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Kinematic Anti-Gravity: A Theoretical Mechanism for Normal Force Generation via Prescribed Constant Speed on Curved Surfaces

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Abstract: Overcoming pervasive forces like gravity typically requires propulsion or medium interaction [1–4]. This theoretical study explores an alternative mechanism where apparent anti-gravity effects emerge from the interplay between prescribed kinematics and curved geometry. We analyze a point mass's dynamics normal to a smooth reference surface, subject to a normal force (e.g., gravity) and, crucially, a **kinematically prescribed constant tangential projection speed** V. Applying Newtonian mechanics within an inertial frame, constrained by this constant V condition, rigorous kinematic differentiation and projection yield the approximate normal acceleration $D'' \approx V^2 k_n - F/M$, where k_n is the normal curvature. This reveals an inherent, geometry-induced inertial term $V^2 k_n$ acting outwards. When $V^2 k_n > F/M$, the model predicts net outward acceleration ($D'' > 0$), signifying a theoretical **"kinematic anti-gravity" effect**. This effect is generally anisotropic but isotropic for spheres ($k_n = 1/R$). The exact spherical case under gravity recovers circular orbit conditions. A **rotating ring** (**$V = \omega r =$ const)** on a sphere illustrates the concept: lift-off against gravity g is predicted if $V^2/R > g$. While predicated on the **acknowledged idealization** of prescribed constant velocity, this work presents a mathematically consistent framework demonstrating how specific kinematic constraints over curved geometries can, in principle, generate inertial forces capable of opposing gravity within the model's defined rules.

Keywords: classical mechanics; differential geometry; kinematic anti-gravity

1. Introduction

Motion relative to curved surfaces is fundamental across scales, from planetary orbits governed by spacetime geometry [1,7] to terrestrial vehicle dynamics [4] and robotic navigation [3]. While interactions involving complex forces or constraints on surfaces are well-studied, understanding motion near surfaces requires disentangling external forces from inertial phenomena arising purely from movement over the curved background itself. A key challenge in physics and engineering remains the control or counteraction of pervasive forces like gravity, typically achieved via direct energy expenditure through propulsion or medium interaction [1–4]. The concept of "anti-gravity" – locally nullifying gravity without conventional means – remains speculative [5]. This paper explores a different, theoretical avenue: can gravity-opposing forces emerge intrinsically from the interaction of inertia, geometry, and specifically prescribed motion?

We investigate this question by constructing a focused theoretical model designed to isolate the effects of imposed kinematics. We consider a point mass M at position $\vec{P}(t)$ moving near a smooth reference surface σ (a geometric tool, not a physical constraint). Its position is related to its unique orthogonal projection $\vec{\sigma}(t)$ onto σ and the outward unit normal $\vec{N}(t)$ at $\vec{\sigma}(t)$ by $\vec{P}(t) = \vec{\sigma}(t) + D(t)\vec{N}(t)$, where $D(t)$ is the normal distance. The mass is subject only to an external force directed normally towards the surface, $\vec{F}(t) = -F(t)\vec{N}(t)$. The defining feature, and **central idealization**, of our model is the imposition of a kinematic condition: the projection point $\vec{\sigma}(t)$ is **mandated to move tangentially along σ with a strictly constant speed** $||\frac{d\vec{\sigma}}{dt}|| = ||\vec{V}_\sigma(t)|| = V =$ constant. This

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constant V condition deviates from standard Newtonian dynamics. We deliberately impose this constraint – assuming unspecified external mechanisms hypothetically maintain this speed – **in order to mathematically isolate and analyze the specific dynamic consequences attributable purely to constant-speed tangential motion over curved geometry**. Our goal is not to model fully realistic motion, but to rigorously understand the outcome of this specific 'what if' kinematic scenario using detailed kinematic analysis and Newton's second law within an inertial frame.

Applying this approach, we derive the approximate governing equation for the normal acceleration D'': $D'' \approx V^2 k_n - F(t)/M$. This reveals a **"kinematic lift acceleration"** term $+V^2 k_n$, emerging as an inertial consequence of the **imposed constant V kinematic constraint interacting with the surface's normal curvature** k_n. When $V^2 k_n > F(t)/M$, the model predicts $D'' > 0$, interpreted as a **theoretical "kinematic anti-gravity" effect**.

Our analysis shows this effect is anisotropic for general surfaces but isotropic for spheres. We validate the framework by deriving the exact equation for the spherical case under gravity, recovering circular orbit conditions. A **rotating ring on a sphere (**$V = \omega r =$ **const)** serves as a primary illustration, demonstrating how the V^2/R kinematic lift can theoretically overcome gravity g. The following sections detail the **step-by-step kinematic derivation under constant V** (Part 2), the derivation and analysis of the governing D'' equation (Part 3), the spherical case study (Part 4), and a concluding discussion (Part 5). The entire analysis is performed consistently within a single inertial frame.

2. (Part 2: Detailed Kinematic Derivation Under Prescribed Constant V)

We begin with the fundamental relationship between the position of the mass $\vec{P}(t)$, its projection $\vec{\sigma}(t)$, the normal distance $D(t)$, and the outward unit normal $\vec{N}(t)$ at $\vec{\sigma}(t)$:

$$\vec{P}(t) = \vec{\sigma}(t) + D(t)\vec{N}(t) \tag{1}$$

Our core kinematic assumption is that the projection $\vec{\sigma}(t)$ moves tangentially on the surface σ with a constant speed V. Let $\vec{V}_\sigma(t) = \frac{d\vec{\sigma}}{dt}$ be the velocity of the projection point. Then $||\vec{V}_\sigma(t)|| = V =$ constant. We define the unit tangent vector in the direction of projection motion as $\vec{T}(t) = \vec{V}_\sigma(t)/V$ (assuming $V \neq 0$).

2.1. *Derivation of Velocity* $\vec{v}_P$

We differentiate Eq. (1) with respect to time t using the product rule for the $D\vec{N}$ term:

$$\vec{v}_P(t) = \frac{d}{dt}[\vec{P}(t)] = \frac{d}{dt}[\vec{\sigma}(t)] + \frac{d}{dt}[D(t)\vec{N}(t)] \tag{2}$$

$$= \vec{V}_\sigma(t) + \left[\frac{dD}{dt}\vec{N}(t) + D(t)\frac{d\vec{N}}{dt}\right] \tag{3}$$

$$= \vec{V}_\sigma(t) + D'(t)\vec{N}(t) + D(t)\frac{d\vec{N}}{dt} \tag{4}$$

where $D'(t) = dD/dt$.

To proceed, we need $d\vec{N}/dt$. The change in the normal vector $\vec{N}$ as its base point $\vec{\sigma}$ moves on the surface with velocity $\vec{V}_\sigma$ is described by the Shape Operator (or Weingarten map) S of the surface σ at $\vec{\sigma}$. The Shape Operator $S\{\vec{\sigma}\}$ maps tangent vectors at $\vec{\sigma}$ to tangent vectors at $\vec{\sigma}$. The fundamental relation is [6,7]:

$$\frac{d\vec{N}}{dt} = -S\{\vec{\sigma}\}(\vec{V}_\sigma(t)) \tag{5}$$

This equation states that the rate of change of the normal vector is tangential and related to the surface curvature encoded in S and the velocity $\vec{V}_\sigma$.

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Substituting Eq. (5) into Eq. (4):

$$\vec{v}_P(t) = \vec{V}_\sigma(t) + D'(t)\vec{N}(t) - D(t)S(\vec{V}_\sigma(t)) \tag{6}$$

This is the velocity vector of the mass M. It has a tangential component $\vec{V}_\sigma - DS(\vec{V}_\sigma)$ (relative to the surface σ at $\vec{\sigma}$) and a normal component $D'\vec{N}$.

2.2. *Derivation of Acceleration $\vec{a}_P$ (Constant V)*

We differentiate the velocity vector $\vec{v}_P$ (Eq. (6)) with respect to time t to find the acceleration $\vec{a}_P$:

$$\vec{a}_P(t) = \frac{d}{dt}[\vec{v}_P(t)] = \frac{d}{dt}[\vec{V}_\sigma(t)] + \frac{d}{dt}[D'(t)\vec{N}(t)] - \frac{d}{dt}[D(t)S(\vec{V}_\sigma(t))] \tag{7}$$

Let's evaluate each term separately, **explicitly using the condition V=constant**:

- **Term 1:** $\frac{d}{dt}[\vec{V}_\sigma(t)]$
  Since $\vec{V}_\sigma = V\vec{T}$ and V is constant:
  $$\frac{d}{dt}[\vec{V}_\sigma(t)] = \frac{d}{dt}[V\vec{T}(t)] = V\frac{d\vec{T}}{dt} \tag{8}$$
  This term is purely due to the change in the direction of the projection velocity.
- **Term 2:** $\frac{d}{dt}[D'(t)\vec{N}(t)]$
  Using the product rule:
  $$\frac{d}{dt}[D'(t)\vec{N}(t)] = D''(t)\vec{N}(t) + D'(t)\frac{d\vec{N}}{dt}$$
  Substituting $d\vec{N}/dt$ from Eq. (5):
  $$\frac{d}{dt}[D'(t)\vec{N}(t)] = D''(t)\vec{N}(t) - D'(t)S(\vec{V}_\sigma(t)) \tag{9}$$
- **Term 3:** $-\frac{d}{dt}[D(t)S(\vec{V}_\sigma(t))]$
  Using the product rule:
  $$-\frac{d}{dt}[D(t)S(\vec{V}_\sigma(t))] = -\left[D'(t)S(\vec{V}_\sigma(t)) + D(t)\frac{d}{dt}(S(\vec{V}_\sigma(t)))\right] \tag{10}$$
  The term $\frac{d}{dt}(S(\vec{V}_\sigma))$ involves the rate of change of the Shape Operator applied to the velocity, encompassing changes in curvature and path direction.

Now, substitute Eqs. (8), (9), and (10) back into Eq. (7):

$$\vec{a}_P(t) = \left(V\frac{d\vec{T}}{dt}\right) + \left(D''\vec{N} - D'S(\vec{V}_\sigma)\right) - \left(D'S(\vec{V}_\sigma) + D\frac{d}{dt}(S(\vec{V}_\sigma))\right)$$

Combine the terms involving $D'S(\vec{V}_\sigma)$:

$$\vec{a}_P(t) = V\frac{d\vec{T}}{dt} + D''(t)\vec{N}(t) - 2D'(t)S(\vec{V}_\sigma(t)) - D(t)\frac{d}{dt}[S(\vec{V}_\sigma(t))] \tag{11}$$

This is the full expression for the acceleration vector of mass M under the constant V condition.

2.3. *Derivation of the Normal Component of Acceleration a_N (Constant V)*

We need the component of $\vec{a}_P$ along the normal direction $\vec{N}$, denoted $a_N = \vec{a}_P \cdot \vec{N}$. We compute the dot product of each term in Eq. (11) with $\vec{N}$:

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- **Term 1:** $(V\frac{d\vec{T}}{dt}) \cdot \vec{N}$
Recall $V\frac{d\vec{T}}{dt} = \frac{d\vec{V}_\sigma}{dt}$. We need the normal component of the acceleration of the projection point σ as it moves at constant speed V. This acceleration is required to keep σ following its path on the curved surface σ. From differential geometry, the normal component of $d\vec{V}_\sigma/dt$ is related to the normal curvature k_n in the direction $\vec{T} = \vec{V}_\sigma/V$. The normal curvature k_n can be defined via the Shape Operator as $k_n = \vec{T} \cdot S(\vec{T})$ [6,7]. Let's relate $(\frac{d\vec{V}_\sigma}{dt}) \cdot \vec{N}$ to k_n. Start with the identity $\vec{V}_\sigma \cdot \vec{N} = 0$. Differentiate wrt time:

$$\frac{d}{dt}(\vec{V}_\sigma \cdot \vec{N}) = \left(\frac{d\vec{V}_\sigma}{dt}\right) \cdot \vec{N} + \vec{V}_\sigma \cdot \left(\frac{d\vec{N}}{dt}\right) = 0$$

So, $(\frac{d\vec{V}_\sigma}{dt}) \cdot \vec{N} = -\vec{V}_\sigma \cdot (\frac{d\vec{N}}{dt})$. Substitute $\frac{d\vec{N}}{dt} = -S(\vec{V}_\sigma)$ (Eq. (5)):

$$\left(\frac{d\vec{V}_\sigma}{dt}\right) \cdot \vec{N} = -\vec{V}_\sigma \cdot [-S(\vec{V}_\sigma)] = \vec{V}_\sigma \cdot S(\vec{V}_\sigma)$$

Now substitute $\vec{V}_\sigma = V\vec{T}$:

$$\left(\frac{d\vec{V}_\sigma}{dt}\right) \cdot \vec{N} = (V\vec{T}) \cdot S(V\vec{T}) = V^2[\vec{T} \cdot S(V\vec{T})]$$

Since S is linear, $S(V\vec{T}) = VS(\vec{T})$.

$$\left(\frac{d\vec{V}_\sigma}{dt}\right) \cdot \vec{N} = V^2[\vec{T} \cdot (VS(\vec{T}))] = V^3[\vec{T} \cdot S(\vec{T})]$$

Using the definition $k_n = \vec{T} \cdot S(\vec{T})$:

$$\left(\frac{d\vec{V}_\sigma}{dt}\right) \cdot \vec{N} = V^2 k_n$$

Sign Convention: We adopt the convention that k_n represents the non-negative magnitude of normal curvature (e.g., $k_n = 1/R$ for a sphere, $R > 0$). The physical acceleration $d\vec{V}_\sigma/dt$ required to follow a path curving "inwards" (towards the center of normal curvature) must have a component along $-\vec{N}$ (since $\vec{N}$ points outwards). Therefore, to align with this physical picture and the non-negative k_n convention, we must write:

$$\left(V\frac{d\vec{T}}{dt}\right) \cdot \vec{N} = -V^2 k_n \tag{12}$$

- **Term 2:** $(D''(t)\vec{N}(t)) \cdot \vec{N}$

$$D''(t)(\vec{N} \cdot \vec{N}) = D''(t) \times 1 = D''(t) \tag{13}$$

- **Term 3:** $(-2D'(t)S(\vec{V}_\sigma(t))) \cdot \vec{N}$
Since $S(\vec{V}_\sigma)$ is a tangent vector, it is orthogonal to $\vec{N}$. $S(\vec{V}_\sigma) \cdot \vec{N} = 0$. Therefore, the contribution is

$$0 \tag{14}$$

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- **Term 4:** $(-D(t)\frac{d}{dt}[S(\vec{V}_\sigma(t))]) \cdot \vec{N}$
  This term $\frac{d}{dt}[S(\vec{V}_\sigma)]$ represents complex effects due to the change in the curvature tensor and path direction along the motion. Its normal component depends on these geometric variations and is proportional to D. We denote this contribution as H.O.T.(D) (Higher Order Terms in D).

$$\text{Contribution} = \text{H.O.T.}(D) \tag{15}$$

Summing the contributions from Eqs. (12), (13), (14), and (15):

$$a_N = \vec{a}_P \cdot \vec{N} = -V^2 k_n + D''(t) + 0 + \text{H.O.T.}(D)$$

$$a_N = D''(t) - V^2 k_n + \text{H.O.T.}(D) \tag{16}$$

2.4. Approximate Kinematic Normal Acceleration (Constant V)

For scenarios where D is small relative to the radii of curvature, or where the surface geometry changes slowly along the path, the H.O.T.(D) term can be neglected. This yields the approximate kinematic normal acceleration under the constant V condition:

$$a_N \approx D''(t) - V^2 k_n \tag{17}$$

This completes the detailed kinematic derivation required for the subsequent dynamic analysis.

3. (Part 3: Detailed Dynamics Derivation and Analysis - Constant V Model)

Having established the kinematic expression for the normal component of acceleration a_N under the prescribed constant V condition, we now incorporate the dynamics dictated by the applied force using Newton's Second Law.

3.1. Newton's Second Law in the Normal Direction

The model assumes the only external force explicitly considered is the normal force $\vec{F}(t) = -F(t)\vec{N}(t)$, where $F(t)$ is the magnitude directed inwards (opposite to $\vec{N}$). Newton's Second Law in vector form is:

$$M\vec{a}_P(t) = \vec{F}(t) \tag{18}$$

$$M\vec{a}_P(t) = -F(t)\vec{N}(t) \tag{19}$$

To isolate the dynamics relevant to the normal motion $D(t)$, we project this vector equation onto the unit normal vector $\vec{N}(t)$. This is achieved by taking the dot product of both sides of Eq. (19) with $\vec{N}(t)$:

$$\vec{N}(t) \cdot [M\vec{a}_P(t)] = \vec{N}(t) \cdot [-F(t)\vec{N}(t)]$$

Since M is a scalar and $\vec{N}$ is a unit vector ($\vec{N} \cdot \vec{N} = 1$), this simplifies to:

$$M[\vec{a}_P(t) \cdot \vec{N}(t)] = -F(t)[\vec{N}(t) \cdot \vec{N}(t)]$$

$$Ma_N(t) = -F(t) \times 1$$

Thus, the dynamic requirement imposed by the applied force on the normal component of acceleration is:

$$a_N(t) = -\frac{F(t)}{M} \tag{20}$$

This equation states that the actual normal acceleration experienced by the mass must be exactly that provided by the applied normal force per unit mass.

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3.2. Deriving the Governing Equation for D'' (Constant V Model)

We now have two expressions for the normal component of acceleration a_N, both derived consistently within the inertial frame:

1. **Approximate Kinematic** a_N (from Part 2, Eq. (17)): Derived from the geometry and the prescribed constant V kinematics, neglecting higher-order terms in D:

$$a_N \approx D''(t) - V^2 k_n$$

2. **Dynamic** a_N (Eq. (20)): Required by Newton's Second Law applied to the assumed normal force:

$$a_N = -\frac{F(t)}{M}$$

Equating these two expressions for a_N yields the relationship governing the normal distance dynamics within the idealized constant V model:

$$D''(t) - V^2 k_n \approx -\frac{F(t)}{M} \tag{21}$$

Rearranging this equation to solve explicitly for the second time derivative of the normal distance, D'':

$$D''(t) \approx V^2 k_n - \frac{F(t)}{M} \tag{22}$$

This ordinary differential equation is the central result of our analysis for the constant V model. It provides an approximate description of how the normal distance D evolves over time under the specific interplay of the imposed kinematics and the applied normal force. The approximation stems from neglecting the H.O.T.(D) term in the kinematic derivation.

3.3. Detailed Analysis of the Governing Equation (22)

Equation (22), $D''(t) \approx V^2 k_n - F(t)/M$, describes the normal acceleration relative to the reference surface σ as determined by the model's assumptions. Let's analyze the terms:

- $D''(t)$: The second derivative of the normal distance D with respect to time. It represents the acceleration of the mass M perpendicularly away from (if $D'' > 0$) or towards (if $D'' < 0$) the reference surface σ, relative to the projection point σ.
- $+V^2 k_n$: The **Kinematic Lift Acceleration** term.
  - V^2: Proportional to the square of the **prescribed constant tangential speed** of the projection σ. Higher imposed speed leads to a larger effect.
  - k_n: The non-negative magnitude of the normal curvature of the reference surface σ in the instantaneous direction of the projection's velocity $\vec{V}_\sigma / V$. Sharper curvature (smaller radius of normal curvature, $1/k_n$) leads to a larger effect. If the surface is flat ($k_n = 0$), this term vanishes.
  - **Origin and Interpretation**: This term arises directly from the inertia of the mass resisting the change in direction mandated by the constant speed V motion along the curved path on σ (as shown by the $-V^2 k_n$ component in the kinematic a_N derivation, Eq. (12)). In the equation for the relative acceleration D'', it appears with a positive sign, signifying an **outward acceleration tendency** relative to the surface. It exists solely due to the combination of inertia, the imposed constant speed V, and the non-zero curvature k_n of the geometric reference.
- $-F(t)/M$: The **Applied Force Acceleration** term.

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- $F(t)$: The magnitude of the applied external normal force, directed inwards (towards σ). This force can vary with time, for instance, through dependence on position D (e.g., gravity $F(D)$).
- M: The mass of the point object.
- **Interpretation**: This term represents the direct inward acceleration caused by the explicitly included external force $\vec{F}$.

3.4. Balance of Effects and the Kinematic Anti-Gravity Condition

The net normal acceleration D'' is the result of the competition between the outward kinematic lift $V^2 k_n$ and the inward applied force acceleration $F(t)/M$.

- **Departure / Kinematic Anti-Gravity ($D'' > 0$)**: Condition: $V^2 k_n > F(t)/M$.
In this case, the outward inertial tendency generated by the prescribed constant V motion over the curved geometry dominates the inward pull of the applied force F. The model predicts the mass will accelerate *away* from the reference surface (in the $+\vec{N}$ direction). This is the mathematical condition defining the theoretical "kinematic anti-gravity" effect within this model.
- **Approach ($D'' < 0$)**: Condition: $V^2 k_n < F(t)/M$.
Here, the inward applied force is stronger than the outward kinematic lift effect. The mass accelerates *towards* the reference surface (in the $-\vec{N}$ direction).
- **Equilibrium ($D'' \approx 0$)**: Condition: $V^2 k_n \approx F(t)/M$.
The outward kinematic lift approximately balances the inward applied force. The net normal acceleration is near zero. If the initial normal velocity $D'(0)$ is also zero, this suggests the mass will tend to maintain a constant normal distance D from the surface, representing a state of dynamic equilibrium relative to the reference surface for the specific prescribed speed V.

3.5. Anisotropy vs. Isotropy

On a general curved surface σ, the normal curvature k_n typically varies depending on the direction $\vec{T} = \vec{V}_\sigma / V$ within the tangent plane (described by Euler's Theorem involving principal curvatures k_1, k_2). Since k_n directly influences the kinematic lift term $V^2 k_n$, the magnitude of this term, and therefore the resulting normal acceleration D'' and the condition for departure, will depend on the **direction** of the prescribed tangential motion $\vec{V}_\sigma$. This leads to **anisotropic** normal dynamics predicted by the model.

In contrast, for a sphere of radius R, the normal curvature $k_n = 1/R$ is the same in all directions. Consequently, the kinematic lift term V^2/R is independent of the direction of motion on the sphere's surface, leading to **isotropic** normal dynamics.

4. (Part 4: Detailed Case Study - Sphere under Gravity and the Rotating Ring)

We now apply the derived framework (based on prescribed constant V) to the specific case of a spherical reference surface under central gravity. This serves to validate the model against known physics (circular orbits) and to provide a clear illustration of the kinematic anti-gravity concept using the rotating ring thought experiment.

4.1. System Definition

- **Reference Geometry**: A sphere of radius R. The normal curvature is constant and isotropic: $k_n = 1/R$. The outward unit normal $\vec{N}$ coincides with the radial unit vector $\hat{r}$.
- **Applied Force**: Central gravity from a central mass M_e. The force on mass M at distance $r = R + D$ from the center is $\vec{F} = -(GM_e M/r^2)\hat{r}$. This matches the model's form $\vec{F} = -F(D)\vec{N}$ with force magnitude $F(D) = GM_e M/(R+D)^2$. The applied inward acceleration is $F(D)/M = GM_e/(R+D)^2$.

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- **Prescribed Kinematics**: The projection $\vec{\sigma}$ moves tangentially on the sphere surface (radius R) with **prescribed constant speed** $||\vec{V}_\sigma|| = V$. This implies a constant angular velocity $\theta' = d\theta/dt = V/R$ relative to the sphere's center.

4.2. Exact Governing Equation for D'' (Spherical Case, Constant V Model)

Due to the high symmetry, we can derive an exact equation without neglecting H.O.T.(D). We use Newtonian mechanics in polar coordinates (r, θ) within an inertial frame, imposing the model's kinematic constraint. The radial coordinate is $r(t) = R + D(t)$, so $\dot{r} = D'$ and $\ddot{r} = D''$. The radial component of acceleration is $a_r = \ddot{r} - r(\dot{\theta})^2$. Newton's Second Law in the radial direction states $Ma_r = F_r$, where $F_r = -F(D) = -GM_eM/(R+D)^2$.

$$M[\ddot{r} - r(\dot{\theta})^2] = -\frac{GM_eM}{r^2}$$

Substitute $r = R + D$ and $\ddot{r} = D''$:

$$M[D'' - (R+D)(\dot{\theta})^2] = -\frac{GM_eM}{(R+D)^2}$$

Now, crucially, substitute the **kinematically prescribed constant angular velocity** from our model, $\dot{\theta} = V/R$:

$$M\left[D'' - (R+D)\left(\frac{V}{R}\right)^2\right] = -\frac{GM_eM}{(R+D)^2}$$

Dividing by M and rearranging gives the exact differential equation for $D(t)$ under the constant V prescription on a sphere:

$$D''(t) = \frac{V^2(R+D)}{R^2} - \frac{GM_e}{(R+D)^2} \tag{23}$$

4.3. Detailed Analysis of the Exact Spherical Equation (23)

- **Structure**: The equation explicitly shows D'' as the sum of two terms:
  - $+\frac{V^2(R+D)}{R^2}$: The outward **kinematic lift acceleration** term, derived exactly for the sphere under the constant $\dot{\theta} = V/R$ condition.
  - $-\frac{GM_e}{(R+D)^2}$: The inward gravitational acceleration.
- **Consistency Check**: For $D \ll R$, $R + D \approx R$. Eq. (23) becomes $D'' \approx \frac{V^2R}{R^2} - \frac{GM_e}{R^2} = \frac{V^2}{R} - g$, where $g = GM_e/R^2$ is the surface gravity. This perfectly matches the general approximate formula Eq. (22) applied to the sphere ($k_n = 1/R$, $F/M = g$). This confirms the validity of the general approximation in this limit and the structural correctness of the exact result.
- **Equilibrium - Circular Orbit Condition ($D'' = 0$)**: Setting $D'' = 0$ requires the prescribed speed V to satisfy the balance:

$$\frac{V^2(R+D)}{R^2} = \frac{GM_e}{(R+D)^2}$$

  Solving for V yields the specific constant projection speed required on the radius R surface to maintain equilibrium (a circular orbit) at altitude D:

$$V_{\text{eq}}(D) = R\sqrt{\frac{GM_e}{(R+D)^3}} \tag{24}$$

  - **Physical Consistency**: We can verify this connects to standard physics. The actual orbital speed required for a circular orbit at radius $r = R + D$ is $v_{\text{orbit}} = \sqrt{GM_e/r} =$

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$\sqrt{GM_e/(R+D)}$. The angular velocity is $\dot{\theta}_{\text{orbit}} = v_{\text{orbit}}/r = \sqrt{GM_e/r^3} = \sqrt{GM_e/(R+D)^3}$. The corresponding projection speed onto radius R would be $V_{\text{proj}} = R\dot{\theta}_{\text{orbit}} = R\sqrt{GM_e/(R+D)^3}$. This exactly matches our $V_{\text{eq}}(D)$. Thus, the equilibrium condition derived from our constant V model precisely corresponds to the condition for a physically realistic circular orbit, validating the model in this specific (constant actual speed) regime.

- **Surface Equilibrium (**$D = 0$**)**: Setting $D = 0$ in Eq. (24) gives $V_{\text{eq}}(0) = R\sqrt{GM_e/R^3} = \sqrt{GM_e/R}$, which is the first cosmic velocity V_1.

4.4. Thought Experiment: The Rotating Ring - Demonstrating Kinematic Anti-Gravity

This experiment provides a direct illustration where the constant V condition is met by definition.

- **Setup**: A thin ring (radius r, mass M_{ring}) rotates flat on the spherical surface (radius R, surface gravity $g = GM_e/R^2$) with **prescribed constant angular velocity** ω. Gravity $dF = g\,dM$ acts normally inwards on each element dM.
- **Constant V Fulfilled**: Each element dM has a tangential speed relative to the underlying sphere $V = \omega r$, which is **constant** due to the prescribed constant ω.
- **Initial Normal Acceleration (**$D = 0$**)**: We apply the exact spherical equation (Eq. (23)) at the initial instant $D = 0$ for each element dM. The gravitational acceleration is g.

$$D''(0) = \frac{V^2(R+0)}{R^2} - \frac{GM_e}{(R+0)^2}$$

$$D''(0) = \frac{V^2}{R} - g$$

Substitute the ring's speed $V = \omega r$:

$$D''(0) = \frac{(\omega r)^2}{R} - g \tag{25}$$

- **Interpretation - Kinematic Lift vs. Gravity**:
  - The term $\frac{(\omega r)^2}{R} = \frac{V^2}{R}$ is the outward **kinematic lift acceleration**.
  - $-g$ is the inward gravitational acceleration.
  - The initial normal acceleration $D''(0)$ of every ring element is the direct sum of these two competing effects.
- **Kinematic Anti-Gravity Lift-off Condition**:
  - If $\frac{(\omega r)^2}{R} > g$, then $D''(0) > 0$.
  - The outward kinematic lift generated by the prescribed constant rotation V exceeds the inward pull of gravity g. Since this applies uniformly to all parts of the ring, the entire ring experiences an initial upward acceleration. The model predicts **lift-off against gravity**.
  - This occurs if the ring's tangential speed $V = \omega r$ exceeds the first cosmic velocity $\sqrt{gR}$ (since $g = GM_e/R^2$, $\sqrt{gR} = \sqrt{GM_e/R}$).
  - This result vividly demonstrates the theoretical "kinematic anti-gravity" effect predicted by the equation $D'' \approx V^2 k_n - F/M$ in a scenario perfectly matching the model's constant V assumption.

5. (Part 5: Synthesis, Discussion, and Conclusion)

5.1. Synthesis: The Kinematic Anti-Gravity Condition

Analyzing dynamics normal to a surface σ under the **idealized condition of prescribed constant tangential projection speed** V and a normal force F, we derived $D''(t) \approx V^2 k_n - F(t)/M$. Key findings within this framework:

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1. An outward **kinematic lift acceleration** $+V^2k_n$ emerges from the imposed constant V kinematics interacting with curvature k_n.
2. A theoretical **"kinematic anti-gravity" effect** (departure, $D'' > 0$) occurs if $V^2k_n > F(t)/M$.
3. The effect is generally anisotropic but isotropic for spheres.
4. Consistency with **circular orbit** physics is established ($V = V_{\mathrm{eq}}(D)$ yields $D'' = 0$).
5. The **rotating ring** ($V = \omega r = \mathrm{const}$) provides a direct illustration: lift-off against gravity g is predicted if the kinematic lift V^2/R exceeds g.

5.2. Discussion

The "kinematic anti-gravity" described is an **inertial effect**, a consequence of the **imposed constant V kinematic constraint** interacting with geometry, rigorously derived within an inertial frame using Newton's laws. It is *not* a modification of gravity. The term V^2k_n quantifies the inertial reaction to this imposed motion.

The **central limitation remains the constant V idealization**. This is non-physical for unconstrained motion determined solely by forces like gravity. Unspecified external mechanisms are implicitly assumed to maintain V. This model explores the consequences of this 'what if' scenario, not its physical achievability. Therefore, dynamics requiring naturally varying speeds (like **elliptical orbits**) are **outside the scope** of the equations derived here.

Despite this, the model offers value by isolating the V^2k_n effect. The mathematical derivation is sound *given the premise*, consistency with circular orbits is shown, and the ring example provides a clear conceptual illustration. It highlights how specifically engineered or constrained motion patterns relative to geometry can lead to significant, potentially counter-intuitive, dynamic effects.

5.3. Conclusion

This work demonstrates mathematically that under the **idealization of prescribed constant tangential projection speed** V, motion over a curved geometry (curvature k_n) can theoretically generate a normal "kinematic lift" acceleration V^2k_n. When this term exceeds the applied normal force acceleration $F(t)/M$, the model predicts net outward acceleration ($D'' > 0$), representing a theoretical "kinematic anti-gravity" effect. The rotating ring thought experiment, where V^2/R can exceed g, exemplifies this prediction. While emphasizing that the **constant V assumption is non-physical for unconstrained systems** and limits the model's scope (excluding phenomena like elliptical orbits), this study provides a rigorous theoretical analysis of the specific dynamic consequences of such a kinematic constraint, offering a distinct perspective on the interplay between inertia, geometry, and prescribed motion near curved surfaces.

References

1. Goldstein, H., Poole, C. P., & Safko, J. L. *Classical Mechanics* (3rd ed., Addison Wesley, 2002).
2. Vallado, D. A. *Fundamentals of Astrodynamics and Applications* (4th ed., Microcosm Press, 2013).
3. Siciliano, B. & Khatib, O. (Eds.). *Springer Handbook of Robotics* (Springer, 2016).
4. Gillespie, T. D. *Fundamentals of Vehicle Dynamics* (SAE International, 1992).
5. Nieto, M. M. & Goldman, T. The arguments against 'antigravity' and the gravitational acceleration of antimatter. *Physics Reports* **205**, 221–281 (1991).
6. do Carmo, M. P. *Differential Geometry of Curves and Surfaces* (Prentice-Hall, 1976).
7. Kreyszig, E. *Differential Geometry* (Dover Publications, 1991).