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Article

A Planck-Thermodynamic Foundation for a Linear-Horizon Cosmological Model and Its Implications for Primordial Lithium

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Abstract

A thermodynamic cosmological model recently proposed by Haug exhibits promising features, including a potential resolution of the primordial lithium problem. The model is characterized by a linearly evolving Hubble radius and a horizon-defined cosmic mass. In this work, we show that the temperature-dependent expansion rate assumed in the model follows uniquely from Planck's law of blackbody radiation combined with the horizon condition. This establishes a first-principles thermodynamic foundation for the model's initial conditions and clarifies the physical origin of the modified nucleosynthesis dynamics responsible for lithium suppression.

Keywords: $R_H = ct$ cosmology; radiation dominant universe; Planck's Law; primordial lithium suppression

1. Introduction

While the standard cosmological model successfully accounts for many large-scale observations, it continues to face persistent challenges in primordial nucleosynthesis, most notably the overproduction of primordial lithium. Motivated by this discrepancy, Haug [1] introduced a thermodynamic cosmological framework in which the Hubble radius evolves linearly with cosmic time. Subsequent nucleosynthesis calculations [2] indicate that this framework naturally suppresses lithium production while preserving agreement with hydrogen and helium abundances.

A key assumption of the model is a temperature-dependent expansion rate. In the present work, we demonstrate that this temperature dependence is not imposed ad hoc but follows directly from Planck's law once the horizon-based mass definition is adopted.

2. Horizon-Based Cosmic Density

The model postulates a Hubble radius evolving as

$$R_{H_t} = ct, \tag{1}$$

This is well known $R_{H_t} = ct$ cosmology that has been discussed as an alternative to $\Lambda - CDM$, there are today a series of different models relaying on this principle, the best known is the Melia [3–6] model. However there are also other $R_{H_t} = ct$ models, where some of them are implying a horizon-defined cosmic mass of:

$$M_t = \frac{c^2 R_{H_t}}{2G}. \tag{2}$$

Distributing this mass within a sphere of radius R_{H_t} yields the corresponding energy density

$$\rho = \frac{M_t c^2}{\frac{4}{3}\pi R_{H_t}^3} = \frac{3c^4}{8\pi G R_{H_t}^2}. \tag{3}$$

Using the identity $H_t = c/R_{H_t}$, this can be written in the critical-energy-density form

$$\rho = \frac{3H_t^2 c^2}{8\pi G}. \quad (4)$$

The defining thermodynamic assumption of the model is that the Hubble parameter scales with temperature as

$$H_t = \Phi T_t^2, \quad (5)$$

leading to

$$\rho = \frac{3\Phi^2 T_t^4}{8\pi G}, \quad (6)$$

where

$$\Phi = \sqrt{\frac{8\pi G a_b}{3c^2 \Omega}}, \quad (7)$$

a_b is the radiation constant and Ω is a the relevant density parameter. In a radiation dominant universe $\Omega = 1$, for the CMB temperature the CMB photon density is $\Omega_\gamma = \frac{1}{5760\pi}$, see [7]. Furthermore we must have:

$$T = \sqrt[4]{\frac{3c^2 \Omega}{8\pi G a_b}} \sqrt{H_t} \quad (8)$$

3. Planck's Law and the Determination of $H(T)$

3.1. Thermal Energy Density from Planck's Law

Planck's law [8,9] for a photon gas in thermal equilibrium implies a spectral energy density

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/(k_B T)} - 1}. \quad (9)$$

Integrating over all frequencies yields the total radiation energy density

$$u(T) = a_b T^4, \quad (10)$$

where

$$a_b = \frac{8\pi^5 k_B^4}{15h^3 c^3} = \frac{4}{c} \sigma. \quad (11)$$

where σ is the Stefan-Boltzmann constant [10,11].

The corresponding mass density is therefore (see Weinberg [12])

$$\rho_m(T) = \frac{u(T)}{c^2} = \frac{a_b}{c^2} T^4. \quad (12)$$

Allowing for the density parameter Ω , the effective thermal density becomes

$$\rho_m(T) = \frac{a_b}{\Omega c^2} T^4. \quad (13)$$

3.2. Unique Determination of the Expansion Rate

Equating the thermodynamic density with the horizon-defined density,

$$\frac{a_b}{\Omega c^2} T_t^4 = \frac{3H_t^2}{8\pi G}, \quad (14)$$

yields

$$H_t(T_t) = \sqrt{\frac{8\pi G a_b}{3c^2 \Omega}} T_t^2 \equiv \Phi T_t^2. \quad (15)$$

Thus, the functional form $H(T) \propto T^2$ is uniquely fixed by Planck thermodynamics combined with the horizon condition, without the need for an independently specified equation of state.

Equation (15) is valid for any cosmic temperature. In the early, radiation-dominated universe, we simply set the density parameter Ω equal to 1. After CMB decoupling, if we are only interested in the CMB temperature, the density parameter should instead be set to $\Omega_\gamma = \frac{1}{5760\pi}$. This yields a CMB temperature given by

$$T_{\text{cmb},t} = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_{H_t}}} \approx 2.725K. \quad (16)$$

where $l_p = \sqrt{\frac{\hbar c}{E_p}}$ is the Planck [8,13] length and $T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \frac{E_p}{k_b}$ is the Planck temperature. This formula for the CMB temperature was presented by Haug and Wojnow [14,15] and derived from the Stefan–Boltzmann law, which is strictly valid only for a black body. Observational studies have demonstrated that the CMB spectrum is very close to that of a perfect black body. Müller et al. [16] state:

Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057$, K.

Earlier, Tatum et al. [17] heuristically proposed a CMB temperature formula that is fully consistent with this result. However, in the present paper we are primarily interested in the universe before CMB decoupling, in which case Equation (15) is required.

4. Consequences for Temperature–Time Evolution

Since the horizon condition implies $H \sim t^{-1}$, the temperature evolution follows directly:

$$T(t) \propto t^{-1/2}. \quad (17)$$

Planck’s law simultaneously fixes the photon number density, it is given by:

$$n_\gamma = \frac{1}{\pi^2} \int_0^\infty \frac{k^2 dk}{e^{k/T} - 1} = \frac{2\zeta(3)}{\pi^2} \left(\frac{k_B T}{\hbar c} \right)^3 \quad (18)$$

This means we have:

$$n_\gamma(T) \propto T^3, \quad (19)$$

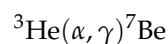
thereby determining the baryon-to-photon ratio and all reaction time scales relevant for primordial nucleosynthesis.

5. Implications for Primordial Lithium Suppression

The suppression of primordial lithium in this framework arises directly from the modified expansion rate $H(T) \propto T^2$. Nuclear reaction freeze-out occurs when reaction rates $\Gamma(T)$ fall below the expansion rate,

$$\Gamma(T) \sim H(T). \quad (20)$$

Because $H(T)$ grows more rapidly with temperature than in standard radiation-dominated cosmology, the window during which lithium-producing reactions such as



can proceed efficiently is significantly shortened.

At the same time, the temperature–time relation $T \propto t^{-1/2}$ alters neutron survival and the deuterium bottleneck timing, reducing the availability of reactants required for lithium synthesis. As a result, reaction flow into the $A = 7$ channel is suppressed without adversely affecting the formation of

hydrogen and helium. This mechanism provides a natural explanation for the reduced primordial lithium abundance reported in [2].

6. Conclusions

We have shown that the thermodynamic cosmological model proposed by [1] admits a rigorous foundation in Planck's law of blackbody radiation. The temperature dependence of the expansion rate follows uniquely from equating the horizon-defined gravitational density with the Planck-derived thermal density. This thermodynamically determined expansion history modifies nucleosynthesis freeze-out conditions in a manner that naturally suppresses primordial lithium production, offering a compelling resolution of the lithium problem.

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