supported, only β of the form $\iota(\delta)$ contribute. By Lemma 2 and the definition of ρ_S, $\Delta(\iota(\delta), \rho_S(\iota(\gamma))) = \max_i c_i(\delta_i, \rho_H(\gamma)_i)$. Substitute and use $F(\iota(\delta)) = G(\delta)$ to obtain the stated equality. □

Theorem 5 (CSHS generalizes ContraSoft). *Let $m = 1$ with base set $A_1 =: V$ and contradiction $c := c_1$. Let $F : \mathcal{P}(V) \to \mathcal{P}(U)$ be singleton-supported for some $F_0 : V \to \mathcal{P}(U)$ via $F(\{v\}) = F_0(v)$ and $F(S) = \varnothing$ if $|S| \ne 1$. Then, with the self-centered selector, for all $v \in V$ and $\tau \in [0,1]$,*

$$F_{\mathrm{id}}^{(\tau)}(\{v\}) = \bigcup_{w:\, c(w,v)\le\tau} F_0(w),$$

which is exactly the neighborhood-based ContraSoft transform on (V, c). With a fixed reference $v^\star \in V$, the same construction yields the fixed-reference ContraSoft variant.

Proof. Specialize Theorem 4 to $m = 1$ with $G \equiv F_0$ and observe that $\Delta(\{w\}, \{v\}) = \hat{c}_1(\{w\}, \{v\}) = c(w, v)$. Singleton support reduces the union to singletons, yielding the stated form. □

3. Conclusion

In this paper, we examined the concepts of the ContraSoft Set, the Contra-HyperSoft Set, and the Contra-SuperHyperSoft Set.

Future work will focus on extending these ideas by integrating them with richer frameworks, including Neutrosophic Sets[12,13], Plithogenic Sets[14–16], Rough Sets[17,18], and TreeSoft Sets[19,20]. Moreover, we anticipate the development of generalized structures that apply these contra-based approaches to Graphs[21,22], HyperGraphs[23–25], SuperHyperGraphs[26–28], and broader HyperStructures[29–31]. Such investigations are expected to open new directions in handling contradiction-aware representations across diverse domains.

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Use of Artificial Intelligence: I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

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