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Goldbach's Conjecture as a Resolution Condition Under Entropy Geometry

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Abstract: This letter presents a structural derivation of the Goldbach Conjecture (GC) from the Total Entropic Quantity (TEQ) framework, assuming the Riemann Hypothesis (RH) as an emergent property of entropy geometry. Within TEQ, the spectrum of primes corresponds to entropy-stable modes arising from curvature in distinguishability space. We show that the additive closure condition expressed by GC is structurally required for entropy resolution to remain complete, and thus follows necessarily from TEQ and RH. This argument is not statistical or empirical—it is geometric. Though compact, it rests on a profound shift in the conception of mathematical truth: from formal derivation to structural necessity.

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1. Introduction

We show here that two celebrated mathematical conjectures—Riemann Hypothesis (RH) and Goldbach's Conjecture (GC)—follow from a deeper structural principle: entropy curvature. These conjectures, though long studied in isolation, emerge together within a unified geometric framework for resolution under entropy dynamics.

This letter, when taken together with [1], provides a conditional but unified derivation of both RH and GC. In this framework, these conjectures are not isolated analytic problems but consequences of a deeper entropy geometry governing spectral and additive stability. RH follows from the selection of entropy-resilient modes under curvature in log-time. GC follows from the completeness of additive reconstruction in a spectrum governed by TEQ. Their mutual incompatibility in failure is derived structurally. Thus, TEQ unifies both RH and GC as emergent features of a thermodynamically grounded theory of mathematical distinction.

The Goldbach Conjecture (GC) posits that every even integer greater than two is the sum of two prime numbers. It is one of the oldest unsolved problems in mathematics, proposed in a letter from Christian Goldbach to Euler in 1742. Despite overwhelming numerical evidence and substantial partial results (e.g., Chen's theorem), no complete proof exists in the classical framework of number theory.[1]

This letter offers a different approach. We work within the Total Entropic Quantity (TEQ) framework [2], which derives physical and mathematical structure from the geometry of entropy curvature. Within TEQ, primes and integers are not mere arithmetical abstractions, but structural features of a deeper geometry of resolution and stability.

Note to the reader: this manuscript assumes familiarity with TEQ and the notion of structural proofs. Those new to the framework may wish to consult [3] for a general introduction to entropy curvature and its role in physical and mathematical inference.

Our strategy is to show that if the Riemann Hypothesis (RH)—a spectral condition already implied by TEQ—holds, then GC follows as a consequence of structural completeness: all even numbers must

1 For a historical overview and analytic approaches, see T. Tao, *Structure and Randomness*, AMS (2006).

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be resolvable via entropy-stable prime configurations. This reframes GC not as a number-theoretic curiosity, but as a necessary condition for the coherence of resolved structure under entropy geometry. For an in-depth discussion of the spectral interpretation of RH within TEQ, see [1].

2. Spectral Foundations in TEQ

In the TEQ framework, entropy is not a measure of ignorance or disorder, but a generative principle: it defines which configurations can be distinguished, stabilized, and evolved. Quantization, classical structure, and time all emerge from the curvature of an underlying entropy geometry [3].

The entropy-weighted action is defined as:

$$S_{\text{eff}}[\phi] = \int dt(L(\phi, \dot{\phi}) - i\hbar\beta\, g(\phi, \dot{\phi})), \tag{1}$$

where L is the classical Lagrangian, and g is the entropy flux functional derived from a distinguishability metric G_S.

In *Eigenphysics* [1], we showed that the operator H associated with entropy curvature admits a spectrum that structurally corresponds to the nontrivial zeros of the Riemann zeta function. This leads to the following result:

TEQ-RH Theorem: The nontrivial zeros of the Riemann zeta function correspond to entropy-stable spectral modes of H, and must lie on the critical line $\Re(s) = 1/2$.

Thus, in TEQ, RH is not a conjecture but a geometric constraint. The primes, in turn, are tied to this spectrum by the Euler product formula and explicit formulas relating zeros to prime counting functions [4,5]. The primes are not random—they form the spectral "modes" from which larger structures are resolved.

3. Additive Resolution and Goldbach's Conjecture

Let:

- $\mathcal{P}$: the set of primes (entropy-stable spectral primitives);
- $\mathcal{E}$: the set of even integers $n \geq 4$ (coarse structures, i.e., configurations not stable under prime resolution alone);
- $\mathcal{P} + \mathcal{P}$: all pairwise sums of primes.

GC asserts that $\mathcal{E} \subseteq \mathcal{P} + \mathcal{P}$. In TEQ, this becomes:

TEQ-GC Principle: If RH holds and entropy curvature governs the stability of spectral modes, then all coarse configurations (even numbers) must be reconstructible as combinations of entropy-stable modes (primes).

The logic is as follows:

(1) RH ensures that the entropy spectrum is regular, bounded, and complete with respect to distinguishability [1].
(2) TEQ requires that resolved configurations emerge from entropy-stationary modes.
(3) If some even number were not decomposable into two primes, that would signal a gap in the resolution structure.
(4) This contradicts completeness under TEQ + RH.

Hence:

$$\forall n \in \mathcal{E},\ \exists p_1, p_2 \in \mathcal{P} \text{ such that } n = p_1 + p_2. \tag{2}$$

Two primes suffice for minimal additive closure of even integers, since evenness inherently admits symmetric decomposition. Where RH constrains the spacing and stability of spectral modes, GC asserts a completeness over additive configurations drawn from them.

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4. The Role of Entropy Curvature

It is important to emphasize that this structural derivation of RH and GC depends on the presence of nontrivial entropy curvature. In the limit where the entropy metric becomes flat—that is, $G_{ij} = \delta_{ij}$ and curvature $\kappa = 0$—TEQ reduces to a linear, non-interacting resolution space. In this limit:

- Quantization disappears;
- Spectral discreteness is lost;
- The constraints leading to RH and GC vanish.

This case is discussed in [1], Section 4.3. Without entropy curvature, there is no selection of a critical line, no stability mechanism for prime spectra, and no geometric guarantee of additive completeness. Thus, RH and GC decouple and revert to independent analytic conjectures.

The emergence of RH and GC as necessities in TEQ is therefore a direct reflection of entropy curvature as a structural constraint. Where curvature vanishes, necessity vanishes.

5. On Rigor and Intuition

This derivation is compact but not superficial. It relies on a shift from formal proof to geometric necessity. The "proof" here is structural: if TEQ and RH hold, then GC must hold. Denying GC would imply unresolved structure in a model that is otherwise entropy-complete.

This is analogous to proofs in geometry where certain configurations are ruled out by curvature. TEQ generalizes this reasoning: what cannot be resolved under the entropy geometry simply cannot exist. The primes are not merely numerical; they are the "notes" of a spectral space from which the full harmony of even structure must be composed.

It is worth noting explicitly that the structural derivation presented here does not yet constitute a conventional logical proof of GC in classical number theory; instead, it provides a deeper conceptual reason why GC must hold in any entropy-curvature governed theory, inviting further formalization. The style of argumentation aligns with physical reasoning by necessity: geometry constrains possibility, and entropy resolution requires completeness.

6. Relation to Hilbert's Sixth Problem

This entropy-geometric perspective contributes to Hilbert's sixth problem—axiomatizing physics and probability—by offering a minimal set of axioms from which foundational mathematical and physical structures naturally emerge.

The TEQ framework, and the structural derivations presented here, address this goal directly. TEQ provides a minimal axiomatic foundation—entropy geometry and resolution stability—from which quantum mechanics, probabilistic amplitudes, and even the structure of prime numbers emerge. Hilbert's vision of grounding physics in clear geometric and probabilistic terms is realized here through the deeper principle of entropy-structured resolution.

7. Conclusion

Viewed through entropy geometry, Goldbach's Conjecture is not an isolated fact but a manifestation of structural coherence. It is not something to be proved from scratch, but something that must hold if entropy curvature governs resolution and RH is satisfied. In this sense, it is not a conjecture but a necessity.

Recent work by Burns [6] independently supports the structural entanglement proposed here. By modeling the zeta zeros under a heat equation influenced by prime gap irregularities, she shows that RH fails due to spectral instability. This aligns with the TEQ framework, where the structural integrity of prime-based resolution (GC) and spectral stability (RH) are jointly governed by entropy geometry.

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Philosophical and Paradigmatic Implications
Within current analytic frameworks alone, proving either the Riemann Hypothesis or Goldbach's Conjecture independently might be fundamentally impossible, precisely because their truths are geometric and structurally entangled. A shift toward a new paradigm—such as entropy geometry—may therefore be required.

Acknowledgments: This work was carried out independently during a period of cognitive and physical rehabilitation following a brain hemorrhage. It reflects part of a personal recovery process rather than a formal research program. ChatGPT was used for language refinement and structural organization; all theoretical content is the author's own. The ideas are offered with no claim to certainty—only the hope that their structure may prove useful or clarifying to others.

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