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Mechanical conversion of the gravitational Einstein's constant κ

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Abstract. This study attempts to answer the question of what space is made of and explores in this objective the analogy between the Einstein's gravitational geometrical theory in one- and two-dimensional linear deformations and a possible space material based on strain measures done on the Ligo or Virgo interferometers. It draws an analogy between the Einstein's gravitational constant κ and the Young's modulus and Poisson's ratio of an elastic material that can constitute the space fabric, in the context of propagation of weak gravitational waves. In this paper, the space is proposed to have an elastic microstructure of 1.566×10^{-35} m grain size as proposed in string theory, with an associated characteristic frequency f. The gravitational constant G is the macroscopic manifestation of the said frequency via the formula $G = \pi f^2/\rho$, where ρ is the density of the space material.

Keywords. Space–time fabric; general relativity; quantum mechanics; Young's modulus; strength of the materials; gravitational waves; gravity Probe B; Hubble's law; space–time curvature; Einstein's constant; dark matter; string theory; graviton.

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1. Introduction

Quantum mechanics and general relativity are the twin pillars of modern physics, but while they have coexisted they have remained broadly irreconcilable.

In order to solve this dilemma, we must go back to the foundations of these two theories to see if something must not be changed at a fundamental level so as to bring them closer.

To date, the general relativity [1–3] clearly dethroned the gravitation according to Newton. It is clear that the concept of Newton's gravitational force is in fact an illusion. General relativity shows indeed that two masses fall against each other not because they attract each other but because they follow the curvature, the deformation of the space–time. But have we really drawn all the consequences of this conceptual error in Newton's gravitation?

In fact, Newton's formula is the basis of the definition of the gravitational constant since the Newtonian gravitational force F is proportional to the gravitational constant G, to the product of the masses M and m, and is inversely proportional to the square of the radius r which separates these two masses (1).

$$F = G \times \frac{M \times m}{r^2}. \tag{1}$$

Considering that this Newton's mathematical expression of the gravitation is only a weak field simplification of general gravitation, that an illusion of force, should we not also consider that the constant G is also an illusion disappearing with the Newton's formula who created it?

Just as it is necessary to abandon Newton's formulation in strong gravitational field, should we not also abandon G as an indivisible universal constant because it is at the basis of this Newtonian formula (1)?

But in this case, is it possible to reconstruct the Einstein's constant κ without going through G but by going through a different theory? Can we separate G from the more fundamental parameters?

To answer these questions it is interesting to compare the strong and weak points of the gravitation according to Newton and Einstein.

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The strong points of Newton's gravitation are

(a) It allowed the discovery, mathematically, of the planet Neptune by Urban Le Verrier. Therefore it works obviously well.
(b) It explains all the effects of gravity on Earth and in the solar system except the Mercury perihelion delay (effect of strong gravitational field near the Sun).

The weak points of Newton's gravitation are:

(a) It only works if the objects have masses. It is not, therefore, possible to predict the curvature of a ray of light tangent at the Sun by the effect of gravitation, though predicted by general relativity and verified by Arthur Eddington on May 29, 1919 [4].
(b) The forces are instantaneous (therefore applied faster than the speed of light) and their mode of transmission is as yet unexplained while the special relativity shows that no phenomenon can be faster than the speed of light.
(c) Its results are imprecise for the action of gravity in strong field. Indeed, the delay in the perihelion of the Mercury planet is defined exactly by the general relativity at 43 arcsec [5] while it is much weaker using the Newton's gravitational approach.
(d) The formulation depends on the parameter r, and if the objects of mass M and m rotate with respect to each other at speeds tending towards the speed of light, the effects of the special relativity change the notion of distance for each observer. Which value of r should be used in calculations in this case?
(e) In this formulation, space–time is a rigid non-deformable object, whereas in general relativity it is precisely the deformation of space–time that generates gravitation giving us the illusion that forces act and attract objects between them.

In view of all these points, it is therefore clear that Newton's concept of force is meaningless. It is an illusion. This formulation is only a simplification of a larger theory, the general relativity. This formulation works in low gravity fields but is false in strong fields.

We can also ask ourselve, why the Universe would depends on a constant G with strange dimensions: inverse of a density by a frequency squared (see (2a) and (2b)). Some authors propose, based on this dimensional equation, that G depends effectively on density and frequency [6].

$$G = 1/(\mathrm{kg/m^3}) \times 1/\mathrm{s}^2 \tag{2a}$$

$$G(\rho, f^2) = cte \times \frac{1}{\rho} f^2. \tag{2b}$$

In general relativity [1], G is introduced because it comes from the use of Poisson's equation (3a) to calibrate the constant κ (eqs (4a) and (4b)) (analysis at the component 00, time t of the different tensors of the Einstein's gravitation formula): see (3b).

Indeed, the Laplacian of the gravitational field, $\Delta\phi$, follows the Poisson's equation

$$\Delta\phi = 4\pi G\rho \tag{3a}$$

and the 00 component of the metric $g_{\mu\nu}$ is then

$$g_{00} \approx 1 + \frac{2\phi}{c^2}. \tag{3b}$$

It is also surprising, in hindsight, to see that Einstein precisely calibrates his equations on the time component of his tensors (3b), while precisely the formulae of Newton (1) and Poisson (3a) that follow are independent of time!

κ (4a) connects the Ricci tensor $R_{\mu\nu}$ (issued of the tensor contraction of the curvature tensor, a function of the metric $g_{\mu\nu}$ and of its second partial derivatives), with the stress energy tensor $T_{\mu\nu}$ (external mass/energy applied at the space–time fabric) in the Einstein's gravitational field equation (5a).

$$\kappa = \frac{8\pi G}{c^4} \tag{4a}$$

$$\kappa = \frac{\frac{\mathrm{m^3}}{\mathrm{kg\ s^2}}}{\frac{\mathrm{m^4}}{\mathrm{s^4}}} = \frac{\mathrm{s}^2}{\mathrm{kg\ m}} = \frac{1}{\mathrm{Newton}} = N^{-1}. \tag{4b}$$

The Einstein's gravitational field equation is

$$G^{\mu\nu} = \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) = -\frac{8\pi G}{c^4} T^{\mu\nu} = -\kappa T^{\mu\nu} \tag{5a}$$

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\ m}}{\mathrm{m}^3}. \tag{5b}$$

We do not show here the cosmological constant Λ as the possible source of dark energy [7]. In addition R is a tensorial contraction of $R_{\mu\nu}$.

Additionally, as the Newton's expression (1) is false for the concept of force (it is the deformation of space–time which gives the illusion of an attractive force between two objects of mass M and m and so there is no attractive force), and as the constant of gravitation is directly related to this concept of force (see Newton's gravitation formulation), is it not necessary to abandon the proportionality factor G associated with this force in general relativity?

Therefore, the question which needs to be asked is the following: If Newton's gravitation did not exist, could Einstein have been able to calibrate κ without going through the Newton's limit, without using the Poisson's equation and the time component (00) of its tensor, but by using directly the spatial components (1,2,3) of its tensors indicated in bold in (5c)?

$$\begin{bmatrix} G^{00} & G^{01} & G^{02} & G^{03} \\ G^{10} & \mathbf{G^{11}} & \mathbf{G^{12}} & \mathbf{G^{13}} \\ G^{20} & \mathbf{G^{21}} & \mathbf{G^{22}} & \mathbf{G^{23}} \\ G^{30} & \mathbf{G^{31}} & \mathbf{G^{32}} & \mathbf{G^{33}} \end{bmatrix} = \kappa \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & \mathbf{T^{11}} & \mathbf{T^{12}} & \mathbf{T^{13}} \\ T^{20} & \mathbf{T^{21}} & \mathbf{T^{22}} & \mathbf{T^{23}} \\ T^{30} & \mathbf{T^{31}} & \mathbf{T^{32}} & \mathbf{T^{33}} \end{bmatrix}. \tag{5c}$$

Or is it possible to find the Einstein constant κ by going through a different theory using the spatial components of the gravitational field tensors?

To define which theory to use, we must now explore the strong and weak points of general relativity. The strong points of the general relativity are:

(a) It addresses all the gaps of Newton's formulation with an extraordinary precision.
(b) It predicts the curvature of a ray of light passing near the Sun during an eclipse.
(c) It introduces time in tensorial writing in four dimensions. The phenomena are no longer instantaneous and respect the special relativity.
(d) It predicts exactly the delay of the perihelion of the Mercury planet, in a strong gravitational field.
(f) It no longer considers space–time as a rigid physical object but as a deformable object, compatible with observations made more than a century ago.

The weak points of the general relativity are:

(a) The Einstein's field equation is a mathematical description of the space–time deformation due to energy density present in this space. Gravitation is therefore mathematically described by the metric of the space–time $g_{\mu\nu}$. This metric is a mathematical description in four dimensions of the space–time deformation.

But this mathematical (geometrical) description of the space said nothing about the physical nature and the possible mechanical properties of this deformed elastic medium constituting the space fabric [8,9].

If we take the analogy of a simple beam in pure bending in Timoshenko's strength of the material, a simplification of the elasticity theory, it is possible to understand the fundamental concept used in Einstein's general relativity (space–time curvature = κ energy density) on the one hand and to assimilate the Mohr's stresses circle in elasticity (see figure 12) at the spin of 2 of the graviton in quantum field theory on the other hand. Indeed, when the facet carrying the stresses makes a complete rotation of one turn on the elastic model in reality, the facet carrying the stresses on the Mohr's circle makes two turns (see Feynman lectures on gravitation – lecture 3, paragraph 3.4, figures 3.3 and 3.4 [10]).

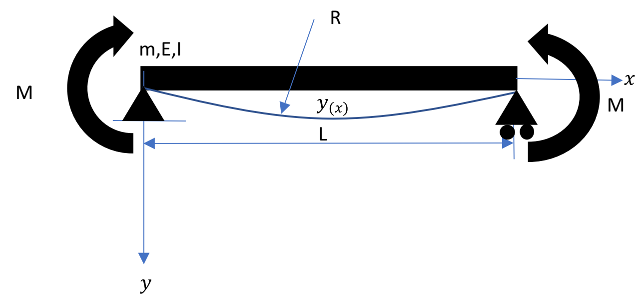

Figure 1. Timoshenko beam, with radius of curvature R, deflection y, loaded by two equal bending moments M.

So, this beam (see figure 1) has a span L (unit m), is made of an elastic material of Young's modulus $Y = E$ (unit MPa $=$ MN/m^2), has a section $S = bh$ (unit m^2), an inertia $I = bh^3/12$ (unit m^4), a mass m (unit kg/m) and a radius of curvature R (unit m).

The equivalent of the curvature equation of this beam analogy, is the equation which defines in elasticity (strength of the material is a simplification of this theory) the deflection of the beam $y_{(x)}$ and the rotation $\theta_{(x)}$ of the beam sections under the influence of two external bending moments M applied at each end which act as the outer mass curving the beam.

The relation to find the equivalent geodesic $y_{(x)}$, depending on the curvature is

$$\frac{\mathrm{d}^2 y_{(x)}}{\mathrm{d}x^2} = \frac{\mathrm{d}\left(\frac{\mathrm{d}y_{(x)}}{\mathrm{d}x}\right)}{\mathrm{d}x} = \frac{\mathrm{d}\theta_{(x)}}{\mathrm{d}x} = \frac{1}{R}. \tag{6}$$

As we are in pure bending, we have ($M_{(x)} = cte$):

$$M_{(x)} = M = -\frac{EI}{R}. \tag{7}$$

The strain energy U (= work done by internal forces) of the beam in pure bending is

$$U = \frac{1}{2}\int_0^L \frac{(M)^2}{EI}\mathrm{d}x = \frac{M^2 L}{2EI}. \tag{8a}$$

In expression (8a), we see the link between the external work, function of M on the right and the strain energy U due to the inner work on the left. By substituting M in

(8a) for its value given in (7), we obtain for the internal forces or internal strain energy of the beam:

$$U = \frac{1}{2}\frac{EIL}{R^2}. \tag{8b}$$

We can reformulate eq. (8b) to extract the curvature term

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2}. \tag{8c}$$

Or to have a similar presentation to Einstein's formalism, we replace $2/EI$ by K and we obtain

$$\frac{1}{R^2} = K\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2}. \tag{8d}$$

When $K = 2/EI$, the coupling constant (8d) is between the curvature, $(\frac{1}{R^2})$, on the left and the linear strain energy density, $(\frac{U}{L})$, on the right. The constant K is therefore identical to flexibility (8c). In addition, we can write the expression of the external work created by the two moments on the beam. The expression of the external work of moment M is

$$W_{\text{ext(1 moment)}} = \frac{1}{2}M\theta. \tag{9a}$$

The rotation θ on each support of our beam under two constant moments is:

$$\theta = \frac{ML}{2EI}. \tag{9b}$$

By referring (9b) to (9a) we obtain for the moment, a new expression of the external work applied on the beam:

$$W_{\text{ext(1 moment)}} = \frac{M^2L}{4EI}. \tag{9c}$$

We can express M^2 according to the total external work of the beam under two constant moments M at each end:

$$W_{\text{ext(total)}} = \frac{M^2L}{2EI}. \tag{9d}$$

So we have

$$M^2 = \frac{2EI}{L}W_{\text{ext(total)}} \tag{9e}$$

That we can substitute in the expression of the inner work U (8c):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right). \tag{10a}$$

Thus, the equation of curvature of a beam in pure bending (10a) can be considered as a one-dimensional analogy of the equation of the Einstein field in four dimensions (5a) (see table 1):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right) = K\left(\frac{W_{\text{ext(total)}}}{L}\right) \tag{10b}$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{\mu\nu}\right) = -\kappa\left(T^{\mu\nu}\right). \tag{10c}$$

For memory, the scalar curvature R of a sphere of radius r, is $2/r^2$. And this parallelism also teaches us that it must exist as an internal work of the fabric of space–time taking into account expression (10d); tensor not developed by Einstein. We shall call $M^{\mu\nu}$ this tensor (see (10e) and (10h)):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right) = K\left(\frac{W_{\text{ext(total)}}}{L}\right) = \frac{2}{EI}\left(\frac{U}{L}\right) \tag{10d}$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} = -\kappa T^{\mu\nu} = -\kappa M^{\mu\nu}. \tag{10e}$$

Indeed, the masses/energies applied within the structure of the space bend the fabric which constitutes it. As a result, its fibres compress, stretch, shear and bend. These actions generate an internal work, a strain energy.

The equation to the dimensions of the fundamental principle linking curvature and strain energy density is (10f)

$$\frac{1}{\text{m}^2} = \frac{\text{s}^2}{\text{kg m}^3} \times \frac{\left(\frac{\text{kgm}^2}{\text{s}^2}\right)}{\text{m}}. \tag{10f}$$

What we can introduce again

$$\frac{1}{\text{m}^2} = \frac{\text{s}^2}{\text{kg m}} \times \frac{\text{U}}{\text{m}^3} = \frac{1}{\text{m}^2} = \frac{1}{\text{N}} \times \frac{\text{U}}{\text{m}^3}. \tag{10g}$$

But the deflection line $y_{(x)}$ of the beam is not the beam itself. There is a physical object in the form of the beam at the start! So if there is a parallelism between the concepts of strength of the material (and by extension elasticity theory) and general relativity (see table 1), it must exist as an elastic substance in the space, a fabric, a sort of frame in correspondence with the beam.

In addition, the beam has a characteristic frequency f (eigenvalue) and a material of density ρ, and so if space is like a beam in each of its three dimensions, it must also intrinsically have these two fundamental characteristics.

In this parallelism, there are only two major differences:

1. The constant K depends on the rigidity of the medium and is expressed from the bending inertia of the beam I and the Young's modulus E characterising the material of the beam which is not the case for κ (in appearance) which is defined definitively according to G and c (non-variable constant value as a function of the space–time fabric characteristics).
2. The strain energy U depends on the beam itself. In general relativity, the stress energy tensor is

Table 1. Parallelism between strength of the material in bending (1D) and general relativity (4D).

Parameters	General relativity (four dimensions)	Unit (s)	Strength of the material (one dimension)	Unit (s)
Principle	$G^{\mu\nu} = \kappa T^{\mu\nu}$	$\frac{1}{\mathrm{m}^2}$	$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{2}{EI}\left(\frac{W_{\mathrm{ext(total)}}}{L}\right)$	$\frac{1}{\mathrm{m}^2}$
Curvature	$G^{\mu\nu}$	$\frac{1}{\mathrm{m}^2}$	$\frac{1}{R^2}$	$\frac{1}{\mathrm{m}^2}$
Strain energy density of the space fabric	Have to be built $M^{\mu\nu}$	$\frac{\mathrm{N\ m}}{\mathrm{m}^3}$	$\left(\frac{U}{L}\right)$	$\frac{\mathrm{N\ m}}{\mathrm{m}}$
External work	$T^{\mu\nu}$	$\frac{\mathrm{N\ m}}{\mathrm{m}^3}$	$\frac{W_{\mathrm{ext(total)}}}{L}$	$\frac{\mathrm{N\ m}}{\mathrm{m}}$
Proportionality factor	$\kappa = \frac{8\pi G}{c^4}$	$\frac{1}{\mathrm{N}}$	$K = \frac{2}{EI}$	$\frac{1}{\mathrm{N\ m}^2}$

associated with the charge (mass) applied to the space, not with the space itself. According to the mechanical principle, external work = internal work (see 10d), we can reconstruct the bridge between the two approaches. Indeed, if the analogy with the beam in 1D (or plate theory in 2D) is exact, it must exist as a tensor of stresses/(normal efforts, bending and torsion moments and shear loads) associated with the curvature of the space. In this case, expression (5a) will become, with a mechanical stress tensor acting within the space–time fabric (interior work) (10h) and κ a coupling constant:

$$\begin{bmatrix} G^{00} & G^{01} & G^{02} & G^{03} \\ G^{10} & G^{11} & G^{12} & G^{13} \\ G^{20} & G^{21} & G^{22} & G^{23} \\ G^{30} & G^{31} & G^{32} & G^{33} \end{bmatrix} = -\kappa \begin{bmatrix} \sigma^{tt} & \tau^{tx} & \tau^{ty} & \tau^{tz} \\ \tau^{xt} & \sigma^{xx} & \tau^{xy} & \tau^{xz} \\ \tau^{yt} & \tau^{yx} & \sigma^{yy} & \tau^{yz} \\ \tau^{zt} & \tau^{zx} & \tau^{zy} & \sigma^{zz} \end{bmatrix}, \tag{10h}$$

where $\sigma^{\mu\nu}$ are the normal stresses in the space–time fabric and $\tau^{\mu\nu}$ are the shear stresses in the space–time fabric both created by the curvature of the space fabric under the external masses.

The constant of proportionality κ is the same as we consider the work of the internal forces ($W_{\mathrm{int}} = U$) or the work of the external forces (W_{ext}), that is to say (K, κ). Later in our reasoning, we shall progress by considering U and therefor the reader must keep in mind expression (10d) connecting the external work to the inner work. We shall therefore make the shortcut associating T with U.

(b) There is a fracture in the quantum field theory that allows to describe the standard model, the vacuum energy, the Casimir force and general relativity. The general relativity wonderfully explains the mechanics of the infinitely large and the quantum mechanics that of the infinitely small. The problem is that these two theories do not overlap. Gravitation has not yet been quantified (independent of h) and is deterministic (not probabilistic as in quantum mechanics)! the graviton, boson vector of the force of gravity and keystone of quantum gravitation in connection with the quantum field theory has not yet been measured.

Nevertheless, we can quote string theory with strings open or closed of 1.0×10^{-35} m length and quantum gravity that try to solve this problem.

Based on the analogy of the beam in pure bending in one dimension, it seems natural to consider an elastic substance to associate with the space fabric. Consequently, a special elastic material should constitute the space described in the Einstein's field equation [8,9].

But, do we have solid proofs of this elastic material and of this space behaviour in weak field which would allow us to study the space following the elasticity theory? The answer is yes and it will be studied in more detail in §3.

Our aim is to explore these issues in five different ways simultaneously by first considering the time and space separately and then reassembling them at the end of the article as a second step:

- Consider that space is made of an elastic substance/material,
- calibrate κ using the spatial components rather than the temporal components of the Einstein's tensor. Thus, the temporal part is studied separately from the spacial part of the gravitation field tensors (see §10): use the elasticity theory, the strains measured on the interferometers Ligo and Virgo, the concept curvature = K energy density, the gravitational waves in weak field,
- replace G by mechanical and physical parameters,
- build a link with the quantum field theory by using the vacuum energy property to physically calibrate G and κ,
- conclude on the possible new characteristics of the space material.

Moving on, we shall propose a new mechanical and physical interpretation of the coupling constant κ in order to provide a different perspective on the understanding of the vacuum and of the potential material constituting the space elastic medium. By handling the nature of time separately from the nature of the space material it is hoped to contribute to the building of a bridge between the general relativity, the quantum field theory and the string theory.

2. Methods

The following methodology has been used focussing on the space part of the gravitational field equation with time being treated separately (see [11]):

(1) Analysis of the behaviour of space from the measurements made for more than 100 years in general relativity and quantum mechanics (vacuum energy) deducing an elastic behaviour.
(2) Research of the elastic theory which capture this behaviour: Hooke's law, Timoshenko's bending theory, beam, plates. Use analogy between the spiral form of the galaxies, the whirlwinds forming during cyclones or whirlpools on the sea as seen during the tsunami of 11 march 2011 in Japan and the potential presence of a space-material fluid at low speed, behaving for objects (gravitational waves) moving at the speed of light like an equivalent solid elastic material (see [8,9]).
(3) Demonstration, in weak gravity field, of the parallelism between the expression of the Einstein's gravity field (curvature $= \kappa \times$ energy density) in four dimensions and the expression obtained in elasticity/strength of the materials in two dimensions (curvature $= K \times$ the elastic strain energy density of two linear elements as the interferometer arms): use elasticity theory and assimilate each void volume of space inside the interferometer arm as a harmonic elastic oscillator made of vacuum full of an equivalent virtual elastic material.
(4) Application of the analogy obtained to the space material included in the two orthogonal tubes of the Ligo/Virgo interferometers: demonstration of the parallelism between the Einstein's field equation (tensorial equation in 4D) and the strain field of these two tubes like that of a strain gauge (tensorial equation in 2D) by using the work principle, $W_{\text{external}} = W_{\text{internal}}$.
(5) Deduction of this parallelism, via the elastic behaviour of the space, of a new mechanical expression of the gravitational constant G and of the Einstein constant κ based on longitudinal, torsional and shear waves in the medium [12,13].
(6) Proposition of numerical values of different parameters of these new expressions of the gravitational constant G and κ from the calculated vacuum energy.
(7) Checking the orders of magnitude of different parameters of these new expressions from the constants of physics.
(8) Proposition of the Young's modulus value $E = Y$ and Poisson's ratio ν for the space medium.
(9) Proposition of a method for measuring the Young's modulus of the space material from the Casimir effect and the possible shear effect in the plane of the interferometers.
(10) Possible reformulation of the Einstein's gravity field equation from this new approach to the constant κ.
(11) Consequences of the nature and characteristics of the space material constituting the microscopic fabric of the Universe (quantification of space and time).

3. Analysis of the space behaviour from the measurements made for more than 100 years in general relativity and quantum mechanics (vacuum energy)

The Einstein's gravitational field equation (5a) has been well verified experimentally. Indeed, all the gravitation tests and measurements carried out for more than 100 years constitute proofs which confirm that space is an elastic deformable physical object, which is not made of 'nothing' but that it is filled with 'something', a field, a material that is elastic, [8,9]. Indeed

(a) The acceleration of the expansion of the Universe has been observed [14]: the galaxies move away from us faster when they are away from us (Hubble's law (11)). In other words, the galaxies are 'motionless' and it is the space between them that dilates like dots on a balloon, which is inflated. So, the space fabric is an elastic body that stretches.

$$v = H_0 d, \tag{11}$$

where v is the recessional velocity of the galaxy, H_0 is the Hubble constant and d is the distance of the galaxy from the observator.
(b) The rays of light tangent at the Sun are curved by the deformation of the space around it. The light rays coming from stars follow the curvatures of an elastic body deformed by the present mass ([4] Eddington

in 1919 measured the deflection angle of the stars around the Sun during an eclipse of 1.75‴).

(c) The space is elastic. As the Sun moves, the space curvature in its wake disappears and becomes flat. The rays of light become straight again.

(d) The rotation of the Earth twists the space fabric around it, indeed gyroscopes placed in orbit at 400 km around the Earth are deflected by this space rotation/torsion/curvature (see Probe B experiment [15]. Thus, a horizontal angle of 0.000010833° has been measured from the gyroscopes placed in vacuum in accordance with the prediction of general relativity).

(e) The space deformations produced by the coalescence, for example, of two black holes in the form of gravitational waves [16–18] were measured for the first time by the Ligo interferometers in 2015 (official announcement of GW150914 detected by Ligo on February 11, 2016). Other observations followed, including a fusion of neutron stars (pulsars) GW170817 also observed in electromagnetic radiation. So the interferometers have effectively measured signals which are moving at the speed of light c inside the frame of the vacuum space medium. So, if there are strains, then there is an elastic physical body that is deformed and we measure these deformations. Consequently, following the elasticity theory, the material of the physical body can be characterised by the Young's modulus E and Poisson's ratio ν. A strain tensor ε_{ij} can thus be established from the space strains $\delta L/L$ measured in the arms of the interferometers. The magnitude of these strains is 10^{-21}. Mechanically speaking, these strains measured are in the principal direction. They correspond to the transverse elastic waves of the medium with a propagation direction perpendicular to the plane build by the two arms of the interferometers. The formulas below describe the space metric g_{ij} built from h_{ij} (12b) a disturbance of the flat metric η_{ij} (12a) in weak gravitational field and the link with the strain tensor ε_{ij}:

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij} \tag{12a}$$

$$h_{ij} = 2\varepsilon_{ij} \tag{12b}$$

$$\frac{\delta L}{L} = \frac{\delta_i}{L} = 10^{-21} = \varepsilon_{ij} = \frac{1}{2}h_{ij}. \tag{12c}$$

(f) And finally, we can consider that the space rings like any elastic material according to the study conducted by Ringermacher and Mead [19].

Based on all these points, it seems logical to consider that space is made of a strange elastic material, a new type of Ether as explained by Einstein himself [20,21].

From the aforementioned observations, it seems that:

(a) The space fabric appears to be physically made up of 'Something' since the galaxies are trained by this 'Something' that is expanding accelerated.

(b) The space bends (analogy with the notion of curvature in elasticity theory see (6)) in the presence of energy density, mass. The sum of the angles of a triangle in the presence of mass is no longer 180° (see [8,9]).

(c) The space fabric is deformed and transmits gravitational waves by elongations and shortenings. So, small strains (interferometers) or angles (Gravity Probe B) are measured when we place it sufficiently far from the said mass/energy (weak field principle).

(d) The space is elastic, the curvature disappears when the mass that created it disappears.

(e) A black hole is a type of yielding of space loaded to the extreme [12,13].

(f) In the application of elasticity theory, space is proposed to have a quantified microstructure with an associated characteristic frequency f. G is the macroscopic manifestation of the said frequency (see eqs (2a) and (2b))

4. Research of the elastic theory which approaches the space behaviour: Hooke's law, Timoshenko's theory

4.1 *Hooke's law*

All these elements suggest that

(a) An elastic material constitutes the space (E, ν).

(b) Space is proposed to have a quantified microstructure of radius r with an associated characteristic frequency f.

(c) The Hooke's law applies to space medium.

See T Damour's book "if Einstein was told to me, chapter 3, the elastic space–time", where Einstein's equation is simplified by $D(g) = K{\cdot}T$ where D is a deformation tensor, T is a tensile tensor and K is a factor of proportionality

Consequently, it seems logical to apply the elasticity parameters to the deformable and elastic medium that constitutes it. This is reflected mathematically by the two expressions below relating to the normal stresses σ (13) and tangential stresses τ (14) connected respectively to the strain ε by the elasticity modulus E (the so-called Young's modulus) and to the shear strain (angle γ) by the shear modulus μ (some times noted as G) (see [22,23]).

$$\sigma = \varepsilon E = \frac{\delta l}{L}E \tag{13}$$

$$\tau = \gamma\mu = \gamma\frac{E}{2(1+\nu)}. \tag{14}$$

It is important to see that

(a) at strain, $\left(\frac{\delta L}{L}\right)$, is associated with Young's modulus E and normal stress σ,
(b) at the shear strain, (angle γ) is associated with a shear modulus μ, Poisson's ratio ν and shear stress τ.

This approach of general relativity by the elasticity theory is not new and has been studied by many researchers (see [11,24–48]). We review now the main points concerning, in particular, the relationship between the stress tensor and the stress–energy tensor (§4.2) on the one hand and between the metric tensor and the strain tensor (§4.3) on the other hand.

4.2 *Similarity between the stress tensor in elasticity and the stress–energy tensor in general relativity*

The Einstein's gravitational field formula (5a) can also be expressed in the following form:

$$T^{\mu\nu} = -\frac{c^4}{8\pi G} \times \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right). \tag{15a}$$

The equation to dimensions is

$$\frac{\frac{\text{kg m}^2}{\text{s}^2}}{\text{m}^3} = \frac{\text{N m}}{\text{m}^3} = \frac{\text{kg}}{\text{m s}^2} = \frac{\left(\frac{\text{m}}{\text{s}}\right)^4}{\frac{\text{m}^3}{\text{kg s}^2}} \times \left(\frac{1}{\text{m}^2}\right) = \frac{\text{kg m}}{\text{s}^2} \times \left(\frac{1}{\text{m}^2}\right). \tag{15b}$$

Or in a compact form, it becomes

$$\frac{\text{N m}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} = \text{N} \times \frac{1}{\text{m}^2}. \tag{15c}$$

We then notice that the energy density has the same dimension as the stress in Newton's classical mechanics. Moreover, it is possible to demonstrate that the stress–energy tensor (16a) is like stress tensor (16b) by replacing in it velocities v_i with four velocities u_μ and moving from a three-dimensional space to a four-dimensional space–time of density ρ (see [49] and Appendix A).

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{16a}$$

$$\sigma_{ij} = \rho v_i v_j. \tag{16b}$$

The stress tensor σ_{ij} in three-dimensional elasticity is written depending on the strain tensor ε_{ij} and two constants according to the Young's modulus of the medium $Y = E$ and the Poisson's ratio ν:

$$\sigma_{ij} = \frac{E}{(1+\nu)}\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right) \tag{16c}$$

or

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}. \tag{16d}$$

With the Lame coefficients:

$$\mu = \frac{E}{2(1+\nu)} \tag{17}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}. \tag{18}$$

In this expression, ε_{kk} is the trace of the strain tensor, δ_{ij} is the Kronecker symbol and ν is the Poisson's ratio.

4.3 *Relationship between the elastic strain tensor and the metric tensor in weak gravitational fields*

4.3.1 *Case of elasticity in four dimensions.* Following [11], Chapter 2.6, formula 2.9, we have in four dimensions:

$$g_{\mu\nu} = (\eta_{\mu\nu} + h_{\mu\nu}) = (\eta_{\mu\nu} + 2\varepsilon_{\mu\nu}). \tag{19}$$

The metric tensor $g_{\mu\nu}$ in a weak field is therefore equivalent to the metric in flat field to which is added a perturbation which is only twice the strain tensor $\varepsilon_{\mu\nu}$. The general structure of the strain tensor ε_{ij} in the theory of three-dimensional elasticity is given in (20a) and (20b).

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}. \tag{20a}$$

With the displacements u_i and u_j we have

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{20b}$$

4.3.2 *Case of gravitational waves*

4.3.2.1 Theoretical aspects. The linearised form of the Einstein's equation in weak gravitational fields is, see (21a) and [50]:

$$\partial^\lambda\partial_\lambda\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}. \tag{21a}$$

In vacuum (case of the gravitational waves), we have

$$\partial^\lambda\partial_\lambda\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = 0. \tag{21b}$$

The d'Alembertian wave operator,

$$\overline{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \tag{22a}$$

$$\overline{h}_{\mu\nu} = A_{\mu\nu}\cos\left(k_\sigma x^\sigma\right) \tag{22b}$$

$k^\sigma = \left(\frac{\omega}{c}; \vec{k}\right)$ quadri vector wave of the plane wave,

$$x^\sigma = (ct, x, y, z) \tag{23}$$

$$\left\|\vec{k}\right\|^2 = \frac{\omega^2}{c^2} \tag{24}$$

ω is the circular frequency of the wave.

$$A_{\mu\nu} = A_+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + A_\times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{26}$$

h is the trace of $h_{\mu\nu}$ and

$$\overline{h} = -h. \tag{27}$$

Using (22a), to have a new expression of (21a) function of ε_{ij}, we obtain a new expression of Einstein's gravitational field (28a)

$$\partial^\lambda \partial_\lambda \left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \right) = \Box \left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \right) = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{28a}$$

By replacing $h_{\mu\nu}$ by $2\varepsilon_{\mu\nu}$ (see (12c)) in (28a) we obtain:

$$\partial^\lambda \partial_\lambda \left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon} \right) = \Box \left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon} \right) = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{28b}$$

After simplification by 2, we obtain the same constant κ in weak field:

$$\partial^\lambda \partial_\lambda \left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{\varepsilon} \right) = \Box \left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{\varepsilon} \right) = -\frac{8\pi G}{c^4} T_{\mu\nu}. \tag{28c}$$

The formulation of particle position variations arranged along a circle during the passage of a gravitational wave perpendicular to the plane xy, allows to find expression (29) by considering, for example, polarisation A_+ (see figure 2).

The movements of these particles correspond always to pure compression or pure traction of the space medium inside the interferometric tube. The deformation of the circle containing the particles is identical but rotates by 45° according to the type of polarisation (A_+ or $A_\times$).

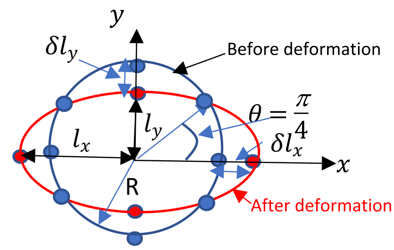

Figure 2. Example of particle coordinates subjected to a gravitational wave polarised A_+ particles propagating perpendicular to the plane xy.

The particle position measured from the centre of the circle is

$$l^2 = R^2 - R^2 A_+(\cos(2\theta))\cos\left(\frac{\omega}{c}(ct - z)\right), \tag{29}$$

where l is the final length after deformation, z is the direction of the wave propagation, t is the time, θ is the angle between the abcissa x and R, the radius of the circle where the particles are positioned, ω is the circular frequency, c is the speed of light, A_+ is the first wave polarisation and $A_\times$ is the second wave polarisation. With the metric $g_{\mu\nu}$ (19) and dimensionless perturbation $h_{\mu\nu}$:

For a polarised wave A_+:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{30a}$$

For a polarised wave $A_\times$:

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{+1} & \mathbf{0} \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{30b}$$

The spatial part of $h_{\mu\nu}$ is indicated in bold. l^2 is the final length squared and R^2 is the initial length squared.

We can calculate the variation in length due to the displacement of the particle.

$$\frac{l^2 - R^2}{R^2} = \frac{(\text{final length})^2 - (\text{initial length})^2}{(\text{initial length})^2} = -A_+(\cos(2\theta))\cos\left(\frac{\omega}{c}(ct - z)\right), \tag{31a}$$

where $L_F = L_i + \delta_i$ is the final length, L_i is the initial length and δ_i is the length variation.

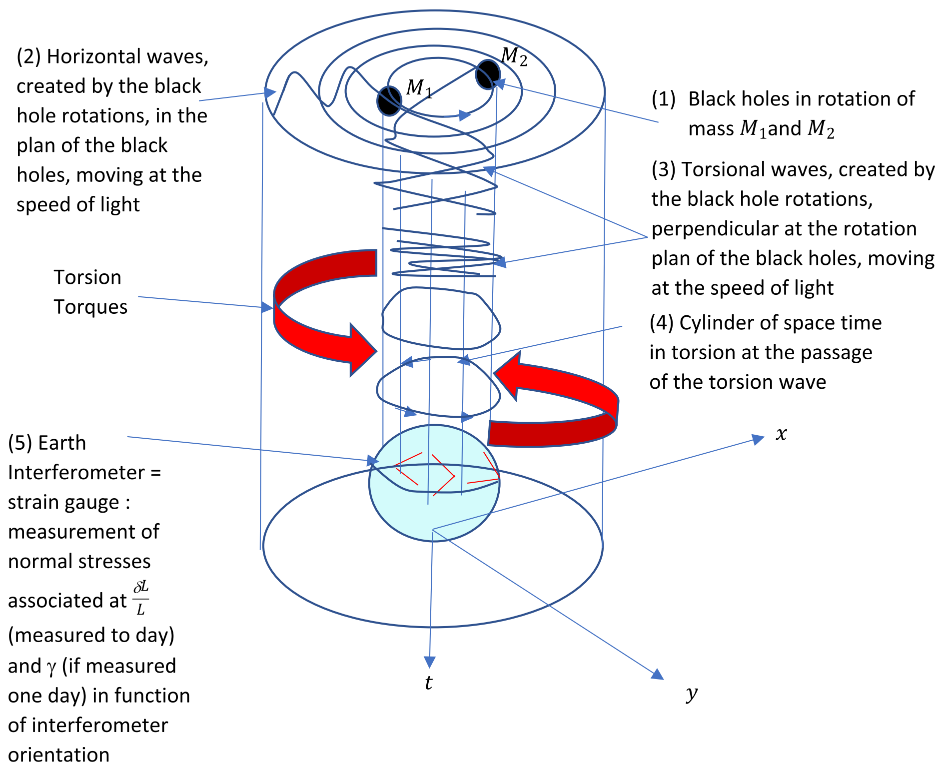

Figure 3. Torsional waves created by a binary system in rotation.

With $\delta_i \ll L_i$

$$\frac{l^2 - R^2}{R^2} = \frac{L_F^2 - L_i^2}{L_i^2} = \frac{L_i^2 + 2L_i\delta_i + \delta_i^2 - L_i^2}{L_i^2}. \quad (31b)$$

With $\delta_i^2 \ll \delta_i$

$$\frac{l^2 - R^2}{R^2} \cong \frac{2\delta_i}{L_i} \quad (31c)$$

and with the strain definition

$$\varepsilon = \frac{L_F - L_i}{L_i} = \frac{L_i + \delta_i - L_i}{L_i} = \frac{\delta_i}{L_i}. \quad (31d)$$

So finally we obtain

$$\frac{l^2 - R^2}{R^2} = \frac{L_F^2 - L_i^2}{L_i^2} \cong \frac{2\delta_i}{L_i} = 2\varepsilon. \quad (31e)$$

To correlate with the Hooke‘s law (13)

$$\frac{l^2 - R^2}{R^2} \cong -A_+(\cos(2\theta)) \cos\left(\frac{\omega}{c}(ct - z)\right) \cong 2\varepsilon. \quad (31f)$$

So, the perturbation $h_{\mu\nu}$ and consequently the metric $g_{\mu\nu}$ are close to $2\varepsilon_{\mu\nu}$. So, in weak gravity field we demonstrate eq. (19) and the relationship between the metric tensor and the strain tensor well. Thus, the metric tensor in the general relativity approach in weak field is assimilated into elasticity to the flat metric tensor to which the strain tensor is added twice (12a).

At the end of this chapter, with the links between the tensors T_{ij} and σ_{ij} on the one hand and between g_{ij}, h_{ij} and ε_{ij} on the other hand, and taking into account the common principle curvature = $K \times$ the energy density, we have all the bridges necessary to interpret general relativity according to the elasticity theory.

4.3.2.2 Interpretation of the results of the calculation of general relativity on gravitational waves in weak field from the angle of elasticity theory. We consider that the rotation of a binary system (like two black holes, for example) creates a sort of ‘torsional waves’, in the space fabric (see figure 3 and [3,12,58]).

Indeed, this point was presented and demonstrated by Professor Kip Thorne (Nobel Prize 2017) during his conference on March 6, 2018 at UCI USA [12] “exploring the universe with gravitational waves from big bang to black holes”. At this conference accessible on the net, he expains with numerical simulations based on the general relativity, the behaviour of two black holes during their coalescence. In fact there is the creation of two types of vortices (turning to right, turning to left) emerging from rotating black holes, dragging the space medium around them, which merge to create a ring which makes these swirl mixtures with compression and tensile tendencies measured by Ligo and Virgo. The consequences are that gravitational waves are not a classical shear wave but a mixture of vortices whose

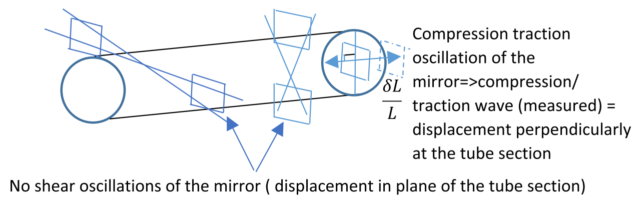

Figure 4. Movements of the laser mirror inside the interferometric tubes formed by the gravitational transversal wave.

results are compression/tensile tendencies measured in the interferometers.

If we compare the results of the general relativity in weak field ($h_{\mu\nu}$) with the elastic strain tensor (ε_{ij}) (see eq. (19) and [10]) we can conclude on the deformation states of the elastic medium in the xy plane of the arms of the interferometer during the passage of a gravitational wave coming from the z direction. Kip Thorne explains also in [13] the geometrodynamics of the space–time made of warped space.

First point:

Therefore, based on expressions (30a) and (30b), it is known that there are two clearly separated types of polarisation of the gravitational wave produced by the coalescence of two massive objects rotating relative to one another: A_+ and $A_\times$ (see [10]).

Second point:

By the relation between $h_{\mu\nu}$ and $\varepsilon_{\mu\nu}$ (see (12c) and (31f) and [11]), there exists for each polarisation of $h_{\mu\nu}$ the equivalent of an associated strain tensor (see [10]):

(a) Based on (30a), (31f) and (12c) associated with the polarisation A_+ we have

$$\varepsilon_{xy(A_+)} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{32a}$$

This state of deformations is obtained in the tubes of the interferometer by analysing the forward and backward motions (δL) or strains ($\delta L/L$) of the laser mirrors in the two tube sections (see figure 4).

(b) Based on (30b), (31f) and (12c) associated with the polarisation $A_\times$ we have:

$$\varepsilon_{xy(A_\times)} = \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{32b}$$

This state of deformations should be obtained in the tubes of the interferometer by analysing the lateral movements of the laser mirrors in the two tube sections (see figures 4 and 5).

These two deformation states (32a) and (32b) certainly prove that $h_{\mu\nu}$ corresponds to a pure shear

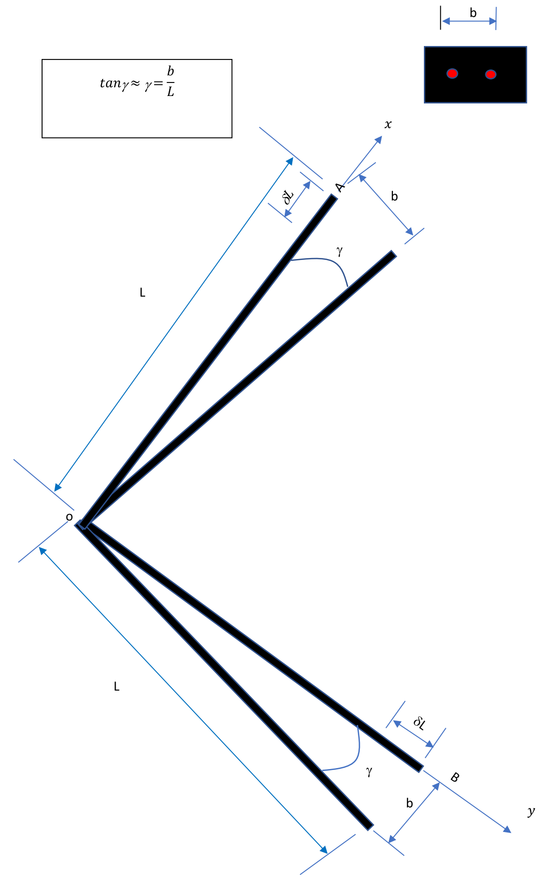

Figure 5. Strains $\frac{\delta L}{L}$ measured and shear strain (angle γ) not measured on the interferometers Ligo and Virgo to this day.

deformation state of the space layers (multisandwich) perpendicular to the direction z of the gravitational wave (see figure 6). It also proves the elastic behaviour of space in perfect correlation with the elastic theory and consequently the existence of an equivalent elastic material in the space vacuum. This fundamental research also proves that it is possible to unify the theory of elasticity with general relativity and thus that it is possible to define an elastic material constituting space.

Third point:

By the theory of elasticity, therefore, there exists, for each deformation tensor, a stress tensor (see [10]):

(a) On the basis of (32a) and (30a), associated with the polarisation A_+, the corresponding stress tensor of pure compression/traction of the space medium in the xy plane is

$$\sigma_{xy(A_+)} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{33a}$$

(b) On the basis of (32b) and (30b), associated with the polarisation $A_\times$, the corresponding stress tensor of

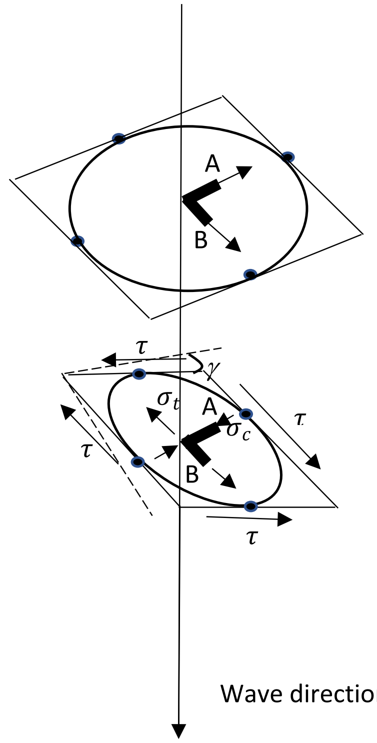

Figure 6. Plane deformations formed by the transverse gravitational wave.

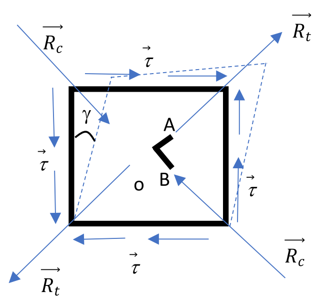

Figure 7. Creation of normal stresses by the combination of shear stresses.

pure shear of the space medium in the xy plane is

$$\sigma_{xy(A_\times)} = \begin{bmatrix} 0 & \tau_{xy} = \sigma_{xx} & 0 \\ \tau_{yx} = \sigma_{yy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{33b}$$

The two stress tensors above, according to the orientation of the facet considered (see figure 8), are characteristic of a pure shear associated with a pure torsion of space following the rotation of two massive objects which merge. Figure 6 shows the plane deformation of the space created perpendicular at the wave direction with the gravitational wave as a transverse wave.

Figure 7 shows how a combination of shear stresses τ create normal stresses σ.

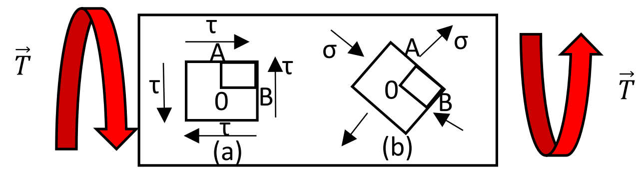

Figure 8. Normal stresses and shear stresses measured on interferometers as a function of their orientations on a space cylinder in torsion.

According to their orientations, the interferometers do not measure anything (Case a) or strains $\frac{\delta L}{L}$ (Case b) as seen in figure 8 and [54].

Fourth point:

Each of its stress states corresponds to a wave velocity characteristic of the elastic medium measured in the interferometric plane by the longitudinal oscillation of the laser mirror.

(a) Associated with the polarisation A_+, a pure longitudinal tensile compression wave velocity in each x or y direction is measured by the laser mirror of the Ligo and Virgo interferometers connected to a tensor according to the expression (33a) (see [53]).

$$\frac{\partial^2 u_{(x,t)}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_{(x,t)}}{\partial t^2} = 0. \tag{34a}$$

The Alembert equation is

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\rho}{E} \times \frac{\partial^2 u(x,t)}{\partial t^2} = 0. \tag{34b}$$

Comparing (34a) and (34b), we have the well-known equation

$$\frac{1}{c^2} = \frac{\rho}{E} \tag{35a}$$

$$c = \sqrt{\frac{E}{\rho}}. \tag{35b}$$

(b) Associated with the polarisation $A_\times$, a pure shear wave velocity, not yet measured by the laser mirror of interferometers Ligo and Virgo (lateral movements of the mirrors (see figures 4 and 5), connected to a tensor according to expression (33b) and a torsion torque [54].

Indeed, if we consider that the fabric which constitutes space has a dynamic behaviour similar to the one we have on Earth in the event of an earthquake but with only transverse waves, we only need to consider shear wave S instead of S and P (pressure) waves. Indeed, in line with what we measure, only strains in the plane xy perpendicular to the direction of the gravitational wave are seen (see figures 4–6).

So, the following expression (36) of elasticity theory applies, with u_i as the component i of the displacement vector $\vec{u}$, t as the time, σ_{ij} as the stress tensor and ρ as the mass density (see formula (3.10), and (3.11), chapter 3.3 of [11]):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial j}. \tag{36}$$

With for the strain tensor ε_{ij}:

$$\varepsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) \tag{37}$$

The classical elastic wave motion is so with (16c) in (36):

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\vec{\nabla}(\mathrm{div}(\vec{u})) - \mu \overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{rot}}(\vec{u})\right) + f_{\mathrm{external}} \tag{38a}$$

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\vec{\nabla}(\mathrm{div}(\vec{u})) - \mu \vec{\nabla}(\mathrm{div}(\vec{u})) + \mu \vec{\Delta}(\vec{u}) + f_{\mathrm{external}}. \tag{38b}$$

After calculation

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)\vec{\nabla}(\mathrm{div}(\vec{u})) + \mu \vec{\Delta}(\vec{u}) + f_{\mathrm{external}} \tag{38c}$$

or again

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)\vec{\nabla}\left(\vec{\nabla} \bullet \vec{u}\right) + \mu \vec{\nabla}^2 \vec{u} + f_{\mathrm{external}}, \tag{38d}$$

where

$$\vec{\nabla}\vec{u} = \overrightarrow{\mathrm{grad}}(\vec{u}) \tag{38e}$$

$$\vec{\nabla}^2 \vec{u} = \vec{\Delta}(\vec{u}). \tag{38f}$$

When $f_{\mathrm{external}} = 0$, the solution of the equation follows the Helmholtz's decomposition that gives two waves that propagate in the elastic medium:

$$\vec{u} = \vec{u}_{\mathrm{pressure,longitudinal}} + \vec{u}_{\mathrm{shear,transversal}}. \tag{39}$$

A pressure with a longitudinal wave of velocity c_{pressure}:

$$c_{\mathrm{pressure}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \tag{40}$$

This hypothesis of compression waves in the direction z is not acceptable because in this case the strains are in the same direction as the propagation of the waves. Of course this is not the case for gravitational waves where the strains are in a plane perpendicular to the wave direction of propagation (z). A shear with a transversal wave of velocity c_{shear}:

$$c_{\mathrm{shear}} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}. \tag{41}$$

This hypothesis of shear waves is also strictly speaking not acceptable, because the strains measured on the interferometer are not shear strains (angles γ) linked at shear waves but, $\frac{\delta L}{L}$, strains that are always linked at eventual compression waves; but we shall consider it because it is possible that shear strain exists and is not already measured.

So strictly speaking, the gravitational waves cannot be assimilated to classical elastic waves in an elastic medium. They have the particularity to create strains perpendicular to the wave propagation direction (transversal shear wave characteristics) but with elongation and shortening (compression/traction wave characteristics); see [12,13]. The proposition of a space medium made up of a multisandwich of thin sheets sheared perpendicularly to the direction of propagation of the gravitational wave therefore seems a reasonable hypothesis (see figure 11).

4.4 *Determination of the Poisson's ratio intensity*

We can conclude from these wave speed equations, on a potential value of the Poisson's ratio ν.

First approach: Analysis of the particle movements on a circle under a gravitational wave (see figure 2)

The analysis of figure 2 from the calculation of general relativity shows that an object on the xy plane positioned perpendicular to the z direction of the propagation of gravitational transverse waves, is simultaneously compressed in one direction and stretched in the perpendicular direction. The strains are equal but of opposite sign: $\varepsilon_{xx} = -\nu\varepsilon_{yy}$.

With the definition of Poisson's ratio we have

$$\nu = \frac{\text{relative transverse shrinkage}}{\text{relative longitudinal elongation}} = 1.$$

Second approach: In the z direction, the gravitational wave is a transverse wave and not a compression wave

There is no compression wave perpendicular to the plane of the interferometer. So eq. (40) must be equal to 0. With eqs (17) and (18) we obtain $\nu = 1$ again.

Third approach: Based on current data (see [11])

Based on the results of ref. [11] and following eq. (31) we have, the Young's modulus $E = Y = 4.4 \times 10^{113}$ Pa (see §3.4, formula 3.13 [11]) and density $\rho = 1.30 \times 10^{96}$ kg/m^3 (see §3.4, formula 3.14 [11]).

$$\nu = \frac{E}{2c^2\rho} - 1. \tag{42}$$

With eq. (42) and the results of [11] we get

$$\nu_z = 0.8829.$$

This last value of the Poisson's ratio remains acceptable taking into account the uncertainty on the intensity of the vacuum energy (very important according to the quantum field theory and very low according to the value measured in the vacuum space) which is the object of many discussions within the international scientific community.

Conclusion: We retain for the Poisson's ratio: $\nu = 1$.

5. Highlighting the parallelism and differences between the strain energy density in elasticity and the mass energy density in general relativity

5.1 *The strain energy density in elasticity*

5.1.1 *Strain energy in general.* The strain energy density in elasticity is [22]

$$U_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}. \tag{43a}$$

By introducing in (43a) the expression of the stress tensor (16c), we obtain the following expression of strain energy density of an elastic body:

$$U_{ij} = \frac{E}{2(1+\nu)}\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} \tag{43b}$$

or

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij}. \tag{43c}$$

In plate theory there are relations between the strain tensor and the curvature tensor (see eqs (78)–(81)) which brings us closer to Einstein's formalism (see 5a) if we consider that the external work produced by the external masses applied on the space fabric is equal to the internal work of the space fabric curved by these masses. A tensorial space approach is also developed in [42].

5.1.2 *Consequences of the parallelism between the elasticity theory and the general relativity.* Thus, to build a bridge between the elasticity theory and general relativity formalism, we have studied the parallelism between the strength of the material and the general relativity about the principle: curvature $= K$ energy density on the one hand (see Introduction) and the transversality of the physical terms (curvature, metric, strain energy, external work, stresses, strains, coefficient of proportionality K and κ) between the general relativity and the elasticity theory on the other hand (see §4.2 and §4.3).

The parallelism between the general relativity and the strength of material formulas on the principle curvature $= K$ energy density is as follows (see "Introduction"):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{ext(total)}}{L}\right) = K\left(\frac{W_{ext(total)}}{L}\right) = \frac{2}{EI}\left(\frac{U}{L}\right) \quad (10d)$$

$$\longrightarrow$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} \quad = -\kappa\ T^{\mu\nu} \quad = -\kappa\, M^{\mu\nu} \quad (10\text{e})$$

The transversality between general relativity and the strength of the material on the key parameters is as follows: (link between ε_{ij}, h_{ij} and g_{ij} (see (31f), (12c) and (19)) and between $T_{\mu\nu}$ and σ_{ij} (see (16a) and (16b) and Appendix A):

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\mathrm{R} = -\frac{8\pi G}{c^4} \quad T^{\mu\nu} \quad (5a)$$

$$\Downarrow$$

$$\partial^{\lambda}\partial_{\lambda}\bar{h}_{\mu\nu} = \Box\,\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} \quad T_{\mu\nu} \quad (21)$$

$$\Downarrow$$

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E} \quad U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij} (43c)$$

From this analysis, it is clear that the constant κ in this case, $\frac{8\pi G}{c^4}$ would be closer to mechanical constant of the space medium function of the Young's modulus E and the Poisson's ratio ν. In other terms κ should be proportional to $\frac{(1+\nu)}{E}$.

To finalise the construction of parallelism between general relativity and the theory of elasticity (or the strength of materials which results from it) and to find a mechanical transposition of the Einstein's constant κ, we must now try to find the expression of κ from the curvature $= K$ energy density principle expressed from simple equations of strength of the materials. The comparison of the two formulas (5a) and (21a) with the formula (43c), terms to terms, allows to compare K and κ and to identify the mechanical correspondences between $(1+\nu)/E$ and the parameters G and c).

The examination of (43c) shows that these equations will be of the form

$$f_{(\varepsilon_{ij^2})} = K_{\left(\frac{1+\nu}{E}\right)}U. \tag{43d}$$

5.2 *Proposition of a tensor equation between curvature and space strain energy based on the strains measured on the interferometers Ligo and Virgo*

5.2.1 *Study of a horizontal space cylinder in one direction solicited by a gravitational wave – without the effect of Poisson's ratio – use of the longitudinal velocity of the correlated compression/traction wave with the* *compression/traction stress tensor in the interferometric tube.* We assume in this section that the coalescence of two black holes, for example, creates by their rotations, a torsion of the space as shown in figure 3 and ref. [12,13].

This twisting of the space sheets creates, in the successive xy planes, tensiles and compressions of the space material sheets (multisandwich) that finally arrive on the Earth in each arm xy of the interferometers (see figures 6 and 11). In weak field, for space only, the metric is given by (12a). Einstein's gravitational equation in a weak field for space is (21b). The result is for a polarised wave A_+, the tensor given in (30a).

Note: We choose this polarisation because it corresponds to the displacements measured in the interferometric arms (compression /traction of the volume which create the advances and retreats of the laser mirrors).

With formula (31f) demonstrated in §4.3.2.1, we have a link between the general relativity (disturbance of the spatial part of the metric h_{ij}) and the theory of elasticity (the strain ε_{ij} of the elastic medium) (see eq. (12c)). So, based on (31f) and (12c), at the spatial perturbation of the metric, $(h_{\mu\nu(A_+)})$ corresponds to the strain tensor ε_{ij} (32a). Thanks to the Hooke's and elasticity formula, it corresponds to the strain tensor ε_{ij} (32a) a stress tensor σ_{ij} (33a). This type of stress tensor (33a) represents a longitudinal pure compression/traction wave (35a) associated with the normal effort N (see figure 9 and [10]).

This section considers therefore a tube of a Ligo/Virgo type interferometer of length L and section S loaded with the normal force N as defined in figure 9. Inside the tube, it is considered that the vacuum consists of a space elastic substance made from very small particles to constitute a granular substance, fluid whose granularity of quantum dimension r.

Note: In this simplified approach we deliberately separate the correlation between the x and y directions of the tubes seen in figure 2. This was done in order to see already if the basic principle curvature $= K \times$ the energy density is respected on the one hand and with the eventual stresses on the perpendicular direction of the transverse wave to be in correlation with the stress plane state on the other hand. The consequence is that in this section we do not take into account the Poisson's ratio to be in agreement with the information of the tensor considered here (see eq. (33a)).

Since there are displacements of the laser mirror in the direction of the tube corresponding to the compression/traction of the space medium (see figure 4 and [53]), there are strains and stresses in this framework and therefore a dynamic normal force N and a pure compression/traction wave inside the tube.

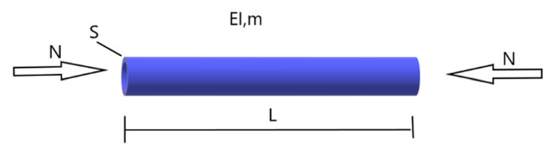

Figure 9. Tube loaded by a normal force N.

The Hooke's law (13) can be written as a function of the displacement $u_{(x)}$:

$$\sigma_{xx} = \varepsilon E = \frac{N}{S} = \left(\frac{u_{(x+dx)} - u_{(x)}}{dx}\right) E = \frac{\delta L}{L} E, \tag{44a}$$

where σ is the normal stress in N/m^2, ε is the strain in %, ($\varepsilon = \frac{N}{ES}$) is measured by Ligo and Virgo, N is the normal force in Newton, S is the tube section in m^2, L is the tube length in m, $V = S \times L$ is the tube volume in m^3, $E = Y$ is the Young's modulus of the material constituting the tube in N/m^2, δL is the length variation in m under the normal force N, $u_{(x)}$ is the longitudinal displacement in m along the longitudinal axis of x.

The stress–displacement relation as a function of rigidity $K = ES/L$, is written as follows:

$$N = \frac{ES}{L}\delta L = K\delta L. \tag{44b}$$

The strain energy U (N m) of the tube in static, when N is constant and taking into account (44b), is

$$U = \frac{1}{2}\int_0^L \frac{N^2}{ES}\mathrm{d}x = \frac{1}{2}\frac{N^2}{K}. \tag{45a}$$

The strain energy of the stiffness spring K (N/m) can be written by substituting N by expression (44b) in (45a):

$$U = \frac{1}{2}K(\delta L)^2. \tag{45b}$$

Moreover, from the strain definition, the displacement variation δL is

$$\varepsilon \times L = \delta L. \tag{46}$$

By introducing (46) in expression (45b) of the strain energy, we obtain

$$(\varepsilon)^2 = \frac{2}{K}\frac{U}{L^2}. \tag{47a}$$

Starting from the rigidity K depending on length L, the Young's modulus E, the tube section S and substituting the expression of L by K in (47a), we obtain

$$(\varepsilon)^2 = 2\frac{K}{E^2}\frac{U}{S^2}. \tag{47b}$$

Considering the tube volume V we extract the section S:

$$\frac{V}{L} = S. \tag{48}$$

By substituting the expression of S (48) in (47b), we obtain the strain energy density U/V:

$$\frac{1}{L^2}(\varepsilon)^2 = 2\frac{K}{V}\frac{1}{E^2} \times \frac{U}{V} \tag{49a}$$

or

$$\frac{U}{V} = \frac{\frac{1}{L^2}(\varepsilon)^2}{2\frac{K}{V}\frac{1}{E^2}}. \tag{49b}$$

We now consider the tube under a dynamic behaviour and with a pure longitudinal wave compression/traction following x in the arm as a consequence of polarised gravitational waves A_+. We note the density ρ as

$$\rho = \frac{m}{V}. \tag{50}$$

The fundamental dynamic equation allows us to find the eigencircular frequency (harmonic oscillator) of the tube made of the space material:

$$U_c + U = \frac{1}{2}m\dot{x}^2_{(t)} + \frac{1}{2}Kx^2_{(t)} = E_0 = 0, \tag{51a}$$

where U_c is the kinetic energy.

By a derivative with respect to t we have

$$\ddot{x}_{(t)} + \frac{K}{m}x_{(t)} = 0 \tag{51b}$$

and of course following the Newton's force definition

$$\text{Force} = m\ddot{x} = -Kx \tag{52}$$

which allows us to express the circular frequency ω according to the tube rigidity K and its mass m:

$$\omega^2 = \frac{K}{m} = (2\pi f)^2 \tag{53a}$$

$$\omega = 2\pi f. \tag{53b}$$

By substituting (53a) in (50) we obtain a new expression for the volume V as

$$V = \frac{m}{\rho} = \frac{K}{\omega^2\rho}, \tag{53c}$$

The total energy density of the system mass-spring U_T is a function of the kinetic energy density U_c and strain energy density U:

$$\frac{U_c}{V} + \frac{U}{V} = \frac{U_T}{V}. \tag{53d}$$

The strain energy density is

$$\frac{U}{V} = \frac{U_T}{V} - \frac{U_c}{V} = T. \tag{53e}$$

By substituting (49b) in (53e) we obtain

$$\frac{\frac{1}{L^2}(\varepsilon)^2}{2\frac{K}{V}\frac{1}{E^2}} = T \tag{54a}$$

or in an equivalent form:

$$\frac{1}{L^2}(\varepsilon)^2 = 2\frac{K}{V}\frac{1}{E^2}T. \tag{54b}$$

By substituting V (53c) in expression (54b) we get

$$\frac{1}{L^2}(\varepsilon)^2 = 2\rho\frac{\omega^2}{E^2} \times T. \tag{54c}$$

By substituting the circular frequency ω (53b) by frequency f in (54c), we obtain

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi^2\rho\frac{f^2}{E^2} \times T. \tag{54d}$$

In the plane of the interferometer, the movements of the mirror are perpendicular to the tube section (see figure 3). Thus, in the plane of the interferometer, the mirror follows the compression/traction movements of the spatial material inside the volume of the tube (, $\frac{\delta L_x}{L_x}$, in connection with , $u_{(x)}$,). These oscillations in the plane of the interferometer are therefore pure compression/traction longitudinal waves correlated to the transversal waves perpendicular to the plane of the interferometers (see [12] and [53]). So in this section, following the type of tensor considered (33a), the velocity of the wave is given in formula (35a) and the Young's modulus $E = Y = \rho c^2$.

The speed is limited because it propagates in an elastic medium of density ρ. A similar experiment would be to pull more or less quickly a metal ball in the middle of a pile of sand. Depending on the density and intensity of sand, the ball will move more or less quickly. In our case, the bullets should be the photons and the medium would consist of a material of extremely fine granulometry (1×10^{-35} m). We come back now at the tube under a gravitational wave. We multiply and divide by ρ the expression (54d):

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi^2 f^2\left(\frac{\rho}{E}\right)^2\frac{1}{\rho} \times T. \tag{55}$$

Finally, we have the relation between the dynamic curvature and the strain in the tube:

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi\left(\frac{\pi f^2}{\rho}\right)\left(\frac{\rho}{E}\right)^2 \times \frac{U}{V}. \tag{56}$$

Taking into account eq. (35a) we obtain

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi\left(\frac{\pi f^2}{\rho}\right)\frac{1}{c^4} \times \frac{U}{V}. \tag{57}$$

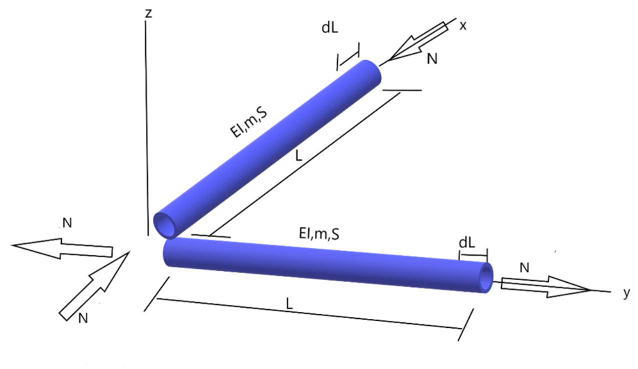

Figure 10. Double perpendicular tube loaded by a normal force.

We notice that the term $\frac{\pi f^2}{\rho}$ has the dimension of the gravitational constant G ($\mathrm{m^3/(kg{\cdot}s^2)}$), the term $\frac{1}{L^2}(\varepsilon)^2$ has the dimension of a curvature ($1/\mathrm{m}^2$) (see eqs (78) and (81)), the term (U/V) has the dimension of the energy density ($\mathrm{N\,m/m^3}$), the term $(\frac{\pi f^2}{\rho})\frac{1}{c^4}$ has the dimension of the inverse of the load (N^{-1}).

The formula (57) thus satisfies the principle curvature $= K$ energy density. Consequently, we learn that the term $(\frac{\pi f^2}{\rho})$ can be identified with the Einstein's constant κ if G

$$G = \frac{\pi f^2}{\rho}, \tag{58}$$

where f is the natural frequency of the spatial material inside the tube and ρ is the density of the spatial material.

In the next section, we take into account the Poisson's ratio.

5.2.2 *Study of two perpendicular horizontal space cylinders solicited by a gravitational wave – effect of the Poisson's ratio – use of the longitudinal velocity of the correlated compression/traction wave with the compression/traction stress tensor in the interferometric tubes.* We assume the same hypothesis on the strain and stress tensors as in §5.2.1. We consider now the two perpendicular tubes undergoing (z direction) a gravitational wave perpendicular to their plane compressing and dilating them simultaneously as shown in figure 10. Therefore, a Poisson's ratio has to be taken into account in this section.

In this case, the two tubes behave like a gigantic stress/strain gauge. Each arm constitutes a principal direction in the xy plane. We can therefore write a tensor expression in two dimensions and consider a strain tensor and a strain energy tensor. We also assume that the xy plane is deconnected from the z direction (elasticity theory: plane stress problem). The consequences is that the spatial material is considered as multisandwiched thin sheets of thickness $2r$ (see [10–13]) successively twisted during the passage of the shear wave (see figure 11).

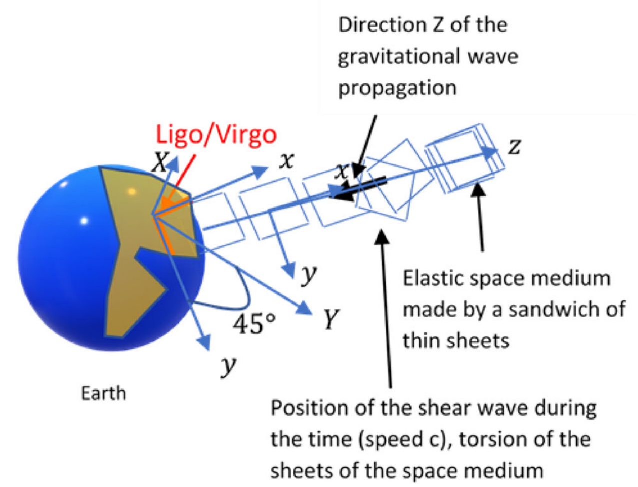

Figure 11. Sandwich structure of the space medium under a gravitational wave.

With the actual measurements made at Ligo and Virgo, we know that when one of the arm is in compression the other is in tension simultaneousely.

First step: Determination of the strains and stresses in the sheet space in torsion

Following the axis $\vec{x}$; $\vec{y}$, we have $\varepsilon_{xx} = -\nu\varepsilon_{yy}$. These two deformations are correlated via the general relativity data by $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ [8,9,11]. The strain tensor according to the axes system $\vec{x}$; $\vec{y}$ is given in (32a). So, the relations between the strains and the stresses are

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) \tag{59}$$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx}). \tag{60}$$

Using the stress tensor definition (16c), we get

$$\sigma_{xx} = \frac{E}{(1+\nu)}\left\{\varepsilon_{xx} + \frac{\nu}{(1-2\nu)}(\varepsilon_{xx} + \varepsilon_{yy})\right\} \tag{61a}$$

$$\sigma_{yy} = \frac{E}{(1+\nu)}\left\{\varepsilon_{yy} + \frac{\nu}{(1-2\nu)}(\varepsilon_{xx} + \varepsilon_{yy})\right\}. \tag{61b}$$

Taking into account that, $\varepsilon_{xx} = -\nu\varepsilon_{yy}$ (with $\nu = 1$, see 4.4), expressions (61a) and (61b) become

$$\sigma_{xx} = \frac{E}{(1+\nu)}\{\varepsilon_{xx}\} \tag{62a}$$

$$\sigma_{yy} = -\frac{E}{(1+\nu)}\{\varepsilon_{yy}\}. \tag{62b}$$

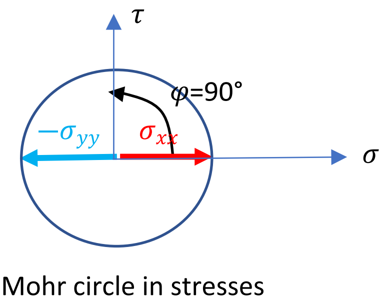

Figure 12. Mohr's circle of the stress state [54].

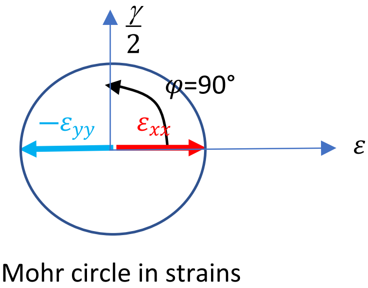

Figure 13. Morh's circle of the strain state [54].

Note: If one of the tube section is in compression ($\vec{x}$ direction) the other one in $\vec{y}$ is in traction (origin of the minus sign).

According to the system of axes $\vec{x}$; $\vec{y}$, the stress tensor is as shown below:

$$\sigma_{xy} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & -\sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{63}$$

Using Mohr's circle (see figure 12) we can confirm the global shear behaviour along a 45° facet (axes system $\vec{X}$; $\vec{Y}$) (see figure 11) and [10].

When we turn 90° on the Mohr's circle, we turn 45° on the real facet (image of the spin 2 of the graviton, see [10]). So on a 45° facet we are in pure shear as shown in figure 7. The strain tensor according to the axes system $\vec{X}$; $\vec{Y}$ is (see figure 13) and [10]:

$$\varepsilon_{XY} = \begin{bmatrix} 0 & \varepsilon_{XY} & 0 \\ \varepsilon_{XY} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{64}$$

The stress tensor according to the axes system $\vec{X}$; $\vec{Y}$ is

$$\sigma_{XY} = \begin{bmatrix} 0 & \tau_{XY} = \sigma_{XX} = \sigma & 0 \\ \tau_{YX} = \sigma_{YY} = \sigma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{65}$$

The shear stress is defined in eq. (11):

$$\tau_{XY} = \frac{E}{2(1+\nu)}\gamma_{XY} = \mu\gamma_{XY}. \tag{66}$$

In addition, we have in elasticity:

$$\varepsilon_{XY} = \frac{1}{2}\gamma_{XY}. \tag{67a}$$

So from (66) and (67a) we get

$$\tau_{XY} = \frac{E}{(1+\nu)}\varepsilon_{XY}. \tag{67b}$$

On the Morh's circle (figure 13) we see that

$$\frac{\gamma_{XY}}{2} = \varepsilon_{XX}. \tag{68}$$

According to this equation, it seems that depending on the orientation of the interferometer in the plane ($\vec{x}$, $\vec{y}$) and ($\vec{X}$, $\vec{Y}$) with respect to the direction of propagation of the gravitational wave ($\vec{z}$), lateral movements of the laser mirrors are possible with the same shape and same wave intensity as the conventional one in compression/traction (see figure 4). It should be interesting to measure these movements especially when the traditional compression/traction motions are not measured because of the position of the interferometer with respect to the direction of propagation of the gravitational wave. From (67a) and (68) we obtain

$$\varepsilon_{XX} = \varepsilon_{XY}. \tag{69}$$

So the deformation of the circle containing the particles is the same (traction in one direction and compression in the other) but the circle rotates by an angle of 45°, see [10].

Second step: Determination of the strain energy of the two connected tubes in the main system of axes x y

We now consider the strain energy of the two tubes connected in traction/compression according to the axes system $\vec{x}\vec{y}$:

Note: The results will be the same if we consider the axes system $\vec{X}$; $\vec{Y}$ because of the equivalence between shear stresses and normal stresses on the one hand (see (65)) and the equivalence between the strains and angles on the other hand (see (64), (68) and (69) and [11]).

The energy density or energy per unit volume is

$$U_{ij} = \frac{U}{V} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}. \tag{70}$$

In the axes system $\vec{x}$; $\vec{y}$, we are in pure compression/traction in the tube, and the total strain energy per unit of volume is

$$\frac{U}{V}=\frac{1}{2}\sigma_{xx}\varepsilon_{xx}+\frac{1}{2}\sigma_{yy}\varepsilon_{yy}. \tag{71}$$

As the section S of the tube is constant and as the stresses and strains are constant on S, for a fixed section S of abcissa (x) or (y), we have

$$\frac{U}{S}=\frac{1}{2}\int_0^L\sigma_{xx}\varepsilon_{xx}\mathrm{d}x+\frac{1}{2}\int_0^L\sigma_{yy}\varepsilon_{yy}\mathrm{d}x. \tag{72}$$

When one arm is in compression the other is in traction and so

$$\varepsilon_{xx}=-\varepsilon_{yy}=\frac{N}{ES} \tag{73}$$

and from the Hooke's law:

$$N_x=N_y=N=\frac{ES}{L}\delta L=K\delta L. \tag{74}$$

The expressions of the stresses are given in (62a) and (62b). We can introduce the expressions of theses stresses in eq. (72):

$$\frac{U}{S}=\frac{E}{2(1+\nu)}\int_0^L\{\varepsilon_{xx}\}^2\mathrm{d}x+\frac{E}{2(1+\nu)}\int_0^L\{\varepsilon_{yy}\}^2\mathrm{d}x. \tag{75a}$$

By replacing the strains by their expressions (73), we obtain

$$U=\frac{E}{2(1+\nu)}\int_0^L\left\{\frac{N}{ES}\right\}^2S\mathrm{d}x+\frac{E}{2(1+\nu)}\int_0^L\left\{\frac{N}{ES}\right\}^2S\mathrm{d}x. \tag{75b}$$

We obtain the generalisation of expression (45a) in two dimensions:

$$U=\frac{1}{2}(1+\nu)\int_0^L\frac{N^2}{ES}\mathrm{d}x+\frac{1}{2(1+\nu)}\int_0^L\frac{N^2}{ES}\mathrm{d}x \tag{75c}$$

or after simplification

$$U=\frac{1}{(1+\nu)}\frac{N^2L}{ES}\ . \tag{75d}$$

The strain energy of a stiffness spring K (N/m) can be written by substituting N by expression (44b) in (75d):

$$U=\frac{1}{(1+\nu)}K(\delta L)^2. \tag{75e}$$

Moreover, according to the strain definition, the displacement variation δL is with $\nu=1$:

$$\varepsilon_{xx}L=-\varepsilon_{yy}L=\pm\varepsilon\times L=\pm\varepsilon_{ii}\times L=\pm\delta L \tag{75f}$$

with $i=x$ or y. Introducing (75f) into (75e), we get

$$U=\frac{1}{(1+\nu)}K(\varepsilon_{ii})^2L^2 \tag{75g}$$

or

$$(\varepsilon_{ii})^2=(1+\nu)\frac{1}{K}\frac{U}{L^2}. \tag{76a}$$

Starting from the rigidity K depending on the length L, the Young's modulus E, the tube section S and substituting the expression of L resulting from K in (76a), we obtain

$$(\varepsilon_{ii})^2=(1+\nu)K\frac{U}{E^2S^2}. \tag{76b}$$

Considering the tube volume V, the section S is extracted (see (48)). Substitute the expression of S (48) in (76b) gives the U/V strain energy density:

$$\frac{1}{L^2}(\varepsilon_{ii})^2=(1+\nu)\frac{K}{V}\frac{1}{E^2}\times\frac{U}{V} \tag{76c}$$

or

$$\frac{U}{V}=\frac{\frac{1}{L^2}(\varepsilon_{ii})^2}{(1+\nu)\frac{K}{V}\frac{1}{E^2}}. \tag{76d}$$

We now consider the tubes under dynamic behaviour and with a compression/traction wave following x and y in both arms simultaneously due to gravitational transversal waves. By proceeding as in §5.2.1 in one dimension we obtain:

$$\frac{1}{L^2}(\varepsilon_{ii})^2=(1+\nu)\rho\frac{\omega^2}{E^2}\times T. \tag{76e}$$

By substituting the circular frequency ω (53b) by frequency f in (76e), we obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2=4\pi^2(1+\nu)\rho\frac{f^2}{E^2}\times T. \tag{76f}$$

We have a pure compression/traction in the interferometer (not in 3D with shear wave), and so the compression/traction wave equation is (see §4.3.2.1 and figure 4) the same as in §5.2.1 (see (35a) and (35b)). We obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2=4\pi^2(1+\nu)\rho\frac{f^2}{c^4\rho^2}\times T. \tag{76g}$$

On the basis of (75f), we can write this equation according to each strain in each arm of the interferometer:

$$\frac{1}{L^2}(\varepsilon_{xx})^2=4(1+\nu)\times\pi\times\frac{\pi f^2}{\rho}\times\frac{1}{c^4}\times\frac{U}{V} \tag{76h}$$

$$\frac{1}{L^2}(\varepsilon_{yy})^2=4(1+\nu)\times\pi\times\frac{\pi f^2}{\rho}\times\frac{1}{c^4}\times\frac{U}{V} \tag{76i}$$

and it is interesting to see that with $\nu = 1$ (see 4.4) and $G = \frac{\pi f^2}{\rho}$ we obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = 8\pi \frac{G}{c^4} \times \frac{U}{V} \tag{76j}$$

or with (75f):

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = \frac{8\pi G}{c^4} \times \frac{U}{V} \tag{76k}$$

$$\frac{1}{L^2}\left(\varepsilon_{yy}\right)^2 = \frac{8\pi G}{c^4} \times \frac{U}{V}. \tag{76l}$$

So we can construct the tensorial expression (76m) on the basis of (76h) and (76i) and following the principle curvature $= K$ strain energy density which is also equal to K times the work of external forces.

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & \left(\varepsilon_{yy}\right)^2 \end{bmatrix} = \frac{8\pi G}{c^4} \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}. \tag{76m}$$

Note: If complementary measurements are carried out on an interferometer and if we can confirm that there are also shear strains (angles γ), then the hypothesis of plate behaviour is confirmed, eq. (76m) will be built with 3×3 matrix completed by Poisson's ratio ν, strains ε_{xy} and ε_{yx} and strain energies T_{xy} and T_{yx}.

By opposition, if measurements are carried out and if there are no shear strains (angles γ), it is necessary to consider the hypothesis of torsional wave cited above. In this case, the mechanical model could not be a plate but a space cylinder in torsion due to the rotation of black holes' binary system (see §5.2.3). But all these elements do not change the κ coefficient subject of this article. We continue so on the basis of the actual data, the $\frac{\delta L}{L}$ measured in each direction of the interferometer arms.

It must be remembered that in weak field there is a relationship between the metric g_{ij} and the strain tensor ε_{ij} (see (12a)) and a relationship between the stress energy tensor T^{ij} and the mechanical stress tensor σ^{ij} (see (16a), (16b) and Appendix A). The parallelism between expression (76m) and the plate bending expression is noted for the diagonal terms of (77c) but not the terms concerning the Poisson's ratio (see above). What is important here, is that the term , $\frac{\varepsilon_{ij}}{L^2}$, can be interpretated as a curvature by comparison with the terms, $\frac{\varepsilon_{ij}}{z^2}$, that we have in a plate case ((77a), (78) and (79)). Indeed, in the case of the plate hypothesis [51,52], the strain energy is with z, the thickness of the plate perpendicular to the plane xy:

$$\frac{1}{z^2}\left[(\varepsilon_{xx})^2+\left(\varepsilon_{yy}\right)^2+2(1-\nu)\frac{1}{4}\left\{\varepsilon_{xy}\right\}^2+2\,\nu\left\{\varepsilon_{xx}\varepsilon_{yy}\right\}\right]$$

$$= \frac{24\left(1-\nu^2\right)}{Eh^3} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}y}. \tag{77a}$$

With the plate bending rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{77b}$$

or

$$\frac{1}{z^2}\left\{\varepsilon_{xx};\varepsilon_{yy};\frac{\varepsilon_{xy}}{2}\right\}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{pmatrix}\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \frac{\varepsilon_{xy}}{2} \end{pmatrix} = \frac{24\left(1-\nu^2\right)}{Eh^2} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}yh} \tag{77c}$$

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\,m}}{\mathrm{m}^3}. \tag{77d}$$

We note that the right term in $\mathrm{N\,m}/m^3$ is like an energy density and the left term in m^{-2} is like a curvature.

We note that $\frac{24(1-\nu^2)}{Eh^2}$ has the same dimension as $\kappa = \frac{8\pi G}{c^4}$ (N^{-1}). We can concider κ as an equivalent flexibility of the space fabric and $1/\kappa$ as an equivalent rigidity of this frame. With the relationships between the curvatures and the second derivatives of z displacements (w) we have

$$\frac{1}{R_x} = \frac{\partial^2 w_{(x,y)}}{\partial x^2}, \quad \frac{1}{R_y} = \frac{\partial^2 w_{(x,y)}}{\partial y^2}, \quad \frac{1}{R_{xy}} = \frac{\partial^2 w_{(x,y)}}{\partial x \partial y} \tag{78}$$

and the relations between strains and curvatures are

$$\varepsilon_{xx} = -\frac{z}{R_x}, \qquad \varepsilon_{yy} = -\frac{z}{R_y},$$

$$\gamma = \varepsilon_{xy} = -2z\frac{\partial^2 w_{(x,y)}}{\partial x \partial y} = -2z\frac{1}{R_{xy}}. \tag{79}$$

The strain tensor ε_{ij} can then be expressed in terms of curvature tensor R_{ij} of the thin plate:

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} = -z\begin{bmatrix} \frac{1}{R_x} & 2\frac{1}{R_{xy}} \\ 2\frac{1}{R_{xy}} & \frac{1}{R_y} \end{bmatrix} = -zR_{ij} \tag{80}$$

or

$$\left\{\frac{1}{R_x};\frac{1}{R_y};\frac{1}{R_{xy}}\right\}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{pmatrix}\begin{pmatrix} \frac{1}{R_x} \\ \frac{1}{R_y} \\ \frac{1}{R_{xy}} \end{pmatrix} = \frac{24\left(1-\nu^2\right)}{Eh^2} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}yh}. \tag{81}$$

Therefore, expression (82) from (76m), can be considered as equivalent at a curvature tensor whose radii of curvature in each direction are infinite ($R \to L$).

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix}. \tag{82}$$

So, we define

$$R_{ij} = \begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix} \tag{83a}$$

and we have tensor of strain energy density as

$$T_{ij} = \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}. \tag{83b}$$

Expression (76k) can therefore be written as

$$R_{ij} = \frac{8\pi G}{c^4} T_{ij}. \tag{84}$$

We consider into this expression (76m) an infinite radius of curvature which therefore implies a zero scalar curvature. Indeed, we assume $\varepsilon_{xx} = -\varepsilon_{yy}$ and after rising the second index of the Ricci tensor we get the diagonal part $(\varepsilon_{xx}; -\varepsilon_{yy})$, for which the trace is zero.

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\,m}}{\mathrm{m}^3}. \tag{85}$$

In this expression (eq. (84)) R_{ij} is the equivalent curvature tensor of the space and T_{ij} is the strain energy tensor of the deformed space.

To reach the parallelism with the Einstein's field equation where $G_{\mu\nu}$ cover the curvature tensor of the space and $T_{\mu\nu}$ cover the energy density out of the space (e.g., the Sun curved the space but is not the space itself), we have to consider in our analogy that the work of the external forces is equal to the work of the internal forces created by the strain energy (see (8d), (5a) and (10e)).

$$T_{\text{external}} = T_{\text{internal}}. \tag{86}$$

In this case, our analogy in 2D (analogy (76m)) is close to the Einstein's field equation in four dimensions (5a).

We thus managed to find κ (see §6), by passing through the mechanical components of the stress tensor (and not via the temporal component of the tensor as Einstein did it to be correlated with the Newton's approach in weak gravitational field) corresponding to the internal work of the space fabric which is equal to the external work of the applied masses.

$$U = W_{\text{int}} = W_{\text{ext}}. \tag{87}$$

5.2.3 *Study of a vertical cylinder in pure torsional space (see figures 5 and 14) – use of the shear velocity of the correlated shear wave with the shear stress tensor* [54]. We assume in this section that the coalescence of two black holes for example, creates by their rotations, a torsion of the space as shown in figure 3 and refs [12,13]. In weak field, for space only, the metric is given in (12a). Einstein's gravitational equation in a weak field for space is

$$\partial^\lambda \partial_\lambda \overline{h}_{ij} = \Box \overline{h}_{ij} = -\frac{16\pi G}{c^4} T_{ij}. \tag{88}$$

The result for a polarised wave $A_\times$ is eq. (30b). With formula (31f) demonstrated in §4.3.2.1, we have a link between the general relativity (disturbance of the spatial part of the metric h_{ij}) and the theory of elasticity (the strain ε_{ij} of the elastic medium) (see 12c). So, based on (30b) and (12c), the spatial perturbation of the metric $h_{\mu\nu(A_\times)}$ is in correspondence with the following strain tensor [10,54]:

$$\varepsilon_{xy} = \begin{bmatrix} 0 & \varepsilon_{xy} = \varepsilon & 0 \\ \varepsilon_{yx} = \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{89}$$

Thanks to the Hooke's and elasticity formula, there is a correspondence with the strain tensor ε_{xy} (89) and the stress tensor σ_{xy} (90):

$$\sigma_{xy} = \begin{bmatrix} 0 & \tau_{yx} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{90}$$

This type of stress tensor is representative of a cylinder in pure torsion, twisted by a torque M_t (see figure 14). For this section, we consider the cube of normal to the facet $\vec{z}$ at a point Q (τ stresses distribution).

Note: The attentive reader will, however, not have failed to notice that the facets subjected to traction and compression are not in the shear plane perpendicular to the direction of the wave (figures 6, 8 and 11), as is the case with gravitational waves, but inclined to 45° (figure 14). This approach is therefore only a simplified model to check the possible value of κ in accordance with the stress tensors (33b) connected to the pure torsion studied according to the elasticity theory [54].

The torsional strain energy is

$$U = \frac{1}{2} \int_0^L \frac{M_t^2}{\mu I_t} \mathrm{d}x = \frac{1}{2} \frac{M_t^2 L}{\mu I_t}. \tag{91a}$$

With θ the angular displacement of the point Q located on the outer surface of the cylinder, we have:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right) = \frac{M_t}{\mu I_t}. \tag{91b}$$

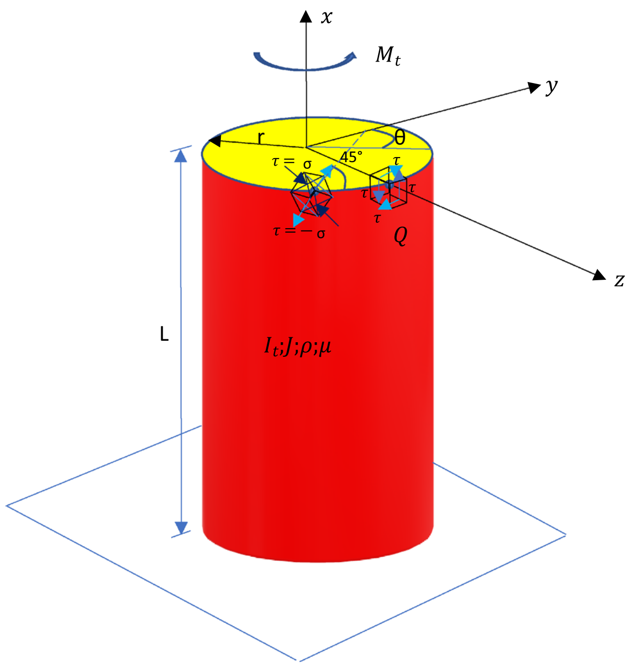

Figure 14. Cylinder clamped in pure torsion [54].

We assume that the torsion torque is constant. Introducing (91b) in (91a) we get

$$U = \frac{1}{2}\mu I_t \left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 L. \tag{92}$$

From expression (92), the equivalent torsional curvature is extracted:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 = \frac{2}{\mu I_t}\frac{U}{L}. \tag{93}$$

The relationship between the torque M_t and the rotation θ gives the torsional stiffness k according to expression (94a):

$$M_t = k\theta. \tag{94a}$$

By integrating (91b) with respect to x, we can extract a new expression for θ:

$$\theta = \frac{M_t L}{\mu I_t} \tag{94b}$$

or

$$M_t = \frac{\mu I_t}{L}\theta. \tag{94c}$$

Comparing (94c) with (94a), the expression of the torsional stiffness is therefore:

$$k = \frac{\mu I_t}{L}. \tag{95}$$

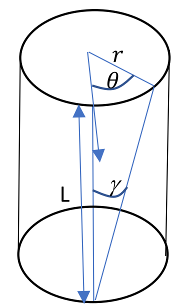

Figure 15. Definition of the shear strain γ.

Introducing (95) into (91a) yields

$$U = \frac{1}{2}\frac{M_t^2 L}{\mu I_t} = \frac{1}{2}\frac{(M_t)^2}{k}. \tag{96a}$$

The introduction of (94a) in (96a) gives

$$U = \frac{1}{2}k\theta^2. \tag{96b}$$

By the Hooke's law (14) we have a relationship between the stress τ and the shear stain γ. Using figure 15, we determine the relationship between the shear strain γ and the angular displacement θ.

We have geometrically:

$$\mathrm{tang}\gamma \approx \gamma = \frac{r\theta}{L} \tag{97a}$$

and the relationship between the stress and the strain (14) is as follows:

$$\tau = G\gamma = G\frac{r\theta}{L}. \tag{97b}$$

By introducing θ from expression (97a) into (96b) we obtain a new formula of the torsional strain energy of the cylinder:

$$U = \frac{1}{2}k\frac{\gamma^2 L^2}{r^2}. \tag{98}$$

That we can rewrite as

$$\frac{\gamma^2}{r^2} = \frac{2U}{kL^2}. \tag{99}$$

We can now replace the torsional stiffness k by its value (95). We extract the length of the cylinder and we obtain

$$L = \frac{\mu I_t}{k}. \tag{100}$$

The introduction of (100) into (99) yields

$$\frac{\gamma^2}{r^2} = \frac{2kU}{\mu^2 I_t^2}. \tag{101}$$

The torsional inertia I_t of a cylinder expressed as a function of S, the surface of the cylinder, is written as

$$I_t = \frac{\pi d^4}{32} = \frac{\pi r^4}{2} = S\frac{r^2}{2}. \tag{102}$$

By introducing I_t in (101) we obtain

$$\gamma^2 r^2 = \frac{8kU}{\mu^2 S^2}. \tag{103}$$

Introducing (104) into (103), we obtain

$$S_c = \frac{V}{L} \tag{104}$$

$$\frac{1}{L^2}\gamma^2 = 8\frac{k}{r^2\mu^2 V} \times \frac{U}{V}\ . \tag{105}$$

We now consider the dynamic behaviour of the cylinder consisting of an elastic substance of density ρ, Young's modulus E, Poisson's ratio ν obtained from of the vacuum energy. The fundamental equation of dynamics is derived from the sum of the kinetic energy and torsional strain energy:

$$U_C + U = \frac{1}{2}J\omega^2 + \frac{1}{2}k\theta^2. \tag{106}$$

For the mechanical characteristics of the cylinder made of a space equivalent material, the following definitions are given:

The moment of inertia of a rotating cylinder is

$$J = \frac{1}{2}\rho\pi r^4 L. \tag{107}$$

The circular frequency is

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = 2\pi f \tag{108}$$

or

$$\omega^2 = \frac{\mu I_t}{JL}. \tag{109}$$

By introducing (108) into (106) we get

$$U_C + U = \frac{1}{2}J\left(\frac{\mathrm{d}\theta_{(t)}}{\mathrm{d}t}\right)^2 + \frac{1}{2}k\theta_{(t)}^2. \tag{110a}$$

By derivative with respect to t (110a) we obtain, looking for the natural frequency of the cylinder:

$$J\frac{\mathrm{d}^2\theta_{(t)}}{\mathrm{d}t^2} + k\theta_{(t)} = 0. \tag{110b}$$

By introducing the formula of k (95) and J (107) in (110b) we obtain

$$\frac{1}{2}\rho\pi r^4 L\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\mu I_t}{L}\theta = 0. \tag{110c}$$

Introducing the expression of I_t (102) in (110c) gives

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{2\mu\frac{\pi r^4}{2}}{L\rho\pi r^4 L}\theta = 0 \tag{110d}$$

and after simplification we get

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\mu}{\rho L^2}\theta = 0. \tag{110e}$$

We verify the dimensional equation of the term before θ:

$$\omega^2 = \frac{k}{J} = \frac{\mu I_t}{JL} = \frac{\mu\frac{\pi r^4}{2}}{\frac{1}{2}\rho\pi r^4 L} = \frac{\mu}{\rho L^2}$$
$$= \frac{\frac{\mathrm{kg\,m}}{\mathrm{s^2 m^2}}}{\frac{\mathrm{kg}}{\mathrm{m^3}}\mathrm{m}^2} = \frac{1}{s^2}. \tag{111a}$$

By replacing the shear modulus μ as a function of k (95) in expression (111a) we obtain

$$\omega^2 = \frac{k}{\rho L I_t}. \tag{111b}$$

In expression (111b), L is extracted and multiplied each side by the surface S of the tube to obtain a new expression of the volume V of the cylinder:

$$L\pi r^2 = V = \frac{k}{\omega^2\rho I_t}\pi r^2. \tag{112a}$$

By introducing V (112a) in (105) we get

$$\frac{1}{L^2}\gamma^2 = 8\frac{k}{r^2\mu^2\left(\frac{k}{\omega^2\rho I_t}\pi r^2\right)} \times \frac{U}{V} \tag{112b}$$

or after simplification

$$\frac{1}{L^2}\gamma^2 = 8\frac{\omega^2\rho I_t}{\pi r^4\mu^2} \times \frac{U}{V}\ . \tag{112c}$$

Introducing the expression of I_t (102) into (112c) we get

$$\frac{1}{L^2}\gamma^2 = 4\rho\left(\frac{\omega}{\mu}\right)^2 \times \frac{U}{V} = 4\rho\left(\frac{\omega}{\mu}\right)^2 \times T. \tag{112d}$$

By replacing ω and the shear modulus μ by their values, we obtain

$$\frac{1}{L^2}\gamma^2 = 4\frac{4\pi^2 f^2\rho}{\left(\frac{E}{2(1+\nu)}\right)^2} \times \frac{U}{V} \tag{112e}$$

or after simplification

$$\frac{1}{L^2}\gamma^2 = 4(1+\nu)^2\frac{16\pi^2 f^2\rho}{E^2} \times \frac{U}{V}. \tag{112f}$$

By multiplying and dividing expression (112f) by ρ we get

$$\frac{1}{L^2}\gamma^2 = 4(1+\nu)^2\frac{16\pi^2 f^2\rho^2}{E^2\rho} \times \frac{U}{V} \tag{112g}$$

or

$$\frac{1}{L^2}\gamma^2 = 64\pi(1+\nu)^2\frac{\pi f^2}{\rho}\left(\frac{\rho}{E}\right)^2 \times \frac{U}{V}. \tag{112h}$$

This time we use the definition of the speed of a transverse wave (torsion wave) acting by shear (see 41) and we obtain

$$\left(\frac{\rho}{E}\right)^2 = \frac{1}{4c^4(1+\nu)^2}. \tag{112i}$$

We introduce (112i) in (112h) and we obtain

$$\frac{1}{L^2}\gamma^2 = 64\pi(1+\nu)^2\frac{\pi f^2}{\rho}\frac{1}{4c^4(1+\nu)^2} \times \frac{U}{V} \tag{112j}$$

or after simplification we get the final result

$$\frac{\gamma^2}{L^2} = 16\pi\frac{\pi f^2}{\rho}\frac{1}{c^4} \times \frac{U}{V}. \tag{112k}$$

Assuming that the external work T is equal to the internal work U, and comparing with (88), we confirm that

$$G = \frac{\pi f^2}{\rho} \tag{113}$$

and

$$\frac{1}{L^2}\gamma^2 = 16\pi\frac{G}{c^4} \times T. \tag{114}$$

That we have to compare (see 28b) with

$$\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}. \tag{115}$$

We therefore have a factor 2 on the left term of (115) (dimension $1/\mathrm{m}^2$) which allows us to find the usual value of κ with a factor 8.

$$\frac{1}{L^2}\gamma^2 => \Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right), \tag{116}$$

with $\Box$ the d'Alembertian.

So, via (112d) and (116), it will be necessary to divide by 2 the factor before U/V in (112d) to obtain κ.

6. Deduction of the parallelism of a new mechanical expression of the gravitational constant G and Einstein's constant κ based on the Ligo and Virgo measurements

6.1 *General*

The parallelism between the Timoshenko approaches (see (57), (76h), (76i) or (114)) and the Einstein's approach (5a) is therefore demonstrated three times (see §5.2). We note that this parallelism is obtained by using the wave propagation in the medium as the transverse gravitational waves in eq. (5a) is a tensor expression (tensor of the curvatures R_{ij} connected to the strain tensor ε_{ij} and strain energy density tensor T_{ij}) (see (84)) and is based on the Hooke's law (13), the spring elastic strain energy and the eigen circular frequency of the spring/mass system assimilating the tube to a stiffness spring k and mass m. In all the calculations carried out, we thus obtain that the gravitational constant G can be expressed as a function of the density ρ and of the natural characteristic frequency f of an elastic microstructure constituting the vacuum space.

$$G = \frac{\pi f^2}{\rho}.$$

On the basis of this equation of G, we can re-express now the Einstein's constant κ (4a) in mechanical terms. As the displacements of the laser mirrors in the interferometers are limited to forward or backward motions, we only consider the associated traction/compression waves (§5.2.1 and §5.2.2) as representatives of that actual data.

6.2 *In the case of §5.2.1: One arm of the interferometer*

Taking into account the following points:

(a) With compression, tensile wave velocities (35a) correlated with the stress tensor (33a) and the polarised wave A_+ [53],
(b) with the definition of the circular frequency ω of the elastic medium, we obtain based on (54c),

we obtain

$$\kappa = \frac{8\pi G}{c^4} = 2\rho\left(\frac{\omega}{E}\right)^2. \tag{117a}$$

We verify the dimensional equation (117b) of κ, indeed:

$$\rho\left(\frac{\omega}{E}\right)^2 = > \frac{\mathrm{kg}}{\mathrm{m}^3}\left(\frac{1}{\mathrm{s}\frac{\mathrm{kg}}{\mathrm{ms}^2}}\right)^2 = \frac{\mathrm{s}^2}{\mathrm{kgm}} = \frac{1}{N}. \tag{117b}$$

6.3 *In the case of §5.2.2 – two arms of the interferometer correlated via the Poisson's ratio ν*

Taking into account the following points:

(a) With compression, tensile wave velocities (35a) are correlated with the stress tensor (33a) and the polarised wave A_+ [53],
(b) with (76f) or (76h) and (76i),
(c) with the definition of the circular frequency ω of the elastic medium,

we obtain

$$\kappa = \frac{8\pi G}{c^4} = (1+\nu)\rho\left(\frac{\omega}{E}\right)^2. \tag{118}$$

Note: We confirm well the parallelism previously demonstrated in §5.1.2. Indeed κ is well proportional to $(1+\nu)$ in the expression, see (43c). In addition, with $\nu = 1$ we obtain (117a).

6.4 *In the case of §5.2.3 – Pure shear and torsion approach*

Taking into account the following points:

(a) With shear wave velocities (41) correlated with the stress tensor (33b) and the polarised wave $A_\times$ [54].
(b) With eqs (112d) and (28b) and taking into account that we have a coefficient 2 on the left term which must be taken into account to reach κ.

Following these hypotheses, we thus obtain from (112d), (115), (116)

$$\kappa = \frac{8\pi G}{c^4} = 2\rho\left(\frac{\omega}{\mu}\right)^2. \tag{119}$$

We obtain a logical result for this case of pure torsion. The formula is the same as that obtained in eqs (117a) and (117b), but this time the shear modulus μ plays the role of Young's modulus E.

7. Physical approach or simple mathematical artefact?

Does all this have such a physical reality, or is it just a mathematical coincidence? Let us take a look at this about the vacuum. The Casimir force [55] in connection with quantum field theory shows that the vacuum has a non-zero ground state, a non-zero energy and therefore, according to the special relativity, a non-zero equivalent mass. The space is therefore a physical object and so is not empty. The measurement of Casimir's force in the vacuum [55] confirms that

1. Particles and virtual antiparticles are created and annihilated spontaneously, generating a force that brings together two parallel plates placed in the vacuum. There is energy in the vacuum.
2. There is a fundamental state different from 0 from the quantified vacuum (QED).
3. There is a scalar field (Brout, Englert, Higgs field) in vacuum space.
4. Emptiness is not a void.

In addition, the vacuum energy has been measured and calculated:

- According to the cosmological constant measurements: $\rho = 1 \times 10^{-29}$ g/cm^3 (1×10^{-26} kg/m^3),
- According to quantum field theory: $\rho = 1.11 \times 10^{93}$ g/cm^3,
- According to the cosmological constant measurements: $T_{\text{vacuum}} = 8.987551787 \times 10^{-10}$ kg m^2/s^2/m^3,
- According to the quantum field theory: $E_{\text{vacuum}} = 1\times10^{113}$ kg m^2/s^2/m^3.

Even if the values are so different, the vacuum energy is not null. That is the fundamental point here.

The best way to know which value is the right one is to measure the Young's modulus via the Casimir test (see §9.4) and to determine the good value of the energy by the formula given in tables 2 and 3. So, following the quantum field theory, the vacuum energy E_v is quantified at the fundamental state:

$$E_v = \frac{1}{2}hf = \frac{1}{2}h\frac{\omega}{2\pi} = \frac{1}{2}\hbar\omega, \tag{120}$$

where E_v is the vacuum energy, h is the Planck's constant and ω is the circular frequency.

In addition, the special relativity laws apply to the vacuum. The energy of emptiness is

$$E_v = mc^2. \tag{121}$$

Using (120) and (121), we deduce the equivalent mass expression m present in vacuum:

$$\frac{1}{2}\frac{\hbar\omega}{c^2} = m \tag{122}$$

and the expression of the vacuum density ρ is

$$\frac{1}{2}\frac{\hbar\omega}{c^2 V} = \frac{m}{V} = \rho. \tag{123}$$

It seems therefore coherent that formula (123) is related to the density ρ and to the specific circular frequency ω (via f) that can be, in this case, those of the vacuum (via vacuum energy). We note also that eqs (113), (117a), (118), (119) are related to ω and ρ.

8. Results

8.1 *Numerical application at the vacuum energy – longitudinal wave in the interferometric tubes*

8.1.1 *Theoretical development.* We note T as a vacuum energy density measured in $\mathrm{J/m^3}$ with V a volume of space elastic material that have to be determined.

$$T \times V = E_v = mc^2 = \rho \times V \times c^2. \tag{124a}$$

So we can extract the density ρ of (124a):

$$\rho = \frac{E_v}{Vc^2}. \tag{124b}$$

Considering eq. (120) we extract the frequency f:

$$f = \frac{2E_v}{h}. \tag{124c}$$

From (120), the energy density of the vacuum E_v/V is therefore written as a function of the circular frequency ω and the Planck's constant h:

$$\frac{E_v}{V} = T = \frac{h\omega}{4\pi V}. \tag{125}$$

We extract from (125) the circular frequency:

$$\omega = \frac{4\pi E_v}{h}. \tag{126}$$

By substituting expressions (124b) and (126) in expression (117a), we obtain the Young's modulus E:

$$E = Y = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}. \tag{127a}$$

By substituting expressions (124b) and (127a) in (35a), we obtain the vacuum energy E_v which depends on volume V:

$$E_v = \rho V c^2 = \frac{Gh^2}{4\pi V c^2} \tag{127b}$$

and from the expression (127b) we determine the vacuum density ρ:

$$\rho = \frac{Gh^2}{4\pi V^2 c^4}. \tag{128}$$

By expressions (35a) and (128) we deduce a new expression of the Young's modulus E:

$$E = \rho c^2 = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2. \tag{129}$$

Expressions (124c) and (127b) draw the expression of f:

$$f = \frac{Gh}{2\pi V c^2} \tag{130}$$

and expressions (130) and (53b) give the circular frequency:

$$\omega = \frac{Gh}{Vc^2}. \tag{131}$$

We must therefore check the small volume hypothesis (quantum approach). According to the quantum field theory, vacuum density $\rho = 1.11 \times 10^{93}\ \mathrm{g/cm^3}$ and the vacuum energy density $T = 1.00 \times 10^{113}\ \mathrm{J/m^3}$. In addition, the fundamental physic constants are:

$$G = 6.67408 \times 10^{-11}\ \mathrm{m^3/(kg\ s^2)}$$
$$h = 6.62607004 \times 10^{-34}\ \mathrm{m^2\ kg/s}$$
$$c = 299\ 792\ 458\ \mathrm{m/s}$$
$$\kappa = 2.0766 \times 10^{-43}\mathrm{N^{-1}}.$$

As all the formulae are functions of volume V, now we are looking for the value of this volume V that allows to satisfy all the physic constants. From this volume V, it will then be possible to determine the r dimension of the space fabric fibres and the Young's modulus E of the space material.

8.1.2 *Magnitude obtained for the new parameters of G based on vacuum energy.* So, the question is therefore what volume V does to simultaneously satisfy expressions (4a), (35a) and (113). Once all iterative calculations have been carried out, we obtain the results given in table 2.

8.2 *Numerical application at the vacuum energy – global approach by a shear torsion wave*

8.2.1 *Theoretical development.* The approach is the same as in §8.1 but the Young's modulus E is replaced throughout the equation by the shear modulus μ. The energy density T follows eqs (124a) and (125). The density ρ follows eq. (124b). The frequency f is defined in eq. (124c). The circular frequency is equal to (126). By substituting expressions (124b) and (126) in expression (119), we obtain the Young's modulus E.

$$\mu = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}\ . \tag{132}$$

With the definition of the shear modulus μ we obtain the Young's modulus E:

$$E = Y = 4(1+\nu)E_v^{\frac{3}{2}}\frac{c}{h}\sqrt{\frac{\pi}{GV}}. \tag{133}$$

By substituting expressions (124b) and (127a) in (41), eq. (127b) is again obtained on the vacuum energy E_v as a function of the volume V considered:

$$E_v = \rho V c^2 = \mu V = \frac{Gh^2}{4\pi V c^2} \tag{134}$$

Table 2. Numerical application (case of Young's modulus approach)

Parameters	Physical objects With $v = 1$ Case of longitudinal waves (pure compression/traction)	Units	Values with $E_{\text{vacuum}} = 1\times10^{113}$ J/m^3
Volume	V	m^3	1.61×10^{-104}
Radius of V (link with the string theory and Planck's length)	r if $V = \frac{4}{3}\pi r^3$	m	1.566×10^{-35}
Vacuum energy	$E_v = \rho V c^2 = \frac{Gh^2}{4\pi V c^2}$	$\frac{\text{kg m}^2}{\text{s}^2} = \text{J}$	1.61×10^{9}
Vacuum energy density (link with the quantum field heory)	$T = \frac{E_v}{V}$	$\frac{\text{kg}}{\text{s}^2\ \text{m}} = \frac{\text{J}}{\text{m}^3}$	1.00×10^{113}
Density	$\rho = \frac{Gh^2}{4\pi V^2 c^4}$	$\frac{\text{kg}}{\text{m}^3}$	1.11×10^{96}
Young's modulus (link with the elasticity theory)	$E = Y = \rho c^2 = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2$ $E = Y = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}$	$\frac{\text{kg}}{\text{s}^2\ \text{m}} = \text{Pa}$	1.00×10^{113}
Speed of light (link with the special relativity)	$c = \sqrt{\frac{E}{\rho}}$	$\frac{\text{m}}{\text{s}}$	299792458
Frequency	$f = \frac{Gh}{2\pi V c^2}$	1/s	4.861×10^{42}
Period (link with the Planck's time)	$T = \frac{1}{f}$	s	2.05684×10^{-43}
Gravitation constant (link with the Newton's gravitation)	$G=\pi f^2 \frac{1}{\rho}$	$\frac{\text{m}^3}{\text{kg s}^2}$	6.67408×10^{-11}
Circular frequency	$\omega = \frac{Gh}{Vc^2} = 2\pi f = \frac{4\pi E_v}{h}$	1/s	3.05×10^{43}
Einstein's constant (link with the general relativity)	$\kappa=2\rho\left(\frac{\omega}{E}\right)^2$	$\frac{1}{\text{Newton}}$	2.07658×10^{-43}

and from expression (127b) the vacuum density ρ (128) is again obtained. By expressions (41) and (128) a new expression of the shear modulus μ is deduced:

$$\mu = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2 \tag{135}$$

and of the Young's modulus:

$$E = \frac{(1+\nu)G}{2\pi}\left(\frac{h}{cV}\right)^2. \tag{136}$$

Expressions (124c) and (134) restore again expression of f (130) and the expressions (130) and (53b) give the circular frequency (131). The space is thus quantified via radius r of the space fibres:

$$r = \left(\frac{9}{64\pi^3} \times \frac{G}{\mu c^2}\right)^{1/6} h^{1/3} \tag{137}$$

or with the new definition of G:

$$r = \left(\frac{9}{64\pi^2} \times \frac{f^2}{\rho\mu c^2}\right)^{1/6} h^{1/3} \tag{138}$$

and the diameter d is

$$d = \left(\frac{9}{\pi^2} \times \frac{f^2}{\rho\mu c^2}\right)^{1/6} h^{1/3} = \left(\frac{9}{\pi^2\rho\mu}\right)^{1/6}\left(\frac{fh}{c}\right)^{1/3}. \tag{139}$$

8.2.2 *Magnitude obtained for the new parameters of G based on the vacuum energy*

So, the question is therefore what volume V does to simultaneously satisfy expressions (4a), (41) and (113). Once all iterative calculations have been carried out, we obtain the results given in table 3.

Table 3. Numerical application (case of shear modulus approach).

Parameters	Physical objects With $\upsilon = 1$ Case of shear waves (torsion)	Units	Values with $E_{\text{vacuum}} = 1\times10^{113}$ J/m^3
Volume	V	m^3	1.61×10^{-104}
Radius of V (link with the string theory and Planck's length)	r if $V = \frac{4}{3}\pi r^3$	m	1.566×10^{-35}
Vacuum energy	$E_v = \rho V c^2 = \mu V = \frac{Gh^2}{4\pi V c^2}$	$\frac{\text{kg m}^2}{\text{s}^2} = \text{J}$	1.61×10^9
Vacuum energy density (link with the quantum field theory)	$T = \frac{E_v}{V}$	$\frac{\text{kg}}{\text{s}^2\text{m}} = \frac{\text{J}}{\text{m}^3}$	1.00×10^{113}
Density	$\rho = \frac{Gh^2}{4\pi V^2 c^4}$	$\frac{\text{kg}}{\text{m}^3}$	1.11×10^{96}
Young's modulus/shear modulus (link with the elasticity theory)	$E = Y = 2\rho c^2(1+\nu)$ $= \frac{(1+\nu)G}{2\pi}\left(\frac{h}{cV}\right)^2$ $\mu = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2$ $E = Y = 4(1+\nu)E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}$	$\frac{\text{kg}}{\text{s}^2\text{ m}} = \text{Pa}$	4.00×10^{113}
Speed of light (link with the special relativity)	$c = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}$ $c = \frac{1}{f}\sqrt{\frac{GE}{2\pi(1+\nu)}}$	$\frac{\text{m}}{\text{s}}$	299792458
Frequency	$f = \frac{Gh}{2\pi V c^2}$	1/s	4.861×10^{42}
Period (link with the Planck's time)	$T = \frac{1}{f}$	s	2.05684×10^{-43}
Gravitation constant (link with the Newton's gravitation)	$G = \pi f^2 \frac{1}{\rho}$	$\frac{\text{m}^3}{\text{kg s}^2}$	6.67408×10^{-11}
Circular frequency	$\omega = \frac{Gh}{Vc^2} = 2\pi f = \frac{4\pi E_v}{h}$	1/s	3.05×10^{43}
Einstein's constant (link with the general relativity)	$\kappa = 2\rho(\frac{\omega}{\mu})^2$	$\frac{1}{\text{Newton}}$	2.07658×10^{-43}

The results are identical to table 2 except for the Young's modulus that is multiplied by 4.

9. Discussion

9.1 *About the numerical values of the results obtained*

Are our numerical results physically acceptable? We obtain radius r of volume V in correlation with the string dimension defined in the string theory: 1.566×10^{-35} m. We obtain Young's modulus as 4.00×10^{113} Pa, which is compatible with the results of ref. [11] (4.4×10^{113} Pa, see §3.4, formula 3.13).

We obtain $\rho = 1.11\times10^{96}$ kg/m^3 compatible with the results of ref. [11] (1.30×10^{96} kg/m^3, see §3.4, formula 3.14). We find a period equal to 2.05684×10^{-43} s near the Planck's time that is logical because at this time all the interactions have to merge. So the magnitude obtained is correct. In addition, with this volume V we satisfy all the constant values of physics: The Planck's time where all the forces merge, G, c, κ. With expression (58) of G and taking into account eqs (129) and (130) we obtain

$$G = \frac{64}{9}\pi^3 \frac{c^4 \rho_v}{h^2} r^6 \tag{140}$$

and the numerical application gives

$$G = \frac{64}{9}\pi^3 \times \frac{(299792458)^4 \times 1.11 \times 10^{96} \times \left(1.566 \times 10^{-35}\right)^6}{\left(6.62607004 \times 10^{-34}\right)^2}.$$

$G = 6.67408000 \times 10^{-11}\ \mathrm{m^3/kg\ s^2}$.

Thus this formulation unites all the theories of the current physics:

(a) quantum mechanics with the Planck constant h,
(b) special relativity with the speed of light c,
(c) quantum field theory with the vacuum energy (vacuum density) ρ_v,
(d) string theory with the size of the medium grain size r (string size),
(e) The gravitation with G.

9.2 *About the two approaches: Longitudinal and shear waves*

Irrespective of whether we adopt the approach following the longitudinal waves of pure compression/traction or pure shear waves (pure torsion) we obtain identical numerical values for all the parameters except Young's modulus recalibrated by the effect of the Poisson's ratio (see tables 2 and 3).

9.3 *Quid de Michelson–Morley and the existence of a certain Ether*

Considering an elastic body for the space medium immediately raises the question of Ether. We recall first the conclusions of Michelson and Morley's [56] which are:

"From all of the above, it seems reasonably certain that if there is a relative movement between the Earth and the luminous Ether, it must be small, small enough to refute the explanation of Fresnel's aberration. ..."

Few comments:

(a) It is important not to confuse the vibration of a medium supposed to be a luminescent Ether that does not exist (light spreads alone without need of medium) with a fabric of curved space whose light follows curvatures (e.g. around the Sun).
(b) Contrary to what is often said, Michelson and Morley did not write that there was no Ether but that it had to be small enough not to be detected,
(c) It is well known that the Higgs field does not interfere with the photon or with the light. It is then obvious that it is impossible to detect this field with light. It is therefore possible that the experience of Michelson and Morley with the light is not the right experience to detect the material that constitutes the elastic space fabric.
(d) We can see the shear strain of space created by the rotation of the Earth measured by the satellite probe B, as the expression of the torsion of space is akin to [15] a certain Ether characterised by $g_{\mu\nu}$. This experience can be considered as the right one to measure the twist of the space and the dynamic state of this new relativistic Ether.

The light follows the curvature of the space but does not need a luminescence medium to spread.

If the Ether of luminescence is dead [56], the relativistic Ether [20,21] appears to be alive.... In Einstein's letter to Lorentz of June 17, 1916 [20], we can read:

"I agree with you that the general theory of relativity is closer to the Ether hypothesis than the special theory. This new theory of Ether, however, would not violate the principle of relativity, because the state of this g_{mn} = The Ether would not be that of the rigid body in an independent state of motion, but each state of motion would depend on the position determined by the material processes".

9.4 *Experimental checking of the Young's modulus of the space medium*

An experimental way to establish the Young's modulus E of vacuum is to perform the Casimir test. This test consists of taking two metal plates spaced at a very short distance L_0, and to position them in the vacuum by simultaneously measuring force F and the horizontal displacements (see figure 16). Generally, the force alone is measured, and the displacement calculated in the Casimir experiment. The result of the test is to draw the stress/strain curve. Of course, the slope of the stress/strain curve corresponds to the Young's modulus E of the space medium according to eq. (142).

9.5 *Experimental checking of the shear of the space*

It will be interesting in the interferometer Ligo/Virgo to measure the eventual lateral movements of the laser beam (or the variation of angles between the two laser beams) to determine the eventual shear strain γ created on the space by the passage of gravitational waves (see figure 5).

10. Consequence on the time of this research

10.1 *Behaviour of time as an elastic material*

In this article, we focussed on the spatial part of Einstein's tensor. Based on the new definition of G, we can now return to the temporal part of the Einstein's tensor (see also [11], chapters 2.4.3 and 2.6). Einstein has demonstrated that the stress energy tensor $T_{\mu\nu}$ curves space but also the time at the location of this mass via a proportionality factor κ. This space–time deformation, mathematically characterised by the metric variation $g_{\mu\nu}$ is the gravitation. Near a black hole the curvature of time is such that time expands endlessly giving the impression that it stops. The question is therefore the following: can we consider time as elastic? For memory, some measured facts are:

(a) At high speed v, time expands: A clock placed in a rapidly moving airplane (t') slowed down compared to a clock stayed on Earth (t) (special relativity principle):

$$\Delta t' = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\Delta t. \tag{141}$$

(b) At high speed the distance contracts:

$$\Delta z' = \sqrt{1-\left(\frac{v}{c}\right)^2}\Delta z. \tag{142}$$

Therefore, time has an elastic behaviour, it lengthens or shortens and time has a behaviour opposite to that of space (dilation = negative contraction) (see (149)). The metric is connected to the interval, $\mathrm{d}s^2$. So in a non-inertial frame of reference for example (if the coordinates according to x, y, z do not vary), we have:

$$\mathrm{d}s^2 = g_{00}c^2\mathrm{d}t^2 \tag{143a}$$

or in the general case, the unknowns of the Einstein's equation are the 10 components of the metric tensor $g_{\mu\nu}$:

$$\begin{aligned}\mathrm{d}s^2 = g_{00}c^2\mathrm{d}t^2 &+ 2g_{10}\mathrm{d}xc\mathrm{d}t\\ &+2g_{20}\mathrm{d}yc\mathrm{d}t + 2g_{30}\mathrm{d}zc\mathrm{d}t\\ &+g_{11}\mathrm{d}x^2 + 2g_{12}\mathrm{d}x\mathrm{d}y + 2g_{13}\mathrm{d}x\mathrm{d}z\\ &+g_{22}\mathrm{d}y^2 + 2g_{23}\mathrm{d}y\mathrm{d}z\\ &+g_{33}\mathrm{d}z^2.\end{aligned} \tag{143b}$$

For this we have to focus on the time component of the metric (3b):

$$g_{00} = \eta_{00} + h_{00} = 1 + 2\varepsilon_{00} \approx 1 + \frac{2\phi}{c^2}. \tag{144}$$

Thus, the expression of the perturbation $h_{\mu\nu}$ for the time component (00 index) is:

$$h_{00} = 2\varepsilon_{00} \approx \frac{2\phi}{c^2}. \tag{145a}$$

The equivalent strain of the time is therefore

$$\varepsilon_{00} \approx \frac{\phi}{c^2}. \tag{145b}$$

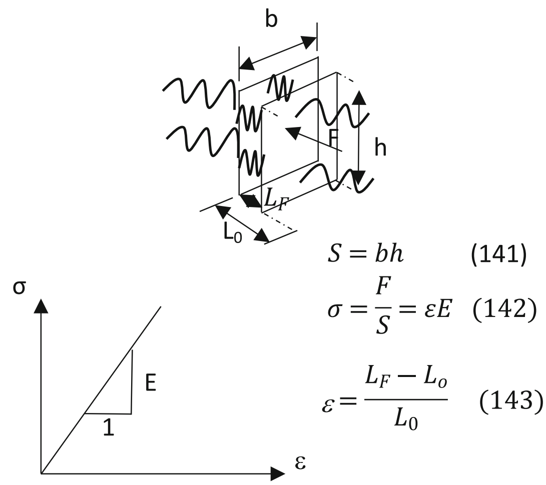

Figure 16. Test to measure the equivalent Young's modulus $E = Y$ of the space medium.

The gravitational potential of a sphere of radius R and mass M, is written as

$$\phi = \frac{GM}{R} = \frac{\frac{\mathrm{m}^3}{\mathrm{kg\ s}^2}\mathrm{kg}}{\mathrm{m}}. \tag{146}$$

By introducing (144) into (143b) we obtain

$$\varepsilon_{00} \approx \frac{GM}{Rc^2}. \tag{147a}$$

Taking into account the new definition of the function G (143) of the density ρ and the frequency f of the space material:

$$\varepsilon_{00} \approx \frac{\pi f^2 M}{\rho R c^2}. \tag{147b}$$

We square the expression (145b):

$${\varepsilon_{00}}^2 \approx \frac{\pi^2 f^4 M^2}{\rho^2 R^2 c^4}. \tag{148a}$$

Taking into account eqs (76a) and (43c), the strain squared is proportional at $(1+\nu)$:

$${\varepsilon_{00}}^2 \approx (1+\nu)\frac{\pi^2 f^4 M^2}{\rho^2 R^2 c^4}. \tag{148b}$$

Given the new definition of G we obtain

$${\varepsilon_{00}}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2 M^2}{R^2\rho}. \tag{148c}$$

We multiply each side of eq. (148c) by $1/R^2$:

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2 M^2}{R^3\rho R}. \tag{148d}$$

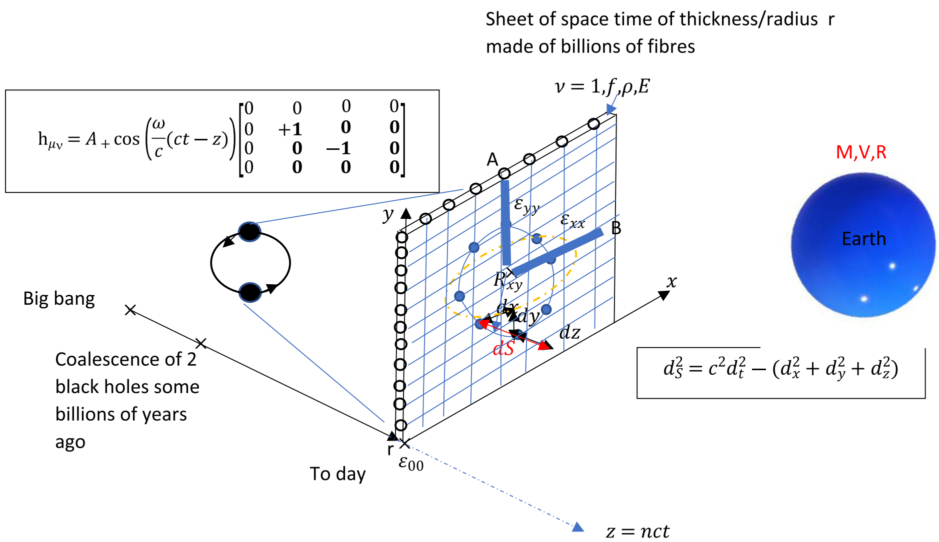

Figure 17. An instantaneous photo (taken at the speed of light) of the xy plane space deformed by the gravitational wave propagating along z direction.

For memory the volume V of a sphere is

$$V = \frac{4}{3}\pi R^3. \tag{148e}$$

We introduce this volume into eq. (147a) via R^3:

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2M^2}{\frac{3V}{4\pi}\rho R} \tag{148f}$$

and we get after some calculations:

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx \frac{4}{3}\pi^2(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2M^2}{V\rho R}\ . \tag{148g}$$

Noting that $\rho V = M$ and the last term of the equation as dimension of energy density U/V (see (148b)):

$$\frac{f^2M}{R} = \frac{\mathrm{kg}}{\mathrm{m\ s^2}} = \frac{\frac{\mathrm{kg\ m^2}}{\mathrm{s^2}}}{\mathrm{m^3}} = \frac{U}{V} \tag{148h}$$

and we obtain

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx \frac{4}{3}\pi\pi(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148i}$$

Taking into account that $3.1 \approx 3$, we obtain

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx 4\pi(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148j}$$

With ($\nu = 1$) (see §4.4):

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx 8\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148k}$$

We obtain, for the temporal component of the disturbance of the metric, an expression similar to that which one obtains by considering a beam in pure compression/traction (see (57)). So, time behaves like an elastic material.

10.2 *Relates time lapses to the thickness of the space fibres*

For memory, some measured facts are

(a) Time slows down when it is immersed in a gravitational field (see general relativity). Therefore,

  (1) Gravitation curves space–time. Time is no longer absolute and become an illusion.
  (2) The more there is gravitation, the more there is curvature and the more there is tension in the material and the more the slow down time expands.

(b) The time acts with a different sign of the space,

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2. \tag{149}$$

For a gravitational wave, the passage of time is a succession of instantaneous pictures following z direction. Figure 17 shows an instantaneous photo of space–time taken during the passage of a gravitational wave.

In figure 17, R_{xy} is the radius of the xy plane, n is the whole number of small quantum distances $c \times t$, $2r$ is

Table 4. Formula obtained in modified general relativity, new expression of κ.

Parameters	Formula	Formula
Strains Polarised wave $h_{\mu\nu}$	$\varepsilon_{ii} = \frac{\delta L}{L}$ $A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\varepsilon_{ij} = \frac{\gamma}{2}$ $A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{+1} & \mathbf{0} \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$
Associated stress tensor	$\sigma_{xy} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\sigma_{xy} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Associated wave in the interferometer tube	Longitudinal compression/traction $c = \sqrt{\frac{E}{\rho}}$	Shear $c = \sqrt{\frac{\mu}{\rho}}$
Curvature = k energy density	$\frac{1}{L^2}(\varepsilon)^2 = 4\pi(1+\nu) \times \left(\frac{\pi f^2}{\rho}\right)\frac{1}{c^4} \times \frac{U}{V}$	$\frac{1}{L^2}(\gamma)^2 = 16\pi \frac{\pi f^2}{\rho}\frac{1}{c^4} \times \frac{U}{V}$
Einstein's gravitational field	$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}$	$\partial^\lambda \partial_\lambda \overline{h}_{ij} = \Box \overline{h}_{ij} = -\frac{16\pi G}{c^4} T_{ij}$
Proposed Izabel's field equation	$G^{\mu\nu} = -(1+\nu)\rho\left(\frac{\omega}{E}\right)^2 T^{\mu\nu}$ $G^{\mu\nu} = -2\rho\left(\frac{\omega}{E}\right)^2 T^{\mu\nu}$	$G^{\mu\nu} = -2\rho\left(\frac{\omega}{\mu}\right)^2 T^{\mu\nu}$
G	$G = \pi\left(\frac{f^2}{E}\right)c^2$ $G = \pi f^2 \frac{1}{\rho} = \frac{\omega^2}{4\pi\rho}$	$G = \pi\left(\frac{f^2}{\mu}\right)c^2$ $G = \pi f^2 \frac{1}{\rho} = \frac{\omega^2}{4\pi\rho}$
Poisson's ratio Young's modulus $E = Y$	$\nu = 1$ $E = Y = \rho c^2$	$\nu = 1$ $E = Y = 2\rho c^2(1+\nu)$ $E = Y = 4\rho c^2$
Speed c	$c = \frac{1}{f}\sqrt{\frac{GE}{\pi}}$	$c = \frac{1}{f}\sqrt{\frac{GE}{2\pi(1+\nu)}}$
Frequency f	$f = \frac{Gh}{2\pi V c^2}$	$f = \frac{Gh}{2\pi V c^2}$
Einstein's constant κ	$\kappa = -(1+\nu)\rho\left(\frac{\omega}{E}\right)^2$ $\kappa = -2\rho\left(\frac{\omega}{E}\right)^2$	$\kappa = -8(1+\nu)^2\rho\left(\frac{\omega}{E}\right)^2$ $\kappa = -2\rho\left(\frac{\omega}{\mu}\right)^2$

the thickness of the space sheet (r is the radius of the fibre of space $= 1 \times 10^{-35}$ m).

The passage of time is the succession of its instantaneous photos following z. We notice that

(1) The perturbation $h_{\mu\nu}$ of the metric depends on the variable , $ct - z$, where ct and $-z$ are at the same level but with opposite sign.
(2) In the $h_{\mu\nu}$ matrix, we see that all the time terms and z terms are correlated at zero (see (30a) and (30b)).

In adddition, as the z-axis is confused with the time axis, we therefore have thin sheets of xy plane space, of thickness $2r$ (2×10^{-35} m) which follow each other in time along the z-axis.Therefore, we propose to relate rate of time lapse to the thickness of the fibre that are said to make up space (following z in the case of gravitational wave). As the gravitational wave passes from one plane of fibres to another (see figure 17), the different time lapses can be counted by these successive passages from one spatial diameter of the fibre to another (quantum of time). Time and space therefore appear to be somewhat quantified:

$$t_q = \frac{2r}{c}. \tag{150}$$

Time has a minimum duration corresponding to the time necessary to transmit information at the speed c from one fibre plane to another (multisandwich sheets). Based on (134) and (17), taking into account the definition V (148e) function of r, we obtain the quantified time lapse t_q:

$$t_q = \frac{2r}{c} = \left[\frac{18}{\pi^3}\frac{G(1+\nu)}{Ec^8}\right]^{1/6} h^{1/3}. \tag{151a}$$

With the new definition of G:

$$t_q = \left[\frac{9}{\pi^2}\frac{f^2}{\rho\mu c^8}\right]^{1/6} h^{1/3} \approx \left[\frac{f^2 h^2}{\rho\mu c^8}\right]^{1/6}. \tag{151b}$$

With the values of table 3:

$t_q = \quad 1.0451 \times 10^{-43}$ s

Value compatible with the Planck time:

$t_p \approx 1.0000 \times 10^{-43}$ s.

If we define $E_t = hf$ the time motor (energy), we get

$$t_q \approx \left[\frac{1}{\rho\mu c^8}\right]^{\frac{1}{6}} E_t^{\frac{1}{3}}. \tag{152}$$

We confirm that if the frequency f is zero, the spatial material disappears and time too disappears. Gravitation can bend time because it bends the fibres of the space fabric which therefore expands. The time passes step by step through each of its fibre, one after the other to transmit information, and so it expands like these fibres. This elasticity of time is a characteristic of its elastic behaviour defined by the strain ε_{00}.

11. Conclusions

In the light of the above, it seems logical to propose a new mechanical and physics expression of the Einstein's constant κ based on:

(1) The elastic behaviour of space and time.
(2) The postulate that space is a substance, an elastic material characterised by its characteristic frequency f, its elastic properties ($E = Y$, ν) and its density ρ (or energy via $E = mc^2$).
(3) The parallelism between the elasticity theory and general relativity (see §5.1.2 and [10]).
(4) The perfect correlation between the two gravitational waves polarisations ($h_{\mu\nu(A_+)}, h_{\mu\nu(A_\times)}$) with the two compression/traction and pure shear tensors of the space medium as a function of the facet considered in elasticity (case of a space medium cylinder twisted by rotation of two massive objects that merge) [10–13] and the link between the graviton of spin 2 with the image of the Mohr's circle in terms of rotation (see [10]).
(5) The intrinsic quantum characteristics of this elastic space medium with a ground state different from 0 (ρ/vacuum energy and circular frequency ω).
(6) The quantified space microstructure with a fibre dimension of the Planck scale, and quantified lapse time.
(7) The new definition of G (macroscopic manifestation of the said frequency f of space medium (see 58)).

We obtain a new set of formulas defined in table 4 as a function of the tensors (polarisations) considered:

Numerically we obtain the following values:

(1) The fibre length constituting the substance texture of the space fabric is 1.566×10^{-35} m compatible with the string dimension defined in the string theory.
(2) The Young's modulus $E = Y = 1.00 \times 10^{113}$ Pa (from the longitudinal wave) and 4.00×10^{113} Pa (from the shear wave) compatible with the results of ref. [11] (4.4×10^{113} Pa, see §3.4, formula 3.13). This value is compatible with the extreme stiffness of space.
(3) The density $\rho = 1.11 \times 10^{96}$ kg/m^3 is also compatible with the results of ref. [11] (1.30×10^{96} kg/m^3, see §3.4, formula 3.14).

As a result:

(1) The gravitational constant G does not seem to be a universal stable constant as it would depend on the characteristics of the material constituting the space elastic fabric (ρ, f) calculated from the vacuum properties.
(2) G could thus have varied in time as the density or the vacuum natural frequency.
(3) The speed limit of light could find an explanation via the limited ratio ρ/E characterising the penetration degree of light inside the elastic space medium (analogy of water that becomes a concrete wall at high speed).

This vision of the medium of space is a new vision of a relativistic Ether [20,21] without any correlation with the luminescent Ether which does not exist. This medium can be made up of infinitesimal beams of small sizes (quantum) in perpetual oscillation forming a three-dimensional frame of stiffness density rather than strings without a bending stiffness of 1×10^{-35} m. This three-dimensional fabric can be characterised by an elastic material itself, characterised by the Young's modulus E, Poisson's ratio ν and density ρ. In this case, we characterise the space by a sort of elastic substance, at the fundamental state (minimum vacuum energy E_v) based on the quantum field theory.

In addition, we have shown that we could consider a new type of wave, space torsion waves, which can generate longitudinal and shear waves. It would therefore be extremely interesting to use current interferometers to measure possible shear strains, that is to say, use the potential lateral movements of the laser in the interferometers to determine the angular shear strain γ and thus the shear modulus μ of space. This will also confirm whether, in addition to transverse shear wave, we should also consider longitudinal waves in the gravitational waves.

We have shown in tables 2 and 3 that it is possible to recalculate the Einstein's constant κ based on the theory of elasticity and volume wave theory. A research conducted by Ringermacher and Mead [19] seems to show that the Universe can sound like a crystal. The analysis of these 'special frequencies' is also an open door to explore and to obtain information about the space fabric structure.

In this study, we focussed on the elastic approach of space in weak fields. The presence of black hole being confirmed, it seems logical that the gravitation in strong field approaches the plasticity of the space material, in which case, it would be interesting to study what becomes of the Einstein's constant in strong fields using the plastic theory of the strength of the materials [12,13].

In this paper we found an expression of the Einstein's constant κ based on the spatial part only of the gravitational field tensors. For this, we used the two-dimensional stress tensor and the Poisson's ratio ν. We recalled that Einstein established its coupling constant κ by using the temporal part of its gravitational field tensor in weak fields (Poisson's formula and Newton's gravitational field equation). It naturally comes from the idea of extending the stress tensor obtained from the elastic theory at four dimensions and to deduce informations about the potential elastic characteristics of time. Such approaches are adopted in [11] and in §10 we showed that it is possible to relate time lapse to the thickness of the space fibres with an elastic time.

Dark energy and the associated cosmological constant Λ are characterised by an accelerated expansion of space. Based on the new definition of $G = \pi f^2/\rho$, we might consider that G varied over time as a function of density ρ. This variation of density can be explained by the intrinsic nature of the space material related to the dark energy (correlation with the expansion coefficient of the elastic material).

Dark matter was proposed to explain why the stars on the periphery of a galaxy were spinning at high speed without being ejected from galaxies (normally speeds decrease when moving from the centre of the disc to its periphery (153)). Considering the elastic deformation of a disc with a hole reamed (analogy of a galaxy with a massive black hole in the centre), made of elastic material (elastic fabric of the space) rotating at high speed, we see by an elastic calculation [57] that the shape taken by it redistributes the material and changes the apparent density ρ of the disc along the radius (thinning near the centre, thickening in the periphery). With the new definition of G as a function of this density ρ (153), it becomes possible to consider a variation of G along the radius of the galaxy (rather to increase the mass M and to look for dark matter) and thus to recalculate different velocities of the stars with a variable value of G. Indeed,we see that the spinning disc gets thinner and stretches. Therefore, the density ρ decreases which, taking into account the new definition of G, implies that $1/\rho$ increases. This can compensate the decrease of $1/R$ when R approaches the periphery of the disc and can thus make it possible to maintain an almost constant speed of rotation of stars in the disc (see also the MOND's approach). In addition, the tensions inside the space fabric disc in rotation allow to maintain the stars in their place, ensuring a global motion of the rotating disc, and a constant speed of the stars in the centre and in the periphery of the disc.

$$v = \sqrt{\frac{M}{R} \times G} = \sqrt{\frac{M}{R} \times \frac{\pi f^2}{\rho}}, \tag{153}$$

where M is the rotating mass and v is the speed of rotation of the stars around the centre of the galaxy.

In addition, if the vacuum is full of fluctuating substance as this article seems to prove via the quantum field theory of the vacuum energy at the fundamental state, it implies that in Young's experience of the double slit experiment, this substance has to be taken into account on the interpretation of the duality wave/particle. Indeed, the fluctuations of the vacuum present everywhere in the medium during the experiment disrupts the trajectory of the particles between the moment it is drawn and the moment it strikes the screen having passed through the two slits.

It seems therefore that in order to make progress in physics today, to renounce the Newton's gravitation concept, as proposed by the Eisntein's general relativity, is not sufficient. As this gravitation force is only an illusion, it seems logical that the universal and constant character of this constant G created with this force also have to be given up. We show in this article that if Newton and his gravitational force had not existed, Einstein could have found his coupling constant κ without going through the temporal component of his theory (μ, $\nu = 0$), but going through the mechanical and spatial parts of its theory (μ, $\nu = 1, 2, 3$) via the elasticity theory, the quantum field theory and the string theory data. If this had been the case, we would not have had the idea of constraining, to a universal constant, the parameters $\pi f^2/\rho$ related to the substance constituting the space that without knowing it Newton had called G in his equation. We also show that by proceeding in this way, and by calculating the parameter G as a function of the vacuum data via quantum field theory, we find elements on the infinitesimal substance constituting the space: the dimension of its granulometry 1×10^{-35} is compatible with string theory; a high Young modulus is necessary considering the very small curvatures generated at the space by extremely massive bodies; $Y = E = 1 \times 10^{113}$; a Poisson's ratio equal to 1. The 'metal' constituting the space is 5×10^{101} times harder than steel at the speed of light (and rather fluid at very low speed)! To advance further in the elastic approach of space, it seems necessary to conduct measurements of Young's modulus (see Casimir test described in this article) and of shear modulus (measurement of the lateral motion of the laser in the large interferometer). This will also make it possible to settle the paradox of the energies of the vacuum resulting from the cosmological constant Λ on the one hand and the quantum field theory on the other hand by quantifying once and for all the energy of the vacuum that have to be considered in the calculation. To summarise, let G vary, and probably many of today's physics problems will in turn disappear. So we shall conclude by quoting one of the first great scientist Alhazen Ibn al-Haytham (965–1040) who said that:

"The search for the truth is difficult, the road that leads to it is full of pitfalls, to find the truth, it is advisable to leave aside its opinions and not to trust the writings of the ancients. You must question them and submit each of their assertions to your critical mind. Trust only logic and experimentation, never the affirmation of one or the other, because every human being is subject to all sorts of imperfections; in our quest for truth, we must also question our own theories, each of our research to avoid succumbing to prejudices and intellectual laziness. Do this and the truth will be revealed to you. "

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Appendix A. Demonstration of the equivalence between the stress tensor $\sigma_{\mu\nu}$ and the stress energy tensor $T_{\mu\nu}$ [49]

In the theory of elasticity, resulting from the continuum mechanics, the relation between the stress tensor σ_{ij}, (with $T_i = \sigma_{ij} n_j$ where $\vec{T}$ is a stress vector attached to the facet of normal vector $\vec{n}$), and the applied force Q_i on a surface S_j can be written as follows:

$$Q_i = \sigma_{ij} S_j. \tag{A1}$$

In the field of variational approach, the stress tensor can be written as follows:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} \quad \text{with } \Delta S_j \to 0, \tag{A2a}$$

where ΔS_i is an area.

So, with m as the mass, ρ as the density of mass energy, V as the volume and a_i as acceleration, we have:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} = \frac{\Delta(m \times a_i)}{\Delta S_j} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j}. \tag{A2b}$$

Assuming that the variation of the force is due only to the variation of volume V as a function of time t we obtain with, $a_i = \frac{v_i}{\Delta t}$,

$$\sigma_{ij} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta V}{\Delta t}\right) v_i. \tag{A3a}$$

Thus, we get with $V = \Delta x_i \times \Delta x_j \times \Delta x_k$

$$\sigma_{ij} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta t}\right) v_i. \tag{A3b}$$

We can replace S_j by its value:

$$\Delta S_j = \Delta x_i \times \Delta x_k. \tag{A4}$$

So the new expression of the stress tensor is

$$\sigma_{ij} = \rho \frac{v_i}{\Delta t}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta x_i \times \Delta x_k}\right). \tag{A5}$$

After simplification we obtain

$$\sigma_{ij} = \rho v_i \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A6}$$

By definition of speed v_j, we have

$$v_j = \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A7}$$

We finally obtain the expression of the stress tensor at low speed as a function of energy density ρ and based on the multiplication of the velocities v_i and v_j:

$$\sigma_{ij} = \rho v_i v_j. \tag{A8}$$

The stress energy tensor results from the product of the energy density and the multiplication of the four-velocities (four dimensions of the space–time) resulting from the general relativity:

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{A9}$$

and for a perfect fluid with P as pressure:

$$T_{\mu\nu} = P g_{\mu\nu} - (\rho + P) u_\mu u_\nu. \tag{A10}$$

For the four-velocities:

$$u_\mu \left| \begin{array}{l} \gamma \times c \\ \gamma \times v_x \\ \gamma \times v_y \\ \gamma \times v_z \end{array} \right. . \tag{A11}$$

The Lorentz factor γ results from the Lorentz transformation which implies that the speed of light remains constant in all the referentials.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{A12}$$

Comparing (A9) and (A8), we can see the similarity between the four-dimensional general relativity stress energy tensor and the three-dimensional elastic stress tensor.

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SOME GEOMETRICAL ASPECTS OF GRAVITATIONAL WAVES USING CONTINUUM MECHANICS ANALOGY: STATE OF THE ART AND POTENTIAL CONSEQUENCES

DAVID IZABEL, YVES RÉMOND AND MATTEO LUCA RUGGIERO

We use an analogy between continuum mechanics and general relativity to investigate, from the perspective of elasticity and crystal plasticity, the deformations of space measured by LIGO/Virgo interferometers during the passage of gravitational waves over Earth. The results of different innovative or existing mechanical models are compared with each other and compared with the observations in the framework of general relativity and Einstein–Cartan theory. Despite limitations, there is a convergence of results: the polarizations of gravitational waves can be viewed as expressions of an equivalent elastic media deformation tensor. Additionally, an anisotropy of space properties is unavoidable at the measurement point of the gravitational wave if we rely on the current first-order general relativity, which predict that gravitational waves generate deformations only in transverse planes. It is demonstrated that the classical polarizations of general relativity can be associated with a state of pure torsion in the analogous elastic medium and acted upon by the rotation of massive bodies such as black holes. This approach involves a transverse isotropic medium composed of independent sheets that deform perpendicularly to the direction of propagation of these waves. Considering geometric torsion in general relativity, associated with plastic crystallography, allows for the examination of complementary polarizations in the direction of wave propagation. This makes it possible to connect these sheets and reconstruct a complete, coherent 3D environment.

1. Introduction

Einstein's theory of general relativity is over 100 years old and is now widely verified. Thus, according to this theory, space-time could be an elastic, deformable physical object. These distortions disappear when the object that created them disappears. Hence the notion of the elasticity of space. Gravitation is thus a manifestation of the geometric deformation of space-time under the effect of the masses or energy density found therein. The manifestations of the deformations of this space-time are now known and measured with great precision in several very specific situations. From a historical perspective, we can mention the variation in the apparent position of stars placed behind the sun during an eclipse measured by Eddington in 1919 [17], the expansion of the universe where galaxies are "fixed" in a space that expands in an increasingly accelerated manner characterized by Hubble's law established in 1929. More recently, we mention the entrainment of the reference frame of space-time by angular distortion by the

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rotation of the Earth (experiment conducted with the satellite gravity probe B carried out with gyroscopes placed in orbit 642 km from the Earth from 2004 to 2010, Lense–Thirring (frame dragging) effects and geodetic precession) [21], the simultaneous deformations of elongation and shortening in each of the arms of the LIGO/Virgo interferometers during the passage of gravitational waves measured for the first time on 15 September 2014 [1; 2]. A thorough overview of 100 years of testing general relativity can be found in [57]. All these manifestations of space-time distortions have led many physicists and mechanics researchers (such as A. Sakharov [48], J. L. Synge [52], C. B. Rayner [44], R. Grot [24], T. G. Tenev and M. F. Horstemeyer [55], P. A. Millette [41], D. Izabel [28; 29] and T. Damour [15]) to consider that the theory of general relativity in the weak field could be considered by analogy as a kind of theory of elasticity, a kind of Hooke's law of a deformable elastic space-time medium.

Thus, either some authors begin with general relativity and attempt to present it within the formalism of continuous mechanics, while other start with 3D continuum mechanics and generalize it to 4D by introducing a "mechanistic" metric that includes the effect of time.

The latest work that, to our knowledge, provides an updated assessment of this topic was giving by [33; 34; 14], based on the seminal works [8; 9; 13; 18; 25; 30; 35; 36; 5; 42; 50; 51; 52].

Summarizing the theoretical context in which our work is situated, we will assume that both general relativity and continuum mechanics are based on the principle of general covariance, which, as B. Kolev aptly summarizes in [34], requires the introduction of three components:

- Lagrangian-type functionals $\mathscr{L}$, depending on the metric g defined on the 4-dimensional universe manifold M, and depending on various fields Ψ;
- Tensors, such as the energy-momentum tensor, dependent on these fields;
- Field equations.

Our approach is therefore placed in the framework of the general covariance of field equations by the group of diffeomorphisms. Or expressed explicitly, a Lagrangian $\mathscr{L}(g, \Psi, ..)$ is general covariant if it satisfies $\mathscr{L}(\varphi^* g, \varphi^* \Psi) = \mathscr{L}(g, \psi)$ for any diffeomorphism φ. Note, still with B. Kolev, that the functional $\mathfrak{H}(g) = \int R_g \mathrm{Vol}_g$, where R_g represents the scalar curvature, is general covariant. Its gradient L^2, the Einstein tensor $G(g)$ is also general covariant. It consequently satisfies $\forall \varphi$, $G_{\varphi^* g} = \varphi^* G(g)$, and $\mathrm{div}^g G(g) = 0$. An energy-momentum tensor $T(g, \Psi)$ satisfying the Einstein equation $G(g) = T(g, \Psi)$ therefore satisfy the mechanics type equation: $\mathrm{div}\, T = 0$. The rest of this work will therefore concern energy-momentum tensors satisfying these two properties. The reader is referred to the work of J. M. Souriau [50; 51] and his successors already cited, for the definitions of the perfect matter field (as a section of a vector bundle) and the conformations, allowing the proposal of relativistic constitutive laws [14].

It is within the framework of this analogy that we mainly place ourselves in this paper. We will not develop further the very theoretical aspects described above. Nor will we address the controversies and debates that these concepts continue to generate, but we will concentrate on the consequences on the mechanical properties of the medium that the experiments induce. Following these preliminary remarks, we can define this "elastic gravitational analogy" based on the three principles of equivalences.

- The perturbations h_{ij} of space in the presence of gravitational waves are linked to the Green–Lagrange covariant tensor: $D = \frac{1}{2}(\varphi^* g - g)$, which will be assimilated in the following, under infinitesimal deformations, as the geometric linearization of the strain tensor $D = \varepsilon = \varepsilon_{ij}$,
- Einstein's equation connects, within the medium, the strain tensor (versus metric perturbation in weak field) to the equivalent stress field, akin to Hooke's law, with the aid of an equivalent compliance matrix.
- The energy density of space-time itself ρc^2 is correlated, through quantum field theory or the Casimir effect, with a nonzero energy density of the vacuum. This vacuum energy density is mirrored in the analogy by the Young's modulus Y of the equivalent elastic medium, given by the equation $Y = \rho c^2$.

In addition to these hypotheses, we must recall the debates on the relativistic equivalent validity of the stress field used in continuum mechanics [22]. Our approach add some further elements to this debate. However, when deformations occur in a vacuum far from the spacetime loading, as is the case with gravitational waves arriving on Earth, the stress-energy tensor is $T = T_{\mu\nu} = 0$.

To maintain a complete Hooke's law, it is necessary to consider an elastic strain energy tensor of the vacuum itself: $T_{e,\mu\nu}$, linked to the deformations correlated with $h_{\mu\nu}$ (the small perturbation of the Minkowski's tensor). This is one way to calculate the gravitational wave energy or to study the spacetime itself as an equivalent elastic medium [54; 7; 53].

We add, however, that continuum mechanics models are based on a primary concept, which is the kinematics of the phenomena. The concept of stress field is a secondary concept which is introduced only after the kinematics choice using the virtual power principle [23; 20], named virtual power theorem if we consider as primary principle, the principle of general covariance. The constitutive laws which link these two concepts, or their time derivatives, are well defined by the local state method [37].

2. Methods

The following methodology was employed to evaluate some geometrical aspects of gravitational waves in linearized general relativity from the perspective of the analogy with continuum mechanics.

(1) Investigation of the discrepancy between the deformations measured during the passage of gravitational waves and the theoretical predictions of linearized general relativity, using deformation measurements made by LIGO/Virgo interferometers.
(2) Analysis of test masses arranged in a circle of the different deformations and associated polarizations according to various versions of general relativity (classical in the first-order, second-order in gravitoelectromagnetism, modified with Einstein–Cartan torsion, and other modified versions).
(3) Study of space deformations during the passage of a gravitational wave by considering either existing or new models, such as interferometer arms, torsional space cylinders, isotropic transverse media of elastic solids.
(4) Comparison of the results from classical first- and second-order or modified theories of general

relativity (Einstein–Cartan and others) with the predictions of various continuum mechanics. Specifically, we study the potential complementary deformations of the equivalent cosmic medium and their consequences on the characteristics of the media. We also explore interactions with the theory of defects in crystalline media regarding the number and types of gravitational wave polarizations.

3. Gravitational waves in relativistic theories of gravity

3.1. ***General relativity.*** According to Einstein's theory of general relativity, gravitation is the geometry of spacetime: specifically, spacetime is a four-dimensional pseudo-riemannian manifold M, which is a pair $(M, g_{\mu\nu})$, where M is a connected four-dimensional Hausdorff manifold and $g_{\mu\nu}$ is the metric tensor.[1] Due to its Riemannian structure, spacetime is endowed with an affine connection compatible with the metric, known as the Levi-Civita connection. We are interested in gravitational waves, which are particular vacuum solutions of Einstein's equations:

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}, \tag{1}$$

where $G^{\mu\nu}$ is the Einstein tensor defined in terms of the Ricci tensor $R^{\mu\nu}$ and scalar curvature R, as $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$ and $T^{\mu\nu}$ is the energy-momentum tensor. To obtain the wave equations, we suppose that the metric tensor in a weak gravitational field can be written in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where $h_{\mu\nu}$ is a small perturbation of the Minkowski tensor $\eta_{\mu\nu}$ of flat spacetime ($h^{\mu\nu} \ll \eta^{\mu\nu}$). By setting $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$, where $h = h^{\mu}_{\mu}$, Einstein's equation can be written in the form

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \tag{3}$$

where $\Box = \partial_\mu \partial^\mu = \nabla^2 - (1/c^2)\partial/\partial t^2$. In a vacuum ($T_{\mu\nu} = \mathbf{0}$) this becomes $\Box \bar{h}_{\mu\nu} = \mathbf{0}$, whose solutions are gravitational waves propagating in empty space, which can be written in the form

$$\bar{h}_{\mu\nu} = A_{\mu\nu} \cos(k_\sigma x^\sigma), \tag{4}$$

$$A_{\mu\nu} = A_+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + A_\times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

where A_+ and $A_\times$ are the amplitude of the wave in the two polarization states, and k^σ is the four-plane wave vector $k^\sigma = (\omega/c; \vec{k})$, where ω is the frequency and $k = \|\vec{k}\| = \omega/c$ is the wave number, with $k^\sigma k_\sigma = 0$. This solution is given in the so-called TT gauge: the deformation caused by the wave is transverse to the propagation direction, and the amplitude tensor $A_{\mu\nu}$ is traceless. For instance, if we consider test masses positioned on a circle of radius R before the passage of the wave, the deformation

[1] Greek indices run from 0 to 3, while Latin indices run from 1 to 3.

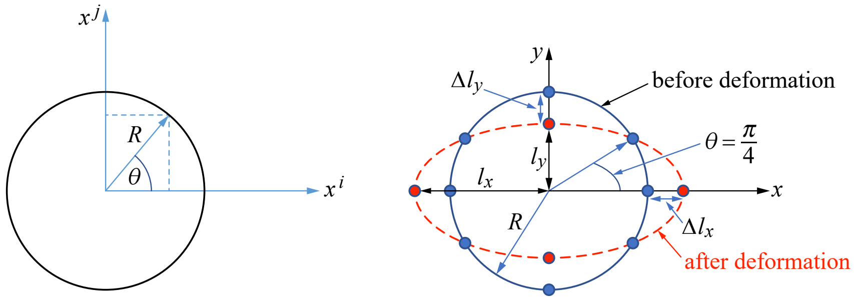

Figure 1. Displacement of test masses in the xy-plane when a gravitational wave propagates in the z-direction.

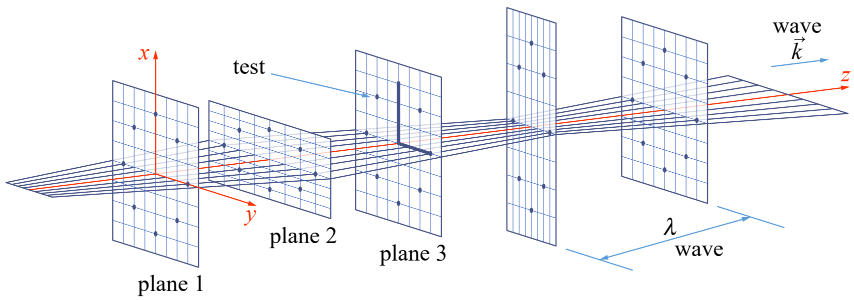

Figure 2. Displacements of test masses when a gravitational wave passes in the direction z following successive transverse planes.

provoked by the A_+ polarization is given by

$$\Delta S = R\left[1 + \tfrac{1}{2} A_+ \cos \omega t \cos 2\theta\right] \tag{6}$$

and it is depicted in Figure 1, for a wave propagating along the z direction. A similar deformation is obtained for polarization $A^\times$, which corresponds to a rotation of Figure 1 by 45 degrees. Additionally, the evolution of the deformation is depicted in Figure 2.

According to this description, the deformations provoked by the wave are in the plane orthogonal to the propagation direction: strictly speaking, this is true if we confine ourselves to the first order with respect to the reference position. Up to second order (see, e.g., D. Baskaran and L. P. Grishchuk [6] and M. L. Ruggiero in 2022 [45]), deformations also occur along the propagation direction. This effect (significantly enlarged) is depicted in Figure 3.

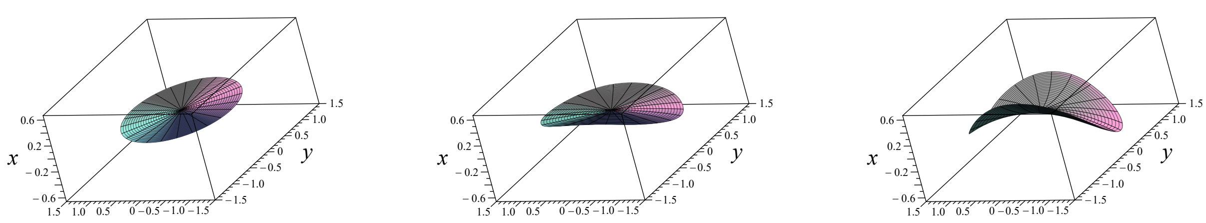

Figure 3. Temporal evolution of test masses due to the effects of A+ polarization with up to second order [45].

3.2. ***Modified theories of gravity.*** Altough general relativity is the most successful model we have for understanding gravitational interactions, there are still unresolved issues regarding the application of GR to large-scale structures, such as the problems of dark matter and dark energy and. Additionally, we do not yet know how to reconcile it with quantum mechanics. Consequently, several proposals have been made to extend Einstein's theory to address these issues. Many of these proposed theories have a richer geometrical structure. For instance, in Einstein–Cartan theory, torsion is present in addition to curvature. Specifically, torsion is related to the spin of the sources of the gravitational field [32]. For our purposes, it is interesting to note that this theory introduces additional polarizations for gravitational waves [19]. Our complementary polarizations appear as shown in Figure 4 and are associated with the polarization matrices

$$P^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P^{(b)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

$$P^{(l)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P^{(xz)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad P^{(yz)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

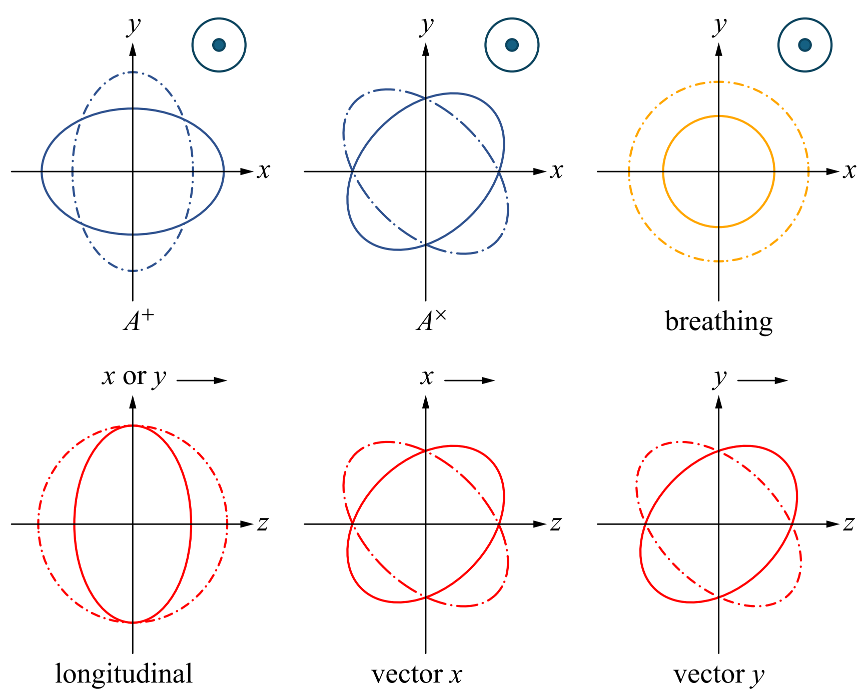

Figure 4. Complementary polarizations that appear in the case of torsional modified general relativity in the case of the Einstein–Cartan–Sciamma–Kibble theory [19].

theory	type of polarization					
	$+$	$\times$	x	y	breathing	longitudinal
general relativity	yes	yes	no	no	no	no
GR in noncompactified 4/6D Minkowski	yes	yes	yes	yes	yes	yes
Einstein/ether	yes	yes	yes	yes	yes	yes
5D Kaluza–Klein	yes	yes	yes	yes	yes	no
Randall–Sundrum braneworld	yes	yes	no	no	no	no
Dvali–Gabadadze–Porrati braneworld	yes	yes	—	—	—	—
Brans–Dicke massive	yes	yes	no	no	yes	yes
Brans–Dicke massless	yes	yes	no	no	yes	no
F(R) metric gravity	yes	yes	no	no	yes	yes
bimetric theory	yes	yes	yes	yes	yes	yes
Palatini gravity	yes	yes	no	no	no	no
scalar tensor theory	yes	yes	no	no	yes	yes

Table 1. Different polarizations associated with different theories of general relativity, according to [40] and [43]. See Figure 4 for the different polarization types. A dash indicates the answer depends on the version of the theory.

The first two matrices in (7) are the classical ones predicted and measured by classical general relativity. The remaining four matrices, $(7)_3$ and (8), result from the introduction of the Einstein–Cartan–Sciamma–Kibble torsion.

More generally, as discussed by L. A. Philippoz [43] and S. Mathur [40], different alternative theories of gravity correspond to specific polarization features for gravitational waves, which are summarized in Table 1. Interestingly, all these modified theories converge on the same idea: that one or more polarizations complementary to A^+ and $A^\times$ should potentially exist. Only advanced measurements of deformations, by coupling various interferometers on Earth or using LISA, are likely to detect such polarizations. We will revisit this at the end of the paper.

Having completed the state of the art of gravitational wave polarizations in classical or modified general relativity, we will now see what the analogy of the continuum mechanics elasticity can help interpreting the results of general relativity (gravitational waves) recalled in the previous paragraphs.

3.3. ***Results of measurements made by LIGO/Virgo interferometers.*** The first direct measurement of a gravitational wave occurred on September 14, 2015 (GW150914) and was presented on February 11, 2016 [1]. Two signals corresponding to the merger of two black holes that occurred 1.3 billion years ago were successively detected by the two LIGO interferometers in the USA.

The nature of these signals is shown in Figure 5, whose lower pane shows the almost perfect superposition of the two signals successively detected by the two interferometers 3000 km apart. Figure 7 shows the superposition of the theoretical curves and measured data after eliminating the noise. General relativity therefore predicts this type of signal remarkably well.

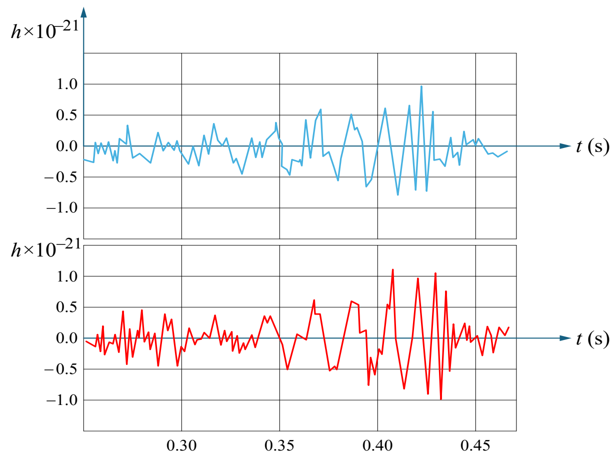

Figure 5. Signal GW150914 picked up by the two LIGO laser interferometers sourced by B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) – First Hanford second Livingstone [1]. The x-axis represents time in seconds, and the y-axis represents the h-deformations of space on the order of $+/-\ 10^{-21}$.

A second interesting signal is GW170817 [2], emanating from the merger of two neutron stars, also known as a kilonova. It was observed simultaneously from both perspectives: gravitational waves and electromagnetic waves. This confirms, if needed, the remarkable efficiency and accuracy of the measurements. There can no longer be any doubt that rigid space deforms extremely little ($h_{xx} = -h_{yy} = 10^{-21}$) when stressed by large masses (black holes, neutron stars, etc.) concentrated in small volumes and rotating relative to each other at high speeds and high acceleration near the final coalescence time.

3.4. ***Discrepancies between LIGO/Virgo interferometer measurements and the classical linearized theory of general relativity.*** By carefully comparing the two GW150914 signals, it is possible to show the deviation in units 10^{-21} between the theory of general relativity and the interferometer measurements (Figure 6). These curves demonstrate the precision of general relativity, and indicate that any theory improving it should explain this few percent difference.

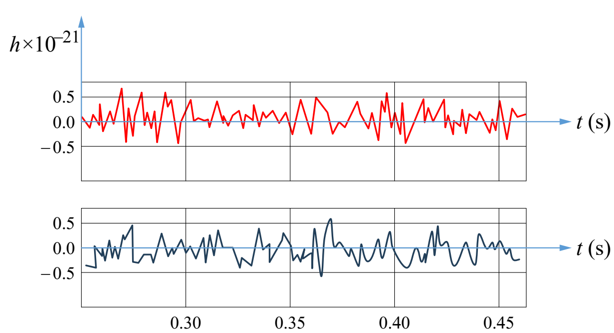

Figure 6. Gap between the theory of general relativity and interferometer measurements for GW150914 source B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) [1].

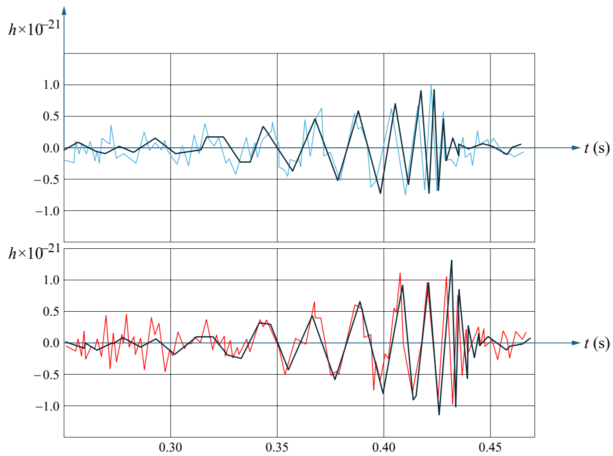

Figure 7. Superposition of the theoretical curves (finite elements) and measured having eliminated the noise and interferometer measurements for GW150914 source B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) [1].

Viewing these results of extremely small deformations of space, physically measured by the two LIGO interferometers, which behave like giant strain gauges — each interferometer arm, traversed by a laser beam reflecting on various mirrors, is 4 km long and operates in a vacuum — one can be led to analogize the vacuum as an elastic medium that deforms under the effect of energy or masses present within it.

Obviously, the continuum mechanics theory of elasticity seems appropriate for modeling these deformations of space by analogy.

In the following sections, we will explore the strengths and limitations of this analogy, its potential contributions to our understanding of general relativity, and whether it can guide us in analyzing classical or modified theories of general relativity that aim to address the few percent difference between measurements and Einstein's version (Figures 6 and 7).

4. Characterization of the deformations of space during the passage of a gravitational wave using different approaches of continuum mechanics

4.1. *Solid-state model for space.*

4.1.1. *Stacking thin sheets in elasticity.* Several important points can be made about this paper:

a- Correspondence between polarizations established in general relativity and deformations established in the mechanics of continuous media.

The hypotheses include, among other things, that in a weak field, the metric is expressed as

$$g^{\mu\nu} = \eta^{\mu\nu} + 2\varepsilon^{\mu\nu}, \tag{9}$$

where $\varepsilon^{\mu\nu}$ is the equivalent of a first-gradient strain tensor comprising spatial and temporal strain components (00, 01, 02, 03).

Comparing expressions (9) and (2), the following equivalence emerges:

$$2\varepsilon^{\mu\nu}(\text{in mechanics}) = h^{\mu\nu}(\text{in GR in weak field}) \tag{10}$$

This correspondence is fundamental for the rest of this paper, as it implies that the components of the polarization matrices of gravitational waves can be interpreted, within the context of the analogy with the elastic medium, as the components of a deformation tensor of the associated elastic cosmic space [55; 28; 29].

b- Relations between longitudinal and transversal strains. Since gravitational waves are transverse, (no longitudinal wave), and the deformations are of equal intensity but opposite sign in each arm of the interferometers, this implies Poisson's ratios of the equivalent anisotropic elastic cosmic medium as:

$$\nu_{xy} = \nu_{yx} = 1. \tag{11}$$

There are two ways to obtain this value. First, measurements are made on the LIGO/Virgo interferometers, where elongation and shortening are measured simultaneously in each of the arms with the same intensity and opposite signs (Figure 5). Alternatively, we can assume an elastic medium and impose the absence of a longitudinal wave (since gravitational waves are transverse according to classical general relativity). Indeed, the velocity of a compression wave in an anisotropic elastic medium can be written as

$$c_{\text{pressure}} = \sqrt{\frac{Y_{xy}(1-\nu_{xy})}{\rho(1+\nu_{xy})(1-2\nu_{xy})}} = 0 \implies \nu_{xy} = 1. \tag{12}$$

The other variation $\nu_{yx} = 1$ is observed in the strains of each arms (Figures 1 and 5). We therefore find that, as interferometers are in the xy-plane, $\nu_{xy} = \nu_{yx} = 1$.

This observation regarding the values of these Poisson's ratios has the following consequences:

- The distortions of space caused in the planes x, y perpendicular to the direction z of propagation of the gravitational waves (Figure 2), associated with polarizations according to the principle described above, are of the same magnitude but of the opposite sign,
- On one hand, the elastic medium associated with space in continuum mechanics is necessarily anisotropic due to the value of these Poisson's ratios. On the other hand, we note the absence of polarization and, thus, complementary deformation in the z direction of wave propagation in classical general relativity.
- According to this result, this equivalent elastic spatial medium would consist of stacking of sheets that deform independently of each other ($\nu_{xz} = \nu_{yz} = 0$). The spatial medium thus consists more in a tailored medium at a very small scale than an equivalent homogeneous medium (Figure 2) [54] where we have strains in three directions of space, not only in two directions (x, y) as in classical general relativity.

Note that in the case of a gravitational wave not perpendicular to the interferometer arms, the angle θ of the observer is taken into account as follows [10]:

$$h_{+(t)} = \frac{4G\mu a^2\omega^2}{Rc^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega t), \quad h_{\times(t)} = \frac{4G\mu a^2\omega^2}{Rc^4}\cos\theta\sin(2\omega t), \tag{13}$$

where $\omega = \sqrt{GM/a^3}$, $M = m_1 + m_2$, $\mu = m_1 m_2/M$, θ is the angle between the normal to the rotation plane of the two bodies and the direction R of the observer, a is the distance between the two rotating bodies (e.g., black holes), and R is the distance between the observer and the system in rotation ($R \gg a$.)

c- The Young's moduli of the equivalent elastic medium can be expressed in terms of fundamental constants. This model was developed by T. G. Tenev and M. F. Horstemeyer [55], among others. In this paper, cosmic space is assumed to consist of ultrathin sheets whose thickness is that of the Planck length, by denoted l_p.

The consequence is that T. G. Tenev, and M. F. Horstemeyer in [55] arrive at these expressions of Young's moduli of cosmic fabric:

$$Y = Y_x = Y_y = E_x = E_y = \frac{6c^7}{2\pi\hbar G^2} = \frac{24}{l_p^2\kappa} \tag{14}$$

(Y_z is not defined in their model).

The cosmic fabric in this paper is an extension of the cosmic medium as seen by [55] with a transversal isotropic behavior.

The numerical application leads to values of the Young's moduli Y_x and Y_y of the cosmic fabric, (in correlation with the energy of the vacuum according to Sakharov [48]) that are outside the usual standards if we assume a stacking of space sheets of the Planck thickness constituting this cosmic fabric.

$$Y_{x\,(\text{vacuum})} = Y_{y\,(\text{vacuum})} = 4.4 \times 10^{113}\ \text{Pa}.$$

This value is associated with a vacuum density of $\rho_{\text{vacuum}} = 1.3 \times 10^{96}\ \text{kg/m}^3$, which is so extreme and lacks a clear understanding.

Two additional remarks emerge from this analysis:

The Hawking black hole radiation equation is one of the few relations that combine the three fundamental constants c, h and G, in addition to Boltzmann's constant k_B. This can be achieved through the analogy of the equivalent elastic medium via the Young's modulus Y with equation (14). Thus, the elastic analogy provides a way to combine these three fundamental physical constants.

By reversing relation (14), the gravitational constant G becomes dependent on the elastic characteristics of the equivalent cosmic elastic medium. In their approaches, T. G. Tenev and M. F. Horstemeyer did not consider an anisotropic medium with several Young's moduli. Thus they propose:

$$G = \sqrt{\frac{6c^7}{hY_k}} \tag{15}$$

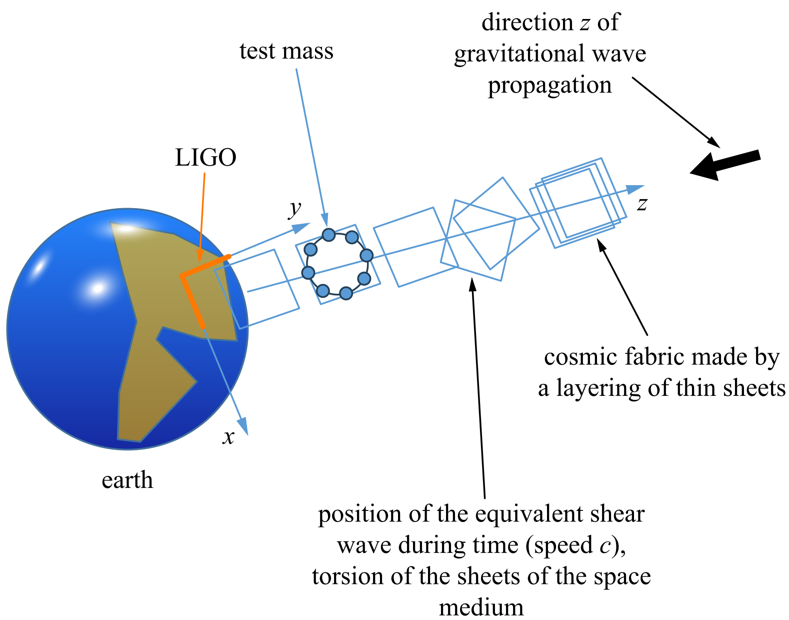

Figure 8. Visualization of the successive deformations of a stack of space sheets during the passage of a gravitational wave.

The elastic medium associated with space in continuum mechanics is necessarily anisotropic due to the values of these Poisson's ratios (transverse isotropic in the plan x, y). It is also anisotropic due to the absence of z-polarization, which is connected with the deformation of space as seen in (10). Y_k is the Young's modulus in the direction k.

4.1.2. *Deformation study of two 90° space tubes containing the interferometer arms.* D. Izabel in [28] and [29] generalized the work of T. G. Tenev, and M. F. Horstemeyer in [55] (Figure 8) regarding a medium consisting of several thin sheets of Planck thickness each characterized by an associated Young's modulus $Y = Y_x = Y_y$ and energy. Izabel sought the analogy with the mechanics of continuous media not by modifying the field equation of general relativity but by introducing mechanical parameters into the constant of proportionality κ, which becomes the flexibility characteristic of the cosmic fabric expressed as a function of Young's modulus. He finds there a formulation analogous to that of a generalized Hooke's law.

Thus, in the case of interferometers positioned in a plane of this cosmic sheet, in each arm he is able to write [28]

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = 4(1+\nu_{xy})\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}, \qquad \frac{1}{L^2}(\varepsilon_{yy})^2 = 4(1+\nu_{yx})\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{16}$$

These amount to a mechanical version of the Einstein's field equation (1) in space.

L is the length of the interferometer arm, U is the elastic deformation energy of the volume V of the arm, f is the natural frequency of vibration of the space in the arm in accordance with the gravitational wave, ρ is the density of the vacuum, ε_{xx} and ε_{yy} are the deformations of the space in the arms, with $\nu = \nu_{xy} = \nu_{yx} = 1$, the Poisson's ratios in the xy-plane.

Assuming equivalence in curvature tensor (the radius of curvature R locally tends to $1/L^2$ so, is quasiflat as for the curvature k of the universe) we can write in the plane of the interferometers:

$$R_{ij} = \begin{bmatrix} 1/L^2 & 0 \\ 0 & 1/L^2 \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix}. \tag{17}$$

Defining $U/V = T_{xx} = T_{yy}$, the components of the strain energy tensor of the cosmic fabric are, with U as the strain energy and V as the volume of the interferometer arms,

$$T_{ij} = \begin{pmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{pmatrix} \tag{18}$$

If we use $\upsilon = \nu_{xy} = \nu_{yx} = 1$, as explained in Section 4.1.1, in the expressions

$$4(1+\nu)\pi \frac{\pi f^2}{\rho} \frac{1}{c^4} = 8\pi \frac{G}{c^4}, \quad G = \frac{\pi f^2}{\rho} = \frac{\pi f^2 c^2}{Y}, \tag{19}$$

we lowercased "We" obtain Einstein's expression (20) in weak field general relativity transposed into an equivalent elastic medium in the plane of the interferometer as

$$R_{ij} = \frac{8\pi G}{c^4} T_{ij} = \kappa T_{ij} \tag{20}$$

Remark. It can be shown [28], [29] that the expression (19) for the cosmic fabric flexibility κ is related to this expression of G as a function of the squared frequency of vibration of the space in the tube and the energy density of the vacuum in quantum field theory. Expression (19) developed in [28] and [29] shows that the Young's modulus can be expressed as a function of G, f and c.

The conclusion of this study is that the expression of Einstein's general relativity can be seen in planes xy transverse to the direction of propagation of the gravitational wave as a Hooke's law, with κ playing the role of the flexibility of the equivalent cosmic fabric structure, but with an anisotropy that remains nonstandard.

4.1.3. *Torsionally stressed space cylinder.* In publications [28] and [29], the author uses the analogy between the perturbations of the metric and the associated distortions of space (see formula (10)). Thus, the two classical polarizations A^+ and $A^\times$ of gravitational waves can be read as follows:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \varepsilon_{xy(A_+)} = \frac{1}{2} A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{21}$$

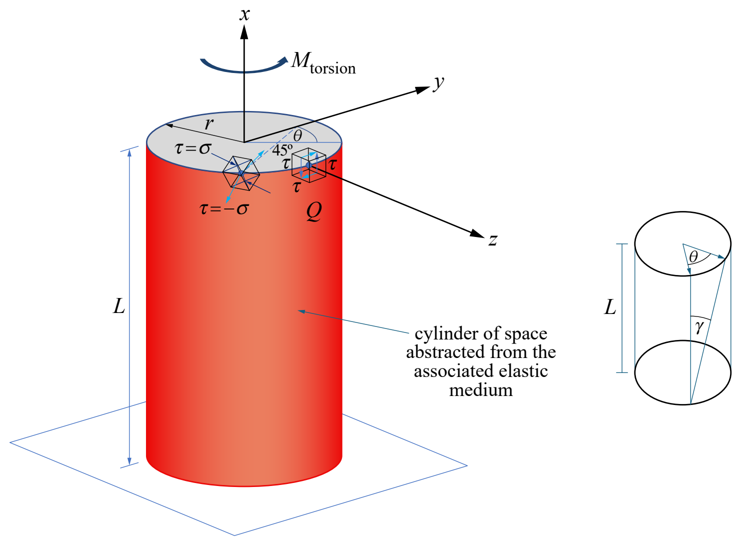

Figure 9. Pure torsion state of an isotropic material in elasticity theory.

These two components of deformations correspond to elongations and shortenings; see formula (10). Moreover,

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & +1 & 0\\ 0 & +1 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \to \varepsilon_{xy(A_\times)} = \frac{1}{2}A_\times \cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix}0 & \varepsilon_{xy} & 0\\ \varepsilon_{yx} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \quad (22)$$

These two components of deformations correspond to distortions, which in elasticity correspond to pure torsion (Figure 9).

Gravitational waves are caused by the rotation of binaries (two black holes, two neutron stars, one black hole and one neutron star, etc.) that can be seen as a twist of space due to their rotation relative to each other (Figure 10).

However, if we use the analogy in the other direction by asking the following question: can an elastic reading of the two polarizations obtained in general relativity offer us additional information in the weak field? By examining the two deformation tensors above, we observe elongations ε_{xx} and ε_{yy} shortenings in the xy-plane associated with the deformations and, already widely measured with the LIGO/Virgo interferometers. Depending on the elasticity, angular distortions in 45° planes would also be possible associated with the deformations ε_{xy} and ε_{yx} (Figure 11) and [28]. It is not possible to measure these distortions by current interferometers because they are not designed for this. This should be possible using the LISA interferometer or the future three-arm triangle Einstein telescope or multiple pulsars as was done for the 2023 detection of the stochastic gravitational wave background:

$$\tan\gamma_{ij} \approx \gamma_{ij} = \frac{b}{L} \quad (23)$$

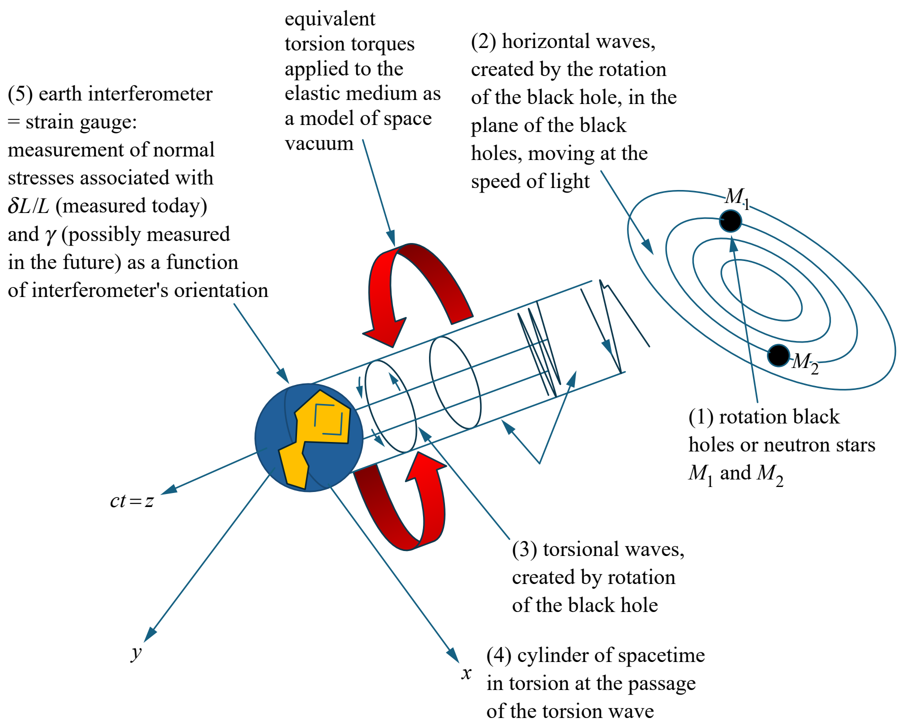

Figure 10. Effect of the rotation of a binary following the analogy of the torsional elastic medium.

Finally, in [28] by studying a cylinder of elastic space twisted, the author finds a mechanical version of general relativity in which it again appears $G = \pi f^2/\rho$. The deformations associated with the curvature are angular distortions, noted as γ in Figure 11, where T is the strain energy U divided by the volume V of the interferometer's arms:

$$U = \frac{1}{2}\int_0^L \frac{M_t^2}{\mu I_t} dx, \qquad \frac{1}{L^2}\gamma^2 = 16\pi\frac{G}{c^4} \times T. \tag{24}$$

This can be compared with (3) by replacing $h_{\mu\nu}$ with (10) using this mechanical expression of general relativity in the weak field:

$$\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\bar{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}. \tag{25}$$

5. Analysis and discussion of the results obtained according to general relativity and the analogy of the elastic medium

5.1. ***Concerning the analogy between the polarizations of gravitational waves and the different forms of the associated deformation tensor of space.*** One of the interesting contributions of the elastic medium analogy is the opportunity to interpret, according to [55; 28; 29], the polarizations of gravitational waves in linearized general relativity as components of a space deformation tensor by applying the analogy of the elastic medium to cosmic space.

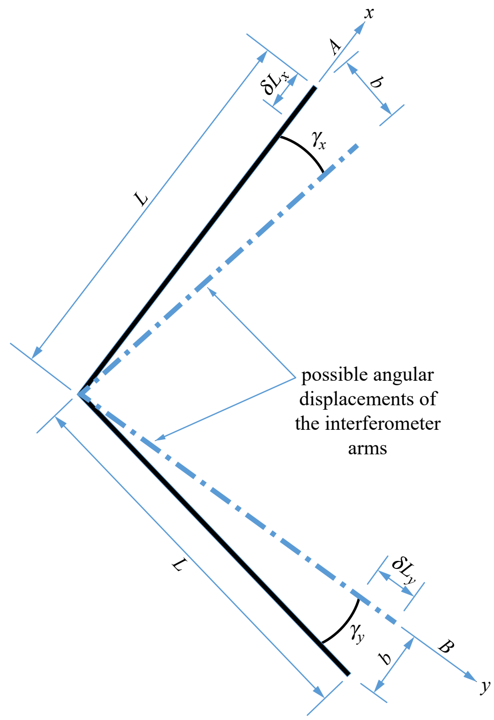

Figure 11. Nonmeasurable angular distortions of the arms interferometers in the xy-plane (top view).

This provides a new illustration of the two polarizations A^+ and $A^\times$. In the analogy of the elastic medium subjected to pure torsion by two massive rotating objects (e.g., GW150914), the strain tensor consists of four components. two diagonal components corresponding to facets stressed in tensile and compressive motion, and two components in the other corresponding diagonal for another facet at 45° of the previous normal stress (Figure 12). Due to the Mohr circle associated with this pure torsion, these shear stresses τ are of the same intensity as the normal stresses σ on the other facet.

5.2. ***Possible lateral deformations of the interferometers.*** According to [28], these potential angular deformations could be easily calculated or measured geometrically using the Mohr circle for two of the arms of the future LISA experiment [26].

The layout is as follows (Figure 12):

1. Plot the deformation ε_{xx} of Arm 1 from the measurement of the elongations of this arm, calculated from the time it takes for the laser beam to travel the distance between Satellite 1 and Satellite 2.
2. Trace the deformation $\varepsilon_{yy} = -\varepsilon_{xx}$.
3. Plot the Mohr circle passing through ε_{xx} and ε_{yy}.
4. The deformation $\varepsilon_{\text{Arm 2}}$ of Arm 2 is reported from the measurement of the shortening of this arm (this is a shortening slightly less than the maximum shortening located at 90° (stated by LIGO and Virgo).

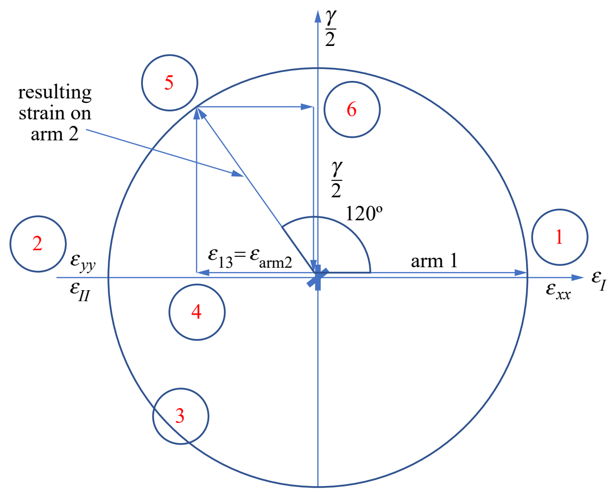

Figure 12. Determination of a potential angular distortion from the future LISA (arms at 60°).

5. Draw the direction of Arm 2 by rotating $-120°$ (due to the angle of $60°$ between the arms of the future LISA interferometer) the direction of the vector associated with the facet of Arm 2.
6. From this, we derive the second component (half distortion $\frac{\gamma}{2}$) for the facet associated with Arm 2 on the vertical axis.

The value of the distortion is intersected by the lengthening and shortening of Arm 3 with the expression α seen between the arms below (Figure 13):

$$cos\alpha_{(t)} = \frac{L_2^2 + L_1^2 - L_3^2}{2L_1L_2} = 60° + \gamma' \tag{26}$$

5.3. ***Concerning the anisotropy of space based solely on current general relativity.*** We have seen that linearized general relativity implies, in the case of gravitational waves, two unique polarizations that can

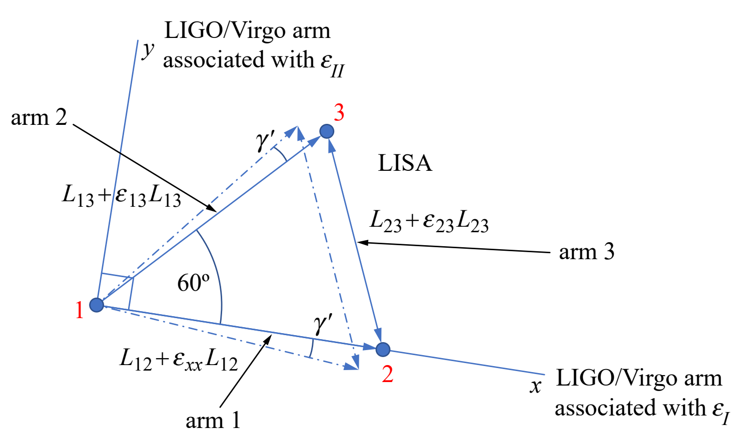

Figure 13. Determination of angles α and γ' from variations in LISA measurements of arms lengths.

be read as two expressions of a strain tensor involving a Poisson's ratio of 1 in these distorted transverse planes [28] and [55]. Since general relativity does not predict a longitudinal wave, the associated space model is found in plane deformations without any deformations in the direction of wave propagation involving zero Poisson's ratios in all planes including the direction of propagation. In this analogy, the elastic medium seems to be made up of transverse plane independent of each others. From a mechanical point of view, it seems clear that, if we adhere to general relativity in its current version, that this blatant anisotropy is contradictory to the hypothesis generally made in physics of a homogeneous and isotropic transverse medium. Clearly, in our analogy, three-dimensional elastic space can no longer be isotropic. We are missing the deformations ε_{xz}, ε_{yz}.

We therefore have, in the case of this local transverse isotropy and the notations of the ASTER code:

For the Young's moduli:

$$Y_L = E_L = Y_T = E_T. \tag{27}$$

For shear moduli:

$$G_{TN} = G_{LN}, \tag{28}$$

$$G_{LT} = \frac{E_L}{2(1+\nu_{LT})}. \tag{29}$$

On the basis of deformations, in the planes perpendicular to the direction of propagation of gravitational waves in weak field general relativity, unmodified, we have

$$\frac{\nu_{LN}}{E_L} = \frac{\nu_{TN}}{E_L} = 0. \tag{30}$$

Therefore

$$\nu_{NT} = \nu_{NL} = 0, \tag{31}$$

$$\nu_{LN} = \nu_{TN} = 0; \tag{32}$$

while in the xy-plane,

$$\nu_{LT} = \nu_{TL} = 1. \tag{33}$$

From the displacements and associated strains imposed at the cosmic fabric by the mass present in space or for vacuum cosmic fabric subjected to the gravitational wave elastic energy [49; 3], it is thus possible to define the equivalent stress field (see introduction) as described in formulas (34), (35), and (36).

Thus, writing the generalized Hooke's law in the frame of reference (L, T, N), where N is he direction of propagation as $\hat{\varepsilon} = K^{-1}\hat{\sigma}$, with:

$\hat{\varepsilon}^T = (\varepsilon_{LL}, \varepsilon_{TT}, \varepsilon_{NN}, 2\varepsilon_{LT}, 2\varepsilon_{LN}, 2\varepsilon_{TN})$ and $\hat{\sigma}^T = (\sigma_{LL}, \sigma_{TT}, \sigma_{NN}, \sigma_{LT}, \sigma_{LN}, \sigma_{TN})$, K^{-1} is the compliance matrix. While the definition of the strain tensor is clear with the space deformation theory, the same cannot be said for the stress tensor $\hat{\sigma}^T$. In our case, we define this stress tensor as the equivalent stress field that induces the observed strain tensor in linearized elasticity under small strain.

The generalized transverse isotropic Hooke's law is

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{Bmatrix} = [K^{-1}] \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{Bmatrix} \tag{34}$$

where $[K^{-1}]$, the classical compliance matrix at the point $M(L, T, N)$ in the frame of reference (L, T, N), is given by

$$\begin{bmatrix} 1/E_L & -\upsilon_{LT}/E_L & -\upsilon_{LN}/E_L & 0 & 0 & 0 \\ -\upsilon_{TL}/E_T & 1/E_T & -\upsilon_{TN}/E_T & 0 & 0 & 0 \\ -\upsilon_{NL}/E_N & -\upsilon_{NT}/E_N & 1/E_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\upsilon_{LT})/E_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{LN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{TN} \end{bmatrix}_{M(L,T,N)} \tag{35}$$

In the case of classical gravitational waves, K^{-1} becomes

$$\begin{bmatrix} 1/E_L & -1/E_L & 0 & 0 & 0 & 0 \\ -1/E_L & 1/E_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/E_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/E_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{LN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{LN} \end{bmatrix}_{M(N,L,T)} \tag{36}$$

To find a spatial and local behavior of the medium, the elastic analogy associated to the modified general relativity suggests the existence of deformations following the direction N ($N = z$, the direction of propagation of the gravitational wave) with Poisson's ratios $\nu_{NT} = \nu_{NL} \neq 0$ associated with equivalent normal stresses σ_N, $(L, T, N) = (x, y, z)$.

5.4. ***Concerning the analogy between geometric torsion and crystal plasticity of the equivalent elastic medium.*** This notion of a complementary vector to close a path on a surface, related to geometric torsion in the Einstein–Cartan theory, is also present in crystal plasticity [49] when, at the atomic level, local plasticization occurs [32]. There are two types of defects: screw dislocations (Figure 15) and edge dislocations (Figure 14, left), sliding by shear effect, and disinclination by forced rotations (Figure 14, right).

In both cases, there is a discontinuity in the network. Mathematically, when we make a path through this dislocation, we must use a closure vector called the Burgers vector, which is equivalent to the geometric torsion explained above; see Figure 15.

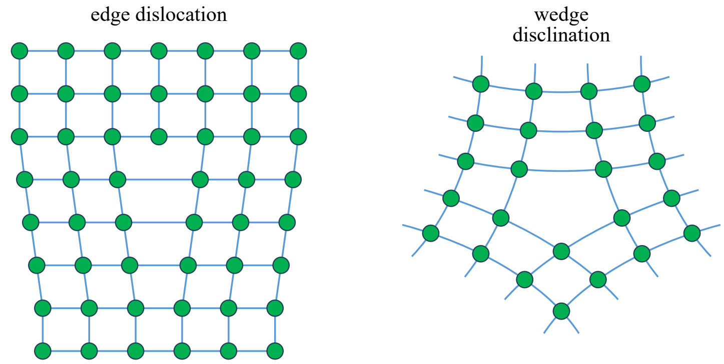

Figure 14. Example defects: edge dislocation and disclination.

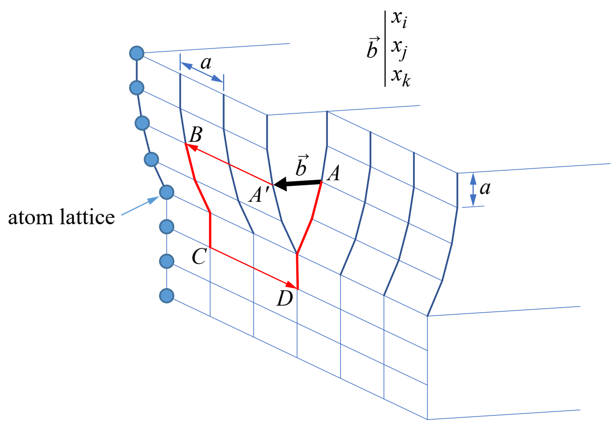

Figure 15. Screw dislocation: the Burgers vector $\vec{b}$, joining the points A and A'.

By writing the path *ABCD* in Figure 15, we can see that mathematically the equivalent Burgers vector is expressed as follows [32]:

$$db^{\mu} = -\Gamma^{\mu}_{\nu\lambda} dA^{\nu\lambda}. \tag{37}$$

$dA^{\nu\lambda}$ being antisymmetric, the symmetric part of the affine bond vanishes, and only the antisymmetric part exists, which is written $\Gamma^{\mu}_{[\nu\lambda]}$. So, the equivalent Burgers vector is

$$db^{\mu} = -\Gamma^{\mu}_{[\nu\lambda]} dA^{\nu\lambda}. \tag{38}$$

The bridge is then made with geometric twisting

$$-T^{\mu}_{\nu\lambda} = -2\Gamma^{\lambda}_{[\mu\nu]}, \tag{39}$$

by defining

$$-[\nu\lambda] = \tfrac{1}{2}(\lambda\nu - \nu\lambda) \tag{40}$$

The theory of defects in crystal plasticity is therefore a mirror of geometric torsion in general relativity [32]. The analogy with two distinct mechanical concepts — elasticity and perfect plasticity — are not

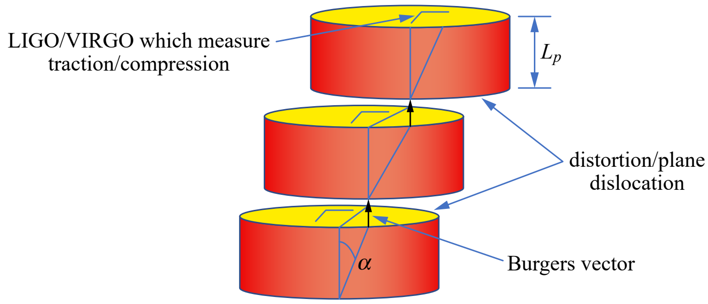

Figure 16. Possible local dislocations between the different layers of the structure of the fabric of spacetime (analogy) during the passage of a gravitational wave.

contradictory here. Indeed, the bridge between the theory of defects and the Einstein–Cartan theory with torsion, as explained above, will imply in the analogy of the elastic medium that planes of space could successively slide locally as a defect during the passage of a gravitational wave, as shown in Figure 16. This is compatible with a possible shear modulus of the medium. There would be no propagation of the torsion as such. Two alternatives are possible: either the space would plasticize locally by shear/distortion, or the analogy would reach its limit in this example.

It is known [11; 47] that the modified Einstein–Cartan theory of general relativity with geometric torsion comprises two field equations, one equivalent to that developed by Einstein (38) and another corresponding to spins of space (39). It has been shown by the authors in [11; 47] that the mathematical formalism associated with this geometric torsion via the Burgers vector (37) to (40) is similar to that in crystal plasticity [38; 47].

It is shown via [19] that polarizations complementary to those predicted in classical general relativity and $(A^+, A^\times)$ appear when we consider this geometric torsion.

We know from [55] that the linearized Einstein equation leads to the squeeze (9) (equivalent to (10)), constituting a bridge that allows us to read polarizations in the elastic domain A^+ and $A^\times$ as components of a strain tensor. This approach has been illustrated in two concrete cases of space twisting in [28] and [29].

However, we have not yet shown that the second spin equation in the Einstein–Cartan theory corresponds to another equation of correspondence between polarizations and deformations in the plastic domain. Is there therefore a transitional formalism equivalent to equation (10) in plasticity associated with defect theory [47] and associated complementary polarizations [11] and [19]? In other words, in plasticity (corresponding to geometric torsion according to [47]), can complementary polarizations also be effectively read as components of a deformation tensor of a four-dimensional space (and no longer only as deformations in successive planes independent of each other as shown in [55] and [28])? The bridge between the components of the polarization tensors of gravitational waves in the case of modified general relativity (Einstein–Cartan) and, by analogy, the deformations linked with an associated elastoplastic medium was made in [12].

The authors thus considered a gravitational wave as a defect (propagation of a Burgers vector) propagating in an equivalent solid medium. The result of their study is again a compression component H in space and shear and distortion components $\pm\sqrt{2}a_i$, as described in expression (41) in four dimensions and (42) in three dimensions.

$$\varepsilon_{\mu\nu} = \frac{1}{2}\begin{pmatrix} H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \\ -\sqrt{2}a_1 & 0 & 0 & -\sqrt{2}a_1 \\ -\sqrt{2}a_2 & 0 & 0 & -\sqrt{2}a_2 \\ H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \end{pmatrix} \tag{41}$$

They also cite the existence of what we called a torsion wave in [28], which they refer to as a J-dependent gyratonic wave (a spin of space), in which the function J is related to the spinning nature of the gyratons. [12]:

$$\varepsilon^{(i)(j)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A} sin\Phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A} cos\Phi \\ \frac{\sqrt{2}J}{\rho A} sin\Phi & -\frac{\sqrt{2}J}{\rho A} cos\Phi & H/A \end{pmatrix} \tag{42}$$

Note that in [12], the authors placed themselves within the framework of nonlinear plane gravitational waves, or parallel-propagating plane front waves (P-P waves) [39]. Indeed, as this study considers the parallelism between crystal plasticity, the implementation of which characterizes a plasticization of the medium by rearrangement of atoms (and therefore a nonlinearity between deformations and stresses) [47], and Einstein–Cartan's nonlinear modified general relativity associated with this theory [47; 11; 19], it makes sense to move away from the realm of traditional elastic waves to consider nonlinear plane waves known as PPs.

Note also that the result they obtain (expressions 43 and 44) for a gravitational wave propagating in the direction z is connected to polarizations A^+ and $A^\times$ gravitational waves in classical general relativity via the following expressions from [39]:

$$H = A_+(u)(x^2 - y^2), \quad H = A_\times(u)xy \tag{43}$$

Recently, in [59; 58; 16; 60], the authors consider a shape memory of space (i.e., a certain residual plasticity of this medium).

The null values in the above two tensors (41) and (42) result from the fact that the authors focused on the geometric torsion part associated with the Burgers vector itself, which is associated with crystal plasticity [32; 47], i.e., the second equation of the Einstein–Cartan theory that presents an analogy with this theory. Thus, as in [19] where complementary polarizations appear because of this geometric torsion in the components (zz), (zx), (zy) and (zt) in 4 dimensions, in [12], complementary deformations appear according to these same components and only for them. However, this publication [12] shows that, unlike the classical equation of general relativity where the bridge between polarization and deformation is direct via expression (10), this time, for the torsion component, the correspondence between the components of the polarization tensors and the deformation tensors in four and three dimensions is no longer direct. The publication [12] explains mathematically how to make this transition.

On the other hand, our paradigm for reading the components of polarizations as components of a strain tensor remains the same.

5.5. ***Concerning the convergence of the different models regarding possible polarizations of gravitational waves in the direction of their propagation.*** In order to analyze the convergence between the different approaches resulting from general relativity, we consider the following:

Concerning the first case: (second-order linearization of $h_{\mu\nu}$ [6; 45], the Einstein–Cartan theory of geometric torsion [47; 11], the theories of modified general relativity [43; 40]) and approaches derived from the analogy of elastic space, we find polarizations complementary to the two classics A^{+} and $A^{\times}$. Concerning the second case: (strong anisotropy of the medium [55], crystal plasticity [32; 47] (junction of transverse planes by successive plastic slips)), we can see by virtue of the analogy between the polarizations of gravitational waves and the deformations of the equivalent elastic medium [55; 28; 29], longitudinal spatial deformations complementing the transverse deformations.

It should be stressed that this convergence aims to continue optimizing the performance of interferometric sensors, to add arms to have a triangular measurement system (Future LISA or Einstein telescope) to possibly detect them and thus settle the question.

5.6. ***Concerning the extreme smallness of the deformations/polarizations in the longitudinal direction of gravitational wave propagation (if they really exist).*** If such longitudinal deformations/polarizations exist, the measurement of the gap between theoretical and real gravitational waves (Figure 5) seems to indicate that they are much smaller than those already measured by the current LIGO/Virgo interferometers [1; 2]. The second-order study of $h_{\mu\nu}$ in gravitoelectromagnetism is more nuanced, the intensity of the out-of-plane deformations depends on the frequency of the gravitational wave [45; 46].

5.7. ***Concerning the importance of excellent coordination of interferometers on Earth complemented by future LISA-type interferometers or the future Einstein telescope or multiple pulsars.*** Publications [3; 27; 4; 40; 57; 56] show that many research teams are currently working to theorize and measure these potential complementary polarizations of gravitational waves, including in the direction of gravitational wave propagation. In [40], the author explains how these different possible polarizations could be studied by an even more efficient interconnection of the different interferometers on Earth.

6. Conclusion

Concerning the convergence between the analogy of space as an elastic medium and the results of general relativity

The analogy of continuum mechanics with general relativity in the weak field is interesting because it allows us to better understand certain aspects of the latter. The metric perturbation tensor is assimilated with twice the strain tensor [55], we notice the similarity between the stress tensor and the energy-momentum tensor [55; 28]. It is possible to consider the parametrizable constant κ as a function of the mechanical characteristics of space [28]. Then, Einstein's equation appears similar to Hooke's law [55; 28; 29], and finally, gravitational waves are similar to medium shear waves [55; 28]. It also provides a new illustration of the origin of the two polarizations instead of one, or six (pure twisting of an elastic medium) [28]. Finally, it suggests distortion (lateral displacement of laser beams of interferometers).

Concerning the divergence between the analogy of space as an elastic medium and the results of general relativity. The analogy of elasticity theory with general relativity in a weak field also reaches its limits and raises questions about its real representativeness, given the extremely high intensity of the associated Young's moduli [55; 41; 28], the value of the Poisson's ratios of 1 in the interferometer plane and 0 out of plane, leading to a strong anisotropy of the medium in any point M and during wave propagation, contrary to the fundamental hypothesis of the homogeneity of the cosmic medium.

Concerning the convergence between current research and what the analogy of space as an elastic medium with the results of general relativity suggests. The discrepancy between the measured and theoretical curves of space deformations during the passage of a gravitational wave (Figure 5) suggests that there is still room for improvement in the general relativity. However, this improvement is a priori extremely small, as this gap is very small [1] and [2].

Research avenues to try to complete this general relativity concern, in particular, potential complementary polarizations of gravitational waves [19; 40; 43].

The modified elastic analogy of general relativity with Einstein–Cartan geometric torsion connected to the theory of defects [11; 19; 47], as well as the approach of general relativity developed in the second order [6; 45], involve additional deformations in the direction of propagation of gravitational waves. Such approaches make it possible to find a coherent spatial behavior and therefore a little less anisotropy of the elastic space associated in our analogy. The geometric torsion, as developed for example by Einstein–Cartan theory can be assimilated at a plasticity between each successive planes via the analogy of the crystallography and defect theory. On this basis, a 3D equivalent anisotropic space medium becomes possible (polarization in three directions and not only on transverse plane, coupled by analogy with strains in three dimensions and not only in two dimensions) [46; 31]. These additional corrections are extremely small [32] and are therefore consistent with what is revealed by the measurements of the deviation between the measured deformation curves associated with gravitational waves and the same deformation curves from unmodified general relativity [1; 2; 3]. According to our paradigm associating these deformations with complementary polarizations, this implies detecting them in order to validate or invalidate these deformations associated with these complementary polarizations.

The multiplicity of recent publications on these potential complementary polarizations in relativistic physics on the one hand [3; 27; 4; 40; 57; 56] and on how to measure them on the other hand indicates (future LISA and Einstein telescope) that this topic is a key research point today. Time will tell whether or not this precise point is a point of convergence between the analogy of the elastic medium and general relativity in the weak field.

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REVIEW

Elastic Medium Analogy of Spacetime: $h_{\mu\nu}$ Metric Perturbation Tensor Analysis and Theoretical Implications

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Abstract
A state-of-the-art review of the different deformations of space–time measured over more than a hundred years within the framework of general relativity in the weak-field regime is presented. The general relativity phenomena considered in this low field context include gravitational waves, the Lense-Thiring effect, gravitational lensing, gravitation around the Earth or the Sun. This overview of various deformations highlights the different active components of the perturbation tensor of the metric $h_{\mu\nu}$. The authors demonstrate that each phenomenon corresponds to one or more distinct components of this tensor. They also show that the various components can be interpreted, within the elastic analogy of space–time, as coherent components of an associated strain tensor $\varepsilon_{\mu\nu}$ in terms of elongation, compression or angular distortion of an equivalent elastic medium modeling the behavior of space–time. Through this synthetic ensemble approach and elastic analogy, it becomes evident—for the first time—that some components of the tensor $h_{\mu\nu}$ remain to be identified and measured potentially corresponding to new phenomena or modified versions of general relativity in the weak-field limit.

Keywords General relativity · Gravitational waves · Frame dragging · Geodetic effect · Lense-Thirring effect · Sun gravitation · Earth gravitation · Gravitational lens · Elasticity · Continuum mechanics

1 Introduction

The theory of general relativity [1, 2] is now more than a century old and has been tested and validated in numerous situations through increasingly precise experiments [3–7], It was proven to be highly successful, and conceptually, gravitation is now understood as a manifestation of deformation or curvature of space–time. This distortion of space–time is associated with the establishment of the metric tensor, $g_{\mu\nu}$, which is determined by solving

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the nonlinear tensor differential equation of the gravitational field formulated by A. Einstein in 1915 [1].

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

In this expression, $R_{\mu\nu}$ is the Ricci curvature tensor, R is the tensor contraction of the Ricci tensor, $T_{\mu\nu}$ the energy–momentum tensor that somehow shapes space–time, G is Newton's gravitational constant, and c is the speed of light.

In weak gravitational fields, the metric $g_{\mu\nu}$ can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2}$$

where $\eta_{\mu\nu}$ is the Minkowski tensor of flat space–time and $h_{\mu\nu}$ a very small perturbation $(\left|h_{\mu\nu}\right| \ll \left|\eta_{\mu\nu}\right|)$.

The purpose of this article is to conduct an in-depth study of the various components of $h_{\mu\nu}$, in the light of the numerous tests of general relativity that have been carried out over the past 100 years.

Some components of $h_{\mu\nu}$ are activated and their intensities are established according to the physical phenomena soliciting space–time. Thus, depending on the type of space–time stress, the activated components, i.e. those necessary for representating the phenomenon under study and the associated deformations concerned, are framed together in Fig. 1. The phenomena of general relativity in low fields targeted are gravitation around the Earth or

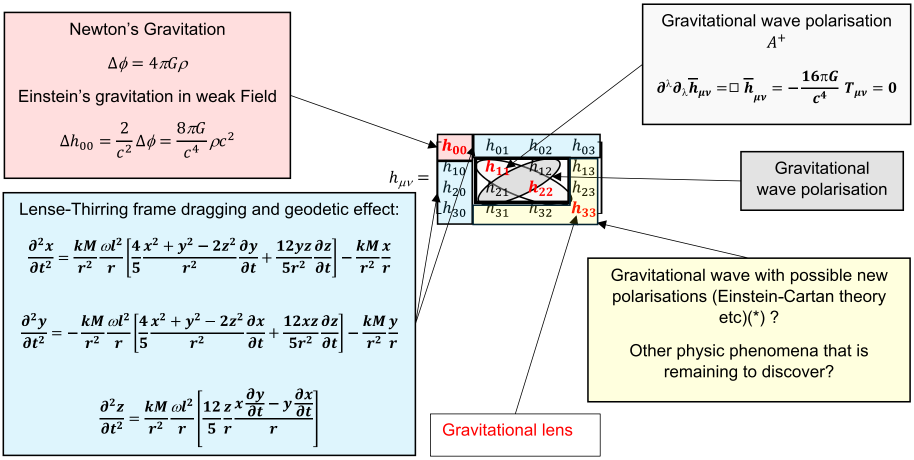

Fig. 1 Physical phenomena associated with each component of the perturbation tensor of the metric $h_{\mu\nu}$—Note (*) It is worth noting that while our analogy is developed within the classical linearized framework of general relativity and the Einstein–Cartan theory, similar correspondences have been observed in the context of exact solutions to Einstein's equations, such as pp-wave spacetimes. In particular, these solutions allow for non-trivial polarization states beyond those derived from the linearized metric perturbation hμν, as the function H(u,x,y) in the pp-wave metric satisfies a two-dimensional Laplace equation. These exact (and non-linearized) solutions to Einstein's equations, such as pp waves (plane-fronted waves with parallel rays), allow for other polarization states. These states are not derived from a linearized version, but from an exact solution, such as that given by the metric: $ds^2 = H(u,x,y)du^2 + dx^2 + dy^2 - 2dudv$ with $\left(\partial_x^2 + \partial_y^2\right)$ H = 0. This opens the possibility of extending the mechanical analogy, especially the link with plastic deformations in crystals, to stronger gravitational regimes. We acknowledge this broader perspective and refer the interested reader to [8, 9] for an example of such a connection in the context of exact solutions

the Sun, gravitational waves, Lense-Thirring effects (frame dragging, geodetic effect). These effects involve different components of $h_{\mu\nu}$ which will now be examined and analyzed through a synthetic overview. Each of these components are particular deformations of space–time (elongations, compression, angular variations) that have been precisely measured via tests carried out in recent years. The objective of this paper is therefore to provide a comprehensive and unified view of the various components of the tensor $h_{\mu\nu}$ and to interpret them within the framework of the elastic analogy, using a four-dimensional elastic strain tensor $\varepsilon_{\mu\nu}$. The aim is to assess wether these two tensors indeed describe analogous mechanical phenomena – sharing the same number of components—and to explore wether certain components remain to be investigated, potentially indicated new physical phenomena or new polarizations associated with additional degrees of freedom in the context of elastic medium analogy.

2 Methods

The methodology followed by the authors is as follows.

- For each phenomenon of general relativity in weak-field regime, identify the active components of the perturbation tensor $h_{\mu\nu}$ of the metric that are involved,
- To construct a unified perspective of the different components of the tensor as a whole, thereby identifying the active components involved in observed phenomena and the inactive ones that may correspond to unexplored or theoretical effects,
- To explicitly construct the correspondences between the specific components of space–time deformations, as positioned within the perturbation tensor of the metric $h_{\mu\nu}$, and the corresponding deformations in the elastic strain tensor $\varepsilon_{\mu\nu}$, within the context of the analogy that models space–time as an elastic medium,
- To use the analogy between the deformation tensor of the equivalent elastic medium and the perturbation tensor of the metric to identify potential new physical phenomena or space–time deformations that have yet to be observed or measured,
- To conclude by exploring the potential implications for the low-energy regime of general relativity, particularly regarding the theoretical extensions or modifications required to incorporate these additional, previously unaccounted-for components of the metric perturbation tensor.

3 Presentation and Structuring of the Different Components of the Perturbation Tensor of the Metric According to the Physical Phenomenon of General Relativity in the Weak Field Studied

3.1 Concerning the Activation of the Component h_{00}– Newton's Classical Gravitation

This framework was developed by Einstein in 1915 [1]. In order for these equations to restore Newtonian gravity in a weak field, Einstein calibrated his passage constant $\kappa = \frac{8\pi G}{c^4}$ based on the components tt (or 00) of the tensors constituting his gravitational field equation [1].

So, the componennt 00 of the metric g_{00} takes the following form. In this expression ϕ represents Newton's gravitational potential.

$$g_{00} = \eta_{00} + h_{00} = 1 + \frac{2\phi}{c^2} \tag{3}$$

Taking the Laplacian noted Δ from the above expression, we obtain:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi \tag{4}$$

Moreover, we know that in a weak field, the gravitational field satisfies the Poisson's equation:

$$\Delta\phi = 4\pi G\rho \tag{5}$$

Thus we find:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2 = \frac{8\pi G}{c^4}T_{00} = \kappa T_{00} \tag{6}$$

This equation is the transposition of Einstein's equation into a weak gravitational field. It gives Newton's gravitation, i.e. the Poisson's equation.

Space–time becomes deformable, it becomes a physical object subject to deformation when it is stressed, just like any material elastic medium that can be studied with the theory of elasticity. It is therefore within the regime of weak gravitational fields, and through the lens of the elastic analogy, that general relativity will be analyzed and interpreted in this paper.

As shown in our publication [10], the analysis of deformations associated with gravitational waves, the Lense-Thirring effect, and the deflection of starlight behind the Sun during a solar eclipse, all falls within the elastic regime, as these deformations are reversible. Spacetime returns to its initial configuration once the deformation energy has dissipated. However, this analysis reveals a foliated medium lacking coherence between layers, especially in the case of gravitational waves (in planes transverse to the direction of wave propagation). In this respect, our model partially aligns in part with those proposed by ADM [11] and by Tenev and Hortemeyer [12]. The issue is that in continuum mechanics, the medium is coherent in all directions. Although anisotropy is permitted, the layers seem to be nonetheless interconnected to some degree. To address this discrepancy and reconstruct a truly three-dimensional—or even four-dimensional -, continuum resembling a crystalline solid, it becomes necessary to introduce geometric torsion into general relativity, for example, through the Einstein–Cartan theory). In solid-state physics, the corresponding concept is the theory of defects [13]. In this context, such defects correspond to local plasticity, specifically, interlayer sliding. The medium thus becomes elasto-plastic: elastic in the transverse planes, and plastic between the planes, along the direction of gravitational wave propagation. In this sense, space–time behaves as an elastic medium within the framework of classical general relativity, but exhibits plastic behavior in the context of modified general relativity. This is what is discussed in publication [10].

To visualize this physical object embedded in transparent space–time—given that the intergalactic vacuum fills the space surrounding the stars—an effective analogy is to consider a two-dimensional membrane represented as a grid-like fabric, on which one can observe how the Sun and the Earth deform it. This membrane or plate approach [14–18] certainly has its limitations, but it allows us to clearly visualize the effects of gravitation, such as the apparent shift of stars behind the sun. This was shown in the publication [19]. The balls thrown onto this surface following the curvature of the canvas will inexorably stick to a heavy ball placed in the center simulating the appearance of a massive object like the sun, giving the impression of a Newtonian force which attracts them towards each other.

In publication [20], the authors demonstrate how a wheeled robot moving on a deformable membrane can accurately replicate the dynamics of curved spacetime. By adjusting its velocity in response to the local curvature of the surface, this active system enablesprecise mapping of radial and orbital trajectories, analogous to those described by general relativity. The study reveals that such active particles do not necessarily follow geodesics in physical space, but rather in a programmable fiducial spacetime, where parameters such as membrane elasticity and instantaneous velocity shape the metric. This framework provides a simple and accessible robophysical model for simulating relativistic effects—such as those near a black hole—and opens new perspectives for robotic exploration in complex terrains and for understanding the dynamics of active matter.

But since gravitation is a three-dimensional phenomenon in Newtonian physics and a four-dimensional phenomenon for Einstein, this analogy necessarily has limit: namely, according to Einstein both space and time are deformed [21]. Moreover, Einstein adjusted his field equation to the temporal component h_{00} of the metric perturbation with Newton's gravitational potential, a correspondence that yields highly accurate results—especially within the context of solar system dynamics [22].

3.2 Concerning the Activation of the Components $h_{00,}h_{xx}$, h_{yy}, h_{zz}, Associated with Newton's Classical Gravitation Applied to Gravitational Lensing – Case on Single Ray of Light

If a mass–energy slightly deforms space [23, 24], a gravitational field arises, and the space–time interval in this weakly perturbed regime is then expressed as follows [25]:

$$ds^2 = -\left(1 + \frac{2\phi}{c^2}\right)c^2dt^2 + \left(1 - \frac{2\phi}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right) \tag{7}$$

The metric of this space–time is then written from the Minkowski tensor and perturbation tensor of the metric:

This metric is used in the case of gravitational lensing [25]. The interval is of the light type $ds^2 = 0$

In this case according to [25], we can obtain the propagation coordinate time:

$$t = \frac{1}{c}\int_S^0 \left(1 - \frac{2\phi}{c^2}\right)dl = \frac{D}{c} - \frac{2}{c^3}\int_S^0 \phi dl \tag{8}$$

So, gravitational lensing depends both on the Newtonian potential h_{00} and on the space curvature h_{ii}.

Thus $h_{\mu\nu}$ can be divided into 3 parts: the temporal components h_{00}, the spatiotemporal components h_{0i} and h_{j0}, and the purely spatial components h_{ij}. Each of these components corresponds to a specific phenomenon in general relativity within the weak-field regime. We will therefore examine these components in detail in the following section.

3.3 Concerning the Components $h_{ij}\left(\mathrm{i,j} \rightarrow x, y\right)$– Gravitational Waves

This was developed by A. Einstein in 1916 [2] and then in 1918 [26]. The components h_{ij} of the metric perturbation tensor are associated with gravitational waves.

In weak-field approximation, Einstein's field equation becomes [26]:

$$\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \tag{9}$$

In the specific case of gravitational waves, when considered in vacuum where: $T_{\mu\nu} = 0$, the field equation reduces to:

$$\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = 0 \tag{10}$$

h is the trace of $h_{\mu\nu}$, $\Box$: the D'Alembert's operator. The gauge condition taken is $\partial^{\lambda}\overline{h}_{\mu\lambda} = 0$.

$\overline{h}_{\mu\nu}$ represents the metric perturbation defined through the following variable transformation:

$$\overline{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \tag{11}$$

The solution of this differential equation is:

$$\overline{h}_{\mu\nu} = -\frac{4G}{c^4}\iiint_{Source}\frac{T_{\mu\nu}\left(x^0 - x_s^0 - \|\vec{x} - \vec{x}_s\|\right)}{\|\vec{x} - \vec{x}_s\|}d^3\vec{x}_s \tag{12}$$

In the case of gravitational waves in vacuum, the solution of the above equation can be writren as:

$$\overline{h}_{\mu\nu} = A_{\mu\nu}cos\left(k_{\sigma}k^{\sigma}\right) \tag{13}$$

With:

$$A_{\mu\nu} = A_{+}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + A_{\times}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

The non-zero terms are therefore at the coordinates xx, yy, xy, yx. Accordingly, we focus on the perturbation of the spatial metric h_{ij}, rather than the spatiotemporal components. Under the imposed gauge conditions, only the spatial part—associated with the deformations caused by gravitational waves—remains. These components are directly related to the sources of gravitational waves [26]; indeed, we have:

$$\overline{h}_{ij(t)} = h_{ij(t)} = \frac{2G}{Rc^4}\frac{d^2}{dt^2}I_{ij}\left(t - \frac{R}{c}\right) = \frac{2G}{Rc^4}\ddot{Q}_{ij}\left(t - \frac{R}{c}\right) \tag{15}$$

Here, R denotes the distance between the observer and the source, I_{ij} is the mass quadrupole moment, $Q_{ij(t)}$ the quadrupole moment. The deformations induces by gravitational waves are thus purely spatial and manifest as elongations and contractions in the (x, y) planes, which are perpendicular to the direction of wave propagation z as considered in this article. For simplicity, we will focus on deformations occurring in the plane throughout this article. Gravitational waves were first directly detected in 2015 through the GW150914 event [6]. Gravitational and electromagnetic waves were observed simultaneously for the first time during the GW170817 event [7]. This dual observation confirmed that gravitational waves propagate at the speed of light.

3.4 Concerning the Off-Diagonal Components $h_{0i}h_{j0}$: Frame-Dragging Effect and Geodesic Precession

This effect was first described by J. Lense and H. Thirring in 1918 [3]. It concerns the dragging of the frame of reference caused by a massive rotating object, such as the Earth or the Sun. In paper [3], formulae 4 (regarding $T_{\mu\nu}$) and 10 (regarding $g_{\mu\nu}$) show that the off-diagonal component h_{0i} and h_{j0} characterize this effect. More recent studies [27] revisit these phenomena using updated notation. The two Lense-Thirring effects—frame dragging and geodetic precession—were measured in the Gravity Probe B experiment [5].

4 Summary and Overview of the Different Components of the Perturbation Tensor of the Metric

As discussed in the previous Chapter 3, in a weak gravitational field, general relativity predicts deformations—such as elongations, shortenings, and angular distortions—both in the plane and perpendicular to it. Figure 1 above illustrates each phenomenon associated with the corresponding components of the metric perturbation tensor $h_{\mu\nu}$ where $(0 = t, 1 = x;2 = y;3 = z)$ studied in this article. By superimposing these different components of $h_{\mu\nu}$ to provide, for the first time, a global and synthetic overview, we observe that some components remain inactive—specifically h_{0z}, h_{iz}, h_{zj}—with respect to the classical phenomena measured and described in chapter 3. Figure 1 above unified so structural interpretation of the components of the metric perturbation tensor $h_{\mu\nu}$, correlating known relativistic phenomena with specific elastic deformations. This diagram outlines both experimentally validated effects (gravity, gravitational waves, frame dragging) and yet-unmeasured components, suggesting new degrees of freedom possibly accessible through extended theories (Einstein–Cartan, teleparallel gravity). This visual framework proposes a structural reading of spacetime curvature and offers a roadmap for future gravitational experiments.

5 Deep Co-Correspondence Between the Deformations of Spacetime from the Perturbation Tensor of the Metric and a Deformation Tensor of an Equivalent Elastic Medium

5.1 Founding Principles and Hooke's Law Associated with the Elastic Analogy of Space–Time in Weak Field

The fundamental principles of equivalence are as follows. They are derived from well-established publications and are reinterpreted here in the context of the elastic analogy of the space–time medium.

5.1.1 Equivalence principle n°1: Correspondence Between the Metric Perturbation Tensor and the Strain Tensor Within the Elastic Medium Analogy

This equivalence principle described in [12, 28] and [29] is as follows:

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu} \tag{16}$$

With:

$h_{\mu\nu}$ the metric perturbation tensor such that defines in the metric of the general relativity from the flat Minkowski metric.

$\varepsilon_{\mu\nu}$ the strain tensor in four dimensions.

Both are symmetric tensors.

In this analogy, the strain elastic tensor $\varepsilon_{\mu\nu}$ corresponds to the space–time metric pertubation tensor $h_{\mu\nu}$ related by a factor of 2.

5.1.2 Equivalence Principle n°2: Equivalence Between the Stress Energy Tensor and the Stress Tensor in Elasticity

This equivalence principle described in [28] is as follows:

In four dimensions, we have $T_{\mu\nu} = \rho_{matter} u_u u_v$. In three dimensions, the stress tensor is given by $\sigma_{ij} = \rho v_i v_j$.

In these expressions, $T_{\mu\nu}$ is the tensor energy–momentum or stress energy tensor, u_u and u_v the four-velocity of the matter distribution, σ_{ij} the stress tensor, v_i, and v_j the components of the velocity vector in space.

In our analogy, the stress tensor σ_{ij}, when extended in four dimensions $\sigma_{\mu\nu}$, corresponds to the stress energy tensor $T_{\mu\nu}$ in general relativity that is equivalent at the loading of space time.

5.1.3 Equivalence principle n°3: Einstein's Constant Characterizes the Flexibility of Space–Time

This equivalence principle, described in [28] and [29], is as follows:

–For an elastic and isotropic Timoshenko bar [30] of length L, with a cross-sectional area S, and Young's modulus Y = E, subjected to a normal force N, the bar undergoes a longitudinal displacements u_x along the direction x and therefore strains,ε_{xx} ,ε_{yy}, and ε_{zz}. If U represents the elastic energy stored in the bar, then we have:

$$\varepsilon_{xx}^2 = \left(\frac{du_x}{dx}\right)^2 = \frac{2}{ES}\frac{U}{L} = \frac{2}{YS}\frac{U}{L};\varepsilon_{yy} = \varepsilon_{zz} = -\nu\varepsilon_{xx} \tag{17}$$

$$U = \frac{1}{2}\int_0^L \frac{N^2}{ES} dx \tag{18}$$

–For an elastic and isotropic Timoshenko bar [30] of length L, with a reduced cross-sectional area S_r due to the shear loading, a shear modulus G = μ, subjected to a shear force V, the bar undergoes rotation of its cross-section. This results in shear strains ε_{xy}, and ε_{yx} associated with angular distortion γ_{xy}, and γ_{yx}:

$$\gamma_{xy}^2 = \gamma_{yx}^2 = (\frac{du_x}{dy})^2 = \left(\frac{du_y}{dx}\right)^2 = \frac{2}{GS_r}\frac{U}{L} = \frac{2}{\mu S_r}\frac{U}{L} \tag{19}$$

$$U = \frac{1}{2}\int_0^L \frac{V^2}{GS_r}dx \tag{20}$$

The classical terms $2/ES$ and $2/GS_r$ are the flexibilities of the bar. They share the same unit (N^{-1}) as the flexibility constant $\kappa = \frac{8\pi G}{c^4}$ (with G the gravitational constant), in four-dimensional space–time, as appears in Einstein's field equation.

In our analogy, the medium's flexibility—proportional to 1/ES – serves as the counterpart to the space–time flexibility κ in general relativity.

Let us generalize Hooke's law for spatial components alone, following reference [12]:

$$U_\xi = \frac{1}{2}\sigma^{ij}\varepsilon_{ij} = \frac{1}{2}C^{ijkl}\varepsilon_{ij}\varepsilon_{kl} \tag{21}$$

With C^{ijkl} the fourth rank elasticity tensor:

$$C^{ijkl} = \frac{Y}{1+\nu}\left(\frac{\nu}{1-2\nu}g^{ij}g^{kl} + g^{ik}g^{jl}\right) \tag{22}$$

The estimated value of the space–time's Young's modulus varies significantly among authors. Tenev and Horstemeyer [31], using quantum field theory, report Y = 10^{113} Pa. M. Beau [32], finds K = 1.6 $\times 10^{109}$ Pa, a magnitude also supported by Izabel in [28]. In contrast, R. Weiss, during his Nobel Prize lecture, derives a value of Y = $10^{20}Y_{steel}$, or approximately 2.1 $\times 10^{31}$ Pa, based on the energy of gravitational waves. Similarly, K. McDonald [33] arrives at a comparable estimate. For the authors of this paper, the Young's modulus is on the order of 10^{44} Pa in the transverse planes of gravitational waves. We have to note that the deformations are extremely small—on the order of 10^{-21}. If one takes the constant 1/κ and assumes it applies per unit area, one obtains a rigidity approximately of 4.8 $\times 10^{42}$ Pa. In publication [34] the authors demonstrate that graphene exhibits mechanical behavior with noteworthy parallels to spacetime, estimating graphene's Young's modulus at 10^{12} Pa. They further propose that certain graphene structures may provide insight into the microstructure of spacetime itself, within the Teleparallel Equivalent of General Relativity (TEGR) framework originally suggested by Einstein in his attempts to unify electromagnetism with gravitation. The authors demonstrate that the coupling constant κ converges toward its value in general relativity, thereby illuminating both the possible microstructure of spacetime and the origin of its intrinsic stiffness. Their approach explicitly incorporates geometric torsion—which, as we showed in [10], is essential for maintaining a coherent stacking of spacetime 'sheets. Their final results are as follows:

> "If the preceding hypothesis is accurate, as substantiated by these findings, and we conceptualize spacetime as a cellular structure comprised of minuscule particles, we can deduce that the force between these particles corresponds to the coupling constant. Expressed in SI units, this equation results in $c\kappa = c^4/16\pi G = 2.41596\times10^{42}$ N. The immense magnitude of this force indicates that spacetime possesses an extremely

high resistance to deformation (about 10^{30} times greater than that of a one-square-meter graphene sheet). Consequently, gravity, which arises from the deformation of spacetime, is a remarkably feeble phenomenon. If the force of deformation is insufficient, meaning that smaller amounts of energy cannot cause considerable deformation of spacetime. This aligns with our understanding of gravity."

5.1.4 Equivalence Principle n°4: In the Weak-Field Limit, Einstein's Field Equations are formally Equivalent to a Generalized Hooke's Law

According to [12, 28, 29], and [35] in the weak-field approximation, Einstein's field equations take the form shown in Eq. 9.

In the transverse gauge, the trace of the perturbation tensor $h_{\mu\nu}$ vanishes, so h = $\overline{h}$= 0 and therefore $\overline{h}_{\mu\nu} = h_{\mu\nu}$= 2 $\varepsilon_{\mu\nu}$, according to principle 1.

Moreover, $T_{\mu\nu}$ is related to the stress tensor in accordance with principle 2.

Since $\frac{16\pi G}{c^4} = 2\kappa$, and by Principle 3, this term is related to the flexibility of the equivalent elastic medium via 1/E = 1/Y.

The entire framework becomes conceptually equivalent to Hooke's law for an elastic medium modeling spacetime [36]:

$$\varepsilon = \frac{1}{E}\sigma \tag{23}$$

$$\gamma = \frac{1}{G}\tau \tag{24}$$

5.1.5 Equivalence Principle n°5: The Young's Modulus of the Elastic Medium can be Related to its Energy Density

In the case of elastic waves, it is shown in [12, 28] and [29] that:

–In the case of longitudinal waves:

$$Y = \rho_{vacuum} c^2 \tag{25a}$$

–In the case of shear waves:

$$\mu = \rho_{vacuum} c_{shear}^2 \tag{25b}$$

Here, ρ denotes the density of the medium (the vacuum, for the purposes of this article). Thus, in our analogy, the Young's modulus of spacetime serves as a bridge to the elastic strain energy density of the vacuum itself. So vacuum as an intrinsic energy as demonstrated by the Casimir force [37–41] that becomes elastic in our model

5.1.6 Principle n°6: The Polarization Modes of Gravitational Waves can be Interpreted as Components of a Strain Tensor in the Vacuum, Modeled as an Elastic Medium

This principle described in [28] is based on principle 1 and is as follows:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \varepsilon_{xy(A_+)} = \frac{1}{2}A_+\cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \varepsilon_{xy(A_\times)} = \frac{1}{2}A_\times\cos\left(\frac{\omega}{c}(ct - z)\right)\begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

From the perspective of deformation—and hence from a mechanical viewpoint—these two expressions describe a state of pure torsion in the elastic medium, manifested as two shear modes oriented at 45° to one another. This formulation is fully consistent with the relativistic origin of gravitational-wave polarizations, namely the orbital motion of a binary black-hole system, which induces a torsional distortion in spacetime [10, 28, 29].

5.1.7 Postulate n°1 The Necessity of Accounting for the Intrinsic Deformation Energy of Vacuum Spacetime Itself

Classical general relativity describes how spacetime responds when "charged" by mass or energy. However, in a true vacuum the classical energy–momentum tensor $T_{\mu\nu}$ vanishes. To retain a Hooke's-law–type relation between stress and strain via a proportionality constant, one may introduce an additional tensor $t_{\mu\nu,el}$ that represents the vacuum's elastic strain energy—an approach developed in [42] and [43]. In this extended framework, Einstein's field equations take the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} + t_{\mu\nu}\right) \tag{27}$$

And in the vacuum:

$$\begin{cases} T_{\mu\nu} = 0 \\ t_{\mu\nu} \neq 0 \end{cases} \tag{28}$$

where $t_{\mu\nu} \sim Y\varepsilon_{\mu\alpha}\varepsilon^{\alpha}_{\nu}$, with Y the effective modulus of the vacuum, and $\varepsilon_{\mu\nu}$, the strain tensor. Assuming the off-diagonal components are negligible compared to the principal strains ε_{tt}, ε_{xx}, ε_{yy}, and ε_{zz}, the trace $t_{\mu\nu}$ may be interpreted as the elastic strain energy:

$$U = \frac{1}{2}\rho(c)^2 = -\frac{1}{2}\varepsilon_{tt}^2 Y_t + \frac{1}{2}\varepsilon_{xx}^2 Y_x + \frac{1}{2}\varepsilon_{yy}^2 Y_y + \frac{1}{2}\varepsilon_{zz}^2 Y_z \tag{29}$$

The tensor $t_{\mu\nu}$ can be interpreted as an internal stress perturbation of the vacuum. It may also be viewed as the Hookean response of an elastic medium to geometric deformations—an idea dating back over a century and echoing Sakharov's induced-gravity concept

[44]. This additional vacuum elastic-energy tensor is thus intimately linked to gravitational energy in empty space, a subject that remains unresolved in modern physics.

Indeed, in classical general relativity the gravitational field lacks a local energy–momentum tensor because of the equivalence principle. Various approaches have been proposed to define gravitational energy. The earliest are the energy–momentum pseudotensors t^{ik} of the gravitational field (Einstein, Landau–Lifshitz) [45], where in chapter 11, one write:

$$(-g)\left(T^{ik}+t^{ik}\right)=\frac{\partial h^{ikl}}{\partial x^{l}}$$

with $t^{ik}=t^{ki}$ the symmetric pseudo-tensor. Later, the ADM energy formalism [46, 47] was developed by decomposing the spacetime metric into three spatial dimensions and one temporal dimension, thereby facilitating the analysis of gravitational dynamics and energy. One writes: $ds^2=-N^2dt^2+\gamma_{ij}\left(dx^i+N^idt\right)(dx^j+N^jdt)$ where N is the lapse function and γ_{ij} is the 3D spatial metric. In this approach, gravitational energy takes the form: $E_{ADM}=\frac{1}{16\pi G}lim_{r\to\infty}\int_{S_r}\left(\partial_j\gamma_{ij}-\partial_i\gamma_{jj}\right)n^idS$.

This direction is consistent with an anisotropic space medium composed of transversely strained sheets under longitudinal gravitational plane waves, as studied by the authors in [10].

We also have the Komar energy (or equivalently, mass integrals) [48]: $m=\int_V\left(2T_{ab}-Tg_{ab}\right)u^a\xi^bdV$ where T_{ab} is the stress-energy tensor, T its trace, u^a a velocity vector, and ξ^b a Killing vector field. Finally, quasi-local energy definitions (Hawking, Penrose, Brown-York) [49, 50], and [51] attempt to define gravitational energy within a finite spacetime region bounded by a closed surface (typically a 2-sphere). These approaches aim to circumvent the fact that gravitational energy cannot be localized (due to the absence of a well-defined energy tensor for gravity) by defining it consistently over a finite domain.

These definitions are either coordinate-dependent (as with pseudo-tensors) or limited to asymptotic or stationary regimes (ADM, Komar) [46–48], and thus do not offer a general covariant local formulation [10, 52].

Therefore, in the context of weak fields and the elastic analogy, we propose a symmetric tensor $t_{\mu\nu}$ to model the intrinsic elastic energy of spacetime deformations—even in vacuum, i.e., in the absence of matter. This tensor plays a role similar to the effective stress-energy content used in Balbus averaging of gravitational waves [53]. Thus, in our elastic gravity framework, we encounter the same foundational challenge as in classical GR regarding the definition of gravitational energy. However, since $Y=\rho c^2$—the vacuum energy density related to mass density ρ and the Young's modulus of the medium—our elastic approach is conceptually closest to the pseudo-tensor formulation.

In the case of the pseudotensor approach—such as when computing the energy of gravitational waves in vacuum—the Einstein field equations take the form:

$$G^{(1)}_{\mu\nu}=-\frac{8\pi G}{c^4}\left(T_{\mu\nu}+t_{\mu\nu}\right) \tag{30}$$

Here, $G^{(1)}_{\mu\nu}$ contains only the term $G_{\mu\nu}$ linear in $h_{\mu\nu}$ (see Eq. 4 of [54]) and the pseudo tensor $t_{\mu\nu}$ is defined by:

$$t_{\mu\nu}=T^{GW}_{\mu\nu}=\frac{c^4}{8\pi G}\left[G^{(2)}_{\mu\nu}+\dots\right] \tag{31}$$

where $G^{(2)}_{\mu\nu}$ comprises the quadratic terms of $h_{\mu\nu}$ (see Eq. 5 of [54]):

5.2 Quantum Beams and the Mechanics of Spacetime: A Unified Structural Framework for the Universe

Basing on the principles definied above, and to illustrate our elastic analogy, imagine that, in the weak-field limit, spacetime is a network of Timoshenko beams [30], each with a thickness on the order of the Planck length [12, 31]—effectively quantum-scale beams much stiffer than simple strings. These beams bear mass (or an energy density $\frac{E}{V} = \frac{Mc^2}{V} = \rho c^2$) at specific nodes that undergo continuous dynamic motion. In 1915–1916, Einstein effectively proposed a mechanical model of this structure, in which the displacements of these nodes are determined by the spacetime metric $g_{\mu\nu}$, and hence by geometry. The interval ds^2 then corresponds to the squared length of an infinitesimal beam element. The constant κ could be interpreted as a measure of flexibility of these bars. This formulation enables the calculation of node displacements through a law analogous to: $E = RC^2$ (Energy density = Rigidity × Curvature^2 for a bending beam, where $\frac{U}{L} = \frac{1}{2}EI\frac{1}{R^2}$).

Notice: The Einstein field equation in general relativity is often written in the form: curvature = κ × energy density, where the unit of curvature is 1/m^2. In this context, curvature refers to Gaussian curvature, originally introduced by Gauss [55] and later generalized by Riemann. It characterizes the intrinsic curvature of a surface as perceived from within and is derived from second derivatives of the metric tensor, hence its unit of 1/m^2. In the mechanical analogy using beams or Timoshenko plates, curvature typically refers to the inverse of the radius of curvature, 1/R. From beam theory, the bending equation $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R}$ relates curvature to the bending moment M, Young's modulus E, and the moment of inertia I. When this is combined with the expression for the elastic strain energy $U = \frac{1}{2}\int_0^L \frac{M^2}{EI}dx$, we obtain a result in one or two dimensions (beam or plate) of the form: $\frac{1}{R^2} = \frac{2}{EI}\frac{U}{L}$. This yields a term on the left-hand side with units of 1/m^2, aligning precisely with the units of curvature in general relativity.

Thus, although curvature is initially expressed differently in the two contexts—1/R in mechanics and via second derivatives of the metric in general relativity—both ultimately converge to a quantity with the same physical units when squared. This formal resemblance is striking and reinforces the validity of the analogy. However, it's important to be aware that the term"curvature"carries different conceptual meanings in the two frameworks, which can sometimes lead to confusion. Thus, the curvature of space–time (expressed via $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ in GR) arises from the load applied (expressed via $T_{\mu\nu}$), just as in beam theory. For example, in general relativity, the scalar curvature of a sphere is exactly $\frac{2}{R^2}$, which directly parallels the mechanical result when squared curvature is involved. From this perspective, general relativity can be interpreted as a displacement method: it allows the calculation of both instantaneous local displacements of nodes in 3D space and delayed displacements—i.e., strains $\varepsilon_{\mu\nu}$ or metric perturbations $h_{\mu\nu}$ —that propagate over time at the speed of light c, governed by the mechanical properties (e.g., stiffness, density of the medium) of the structural elements of spacetime that constitute a sort of cristal [34, 56, 57]. That is why GR is well exprimed in 4 dimensions to take into account of these two types of deformations instantaneous and differed. In this elastic analogy, $t_{\mu\nu}$ represents the internal stresses associated with deformations and displacements. It includes contributions from the self-weight or intrinsicstress of the structure itself—identified here with the quantum vacuum, characterized by a vacuum energy density ρ_{vacuum}. including those due to the self-weight of the structure—namely, the quantum vacuum with density ρ_{vacuum}.

This structural model would also be sensitive to thermal gradients between the cosmic web—a relatively warm, matter-rich environment—and the much colder quantum vacuum.

Such gradients could drive the expansion of space and result in a residual thermal curvature present throughout the universe. This effect could manifest as a form of mysterious energy that evolves over time, consistent with recent observations made by the Dark Energy Spectroscopic Instrument (DESI) in 2025 [58]. The resulting temperature differential between the warm cosmic web and the cold vacuum introduces a form of thermally driven deformation in the spacetime structure.

Moreover, under long-term stress, this spacetime"material"would exhibit creep behavior—a slow, continuous deformation under constant loading. This phenomenon may offer a physical analogy for the presence of dark matter, whose gravitational effects imply more deformation (curvature) than can be explained by visible matter alone [59]. In this framework, dark matter emerges not as an unknown particle, but as a manifestation of structural response—a deeper deformation within the elastic medium of spacetime.

This structure may also exhibit defects and localized plastic behavior, much like a crystal, resulting in anisotropic responses under specific loading conditions. This behavior is analogous to the presence of geometric torsion and defect structures in the Einstein–Cartan theory of gravity [10, 13] with all associated polarizations longitudinally (remaining to measure) and transversally (already measured).

Furthermore, the eigenfrequencies and eigenmodes of these quantized quantum beams could be directly related to the energy levels and wavefunctions of associated quantum wells [60]. In this context, wave propagation through the structure—triggered by violent, localized disturbances—mirrors the dynamic response of beam-column systems under seismic loading. This is the mechanical analogue of gravitational waves, which are generated by cataclysmic astrophysical events such as black hole mergers [10, 12, 28].

If this structure becomes locally overloaded at a given node, plastic hinges may form at that location. In such cases, the connected beams behave like a mechanism, allowing relative rotations. As loading continues, the internal fibers may exceed their elastic limit, and the characteristic triangular stress distribution transitions to a bi-rectangular profile, indicating the onset of plasticity. The hinge localizes into a single point, where a singularity emerges in the structure. This phenomenon serves as a mechanical analogue of black holes, as described in [29].

Moreover, when the beams are subjected to high compressive forces, they may undergo buckling. There exists a compelling analogy between buckling phenomena in mechanics and spontaneous symmetry breaking in quantum field theory, as discussed by J. Iliopoulos in the context of the Higgs boson [61].

So, quantum mechanics emerges so as the dynamic theory of an elastic microstructure of spacetime: energy levels correspond to vibrational modes of quantum-scale beams, wavefunctions to deformation shapes, and spontaneous symmetry breaking to mechanical buckling. Quantization is not imposed axiomatically but arises naturally as the modal behavior of a structured medium.

Finally, in this framework, all fundamental physical phenomena emerge and are unified from the mechanical behavior of a quantum-scale beam network structuring spacetime. General relativity becomes the static deformation of this lattice—its curvature representing gravitational effects. Gravitational waves correspond to vibrational modes propagating through the beam structure. Quantum mechanics arises naturally as the quantization of these vibrational states, with energy levels corresponding to modal eigenfrequencies. Quantum field theory is interpreted as the study of instability and buckling within this network, leading to symmetry breaking as observed in the Higgs mechanism. Dark energy appears as a global thermal curvature—analogous to a thermal expansion of the spacetime framework—while dark matter results from the long-term viscoelastic creep of the medium. Finally, black holes are understood as plastic hinges, rupture zones where the structural continuum collapses. This unified vision reframes the universe not as abstract geometry, but as a structured, dynamic, and responsive mechanical entity. Ultimately, this perspective suggests that the universe is not a mere abstract mathematical construct, but rather an immense feat of engineering, a sort of cosmic Eurocode — a vast, structured edifice whose fabric, dynamics, and evolution obey the deep laws of mechanics. So, in our analogy, The Fabric of the Universe become an Elastic Lattice made of quantum beams: A New Paradigm.

5.3 The Mechanical Constitution of the Equivalent Elastic Medium Behaving like Space–Time

The literature surrounding the definition of this equivalent elastic medium is extensive and multifaceted. Notable contributions include the works of Tenev and Horstemeyer [12], Izabel [28, 29, 58], and many others [10, 35, 44, 62–66]. Generally, two principal estimations of the Young's modulus of spacetime emerge. The first, derived from quantum field theory and the vacuum energy density, places it around 10^{113} Pa [12, 28, 29, 58, 62]. The second, based on the elastic energy associated with spacetime deformations due to gravitational waves, reaches much lower values—on the order of 10^{31} Pa [67].

It is also important to highlight that the mechanistic approach to spacetime—constructed as a network of quantum-scale beams—not only provides a structural framework for interpreting general relativity and quantum mechanics, but also offers a natural convergence point for the fundamental constants of physics. In this perspective, key constants such as the speed of light c, Planck's constant $\hbar$, and Newton's gravitational constant G become interrelated through the mechanical properties of spacetime itself. For instance, in the work of Tenev and Horstemeyer [12, 31] confirmed by Izabel [28, 29] the elastic modulus of the quantum vacuum is expressed as $Y = \frac{6c^7}{2\pi\hbar G^2}$, an equation remarkably similar in structure to the Hawking temperature formula for black hole radiation. This convergence suggests that mechanical properties such as stiffness, energy density, and curvature are not merely analogues but are physically and dimensionally linked to the constants that govern the deepest layers of reality. Thus, the structural model not only unifies physical theories but also binds together the constants that define them.

Taking into account that the Young's modulus Y can be related to an energy density through the expression $Y = \rho c^2$, we encounter a situation analogous to the well-known discrepancy between the quantum vacuum energy predicted by quantum field theory and the energy density associated with the cosmological constant Λ. This is the infamous"vacuum catastrophe,"where the theoretical prediction exceeds the observed value by approximately 10^{120} orders of magnitude [68]. However, within the elastic analogy framework, this discrepancy is somewhat reduced: the gap is on the order of 10^{80}, which—while still enormous—represents a narrowing of the problem when interpreted through the lens of mechanical elasticity.

The literature [12, 28, 29, 58] consistently reports a Poisson's ratio of 1 when analyzing the characteristic deformations induced by gravitational waves. This reflects a behavior in which space deforms transversely without any longitudinal contraction or expansion, a hallmark of incompressibility under transverse strain. Additionally, the"particle size"of the equivalent elastic medium—often interpreted as a fundamental scale of granularity—tends toward the Planck length, as discussed in [12, 28, 29, 58, 62], and [69].

A more detailed mechanical analysis of gravitational waves [10] reveals that this medium exhibits anisotropic behavior. Specifically, deformations occur primarily in planes perpendicular to the wave propagation direction, while there is virtually no deformation along the direction of propagation. This dynamic response suggests that spacetime behaves as if composed of layered sheets—or"leaves"—which lack cohesion in the longitudinal direction. Such behavior is indicative of a laminated or stratified medium with weak coupling between layers, echoing models in elasticity theory where out-of-plane coherence is minimal or absent.

When geometric torsion is added to the Riemann curvature tensor, the framework of general relativity is modified. This extension gives rise to additional gravitational wave polarizations and, following Principle 6, leads to complementary deformations between the spatial sheets that make up spacetime [67, 70, 71]. Such phenomena are also observed in gravito-electromagnetism, when general relativity is expanded to second-order approximations [13, 72], or when spacetime is modeled as a fluid medium, with light behaving analogously to acoustic waves in hydrodynamics [73].

These approaches collectively allow us to reconstruct a coherent elastic medium in three dimensions, capable of transmitting not only curvature-induced deformations but also shear and torsional effects—attributes that classical general relativity (which neglects torsion) does not capture.

Notably, the mathematical formalism of geometric torsion closely mirrors that of defect theory and plastic crystallography [13, 67, 70–72], suggesting that spacetime could be understood as a medium with internal defects and dislocations—much like a crystalline solid under stress.

In this context, Ref. [74] offers a rigorous geometric interpretation of dislocations and disclinations within the framework of Riemann–Cartan geometry. In this formalism, torsion and curvature serve as the geometric analogs of Burgers vectors and Frank vectors, respectively, interpreted as surface densities. This geometric correspondence reinforces the analogy with defect theory in solid-state physics, where dislocations and disclinations represent discontinuities in the lattice structure.

Moreover, these results confirm that asymmetric elasticity theories and chiral field models can arise naturally from such a geometric framework. This supports the idea that torsion should be incorporated into the geometry of spacetime—not merely as a mathematical extension, but as a physically motivated necessity for describing microscopic deformations and interactions beyond the linear regime of classical general relativity.

Based on this state of the art, it is therefore reasonable to consider that spacetime can be modeled as an elastic medium which, in the regime of weak gravitational fields, obeys a form of Hooke's law. However, this medium exhibits extremely low—virtually negligible—flexibility, corresponding to an almost perfectly rigid behavior. This extreme stiffness is consistent with the very high equivalent Young's modulus inferred from vacuum energy densities and the observed dynamics of gravitational waves. Such a model provides a coherent mechanical analogy for understanding spacetime deformations, while also capturing the geometric richness of general relativity.

5.4 Systematic Correspondences Between Measured Weak-Field Space–Time Deformations and those of an Equivalent Strain Tensor

It is well known in elasticity theory that the diagonal components of the strain tensor are related to elongations and shortening of an elementary volume and that the other components are related to angular deformations.

If we superimpose the various components of the tensor $h_{\mu\nu}$ with the tensor $\varepsilon_{\mu\nu}$, (see Fig. 1), we observe the following:

h_{00} associated with the product $c.t$, is the first diagonal term related to the time component of the tensor. It corresponds to an isotropic compression in all directions—representing classical gravitation, which binds together the Earth, planets, stars, and other celestial bodies,

$h_{ii} = -h_{jj}$: These diagonal terms associated with the spatial components of the tensor and the A^+ polarization, correspond to the spatial elongations and contractions measured by interferometers such as Ligo and Virgo during the passage of a gravitational waves,

$h_{ij} = h_{ji}$: These are the off-diagonal transverse spatial terms, associated with the $A^\times$ polarization. They correspond to shear (or torsional) deformations depending on the orientation of the facets (typically at 45°). These angular distortions are still to be measured by interferometers during the passage of a gravitational wave.

h_{0i};h_{j0}: These time–space components correspond to angular distortions generated both within the orbital plane and perpendicular to it by the Lense–Thirring effect—namely, frame dragging and the geodetic effect caused by a rotating massive body.

Note: We consider here various components of the metric perturbation. However, since they are expressed in different reference frames—such as the Earth's center for the Lense–Thirring effect, and the transverse-traceless (TT) gauge for gravitational waves—they cannot be directly compared within a single tensor without first being transformed into a common coordinate system. Nevertheless, the key point is that the active components of a general $h_{\mu\nu}$ tensor, once expressed within a unified and consistent framework encompassing all relevant phenomena, can be meaningfully associated with specific mechanical deformations. This allows for an analogy with a four-dimensional strain tensor of an elastic spacetime medium.

$$\begin{bmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{bmatrix} \rightarrow \begin{bmatrix} GravitationorGLense & Lense-Thirring & Lense-Thirring & Lense-Thirring \\ Lense-Thirring & GWLIGOorGLense & GW & NewGW? \\ Lense-Thirring & GW & GWLIGOorGLense & NewGW? \\ Lense-Thirring & NewGW? & NewGW? & NewGW?orGLense \end{bmatrix} \tag{32a}$$

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$$\begin{bmatrix} h_{tt} & h_{tx} & h_{ty} & h_{tz} \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} \\ h_{yt} & h_{yx} & h_{yy} & h_{yz} \\ h_{zt} & h_{zx} & h_{zy} & h_{zz} \end{bmatrix} \rightarrow 2 \times \begin{bmatrix} \varepsilon_{tt} & \frac{1}{2}\gamma_{tx} & \frac{1}{2}\gamma_{ty} & \frac{1}{2}\gamma_{tz} \\ \frac{1}{2}\gamma_{xt} & \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yt} & \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zt} & \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{bmatrix} \tag{32b}$$

Physics and Mechanical Analogy: Elastic Medium Behavior in Weak Gravitational Fields

6 Consequences of Analogy—Identification of Components not yet Associated with Physical Phenomena in General Relativity and Possible New Deformations, Degrees of Freedom of Space, Time and Polarization

As demonstrated in publication [28], the components h_{ij}, by analogy with the presumed distortion tensor associated with torsion of space caused by the rotation of two stars orbiting each other, correspond to angular movements in the planes of the interferometer arms. These angular deformations have yet to be measured and are expected to be detectable with next-generation interferometers such as LISA.

The components h_{0z}, h_{xz}, h_{yz} do not correspond to deformations that have been identified or measured within classical general relativity.

It is also important to remember that the degrees of freedom associated with gravitational waves deformations correspond physically to the polarizations of those waves. Thus, if the $h_{0z}, h_{xz}, h_{yz}, h_{zz}$ are linked to complementary polarizations, this would imply additional degrees of freedom in the mechanical analogy—specifically, components of angular deformation (for $h_{0z}, h_{xz}, h_{yz}, h_{zz}$) as well as elongation or contraction along the direction of wave propagation, as demonstrated in [10].

These complementary polarizations have not yet been measured and therefore remain speculative at this stage. It should be remembered that they do not emerge within the framework of classical general relativity but arise only in modified theories such as Einstein-Cartan gravity or related extensions.

Summarizing the theoretical context in which our work is situated, we will assume that both general relativity and continuum mechanics are based on the principle of general covariance, which, as B. Kolev aptly summarizes in [75], requires the introduction of three components:

- Lagrangian-type functionals $\mathcal{L}$, depending on the metric g defined on the 4-dimensional universe manifold M, and depending on various fields Ψ;
- Tensors, such as the energy–momentum tensor, dependent on these fields;
- Field equations.

Our approach is therefore placed in the framework of the general covariance of field equations by the group of diffeomorphisms. Or expressed explicitly, a Lagrangian $\mathcal{L}(g, \Psi, ..)$ is general covariant if it verifies $\mathcal{L}(\boldsymbol{\varphi}^* g, \boldsymbol{\varphi}^* \Psi) = \mathcal{L}(g, \psi)$ for any diffeomorphism $\boldsymbol{\varphi}$. Note, still with B. Kolev, that the functional$\mathfrak{H}(g) = \int R_g Vol_g$, where R_g represents the scalar curvature, is general covariant. Its gradient L^2, the Einstein tensor $G(g)$ is also general covariant. It consequently verifies:$\forall \boldsymbol{\varphi}, G_{\boldsymbol{\varphi}^* g} = \boldsymbol{\varphi}^* G(g)$, and $div^g G(g) = 0$. An energy–momentum tensor $T(g, \Psi)$ verifying the Einstein equation $G(g) = T(g, \Psi)$ therefore verify the mechanics type equation:$divT = 0$. The rest of this work will therefore concern energy–momentum tensors verifying these two properties. The reader is referred to the work of J. M. Souriau [76, 77] and his successors already cited, for the definitions of the perfect matter field (as a section of a vector bundle) and the conformations, allowing the proposal of relativistic constitutive laws [78].

The problem of defining gravitational energy in general relativity remains open. While matter and non-gravitational fields are described by $T_{\mu\nu}$, the gravitational field itself resists a covariant and local description. The introduction of a vacuum energy tensor $t_{\mu\nu}$ is motivated by this gap, and aims to provide a mechanical representation of spacetime response, within the elastic analogy, beyond the constraints of pseudo-tensors. This analogy aligns with the spirit of quasi-local definitions [52] and extends it to a tensorial framework inspired by elasticity.

7 Possible Experimental Validation: Optimizing Gravitational Wave Detection to Reveal Additional Polarizations and Associated Strain and Metric Perturbations

To complete the description of the components of the metric perturbation tensor—particularly h_{0z},h_{xz}, h_{yz}, and their symmetric counterparts—several avenues can be explored.

- Complementary polarizations in the direction of gravitational wave propagation of that have not yet been experimentally observed [12] to [10, 58].

- Hypothetical pure shear stresses within the fabric of space–time, suggesting new forms of deformation beyond those currently accounted for in classical general relativity.

In any case, as Paul Langevin aptly noted, “tensors are always one step ahead of physicists”—the implication being that the currently unpopulated components of the metric perturbation tensor are unlikely to remain empty forever. It is highly plausible that physical phenomena not yet described by classical or modified general relativity remain to be discovered, and these could naturally fill in the missing components.

Next-generation instruments such as the LISA interferometer, the Einstein Telescope, and pulsar timing arrays are designed with the sensitivity required to detect these potential new gravitational wave polarizations or complementary spacetime distortions. In particular, they may also enable the detection of lateral angular deviations of the interferometer arms—an effect predicted by analogy with torsional deformations [10, 28].

8 Conclusion

We show that a simultaneous analysis of the different components of the metric perturbation tensor, $h_{\mu\nu}$ which corresponds—within the elastic analogy of gravitation – to twice the elastic strain tensor $\varepsilon_{\mu\nu}$, provides a deeper understanding of general relativity through the lens of the continuum mechanics in the weak-field regime.

The components h_{00}, h_{0i}, h_{j0}, $h_{ij}(x, y)$ enable the characterization of physical effect—such as linear elongations, compressions or angular distorsions—that are fully consistent, under the elastic analogy, with the behavior of a strain tensor describing space–time as an elastic medium in the weak-field approximation, provided that all relevant components of $h_{\mu\nu}$. are consistently expressed within the same reference frame. Einstein’s field equation can be interpreted as a generalized form of Hooke’s law in four equivalent spatial dimensions, since in the interval ds^2, time is scaled by the speed of light, effectively reconstructing a fourth dimension with the squared length term c^2dt^2. The speed of light acquires its value as the square root of the ratio between the effective Young’s modulus of the space–time fabric and the density of the equivalent medium. Since space–time is a dynamic entity, instantaneous and absolute deformations are not physically possible. Some deformations at a given point can be thought of as’in transit’—still propagating through the medium [79, 80]. This highlights the necessity of treating space–time as a genuinely four-dimensional mechanical system, rather than a merely three-dimensional one, to accurately describe its behavior [81, 82]. In classical general relativity, gravitational waves exhibit two polarization modes, corresponding to the two expressions of the strain tensor under pure shear—each oriented along facets rotated 45° with respect to the other. This could potentially give rise to complementary lateral displacements of the interferometer arms—undetectable by LIGO or VIRGO, but possibly observable with next-generation detectors such as the Einstein Telescope or LISA. The interval of special relativity can be interpreted as an elastic strain equation that incorporates both aspects: the spatial deformations that have already arrived at a given point and time, and those that are still in transit. In this sense, what we measure are local spatial deformations minus those that have not yet reached the observer—leading to differences in perception and quantification depending on the observer’s position. Space–time exhibits mechanical properties fundamentally different from those of conventional terrestrial materials [83]. Its effective Young's modulus varies significantly depending on the direction of deformation: estimates suggest values on the order of 10^{20} Pa for deformations perpendicular to the plane of propagation and 10^{40} Pa within the plane itself [29, 58]. A Poisson's ratio of 1 is observed in-plane. These anisotropic characteristics imply that the'engine'driving time differs from that governing spatial behavior—if one accepts the correlation between Young's modulus and the energy density of the medium.

This analysis of the perturbation tensor of the weak-field metric—based on well-documented experimental results in general relativity (such as gravitational waves GW150914 and GW170817, Gravity Probe B, and Earth- or Sun-based gravitation [84])—also demonstrates that the elastic analogy of gravitation plays a predictive role. It enables the consideration of additional space–time deformations, extremely small in magnitude (on the order of 10^{-21} for gravitational waves), which classical general relativity without geometric torsion appears to overlook. In particular, additional polarization modes seem necessary beyond the two canonical ones, involving components such as $h_{0z}, h_{xz}, h_{yz}, h_{zz}$ and their symmetric counterparts. These suggest a complementary set of strain tensor components, analogous not to a string model but rather to a Timoshenko bar under pure compression [30], incorporating shear and angular distortions.

According to our elastic analogy, the weak-field deformations of space–time, represented by the metric perturbation tensor $h_{\mu\nu}$ can be interpreted as those of a four-dimensional elastic medium governed by a strain tensor $\varepsilon_{\mu\nu}$.This tensor captures longitudinal deformations—such as elongation and compression—through its diagonal terms, and shear or angular distortions through its off-diagonal terms. Some components of the perturbation tensor have already been measured and experimentally verified – such as h_{00} for gravitation, h_{ij} for gravitational waves, and h_{0i}, h_{j0} for frame-dragging effects. Others, however, remain to be discovered, particularly those associated with the direction of gravitational wave propagation – namely $h_{0z}, h_{xz}, h_{yz}, h_{zz}$. Such is the central message of this article, assuming the validity of the proposed elastic analogy. By establishing a direct correspondence between the metric perturbation tensor and the elastic strain tensor—$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$— the elastic analogy in weak gravitational fields provides theoretical support for ongoing investigations into the existence of additional space–time polarizations, as suggested by several advanced extensions of general relativity [85–90].

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Analogy of spacetime as an elastic medium—Can we establish a thermal expansion coefficient of space from the cosmological constant Λ?

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This paper advances the state-of-the-art by extending the study of the analogy between the fabric of spacetime and elasticity. As no prior work exists about a potential spacetime thermal expansion coefficient α, we explore the analogy of general relativity with the theory of elasticity by considering the cosmological constant Λ as an additional space curvature of the structure of space due to a thermal gradient coming from the cosmic web and the cold vacuum and we propose $(\frac{\alpha_S \Delta T}{e})^2 = (\frac{1}{R_0})^2 = \Lambda$ with R_0 being the curvature radius of the space fabric. It follows from this analogy and from the supposed space model consisting of thin sheets of Planck thickness l_p curved by this thermal gradient ΔT a possible thermal expansion coefficient of the equivalent elastic medium modeling the space $\alpha_S = \frac{l_p \sqrt{\Lambda}}{\Delta T}$ of the order of $\alpha_{\text{space-QFT}} = 1.16 \times 10^{-6}\,\text{K}^{-1}$. As spacetime and not only space must be considered in general relativity, this paper also proposes an innovative approach which consists in introducing into the interval ds^2 of special relativity a temperature effect $T : d_s^2 = (1 \pm \alpha_t T)^2 c^2 dt^2 - (1 \pm \alpha_s T)^2 [dx^2 + dy^2 + dz^2]$ (entropy variations correlated with time laps, based on temperature variations affecting always physically the clocks) based on different thermal expansion coefficients for space and time with for the flow of time $t : \frac{ct}{cn\tau} = \frac{k_B t}{nh} \times \Delta T = \alpha_t \Delta T$. With $T \approx 10^6\,\text{K}$, $n = 1$, the associate time interval is $4.8 \times 10^{-17}\,\text{s}$ and $\alpha_t = 1.0 \times 10^{-6}\,K^{-1}$. The consequence of this hypothesis is that dark energy potentially becomes a thermal spacetime curvature $(\frac{\alpha_f T}{l_p})^2$ with f equal to s or t depending of the temperature, the thermal entropy variation of the universe, the Planck thickness and time, that increases since the Big bang, depending on thermal expansion coefficients for spacetime α_s and α_t as a function, respectively, of Λ, $\frac{k_B}{h} \times t$, in opposition to spacetime curvature gravity due to mass/energy density as described in general relativity.

Keywords: Spacetime fabric; general relativity; elasticity theory; quantum mechanics; expansion coefficient; cosmological constant; dark energy; time.

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1. Introduction

1.1. *Measured strains of space and analogy of the general relativity theory with the elasticity theory*

Einstein's theory of general relativity is over 100 years old and is now widely verified. Thus, spacetime is according to this theory a deformable elastic physical object. Gravitation is thus a manifestation of the geometric deformation of spacetime under the effect of the masses or energy density therein. The manifestations of the deformations of this spacetime are now known and measured with great precision. We can quote the apparent position variation of stars placed behind the sun during an eclipse measured by Edington,[1] the frame dragging of spacetime by angular distortion by the rotation of the earth (experiment prob B, Lense-Thirring and frame dragging effects),[2] the simultaneous lengthening and shortening deformations in each of the arms of Ligo/Virgo type interferometers during the passage of gravitational waves,[3,4] gravitational lenses or substantial masses located between a galaxy and our field sighting on earth distorts space to the point of making it appear to us in the shape of a circle (a bit like a candle placed behind the flat circle of a stemmed wine glass appears circular by its transparency reflection), and finally the expansion of the universe where the galaxies are "fixed" in a space which expands in an increasingly accelerated way characterized by Hubble's law. All these manifestations of the deformations of spacetime have led many physicists like Sakharov,[5] Synge,[6] Rayner,[7] Grot,[8] Vasilev and Fedorov,[9,10] Brown,[11] Tenev and Horstemeyer,[12] Millette[13] and many others as Damour in its conferences and books consider that the theory of general relativity is a kind of theory of the elasticity of a deformable elastic spacetime medium. We then speak of "elastic metric" or "elastic theory of gravitation". It is within the framework of this analogy that we place ourselves in this paper.

If by analogy, therefore, spacetime is considered as an equivalent four-dimensional deformable elastic medium, where a simplistic two-dimensional image is a heavy ball put on a rubber sheet deformed by this ball, they are two consequences. We will study it in the next two paragraphs.

1.2. *The mechanical characteristics of the equivalent elastic medium in the field of the analogy of the elasticity theory with the General Relativity—review of the state-of-the-art*

This equivalent elastic medium must therefore be characterized with the usual parameters linked to all elastic mediums and to the elasticity theory (Young's modulus $Y = E$, Poisson's ratio ν, density ρ, etc). Thus various authors have sought to establish an equivalent Young's modulus of the space noted Y_{space}. We can quote R.Weiss during his nobel prize speech about gravitationnal waves I quote "*In other words, it takes enormous amounts of energy to distort space. One way to say it is,* *the stifness (Young's modulus) of space at a distortion frequency of* 100 Hz *is* 10^{20} *larger than steel*".

Tenev and Horstemeyer[12] who propose by considering the spacetime made up of thin elastic sheets of the thickness of Planck the following formulation (1) giving $Y_{\text{space}} = 4.4 \times 10^{113}\,\text{N/m}^2$:

$$Y_{\text{space}} = \frac{6c^7}{2\pi\hbar G^2} = \frac{24}{l_p^2 \kappa}. \tag{1}$$

In this expression, c is the speed of light, G is the gravitational constant, $\hbar$ is the reduced Planck's constant, l_p is Planck's length (thicknesses of the thin sheets supposed to constitute the space fabric in Ref. 12) and κ is Einstein's gravitational constant ($\kappa = \frac{8\pi G}{c^4}$).

Beau proposes a space bulk modulus[14] and arrives at $K_{\text{space}} = 1.64 \times 10^{109}\,\text{N/m}^2$.

McDonald[15] proposes another expression of the Young's modulus (2) based on dimensional equations and obtain:

$$Y_{\text{space}} = \frac{c^2 f^2}{G}, \tag{2}$$

where f is the frequency of the gravitational wave. He thus obtains for a gravitational wave of 100 Hz $Y_{\text{space}} = 10^{20}\ \text{Y}_{\text{steel}}$ so, $Y_{\text{space}} = 4.5 \times 10^{31}$ Pa as R. Weiss.

Izabel in Ref. 16 arrives at an expression similar to Mc Donald by studying the elastic deformations in space dynamics located in the arms of the Ligo/Virgo laser interferometers and by studying the elastic deformations of a space cylinder twisted by the rotation of two black holes. He thus obtains the expression (3) of Young's modulus of space similar to McDonald at the factor π close:

$$Y_{\text{space}} = \frac{\pi f^2 c^2}{G}. \tag{3}$$

This leads by considering a density ρ from quantum field theory (hypothesis similar to Tenev and Horstemeyer[12]) at $Y_{\text{space}} = 1$ at 4.0×10^{113} Pa.

Melissinos[17] considers vibrating plates in the planes of the arms of the interferometers and arrives at the following expression (4) of Young's modulus of space:

$$Y_{\text{space}} < \frac{\pi c^2 f^2}{4G} \times \frac{c\Delta\tau}{\Delta z}. \tag{4}$$

In this expression, $\Delta\tau$ is the length of the gravitational wave burst and the total path length traversed by the GW is designated by Δz. The numerical application is done with $\Delta\tau \approx 1$ s and $\Delta z \approx 400$ Mpc and leads it to a next value of Young's modulus $Y_{\text{space}} < 2.5 \times 10^{-17}(c^2 f^2/G)$.

Finally, let us quote Hwang in Ref. 18 which by comparing the energy of gravitational waves for different frequencies (35 to 100 Hz and different GW) with the deformation energy of a spring modeling the lengthening and shortening of the arms of the interferometers arrives at values of the Young's modulus of space time $Y_{\text{space}} = 1.0 \times 10^{36}$ at 1.0×10^{54} Pa.

Another key parameter of any elastic medium is of course the coefficient of transverse deformation, the space Poisson's ratio ν. Again, many authors have proposed values. From the simultaneous deformations of the arms of the Ligo/Virgo interferometers (while one arm is shortened by a deformation by shortening of the order of -10^{-21} the other lengthens by the same amount of $+10^{-21}$) Tenev and Horstemeyer[12] propose $\nu = 1$ which presupposes a certain anisotropy of space which behaves like a kind of thousand leaves during the passage of a gravitational wave, each plane deforming successively according to the polarizations A^{+} and $A^{\times}$. The Poisson's ratio being close to zero in the direction of propagation of the wave and being equal to 1 in the plane perpendicular to the direction of propagation.

Izabel in Ref. 16 arrives at the same conclusion on this Poisson's ratio.

Concerning the equivalent density of the medium space ρ, Sakharov[5] shows that quantum considerations of space (quantum field theory) make it possible to go back to an elastic theory of space (5). We quote it below:

"In Einstein's theory of gravitation, it is postulated that the action of spacetime depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \tag{5}$$

The presence of action (1) results in a "metric elasticity" of space, that is, the generalized forces that oppose the curve of space."

Tenev and Horstemeyer[12] and Izabel[16] also follow this path which leads them by considering the minimum nonzero energy of the vacuum to the following expression (6):

$$\rho = \frac{Y_{\text{space}}}{4c^2} = 1.3 \times 10^{96}\,\frac{\text{kg}}{\text{m}^3}. \tag{6}$$

Millette in Ref. 13 (formula (19.36)) proposes the following expression (7) for the density of space:

$$\bar{\rho}_0 = \frac{32c^5}{\hbar G^2} = 1.7 \times \frac{10^{98}\,\text{kg}}{\text{m}^3}. \tag{7}$$

Finally concerning the shear modulus of the middle space Millette always in Ref. 13 (formulae (19.14) and (19.22)) proposes the following expression (8a):

$$\bar{\mu}_0 = \mu = G = \frac{Y}{2(1+\nu)} = \frac{32c^7}{\hbar G^2}1.5 \times 10^{115}\,\text{N/m}^2. \tag{8a}$$

For bulk modulus, (formula (19.21) of Ref. 13) the following equation can be seen:

$$\bar{\kappa}_0 = K = \frac{\bar{\mu}_0}{32} = \frac{c^7}{\hbar G^2} = \lambda + \frac{2}{3}\mu = \frac{Y}{3(1-2\nu)} = 0.046875 \times 10^{115}\,\text{N/m}^2. \tag{8b}$$

That concludes this review of the characteristics of space following the analogy of it as an equivalent elastic medium. Two conclusions emerge from this state-of-the-art.

First, the mechanical parameters of the equivalent elastic material constituting in our analogy the comic fabric are not of the same magnitude as those found on Earth given the orders of magnitude extremely small of the strains of (10^{-21}), extremely large of Young's modulus (10^{113} Pa), the Poisson's ratio outside the usual standards ($\nu = 1$), the density ρ of the medium also extremly large (10^{98} kg/m^3) and its associated anisotropy linked to Poisson's ratio if it is 1 in a direction perpendicular to the propagation of the wave.

Second, and this is partly the subject of this paper, no publication to our knowledge deals with a possible expansion coefficient α of this space medium. We are therefore going to determine an original and innovative approach in this paper to try to propose a mechanical expression and a numerical value of this possible expansion coefficient α of the equivalent spatial fabric on the one hand and to study the consequences on time on the other hand.

1.3. *Analogy about the behavior law of the space following the general relativity with the Hooke's law in elasticity without and with cosmological constant* $\boldsymbol{\Lambda}$*—thermal gradient implication about the different curvatures that have to be considered*

The second aspect about this analogy is the law governing this equivalent elastic medium, this very special space fabric. Indeed, it is well known that Einstein's equation of general relativity without cosmological constant Λ relates the curvature of space $G_{\mu\nu}$ to the density of energy which deforms it $T_{\mu\nu}$ see Eq. (9).

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}. \tag{9}$$

In this expression $R_{\mu\nu}$ is the Ricci tensor resulting from the contraction of the Riemann tensor, R is the scalar curvature resulting from the contraction of the Ricci tensor and $T_{\mu\nu}$ the momentum energy tensor. $\mu\nu$ varying from 0 for time to 3 for the three dimensions of space.

Izabel showed in Ref. 16 that the general relativity equation in four dimensions presents an analogy with, respectively, the expressions in one and two dimensions of the curvature of beam (10), (11) and plate (12) in pure bending under two moments applied at each extremity (see Fig. 1) according to Timoshenko's strength of materials theory issued of the elasticity theory.

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right) = K\left(\frac{W_{\text{ext(total)}}}{L}\right). \tag{10}$$

In analogy with:

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}(T^{\mu\nu}) = -\kappa(T^{\mu\nu}). \tag{11}$$

In these expressions, R is the radius of curvature of a beam of span L, moment of inertia I and Young's modulus $E = Y$ associated with a work of the external forces W. K is the mechanical coupling constant between the curvature and the strain energy U of the beam which is equal to the work applied external forces W.

Or in theory of the plates of thickness h following[19] with R_x, R_y, R_{xy}, the curvature radii of the plate in bending in the different directions, $\Delta x \cdot \Delta y \cdot t$ an elementary volume of the plate of thikness t, ν the Poisson's ratio and E Young's modulus of the material constituting the plate.

$$\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^{.2} + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] = \frac{24(1-\nu^2)}{Et^2} \times \frac{\Delta U}{t\Delta x \Delta y}. \tag{12}$$

Tenev and Horstemeyer[12] but also Damour in his book "if Einstien was told to me", showed this analogy from a tensor point of view in terms of Hooke's law (13) and (14) with σ^{kl} the stress tensor, ε^{ij} the strain tensor, Y is Young's modulus and g^{kl} a metric.

$$\sigma^{kl} = \frac{Y}{1+\nu}\left(\frac{\nu}{1-2\nu}g^{ij}g^{kl} + g^{ik}g^{il}\right)\varepsilon_{ij}, \tag{13}$$

$$T_{\mu\nu} = \frac{1}{\kappa}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right). \tag{14}$$

In the space fabric model of Tenev and Horstemeyer,[12] space is assumed to be made up of ultra-thin sheets of Planck thickness, having an elastic behavior. It is in this space model that we will place ourselves in the rest of this publication.

But the tensorial equation of Einstein (9) can be also written by considering this time the cosmological constant Λ as a materialization of a certain repulsive dark energy. It is written as follows (see formula (15) and (16)):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} - \Lambda g_{\mu\nu}. \tag{15}$$

Or by factoring Einstein's constant κ:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{c^4\Lambda}{8\pi G}g_{\mu\nu}\right]. \tag{16}$$

But Einstein's tensorial equation (9) considering the cosmological constant Λ can be also written as an additional curvature present in all space (frame of this paper). It is written as follows formula (17):

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}. \tag{17}$$

If we follow completely the analogy of the beam or the plate in elasticity describes before it exists not one but two sources of curvature (see Chap. 3): one under the applied masses developed in Refs. 12 and 16 and one under the temperature gradient (so with 0 mass), hence our idea of associating the second with the cosmological constant if our analogy is correct, the first one having already been partially demonstrated in Ref. 16.

So, we come to the subject of this publication. In the same way that we have shown that Einstein's constant κ, by analogy with an elastic medium made of thin sheets of thiknesses l_p, (plate theory) could be expressed in terms of mechanical constants[12,16] see expression (12) ($\frac{24(1-\nu^2)}{Et^2} \to \kappa \to Y = \frac{24}{l_p^2\kappa}$), the cosmological constant Λ which is generally associated with a dark energy[20] opposed to gravitation (15) and (16), can it not also be expressed in terms of mechanical parameters as an additional curvature present in all space (17) due at a thermal gradient applied at these thin sheets of thicknesses l_p?

This constant Λ generally associated with an expansion of space[21] "Dark energy as a curvature of spacetime induced by quantum vacuum fluctuations", can it not be correlated with a hidden mechanical behavior of space via a parameter missing from the state-of-the-art cited above in the analogy of space as a elastic medium, namely a thermal expansion coefficient α of it? Does not the phenomenal temperature difference between the cosmic web that fills the entire universe and the icy vacuum at $-2.73\,\mathrm{K}$ constitute such a thermal gradient sufficient to impose on the sheets[12] constituting the space of a thickness of Planck in the framework of our model an additional source curvature of this cosmological constant? It is these hypothesis that we will study in this paper, thus placing ourselves in the continuity of the publications and theoretical models of Tenev and Horstemeyer[12] and Izabel.[16]

2. Methods

The following methodology has been implemented to estimate, within the framework of the analogy of space as an equivalent elastic medium, the value of a possible thermal expansion coefficient α of space in line with the cosmology constant Λ associated with an additional thermal space curvature in the framework of a Planck's thickness sheet space model as considered by Tenev and Horstemeyer:[12]

(1) Within the framework of the analogy of an elastic medium to model the deformations of space in the presence of mass energy, restructure the simplified and appropriate mechanical models among those already developed to evaluate a possible generalized thermal curvature of this one.
(2) Search for certain scientific data that can feed this mechanical model of curved space under the effect of a thermal gradient between the cosmic web and the space vacuum.
(3) Consider the cosmological constant Λ not as a gravitationally repulsive dark energy (if placed to the right of Eqs. (15) and (16) but as an additional curvature present in all space (if placed to the left of Eq. (17)) and analysis of the consequences of this approach.
(4) Extract from the mechanical model of curvature of space under thermal gradient and from the scientific data available in connection with this model by

considering the cosmological constant Λ, a possible thermal expansion coefficient α of the space medium.

(5) Discuss the representativeness of this model by varying the assumptions (values of Λ) and seeing the consequences on the value of the thermal expansion coefficient of the associated elastic space fabric.

(6) Discuss the additional verifications necessary to confirm this approach.

(7) Investigate by the state-of-the-art analysis, the potential effect of temperature on time via entropy variation and its profound implication on the interval $\mathbf{ds^2}$ in special relativity, value of Λ in general relativity expressed in terms of curvature of space and time.

(8) Presentation of the possible consequences of this model concerning the interpretation of dark energy.

(9) Proposition of a testing way to verify this theory.

3. Search for a Simplified Mechanical Model of Space Allowing to Represent a Thermal Curvature of this One

3.1. *Analogy of the thermal curvature of an elastic medium with a simplified approach of resistance of materials in one dimension—beam under thermal gradient*

According to the strength of materials theory, two phenomena and only two can create a curvature of a physical object (beam, plate, shell, 3 dimensions structure)

- masses supported by the object,
- a thermal gradient applied to the object.

We studied the first type of curvature in Ref. 16 and showed that masses placed on a beam create curvature (18). Thus we have shown in Ref. 16 that the curvature of a beam in pure bending (solicited by two moments M at each end) (18) takes a form similar to Einstein's equation from the point of view of the analogy of space as an elastic medium in the form curvature = K × a linear strain energy density U/L:

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right), \tag{18}$$

where U is the strain energy of the beam, L is the span, e is the height of the beam, E is Young's modulus of the material, $I = \frac{be^3}{12}$, is the moment of inertia, $(b \times e)$ is the section and R is the curvature radius (see Fig. 1).

The second type of curvature, we study it in this paper, is the curvature created by the difference in temperature between the two extreme fibers of the beam. The beam is then subjected to a thermal gradient ΔT which leads to an elongation of the heated fibers and a shortening of the cooled fibers. These differences in elongation create a curvature without internal forces of the beam if this one is not constrained in displacement somewhere along its surface.

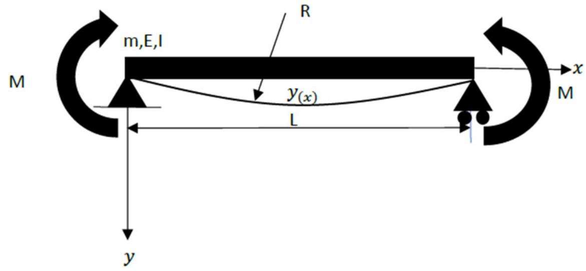

Fig. 1. Definition of a Timoshenko's beam in pure bending.

Damour tells us in his various conferences and publications that curvature in the Einstein sense has the dimension of an angle divided by a surface (see also the first definition of curvature and differential geometry in Gauss work[22]). We will show that is indeed the case of a beam loaded by a thermal gradient (temperature difference T between the lower and upper fibers of the beam $\Delta_T = T_{\text{ext}} - T_{\text{int}}$) with T_{ext} and T_{int} the temperature of each side of the sheet considered.

Let us now consider the case of an identical beam in pure bending undergoing a thermal gradient (see Fig. 2):

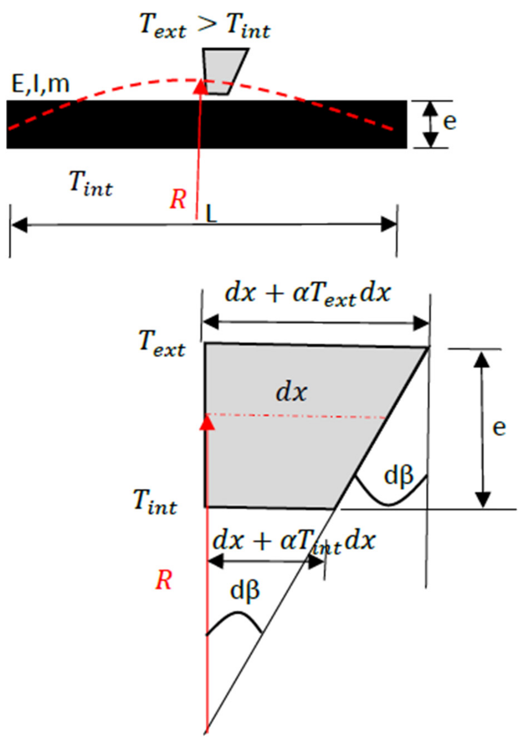

Fig. 2. Strain of a beam in bending under a thermal gradient ΔT.

The strain energy U (19) is always with M a bending moment for a beam of span L, thickness e and rigidity EI:

$$U = \frac{1}{2}\int_0^L \frac{M^2}{EI}dx. \tag{19}$$

Figure 2 and formula (20) allow us to write the following geometric relations between the angle β (radiant) and the curvature $(1/R)$ of the beam with e the height of the beam, α its expansion coefficient and ΔT the thermal gradient applied to this beam:

$$\begin{aligned}\tan(d\beta) &= \frac{dx}{R} = \frac{(dx + \alpha T_{\text{ext}}dx) - (dx + \alpha T_{\text{int}}dx)}{e} \\ &= \frac{(\alpha T_{\text{ext}} - \alpha T_{\text{int}})dx}{e} = \frac{\alpha \Delta T dx}{e}.\end{aligned} \tag{20}$$

Given Fig. 2, from the relation between the curvature of a beam under a thermal gradient and the second derivative of its displacement equation $y(x)$, we obtain the expression (21):

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R} = \frac{\tan(d\beta)}{dx} = \frac{d\beta}{dx} = \frac{\alpha\Delta T}{e}. \tag{21}$$

Considering the new expression of the moment M from (21), we have

$$M = EI\frac{d\beta}{dx}. \tag{22}$$

By replacing the moment M by its expression above (22) in the expression of the strain energy U of the beam (19), we then obtain (23):

$$U = \frac{1}{2}\int_0^L \frac{\left(EI\frac{d\beta}{dx}\right)^2}{EI}dx. \tag{23}$$

So, after simplification (EI = constant allong the beam) we obtain (24):

$$U = \frac{EI}{2}\left(\frac{d\beta}{dx}\right)^2 L. \tag{24}$$

After some mathematical calculations, the final result is Eq. (25):

$$\boldsymbol{\left(\frac{d\beta}{dx}\right)^2 = \frac{2}{EI}\frac{U}{L}}. \tag{25}$$

That can be compared with the expression recalled above in the case of the beam in pure bending (18).

We therefore have an angle divided by an area as the definition of the curvature $(1/R)$ squared of the beam (26).

$$\boldsymbol{\left(\frac{d\beta}{dx}\right)^2 = \frac{d\beta^2}{dx \times dx} = \left(\frac{1}{R}\right)^2}. \tag{26}$$

This corroborates the expression of Damour from the differential geometry of Gauss developed by Riemann[22] and given again in the formula (27):

$$\text{Curvature} = \frac{\alpha + \beta + \gamma - 180°}{\text{Area}}. \tag{27}$$

The strain energy of a beam of span L, rigidity EI=YI for a constant thermal gradient ΔT is given by formula (28):

$$U_{\Delta T} = \frac{EI}{2}\left(\frac{d\beta}{dx}\right)^2 L = \frac{EI}{2}\left(\frac{\alpha\Delta T}{e}\right)^2 L. \tag{28}$$

So :

$$\left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}} = \boldsymbol{\frac{2}{EI}\frac{U_{\Delta T}}{L}}. \tag{29}$$

As our analogy is based on the elasticity theory, we can superimpose the load cases and thus superimpose the case of the beam in pure bending (due to the two moments M due at masses at each extremity of the beam, see Fig. 1) with the case of the beam under thermal gradient uniform constant ΔT (see Fig. 2), we then obtain the formula (30) of the generalized curvature of a beam under load and thermal gradient with $W_{\text{ext,total}(M+\Delta T)}$ the total external work of the external forces under moment and thermal gradient:

$$\frac{1}{R^2} + \left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}} = \frac{2}{EI}\left(\frac{U_M}{L}\right) + \boldsymbol{\frac{2}{EI}}\left(\boldsymbol{\frac{U_{\Delta T}}{L}}\right) = \frac{2}{EI}\left(\frac{W_{\text{ext,total}(M+\Delta T)}}{L}\right). \tag{30}$$

This expression is compatible from the point of view of the elastic analogy with the expression (17) of Einstein's field equation with cosmological constant Λ according to the correspondences (31) and (32) if one assumes (hypothesis of this article) that the cosmological constant Λ is by analogy correlated with a thermal gradient applied in all space:

- For the masses curvature the analogy between general relativity and strength of material/elasticity is:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \rightarrow \frac{1}{R^2}. \tag{31}$$

- For the thermal curvature the analogy between general relativity and strength of material/elasticity is:

$$\Lambda g_{\mu\nu} \rightarrow \left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}}. \tag{32}$$

About the formula (31), indeed, for memory, it can be proved that the Ricci tensor for a classical 2sphere is $2/R^2$ (see book what is space time made of? of Izabel).

3.2. *Analogy of space as an elastic medium with the thermal curvature of a thin plate associated according to Timoshenko's theory*

Considering space as a fabric made up of thin sheets has already been explored as we have said above by many authors. Let us quote Melissinos,[17] Tenev and Horstemeyer,[12] Izabel,[16] and Perko.[23]

It is therefore quite natural that we take up this hypothesis on the structure of the fabric of space.

Moreover, if we take a piece of the universe locally, its surface will be considered almost flat according to the value of the cosmological curvature $k = 0$ obtained by measurements from the Planck satellite.[24] In these publications, it is proven that the joint constraint with BAO measurements on space curvature is consistent with a flat universe

$$\Omega k = -kc^2(r_0 H_0)^{-2} = -0.0010 + 0.0018/-0.0019$$

As a reminder, the space expansion scale factor is written $R_{(t)} = a_{(t)} = \frac{r_{(t)}}{r_0}$. r_0 is the radius of reference and $r_{(t)}$,the radius of the 3 sphere in the metric of Friedmann–Lemaitre–Robertson–Walker.

The Hubble constant squared ($1/\mathrm{s}^2$) is :$H_0^2 = \frac{8\pi G}{3}\rho_c$ where ρ_c is the critical density of the medium.

So, the curvature k (unit $1/\mathrm{m}^2$) of the univers can be considered as flat (33).

$$\Omega_k = -kc^2(r_0 H_0)^{-2} = -\frac{3kc^2}{8\pi G\rho_C a^2}, \tag{33}$$

where Ω_k is therefore the dimensionless curvature parameter of space (34):

$$\frac{\frac{1}{m^2} \times \frac{m^2}{s^2}}{\frac{m^3}{kgs^2} \times \frac{kg}{m^3} \times 1} = 1. \tag{34}$$

It is therefore not absurd to consider space as an infinitely long thin plates superposition given the very large radius of curvature of the universe associated with its gigantic size.[25]

In this case the curvature of a thin plate under a thermal gradient is well known and is given in Timoshenko in his book[19] in Chap. 14, formula (50). We give the expression below (formula (35)) similar to that obtained in the case of a beam :

$$\frac{\alpha\Delta T}{e} = \frac{1}{R}. \tag{35a}$$

Or squared to stay consistent with the previous paragraph regarding beam theory (35b):

$$\left(\frac{\alpha\Delta T}{e}\right)^2 = \left(\frac{1}{R}\right)^2. \tag{35b}$$

In these two expressions, the curvature squared of the plate $(\frac{1}{R})^2$ is therefore linked to the thickness of the considered plate e, to the thermal expansion coefficient α associated with the elastic material and to the thermal temperature gradient ΔT between the extreme fibers of the plate.

It is therefore this model that we will consider later in the continuity of the authors cited above.[12,16] We must therefore establish the different parameters involved in this model. Namely, what thickness e of the sheets? what intensity of the thermal gradient ΔT? what value of the curvature $1/R$? This is what we will study in the next section.

4. Search for Certain Scientific Data that can Feed this Mechanical Model of Thermal Curvature of the Space Fabric

4.1. *What plate thickness consider?*

We know from Sakharov[5] that the potential quantum nature of vacuum can generate an elastic metric of space. Tenev and Horstemeyer in Ref. 12 consider Planck thickness sheets. Izabel in Ref. 16 manages to find Young's moduli of the different authors by considering also a Planck length. Millette[13] does the same.

Consequently, we will consider as in Ref. 12 within the framework of this publication, a thickness e of plate or fiber of the elastic fabric of space equal to l_p the length of Planck (36).

$$e = e_p = l_p = \sqrt{\frac{\hbar G}{c^3}}. \tag{36}$$

Remark 1. This model of elastic fabric of thickness l_p has been studied in the thesis of Tenev to modelize the sun gravity. There is a perfect accordance between theory and model.[12,26]

4.2. *Which thermal gradient consider?*

New data show a temperature gradient between the absolute vacuum at 2.73 K and the cosmic web see Ref. 27, "The Cosmic Thermal History Probed by Sunyaev–Zeldovich Effect Tomography". In this paper, we can read:

"We estimate Te, the density-weighted electron temperature of the universe, which goes from 7×10^5 K at $z = 1$ to 2×10^6 K today" The cosmic thermal history probed by Sunyaev-Zeldovich effect tomography YI-KUAN CHIANG , RYU MAKIYA, BRICE MÉNARD ET EIICHIRO KOMATSU".[27]

In this paper, the authors therefore highlight a certain thermal gradient between the cold zones of the universe and the very hot zones at the level of the cosmic web. So, this recent paper suggests that the average temperature of the gas present in the large structures of the observable Universe has been multiplied by 10 during the last 10 billion years to reach about two million Kelvin today.

We will therefore retain this hypothesis: $\Delta T = 2,000,000°K$.

4.3. *What curvature of space associated with this thermal gradient consider?*

Peebles in his paper[20] review the different approaches for linking the cosmological constant and dark energy.

Santos in Ref. 21 study the dark energy induced by the curvature of spacetime by quantum vacuum fluctuations.

On the basis of these two papers in particular, we will therefore postulate that the cosmological constant is the source of a curvature of space linked to a thermal gradient acting on the thin sheets of space of quantum thickness equal to the length of Planck.

To be consistent with the approaches of Sakharov[5] and Tenev and Horstemeyer,[12] we consider in this study the cosmological constant resulting from vacuum fluctuations (quantum field theory). We will see in Chap. 6 how to explore all the scientific options, what gives other values of the cosmological constant Λ- resulting from cosmological observations.

We make this hypothesis based on quantum field theory taking into account all of the previous paragraphs:

(1) the analogy of the space fabric as an elastic medium works well with the general relativity equation for mass/energy,[5–17]
(2) the elastic analogy suggests two possible curvatures and only two: one resulting from the masses/energy acting in the medium and one resulting from the thermal gradient acting on the medium,[16]
(3) there is obviously a thermal gradient present in space following Ref. 27,
(4) many authors[12,16,23] and measurements of gravitational waves[3,4] suggest the presence of plane deformations of elastic media (spatial fabric),
(5) Sakharov[5] showed that general relativity is correlated with an elastic metric resulting from quantum fluctuation of space.

4.4. *Final model retained to estimate a coefficient of thermal expansion of the fabric of the equivalent space*

Considering the previous paragraphs, we therefore arrive at the simplified model given in Fig. 3.

Thus, the almost flat thin sheets of space ($k = 0, T$ of space curvature tending towards infinity[12,24,25]) of Planck's thickness (l_p) according to the reasoning by Sakharov[5] and Tenev and Horstemeyer[12] undergo a thermal gradient resulting from the differences in average temperature between the cosmic web and the space vacuum.[27] This causes their curvature. It is this curvature that the cosmological constant Λ[20,21] can represent. By placing ourselves in the analogy of space functioning as a medium, an elastic fabric, the mechanics of continuous media by

Timoshenko's plate theory[19] allows us to model its behavior in a simplified way and to extract a thermal expansion coefficient α of the space fabric.

Remark 2. The thermal conductivity λ of the vacuum is by definition $0\,\mathrm{W/m^2 \cdot K}$. This allows us to have a thermal gradient between the two faces of thickness $e = l_p$.

5. Consequence of Considering the Cosmological Constant as a Generalized Thermal Curvature

5.1. *Determination of the space fabric expansion coefficient*

From Timoshenko's expression (36) (see also Ref. 5) of a spatial plate curvature ($k = 0$) see Refs. 24 and 25 in pure bending loaded by a thermal gradient ΔT[27] and in considering the cosmological constant Λ as an additional curvature (and not a dark energy) associated with the thermal gradient acting in space[20,21] we postulate Eq. (37):

$$\left(\frac{\alpha_S \Delta T}{e}\right)^2 = \left(\frac{1}{R_0}\right)^2 = \Lambda, \tag{37}$$

where R_0 is the curvature radius of one of the thin sheet supposed constitute the space fabric[12] (see Fig. 3).

We deduce from the above expression the thermal expansion coefficient α_S of the spatial fabric:

$$\alpha_S = \frac{\boldsymbol{e}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}. \tag{38}$$

By definition of the comological constant with ρ_{vacuum} the vacuum density and c the speed of light we obtain (39):

$$\frac{c^4 \Lambda}{8\pi G} = \rho_{\text{vacuum}} c^2. \tag{39}$$

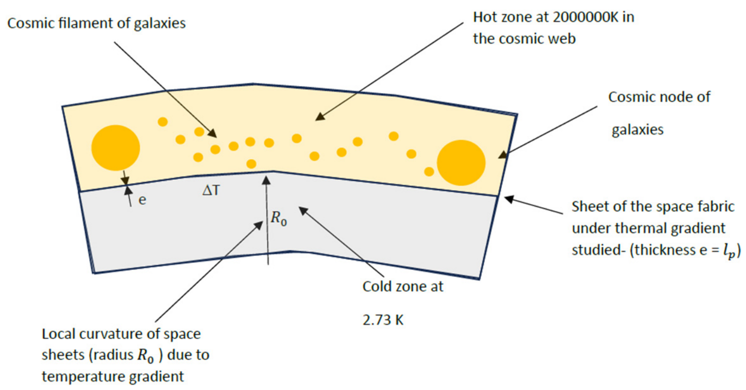

Fig. 3. Simplified mechanical model of elastic space undergoing a curvature by a thermal gradient between a very hot zone and a very cold zone of the universe.

So, after some mathematic calculations (40):

$$\Lambda = \frac{8\pi G\rho_{\text{vacuum}}}{c^2}. \quad (40)$$

By transferring this expression to the formula of the thermal expansion coefficient of the space fabric (38), we obtain the expression for the space thermal expansion coefficient α_S from the density of the vacuum (41):

$$\alpha_S = \frac{\boldsymbol{e}\sqrt{\frac{\boldsymbol{8\pi G\rho_{\text{vacuum}}}}{\boldsymbol{c^2}}}}{\boldsymbol{\Delta T}} = \frac{\boldsymbol{e}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}. \quad (41)$$

If we consider a space plate thickness e of Planck dimension (35) like Tenev and Horstemeyer[12] and the associated vacuum energy, we obtain (42) a first approximation of this thermal expansion coefficient α_S.

$$\alpha_S = \frac{\sqrt{\frac{hG}{2\pi c^3}}\sqrt{\frac{8\pi G\rho_{\text{vacuum,QFT}}}{c^2}}}{\Delta T}. \quad (42)$$

So, all calculations performed give (43):

$$\alpha_S = \frac{\boldsymbol{2G}\sqrt{\frac{\boldsymbol{h\rho_{\text{vacuum,QFT}}}}{\boldsymbol{c^5}}}}{\boldsymbol{\Delta T}}, \quad (43)$$

where $\hbar$ is the reduced Planck's constant $(h/2\pi)$, ρ is the quantum vacuum density according to quantum field theory, G is the gravitational constant, c is the speed of light and ΔT is the thermal gradient applied to these sheets of space.

Consider the expression of the thermal expansion coefficient α_S:

We check that the dimensional equation (44) is satisfied:

$$\alpha_S = \frac{\frac{m^3}{kgs^2}\sqrt{\frac{\frac{kgm^2}{s}\times\frac{kg}{m^3}}{\frac{m^5}{s^5}}}}{K} = K^{-1}. \quad (44)$$

Numerical application using the constants of physics and using the same assumptions as Millette,[13] and Tenev and Horstemeyer[12] gives:

$$G = 6.6743015 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2,$$

$$h = 6.62607004 \times 10^{-34}\,\text{m}^2\cdot\text{kg/s},$$

$$\rho_{\text{vacuum,QFT}} = 1.11 \times 10^{96}\,\text{kg/m}^3,$$

$$\Delta T = 2000000\,K,$$

$$c = 299,792,458\,\text{m/s}.$$

We obtain for the thermal expansion coefficient of the space fabric:

$$\alpha_{S,QFT} = 1.16317 \times 10^{-6}\,\text{K}^{-1}.$$

For memory, the expansion coefficient of steel is worth $12.0 \times 10^{-6}\,\text{K}^{-1}$.

We therefore obtain a result that seems realistic since the space is rather rigid if we refer to the value of κ which represents the flexibility (1/rigidity) of the space and which is equal to $2.0766\times10^{-43}\,\mathrm{N}^{-1}$.

We find this result directly from Eq. (41) with the following data by considering Λ from quantum field theory:

$$\Lambda_{\mathrm{QFT}} = 2.0717\times10^{70}\,\mathrm{m}^{-2},$$
$$e = l_p = 1.61626\times10^{-35}\,\mathrm{m},$$
$$\Delta T = 2000000\,\mathrm{K}.$$

6. Discussion of the Representativeness of the Mechanical Model of Thermal Curvature of Space

It is well known that one of the greatest challenges of physics today is this problem of the cosmological constant Λ which, depending on the hypothesis adopted to establish it (vacuum energy resulting from quantum fluctuations in the ground state provided by quantum field theory or cosmological observations via the Planck satellite in particular) leads to a ratio of 10^{120} between the two values of Λ! We do not claim in this paper to solve this problem, simply from a scientific point of view it is important to explore what these two values of the cosmological constant imply on the coefficient of potential thermal expansion of the fabric of space.

In this case, the application of formula (41) with the following numerical values:

$$l_p = e = 1.61626\times10^{-35}\,\mathrm{m},$$
$$\Lambda = 1.088\times10^{-52}\,\mathrm{m}^{-2},$$
$$\Delta T = 2000000\,\mathrm{K}.$$

It leads to an extremely very small coefficient of thermal expansion...

$$\alpha_{S,\mathrm{astrophysics}} = 8.42936\times10^{-68}\,\mathrm{K}^{-1}.$$

7. Discussion of the additional Verifications Necessary to Validate or not this Model

To really validate our model, it would of course be necessary to solve this problem of the possible variation of values of the cosmological constant Λ. A model of the universe reproducing the construction of the cosmic web from general relativity and observations exists, however these models do not integrate any mechanical behavior such as the analogy of space as a deformable elastic medium.

It would undoubtedly be necessary to model the space, containing and contained in a sheet structure, to set up the cosmic web and impose the thermal gradient between the hot and cold zones to check whether or not a curvature of thermal origin appears on average. On the entire universe, find out what its intensity is and therefore take advantage of it to evaluate the true value of the cosmological constant and therefore the true value of the thermal expansion coefficient of the space fabric.

8. Consequence of a Thermal Expansion Coefficient on Time for Special Relativity, General Relativity and Quantum Gravity

8.1. *Necessary transition from space to spacetime in the consideration of temperature*

We have seen in the previous chapters that the analogy with an elastic continuous medium proposes a kind of curvature of zero mass of space related to a temperature effect. But since general relativity is aimed at the curvature of spacetime, and not only space, it is necessary to specify how this temperature effect could act on time.

8.2. *Effect of temperature on time*

8.2.1. *Generalities on clocks and their physical sensitivities to temperature*

It is well known that temperature variations can affect clocks in different ways, especially in mechanical and electronic clocks.

In mechanical clocks, metal parts can expand or contract depending on temperature. This can result in minimal variations in component dimensions, which in turn can affect the accuracy of movement. The materials used in the mechanisms may react differently to temperature changes, which can cause shifts in the clock rhythm. So we have a spatial effect.

In electronic clocks, temperature can influence the frequency of quartz oscillators used to measure time. Quartz crystals have an electromechanical resonance that is sensitive to temperature. Temperature variations can slightly alter the frequency at which the quartz oscillates, which can lead to deviations in the time count. If the equivalent of quartz is very small, see the case of atomic clocks where it is the frequency of an atom that intervenes, we have a quantum effect.

When it comes to electronic processors, temperature variations can also impact their performance. When a processor heats up, its components can expand slightly, which could potentially affect the speed of electrical signals through the circuit. However, modern processor designs usually incorporate temperature control mechanisms to minimize this effect.

So, by thinking about it, the temperature actually influences time by playing on dimensional variations of space or on the vibrations of quartz, so more deeply at the quantum level via $E = h\nu$.

So, to measure time, we need clocks which, whatever the physics and the technology used, will be systematically sensitive to the temperature of the environment in which they are immersed. This is the conclusion of this paragraph.

The following question then arises: while it is clear that temperature affects time measuring devices, does it also fundamentally affect time, which is an abstract entity in physics independent from any measuring instruments?

We will see in the following section by reviewing the state-of-the-art on this question, how different authors have approached this question by focusing on time as a change in entropy, that is to say at a increased disorder, which in turn is linked to a temperature effect.

8.2.2. *State-of-the-art on the link between time, a clock, the entropy variation and the temperature in a black body*

In Refs. 28 and 29, the authors study what is a clock? what kind of measurements can be done? how temperature can affect the time fundamentaly and they explain how time passes according to the increase in entropy and therefore temperature.

In these papers, it is clear that the time itself cannot be defined without a physical process to measure it. I quote[28]:

"Einstein pointed out that the definition of time must be based on the clock measure, but it has been pointed out the need of physical meaning for time coordinates. In the analysis of clocks, a timeclock relation has been introduced; it states that there is a conceptually necessary relation between time and a physical process which functions as the core of a clock. This time-clock relation implies that a physical process must exist as the basis of a clock. Time and the physical process cannot be defined independently. The time-clock relation involves also a reference to a physical process in conformity with physical laws. Consequently, a well-defined use of time requires that time has aphysical basis."

So, from this reflection, time is inseparable from a physical process and as in Sec. 8.2.1 we recalled that all physical processes are subject to temperature effects, so time should also be affected by temperature in addition to what Einstein showed in special relativity, namely that time depends on the speed of the observer.

Therefore, the authors in Ref. 28 and formula (1) in Ref. 29 introduce the definition of the time interval in relation to the local entropy S and the local entropy rate $\dot{S}$, as follows:

$$t = \frac{S}{\dot{S}}. \tag{45}$$

From the entropy variation the authors obtain in Ref. 28 formula (12) and Ref. 29 a variation τ function of the frequency ν of the physical object used to construct a physical clock as required by Einstein. However this time this clock depends on the temperature T, the Planck constant h and the Botzman constant k_B. They obtain then the fundamental expression (46):

$$\tau_{(T)} = \frac{1}{\nu} = \frac{h}{k_B} \times \frac{1}{T} = \frac{4.799243 \times 10^{-11}[\mathrm{K}\cdot\mathrm{s}]}{T[K]}. \tag{46}$$

An finally according to Ref. 28, formula (13) and Ref. 29 in the field of a black body, there is a relation (47) giving the variation of the flow of time t according to the temperature T, the periodic time of oscillation τ and n a natural number.

$$t_{(T)} = n\tau = n\frac{h}{k_B} \times \frac{1}{T} = n \cdot \frac{4.799243 \times 10^{-11}[K.s]}{T} \approx n\frac{4.80 \times 10^{-11}[Ks]}{T}. \quad (47)$$

Remark 3. In Ref. 28, Table 1, we can see that the effect of the temperature on time is very small.

For $T = 10^6$ K, n =1 the time variation interval linked to this temperature is 4.8×10^{-17} s.

Remark 4. If we take the inverse of the expression (47) and multiply by the time t each side of the equation we obtain (48) that allows to define a time-dependant thermal expansion coefficient:

$$\frac{t}{n\tau} = \frac{k_B t}{nh} \times T = \alpha_t T. \quad (48)$$

Thus, the term $\frac{k_B t}{nh}$ has the dimension of an expansion coefficient (K^{-1}) depending on time which will note α_t:

So, the time should be influenced at a quantum level function of the temperature T, the Planck constant h and the Boltzmann constant k_B and thus by the entropy of the medium S. The time is quantified ($n \times h$).

Thus, associated with $T = 1 \times 10^6$ K (see order of magnitude of the temperature of the cosmic web[27]), $n = 1$ and considering a time interval of 4.8×10^{-17} s (following Table 1 of Ref. 28), we obtain with (48) an evaluation of the amplitude of this time-dependent thermal expansion coefficient:

$$\alpha_t = 1.0{\times}10^{-6}\mathrm{K}^{-1}.$$

So, an order of magnitude for α_t similar to that which we obtained for α_s in the first part of our article based on the comological constant Λ as a value of the thermal curvature of the fabric of space.

Remark 5. In the Cosmic fabric model,[12] Tenev and Horstemeyer related time lapse to the speed of signal propagation within the fabric.

Thus the variation of the time lapse (49) is written as follows:

$$\frac{d\tau}{dt} = \frac{1}{(1 + \varepsilon^{3D})}. \quad (49)$$

With the formula (50), we have

$$\varepsilon^{3D}_{,kk} = c^2\kappa\rho, \quad (50)$$

where $\varepsilon^{3D}_{,kk} \equiv \nabla^2\varepsilon^{3D}$ is the Laplacian of the volumetric strain, c is the speed of light, κ is the Einstein constant, and ρ is the density of matter-energy.

In this expression, ε^{3D} is a scalar field that represents the fractional increase of the fabric's mid-hypersurface volume (51):

$$\varepsilon^{3D} \equiv \varepsilon_i^i. \tag{51}$$

The habitual strain tensor is as follows:

$$\varepsilon_{ij} = \frac{1}{2}(g_{ij} - \delta_{ij}). \tag{52}$$

Thus the time depends on a variation of volume due to the stress energy tensor that generate strain on the elastic medium (53):

$$\frac{dV}{d\bar{V}} = (1 + \varepsilon^{3D}). \tag{53}$$

But to come back at the complete analogy with the elastic medium, this varaition of volume can also come from temperature (54):

$$\frac{dV}{d\bar{V}} = \alpha_S \Delta T. \tag{54}$$

Finally in Eq. (48) we can transpose this to the time as a ratio of length of the space fabric (we see thus the importance of the speed of light as a intrisic characteristic of the space fabric) see (55):

$$\frac{ct}{cn\tau} = \frac{k_B t}{nh} \times \Delta T. \tag{55}$$

8.2.3. *State-of-the-art about cosmological time, coordinate time linked to entropy and the dynamics of the expansion of the universe*

In Ref. 30, "Cosmological Time, Entropy and Infinity" the authors go in the same direction as Refs. 28 and 29 by linking time to entropy but go further by associating this variation of entropy with the dynamic temporal evolution of the universe in its entirety from the big bang until now.

I quote "Time is a parameter playing a central role in our most fundamental modelling of natural laws. Relativity theory shows that the comparison of times measured by different clocks depends on their relative motion and on the strength of the gravitational field in which they are embedded. In standard cosmology, the time parameter is the one measured by fundamental clocks (i.e. clocks at rest with respect to the expanding space). This proper time is assumed to flow at a constant rate throughout the whole history of the universe. We make the alternative hypothesis that the rate at which the cosmological time flows depends on the dynamical state of the universe. In thermodynamics, the arrow of time is strongly related to the second law, which states that the entropy of an isolated system will always increase with time or, at best, stay constant. Hence, we assume that the time measured by fundamental clocks is proportional to the entropy of the region of the universe that is causally connected to them. Under that simple assumption, we find it possible to build toy cosmological models that present an acceleration of their expansion without any need for dark energy while being spatially closed and finite, avoiding the need to deal with infinite values."

In this paper, the following hypothesis is made, I quote again "the cosmological time t measured by such observers is proportional to the entropy of the region of the universe that is causally connected to them."

The authors propose so the following expressions of the time function of the entropy at the univers level (see the 4 points below described in Ref. 30):

- First the link between the universe entropy and the space temperature T:

Based on the CMB photon gas which the authors[31] assume is very close to thermodynamic equilibrium, the entropy S is written as follows (see (56)):

$$S = \frac{4\pi^2 k_B^4}{45c^3\hbar^3} V T^3, \tag{56}$$

where V is the volume considered (e.g. horizon V_{horiz}), $\hbar$ is the reduced Planck's constant, c is the speed of light, k_B is the Boltzmann's constant and T is the temperature associed.

- Second, the link between the temporal variation of horizon entropy S_{horiz} (57) and universe dynamic expansion:

$$\frac{dS_{\mathrm{horiz}}}{dt} = \frac{64\pi^3 k_B^4}{45\hbar^3} T_0^3 \frac{1}{H_{0,t}^2 \Omega_{0,t}^2} (\Omega_{0,t} + R(1 - \Omega_{0,t})), \tag{57}$$

where T_0 is the present CMB temperature ($T \times R = T_0 \times R_0$), R is the classical scale factor (sometime noted a also), k is the curvature, the Robertson walker metric, t is the classical time variable, $H_{0,t}$ (58) is the Hubble constant, $\Omega_{0,t}$ (59) is the matter density paremeter.

$$H_{0,t} = \frac{1}{R_0} \left. \frac{dR}{dt} \right|_{t=t_0}, \tag{58}$$

$$\Omega_{0,t} = \frac{8\pi G \rho_0}{3H_{0,t}^2}. \tag{59}$$

- Thirdly the link between the entropy variation and the cosmic time dτ (see (60)):

$$\frac{d\tau}{dt} = \frac{dS_{\mathrm{horiz}}/dt}{dS_{\mathrm{horiz}}/dt|_{BB}} = 1 + R\left(\frac{1}{\Omega_{0,t}} - 1\right). \tag{60}$$

With $dS_{\mathrm{horiz}}/dt|_{BB}$ for the temporal variation of the entropy in the causally connected volume at the BigBang (the coordinate time t (s) that flow at constant rate is equal to τ the cosmological time at the Big Bang of which the unit is the varying length).

- Fourth, the curvature of space which is linked to the interaction of variation of entropy, itself linked to time and temperature:

If $\frac{d\tau}{dt} = 1$ for a flat universe ($\Omega_{0,t} = 1$) the cosmological time flow at a constant rate. It is not the case in the curved space we have the following:

$$\Omega_{0,\tau} = \frac{1}{\Omega_0, t}. \tag{61}$$

The curvature k is for $R_0 = 1$ given in the following equation:

$$k = \frac{H_{0,t}^2(\Omega_{0,t} - 1)}{c^2}. \tag{62}$$

In conclusion of this state-of-the-art, we find with Refs. 30 and 31, equation by equation, a direct link between temperature, the evolution of entropy S, the evolution of time t and the associated curvatures k of the universe. We will focus in the following paragraphs on this direct link between temperature and time via an equivalent thermal expansion coefficient of time to be inline with our mechanical analogy of the spacetime. Of course, based on Refs. 28–31 this link between time and temperature passes in general relativity by the notion of spacetime associated this time with curvature of time but also of associated space therefore to a temperature effect. This therefore goes somewhere in the direction of a confirmation of our initial idea of a cosmological constant linked to the thermal curvature of space.

8.3. *Generalization of the special relativity interval taking into account temperature*

Basing of the precendent section and formula (48) or (55), this invites us to go far and to postulate a rewriting of the interval ds^2 of special relativity to introduce a mechanical thermal effect which gives Eq. (63) with α_t a potentiel dilatation coefficient of the time and α_s a potential expansion coefficient of space supposed homogenous:

$$d_s^2 = c^2(dt \pm \alpha_t T dt)^2 - (dx \pm \alpha_s T dx)^2 - (dy \pm \alpha_s T dy)^2 - (dz \pm \alpha_s T dz)^2. \tag{63}$$

We postulate that α_s is connected with the cosmological constant Λ of space by the expression $\alpha_s = \frac{\boldsymbol{l_p}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}$.

α_t is connected with the quantum mechanic and thermodynamic and time following Refs. 28 and 29 by the expression $\alpha_t \approx \frac{k_B t}{nh}$.

Indeed, given the special character and the unknown nature of time, there is no reason in first approach for a coefficient of time expansion to be identical to that of space.

Thus, the expression (63) becomes expression (64) if we separate the spatial part of the time part as

$$d_s^2 = (1 \pm \alpha_t T)^2 c^2 dt^2 - (1 \pm \alpha_s T)^2 [dx^2 + dy^2 + dz^2]. \tag{64}$$

Some important points arises from this expression.

(a) The two quantities in Eq. (64) above have an opposite sign so a potential contradictory effect.
(b) According to the Big Bang theory, space and time are created at the same time. According to general relativity we arrive at a sungularity at the beginning of the universe where the energy (and therefore the associated mass) is infinitely large and therefore the curvature is infinitely large in an infinitely small volume which requires the introduction of quantum mechanics and therefore the creation of a theory of quantum gravity. But we also often forget, at this moment in the universe the temperature is also infinite and therefore constitutes from the beginning a variable of the problem potentially influencing spacetime, therefore space and time in our fabric model cosmic are influenced by temperature. So, it's logical for us to introduce it in the interval formula.
(c) Of course, if there is no thermal effect, the interval reverts to the classical interval in special relativity.

8.4. *Can curvature, time and temperature be related in general relativity?*

The theory of general relativity in its current version does not directly take into account a slowing effect of time due to heat or temperature, nor does it specifically propose a "thermal curvature of time". This is what we deduce from our previous reflections in Sec. 8.3 based on Refs. 26,29–31. Indeed, from (64) we have rather a simple dilatation effect and not a curvature effect in potentially all the directions.

However, under certain circumstances, heat can indirectly have an effect on time by altering the curvature of spacetime. For example, in very massive and hot astrophysical objects, such as neutron stars or black holes, the effects of heat and pressure can influence the curvature of spacetime and therefore the trajectories of moving objects but also the time (time and space become as reversed) (see Ref. 32). The Hawking's temperature of black hole, that is the first quantum gravity equation depending on c, h, k_B and G and M the black hole Mass, goes in this direction.

There are also related concepts in theoretical physics, such as string theory and quantum gravity, that attempt to unify general relativity with quantum mechanics. In these theories, it is possible that temperature-related phenomena could have implications for the nature of spacetime, but this remains an ongoing area of research and exact specifics are not yet well established.

Thus, based on (64) for the new definition of the interval, taking into account[28] "Time & clocks: A thermodynamic approach" and[30] "Time and Thermodynamics Extended Discussion on Time & clocks : A thermodynamic approach" see https://www.sciencedirect.com/science/article/pii/S2211379720311013 for the temporal variation as a function of temperature and the associated temporal expansion coefficient α_t, from Eq. (37) for the relationship between the cosmological constant Λ and the space expansion coefficient α_s, from Ref. 12 for the thin sheet structure of thickness l_p, from Refs. 30 and 31 for the relationship between the temperature, the temporal variation, the entropic variation linked to the expansion of the universe and the associated curvature, we therefore have the following modification of Einstein's equation (17) expressed in terms of mechanical curvature:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \left(\frac{\alpha_f T}{l_p}\right)^2 g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{65}$$

Thus the cosmological constant Λ introduced by Einstein, considered as potentially the source of dark energy, becomes in our paper an additional thermal curvature $(\alpha_f T/l_p)^2$, connected for space and time to two complementary mechanical parameters linked to the spacetime fabric, namely its coefficient of thermal expansion of space $(f = s) \to \alpha_s$ and its coefficient of thermal expansion of time $(f = t) \to \alpha_t$ as well as the sandwich texture of it (multilayer of thickness l_p) all depending on the temperature $T(\to \Delta T)$ acting in connection with the variation of entropy during the temporal evolution of the universe.[30]

Remark 6. We are not certain today that the dark energy translated into our elastic model in the form of thermal curvature must be strictly constant or not.

Thus, in Einstein's equation the cosmological constant is constant, but in our approach it can potentially vary depending on the temperature or the temperature gradient.

Remark 7. The thermal expansion coefficient of time depends on t if we start from the approaches of Refs. 28 and 29.

8.5. *Synthesis on spatio-temporal approach to consider the temperature on the curvature of spacetime fabric*

First of all we recall our hypothesis resulting from the analysis of gravitational waves. The deformations of the space seem to be decoupled between the transverse deformations in the (xy) plans and the distance z of the waves propagation. Thus (ct and z) act at a same level (in the same direction) in a TT gauge in the expression of the deformations of gravitational waves. What leads Tenev and Horstemeyer assumed the space consisting of thin sheets thick of Planck. I quote.[12]

"The thickness must be very small so that the fabric can behave as an essentially 3D object at ordinary lengthscales and be an appropriate analogy of 3D physical space. The thickness itself defines a microscopic lengthscale at which the behavior of the physical world would have to differ significantly from our ordinary experience. A value equal or comparable to Planck's length l_p meets this criteria. However, the exact value of the thickness is not essential to the model as long as it is small but notvanishingly so".

We place ourselves at a distance from the cosmic web so as to consider on a given sheet of space a constant average temperature applied to said sheet (it is actually necessary to dissociate the quantum, local and global thermal effect associated with the scale factor, great thermal curvature of space) which allows us to guard against the inevitable thermal variations at smaller scales associated with the cosmic web.

These two postulates in place we have:

A first effect associated with our space thermal expansion coefficient which implies a thermal expansion of the space in the plane xy (see Fig. 4). The curvature of the space k being almost zero, it appears to us as a leaf that expands in part because of the average temperature of the cosmic web.

A second effect associated with our time thermal expansion coefficient involves this time a following thermal curvature linked this time between the temperature difference between two layers of Planck thickness space folowing the z direction.

Figure 4 gives an idea of the concepts developed in this paper.

So in our model illustrated in Fig. 4 in a way near to Ref. 30:

(1) Spacetime expands as time progresses every moment since the Big Bang.
(2) The engine of time, the fact that each present moment is renewed indefinitely is correlated with entropy variation which can only increase over time: $t = \frac{S}{\dot{S}}$.
(3) Time function of temperature because linked to entropy variation itself is quantified by: $t_{(T)} = n\tau = n\frac{h}{k_B} \times \frac{1}{T}$.
(4) The entropy S is linked to a temperature effect by the Bolzmann constant and by Ω different microstates by ($S = k_B \log \Omega$).

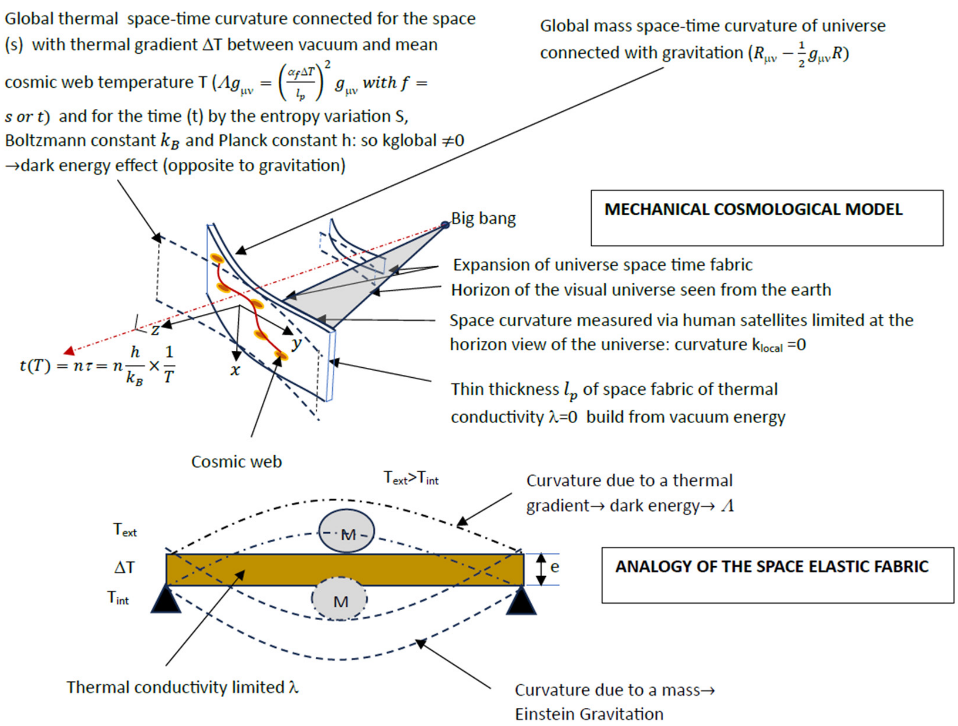

Fig. 4. Overview of the principles and concepts developped in this paper (thermal curvature of the spacetime connected to Λ, time, entropy variation and temperature).

(5) Space expands and via the interval ds^2 curves not only in the presence of mass (by general relativity) but also as a function of temperature or temperature delta (contribution of the analogy of general relativity with the theory of elasticity) which we propose to correlate with the cosmological constant Λ as thermal curvature of quantum space sheets of thickness l_p.
(6) By general relativity not only is space curved, but so is spacetime, so seeing what precedes time ($\times c$) expands and curves by the effect of variation in entropy itself linked to an evolution of temperature.[28–30]
(7) The curvature of spacetime due to mass is opposed to the curvature of spacetime due to temperature namely the curvature contribution that is identified by the cosmological constant Λ.

9. Presentation of the Possible Consequences of this Model Concerning the Interpretation of Dark Energy

The publications of Refs. 20 and 21 in particular and many others seem to indicate that the dark energy source of expansion of the universe is connected to the cosmological constant in particular when this one is positioned on the right of Eqs. (15) and (16). We are interested in this paper in a cosmological constant placed on the left of Eq. (17) that is to say in an additional curvature present in all spacetime. This more mechanistic approach makes it possible to no longer having recourse to an energy of unknown origin and is closer to a functioning of the universe as a structure "charged" within it by the cosmic web and undergoing the thermal gradient of this one.

But this approach raises other questions: which thermal gradient considered? does space really have a sheets structure deforming relative to each other as the deformations associated with the polarizations $\mathbf{A}^+$ and $\mathbf{A}^\times$ that seem to suggest the gravitational waves? What is the real thickness of these sheets?how to integrate the anisotropy of space suggested by a Poisson's ratio of 1?The quantum field theory approach seems more coherent because it leads to a coeffcient of thermal expansion compatible with materials on Earth, nevertheless the difference with the astrophysical value of the cosmological constant raises a real fundamental question.Moreover the values of the Young's moduli that we recalled at the beginning of the paper of on the one hand and the vacuum energy densities on the other hand being so outside the usual values of elastic materials on Earth that it is advisable to be particularly careful with respect to this "quasi-normal" value of the coefficient of expansion thermal that we propose in this study.

Another consequence of our study is that given that general relativity applies to space time, introducing a link between the cosmological constant and a temperature effect implies that temperature influences also time.

We therefore have by interpreting the expression (64), on the right a spatial part which only varies as a function of the temperature T and the coefficient of spatial thermal expansion $\alpha_{\boldsymbol{S}}$ which nowadays is that of the vacuum (cold) in interaction with the temperature of the cosmic web (warm) which creates thermal gradient and curvature (so rather stable to day in mean), while the left part linked to time and to the coefficient of time thermal expansion α_t has only increased since the Big Bang. Thus the curvature term in (65) $-(\alpha_f T/l_p)^2$ represents the thermal curvature which is opposed at the gravitational curvature due to mass.

The part of the thermal curvature associated with time could have an apparent effect of accelerating the expansion of space. In (65) the global thermal space time curvature is so in our model in the opposite direction of mass curvature to play the game of the dark energy.

The temperature of the cosmic web also reflected an expansion of space in its **xy** plane (see Fig. 4) with (k apparent $= 0$).

So, our approach is similar to Ref. 30 but using a mechanical formalism based on thermal expansion coefficient of the spacetime fabric.

Following our reflection, we could have a general curvature of spacetime linked to temperature which is broken down into two parts. A first spatial part due to a thermal gradient applied to the thin sheets of space fabric which could be linked to the cosmological constant Λ (expansion of the universe and positive curvature k). A second resulting from a time/entropy/temperature effect. Both creating a total negative thermal curvature k opposite to the curvature linked to mass and therefore to gravitation.

10. How to Verify this Effect of the Temperature on the Time

Measuring the variation of time as a function of temperature on atomic clocks is a good way to test the ideas of this paper.

The idea is to put two strictly identical atomic clocks under the same height conditions in the earth's gravity field and to vary the temperature for one of them in order to measure the possible time lag with respect to the remaining one at the initial temperature.

11. Conclusions

We explore in this study the analogy of space as an elastic medium by focusing on the mechanical parameters associated with any elastic four-dimensional fabric, its coefficients of thermal expansion α_S for space and α_t for the time. The state-of-the-art is rather poor or even nonexistent on their definitions and their intensities within the framework of an elastic model of space. Some research exists for the time dilatation function of temperature. By taking inspiration from models of space fabric in the form of thin sheets, placing ourselves within the framework of quantum field theory both on the value of the cosmological constant Λ and on the thickness of these sheets of the order of the Planck's length, considering the recent measurements making it possible to establish the orders of magnitude of the space thermal gradient between the hot and cold zones of the universe and finally considering that these sheets bend under this thermal gradient by analogy with mechanics plates, we propose a formulation and a value of the expansion coefficients of the space fabric.

Since general relativity is built on spacetime, this thermal curvature approach implies that not only space but also time could vary with temperature. We therefore propose an expression for the temporal expansion coefficient $\alpha_\mathbf{t}$ based on the ratio between Botzmann's constant and Planck's constant in connection with a variation in entropy of the universe and therefore its temperature. Since general relativity is itself based on special relativity, this results in the need to take into account variations in distance and time as a function of temperature. We therefore propose a way to modify the definition of the spacetime invariant $\mathbf{ds^2}$ by introducing both a spacetime coefficient and a temporal expansion coefficient.

When we return to the simplistic analogy cited at the beginning of the stretched fabric supporting a heavy ball, it indeed appears logical that the fabric supporting this ball elongates more or less depending on its temperature in addition to spatial flexibility and mass/energy which distorts it.

Thus gravitation influences space and time but would also be linked to temperature. Temperature would influence spacetime curvature at the quantum scale (possible nature of the time expansion coefficient) and at large scale (possible cosmological constant effect in link with the cosmic web). The answer in our elastic medium analogy would be of two types. A first on a lengthening and shortening of spacetime, a second on a thermal curvature of spacetime connected with the dark energy.

This study remains a initial pproach which somehow clears this path given the great variability of the intensity of the cosmological constant depending on whether one considers its value from quantum field theory or its value from cosmological measurements. Other modelings are certainly necessary to fine-tune the value of these thermal expansion coefficients of the spacetime fabric and this study remains only a first approach.

Einstein said that God is not playing dice when he talks about quantum mechanics, but perhaps God is playing structural engineer with a cosmic structure driven by a thermal gradient manifesting as dark energy acting throughout the universe—at least that's the question this paper poses.

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Analogy of spacetime as an elastic medium — Estimation of a creep coefficient of space from space data via the MOND theory and the gravitational lensing effect — the ball cluster and via time data from the GPS effect — comparison, discussion and implication of the results for dark matter and Einstein's field equation

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After recalling the principles that allow spacetime to be considered by analogy as an elastic medium, we show how the modified gravity according to the MOND theory concerning the anomaly of the velocities of stars at the periphery of galaxies can be seen as a creep of space acting on the radius of galaxies that give a creep coefficient of $\varphi_{\text{space}} = \frac{a_0}{a}\frac{\rho_{\text{local}}}{\rho_{\text{mean}}} - 1$. The values vary between 0.2 and 9 depending on the type of galaxy and density distribution. Considering the gravitational lensing effect of the ball cluster we obtain a creep coefficient $\varphi_{\text{space}} = \frac{1-p_v}{p_v}$ with p_v the percentage of visible matter and p_{DM} the percentage of dark matter from the global mass ($p_v + p_{\text{DM}} = 1$). The values vary between 0.66 and 4 for this cluster. This paper therefore raises the question, via these creep coefficients, of the possible granular nature of the vacuum and therefore of space fabric on the one hand and proposes another dark matter-free approach based on the creep of the texture of space to explain gravitational anomalies on the other hand.

Keywords: Creep; dark matter; spacetime; quantum cosmology; general relativity; MOND theory; elasticity theory; elastic medium.

PACS numbers: 04.50.Kd, 46.90.+s

1. Introduction

Cosmology is the study of the behavior of spacetime constituting the universe as a whole characterized by Einstein's gravitational field equation[1,2] on the one hand using the Friedmann-Lemaitre–Robertson–Walker metric on the other hand. However, some gravitational phenomena can only be explained by using a mysterious mass or dark matter.

The first phenomenon is the speed of stars on the outskirts of galaxies. Zwicky in the 1930s suggested that a certain dark matter could explain the discrepancy between the velocities of stars predicted according to Newton's theory, which is supposed to decrease as we move away from the center of galaxies, and those actually measured, where the rather high speed remains almost constant (see Fig. 1).

But for more than 50 years, the mysterious dark matter particles have been untraceable, regardless of the detectors and the sensitivity of the detectors. The ontological approach therefore fails for the moment. This leaves the legislative approach, which consists of changing the law of gravitation, at least in part, in certain acceleration ranges. This is the goal of the modified gravity theory developed by Mordehai Milgrom[3] and supplemented by other scientists.[4]

The second phenomenon occurs at the level of the cosmic microwave background, where dark matter is again needed to create the small variations in density of the original plasma that led to galaxies today.

The third is at the level of gravitational lensing, where the mass of galaxy clusters alone is insufficient to explain the amplitude of the observed light distortions.

In this paper, we propose to approach the problem of dark matter through the prism of the spacetime analogy with an elastic medium, in the specific case of the anomaly of the velocities of stars at the periphery of galaxies for space, and to see the consequences and interpretations for time based on the feedback in this field, namely the GPS shift and the desynchronization of clocks in the Hafele and Keating experiment. Considering that Einstein's equation replaces gravitation as a force with a deformation of spacetime, it implies modeling spacetime as a deformable elastic medium. This has been done by many authors in subsequent publications.[5–16] Like any elastic medium, it is then necessary to define the characteristics of the medium, its Young's modulus between 10^{31} and 10^{113} Pa according to Refs. 5–7 and 10, its Poisson ratio,[5–7,10] of the order of 1, its coefficient of thermal expansion[8] of the order

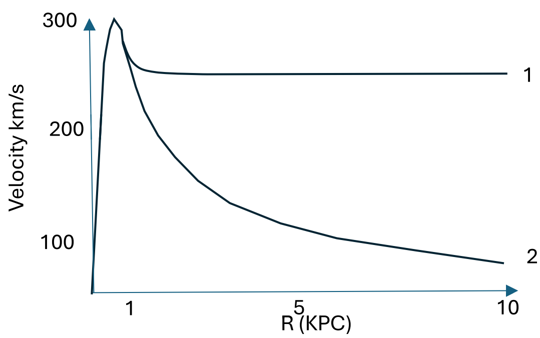

Fig. 1. Velocity of stars in galaxies of radius r according to Newton's theory (2) and observed (1).

of that of steel and therefore, to be complete, its creep coefficient if the medium is sensitive to this phenomenon, which is well known in mechanics.

The state of the art on the creep of spacetime is very limited but not zero. We found only two papers of Refs. 17 and 18. In Ref. 17, the author shows that the equation that manages the propagation of dislocations can be seen as a creep of atoms in a crystal and that a similar expression can be written in cosmology for the vacuum from the cosmological constant Λ. Thus, in formula 24 of Ref. 17, the dislocation density ρ is a function of the volume density of the elastic energy $U_{\sigma i}$ of the solid crystal:

$$\rho = \frac{4(1+\nu)}{\alpha^2 M^2 \mu_m b^2} U_{\sigma i}, \tag{1}$$

where α is a constant that characterizes the interaction of dislocations and depends on the geometric arrangement of the dislocation structure ($\alpha \cong 0.5$):

$$\mu_m = \frac{E}{2(1+\nu)}, \tag{2}$$

where μ_m is the usual shear modulus, M is a geometric factor called the Taylor factor, b is the modulus of the Burgers vector and ν is the Poisson coefficient.

In formula 31 of Ref. 17, the cosmological constant Λ is a function of the density of the energy of the vacuum U_v on the other hand,

$$\Lambda = \frac{8\pi G}{c^4} \times U_v. \tag{3}$$

Thus, U_σ for dislocation creep plays the role of U_v in relativistic cosmology. This point, based on Ref. 17, is essential for making the case that space can be assumed as a crystalline structure and therefore is subject to creep.

In Ref. 17, the author seriously considers the hypothesis that the vacuum is a crystalline structure subject to everything that occurs in this type of media, namely dislocations and creep. Therefore, considering a possible creep of the vacuum associated with the nature of space and time within spacetime becomes conceivable if it has a granular and/or crystalline structure. Finally, in special relativity, time is associated with c which squared gives the dimension of a squared length with an opposite sign to the Pythagorean sign of space. By definition, any creep characterizes, depending on the nature of a material, the ability of the latter to deform under constant load over time and therefore to vary in length by displacements over time under a constant load without adding or modifying the initial mass that deforms and curves it. Therefore, considering a creep of space from space and time data becomes possible in general relativity.

At the same time, feedback from general relativity and cosmology shows that spacetime is clearly subject to more gravitational effects than the visible mass alone allowing us to envisage in view of what we have stated in the previous paragraphs. However, according to Einstein, gravitation is in fact a geometric phenomenon.

So, the first approach to explain this observation is to solve the paradox (more gravitation than mass to cause it) by fictitiously increasing the mass that generates this gravitation. Since we do not see it, it has been called dark matter. The problem is that it is not made of traditional matter, is invisible, interacts with classical matter in the sense of quantum field theory and standard model, has a major gravitational effect and remains to this day absolutely undetectable other than by the distortions of spacetime that it creates.

The second approach, as we have seen, is to modify Newton's law, as Mordechai Milgrom did. But in this case, why would nature need two laws of dynamics? So, does not Ockham's razor apply to spacetime?

It is from this point of view that Ref. 18 proposes to reinterpret this modification of Newton's law by considering that the anomalous gravitational effects currently attributed to dark matter can alternatively be explained as a manifestation of the inherent structure of space at the galactic level by an effect of length scales. Specifically, the authors show that the inherent curvature of space amplifies the gravity of ordinary matter such that the effect resembles the presence of the hypothetical hidden mass. Their study is conducted in a context of low gravity, quasi-static conditions, and spherically symmetrical configuration, and exploits the Cosmic Fabric model of space.[5] It is in this same philosophy that this paper is placed, considering that it is the very texture of the fabric of space that creeps under the permanent effect of the internal constraints generated by ordinary matter.

Other approaches are based on in binding of cosmological structures by massless topological defects.[19] So, no need for dark matter at all.

Finally, our innovative approach, in link with Ref. 18, is to rely on the analogy of the elastic medium and the well-known behavior of materials such as concrete or polyurethane, for example. The creep we described above. Indeed, the data on the problem are clear. We observe a gravitational effect greater than there is mass to cause it or translate into an Einsteinian approach an amplification of the distortions of spacetime that do not correlate with the visible mass that causes it, which remains constant (visible matter). The only phenomenon in mechanics that allows deformations to be amplified over time without modifying the load is the creep of the medium or material that constitutes the structure itself. In a way, when the deformation under self-weight of a concrete slab due to the loss of water during drying or of a polyurethane-core sandwich panel due to the reorganization of the polyurethane polygonal cells increases over time, it is as if a dark mass (rather transparent, invisible) which would charge it giving more deformation over time than the initial one under its mass alone. Seen from the outside, one would therefore have the impression that this invisible mass loads the structure more than there is visible mass, since the deformations increase slowly over time compared to the initial deformation under the sole mass of the structure.

We now need to understand how physically a space creep could be in action to replicate the effects of dark matter, which accounts for about 30% of the elements at play in the universe, the rest being dark energy around 65% or visible matter 5%.

Let us first recall how creep works and manifests itself on Earth with well-known materials.

Creep is a physical phenomenon that occurs in materials when they are subjected to constant stress over a long period of time. This process leads to progressive deformation, even if the stress remains unchanged. On Earth, this phenomenon is particularly observed in certain materials under specific conditions, including metals, plastics, rocks and some ceramics, at high temperatures or under high forces or for reconstituted materials such as concrete or polyurethane.

In the case of metals, creep occurs mainly at temperatures close to 30–50% of their melting temperature. For example, in aircraft turbines or high-performance engines, metals subjected to high temperatures can gradually deform under mechanical stress. In their cases, atomic scattering is the main cause of high-temperature creeping. Under the effect of heat, the atoms in the metal structure have more energy and can move, thus causing the material to deform.

In the case of plastics and polymers, creeps are common at ambient or slightly elevated temperatures. For example, plastic materials that are subject to static stress, such as pressure pipes or plastic objects that support constant weights, can deform over time. In their cases, creep is due to the reorganization of polymer chains in the material, where the molecules gradually readjust under the load.

In the case of geological materials, such as rocks and ice, creep is observed over long time scales. For example, the creep of ice in glaciers or that of rocks in the Earth's crust under strong tectonic pressures. In their cases, the creep is due to the internal reorganization of mineral or molecular grains under the effect of internal forces in the Earth. In geology, this can take millennia, as in the case of tectonic movements.

In the case of composite materials, water loss in the case of concrete is the source of creeping. See "Creep and relaxation Poisson's ratio: Back to the foundations of linear viscoelasticity. Application to concrete."[20]

Finally, ceramics, which are generally very rigid, can be creeped at very high temperatures, for example in reactors or combustion systems, where the temperature is extreme. In their case, the creep mechanism is often due to diffusion processes similar to those seen in metals.

In summary, the parameters that influence creep are therefore high temperatures, prolonged stresses and the nature of the media, grains, and polymer with deformable geometric shape connected with Poisson's ratio.[20]

We must therefore see to what extent the spacetime medium could be affected by this phenomenon.

First of all, all research considers the extremely fine texture of Planck's size. Quantum gravity considers the granularity of spacetime.[17,21,22] In Ref. 5 hyperspace sheets have the characteristics of a fabric or connected fibers with a Poisson ratio of 1. In Ref. 5 the authors cite two publications of Refs. 23 and 24, where Poisson's coefficients of 1 and where the texture of the medium is fibrous or in the form of a polygon that deforms. So, Poisson's ratio of 1 implies somewhere a fine-grained or fiber-shaped or polygonal deformable structure, all of which are polymer-like creep source parameters.

If we now look at Young's modulus of the space fabric, it turns out to be very high and greater than 10^{20} times that of steel, in which case only temperature could make it flow. Here too between the hot zones of the cosmic web, temperatures of several million degrees could be enough to generate metal-like creep.

In conclusion, Poisson's ratio of 1 is a strong indication of the potential nature of the elastic medium that in the analogy could flow, i.e. fine grains or fibers or polygonal meshes of a fabric that can all be sensitive to creeping.

Creep is therefore the result of mechanisms at the atomic or molecular scale that allow the particles of the material to gradually rearrange themselves under the effect of constant stress and a given temperature. The spacetime medium being charged by the entire cosmic web for more than 13.7 billion years, we can therefore understand if the nature of the elastic medium allows it to undergo a creep effect, i.e. constant deformations, therefore constant additional equivalent gravitation, without adding mass. Consequently, granular materials are subject to creep in compression[25] or shear.[26]

We therefore propose considering an identical phenomenon for rotating spacetime in the case of a galaxy (the quantity of stars inside in rotation loads the space fabric to continue for a long time) or deformed by a cluster of galaxies during gravitational lensing or even during the hot big bang. We will therefore see in this paper whether finally the modification of Newtonian dynamics proposed by MOND's theory, which works but does not go in the direction of simplification, does not amount to considering a creep coefficient to apply κ Einstein's constant which characterizes the flexibility of spacetime itself and can therefore be correlated from the point of view of the analogy of spacetime to a correction as a function of the time of Young's modulus of the latter. In fact, we can consider that the creep has these two effects: (1) it affects the constitutive properties of space resulting in an effective gravitational constant that is different from the conventional one and (2) it modifies the geometry of space resulting in additional curvature. Then point out that we are focusing on (1) and that the consequences of (2) are covered in Ref. 18. Finally, if we modify the constant κ to incorporate these creep effects, then it implies that not only space but also time is impacted by this effect. We will therefore try to establish a creep coefficient also for time and compare it with the one obtained for space with the MOND theory and gravitational lensing (ball's cluster) to see if all these values are consistent.

2. Method

The following methodology has been followed in this paper.

(1) Reminder of the equations of general relativity and the analogy with the mechanics of continuous media,
(2) Reminder of the notions of creep in mechanics,

(3) Reminder of the theory and the MOND equations associated with the speed of stars on the periphery of galaxies,
(4) Re-expression of the MOND equations and Newtonian gravitation in the case of the problem of the velocities of stars at the periphery of galaxies in terms of variation of equivalent geometric radii of the latter,
(5) Integration of creep in general and Newtonian relativity on the gravitational constant G,
(6) Evaluation of the creep coefficient of space φ_{space},
(7) Evaluation of the creep coefficient of space from another characteristic source, the ball cluster,
(8) Expression of a creep coefficient in relation to time fluctuations and data in a weak gravitational field,
(9) Evaluation of the creep coefficient of time from GPS within the vicinity of the Earth,
(10) Discussion, comparison of the results obtained with other properties of space-time compared with other phenomena and other theories.

3. Reminder of the Equations of General Relativity and the Connections with the Analogy of the Elastic Medium

The law of gravitation according to general relativity is written with $R_{\mu\nu}$ the Ricci tensor, $g_{\mu\nu}$ the metric tensor, R the scalar curvature, G the gravitational constant and c the speed of light:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (4)$$

In a weak field it becomes

$$\Box \bar{h}_{\mu\nu} = \frac{16\pi G}{c^4} T_{\mu\nu} \quad (5)$$

with

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2} \eta_{\mu\nu} \bar{h}, \quad (6)$$

where $\bar{h}$ the trace of $\bar{h}_{\mu\nu}$.

We have shown in Refs. 5–7 that the perturbation tensor of the metric $h_{\mu\nu}$ is equivalent to a strain tensor in four dimensions $\varepsilon_{\mu\nu}$ by a factor of 2:

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu}. \quad (7)$$

It has been shown[6] that the stress-energy tensor $T_{\mu\nu}$ is equivalent to a stress tensor σ_{ij} with the velocity vectors v_i and v_j, the four-velocity vectors u_μ and u_ν.

$$\sigma_{ij} = \rho v_i v_j, \quad (8)$$

$$T_{\mu\nu} = \rho u_\mu u_\nu. \tag{9}$$

Finally, it is well known[5–7] that Einstein's constant κ is equivalent to the flexibility of spacetime for a unit surface in N^{-1} with Y the Young modulus of spacetime and S a unit surface:

$$\kappa = \frac{8\pi G}{c^4} \to \frac{1}{YS}. \tag{10}$$

This is why Einstein's equation is often considered to be Hooke's law with ε a deformation, E the Young modulus of the elastic material and σ a stress:

$$\varepsilon = \frac{1}{E}\sigma. \tag{11}$$

This leads according to Refs. 5–8, 10, and 27 to consider that the gravitational constant G is related to Young's modulus of spacetime via a frequency f of eigen vibration to the vacuum and ρ a density of the vacuum:

$$G = \frac{\pi f^2}{\rho}, \tag{12}$$

which becomes with $Y = \rho c^2$ the expression that presents the propagation of compression waves in an elastic medium:

$$G = \frac{\pi f^2 c^2}{Y}. \tag{13}$$

Let be a κ mechanized constant[6,7,17]:

$$\kappa = \frac{8\pi G}{c^4} \to \frac{1}{YS} \tag{14}$$

or

$$\kappa = \frac{8\pi^2 f^2 c^2}{Yc^4} = \frac{8\pi^2 f^2}{Yc^2}. \tag{15}$$

So, κ corresponds to the flexibility of the unit area ($S = 1\mathrm{m}^2$) of spacetime and is a function of $1/E = 1/Y$.

4. Reminder of the Concepts of Creep in Mechanics

If space is modeled as an elastic medium, then this typology of medium could be subject to creep via a creep coefficient φ, if by analogy it is comparable to a granular or crystalline structure. We recall that in quantum gravity we assume that spacetime is grainy. In sandwich theory, creep corresponds to the polygonal cells of polyurethane that deform under loading, influencing the shear modulus μ of this core. In concrete steel flooring, for example, the concrete creep is due to water loss and therefore when the bending deformations of the slabs are calculated, the cracked and uncracked concrete sections are calculated with Young's modulus E that fluctuates as a function of time.

Concretely, in mechanics, creep is introduced by a coefficient φ by varying Young's modulus or shear modulus in the following way with E_{st} the short-term Young modulus and E_{Lt} the long-term Young modulus:

$$E_{\mathrm{Lt}} = \frac{E_{\mathrm{St}}}{(1+\varphi)}. \tag{16}$$

In the case of the shear modulus, we have the short-term shear modulus μ_{st} and the long-term shear modulus μ_{Lt} in the same way:

$$\mu_{\mathrm{Lt}} = \frac{\mu_{\mathrm{St}}}{(1+\varphi)}. \tag{17}$$

In both cases, the objective is to increase the deflection of the beam by decreasing Young's modulus.

For example, in the case of a beam of inertia I on two supports, with a span L, uniformly loaded by p, with a bending creep-sensitive, the vertical deflection is written with $\varphi > 0$:
without creep,

$$f_{\text{without creep}} = \frac{5pL^4}{384EI} \tag{18a}$$

with creep:

$$f_{\text{with creep}} = \frac{5pL^4}{384EI}(1+\varphi). \tag{18b}$$

So, a ratio of deflection

$$\frac{f_{\text{without creep}}}{f_{\text{with creep}}} = \frac{1}{(1+\varphi)}. \tag{18c}$$

If spacetime is subject to creep, the constant κ that represents the flexibility of creep could vary as a function of G that can be traced back to Young's modulus of spacetime that would vary over time as seen in paragraph 3. We would then obtain for the creep flexibility of spacetime:

$$\kappa = \frac{8\pi G}{c^4} \to \frac{8\pi^2 f^2 c^2}{\frac{Y}{(1+\varphi)} c^4} = \frac{8\pi^2 f^2 (1+\varphi)}{Yc^2}. \tag{19a}$$

Or with a gravity modified by creep effect:

$$G_{\mathrm{Lt}} = \frac{\pi f^2 c^2}{Y}(1+\varphi) = G(1+\varphi). \tag{19b}$$

A new κ expression corrected by creep is therefore written as

$$\kappa = \frac{8\pi^2 f^2 c^2}{\frac{Y}{(1+\varphi)} c^4} = \frac{8\pi G(1+\varphi)}{c^4}. \tag{20}$$

Thus, with $\varphi > 0$, the flexibility of space will increase, and the rigidity will decrease. So, the radius of curvature and deflection will increase and the ratio $1/R^2$ will decrease.

5. Reminder of MOND's Theory and Equations

It is well known that the curve of the rotation speeds of stars in spiral galaxies does not follow the classical Newton's law. The velocities remain radially constant as one moves away from the center of the galaxy (see Fig. 1). The MOND theory (Modified Newton Dynamic) proposed by Mordehai Milgrom[3,4] makes it possible to reproduce this velocity curve precisely without using dark matter but does not explain why this is so.

Thus, the expression according to Newton's law with M the mass of the galaxy and r the radius of calculation of the speed v of the stars with respect to its center and G the gravitational constant is written as

$$v = \sqrt{\frac{GM}{r}}. \tag{21}$$

Dimensional analysis indeed produces units of a velocity:

$$\left(\frac{\frac{\mathrm{m}^3}{\mathrm{kgs}^2}\mathrm{kg}}{\mathrm{m}}\right)^{1/2} = \frac{\mathrm{m}}{\mathrm{s}}. \tag{22}$$

This expression of velocities v according to Newton is replaced according to Refs. 3 and 4 by a constant value that no longer depends on the radius r of the galaxy (see demonstration below):

$$v = \sqrt[4]{GMa_0}. \tag{23}$$

The equation of dimensions allows us to verify that it is indeed a velocity:

$$\left(\frac{\mathrm{m}^3}{\mathrm{kgs}^2}\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)^{1/4} = \frac{\mathrm{m}}{\mathrm{s}} \tag{24}$$

With a_0 a new constant in physics[28]:

$$a_0 = 1.2 \times 10^{-10}\,\mathrm{m/s}^2. \tag{25}$$

We can therefore see that according to this MOND approach, the gravitational constant is corrected by a factor a_0 that we will somehow transform to an equivalent concept within the domain of Mechanics to deduce a creep coefficient φ_{space} for the geometry of space.

MOND's new law of dynamics,[3,4,29] i.e. the force F of gravitation in the case of galaxies, is written with μ a function of $\frac{a}{a_0}$ which modifies the classical acceleration a:

$$F = \frac{GMm}{r^2} = \mu m\left(\frac{a}{a_0}\right)a. \tag{26}$$

Or according to Refs. 4 and 29:

$$a = \sqrt{F\frac{a_0}{m}}. \tag{27}$$

This makes it possible to write by simplifying by m:

$$F = \frac{GM}{r^2} = \mu\left(\frac{a}{a_0}\right)a. \tag{28}$$

When r of the galaxy is very large, a is smaller than a_0 and we therefore obtain $\mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}$ which give

$$\frac{GM}{r^2} = \frac{a^2}{a_0}. \tag{29}$$

With the radial acceleration which is classically equal to

$$a = \frac{v^2}{r}. \tag{30}$$

Let us transfer this acceleration to MOND's expression:

$$\frac{GM}{r^2} = \frac{v^4}{r^2 a_0}. \tag{31}$$

Let be the constant speed of the stars in the galaxy:

$$v^4 = GMa_0. \tag{32}$$

Let be the final expression of the speed of stars in a galaxy[28,29]

$$v = \sqrt[4]{GMa_0}. \tag{33}$$

It can therefore be seen that MOND's solution makes it possible to give a constant velocity of the stars on the periphery of galaxies independent of the radius r of the latter, which corresponds well to what has been measured experimentally.

From a mechanical point of view, it is as if the galaxy were caught in a space disk constituting a rotating elastic medium in which all the stars trapped in it are driven at the same speed.

6. Re-Expression of the MOND Equations in Terms of Geometric Variations of the Radius r of an Elastic Disk Constituting the Galaxy and Comparison with the Same Radius in the Case of the Classical Newtonian Approach

From the expression of the velocities established in the MOND theory, we obtain the expression of the velocity squared of the stars:

$$v^2 = \sqrt{GMa_0} \tag{34}$$

with $a = \frac{v^2}{r}$ for the acceleration and replacing v^2 by this acceleration a in the above expression of the velocities of the stars in the galaxy according to the MOND's theory, we obtain an equivalent radius of the galaxy that we denominate r_{mond}:

$$ar = \sqrt{GMa_0}, \tag{35}$$

we deduce an equivalent radius of the galaxy according to the MOND theory:

$$r_{\text{mond}} = \sqrt{\frac{GM}{a}\frac{a_0}{a}}. \tag{36}$$

We can also reformulate the radius of the galaxy in the case of classical Newtonian gravitation from $F = \frac{GMm}{r^2} = ma$. Let with $a = \frac{GM}{r^2}$, we get

$$r_{\text{Newton}} = \sqrt{\frac{GM}{a}}. \tag{37}$$

So, finally the MOND theory can be seen as a modification of the radius of the galaxy's disk by a ratio of acceleration a of it with respect to a reference acceleration a_0:

$$r_{\text{Mond}} = \sqrt{\frac{GM}{a}\frac{a_0}{a}} = r_{\text{Newton}}\sqrt{\frac{a_0}{a}}. \tag{38}$$

By comparing the expression of the equivalent radius of the galaxy according to the MOND theory with that of the radius of the galaxy according to Newton member by member, it naturally becomes

$$r_{\text{Mond}} = \sqrt{\frac{M}{a}\left(G\frac{a_0}{a}\right)} = r_{\text{Newton}}\sqrt{\frac{a_0}{a}} \tag{39}$$

or

$$r^2_{\text{Mond}} = \frac{M}{a}\left(G\frac{a_0}{a}\right) = r^2_{\text{Newton}}\frac{a_0}{a}. \tag{40}$$

So, if a is smaller than a_0 (initial assumptions are taken to simplify $\mu(\frac{a}{a_0})$), the equivalent radius of the galaxy according to the MOND theory becomes larger. It is as if the radius of the galaxy in the context of the analogy of the elastic medium had creeped! The analogy of the elastic medium gives a mechanical visualization of the structures of the modification of Newton's dynamics so that the velocities of the stars measured in the galaxy correspond to that of the MOND theory.

This constant a_0 represents, according to Milgrom, I quote: "It is roughly the acceleration that will take an object from rest to the speed of light in the lifetime of the universe. It is also of the order of the recently discovered acceleration of the universe.[30]" We can therefore see an increase in the creep of the universe over time in the sense of the analogy of the elastic medium.

7. Integration of Creep in General Relativity and in Newtonian Gravitation in the Framework of the Analogy of the Elastic Medium

7.1. *Expression of the creep coefficient linked to space*

The mass M and acceleration a of the galaxy being known, we can therefore postulate that the ratio $\frac{a_0}{a}$ affects the gravitational constant as seen in the r_{mond} formulas above. Moreover, we have seen that the constant κ in the framework of elastic analogy is related to the flexibility of spacetime in $1/E = 1/Y$ and therefore with creep in $1/(\frac{E}{1+\varphi})$. Using the analogy of the elastic medium, we can therefore compare the value of the short-term gravitational constant with a long-term gravitational constant corrected by a creep effect associated with its Young's modulus Y via a coefficient $(1+\varphi)$. We therefore obtain by comparing G long-term denoted G_{Lt} with short-term G:

$$G_{\mathrm{Lt}} = \frac{\pi f^2 c^2}{Y}(1+\varphi) = G(1+\varphi). \tag{41}$$

The relationship between Newton's short-term constant G and long-term G_{LT}:

$$G_{\mathrm{Lt}} = G(1+\varphi). \tag{42}$$

We therefore postulate that the MOND's theory effect on the equivalent radii of the galaxy is due to a variation of G by the creep effect of space (since G is associated with Young's modulus Y in the elastic medium analogy):

$$G(1+\varphi) = \left(G\frac{a_0}{a}\right). \tag{43}$$

Let us consider the following relation which allows us to evaluate the creep coefficient of space considered by analogy as an equivalent elastic medium:

$$\frac{a_0}{a} = (1+\varphi). \tag{44}$$

Let the following expression of the creep coefficient of space which, according to the MOND theory seen under the analogy of the elastic medium, implies a variation in the radius of the galaxy

$$\varphi_{\mathrm{space}} = \frac{a_0}{a} - 1. \tag{45a}$$

7.2. *Numerical evaluation of the creep coefficient of the equivalent elastic medium from the MOND's equations explaining the almost constant rotation of stars at the periphery of galaxies*

From the above expression, we can proceed to the numerical application to evaluate the value of the creep coefficient φ. We will use for that the data given in Ref. 18.

Table 1. Estimation of the creep coefficient from MOND approach for several galaxies based on G affected by creeping.

Case	Galaxy	Type of galaxy	Mass M [10^9] solar mass n	Radius r [10^3] light years	$\varphi_{\text{space}} = \frac{1.2\times10^{-10}}{a_{(n,r)}} - 1$
1	Milky way	Barred spiral	1000 at 1500	50	−0, 54922192
	With all the 160 globular cluster around milky way[31]	S(B)bc I-II	1500	100	−0.26388247
			1500	520[31]	2.827
2	LMC (*Large Magellanic Cloud)*	Dwarf spiral SB(s) m	10	7	−0,36891068
3	SMC	Dwarf spiral SB(s) m	7	3.5	−0,622852
4	Andromeda	Spiral SA(s)b Interaction with M32 and M110	1000	110	−0, 00828822
5	M33	Spiral SA(s) cd	50	30	0,209564525
6	Pinwheel M101 (NGC 5457)	Spiral intermediate SA B(rs)cd	100	85	1,423325287
7	Wirlpool M51 (NGC 5194)	Spiral SA(s)bc pec	160	30	−0,32383287
8	SunFlower messier 63	Spiral SAbc	140	49	0,180660002
9	M77	Spiral (R)SA(rs)b Sb/PSb	1000	85	−0, 23367726
10	Condor NGC 6872	Barred spiral is interacting with the lenticular galaxy IC 4970, SB(s)b pec	100	261	6,441034116
11	Carwheel	Lenticular	4	75	9,691140972
12	Malin 1 (PGC 42102)	Galaxy geante	1000	325	1,930057542
13	Phonix cluster	Cluster of galaxy	2000000	550	−0,88912325
14	GN-z11 (the furthest in 2016 at 13.4 year light)	Unknown	1	1.5	−0,57235436

Notes: Sb-like galaxies can be distinguished from Sc galaxies by a less open spiral structure and a more prominent bulge. Sa-type galaxies have an even less open spiral structure and an even more prominent bulge. Presence of a bar (SB) and open arms with a weak bulb (SBc).

For the numerical application we have with r in year light and M in n solar masses:

$$a_0 = 1.2 \times 10^{-10}\,\mathrm{m/s^2},$$
$$a_{(n,r)} = \frac{\sqrt{GMa_0}}{r} = \frac{\sqrt{6.674 \times 10^{-11} \times n \times 10^9 \times 1.98 \times 10^{30} \times 1.2 \times 10^{-10}}}{r \times 1000 \times 365 \times 24 \times 60 \times 60 \times 300000000}.$$

Hence the creep coefficient related to space:

$$\varphi_{\text{space}} = \frac{1.2 \times 10^{-10}}{a_{(n,r)}} - 1. \tag{45b}$$

In Table 1, concerning several galaxies, we are looking at the effects of creep as it relates to how it changes the effective gravitational constant and without considering the additional geometric effects of the creep.

7.3. *Discussion of the results obtained*

We see that certain values are positive (half of the cases), that is, logical but that also for the other half the values are negative. For memory, creep has these two effects: (1) it affects the constitutive properties of space, such that the long-term value of G changes (analogous to the long-term value of Young's modulus $Y_{\text{Long term}}$) and (2) it affects the geometry of space, where it may lead to additional curvature.

Since here we are only concerned with (1), it is possible that the negative values indicate that (2) has a significant effect that needs to be taken into account.

For these negative values, we first recall the simplifying assumption that has been adopted, which is that the radius r must be very large, for the acceleration a *to* be much smaller than a_0 and therefore the expression of $\mu(\frac{a}{a_0})$ becomes equal to $\frac{a}{a_0}$. So, for all dwarf galaxies (cases 2 and 3) this is no longer the case because of their smallness. The assumption is no longer true and therefore the above approach does not apply.

Then for all galaxies in clusters or interacting with other galaxies, we are not in the simple case of an isolated MOND galaxy either, so for cases 4 and 13 the method does not apply either.

For the galaxy 13.7 billion light-years away (case 14), the dwarf or nondwarf nature is unknown.

Cases 9, 7 and 1 therefore remain in negative values.

For case 1, it turns out that the real radius, taking into account globular clusters, is much larger than those commonly established in the first approach, see Ref. 31. It can be seen that the value of φ becomes positive again when taking into account the peripheral globular clusters.

As cases 7 and 9 are of the same type as case 1, a more accurate knowledge of these globular clusters will probably settle this negative value.

Table 2. Issued of Ref. 18 effect of inherent curvature of space with $s = \sqrt{\frac{GM}{a_0}}$.

Galaxy	Mass M [10^9] solar mass n	Radius r [10^3] light years	Ratio $\frac{R}{s} = \frac{f_{\text{without creep}}}{f_{\text{with creep}}} = \frac{1}{(1+\varphi)}$ see formula (18c)	φ
Milky way	1000	50	0.45	1,22222222
LMC	10	7	0.63	0,58730159
SMC	7	3.5	0.38	1,63157895
Andromeda	1000	110	0.99	0,01010101
M33	50	30	1.21	0,17355372
Pinwheel	100	85	2.42	0,58677686
Wirlpool	160	30	0.67	0,49253731
SunFlower	140	49	1.18	0,15254237
M77	1000	85	0.76	0,31578947
Condor	100	261	7.42	0,86522911
Carwheel	4	75	10.67	0,90627929
Malin 1	1000	325	2.92	0,65753425
Phonix Cluster	2000000	550	0.11	8,09090909
GN-z11	1	1.5	0.43	1,3255814

So, there are still values of φ varying from 0.18 to 9.7 such as the one in Table 2 established from Ref. 18. How can such discrepancies be explained? Probably three other parameters come into play.

— The mass/density distribution that could increase creep in certain areas than in others created globally.
— The time of loading between the center of the galaxy with more gravity than in the periphery.
— The temperature that can impact the Earth material is the creep value (see Ref. 32 on the possible thermal expansion coefficient of the spacetime fabric).

Concerning the first parameter, we can see from the reading of Table 1 that several galaxies interact with others, some are barred spiral or not, and others are giant or dwarf, therefore of different shapes, which necessarily implies a distribution of nonuniform matter that more or less overloads the space fabric and therefore gives by neglecting this phenomenon large variations in the value of the creep coefficient. In our opinion, in order to refine our model, it would be necessary to calibrate the creep coefficient as a function of the mass density at the place, where the creep is calculated ρ_{local} with respect to the mean value ρ_{mean}:

$$\varphi_{\text{space}} = \frac{a_0}{a}\frac{\rho_{\text{local}}}{\rho_{\text{mean}}} - 1. \tag{45c}$$

Regarding the second parameter, the loading time should influence the creep intensity.

Based on the above database, we see that the Milky Way, which is rather recent, and GN-Z11, which is roughly the age of the universe, lead to the same value of the creep coefficient. Therefore, more data would be needed to confirm or refute this stability over time.

Regarding the third parameter, the effect of temperature, we would need to have the temperature variations of each of the galaxies above to quantify how it would influence the fabric of spacetime.

Thus, if we are interested in the shape of galaxies and their mass distribution, we could explain these differences.

To verify the magnitude of our results, we can compare them with those obtained in Ref. 18, where the authors calculated the variation of the radius of the galaxy with an effect inherent to the fabric of space itself. Thus, the variation in radius of the galaxy expressed in the form of the ratio R/s can be interpreted according to our approach as the direct value of the creep linked to the variation in horizontal arrow of the galaxy in its plane.

We find that the results are consistent with our approach in many cases (see Table 2).

Reference 18 and Table 2 describe a geometric effect of creep (modification of shape and so curvature of the geometric structure of the space fabric) that may be complementary to the effect described in Table 1 (intrinsic elastic effect of the nature of the space fabric itself). If $G_{\text{Long term}} > G$ is considered as causing the dark matter effect, creep could have caused the extra space "dimples" described in Ref. 18 that would further amplify gravity.

Additionally, the thermal gradient between the cosmic fabric and galaxies and the vacuum can also create additional curvature that changes the geometry and that can also influence the intrinsic creep of the elastic fabric.[32]

At this stage, by this approach we arrive at creep coefficients for space between 0.2 and 9.

8. Evaluation of the Creep Coefficient of Space from a Representative and Documented Gravitational Lensing Effect — The Ball Cluster

Dark matter also manifests itself through gravitational lensing. We then measure deviations of light that are greater than that which should be measured with the effect of visible matter alone.

As a reminder, the angular deviations by gravitational lensing obtained from general relativity are worth[33]:

$$\alpha = \frac{4GM}{c^2R}, \tag{46}$$

where R is the minimum distance between the light ray and the massive object, M is the mass of the object, c is the speed of light, and G is the gravitational constant.

The galaxy cluster that most highlights the effect of dark matter is the ball cluster.

According to Ref. 34, the mass share due to visible mass and dark matter mass varies from 60% near the center of the cluster to 20% at the periphery (Fig. 12 of Ref. 34). So roughly 20% of mass bound to visible matter and 80% mass bound to dark matter.

From these data, we can make an estimate of the associated creep coefficient.

We first write that the percentages of mass bound to visible matter p_v and bound to dark matter p_{DM} are 1:

$$p_v + p_{\mathrm{DM}} = 1, \tag{47}$$

which translates based on formula 46 into percentage of deflection angles for the visible mass α_v and the dark matter mass α_{DM} given the proportionality between the deflection angle α and the mass M that causes it[33] by

$$\alpha_v + \alpha_{\mathrm{DM}} = 1. \tag{48a}$$

The creep can be seen as an increase of the angle due to the visible matter α_v by a factor φ:

$$\alpha_v + \varphi\alpha_v = 1, \tag{48b}$$

$$\alpha_v(1+\varphi) = 1. \tag{48c}$$

We express how much the total deviation is greater than the deviation related to the visible mass α_v by a factor k. Based on proportionality between α and M and so between α_v and p_v we obtain

$$k\alpha_v = kp_v = 1, \tag{49}$$

$$k = \frac{1}{\alpha_v} = \frac{1}{p_v}. \tag{50}$$

This amplification is expressed by a creep effect by the amplification of the strains of space, based on (48c) and (49) we obtain

$$k = 1 + \varphi. \tag{51}$$

This therefore implies for the creep coefficient φ:

$$\varphi = k - 1. \tag{52}$$

With the expression of k as a function of the mass percentages and

$$\varphi_{\mathrm{space}} = \frac{1}{\alpha_v} - 1 = \frac{1-\alpha_v}{\alpha_v} = \frac{1-p_v}{p_v}. \tag{53}$$

8.1. *Numerical application*

With the data of Ref. 34 we obtain the following:

— For a radius close to the center of the galaxy $p_v = 0.6$ gives $\varphi = 0.66$
— For a radius far from the center of the galaxy gives $p_v = 0.2$ gives $\varphi = 4$.

The values are therefore consistent with those obtained from the MOND law close to the center of the galaxy but far from it for large radii.

9. Expression of a Creep Coefficient in Relation to Time Fluctuations in Weak Gravitational Fields

9.1. *Expression of the time creep coefficient*

We have seen in the previous paragraphs that we can replace the effects of dark matter in space in the case of the rotation of galaxies by an equivalent creep of the latter.

But since general relativity implies that in connection with special relativity we must also consider time, we must therefore evaluate from the experiments carried out on the lengthening and shortening of time (effect of gravitation and special relativity around the Earth by GPS effect, effect of desynchronization of clocks in reverse motion around the Earth in airplanes compared to clocks on Earth) what this creep coefficient would be time.

In the case of space (see Damour lecture[35]) it can be shown that curvature results in a satisfactory variation of angles:

$$\hat{A} + \hat{B} + \hat{C} \approx \pi\left(1 + \frac{2GM}{rc^2}\right). \tag{54}$$

General relativity for gravitation and special relativity tells us that time lapse varies. Thus, if the tic – tac translates this duration dt of time, it lengthens and shortens in general relativity according to the following formula for the effect of time between the time on the surface of the Earth and the Earth away from the gravitational field[35]:

On the Earth surface, we have so

$$\Delta t_{\text{Surface}} = d\tau = \Delta t_\infty \sqrt{1 - \frac{2GM}{rc^2}}. \tag{55}$$

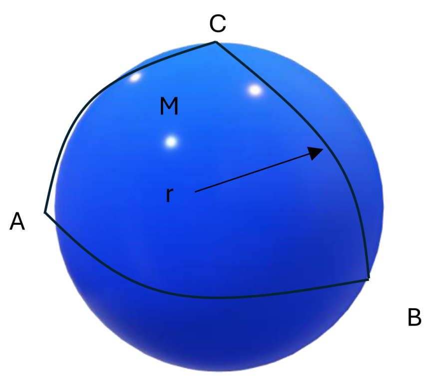

Fig. 2. Effect of the curvature of space on Earth.

The interval is written in the case of Schwarzschild's metric for the time with $d\tau$ the proper time:

$$d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right)dt^2. \tag{56}$$

That can be written with a limited development in weak field:

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{\left(1 - \frac{2GM}{rc^2}\right)}} \approx \frac{1}{1 - \frac{GM}{rc^2}}. \tag{57}$$

We see that the lapse of time $\tau d < dt$, on the Earth surface, the lapse of time is smaller than at a distance r of the center of gravity mass, so the time $(t+d\tau)$ passes less quickly than at a distance r with $t + dt$, so there is a time dilatation on Earth by gravitation

$$t + d\tau < t + dt. \tag{58}$$

We know from Ref. 5 that the ratio between the proper time and the time of the observer is connected with the volumetric stress ε^{3D} Mathematically, the following variational expression where $d\tau$ for the proper time variation and dt for the time variation applies

$$\frac{d\tau}{dt} = \frac{1}{(1 + \varepsilon^{3D})}. \tag{59}$$

In Ref. 5 the author explains the link between the time component and the volumetric stress noting $\varepsilon^{\mathrm{3D}} = \varepsilon_i^i$. This volumetric stress is also connected with the shear modulus μ by the expression as follows:

$$\mu = \frac{1}{(1 + \varepsilon^{\mathrm{3D}})^3}\mu_0, \tag{60}$$

where μ_0 is the reference modulus of the undeformed fabric.

We know from Ref. 5 that the shear modulus μ is connected to Poisson's ratio that is 1 for space and time in our model based on the gravitational wave displacement and associated strains measured by the interferometer Ligo/Virgo:

$$\mu = \frac{Y}{2(1 + \nu)}. \tag{61}$$

We know also with Refs. 23 and 24 (as the explanation given in Ref. 5 about the Poisson ratio of 1) that this value of ν implies a particular shape of the spacetime as small grains or fibers that can be distorted (fibers, polygonal form grain, etc. see first part of this paper).

This possible grain form of the time associated with spacetime granularity is developed in quantum loop gravity[21] and also explained in Refs. 17 and 22 "Granular Space–time: The Nature of Time Carlton Frederick of 2022." In this publication,

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online with a granular time, the author considers that to define time we need not 1 parameter but 2. I quote “a coordinate (t) from minus to plus infinity (or from the big bang to some end of time), and a sequencer (v) (a measure of something coming before or after something else, and the interval between them), an ordering schema as described by H. Reichenbach[36] determining the direction and ‘speed’ of time in link with the arrow of time.” Based on Ref. 36, these schema have the same form that the typical form source of creep in space defined in the first part (e.g. rectangular hexagon following Ref. 22 a sort of “time polymer shape” we can say) that can be so distorted and thus can create creep of sequence of the time (rather that the time itself). But in the end the variation of time is concerned by this potential sequencer variation that will be studied below.

Thus, we arrive at the same hypothesis given as the basis of the quantum gravity that the granularity of time[22] is also the base of the quantum loop gravity[21] and interweaving of loops. And this type of fabric is sensible at creep. Spacetime is considered as well as fabric both for space and time as done in Ref. 5. So, we have to find the formula to connect the time creep and the general relativity information of the time elasticity. In the weak field we have the following[5]:

$$1 + 2\varepsilon_{00} \approx g_{00}. \tag{62a}$$

The proper time takes the following expression with convention for the interval (+---):

$$d\tau = g_{00} dt. \tag{62b}$$

That becomes

$$\frac{d\tau}{dt} = (1 + 2\varepsilon_{00}). \tag{63}$$

That we compare with Eq. (56) reformulated

$$\frac{d\tau}{dt} = \left(1 - \frac{GM}{rc^2}\right), \tag{64}$$

where $d\tau$ is smaller than dt.

By comparison with (57) and keeping the first term as ε_{00} is small:

$$\varepsilon_{00} = -\frac{GM}{2rc^2}. \tag{65}$$

So, this strain about time becomes from Eq. (64):

$$\frac{d\tau - dt}{dt} = -\frac{GM}{rc^2}. \tag{66}$$

Taking into account that creep implies an increase of G by a factor $(1 + \varphi)$ we obtain

$$\frac{d\tau - dt}{dt} = -\frac{GM(1 + \varphi)}{rc^2}. \tag{67}$$

We can extract φ from (67):

$$-\left(\frac{d\tau - dt}{dt}\right)\frac{rc^2}{GM} - 1 = \varphi \tag{68}$$

or equivalently

$$\left(\frac{dt - d\tau}{dt}\right)\frac{rc^2}{GM} - 1 = \frac{\delta t}{\delta t_0}\,\frac{rc^2}{GM} - 1 = \left(\frac{\frac{\delta t}{\delta t_0}}{\frac{GM}{rc^2}} - 1\right) = \varphi. \tag{69}$$

9.2. *Numerical application to the effect on Earth in the case of time offset with GPS satellites*

It is well known that Newtonian gravitation works well in weak fields. Therefore, any creep effect around the Earth at a very short distance must be negligible. Therefore, the effects of dark matter must be insignificant on spacetime around the Earth. This is what we will try to verify in this paragraph.

The value of the deformation of time in the case of the Earth is first determined $\frac{c\delta t}{c\delta t_0}$.

At $r = 20\,000\,\text{km}$ from the Earth, the GPS shift measured per day from 38 to 45 μs.[37]

But the part due to the strict gravitation effect is $+45.9\,\mu$s and $-7\,\mu$s for special relativity. So, we keep the gravity part to isolate the gravity effect alone:

$$\frac{\delta t}{\delta t_0} = \frac{45.9 \times 10^{-6}}{24 \times 60 \times 60} = 5.3125 \times 10^{-10}.$$

We time dilation ($\varepsilon_{00,\text{satellite}}$) of the satellite level is

$$\frac{GM}{rc^2} = \frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{26371000 \times 299792458^2} = 1.681166 \times 10^{-10}.$$

At the surface of the Earth the time dilatation $\varepsilon_{00(\text{Earth})}$ is

$$\frac{GM}{rc^2} = \frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{6371000 \times 299792458^2} = 6.96077 \times 10^{-10}.$$

So, the strain variation between the satellite level and the Earth surface is

$$6.96077 \times 10^{-10} - 1.681166 \times 10^{-10} = 5.279604 \times 10^{-10}.$$

Let us have a creep coefficient of: φ_{time}:
$\varphi_{\text{time}} = \left(\frac{5.3125\times10^{-10}}{5.27\times10^{-10}}\right) - 1 = 0.00632568.$

So in fact we obtain that the creep effect is quasi-null around the Earth, which is normal because Newton's law works pretty well!

10. Discussion of the Results

Thus, such creeps tend to show that space behaves like a granular medium. This overlaps Refs. 16 and 17 on the transverse isotropy of spacetime with regard to its behavior under gravitational waves.

Of course, this modification of the gravitational constant in $\frac{a_0}{a}$ is linked to only MOND's theory for the constancy of the speed of stars in the galaxy. As it potentially also affects Einstein's constant κ, and as it is supposed to overcome the problem of the absence of detection of dark matter as a particle but that gravitational anomalies are very real, this modification should also be tested wherever the presence of dark matter is required (gravitational lensing, distribution of dark matter at the time of the big bang, etc.) to be sure that this interpretation in terms of creep given by the elastic analogy is the right one. Additional tests should also be carried out for the temporal approach in a large gravitational field to confirm this value for space of the creep coefficient φ of the order of magnitude between 0.2 and 9.

It is well known that dark matter does not have a small effect because in gravitation this unknown mass effect represents about 30% of the overall part that constitutes the universe. So, while we are trying to replace dark matter with a creep effect, this creep effect is also not minimal but corrects an important part of general relativity at the cosmological level.

From a more general point of view, this creep approach significantly modifies Einstein's field equation which becomes

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \frac{8\pi G}{c^4}(1+\varphi) t_{e,\mu\nu}, \tag{70}$$

where $t_{e,\mu\nu}$ is the stress energy tensor connected with the space fabric itself as an elastic medium proposed by Tartaglia in Ref. 38 and Beau in Ref. 39.

This additional stress-energy tensor is also activated to explain the energy of the gravitation wave in vacuum.[40] The same approach is followed when we want to establish the energy of gravitational waves in a vacuum. The Einstein field equation becomes

$$G^{(1)}_{\mu\nu} = -\frac{8\pi G}{c^4}(T_{\mu\nu} + t_{\mu\nu}), \tag{71}$$

where $G^{(1)}_{\mu\nu}$ is constructed from $G_{\mu\nu}$ terms which are linear in $h_{\mu\nu}$ (see formula 4 of Ref. 40):

$$t_{\mu\nu} = T^{GW}_{\mu\nu} = \frac{c^4}{8\pi G}[G^{(2)}_{\mu\nu} + \cdots], \tag{72}$$

where $G^{(2)}_{\mu\nu}$ is constructed from quadratic terms of $h_{\mu\nu}$ (see formula 5 of Ref. 40).

Or more in accordance with our approach of additional curvature or amplification of the curvature due to the only visible mass/energy that does not change, the effects of dark matter (gravitational amplification) on vacuum are thus taken into account without the need for additional dark mass $\varphi > 0$). So, in vacuum we have

$$\frac{1}{1+\varphi}\left\{R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R\right\} = \frac{8\pi G}{c^4} t_{e,\mu\nu}. \tag{73a}$$

We verify well that as $\varphi > 0$ then $1/(1+\varphi) < 1$, so the curvature is reduced, so, the equivalent R radius increases and deflection too.

$$\left\{R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R\right\} = \frac{8\pi G}{c^4}(1+\varphi) t_{e,\mu\nu}. \tag{73b}$$

As $\varphi > 0$, we verify well that flexibility increases, so the rigidity decreases, and so the strains increase and the gravitation increases. $(1+\varphi)t_{e,\mu\nu}$ can be seen also as an additional dark matter that increases the gravitational effects.

11. Conclusion

We have therefore shown that within the framework of the elastic medium analogy, the effects of dark matter can be interpreted in the case of the anomaly of the star velocities around galaxies via the MOND theory or gravitational lens as bullet cluster or from time dilatation data as a creep of the space medium.

This kind of behavior is typical of granular crystal[17] and therefore overlaps with other clues about the behavior of space:

- The regular repetition of the peaks in the case of X-ray diffractograms of lamellar crystal or clay in sheets as a function of the angles of refraction[41] and the peaks of the cosmic microwave background of the power spectra in temperature and polarizations when the photons passed through the Cosmic plasma at the time of the big bang giving the cosmic microwave background[42]).
- The great plasticity and creep effect of the crystal or granular medium under shear,[26] and their ability to self-clog are comparable to the absence of residual tears in spacetime behind rotating black holes.
- The anisotropic Poisson coefficients 1 in one direction[5] and very weak in the other.

Quantum gravity, even if it has not yet been validated by experiment, also leads to quantification of spacetime in small grains of the size of Planck. Our result is therefore consistent with the trend in current physics of spacetime granularization. It remains to test this creep coefficient on the other physical phenomena where dark matter is involved to confirm or deny its value on the one hand and to verify whether we really need this dark matter to explain the discrepancy between the observed gravitation and the only visible masses that cause it or if, on the contrary, it is a decoy in the sense that spacetime would flow under the only clearly visible masses. Finally, in the latter case, still, according to the analogy of the elastic medium, space would behave like a crystal-type material generating deformations by creep and therefore gravitation in time under constant loading, i.e. a well-visible, classical and identified mass. But like any resolution of one mystery, another appears, if spacetime is subject to creep, what is it constituted to have such a behavior? grains of spacetime as proposed by loop quantum gravity? The question remains open.

Moreover, if dark matter is only a creep effect, time necessarily intervenes in the evolution of deformations. It is therefore necessary to study how dark matter has evolved over time. If it is stable, then it cannot be a creep effect. If, on the other hand, it varies and the gravitational effects increase like the distortions of spacetime over time (a little like the small fluctuations in the density of the original plasma, we lead 13.7 billion years later to a cosmic web with bubbles of very large dimensions where on the surface there are galaxies and voids within the breasts of its bubbles) then the hypothesis of the creep of spacetime instead of and the place of a dark matter that remains invisible to detection to this day remains credible. Time and experience will give us the answer if the creep coefficient for space φ is effectively in the range $+0.2/+9$.

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