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The Upgraded Planck System of Units

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Simple Summary: We connect the Planck scale to the subatomic world; we explain the physical meaning of the famous dimensionless constant “137;” we derive the masses of the quarks, leptons, and vector bosons from the Higgs field; and we deduce the origin of Koide’s $K = 2/3$ enigmatic constant from first principles.

Abstract: Natural systems of units $\{U_i\}$ need to be overhauled to include the dimensionless coupling constants $\{\alpha_{U_i}\}$ of the natural forces. Otherwise, they cannot quantify all forces in a unified manner. Thus, each force must furnish a system with at least one dimensional and one dimensionless constant. We revisit three natural systems of units (atomic, cosmological, and Planck). The Planck system is easier to rectify, and we do so in this work. The atomic system discounts $\{G, \alpha_G\}$, thus it cannot account for gravitation. The cosmological system discounts $\{\hbar, \alpha_\hbar\}$, thus it cannot account for quantum physics. Here, the symbols have their usual meanings; in particular, α_G is the gravitational coupling constant and α_\hbar is the fine-structure constant. The speed of light c and the impedance of free space Z_0 are resistive properties of the vacuum, thus they must be present in all systems of units. The upgraded Planck system

$$\text{UPS} := \{c, Z_0, G, \alpha_G, \hbar, \alpha_\hbar, \dots\},$$

describes all physical scales in the universe—it is nature’s system of units. As such, it reveals a number of properties, most of which have been encountered previously in seemingly disjoint parts of physics, and some of which have been designated as mere coincidences. Based on the UPS results, that relate (sub)atomic scales to the Planck scale and the fine-structure constant to the Higgs field, we can state with confidence that no observed/measured physical properties are coincidental in this universe.

Keywords: Atomic processes; Cosmological parameters; Cosmology: theory; Early Universe; Elementary particles; Galaxies: kinematics and dynamics; Gravitation.

1. Introduction and Motivation

1.1. Three Major Systems of Units Considered

In a recent paper [1], we used a cosmological system of units based on the speed of light c , Newton’s gravitational constant G , and MOND’s characteristic acceleration a_0 [2–4]. Since G is a building block of this system, then a_0 can substitute for the universal MOND unit, the mysterious $\mathcal{A}_0 = a_0 G$. Besides \mathcal{A}_0 , pairs of the fundamental units produced two more *de facto* important dynamical units: the surface density $\sigma_0 = a_0/G$, and the force $F_0 = c^4/G$. Notice the breaking of the symmetry: the term $c^4 G$ only produces a composite unit of no particular interest with dimensions $[M]^{-1}[L]^7[T]^{-6} = c^8/F_0$. This deviation singles out the unit of force $F_0 = c^4/G$ as an important component of the cosmological system, but there is not much more one can do with it at this point, besides noting that the

same unit of force appears in the Planck system of units as well, and that F_0 is a geometry-independent quantity, since both c and G do not carry an imprint of the geometry of the three-dimensional space.

The above system of units does not use Planck's constant h [5,6], which turns out to be a derived unit of no particular interest, but current thinking forgives the oversight—after all, this is a cosmological system designed for measurements in universal scales. Nevertheless, we were drawn into comparisons with the Planck system which now uses Dirac's $\hbar = h/(2\pi)$ as a fundamental unit [7–9]; and, soon enough, we also added Hartree's atomic system of units [10], which paradoxically does not use c (or G , for that matter) as a fundamental unit. So, we ended up juggling three different systems of units, comparing and contrasting their building blocks and the assumptions that went into their architectures.

1.2. Dirac's Constant \hbar and the Three Atomic Radii

The modern atomic and Planck systems use $\hbar = h/(2\pi)$ [7] instead of Planck's original constant h [5]. This modification is not trivial because the 2π carries the unit of radians. This unit has been dropped by many authors and also by the SI system of units, leading to a widespread misunderstanding that \hbar simply absorbs a numerical value of 2π with no further ramifications. The inconsistency was noted by Bunker et al. [11] who inserted the unit of radian in the definition of \hbar and the unit of cycle in the definition of h . The SI system must reinstate at least the radian unit, because it alerts us to the presence of geometry.

Dirac believed that \hbar is the true universal constant, and we can only guess the reason why: the 2π in \hbar has introduced 2-D geometry into the constant, so, unlike Planck's h , the constant \hbar is not purely physical, it is a composite constant. This fact was effectively proven by Leblanc et al. [12] who showed that the Compton radius $r_c (\propto \hbar)$ also includes a geometric component. This result has important consequences in physics that become visible when we write side-by-side the three electronic radii of the atomic world:

$$\begin{aligned} \text{Classical radius : } r_e &= e^2/(\epsilon_0 m_e c^2) = r_c \alpha_{\hbar} \\ \text{Compton radius : } r_c &= \hbar/(m_e c) \\ \text{Bohr radius : } r_b &= \epsilon_0 \hbar^2/(m_e e^2) = r_c/\alpha_{\hbar} \end{aligned} \quad , \quad (1)$$

where α_{\hbar} is the fine-structure constant and ϵ_0 is the reduced vacuum permittivity defined by $\epsilon_0 \equiv 4\pi\epsilon_0$, an equation that shows how the stereometry of space modifies the physical unit ϵ_0 of the vacuum. Therefore, we have an SI unit problem here too, just as Bunker et al. [11] discovered for \hbar . The vacuum is a three-dimensional space, hence the stereometric 4π ; thus, the units of ϵ_0 must also include steradians.¹ Now, these geometric considerations show why three different radii exist in atomic physics: although they have the same physical dimension, they capture entirely different geometries; the electrons in the atoms venture in 3-D space (hence, the $1/(4\pi)$ in r_e); the emitted photons only “see” two dimensions (see footnote 1); and the electrons in the Bohr model of the atom are quantized and see only discrete sectors embedded in 3-D space (hence, the $1/\pi$ in r_b). The factor of $1/4$ “missing” from the $1/\pi$ is however applied to the energy levels, because this factor is included in the Rydberg energy E_R (see below).

Deriving the geometric pattern of the quantized radii r_n of the Bohr model is a little harder, but within our grasp.² In any case, the $1/4$ is necessarily missing from r_b , so that the quantized angular momentum \mathcal{L}_n is truly a 2-D quantity ($\mathcal{L}_n \propto \hbar \propto 1/(2\pi)$), and

¹ Similarly, $\mu_0 \equiv \mu_0/(4\pi)$ is the reduced vacuum permeability, and then, $\epsilon_0\mu_0 = 1/c^2$. The stereometric 4π terms cancel out nicely to produce the “definition” of c , which is a purely physical quantity. Then, Dirac's 2π term in \hbar tells us that Planck's free photons only “see” two dimensions, no matter how they move in stereometric (3-D) space (in lines, or circles, or ellipses, etc.). We also learn that the fundamental natural constant c is produced by the vacuum itself, and it is the geometric mean of two smooth *inverse Lie mappings* of ϵ_0 and μ_0 [13].

² In Bohr's model, the (nongeometric) number-parts of energies E_n and radii r_n are related by $E_n \propto 1/r_n = 1/n^2$. So, for pure numbers, we see that $\sqrt{r_n} = n$, and the coefficients of the quantized radii are essentially produced by geometric averaging, viz., $r_{n+1} = \sqrt{r_n r_{n+2}} + 1$. The +1 extends the sequence back to $n = 0$, $r_0 = 0$.

the associated Rydberg energy E_R is independent of geometry (although the abolished geometry contributes a unitless constant of $1/4$, i.e., $E_R \propto 1/(\epsilon_0 \hbar)^2 \propto 1/[4\pi/(2\pi)]^2 = 1/4$, the same factor as that “missing” from r_b).

Lastly, the Bohr radius is the fundamental unit of length in the atomic system, but we argue that the Compton radius is actually the basic unit because its definition in equation (1) does not contain α_H . In addition, there is more circumstantial evidence that r_c is important among the three radii of equation (1): r_c is the geometric mean of r_e and r_b (i.e., $r_c = \sqrt{r_e r_b}$), and this implies that the Compton radius is also the geometric mean of all three length scales, viz.

$$r_c = \sqrt[3]{r_e r_c r_b}. \quad (2)$$

Thus, r_c is singled out among the three electronic radii for further duty in all systems of units (but probably without the 2π term, to remove the artificially inserted 2-D geometry). Furthermore, it may not be as obvious yet, but geometric averaging plays a huge role in nature. The above geometric means (hereafter, G-Ms, to avoid confusion with “GM”) are only a prelude to their ubiquitous appearances in many combinations of natural constants and physical quantities.

1.3. Dimensionless Constants

The general notion about constructing a system of units is that one is free to choose any units to be the building blocks. Dimensionless constants do not have units to offer, thus they are not chosen as building blocks. They remain as *passive* invariants in any adopted system of units, and they serve mostly as cross-checks of the various calculations performed between dimensional quantities. The current thinking is summarized in the following excerpt from Zeidler [14]:

“A special role is played by those physical quantities which are dimensionless in the SI system. We expect that such quantities are related to important physical effects. The experience of physicists confirms this.”

So, we suspect that such constants are important in physics, but we do not really know what to do with them, beyond their confirmed invariance, because they lack units.

The general notion about constructing a system of units is wrong on two counts. (1) Although unitless constants do not have units to offer, they must be *actively* included in systems of units, because they introduce the natural forces that cause the important physical effects. (2) We are not free to choose any dimensional units as building blocks; we must choose wisely the units that measure the fundamental forces and, in addition, those units dictated by the vacuum itself. In particular, choosing a favorite particle to supply its properties for building blocks (as in the three atomic radii) is a bad idea and the reason is that such favoritism would violate a principle of fairness in this world. As will be seen below, nature does not at all favor or neglect any particle or force field, not even the “very large” ones against the “very small” ones, and vice versa.

So, one or both defects have crept into our systems of units, where they selectively crippled or eliminated entirely some fundamental forces. A force is crippled when its dimensional or its dimensionless constant is not included as a building block; and a force is eliminated entirely when both of its defining constants are not included in a system of units.

1.4. Outline

In this work, we construct a self-consistent system of units that does not suffer from the above defects and that includes gravity, electromagnetism, and the weak interaction. Since all forces besides gravity have been unified, leaving out the coupling constant of the strong interaction may not hurt the effort too much—and we are going to test our system’s performance in atomic and subatomic scales, and in Planck and macroscopic scales as well.

It turns out that the Planck system is easier to upgrade, because it already includes the appropriate dimensional constants $\{c, Z_0, G, \hbar\}$, although the impedance of free space Z_0 has so far been sidelined. So, what we need to do for the upgrade is to activate the unitless coupling constants α_G (gravitational coupling constant) and α_\hbar (fine-structure constant); and to repair the damage that \hbar has caused by inadvertently introducing geometry in them (see footnote 6 for details), besides the well-intended quantum forces.

In § 2, we describe the building blocks of the upgraded Planck system of units. In § 3, we collect the new results concerning masses, charges, and lengths in the new system. In § 4, we discuss the results, and, in § 5, we summarize potential issues still lingering in this system of units, as well as some future research prospects.

Finally, in § 6, we list the most important highlights of our investigation, including the results obtained in the appendices. In Appendix A, we derive the long-sought physical significance of Koide's lepton constant [15] of atomic physics, and in Appendix B, we discuss the universality of the Tully-Fisher/Faber-Jackson relation [16,17] discovered in spiral and elliptical galaxies, respectively.

2. The Building Blocks of the Upgraded Planck System

The upgraded Planck system (UPS) includes the following building blocks at the very least:

$$\text{UPS} := \{c, Z_0, G, \alpha_G, \hbar, \alpha_\hbar\}, \quad (3)$$

where $\{c, Z_0, G, \hbar\}$ are dimensional units and $\{\alpha_G, \alpha_\hbar\}$ are dimensionless units. We use a slash to indicate the presence of geometric units (which have been inadvertently dropped from the systems of units), but the opposite applies to the coupling constants, in which the role of geometry has been reversed due to the very old use of \hbar in their definitions (see below). This is a problem with which we have to contend throughout this paper.

2.1. Dimensional Units

For future reference, we need to recall and emphasize a gem of natural units, the fundamental dimensional relation between gravity (supplying G) and electromagnetic (EM) forces (supplying also the vacuum's constant $(4\pi\epsilon_0)^{-1}$). This is obtained by equating the dimensions of the forces in Newton's gravitational law and Coulomb's law for two electrons. We find that

$$G m_e^2 \sim (4\pi\epsilon_0)^{-1} e^2, \quad (4)$$

where m_e is the mass of the electron and e is the elementary positive charge. Here, we write down explicitly the vacuum permittivity ϵ_0 to ensure that its geometric content (the 4π in the EM term) is clearly visible.

Each side of equation (4) becomes unitless when divided by $\hbar c$, as is done separately in the definitions of the two fundamental coupling constants. Unfortunately, the \hbar introduces then geometry into the gravitational part and eliminates geometry from the EM part, clearly altering the original geometrical characteristics of the two coupling constants (see § 2.1.2 below). The unit of $[\text{rad}]^{-1}$ has been dropped from \hbar by international agreement, so this intrusion of geometry is no longer visible [11]; going as far back as Schrödinger [18], our community is under the mistaken impression that it is only a pure numerical factor of 2π which has been absorbed in the definition of \hbar .

2.1.1. Constants Imposed by the Vacuum

The speed of light is not related to a natural force. It is indirectly produced by the vacuum to dictate the speed of EM waves and also sets an upper limit to the motion of material objects possessing mass. The vacuum does that incidentally by providing the smallest natural resistance to motion. The magnitude of c is set by the G-M of two inverse properties of the vacuum, viz.

$$c = \sqrt{\epsilon_0^{-1} \mu_0^{-1}}. \quad (5)$$

The SI value of c is $c = 2.9979 \times 10^8 \text{ m s}^{-1}$, and its dimensions are $[\text{length}][\text{time}]^{-1}$ [19].

In equation (5), the reduced values of vacuum permittivity and vacuum permeability combine in a way that removes geometric constraints from this speed (see also footnote 1); the maximum permitted velocity of a combined EM wave or a massive object must be the same in any direction of motion. In contrast, it is understood that a static electric field in vacuum must adjust to the geometric constraint imposed by ϵ_0^{-1} , and this is why the vacuum's inverse permittivity appears in Coulomb's law. In fact, ϵ_0^{-1} is the slope between the electric field \mathcal{E} and the surface charge density e/r^2 , where r represents distance (see also Appendix B for the role that various surface densities play in disjoint parts of physics).

Since equation (5) can be written in the equivalent form

$$c = \sqrt{\epsilon_0^{-1} \mu_0^{-1}}, \quad (6)$$

we can surmise that the physical quantities ϵ_0 and μ_0 are geometry-free. Indeed, after some manipulations involving the also geometry-free fine-structure constant α_{\hbar} (as this was inadvertently defined long ago, using Dirac's \hbar ; see below), we find that

$$\epsilon_0^{-1} = c \left(\frac{2h\alpha_{\hbar}}{e^2} \right), \quad (7)$$

and

$$\mu_0^{-1} = c \left(\frac{e^2}{2h\alpha_{\hbar}} \right). \quad (8)$$

There is no geometric influence on the right-hand sides of these equations. The quantity that is inverted from one equation to the other, $h/e^2 = \mu_0 c / (2\alpha_{\hbar})$, is proportional to the impedance of free space $Z_0 = \sqrt{\mu_0 / \epsilon_0} = 376.730 \Omega$, which is the G-M of μ_0 and ϵ_0^{-1} ; thus, h/e^2 has dimension of [ohmic resistance] (see [20] and footnote 20 below). Thus, ϵ_0^{-1} and μ_0^{-1} can effectively be expressed as G-Ms involving the squares of c and Z_0 (i.e., $\sqrt{c^2 Z_0^2}$ and $\sqrt{c^2 (1/Z_0^2)}$, respectively); the first G-M involves a direct multiplication of the two constants involved, whereas the second G-M uses the Lie-type inversion of one of the two constants [13]. We will pick up again this important inference in § 4.2 below.

2.1.2. Dirac's Constant \hbar

Dirac's constant is the slope between the energy E carried by a single photon and its angular frequency ω , viz.

$$E = \hbar \omega. \quad (9)$$

Its SI value is $\hbar = 1.0546 \times 10^{-34} \text{ J s rad}^{-1}$ [11,19], and its dimensions are $[\text{action}][\text{rad}]^{-1}$ or, equivalently, $[\text{moment of inertia}][\text{second}]^{-1}[\text{rad}]^{-1}$.

This new awareness, that inertia is built into \hbar (and Planck's h), may be the spark we need to theorize that the weak equivalence principle [21] is embedded into the microcosm as well, where gravity is not important. Action integrals [22], in particular, may be viewed as carrying the units of $[\text{moment of inertia}][\text{second}]^{-1}$, thus each action is a measure of the rate of change of moment of inertia at all scales of the universe (see also § 3.5 below).

In the spirit of equations (7) and (8), Planck's reduced constant may also be split into a product of two G-Ms, viz.

$$\hbar = \sqrt{\hbar(\epsilon_0 c)} \sqrt{\hbar \left(\frac{1}{\epsilon_0 c} \right)} = e \left(\frac{\hbar}{e} \right); \quad (10)$$

the first G-M on the right-hand side is geometry-independent; the next G-M is influenced by 2-D geometry since it is directly proportional to

$$\sqrt{\hbar(\epsilon_0^{-1})} \propto \sqrt{(4\pi^2)^{-1}(2\epsilon_0)^{-1}} \propto (2\pi)^{-1}.$$

This G-M that reduces to (\hbar/e) has dimensions of [magnetic flux] = [magnetic field][area].³ It is understood from the G-M decomposition (10) that the vacuum quantity $\epsilon_0 c = 4\pi/Z_0$ can couple to \hbar , and thus influence quantum phenomena; and it does so in the definition of the fine-structure constant (§ 2.2.1).

2.1.3. Newton's Gravitational Constant G

Newton's gravitational constant G is the slope between the gravitational field $a(r)$ and the surface mass density $\sigma(r) \equiv M(r)/r^2$ [1] on the surface of a sphere of radius r enclosing a total mass of $M(r)$, viz.

$$a(r) = G \sigma(r). \quad (11)$$

Its SI value is $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ [19], with dimensions of [acceleration] [surface density]⁻¹.

In the spirit of equation (10), Newton's gravitational constant can also be split into a product of two G-Ms, viz.

$$G = \sqrt{G M^2} \sqrt{G \left(\frac{1}{M^2} \right)}, \quad (12)$$

which shows the potential of mass M to couple to G , and thus influence gravitation. The M^2 term does precisely that in the definition of the gravitational coupling constant (§ 2.2.2). Here, the M^2 is meant to include both the inertial and the gravitational mass on an equal footing.

2.2. Dimensionless Units

We now come to the operations and the properties of the unitless coupling constants $\{\alpha_G, \alpha_\hbar\}$. We show how these units rectify the Planck system of units and make it functional over all non-nuclear scales of the universe, including atomic and subatomic scales as well.

2.2.1. Fine-Structure Constant α_\hbar

The fine-structure constant has been defined as

$$\alpha_\hbar = \frac{e^2}{\epsilon_0 \hbar c}. \quad (13)$$

Its value has been measured [23] to be very close to $(137.036)^{-1}$ (or $\alpha_\hbar = (861.022576)^{-1}$ for the wiser choice $h \rightarrow \hbar$ in the definition). Other than that, α_\hbar brings no geometry and no units into the system of units. In particular, the geometry embedded in the electric field (and carried on by ϵ_0) has been inadvertently eliminated by the insertion of \hbar in the modern definition (13).

Nevertheless, definition (13) provides a powerful tool (§ 2.2.3), which we have not taken advantage of in the past: being a measurable constant, α_\hbar may serve as the reference UPS unit against which we can quantify all the other unitless coupling constants. For instance, the gravitational coupling constant α_G , which we describe next, acquires a quantitative meaning only by comparison to α_\hbar in the ratio (α_G/α_\hbar) (although, unfortunately,

³ The unit of [area] justifies the deduced geometric factor $1/(2\pi)$ in the second G-M of equation (10). The additional factor of 2 attached to ϵ_0 is a unitless imprint, but it has a geometric origin. This type of imprinting is difficult to track down in the various equations of physics when they are presented in reduced, simplified form (see also the discussion in § 1.2 about the numerical factor of 1/4 imprinted by geometry to the Rydberg energy).

the reference value α_{\hbar} introduces a unit of [rad] in this ratio and an artificial dependence on geometry).

2.2.2. Gravitational Coupling Constant α_G

Using G and the electron mass m_e , the gravitational coupling constant has been defined as

$$\alpha_G = \frac{Gm_e^2}{\hbar c}. \quad (14)$$

Its value is 1.7518×10^{-45} (or $\alpha_G = 2.7881 \times 10^{-46}$, the geometry-free value obtained for $h \rightarrow \hbar$ in the definition), as determined by calculation.

Comparing the definitions (14) and (13), we see that α_G is, unfortunately, geometry-dependent. This problem did not exist during Max Planck's heydays, when h was in use and \hbar did not exist. In general, the problem with the modern definitions of constants and variables is that \hbar necessarily introduces 2-D geometry and a [rad] measure, in addition to the intended physical constant h .

The geometry dependence so artificially inserted in α_G will be taken out entirely in the calculations that follow. We must emphasize up front that reinstating the true nature of α_G (and α_{\hbar}) is necessary for the successful repair of the modern Planck system, and it leads to the determination of natural scales of mass, length, and charge for the chosen mass-to-charge ratio (m_e/e) of the electron (§ 2.2.3), or any other chosen particle for that matter (see footnote 7 for details).

2.2.3. Relative Strength of Gravitational Coupling β_G

Leaving aside the unfortunate introduction of [rad] in the above coupling constants, we come now to the only known method of *actively using* such dimensionless (pure) numbers. Being pure numbers, these constants have absolutely no meaning or practical use, but they become useful in ratios, in which their strengths are compared; then, these ratios acquire meaning by comparisons, and their relative strengths are, for all practical purposes, measurements of the same stature and importance as dimensional quantities (which, incidentally, are also measured by comparisons to international standards). One unitless coupling constant should however be included in the system in absolute terms in order to provide the reference value for comparisons.⁴

For the UPS, we choose α_{\hbar} for this duty because it has been measured by experiment [23], and its physical meaning has now become clear (see § 3.1 and Table A1 below): the factor $\sqrt{\alpha_{\hbar}} \simeq 1/30$ (with $h \rightarrow \hbar$ in equation (13)) is a fundamental scale factor used by the Higgs field to assign mass to the bottom quark, which then becomes the gateway for the assignments of all lower particle masses.

In this study, we assume that the above two coupling constants do not vary in space. Then, their invariant nature tells us that, if we determine their relative strength for a particular particle or object (an electron in the UPS), then this ratio will be the same at all scales within the utilized system of units. Thus, we calculate the universal UPS ratio

$$\beta_G \equiv \frac{\alpha_G}{\alpha_{\hbar}} = \frac{\epsilon_0 G m_e^2}{e^2} = 2.4006 \times 10^{-43}, \quad (15)$$

a pure comparative number which is independent of \hbar and h (and c , for that matter—as expected, the vacuum does not at all contribute to such a ratio of forces). This “measurement” of β_G represents the strength of gravitational coupling relative to that of the EM coupling obtained from electrons (see footnote 7 for considering protons instead, and finding another UPS with different scales but with the same particles).

⁴ The reference unitless constant (α_{\hbar} here) plays the exact same role that the standard 1-meter ruler and the standard 1-kilogram cylinder play in the SI system of units of length and mass, respectively.

Compared to the relation between units shown in equation (4), β_G carries a lot more weight because it can be used in quantitative calculations (although it was equation (4) that gave us a reason to define β_G). The numerical value obtained in equation (15) does not tell us that gravity is weak and the EM force is strong;⁵ it only tells us about the relative couplings of these forces in the particular system of units that includes β_G as a building block. Gravity is attractive and has always had a chance to grow past the other forces in extraordinarily massive settings ($M \gg m_e$)—something that is actively occurring in many places in the present universe. In contrast, the Coulomb force cannot do the same, because its attraction brings together charges of opposite signs that cancel each other out [24].

Also, equation (15) does not tell us that, in the distant past, gravity could have been much stronger in the atomic world, and it only got weaker going forward in time [8,9] because of the expansion of the universe. The gravitational force has always been weak in the atomic world because the characteristic atomic masses are too small (much smaller than the Planck mass). So, if it should come to a bout of conjectures, probably the safest choice is the assumption that these constants do not vary in time either, just as they do not vary in space.

2.3. Determining a New Atomic Mass Scale

Definitions (13) and (14) have both incorporated \hbar , thus the coupling constants have been defined in the microcosm. Here, we use the above results to establish a new atomic mass scale, after correcting for the unintended insertion of planar geometry into the coupling constants.

Looking at equation (14), we see two problems that need to be addressed: (a) Despite the apparent lack of units (not entirely true, since \hbar also carries radians), α_G is not influenced by EM coupling (there is no e in the definition, only mass m_e , and the two forces are not linked to one another, although they do so in the real world). (b) The unfortunate use of \hbar has had the unforeseen consequence of arbitrarily adding more geometry (and a [rad] unit) into the dimensionless mix.⁶

We can solve both problems by adopting equation (15) to help us define a new atomic mass scale M_A in the UPS. The relative ratio β_G carries both forces, and the composite unit \hbar , which was not appropriate in the first place, has been eliminated (correcting thus a century-old oversight). One unavoidable conclusion is that the geometry of the vacuum (the ϵ_0) is still present in β_G . This comes from the geometric dependence of the electric field, which will now overtly influence the new mass scale M_A (see also § 5 below). This appears to be a realistic relationship since mass also carries charge and the carried electric field is influenced by the vacuum's ϵ_0 .

Based on these considerations, we return to equation (14), and we rewrite this definition by making the following substitutions: $\beta_G \rightarrow \alpha_G$, $h \rightarrow \hbar$, and $M_A \rightarrow m_e$. We find a new equation, viz.

$$\beta_G = GM_A^2 / (hc), \quad (16)$$

in which both sides are dimensionless; and substituting for β_G from equation (15), we obtain the new atomic mass scale

$$M_A = \sqrt{\epsilon_0 \hbar c} \left(\frac{m_e}{e} \right) = 2.6730 \times 10^{-29} \text{ kg}. \quad (17)$$

We repeat here the two ingredients that form the physical basis for this mass: (i) the unitless ratio β_G in equation (15) has no dependence on \hbar , or h , or c ; and (ii) the substitution $h \rightarrow \hbar$ produces a truly unitless equation (16); there are no loose [rad] units in this equation,

⁵ Had we used the mass and the charge of a supermassive black hole (e.g., [24]) in equation (15), we would have obtained a relative strength of couplings $\beta_G \gg 1$ and a different system of units, which would be hard to relate to the Planck scale and even harder to use in the atomic world.

⁶ We point out again that using \hbar in definitions (13) and (14) reverses what nature intended. It makes α_\hbar be a geometry-free value, although the geometry should have been that of ϵ_0 coming from the electric field), and α_G ends up with 2π radians, although it should have been geometry-free.

covertly suppressed by SI conventional practices (although here the “cycle” unit has been suppressed in h , since it does not signify insertion of geometry; see also [11]).

It is quite interesting that only the ratio (m_e/e) of the characteristic parameters of the electron ends up being a building block of the new mass scale M_A . The reciprocal ratio, i.e., $e/m_e = 1.7588 \times 10^{11} \text{ C kg}^{-1}$, was first measured by J. J. Thomson [25], years before the electronic charge itself was finally measured by experiment (see also § 4.2).

The presence of ϵ_0 (coupled to h , as shown in the G-Ms given in § 2.1.2) in the new mass scale M_A is necessary (the vacuum’s ϵ_0 is a building block of the electrostatic field): after some algebraic manipulations, we recast equation (17) (or equation (16)) to the equivalent form

$$M_A = \sqrt{\left(\frac{hc}{G}\right) \left(\frac{E_{\text{grav}}}{E_{\text{elec}}}\right)}, \quad (18)$$

where the ratio of energies,

$$\frac{E_{\text{grav}}}{E_{\text{elec}}} = \beta_G,$$

was determined from the corresponding forces acting between two interacting electrons.⁷ In dimensional analysis, this ratio is 1 (see equation (4) above), but here, β_G plays an important quantitative role: the unitless factor

$$\sqrt{\beta_G} = 4.900 \times 10^{-22} \quad (19)$$

scales the original Planck mass [5] ($M_p = \sqrt{hc/G} = 5.4555 \times 10^{-8} \text{ kg}$) down to the atomic world. This scaling is a significant result of our work, as it connects the original Planck mass scale with the M_A scale of the atomic world,⁸ viz.

$$M_A = M_p \sqrt{\beta_G}. \quad (20)$$

We note that M_A and M_p are mass scales related by this equation, and they do not (cannot) correspond to any real particle or object in nature (see also § 4.3).

Notice the complete absence of \hbar from equations (15)–(18). The only geometric dependence entering these equations is that which is imposed by the vacuum on to the electrostatic field (hence, $M_A \propto \epsilon_0^{1/2} \propto 2\sqrt{\pi}$). The $\sqrt{\pi}$ does not carry any angular units since $E_{\text{elec}} \sim e^2/\epsilon_0 \sim [\text{Joule}]$ in equation (18)—just like the $1/\pi$ in the Bohr radius and the factor of $1/4$ in the Rydberg energy (see the analysis following equation (1) in § 1.2). Therefore, besides introducing the speed of light, the vacuum manages to imprint M_A only with a unitless, purely numerical constant⁹ (see also footnote 3).

⁷ An alternative choice, such as of two interacting protons with masses m_p , leads to another complete system of units, say UPS'. In this case, we find that $M'_A = 27.5 \text{ GeV}/c^2$ and $(\beta'_G)^{1/2} = 9.00 \times 10^{-19}$, but equation (18) is still valid, and connects M'_A with the original Planck mass $M_p = \sqrt{hc/G}$. Also, the scaling $M'_A/M_A = m_p/m_e$ holds precisely between the two systems of units; and the relation $M_A M'_A = (m_e m_p)/\alpha_f$ is exact as well. Finally, referring to the upcoming UPS results in § 3.1 below, the relation $M'_A = \sqrt{m_t m_b}$ holds to within 2.5% in the UPS', where m_t and m_b are the masses of the top and bottom quarks, respectively; thus, M'_A actively participates in the mass ladder of the UPS', just as M_A does in the UPS mass ladder of § 3.1 and Appendix A.

⁸ We knew that a rescaling of the Planck mass $M_p = \sqrt{hc/G}$ by some power of β_G would produce an atomic mass. But we did not know which power is appropriate to use. Here, we have shown that the appropriate coefficient of M_p is the G-M of E_{grav} and $(E_{\text{elec}})^{-1}$, if we are scaling M_p down to lower masses. If we are scaling up, then the -1 exponent naturally moves on top of E_{grav} in the G-M (see § 3.4 below). In retrospect, these two G-Ms make perfect sense in a “fair” world dominated by the pervasive G-M averaging of pairs of fundamental physical quantities.

⁹ The realization that the vacuum also leaves unitless numerical imprints (in addition to its dimensional constants $\epsilon_0, \mu_0, c, Z_0$) is new, unexpected, and it may prove very important in future work. Sooner or later, we will have to investigate such imprints of the vacuum to the nuclear world, especially in the strong interactions and the so-called *beta functions* [26].

3. Results within the UPS Realm

3.1. Subatomic Masses

This new UPS mass scale (17) corresponds to

$$M_A = 15.0 \text{ MeV}/c^2, \quad (21)$$

thus it lands near the subatomic world of the low-mass up and down quarks, with corresponding masses $m_u = 2.16 \text{ MeV}/c^2$ and $m_d = 4.67 \text{ MeV}/c^2$ [27]; and it is smaller than the G-M defined for the hydrogen atom

$$\sqrt{m_e m_p} = 21.9 \text{ MeV}/c^2, \quad (22)$$

where m_p is the proton mass.

The new mass scale M_A appears to be important for the standard model of particle physics, and it should be investigated further theoretically (there is no elementary particle corresponding to this energy). So far, we have derived the following useful relations (sufficient to lead us to a clear physical interpretation of Koide's enigmatic constant and other constants of the same type; for details, see Appendix A):

- (1) The mismatch between M_A and $\sqrt{m_e m_p}$ may be related to Koide's K -constant, $K = 2/3$ [15], viz.

$$M_A / \sqrt{m_e m_p} = 0.685, \quad (23)$$

connecting thus the masses of leptons to the atomic constants M_A and m_p .

- (2) Using the above values of first-generation quark masses and the mass of the strange quark, $m_s = 93.4 \text{ MeV}/c^2$ [27], we find that

$$\sqrt{m_u m_s} / M_A \simeq 0.95, \quad (24)$$

and

$$\sqrt{m_d m_s} / \sqrt{m_e m_p} \simeq 0.95, \quad (25)$$

showing only a 5% deviation of both quark G-Ms from the two atomic mass constants. The results indicate that the mass of the second-generation strange quark is connected to both M_A and the masses of the first-generation quarks. Thus, a connection should exist for the charm quark too,¹⁰ and so on for the third generation of quarks as well.

- (3) It certainly appears that there exists a ladder-type mechanism that uses G-Ms (and some scaling coefficients) to relate various particle masses (see also Table A1 in Appendix A below). Some examples (and their corresponding deviations from experiment) are:

$$m_s = \sqrt{m_d m_\tau} \quad (2.5\%), \quad (26)$$

where $m_\tau = 1.777 \text{ GeV}/c^2$ is the tauon mass;

$$m_s = \sqrt{m_u m_b} \quad (1.7\%); \quad (27)$$

$$m_c = \sqrt{m_p m_\tau} \quad (1.7\%); \quad (28)$$

$$m_c = \sqrt{2 m_d m_t} \quad (0.054\%), \quad (29)$$

where $m_t = 172.5 \text{ GeV}/c^2$ is the top quark mass;

$$m_u = \sqrt{2 m_d m_e} \quad (1.1\%); \quad (30)$$

¹⁰ The charm and bottom quarks have masses of $m_c = 1270 \text{ MeV}/c^2$ and $m_b = 4180 \text{ MeV}/c^2$, respectively [27]. At such high masses, something must be changing in the dynamics: for the ordered by mass triplet s-c-b, we find, to within a 1.6% accuracy, that $m_c = 2\sqrt{m_s m_b}$. We also find that the charm quark participates rather "reluctantly" in just one pure/unscaled G-M (equation (28), referring to the compact triplet p-c- τ); and even that one is unusual, as it involves the proton mass m_p .

$$m_p = \sqrt{2m_\mu m_b} \quad (0.17\%), \quad (31)$$

where $m_\mu = 105.66 \text{ MeV}/c^2$ is the muon mass;

$$m_b = \sqrt{m_\mu m_t} \quad (2.1\%); \quad (32)$$

and

$$M_A = \sqrt{m_\mu m_u} \quad (0.71\%), \quad (33)$$

$$M_A = \sqrt{\sqrt{m_e m_\mu} \sqrt{m_e m_\tau}} \quad (0.80\%). \quad (34)$$

- (4) The Higgs boson ($m_H = 125.25 \text{ GeV}/c^2$) is certainly special, although unavoidably a part of the mass ladder. This is the only particle that is not involved in simple G-Ms with the low-mass particles. Two of its complex relations are the following:

$$m_b = \sqrt{m_s (m_H/K)} \quad (0.21\%), \quad (35)$$

where $K = 2/3$ [15]; and

$$\frac{m_H}{m_b} = 30.0 \simeq \frac{M_A}{m_e} \quad (2.0\%). \quad (36)$$

This relation shows how the Higgs boson manages to assign mass to the much lower-mass bottom quark by using a novel mechanism, not related to a G-M or Koide's scale factor (see below).

- (5) The vacuum expectation value (VEV) of the Higgs field is $v = 246.22 \text{ GeV}/c^2$ [28]. To within a deviation of 1.8%, we find for the compact¹¹ triplet H-t- v that

$$m_t = \sqrt{m_H v}, \quad (37)$$

which shows exactly where the most massive quark is located at the top of the mass ladder. Furthermore, the Higgs mass is the G-M of the top quark mass and the mass of the Z^0 boson $m_{Z^0} = 91.1876 \text{ GeV}/c^2$ (a deviation of only 0.13%), viz.

$$m_H = \sqrt{m_t m_{Z^0}}. \quad (38)$$

Obviously, the top quark receives its mass from the Higgs field, and then it participates in the G-Ms that define the masses of the other particles (see Table A1 in Appendix A). The high-mass geometric sequence Z^0 -H-t- v appears to be very compact indeed (footnote 11), and its common ratio is about 1.38.¹² We note that W^\pm (mass $m_{W^\pm} = 80.377 \text{ GeV}/c^2$) is not a member of this sequence since $m_{Z^0}/m_{W^\pm} = K^{-1/4} \simeq 1.11$.¹³ This relation provides another definition of Koide's K in terms of the decay products of the Higgs boson (deviation 2.5%).

- (6) On the other hand, the G-M of m_H and m_{W^\pm} is 10% larger than m_{Z^0} ; but using empirically Koide's constant, we find that

$$m_{Z^0} = \sqrt{(K^{1/2} m_H) m_{W^\pm}}, \quad (39)$$

¹¹ No other available particle slots in the domain.

¹² It will become apparent in Appendix A that the ratio 1.38 approximates $C_F = 4/3$ (to within a deviation of 3.5%), where C_F is the quadratic Casimir charge of the SU(3) fundamental representation of the quark potential (equation (4.45) in Reference [29]).

¹³ Eliminate m_H between equations (39) and (40).

an important relation with a deviation of the G-M from the measured m_{Z^0} value of only 0.57%. Furthermore, the relation $m_{W^\pm} = \sqrt{K m_H m_{W^\pm}}$ also appears to hold (1.9% deviation), which then implies that

$$m_{W^\pm} = K m_H. \quad (40)$$

This relation helps us understand the important role of the exact constant $K = 2/3$ [15]: K is a numerical scale factor that relates some close pairs of particle masses. Here, the Higgs field connects to Z^0 by an inverse-mapping G-M,¹⁴ viz.

$$m_{Z^0} = \sqrt{m_H^3 (1/v)},$$

and to W^\pm by the simple scale factor K , as seen in equation (40). In hindsight, the Higgs field could not assign two different (but comparable) masses to Z^0 and W^\pm , both by using G-M averages, so it used two different methods.

(7) Returning now to equation (35), we see the Higgs mass is scaled inversely (by $1/K$) to participate in a G-M with m_s and m_b . Although we have only a partial view of the dynamics of the Higgs field in the above equations, it is obvious that it follows a set of scaling rules in addition to participating in G-Ms. The origin of these scaling rules is unknown to us at this moment, but we feel confident that we have made a step in the right direction with this analysis. The relevant scaling factors may not be visible in the action integrals, but they should then appear in the solutions (a manifestation of symmetry breaking).

(8) The next and considerably more difficult step concerns the assignment of mass to the bottom quark, whose mass is much lower than the Higgs mass and the masses of its decay products. We were surprised to find that yet another method is used by the Higgs boson for this assignment.¹⁵

Notice the unitless factor of 30.0 in equation (36). This equation suggests that the mass scale M_A and the electron mass m_e are related to the mass ratio m_H/m_b . But m_e is not a mass scale, so the proportion M_A/m_e in equation (36) is at least obscure, if not superficial (it does not represent a ratio of mass scales or a ratio of particle masses). Using equation (58) derived below and the equations of § 2.3, we rewrite this proportion (equation (36)) in a palatable (physical) form, viz.

$$\frac{m_b}{m_H} = \frac{1}{30} \simeq \sqrt{\alpha_h} \quad (2.1\%), \quad (41)$$

where $\alpha_h = (861.022576)^{-1}$ is given by equation (13) with the corrective substitution $h \rightarrow \hbar$ [5,6].

Thus, the mass of the bottom quark m_b , which is 30 times lower than m_H , is determined self-consistently from this scaling equation by effectively using the ratio of scales M_p/M_A in intermediate steps and the Planckian fine-structure constant¹⁶ $\alpha_h = e^2/(\epsilon_0 \hbar c)$ in the final step. This is the third method employed by the Higgs boson to assign mass. In particular, it uses this $1/30$ scaling to get down to the bottom quark and, then, into the regime of the lower particle masses (see also Table A1 below). If the m_b assignment also involves the W^- boson (which carries Koide's scale factor K) to deliver charge to the bottom quark, then equations (40) and (41) combine to show

¹⁴ Eliminate m_t between equations (37) and (38).

¹⁵ There is no way for the Higgs boson or its decay products to reach down to the bottom quark mass by G-Ms because of the barrier set by v . When we run the v - W^\pm ... G-M toward lower masses, i.e., $(m_{W^\pm})^2/v$, it reaches a lowest possible mass slot that is $\simeq 2\pi$ times higher than m_b . There is no justification for adopting yet another scaling factor of 2π , as it does not appear in any other G-M relation. Furthermore, introducing geometry in mass assignments does not appear to be an appropriate practice.

¹⁶ We cannot help but wonder—if A. Sommerfeld, W. Pauli, C. Jung, R. Feynman, et al. [30] became familiar with this result, would they show the same fascination for number 861?

that $m_b = (m_{W^-})(\sqrt{\alpha_h}/K) \simeq 0.05(m_{W^-})$. The physical significance of Koide's factor for the high-mass quarks is discussed in detail in Appendix A.2.

3.2. The Planck Charge

The Planck charge q_p is a prime example of the state of confusion in the field: not understanding the significance of the meddling of geometry in the modern Planck units, people adopted different definitions of q_p by arbitrarily choosing between ϵ_0 and ϵ_0 and between \hbar and h . In the end, this unit, along with the Planck units of magnetic flux $[\hbar/(\epsilon_0 c)]^{1/2}$ and ohmic resistance $(\epsilon_0 c)^{-1}$, fell out of favor.¹⁷

Now, we know better. The definition of the Planck charge q_p must be geometry-free, viz.

$$q_p \equiv \sqrt{\epsilon_0 \hbar c} = \sqrt{2\epsilon_0 \hbar c}. \quad (42)$$

Absence of geometry is required, first because this is a unit of charge, and second because q_p provides an alternative definition of the fine-structure constant (which is geometry-independent in its current definition (13)), viz.

$$\alpha_{\hbar} = \left(\frac{e}{q_p} \right)^2. \quad (43)$$

We find that $q_p = 1.8755 \times 10^{-18} \text{ C} = 11.7062e$ (where $11.7062 = \sqrt{137.036}$). Once again, nature shows us here her principle of fairness (or impartiality). As in the case of the electron mass m_e , the elementary charge e here is not related to the fundamental unit of charge q_p by a rational numerical factor; instead, q_p is chosen as the UPS scale of charge that does not correspond to the charge multiple of any specific particle or field.

3.3. A New Atomic Length Scale

Equation (1) can help us determine a new length scale for the UPS, a scale that certainly does not correspond to any of the three atomic radii in equation (1): based on nature's apparent principle of fairness, we understand that none of the known electronic radii can be the fundamental unit of length. But we know that scale values generally fall between particle values and vice versa. To proceed then, we use the G-M of r_e and r_c to determine a new atomic length scale L_A .¹⁸

The G-M of r_e and r_c gives

$$L_A = r_c \sqrt{\alpha_{\hbar}} = \sqrt{\frac{\hbar}{\epsilon_0 c^3}} \left(\frac{e}{m_e} \right), \quad (44)$$

and $L_A = 3.2987 \times 10^{-14} \text{ m} = r_c/11.7062$. The numerical value 11.7062 is the same with that found for the ratio q_p/e (equation (43)) because

$$\alpha_{\hbar} = (L_A/r_c)^2, \quad (45)$$

and then, the following proportion (cross-multiplied) holds exactly:

$$L_A q_p = r_c e. \quad (46)$$

This relation implies that the G-M of the new scales L_A and q_p is equal to the G-M of the traditional and widely-used electronic constants r_c and e ; and it brings to light a previously

¹⁷ See Reference [31] and also article https://en.wikipedia.org/wiki/Planck_units.

¹⁸ The remaining choice, the G-M of r_b and r_c , would give an equivalent result, scaled by a different power of α_{\hbar} ($L'_A = L_A/\alpha_{\hbar}$), such that $r_c = \sqrt{L_A L'_A}$.

unused combination of units with dimensions of [length][charge]. These dimensions are equivalent to

$$\frac{[\text{momentum flux}]}{[\text{magnetic flux}]} = \frac{[\text{momentum}]}{[\text{magnetic field}]} = \frac{[\text{energy}]}{[\text{electric field}]},$$

these interesting units compare mass flows (“matter waves”) to EM waves (“energy flows”), and energy/momentum to EM field components. These quotients also indicate a close correspondence between the relativistic energy-momentum (E - p) equation

$$E = cp, \quad (47)$$

and Maxwell’s EM amplitudes ($\mathcal{E}_0, \mathcal{B}_0$; [32]) relation

$$\mathcal{E}_0 = c\mathcal{B}_0. \quad (48)$$

The above dimensional ratios of units are obtained easily by dividing these two equations. We see then that \mathcal{B}_0 (current flow) is to EM waves what momentum p (mass flow) is to dynamics, and similarly for amplitude \mathcal{E}_0 and energy E .

Length L_A is much larger than the modern Planck length $L_P = \sqrt{\hbar G/c^3} = 1.6163 \times 10^{-35}$ m. (The modern definition of L_P must be used here, because r_c in equation (44) brought its 2-D geometry into L_A , and α_{\hbar} is accidentally geometry-free.) In this case, L_A must be scaled down to produce L_P ; thus, $L_P = L_A\sqrt{\beta_G}$. This scaling-down of L_A should be contrasted to the scaling-up of M_A to produce the original Planck mass M_P (i.e., $M_P = M_A/\sqrt{\beta_G}$; see § 2.3).

3.4. Cosmological Scales and Some Ambivalent Supratomic Particles

In §§ 2.3 and 3.3 above, we rescaled the fundamental scales of the UPS to obtain the corresponding Planck scales. These “A” and “p” values do not describe any specific particle or object in the universe; that would not be a fair choice by nature. But we can extend both scales into the macrocosm by running the G-Ms toward larger masses and lengths.

(a) *Cosmological Mass Scales.*—We evaluate a geometric progression that starts with scales M_A and M_P , and moves on to larger mass scales:

$$\{M_B, M_C, M_D\} = \{1.113 \times 10^{14}, 2.271 \times 10^{35}, 4.633 \times 10^{56}\} \text{ kg}. \quad (49)$$

Mass scale M_D is 2-3 orders of magnitude larger than the current estimates of the mass of the universe [1], so we can halt the sequence at M_D . The common ratio of the geometric progression is $M_P/M_A = 1/\sqrt{\beta_G} = 2.041 \times 10^{21}$. The G-M of M_B and M_C is equal to 0.84 earth masses; and the G-M of M_C and M_D is 5×10^{15} solar masses and identifies universal structures much larger than individual galaxies (e.g., galaxy clusters).

(b) *Cosmological Length Scales.*—We evaluate a geometric progression that starts with scales L_P and L_A , and moves on to longer length scales:

$$\{L_B, L_C\} = \{6.730 \times 10^7, 1.373 \times 10^{29}\} \text{ m}. \quad (50)$$

Length scale L_C is 2-3 orders of magnitude larger than the current estimates of the size of the universe [1], so we can halt the sequence at L_C . The common ratio of this geometric progression is $L_A/L_P = 1/\sqrt{\beta_G} = 2.041 \times 10^{21}$, the same as the common mass ratio given in item (a) above. The G-M of L_B and L_C is equal to 98.5 parsecs, a value typical of giant molecular cloud complexes in spiral galaxies.

(c) *Cosmic Microwave Background (CMB).*—We convert the temperature of the CMB, $T_{\text{CMB}} = 2.7255$ K, to an equivalent mass $m_{\text{CMB}} = 3.52 \times 10^{-10}$ MeV/ c^2 . Since $m_{\text{CMB}} \ll M_A$, we need to extend the geometric progression of mass scales to much lower masses as well.

At the low-mass end of the geometric sequence $\{M_0, M_A, M_P\}$, the tiny mass scale M_0 is found to be $M_0 = 7.35 \times 10^{-21} \text{ MeV}/c^2$. Then, the G-M relation

$$m_{\text{CMB}} = \sqrt{M_0 M_A}, \quad (51)$$

holds to within a 5.7% deviation between the two sides. This deviation is relatively small, given the enormous difference in scales (by 21.31 orders of magnitude) involved on the right-hand side of equation (51).

(d) *A Superatomic Particle Near the Planck Mass?*—The equivalent mass of the CMB photons is so low, that extending the geometric sequence of $\{m_{\text{CMB}}, m_H\}$ to higher masses, we obtain a potential particle mass of $M_S = m_H^2/m_{\text{CMB}} = 4.453 \times 10^{16} \text{ GeV}/c^2 \simeq 1.455 \times 10^{-3} M_P$, which is at the scales where the strong force supposedly joins in with the other forces [33]. Since the Higgs mass is $m_H = 125.25 \text{ GeV}/c^2$, then the energy ratio $\sqrt{\beta_W}$ (analogous to $\sqrt{\beta_G}$ in § 2.3) that scales the strong interaction down to the weak interaction is

$$\sqrt{\beta_W} = \sqrt{\frac{E_W}{E_S}} = \sqrt{\frac{m_H}{M_S}} = 5.30 \times 10^{-8}. \quad (52)$$

This value is smaller by a factor of 20 compared to the usually quoted coupling constant ratio of the weak to the strong interaction. One reason is that the quoted estimates of this ratio in particle physics depend on microphysics [33]; these values are not really constants, since they show some secular dependence on particle energy [34,35]. In any case, it is doubtful that the Higgs field can assign masses above its VEV of $246.22 \text{ GeV}/c^2$ [36].

(e) *Sub-TeV Particles?*—In the atomic world, the Higgs VEV appears to be a barrier against growing more massive nuclei and particles. Nevertheless, researchers are searching the TeV scales in hopes of discovering such particles [37]. If there is a way to jump across the Higgs VEV (which we do not currently see; see also [36]), then the next few particle slots generated by the high-mass geometric progression $Z^0\text{-H-t-}v \dots$ will have energies of 0.351, 0.502, 0.716, and 1.022 TeV.

3.5. Units Jumping to the Forefront

Consider the units of mass M and area R^2 ; think of them, in abstract terms, as gravity and 2-D geometry. It has long been known that one can form two new units from them, the surface density $\sigma = M/R^2$ and the moment of inertia $I = MR^2$. These have been used in physics applications, but they are not very popular, certainly not as popular as the unit of volume density we get from M and R^3 . It has become clear to us that σ and I have not been given due attention in the past, but now they seem to come to the forefront in our analyses of various astrophysical problems.

Surface density σ plays an important role in our recent work on varying- G gravity [1], and provided the cosmological system of units with an important relation used in unit reductions and in the sourcing of the gravitational field: $\sigma \sim a/G$, where a is acceleration (or, equivalently, the gravitational field), so that

$$[\text{surface density}] = \frac{[\text{grav. field}]}{[\text{grav. constant}]},$$

indicating that the surface density $\sigma(r) = M(r)/r^2$ is a dynamical quantity of considerable influence. Moment of inertia I has already played a role in our calculations of units, as we hinted in § 2.1.2. Here, we discuss its potential significance for inertial masses and the weak equivalence principle [21]. We use dimensional analysis throughout this subsection, so all symbols (having their usual meanings) represent units.

Bunker et al. [11] righted a wrong in physics when they pointed out that, in units, $\hbar = E/\Omega$. (In this notation, $[\text{rad}]$ is a unit carried by $\Omega = [\text{rad}][\text{second}]^{-1}$.) The current SI system suffers from this inconsistency, and it should be repaired as soon as possible. The problem that concerns us here is the following: in the original Planck system, one of the

fundamental units was h , and action integrals had dimensions of $h = ET = MR^2T^{-1} = \mathcal{L}$, where \mathcal{L} denotes angular momentum. This description has been in the books since Dirac and Landau's times circa 1930 [8,9,22].

In their dimensional analysis of Newton's G , Landau & Lifshitz [22] used only basic cgs-system units, and they did not mention any interpretation of the units of action. Other authors have mentioned that the action integrals \mathcal{S} appear to have the same unit as angular momentum \mathcal{L} , so $\mathcal{S} = \mathcal{L} = ET$. This is an interesting conclusion, but not a fundamental one, since there exist objects and fields in nature that neither rotate nor revolve (e.g., free spin-0 particles in vacuum and tenuous gases in galaxy clusters).

We have a different interpretation of the units of action \mathcal{S} , which appears to be fundamental, viz.

$$\mathcal{S} = MR^2/T = I/T;$$

that is, \mathcal{S} describes the rate of change of the moment of inertia I/T , where $I = MR^2$, and M is *bona fide* inertial mass. We believe that this interpretation has important ramifications going forward, because it introduces the concept of inertia into the units of all action integrals, including those of non-gravitating (atomic and nuclear) fields. But if the massless fields know about inertia in their variations, then the weak equivalence principle (and its inertial mass) may have already permeated all other physical processes besides gravity. In other words, macroscopic mass is, in a major way, gravity; but its inertia appears to be a universal property,¹⁹ communicated by the vacuum itself (i.e., by the "resisting" properties c and Z_0) in the same way that geometry-dependent vacuum properties (ϵ_0 and μ_0) are also installed in the universe.

To conclude, we believe that force actions cannot be unified before the influence of the vacuum on every single field is delineated and becomes fully understood. This is not going to be an easy task; as we mentioned above, the vacuum interferes in the universe by three distinct ways: dimensional geometry-dependent constants (ϵ_0, μ_0), dimensional geometry-free constants (c, Z_0), and purely numerical imprinted values. (We note that it is this vacuum "activities list" that prompted us to include Z_0 as another building block of the UPS, as shown in equation (3).)

4. Discussion

4.1. Pairs of Fundamental Dimensional Units

Equation (45) shows that two lengths are needed to produce the fine-structure constant α_\hbar in any system of units: the fundamental scale L_A and a Compton-type scale such as r_c . This subsidiary scale cannot be defined by using the fundamental mass scale (then, one gets $\alpha_\hbar = 1$). Therefore, equation (45) defines r_c independently of mass M_A . In our case, this definition is obtained easier from equation (46): $r_c = L_A(q_p/e)$. Using the definition of the fine-structure constant is an integral part of the above derivation of r_c ; and this example justifies our statement that all systems besides the UPS are incomplete, missing at least the unitless coupling constants, and thus, they are incapable of describing all scales and forces in the universe.

Next, we consider Planck's original set of dimensional units $\{c, G, h\}$, with h in place of \hbar to avoid misunderstandings from the introduction of geometry into the units. The speed of light barrier is applicable to all systems of units, but h is not fundamental in the cosmological system and G is not fundamental in the atomic system for "obvious" (now known to be obviously wrong) reasons: negligibly weak influences should not be building blocks at the core of a system. We realize now that all three constants are necessary building blocks, and that the vacuum-force pairs $\{c, G\}$ and $\{c, h\}$ serve two different (but complementary) functions within the UPS:

¹⁹ In the absence of microcosmic inertia, a delta-function impulse delivering specific energy $E/M > c^2$ to an elementary particle would easily achieve motion with speed $v > c$.

(a) The pair of constants $\{c, G\}$ with its universal unit of force²⁰ $F_0 = c^4/G$ and the corresponding unit imprint of the famous Tully-Fisher/Faber-Jackson relation [16,17] $c^4 = GMa_0$ (where $F_0 = Ma_0$; [2–4]) was analyzed in our companion paper [1] in the cosmological system of units. (We discuss the universality of this relation in Appendix B.) Combined with Newton's G , powers of c define units whose purpose is to monitor the effectiveness of forces F in producing motion (speed v). Some of these units are very well-known: $c^2/G \sim F/v^2 = M/R_S$, $c^3/G \sim F/v = Z_m$, $c^4/G \sim F$, and $c^5/G \sim Fv = P$. Here, M is mass, R_S is (Schwarzschild) radius, Z_m is mechanical impedance, and P is power.

(b) With the notable exceptions of $\sqrt{\epsilon_0 hc} \sim q$ (charge) and $\sqrt{h/(\epsilon_0 c)} \sim \Phi_B$ (magnetic flux) (§ 2.1.2), the pair of constants $\{c, h\}$ can only generate composite units, which cannot be viewed as fundamental units in the physical world; although these units do afford some interesting symmetries. For instance, examine the sequence of units $hc \rightarrow [E][L]$, $h \rightarrow [E][T]$, $h/c \rightarrow [M][L]$, and $h/c^2 \rightarrow [M][T]$, before the next powers of c generate some lower-level subsidiary units, e.g., $h/c^3 \rightarrow [M][a]^{-1}$. Combining powers of c with Planck's h , these units are designed to monitor the action integral \mathcal{S} (i.e., integrated energy in time) during motion, although they are not as well-known: $h/c^3 \sim \mathcal{S}/v^3$, $h/c^2 \sim \mathcal{S}/v^2$, $h/c \sim \mathcal{S}/v$, $h \sim \mathcal{S}$, and $hc \sim \mathcal{S}v$. Since action \mathcal{S} determines both speed v and acceleration a , this sequence of units can also be interpreted as: $h/c^3 \sim (E/v^2)/a = M/a$, $h/c^2 \sim (E/v)/a = p/a$, $h/c \sim E/a$, $h \sim (Ev)/a$, and $hc \sim (Ev^2)/a$, where E represents energy and p represents momentum.

The above symmetries are naturally propagated also to derivative units. As a typical case, we discuss the sequence of composite units M/T^n (for integer n) generated by the widely-used pair of units of mass and time $\{M, T\}$, because this sequence holds some surprises. These units apparently measure resistive properties in the material world:

$$\begin{aligned} M/T &\sim F/(R/T) = Z_m \text{ [mechanical impedance]} \\ M/T^2 &\sim F/R = S_m \text{ [mechanical stiffness]} \\ M/T^3 &\sim F/(RT) = \sigma_P \text{ [power][area]}^{-1} \end{aligned} \quad (53)$$

where R represents length and index P represents power. It is surprising that the unit M/T (of the ubiquitous \dot{M} of accretion physics) turns out to be a resistive property of inflowing matter. It is also quite surprising that the “power surface density” σ_P is a member of this sequence of units that describe the various types of mechanical resistance. In Appendix B, we find that power surface density is a universal dynamical quantity, although it appears prominently only in the Stefan-Boltzmann law [38,39]. Its resistive character becomes apparent when we rewrite it in terms of force F and moment of inertia I , viz.

$$\sigma_P = F^2/(I/T), \quad (54)$$

where (I/T) represents resistance due to the rate of change of the moment of inertia. In this equation, we recognize the importance of the force squared F^2 in $\sigma_P \sim M/T^3$. Coming full circle to expressing the resistances in terms of F^2 , we find for the impedance and the stiffness that $Z_m = F^2/P$ and $S_m = F^2/E$, respectively, where E represents energy. Therefore, the magnitude of F^2 appears to be regulated by power in impedance, by energy in stiffness, and by inertial changes in power surface density.

Furthermore, the inertial magnitude itself appears in the next term of the sequence (53), i.e., $M/T^4 \sim F^2/I$, and the integrated quantity (IT) appears next in $M/T^5 \sim F^2/(IT)$. Obviously then, the units of the sequence M/T^n describe resistive properties in which F^2 is regulated by the temporal variations of inertia.

²⁰ Besides combining with G to produce the units of force and power in the cosmological and Planck systems, c does something else that is notable: it combines with ϵ_0 to produce a surprise unit for ohmic resistance: $1/(\epsilon_0 c) = Z_0 = h/e^2$ (see § 2.1.1).

4.2. The Varied Contributions of the Vacuum

The free space known as the vacuum is described by four interdependent constants (ϵ_0 , μ_0 , $c = 1/\sqrt{\epsilon_0\mu_0}$, $Z_0 = \sqrt{\mu_0/\epsilon_0}$). When the vacuum wishes to also promote geometry in some parts of the natural world, then it introduces either $\epsilon_0 \equiv 4\pi\epsilon_0$ or $\mu_0 \equiv \mu_0/(4\pi)$ or both, provided they are not introduced in a product (there is no geometry in $\epsilon_0\mu_0 = 1/c^2$).

From the nongeometric vacuum quantities ϵ_0 and μ_0 , only two additional purely physical quantities can be constructed by G-Ms, the speed of light c and the impedance of free space Z_0 (§ 2.1.1). They both represent upper limits in nature, the only known upper limits communicated by the vacuum to all scales and in all directions within the universe. Their origin is the least (but nonzero) resistance that the vacuum mounts passively against all motions in the material world (see also § 5 below).

Next, we wish to track down the geometry that is imposed selectively by the vacuum, so we rewrite the fundamental G-Ms discussed in § 2.1.1 as follows:

$$\sqrt{\epsilon_0^{-1}\mu_0^{-1}} = c, \quad (55)$$

and

$$\sqrt{\epsilon_0^{-1}\left(\frac{1}{\mu_0^{-1}}\right)} = \frac{Z_0}{4\pi}. \quad (56)$$

The G-M (55) is clearly geometry-free, whereas the G-M (56) attaches the 4π of 3-D space to the geometry-free impedance of free space Z_0 .²¹ This is an important conclusion: when ϵ_0^{-1} or μ_0^{-1} appear on their own in equations, or in any combination other than their product (55), they do carry geometry with them. These composite vacuum constants show us how free space manages to interfere in the construction and evolution of additional (ready-to-interact with one other) physical entities, such as mass and electric charge, that characterize the underlying force fields.

We emphasize here that mass and charge are not actually fundamental quantities, as is widely believed; they can only be derived and clearly understood, if the contributions of the vacuum and the unitless coupling constants are also taken into account. We demonstrate this point here, with exact calculations:

(a) Consider first equation (13). Solving for the charge e , we obtain a scaled-down G-M relation of the form

$$e = \alpha_{\mu}^{1/2} \sqrt{h(2\epsilon_0 c)} = \alpha_{\mu}^{1/2} q_p. \quad (57)$$

So, Planck's physical constant h and the vacuum's combination of $(2\epsilon_0 c)$ determine e as a geometry-free, G-M quantity. From this point of view, we can also see how dimensionless constants resize properties of the material world: the G-M is scaled down by the geometry-free factor of $\alpha_{\mu}^{1/2} \simeq 1/\sqrt{137}$ (see also § 3.2).

(b) Consider next equation (14). Solving for the mass m_e , we obtain a G-M relation of the form

$$m_e = \left(\frac{\alpha_G}{2\pi}\right)^{1/2} \sqrt{\left(\frac{h}{G}\right)c} = \left(\frac{\alpha_G}{2\pi}\right)^{1/2} M_p. \quad (58)$$

In this case, m_e is determined by the G-M of the composite physical constant h/G and the vacuum's c . (G participates because this is a mass assignment.) The G-M is scaled down by a factor of $[\alpha_G/(2\pi)]^{1/2} = 1.670 \times 10^{-23}$ relative to M_p . Because of the inclusion of 2π , this factor is geometry-free, and so is m_e .

²¹ However, we are not aware of a system in which the geometry-dependent term $Z_0/(4\pi)$ is introduced. The impedance of EM modes in waveguides and in ideal dielectrics is a multiple of Z_0 [32] that does not involve the factor of 4π .

By dividing equations (57) and (58), and neglecting for the moment the dimensionless, geometry-free factor $(4\pi/\beta_G)^{1/2} = 7.235 \times 10^{21}$, we obtain a geometry-independent G-M for the electron's charge-to-mass ratio, viz.

$$\frac{e}{m_e} \propto \sqrt{\epsilon_0 G}. \quad (59)$$

Thus, the ratio e/m_e is determined mainly by the G-M of the nongeometric constants ϵ_0 and G (vacuum and gravity, respectively); and the neglected scale factor carries the relative strength of the two unitless coupling constants ($\sqrt{4\pi/\beta_G} \propto \sqrt{\alpha_h/\alpha_G}$) with the geometry taken out of the ratio β_G .

4.3. Sparse Geometric Averaging in Nature

We think we understand why virtually all pairs of constants and units (U_1, U_2) combine in G-Ms²² involving the direct form $U_1 U_2$ or the inversion form $U_1 U_2^{-1}$ (or $U_1^{-1} U_2$). Physically, two basic G-M quantities can be derived from each pair of units. Mathematically, these two operations result in mappings that are always "smooth" since they involve constants; thus, the units of a system of units form a Lie group [13], and the associated Lie algebra can be carried out with ease.

One remaining question is why there are also square roots on top of these unit combinations, establishing thus G-Ms. We fall back to what is already known about G-Ms: compared to the commonly used arithmetic means, G-Ms place significant more weight to the smaller of the two values. Thus, the most obvious property of the geometric averages $\sqrt{U_1 U_2^{\pm 1}}$ is that they assist the smaller physical constants to leave their indelible marks in their combinations with larger constants. In a sense, by not letting small constants become negligible (or dominant) when they are combined with large constants,²³ nature seems to follow a principle of fairness or impartiality at all scales of the universe. The degree of support for the small constants can be quite dramatic for much differing constants, as equations (15) and (19) vividly demonstrate: the G-M $\sqrt{\beta_G}$ gains 21.31 orders of magnitude relative to the ratio β_G , and it connects the Planck scale with the atomic world.

Consider now the subatomic particles discussed in § 3.1. Nature did not make a particle in each individual G-M slot. The mass ladder is mostly empty, and just a few actual particles have materialized in the subatomic scales of the universe [33]. So, there are additional selection criteria (scaling laws) on top of the G-Ms that regulate the creation of particles. Besides the factors of 2 and $\sqrt{2}$ in the equations of § 3.1, we have seen that the Higgs boson does not rely on pure G-Ms to reach down to lower masses; it uses, in addition, two different scale factors, Koide's $K = 2/3$ and $\sqrt{\alpha_h} \simeq 1/30$ (equation (41)), to bypass many available particle slots (see also Table A1 below). In particular, the dramatic drop from the Higgs mass to the mass of the bottom quark can only be described as a deflation of particle mass that bypasses 10 G-M particle slots intervening between m_{Z^0} and m_b .

5. Lingering Issues, Future Prospects, and a Brief Summary

The UPS was summarized in equation (3). The system is not flawless yet, and several issues must be investigated and resolved in the future (see, e.g., footnote 24 below). These issues can be traced to Dirac's introduction of $\hbar = h/(2\pi)$ in place of Planck's h .

It is certainly true that in quantum mechanics, Dirac's composite constant $\hbar/(2\pi)$ always appears in form, and this also prompted Schrödinger [11,18] to absorb the 2π into a convenient new constant K . This tactic tells us that Schrödinger was not aware that he

²² Note that even actual planetary orbits [40] and also theoretical orbits in the virtual Hooke potential [41] show G-M averaging in many of their properties [42,43]. The two types of elliptical orbits have fundamentally different centers, but this is not enough to suppress or modify the ubiquitous geometric averaging that is so obvious in the parameters of the two sets of ellipses.

²³ Arithmetic averaging would favor the large constant, whereas harmonic averaging would turn the tables and clearly favor the small constant. Compared to G-Ms, either one of these extreme averages treats "unfairly" one or the other participant.

was including geometry into his constant K . Dirac [7–9], on the other hand, believed that $\hbar = K$ is the true constant (not h), so we can guess that he sensed that the two constants are fundamentally different in their makeup (see § 1.2 for more details).

Dirac's reform has modified quite substantially the systems of units that have adopted \hbar , but this modification came with a heavy price. Planck's purely physical constant h cannot be dropped so nimbly, because then, we introduce errors in the definitions of the coupling constants. Dimensionless coupling constants should not include geometric dependencies other than ϵ_0 or μ_0 (and these enter only via EM terms); geometry would give the constants an additional unit of [rad] and it would alter their nature. On the dimensional side of vacuum units, c and Z_0 (§ 4.2) are also geometry-free constants for a good reason: they represent upper limits set by the vacuum to be applicable in any direction of space irrespective of the dimensionality of space.

We note another issue concerning \hbar : In the dimensional part of the UPS, the constant \hbar is the only fundamental dimensional unit that introduces geometry in the physical units. This is unusual and a singular property. Although we were inclined to adopt Planck's h in place of \hbar , we did not do so, because we do not know how to choose between the two constants. It seems from the calculations above that the use of h in the definitions of scales (Planck units, coupling constants) is mandatory, but then \hbar may be more appropriate to be retained for particles and fields, as Dirac [7–9] also thought. Perhaps, both constants should be retained in a modified UPS, along with α_h and β_G (see the UPS as described in footnote 24).

Examining now the definitions of the dimensionless units that we summarized in § 2.2 (equations (13) and (14)), we see that α_\hbar is indeed geometry-free ($\epsilon_0 \hbar = 2\epsilon_0 h$), but α_G is not ($\alpha_G \propto 1/\hbar$). We think this is an enormous oversight flying undercover at least since Dirac [8] introduced his “large numbers hypothesis;” and it has prevented physicists from defining an atomic mass scale in the modern Planck system, creating thus an insurmountable obstacle to force unification. The state of confusion can best be seen in the widespread misconception “that G carries units into the action of general relativity, thus gravity is not like the other forces of nature,” taught to thousands upon thousands of physics students for nearly a century. We now understand that gravity is just like the other forces, and it enters the “ring” with one dimensionless (α_G) and one dimensional (G) constant, just as the EM forces do too.

Owing to the omnidirectional nature of the gravitational force, both of its constants should be geometry-free. For this reason, we tried to bypass the problem with the definition (14) of α_G (it effectively carries a unit of [rad], thus it cannot be utilized) and to define new consistent atomic units within the UPS. First, we created a dimensionless ratio $\beta_G = \alpha_G / \alpha_\hbar$ of the coupling constants that describes their relative strength; the \hbar does not partake in this ratio, and the only geometric influence left comes from the EM field. But this does not affect the makeup of the relative strength β_G , since β_G is expressed as a ratio of energies.²⁴

Next, we created a dimensionless geometry-free combination of fundamental units to attach to β_G , viz.

$$\beta_G = GM_A^2 / (hc) = (M_A / M_p)^2,$$

where M_p is the mass scale of the original [5] system of units. Finally, the new atomic mass scale M_A was derived from the known values of β_G and M_p , viz.

$$M_A = M_p \sqrt{\beta_G}.$$

²⁴ We also timidly attempted a preliminary calculation of the scaling between weak and strong interactions, as a ratio of energies β_W (equation (52)). It seems that such energy ratios/comparisons are the way to incorporate consistently the dimensionless constants into the UPS. We can then imagine a complete UPS := $\{c, Z_0, G, h, \alpha_h, \beta_G, \beta_W, \dots; \epsilon_0, \mu_0, \hbar\}$ that includes geometry-free and a set of geometry-dependent units, along with relative β -ratios $\ll 1$ of unitless constants.

The interpretation of this relation is straightforward: the ratio of the two widely different mass scales $M_A / M_p = 4.9 \times 10^{-22}$ is precisely equal to the square root of the relative ratio of the two coupling constants $\beta_G = \alpha_G / \alpha_{\hbar} = 2.4 \times 10^{-43}$.

In § 3, we tested the influence of this mass scale in the atomic and subatomic world, and the results appear to be strong. The mass constant M_A has no trouble meddling in G-Ms (§ 3.1) along with particle (sub)atomic masses that have been measured by experiment [19,27] (but see also footnote 7 for UPS', an alternative system of units based on proton's parameters). In the process, we also clarified the confusion surrounding the so-called Planck charge (§ 3.2), and we also derived a new length scale that had no trouble meshing in G-M calculations with the already-known atomic radii (§ 3.3 and § 1.2).

In §§ 3.1 and 3.4, we calculated both mass scales and actual particle masses at practically all scales of the universe. The Higgs field uses a multitude of methods to distribute masses to selected atomic particles. This diversity of methods is, in part, responsible for hindering progress in the effort to unify the four fundamental forces of nature. The other part concerns the role of the vacuum (§§ 2.1.1 and 4.2). The behavior of the vacuum is not at all what our books describe [e.g., 26,29,33]. As far as we can see, the vacuum is not subject to forcing of any kind, and it seems to be unresponsive to quantum fluctuations, which occur exclusively in fields. By and large, the vacuum appears to be a passive independent entity, with no physical properties of its own, that imposes implicitly certain rules (by resisting the least) to the material world, that all inhabitants must necessarily observe and obey (to within the bounds of the uncertainty principle of course; see also Appendix B.2). Under these circumstances, there is no back reaction from the material world on to the vacuum itself. In hindsight, this conclusion makes sense—how can anything tangible manage to tangle up that which is the epitome of nothingness?

6. Highlights

6.1. Summary of Results

- (1) Current systems of units are incomplete and incapable of describing all aspects of this universe. They do not include some of the fundamental dimensional constants, the dimensionless coupling constants, and all the restrictions installed by the vacuum itself to the material world.
- (2) Each force of nature must be represented in a system of units with a dimensional and a dimensionless coupling constant. If Planck's h is dropped, then the system cannot measure quantities related to quantum phenomena. If Newton's G is dropped, then the system does not include gravity.
- (3a) The fine-structure constant $\alpha_{\hbar} = (861.022576)^{-1}$, not multiplied by 2π , is the only coupling constant that must be included in absolute terms. It has been measured by experiment, and it provides the scale factor $\sqrt{\alpha_{\hbar}} \simeq 1/30$ used by the Higgs boson to deflate and assign mass to the bottom quark; and then, to reach down to all the other lower-mass particles (Table A1 below). On the other hand, the Higgs boson creates the masses of the Z^0 and W^{\pm} bosons and the other quarks by G-M averaging and by using Koide's scale of 2/3 in various incarnations.
- (3b) All other unitless constants must be included in relative terms because only ratios of these coupling constants have physical meaning—they provide relative strengths, just as the dimensional quantities and the units also do.
- (3c) The modern definitions of the unitless coupling constants are incorrect because \hbar was used instead of Planck's physical constant h . Dirac's \hbar is a composite constant that also carries planar 2-D geometry and a unit of $[\text{rad}]^{-1}$; the 2π radians in \hbar have inadvertently reversed the influence of geometry on to the coupling constants.
- (4) The vacuum is a passive entity and not subject to forcing of any kind by the material world. By providing the least (but nonzero) resistance to all motions that occur in its

domain, the vacuum installs upper limits to the material world (c and Z_0 in nearly perfect dielectrics), which must then be included in systems of units as well. These two geometry-free constants also bring $4\pi\epsilon_0$ and $\mu_0/(4\pi)$ with them, in which the influence of 3-D geometry is apparent. (Here, the vacuum's ϵ_0 and μ_0 are both lower limits.)

- (5) The “dark energy” that permeates the universe and that drives its accelerated expansion is not produced by quantum gravity. The vacuum that we described possesses no physical property that could possibly oscillate, and it cannot produce a discrepancy of 120 orders of magnitude in energy density. Furthermore, in recent work, we showed that the spatial variation of Newton's G , coupled to mass surface density, can apparently create positive dark pressure whose magnitude is comparable to the observed value and whose gradient accelerates the expansion of the universe [1].
- (6) The rate of change of the moment of inertia is the universal unit of all action integrals, so it appears that inertia and the weak equivalence principle have been built into all scales of the universe, large and small, gravitating or not.
- (7a) There exists a new atomic mass scale $M_A = 15.0 \text{ MeV}/c^2$ that we found by deflating the original Planck mass M_P by $\sqrt{\beta_G} = 4.900 \times 10^{-22}$, where β_G is the relative ratio of the coupling constants of gravity and fine structure. Of course, in our expanding universe, the event took place in reverse ($M_A/\sqrt{\beta_G} \rightarrow M_P$). This inflation of scale accounted for 21.31 orders of magnitude in mass, and explains how the Planck scale is connected to the atomic world. (At the same time, the atomic scale of length was deflated by precisely the same amount to produce the tiny Planck length.)
- (7b) No (sub)atomic particle is found to occupy a scale value, and the measured masses in the atomic world are connected mostly by G-M averaging. By using G-M averaging, nature (a) remains impartial to designating any one of the particles as being more significant than the other particles; and (b) assigns more weight to the smaller participant in the G-M, thereby assisting smaller forces to leave their marks on the universe.
- (7c) We can relate characteristic atomic constants (charge e , mass m_e , G-M $\sqrt{m_e m_p}$, Compton radius r_c) to scale values (q_p, M_P, M_A, L_A , respectively), but this is not how these physical entities were created; they were created by the Higgs scalings (1/30 and 2/3) and by G-M averaging of other nearby physical entities.
- (8) Leptons, quarks, and bosons get their masses from the Higgs field. The boson-quark mass ladder is shown in Table A1 below. How the Higgs field acquires its mass m_H and its vacuum expectation value v remains a mystery; the only hint in the known masses [27] is that $m_H \simeq v/2$ (to within a deviation of 1.7%).
- (9) Koide's lepton constant $K = 2/3$ is one of the scaling constants used by the Higgs field. We derived it from first principles in Appendix A, and it is also applicable to the high-mass quark triplet c-b-t. We also derived two additional Koide-type constants, $J = 4/7$ (from the low-mass quark triplet u-d-s) and $B = 0.336$ (from the Higgs bosons W^\pm - Z^0 -H). Constant B is barely 0.8% larger than the absolute minimum value of 1/3 that occurs for three equal masses.
- (10) In Appendix B, we pointed out four instances of a universal law that has the general form

$$(\text{a surface density}) \propto (\text{a kinetic scalar quantity})^4,$$

in which the power of 4 is the sum of the 3 spatial degrees of freedom and 1 additional degree of freedom for the scale of the underlying scalar quantity. The three types of surface density involved describe force F , power P , and moment of inertia I , all divided by surface area A . Pressure F/A appears in the Higgs field and the Casimir effect; intensity P/A appears in the Stefan-Boltzmann law; and mass I/A appears

in the Tully-Fisher/Faber-Jackson relation in spiral/elliptical galaxies. It certainly appears that the dynamics of the present universe is driven by the surface densities of various fundamental quantities (see also Appendix B.2).

6.2. Critical Questions and Answers

(Q1) How does Planck mass relate to the atomic world?

—The atomic mass scale $M_A = 15 \text{ MeV}/c^2$ inflates to the Planck mass: $M_A/\sqrt{\beta_G} \rightarrow M_P$, where $\sqrt{\beta_G} = 4.900 \times 10^{-22}$.

(Q2) What is the physical meaning of number 137?

—Number $137 = 861/(2\pi)$, where the 2π is a geometric term carrying the unit of [rad]; so, 137 is not unitless. The actual unitless constant is 861, and the scale factor $\sqrt{861} \simeq 30$ is used by the Higgs boson to assign masses to much lighter particles, starting with the bottom quark and moving on down the mass ladder (Table A1).

(Q3) What is the physical meaning of Koide's constant?

—Koide's $K = 2/3$ is another scale factor used by the Higgs boson to assign masses to lighter vector bosons (the particles W^\pm and Z^0). Koide's formula holds exactly for the leptons $e-\mu-\tau$ and for the high-mass quarks $c-b-t$.

(Q4) How does the top quark get its mass?

—The top quark mass is the geometric mean of the Higgs mass and the Higgs field vacuum expectation value $v = 246.22 \text{ GeV}/c^2$, so that $m_t = \sqrt{m_H v}$.

(Q5) How do Higgs vector bosons get their masses?

—By two different mechanisms: In the ordered high-mass triplet, $Z^0\text{-H-}t$, the Higgs mass is the geometric mean of the Z^0 mass and the top quark mass, so $m_{Z^0} = m_H^2/m_t$. In contrast, W^\pm gets its mass by Koide scaling: $W^\pm = Km_H$.

(Q6) How does the bottom quark get its mass?

—By a third mechanism: The Higgs mass is scaled down by the factor of 30 derived in (Q2) above, so that $m_b = m_H/30$.

(Q7) Is \hbar , rather than h , the true universal constant?

—They both are, but Planck's h is a pure physical constant, whereas Dirac's $\hbar = h/(2\pi)$ is composite and includes also the geometric term 2π that carries the unit of [rad]. Because of this geometric content, an error was committed in the post-Planckian era when \hbar was adopted for the modern definitions of the fine-structure constant and the gravitational coupling constant.

(Q8) Is dark energy a quantum phenomenon?

—No, it is not. The vacuum possesses no physical or tangible property, it only provides the least (albeit nonzero) resistance to all motions of its inhabitants, thereby indirectly installing upper limits to the flow speeds of matter/energy and electric currents (speed c and impedance Z_0 , respectively). According to this interpretation, and in agreement with recent observations and current cosmological models of accelerated universal expansion, we have obtained a reasonable estimate of the dark energy density ($U_0 = 1.35 \text{ GeV m}^{-3}$ corresponding to $\rho_0 = 2.40 \times 10^{-27} \text{ kg m}^{-3}$) created in empty space by the spatial variation of Newton's G [1].

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Abbreviations

The following abbreviations are used in this manuscript:

G-M	Geometric-Mean
MOND	Modified Newtonian Dynamics
UPS	Upgraded Planck System

Appendix A. The Physical Meaning of Koide's Lepton Constant

Koide's constant of 2/3 involves the three lepton masses and it is a puzzle in particle physics [15]. This constant is not a mere numerical coincidence, it is a fundamental scale that the Higgs boson uses to create the W^\pm bosons, and then it is propagated to lower masses according to the relations given in § 3.1.

Koide's constant is defined for the three leptons e-μ-τ as

$$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \equiv 2/3. \quad (\text{A1})$$

Mathematically, this relation is equivalent to an equation involving arithmetic means and G-Ms of paired quantities, viz.

$$f \equiv (m_e + m_\mu + m_\tau) / (\sqrt{m_e m_\mu} + \sqrt{m_e m_\tau} + \sqrt{m_\mu m_\tau}) = 4, \quad (\text{A2})$$

where the numerator should be viewed as the triple sum of arithmetic means of paired quantities, viz.

$$m_e + m_\mu + m_\tau = \frac{1}{2}(m_e + m_\mu) + \frac{1}{2}(m_e + m_\tau) + \frac{1}{2}(m_\mu + m_\tau).$$

Equation (A2) reveals a special relationship between the three arithmetic means in the numerator and the three G-Ms in the denominator: the ratio of their averages must be equal to 4; then, equation (A1) is an identity, as Koide [15] discovered. This special relationship is recovered from the equations of § 3.1. We carried out several reductions²⁵ by evaluating lower masses starting from the Higgs mass and the masses of m_{W^\pm} , m_{Z^0} , and m_t . The endpoint is a pair of inversion G-M relations between lepton masses, viz.

$$\sqrt{m_\mu/m_e} = 22K, \quad (\text{A3})$$

and

$$\sqrt{m_\tau/m_e} = 40K^{-1}. \quad (\text{A4})$$

Their product is independent of K , and its value (880) carries an error of 4% as compared to the experimental value of 848. The coefficients in these ratios were rounded off to obtain a 2% accuracy in each individual ratio (not rounded, the coefficient in equation (A4) has a deviation of 5%, sufficiently large to produce a higher K -value of 0.70).

Substituting the above ratios into equation (A2), we find a fourth-order polynomial equation of the form

$$484K^4 - 88K^3 - 3519K^2 - 160K + 1600 = 0, \quad (\text{A5})$$

²⁵ From (35) and (36), we get $m_b = 30m_s/K$ (#1). From (32), (38)-(40), and (36), we get $m_b = 30K^{-3/4}m_\mu$ (#2). From (#1) and (#2), we get $m_s = K^{1/4}m_\mu$ (#3). From (27), (#3), and (#2), we get $m_\mu = 30K^{-5/4}m_u$ (#4). From (26), (#3), (#4), and (30), we get equation (A4). Finally, from (34), (36), and (A4), we get equation (A3). Equations (28), (31), (33), and (37) were not used.

The solutions of the fourth-order equation can be obtained analytically, and they are all real. The two positive roots are

$$K_1 = 0.66641 \quad \text{and} \quad K_2 = 2.7283. \quad (\text{A6})$$

Root K_1 is Koide's constant; it determines the lepton mass ratios, and they, in turn, satisfy equation (A1) to within an error of 0.04% (Koide's $K = 2/3$ is an extremely robust physical constant, as experimenters have discovered).

Root K_2 is rejected based on the experimental results. Substituting K_2 into the equations of the lepton mass ratios reverses the two values, and results in $m_\mu > m_\tau$. This solution is obviously incompatible with the measured masses of these two particles [27].

Despite the rejection, K_2 is of some theoretical interest: the ratio $K_2/K_1 = 4$, the same 4 that appears in equation (A2). Then, we find that the $f = 4$ factor in equation (A2) is the raw physical constant, and Koide's K is derived from it:

$$K = \frac{f}{f+2} = \frac{2}{3}. \quad (\text{A7})$$

This relation is derived by substituting equation (A2) into equation (A1).

Appendix A.1. Physical Interpretation

The physical interpretation of nature's choice of $f = 4$ is deduced from equation (A2), rewritten in the accessible form

$$\frac{1}{3}(m_e + m_\mu + m_\tau) = \frac{4}{3}(\sqrt{m_e m_\mu} + \sqrt{m_e m_\tau} + \sqrt{m_\mu m_\tau}). \quad (\text{A8})$$

The factor of $1/3$ indicates that the left-hand side is the arithmetic mean of the lepton masses. The factor of $4/3$ on the right-hand side (also the G-M $\sqrt{K_1 K_2}$ to within 1%) is C_F , the quadratic Casimir charge of the SU(3) fundamental representation [29]. It seems then that the assignment of masses to the leptons is constrained also by the delivery of charge, and this is why

$$f = 3C_F, \quad (\text{A9})$$

appears in the right-hand side of equation (A2). This last equation is a special case of the general formula ([29], section 4.5) of quantum chromodynamics, viz.

$$\frac{N_A}{2} = NC_F, \quad (\text{A10})$$

as applied to SU(3), where $N = 3$ dimensions, the indices $1, 2, \dots, N_A$ label the $N_A \times N_A$ color generators in the "octet" quark-antiquark state, $N_A = N^2 - 1 = 8$, and $f = N_A/2$; thus, we find that $f = 4$ and $C_F = 4/3$ in SU(3). Finally, Koide's constant turns out to depend only on the SU(3) octet number N_A , viz.

$$K = \frac{N_A}{N_A + 4}. \quad (\text{A11})$$

Appendix A.2. Additional Koide-type Constants

The G-M relations of § 3.1 can help us make physical sense of various other triple combinations of masses involving quarks or bosons that have been previously derived by numerology. Here, we summarize the calculations for three triplets that naturally come to mind, the quarks c-b-t and u-d-s, and the bosons W^\pm - Z^0 -H:

High-mass quarks c-b-t.—Based on experimental masses, equation (A1) with c-b-t values in place of e- μ - τ values produces a constant of 0.669 on the right-hand side, only 0.35% higher than $K = 2/3$. This is a solid physical result. From the equations of § 3.1, we find

Table A1. Boson-Quark mass ladder in terms of the Higgs mass m_H . Two scales are used, Koide’s $K = 2/3$ and the $1/30$ deflation of m_H down to the bottom quark mass m_b .

Particle	Mass Relation	Deviation* (%)
BOSONS		
Z^0	$m_{Z^0} = K^{3/4}m_H$	+1.3
W^\pm	$m_{W^\pm} = Km_H$	+3.9
QUARKS		
top	$m_t = K^{-3/4}m_H$	−1.6
bottom	$m_b = m_H/30$	−0.12
charm	$m_c = 2\sqrt{K/30^3}m_H$	−2.0
strange	$m_s = (K/30^2)m_H$	−0.67
down	$m_d = 2(K^{7/4}/30^3)m_H$	−2.3
up	$m_u = (K^2/30^3)m_H$	−4.6

* Deviation = [(right-to-left side) − 1] × 100%.

that $m_t = 30m_bK^{-3/4}$, $m_c = 2m_b\sqrt{K/30}$, and a corresponding constant of 0.668 with a deviation of 0.20% from $K = 2/3$.

Low-mass quarks u-d-s.—Koide’s K is not produced by the masses of the u-d-s triplet. Based on their experimental masses, equation (A1) with u-d-s in place of e-μ-τ produces the constant $J = 0.567$ on the right-hand side, probably a value of no interest to numerology. We, on the other hand, have derived this constant analytically by utilizing the G-M relations of § 3.1 and by expressing the u-d-s quark masses in terms of the Higgs scales $m_b/m_H = 1/30$ (deflation) and $K = 2/3$ (Koide), used in assignments of masses lower than m_H . It turns out that the entire boson-quark mass ladder has to be calculated in the process. The results of our calculations are listed in Table A1. Using the values obtained for the u-d-s masses at the bottom of the mass ladder, we find that

$$(m_u + m_d + m_s)/(\sqrt{m_u} + \sqrt{m_d} + \sqrt{m_s})^2 = 0.570, \tag{A12}$$

a constant that deviates only by 0.53% from the experimental value of $J = 0.567$. In this case, we find that $0.570 \rightarrow 4/7$, $f = 2C_F = N_A/N = 8/3$, and $J = N_A/(N_A + 2N) = 4/7$ in SU(3) (in place of equations (A9)-(A11)).

Higgs bosons W^\pm - Z^0 -H.—Based on experimental masses, equation (A1) with W^\pm - Z^0 -H masses in place of e-μ-τ masses produces a constant of $B = 0.336$ on the right-hand side, only 0.80% higher than the lowest attainable value of $1/3$ obtained for three equal masses. From equations (39) and (40), we find that $m_{Z^0} = K^{3/4}m_H$ and $m_{W^\pm} = Km_H$ (see also Table A1), and then, equation (A1) for the W^\pm - Z^0 -H triplet is transformed to

$$1 + K^{3/4} + K = 0.336\left(1 + K^{3/8} + K^{1/2}\right)^2, \tag{A13}$$

with an accepted root at $K_1 = 0.662$ (deviation 0.70% from $K = 2/3$) and a rejected root at $K_2 = 1.541$. We conclude that K_1 is Koide’s constant, in which case, $f = 1.012$ on the right-hand side of equation (A2) and $B = f/(f + 2)$ (let $B \rightarrow K$ in equation (A7)). The quark-antiquark color octet number N_A and the quadratic Casimir charge C_F are not involved in these calculations (equations (A9)-(A11) are not applicable to bosons).

Appendix B. A Universal Natural Law Discovered in Widely Distant Scales

The work that we presented in this paper was triggered by the realization that the unit of force $F_0 = c^4/G$ is precisely the same in the cosmological and the Planck systems of units. Furthermore, this unit takes the form of the [speed]⁴ Tully-Fisher/Faber-Jackson

relation [16,17] discovered in galaxies (§ 4.1). This is astonishing, given the tremendous difference in scales between the two systems of units.

The implication is that such a fundamental relation ought to exist also in the Planck scale, and presumably in the microcosmic scales as well. Indeed, they do exist, and the first such relation dates back to Stefan [38], who discovered the famous [temperature]⁴ dependence of an emitting blackbody's intensity or power surface density with units [power][area]⁻¹. We searched and found that such a universal law has been discovered in all of the above scales, but a connection has not been previously made because the corresponding subfields of physics are presently disjoint.

The universal law involves *the surface densities* of various fundamental quantities. (Such surface-density dynamical variables have become of primary importance in the work presented in our recent paper [1].) These surface densities are all related to the fourth power of kinetic terms which are limited by the various small resistances imposed by the vacuum. Specifically:

- (1) In quantum gravity, the energy-density shift of the Higgs field U_H resulting from spontaneous symmetry breaking (that prevents ultraviolet divergence) is $U_H \propto v^4$, where v is the Higgs vacuum expectation value [26]. This relation is equivalent to

$$\sigma_F \propto v^4, \quad (\text{A14})$$

where the "force surface density" $\sigma_F \equiv F/A$ (where F is force, A is area, and $U_H = F/A$ has dimensions of [pressure]).

- (2) In the macroscopic realization of the Casimir effect, the same force per unit area is proportional to the fourth power of the reciprocal of distance D between parallel plates [44,45], viz.

$$\sigma_F \propto (1/D)^4. \quad (\text{A15})$$

The units agree in the last two relations, since v above has dimensions of [distance]⁻¹ [26].

- (3) In atomic physics, the celebrated Stefan-Boltzmann law [38,39] takes the equivalent form

$$\sigma_P \propto \Theta^4, \quad (\text{A16})$$

where the "power surface density" (intensity) $\sigma_P \equiv P/A$, P is power, and Θ is mean temperature.

- (4) In astrophysics, galaxies obey the relation $M \propto V^4$, where M is mass and V is rotational speed or stellar velocity dispersion in spiral [16,46–48] and elliptical [17,49,50] galaxies, respectively. This relation is equivalent to

$$\sigma_I \propto V^4, \quad (\text{A17})$$

where the "moment-of-inertia surface density" $\sigma_I \equiv I/A$ (where I is moment of inertia and I/A has dimensions of [mass]).

Appendix B.1. Dimensional Analysis of Surface Densities

Dimensional analysis can help us understand the meaning of these surface densities, but not by reducing their definitions to the fundamental units of the UPS. We have to search a little harder to find any common properties between these quantities. We begin with the power surface density (wave intensity) σ_P that assumes the simplest form among the surface densities:

$$\sigma_P = \frac{F^2}{(I/T)}. \quad (\text{A18})$$

In EM interactions, the rate of change of moment of inertia can be replaced by

$$I/T = q^2 Z_0, \quad (\text{A19})$$

where q is charge and $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is vacuum impedance; we find that

$$\sigma_P = Z_0^{-1} \mathcal{E}^2, \quad (\text{A20})$$

where the electric field is given by $\mathcal{E} = F/q$.

For gravitational power, the force in equation (A18) is also modified by the vacuum, but only by coupling to Newton's G . Rewriting equation (A18) in terms of the gravitational field (acceleration) a , we find that

$$\sigma_P = \left(\frac{c}{G}\right) a^2. \quad (\text{A21})$$

Force is $F = P/c$ in terms of power P , and the force surface density σ_F takes the corresponding forms

$$\sigma_F = \frac{\sigma_P}{c} = \epsilon_0 \mathcal{E}^2 = G^{-1} a^2, \quad (\text{A22})$$

where the vacuum's c drops out from gravity's σ_F . This is a fundamental difference as compared to the EM field's σ_F , in which the vacuum (the ϵ_0 here) is permanently attached.

Finally, as was probably expected, the moment-of-inertia surface density $\sigma_I = M$ does not quite conform to the above picture. We find that

$$\sigma_I = \frac{F^2}{(I/T^4)} = \frac{\mathcal{J}^2}{(I/T^2)}, \quad (\text{A23})$$

where \mathcal{J} is impulse and I/T^2 is energy. Mass is already built with inertia, and it is not surprising that it does not scale as $(I/T)^{-1}$, as the other densities do. To find out how this force squared and impulse squared are regulated, we rewrite the terms in the denominators. It turns out that

$$I/T^3 = P, \quad (\text{A24})$$

so that equation (A23) then takes the form

$$\sigma_I = \frac{F^2}{(P/T)} = \frac{\mathcal{J}^2}{PT}, \quad (\text{A25})$$

where the integrated quantity PT represents energy E . Thus, F^2 is regulated by the rate of change of power P/T , and \mathcal{J}^2 is regulated by energy PT , both of which are restricted by the speed of light.

Appendix B.2. Physical Properties of Surface Densities

We conclude with a summary of properties of the above three surface densities:

- (a) The densities σ_P and σ_F (wave intensity and pressure, respectively) are both modified by the rate of change of inertia (I/T) at all scales (equations (A18) and (A22)).
- (b) Density σ_I (i.e., mass) is not modified by inertia, it is inertia; instead, we can say that mass is force squared F^2 regulated by the rate of change of power (P/T), or impulse squared \mathcal{J}^2 regulated by energy E (equation (A25)), where E should be viewed here as the rate of change of the action integral, i.e., (S/T) .
- (c) Vacuum constants are explicitly present in σ_P (equations (A20) and (A21), where σ_P is written in terms of the force fields squared \mathcal{E}^2 and a^2 , respectively).
- (d) The vacuum remains present in the σ_F of the EM field, but it drops out from the σ_F of the gravitational field (both behaviors are shown in equation (A22)).

- (e) The force surface density σ_F (equation (A22)) represents the conventional energy density of the force fields, whereas σ_P (equation (A18)) shows that inertia is present during the action of all forces.
- (f) Both sides of equation (A19) have dimensions of Planck's constant h , thus $I/T \sim q^2 Z_0 = [\text{action}]$. Higher powers of T in I/T^n ($n = 2, 3$) are also physically quite important: $I/T^2 = [\text{energy}]$ and $I/T^3 = [\text{power}]$. Equation (A24) for I/T^3 then implies that power stems from the third time derivative of the moment of inertia, a property that is fundamental for the emission of gravitational waves. The same relation, applied to EM waves, produces the ohmic power with dimensions of $[\text{electric current}]^2 [\text{ohmic resistance}]$.
- (g) Equations (A14)-(A17) all have the characteristic form

$$[\text{a surface density}] \propto [\text{a kinetic scalar quantity}]^4,$$

in which the power of 4 represents $N + 1$ degrees of freedom, with $N = 3$ for the spatial dimensions, plus 1 degree of freedom for the scale of the underlying scalar quantity.

- (h) The intensity σ_P and the pressure σ_F are proportional to kinetic terms that express $[\text{specific energy}]^4$. The Tully-Fisher/Faber-Jackson relation $M \sim V^4 \sim [\text{specific energy}]^2$ [16,17] then indicates that, in units where $c = G = 1$ ([26], p. xix), it is the square of the mass

$$M^2 = (\sigma_I)^2 \sim [\text{specific energy}]^4,$$

that falls in the same category. Thus, we believe that the M^2 term here is meant to signify that the inertial and the gravitational mass are included on an equal footing (just as in equation (12) discussed in § 2.1.3).

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