VTT–Orbital Genesis: The Electron as a Residual Geometry of Informational Curvature

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Article

VTT–Orbital Genesis: The Electron as a Residual Geometry of Informational Curvature

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Abstract

In standard quantum mechanics, the electron is treated as a fundamental particle whose wavefunction describes a spatial probability distribution. While this formalism provides extremely accurate predictions, the conceptual relationship between orbital geometry, particle localization, and wave–particle duality remains interpretatively open. In this work, we propose a geometric reinterpretation within the framework of Viscous Time Theory (VTT). In this view, atomic orbitals arise as stabilized basins of informational curvature within a viscous informational manifold, and the electron emerges as the undissipated residual of this geometric formation. By introducing the Informational Hessian as the curvature tensor associated with coherence deviation ΔC, orbital stability can be formulated as a positive-definite curvature condition over the informational manifold. Within this framework, electron mass is reinterpreted as an integrated curvature excess associated with stabilized orbital geometry. This approach provides: (i) a geometric interpretation of wave–particle duality as periodic coherence recall, (ii) a reinterpretation of excited states as metastable curvature attractors, and (iii) a potential structural mechanism for residual mass generation within stabilized informational structures. The proposed framework is presented as a constructive extension compatible with Schrödinger dynamics. Rather than replacing the standard formalism, we suggest the existence of a deeper geometric layer whose implications invite further mathematical and physical investigation.

Keywords: Viscous Time Theory (VTT); informational curvature; informational hessian; orbital geometry; electron mass; spectral decomposition; Laplacian eigenmodes; residual functional; curvature invariants; wave–particle duality; metastability; geometric interpretation of quantum mechanics; informational viscosity; coherence transport

1. Introduction

The conceptual ontology of the electron remains one of the central interpretative questions in modern physics. In standard quantum mechanics, the electron is treated as a fundamental particle whose behavior is described by a wavefunction ψ encoding probability amplitudes across space [1]. The solutions of the Schrödinger equation determine the allowed orbital structures in atoms, and these solutions successfully predict spectral lines, chemical structure, and numerous quantum phenomena with remarkable precision [2].

Despite this predictive success, several conceptual tensions persist. First, wave–particle duality remains interpretatively unresolved. The wavefunction provides a probabilistic description of measurement outcomes, yet the physical status of the electron between measurements continues to generate debate across multiple interpretations of quantum mechanics [3–5]. Second, the origin of the electron's mass is not explained within the non-relativistic Schrödinger framework and is instead introduced as a fixed parameter inherited from deeper field-theoretic models [6]. Third, the mathematical structure of atomic orbitals appears to precede the localization of the electron itself: orbital geometry arises as a solution of the governing equations, while the particle is typically interpreted as occupying these pre-existing probability distributions.

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These observations motivate the exploration of alternative geometric interpretations of quantum structure. Several approaches in modern theoretical physics have proposed that physical particles may emerge from deeper informational or geometric structures of space–time or fields [7–9]. In particular, recent developments in informational physics suggest that physical systems can be interpreted as manifestations of underlying informational or coherence-based structures [10–12].

In this work we explore a related but distinct perspective within the framework of Viscous Time Theory (VTT). Within VTT, physical structures emerge from coherence folding processes occurring within a viscous informational manifold. Instead of treating the electron as the primary object generating an orbital probability cloud, we propose a reversal of causal interpretation: the orbital geometry arises first as a stabilized basin of informational curvature, and the electron emerges as the undissipated residual density associated with this stabilized geometric structure.

Under this interpretation, orbital formation corresponds to a curvature-stabilization process in the informational manifold. The stability of such structures can be analyzed through the curvature properties of a coherence deviation field ΔC. We introduce the Informational Hessian as the curvature tensor of this field and show that orbital stability corresponds to a positive-definite curvature condition analogous to basin stability in geometric systems.

Importantly, this approach does not attempt to replace the standard quantum mechanical formalism. Instead, it proposes a geometric layer underlying the conventional description, within which orbital solutions can be interpreted as stabilized curvature structures and the electron as a residual geometric manifestation of this stabilization process.

The remainder of this work develops this framework in detail. Section 2 introduces the concept of informational curvature and defines the Informational Hessian used to characterize orbital stability. Section 3 analyzes the geometric conditions under which stabilized orbital basins produce localized residual structures interpreted as electrons. Section 4 discusses the implications of this framework for wave–particle duality, excited states, and residual mass generation. Finally, Section 5 summarizes the results and outlines directions for further theoretical investigation.

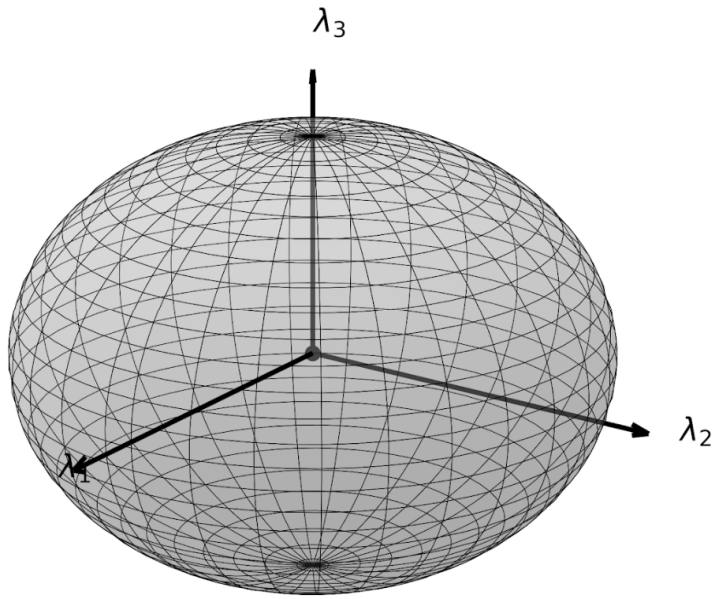

Figure 1. Spectral decomposition of the Informational Hessian. Principal curvatures λ_k determine basin stability and classify orbital "residences" versus metastable configurations.

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2. Material and Methods

Section 2 introduces the geometric framework used to reinterpret atomic orbital structure within the VTT informational manifold. We first define the curvature properties of the coherence deviation field ΔC and show how stabilized curvature basins correspond to orbital geometries. We then interpret electron mass as a residual curvature excess and analyze the dynamical implications of this structure for wave–particle duality and excited states.

2.1. Informational Curvature and Orbital Formation

We begin by introducing the curvature structure of the informational manifold. Within the VTT framework, atomic orbitals are interpreted as stabilized basins of informational curvature generated by the coherence deviation field ΔC.

Let $\Delta C(x)$ denote the local coherence deviation field over spatial manifold $\Sigma \subset \mathbb{R}^3$.

We define the Informational Hessian:

$$\mathcal{H}_{ij} = \frac{\partial^2(\Delta C)}{\partial x_i \, \partial x_j}. \qquad (1)$$

The Hessian characterizes local curvature of the coherence field.

An orbital configuration corresponds to a stabilized basin of informational curvature satisfying:

Stationarity of gradient:

$$\nabla \Delta C = 0, \qquad (2)$$

Positive-definite curvature:

$$\lambda_k(\mathcal{H}) > 0 \forall k, \qquad (3)$$

ensuring that coherence is trapped within a locally convex informational topology.

Thus, the orbital is interpreted as a **primary curvature residence** — a geometry capable of sustaining bounded coherence flow under viscous dissipation.

Importantly, this curvature condition mirrors the stability requirement for potential wells in classical mechanics, but here arises from intrinsic informational geometry rather than external potential imposition [13].

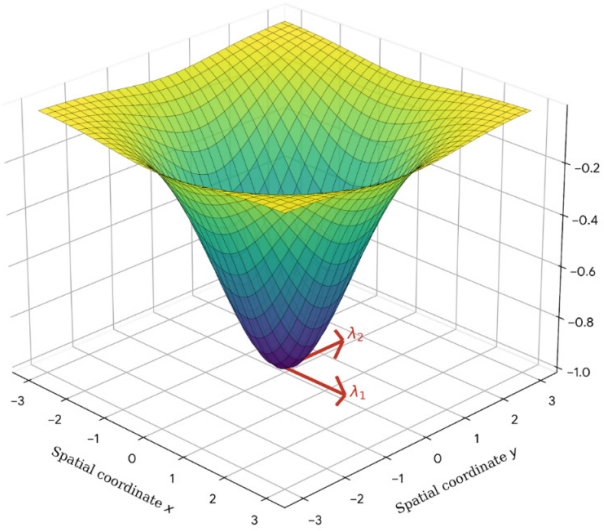

Figure 2. Example of a stabilized informational curvature basin representing an orbital manifold Σ_{orb}. The convex region corresponds to a positive-definite Informational Hessian, ensuring geometric stability of the curvature basin. Electron localization is interpreted as emerging from curvature stabilization rather than external potential confinement.

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2.2. Electron Mass as Curvature Residual

Once orbital stabilization is established as a curvature phenomenon, the next question concerns the origin of localized mass within this geometry. In the proposed framework, mass does not arise as an independent parameter but as a residual curvature density that remains after curvature stabilization**.**

If curvature stabilization were perfectly dissipative, the system would contain only geometry and no localized mass.

However, stabilization occurs through an admissibility gate and finite viscosity. A residual coherence density remains that cannot be fully absorbed into smooth curvature.

We formalize the electron mass as a curvature excess functional:

$$m_e = \kappa \int_{\Sigma_{orb}} (\mathrm{Tr}(\mathcal{H}) - \langle \mathrm{Tr}(\mathcal{H}) \rangle)_+ \, d\Sigma, \qquad (4)$$

Here $\Sigma_{orb} \subset \Sigma$ denotes the stabilized orbital basin associated with the curvature minimum of the coherence deviation field ΔC. In geometric terms, Σ_{orb} represents the compact region of the informational manifold in which the Informational Hessian remains positive-definite and coherence transport is locally confined.

where:

- κ is a dimensional coupling constant,
- $\mathrm{Tr}(\mathcal{H}) = \nabla^2 \Delta C$,
- $\langle \cdot \rangle$denotes mean curvature over the orbital basin,
- $(\quad \cdot)_+$selects curvature excess above equilibrium.

Under this interpretation:

- The majority of the field energy contributes to the formation of the orbital geometry itself.
- The electron's small mass corresponds to the residual curvature that cannot be smoothed by viscous transport.
- The electron is thus the **localized remainder of a curvature construction process**.

This construction does not contradict the Schrödinger equation; rather, it proposes that Schrödinger solutions describe the stable curvature modes of the ΔC-field.

2.3. Periodic Coherence Recall and Wave–Particle Duality

The curvature interpretation of orbitals also suggests a dynamical reinterpretation of wave–particle duality. Stabilized curvature basins do not imply static geometry; rather, coherence transport within the basin produces periodic curvature modulation.

Within the proposed framework, orbital stabilization does not imply static geometry.
Coherence transport remains active along admissible corridors, generating periodic modulation of curvature density.

Let the informational derivative along viscous time be defined as:

$$\frac{D}{D\tau}(\Delta C). \qquad (5)$$

Stationary orbitals correspond to configurations where:

$$\frac{D}{D\tau}(\Delta C) = 0 \qquad (6)$$

in the averaged sense, while permitting bounded oscillatory fluctuations.

These bounded oscillations can be represented as:

$$\Delta C(x,\tau) = \Delta C_0(x) + \epsilon(x) e^{-i\omega\tau}. \qquad (7)$$

This structure mirrors the stationary-state solution of the Schrödinger equation:

$$\psi(x,t) = \phi(x) e^{-iEt/\hbar}. \qquad (8)$$

In this reading:

- The exponential time factor corresponds to periodic curvature recall.
- The spatial eigenfunction corresponds to the stabilized curvature basin.

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- Localization events correspond to admissibility-triggered collapse of curvature oscillation into density excess.

Wave–particle duality is thus interpreted as:

Oscillatory coherence modulation around a stabilized geometric basin.

The "particle" appears when oscillatory curvature residual crosses admissibility threshold.

This interpretation preserves standard quantum predictions while providing geometric interpretation.

2.4. Excited States as Metastable Curvature Attractors

The curvature framework further provides a natural interpretation of excited atomic states. Higher-energy orbitals correspond to metastable curvature attractors whose stability is maintained by viscous coherence transport.

Higher-energy orbitals correspond to higher-order curvature basins satisfying:

$$\nabla\Delta C = 0, \text{with mixed eigenvalue structure.} \quad (9)$$

Unlike the ground state, excited states may contain saddle-like directions that remain dynamically stabilized by viscous transport.

Metastability condition:

$$\lambda_{min}(\mathcal{H}) > -\eta_I, \quad (10)$$

where η_I is informational viscosity.

This implies:

- Excited orbitals are not purely potential wells.
- They are curvature attractors sustained by coherence transport.
- Transition between orbitals corresponds to topological reconfiguration of the curvature basin.

Spontaneous emission may then be interpreted as:

Curvature collapse from higher-order metastable basin to lower-energy stable basin, releasing coherence excess.

2.5. Residual Curvature Functional

To formalize the interpretation of electron mass as a residual curvature phenomenon, we introduce a functional that measures curvature excess within a stabilized orbital basin. The purpose of this section is to define this residual curvature functional in a mathematically consistent manner under minimal geometric assumptions on the coherence deviation field ΔC.

We first specify the regularity conditions and stability properties required for the informational curvature basin, and then construct the functional that extracts the curvature excess associated with localized mass.

We begin by specifying the regularity and stability conditions for the informational curvature field within the orbital basin.

2.5.1. Preliminaries

Let

$$\Delta C: \Sigma \subset \mathbb{R}^3 \to \mathbb{R} \quad (11)$$

be a twice continuously differentiable coherence deviation field:

$$\Delta C \in C^2(\Sigma). \quad (12)$$

Assume:

1. **Compact orbital basin**

$\Sigma_{orb} \subset \Sigma$ is compact and bounded.

2. **Stationary point condition**

$$\nabla\Delta C = 0 \text{on } \Sigma_{orb}. \quad (13)$$

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3. **Stability condition**

$$\lambda_k(\mathcal{H}) \geq 0 \text{for all eigenvalues.} \quad (14)$$

The Informational Hessian is:

$$\mathcal{H}_{ij} = \frac{\partial^2(\Delta C)}{\partial x_i \partial x_j}. \quad (15)$$

Its trace satisfies:

$$\mathrm{Tr}(\mathcal{H}) = \nabla^2 \Delta C. \quad (16)$$

2.5.2. Definition of the Residual Functional

We begin by specifying the regularity and stability conditions for the informational curvature field within the orbital basin.

Define the curvature excess:

$$R(x) = (\mathrm{Tr}(\mathcal{H})(x) - \langle \mathrm{Tr}(\mathcal{H}) \rangle)_+, \quad (17)$$

where

$$\langle \mathrm{Tr}(\mathcal{H}) \rangle = \frac{1}{|\Sigma_{orb}|} \int_{\Sigma_{orb}} \mathrm{Tr}(\mathcal{H}) d\Sigma. \quad (18)$$

Then define the residual mass functional:

$$\mathcal{M}[\Delta C] = \kappa \int_{\Sigma_{orb}} R(x)\, d\Sigma. \quad (19)$$

The residual curvature functional defined above provides a quantitative measure of localized curvature density within a stabilized orbital basin. In this interpretation, electron mass corresponds to the integrated curvature excess that cannot be smoothed by viscous coherence transport.

The mathematical properties of this functional, including its non-negativity, existence under non-uniform curvature conditions, and stability under small perturbations, are presented in Appendix A.

Having established the definition of the residual functional, we now examine the spectral structure of the Informational Hessian and the role of curvature eigenmodes in determining the spatial localization of curvature residuals. This analysis forms the basis of the spectral framework developed in Section 3.

3. Results

Having introduced the geometric interpretation of orbital basins and the residual curvature functional, we now analyze the spectral structure of the Informational Hessian. The eigenvalues of this operator encode the principal curvature directions of the coherence deviation field ΔC and therefore provide a natural coordinate-free characterization of orbital stability, metastability, and curvature localization. This section develops the spectral framework that connects orbital geometry, curvature invariants, and the localization of residual curvature density.

3.1. Spectral Structure of the Informational Hessian and Curvature Invariants

3.1.1. Hessian Spectrum and Principal Curvatures

Let $\Delta C \in C^2(\Sigma)$. The Informational Hessian is

$$\mathcal{H}(x) = (\mathcal{H}_{ij}(x))_{i,j=1}^3 = \left(\frac{\partial^2(\Delta C)}{\partial x_i \partial x_j}\right). \quad (20)$$

Since $\mathcal{H}(x)$ is a real symmetric matrix, it admits an orthonormal eigen-decomposition:

$$\mathcal{H}(x) = Q(x)\, \Lambda(x)\, Q(x)^{\mathsf{T}}, \quad (21)$$

where $Q(x) \in SO(3)$ and

$$\Lambda(x) = \mathrm{diag}(\lambda_1(x), \lambda_2(x), \lambda_3(x)). \quad (22)$$

The eigenvalues $\lambda_k(x)$ are interpreted as **principal informational curvatures** of the coherence deviation field and therefore encode the local geometric structure of the orbital basin.

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3.1.2. Curvature Invariants (Coordinate-Free Quantities)

While the eigenvalues depend on the chosen coordinate frame, several scalar combinations of the spectrum remain invariant under rotations. These curvature invariants provide coordinate-independent descriptors of orbital geometry.

The spectral decomposition yields canonical scalar invariants:

1. **Mean curvature density (trace / Laplacian)**

$$I_1(x) = \mathrm{Tr}(\mathcal{H}(x)) = \lambda_1 + \lambda_2 + \lambda_3 = \nabla^2(\Delta C). \quad (23)$$

2. **Quadratic curvature energy**

$$I_2(x) = \mathrm{Tr}(\mathcal{H}(x)^2) = \lambda_1^2 + \lambda_2^2 + \lambda_3^2. \quad (24)$$

3. **Determinant (volumetric curvature invariant)**

$$I_3(x) = \det(\mathcal{H}(x)) = \lambda_1\lambda_2\lambda_3. \quad (25)$$

These invariants are independent of coordinate choice and therefore natural objects for orbital characterization.

3.1.3. Stability and Basin Classification via Eigenvalue Signatures

In the orbital interpretation, Σ_{orb} is a stabilized basin region. Locally:

- **Stable basin (convex curvature well):**

$$\lambda_k(x) > 0 \ \forall k \Rightarrow \mathcal{H}(x) \succ 0. \quad (26)$$

- **Metastable basin (viscosity-stabilized saddle):**

$$\lambda_{\min}(x) > -\eta_I, \quad (27)$$

which permits limited negative curvature directions provided viscosity (coherence transport resistance) prevents runaway deformation.

This motivates the classification of orbital-like structures by the **Hessian signature**, a standard tool in differential topology and stability analysis.

This corresponds to a curvature saddle that would be unstable in a purely conservative system but becomes dynamically stabilized by viscous coherence transport.

3.1.4. Orbital "Residence" vs "Metastable Residence" as Spectral Geometry

The spectral structure of the Informational Hessian also provides a precise interpretation of the intuitive notion of orbital "residence" introduced in Section 2.

The intuitive "residence" language can be expressed precisely:

- A **primary residence basin** corresponds to a region where $\mathcal{H}$ is predominantly positive definite and the invariants satisfy:

$$I_1 > 0, I_3 > 0, I_2 \text{ large enough to enforce confinement.}$$

- A **metastable residence (excited-like)** corresponds to mixed signature with viscosity-controlled admissibility:

$$I_3 < 0 \text{ possible}, \lambda_{\min} > -\eta_I. \quad (28)$$

Thus, the qualitative occupancy concept is grounded in the spectral structure of $\mathcal{H}$.

3.1.5. Curvature Residual as an Invariant-Based Functional

The residual functional used in the main text was defined using $I_1 = \mathrm{Tr}(\mathcal{H})$. One may equivalently define residual density using other invariants (for robustness checks):

- Trace-based residual:

$$R_1(x) = (I_1(x) - \langle I_1 \rangle)_+. \quad (29)$$

- Quadratic-energy residual:

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$$R_2(x) = (I_2(x) - \langle I_2 \rangle)_+. \quad (30)$$

- Determinant-based residual (highly selective):

$$R_3(x) = (I_3(x) - \langle I_3 \rangle)_+. \quad (31)$$

Each choice defines a corresponding mass-like functional

$$\mathcal{M}_k[\Delta C] = \kappa_k \int_{\Sigma_{orb}} R_k(x)\, d\Sigma. \quad (32)$$

This provides an empirical program: if the framework is correct, residual characterization should be consistent across invariant choices (up to scaling), or else indicate which invariant best encodes "residual localization".

Because the residual functional depends only on curvature excess relative to the orbital basin average, alternative invariant definitions provide robustness checks for curvature localization.

3.1.6. Conservative Bridge to Eigenmodes (Schrödinger Compatibility)

An important observation concerns the relationship between the curvature operator introduced here and the Laplacian appearing in quantum mechanics. The trace of the Informational Hessian satisfies

$$I_1(x) = \nabla^2(\Delta C). \quad (33)$$

In this sense, Schrödinger eigenmodes impose spectral constraints on the wavefunction, while VTT orbital basins impose spectral constraints on the curvature operator $I_1 = \nabla^2(\Delta C)$. This suggests that orbital quantization may be interpreted as a discretization of stable curvature modes of ΔC, while remaining fully compatible with the conventional eigenvalue formalism.

An important observation concerns the relationship between the curvature operator introduced here and the Laplacian appearing in quantum mechanics. In conventional quantum theory, stationary states satisfy an eigenvalue equation involving the Laplacian operator $\nabla^2\psi$. While the present framework does not identify ΔC with the quantum wavefunction ψ, a structural compatibility emerges: both descriptions impose spectral constraints on Laplacian-type operators governing spatial structure.

Without asserting identity $\Delta C = \psi$, we note the following structural correspondence:

- Schrödinger eigenmodes impose spectral constraints on a Laplacian operator acting on the wavefunction.
- VTT orbital basins impose spectral constraints on the curvature operator $I_1 = \nabla^2(\Delta C)$.

This correspondence suggests that orbital quantization may be interpreted as a discretization of stable curvature modes of ΔC, while remaining fully compatible with the conventional eigenvalue formalism.

Summary : This section provides:

1. A coordinate-free spectral decomposition of the Informational Hessian,
2. Canonical invariants (I_1, I_2, I_3) for orbital classification,
3. Stability and metastability criteria grounded in eigenvalue signatures,
4. A robust family of residual functionals,
5. A conservative, operator-level bridge to Schrödinger eigenmodes.

3.2. Spectral Selection: Residual Functional Prefers High-Curvature Components

To understand how residual curvature localizes within an orbital basin, we analyze the spectral structure of the Laplacian operator on the orbital domain. This allows the curvature residual functional to be expressed in terms of Laplacian eigenmodes.

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3.2.1. Setup: Laplacian Eigenbasis on the Orbital Domain

Let $\Sigma_{orb} \subset \mathbb{R}^3$be a bounded domain with sufficiently regular boundary $\partial\Sigma_{orb}$. Consider the Laplacian eigenvalue problem (Dirichlet or Neumann boundary conditions) [14]:

$$-\nabla^2\varphi_n = \lambda_n\varphi_n, \lambda_n \geq 0, \quad (34)$$

with $\{\varphi_n\}_{n\geq 1}$forming an orthonormal basis of $L^2(\Sigma_{orb})$.

The eigenfunctions $\{\varphi_n\}$ provide a natural spectral basis for decomposing curvature fluctuations within the orbital domain.

Expand the coherence deviation field as:

$$\Delta C(x) = \sum_{n=1}^{\infty} a_n\, \varphi_n(x). \quad (35)$$

Then:

$$\nabla^2(\Delta C)(x) = -\sum_{n=1}^{\infty} \lambda_n\, a_n\varphi_n(x). \quad (36)$$

Since the trace of the Informational Hessian satisfies Tr(H) = ∇²(ΔC), the trace-curvature can be expressed through the Laplacian spectral expansion.

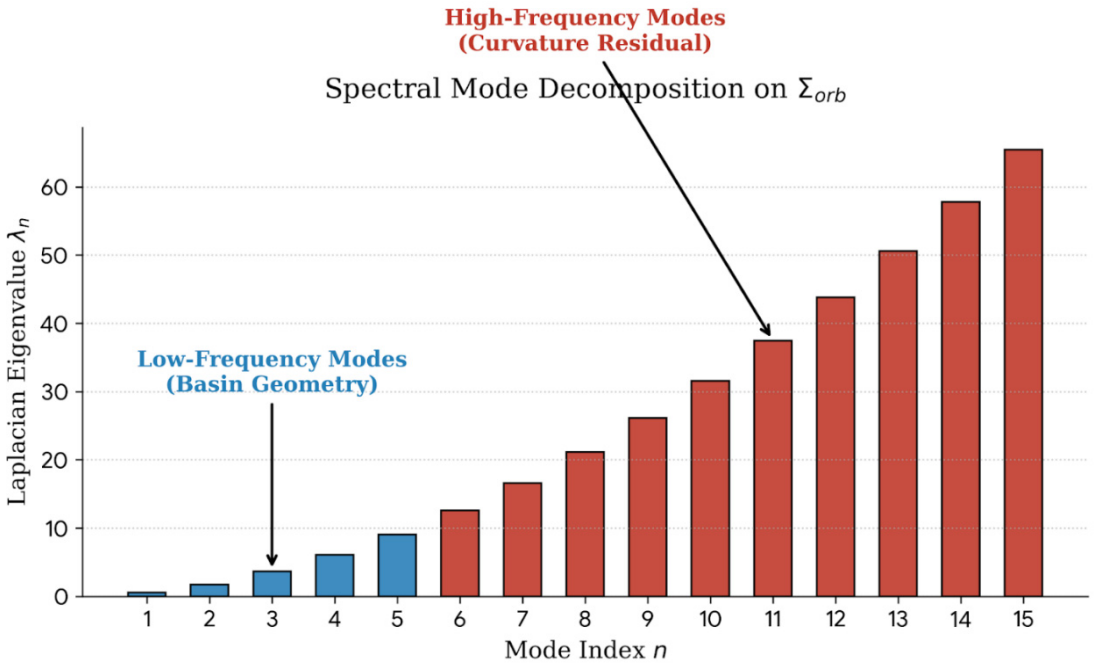

Figure 3(a): Spectral decomposition of Laplacian modes on the orbital domain Σ_{orb}. (conceptual illustration)

Conceptual illustration distinguishing low-frequency modes that define the stabilized basin geometry from higher-frequency modes that contribute to curvature-residual structure.

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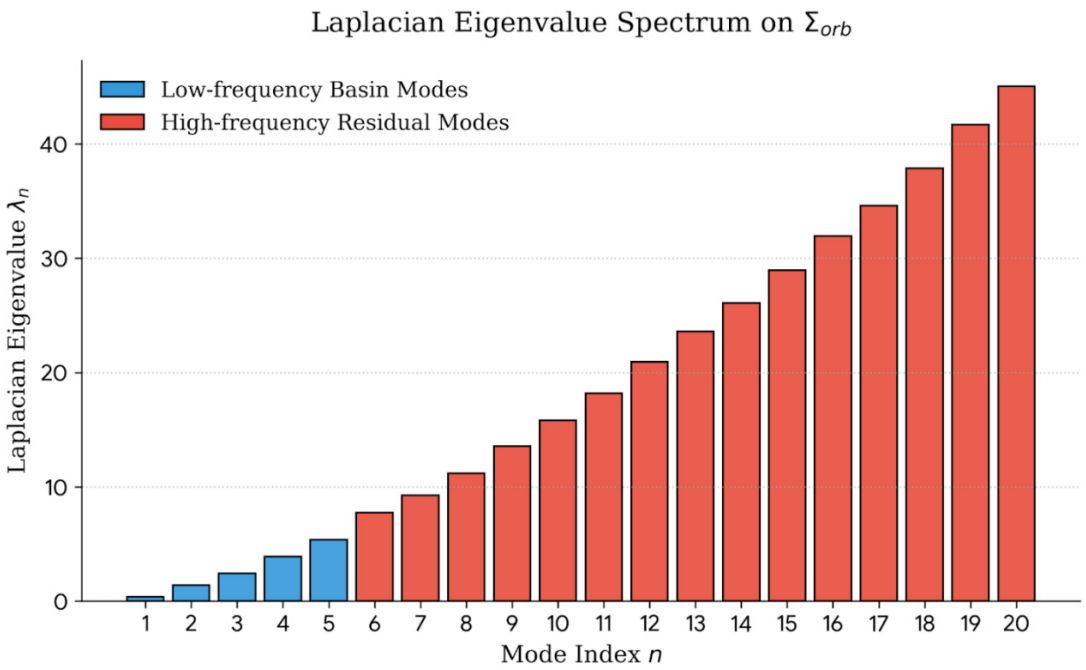

Figure 3(b): Spectral decomposition of Laplacian eigenvalue spectrum on $\boldsymbol{\Sigma_{orb}}$

Numerical example of the Laplacian eigenvalue spectrum. Low eigenvalue modes primarily describe the large-scale basin geometry, whereas higher eigenvalue modes contribute more strongly to curvature-residual generation.

3.2.2. Proposition (High-Curvature Preference)

Proposition 4 (Residual Selection Bias Toward Large $\boldsymbol{\lambda_n}$)

Assume $\Delta C \in H^2(\Sigma_{orb})$and define the trace-based curvature residual:

$$R_1(x) = (\nabla^2(\Delta C)(x) - \langle \nabla^2(\Delta C) \rangle)_+. \quad (37)$$

Then, among all fields ΔC with fixed L² energy $\| \Delta C \|_{L^2}$(i.e., fixed $\sum a_n^2$), the magnitude of curvature fluctuations $\| \nabla^2(\Delta C) \|_{L^2}$ is maximized when spectral energy is concentrated in modes with larger eigenvalues λ_n

In particular,

$$\| \nabla^2(\Delta C) \|_{L^2}^2 = \sum_{n=1}^{\infty} \lambda_n^2 a_n^2, \quad (38)$$

so contributions from modes with large λ_n are amplified quadratically.

Consequently, any residual functional built from curvature excess (including R_1) preferentially weights high-λ_n components, i.e. high-curvature / high-frequency structure.

Proof

From orthonormality:

$$\| \nabla^2(\Delta C) \|_{L^2}^2 = \int_{\Sigma_{orb}} (\sum_{n=1}^{\infty} \lambda_n a_n \varphi_n(x))^2 \, dx = \sum_{n=1}^{\infty} \lambda_n^2 a_n^2. \quad (39)$$

For fixed spectral energy $\| \Delta C \|_{L^2}^2 = \sum a_n^2$, the weighted sum $\sum \lambda_n^2 a_n^2$ increases when a larger portion of the spectral energy is allocated to modes with larger eigenvalues λ_n.

Since $R_1(x)$ is a positive-part transform of $\nabla^2(\Delta C)$ relative to its mean, larger curvature fluctuations generally enlarge regions where $\nabla^2(\Delta C)$ exceeds its mean, thereby increasing the integral of R_1(in the generic non-degenerate case).

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3.2.3. Interpretation (Conservative, Paper-Appropriate)

The above proposition formalizes an important structural feature of the curvature-residual mechanism:

- Orbital basins correspond primarily to low-frequency spectral modes describing stabilized geometry.
- Residual localization arises from curvature excess relative to the basin average.
- Curvature excess naturally emphasizes high-λ_n components.

Thus, within the VTT reinterpretation, the "electron residual" is mathematically consistent with the selection of higher-curvature spectral structure superimposed on a stabilized basin.

No modification of Schrödinger theory is required for this statement: the result follows from a spectral property of curvature-based residual functionals acting on the same Laplacian structure.

3.3. Numerical Recipe for Curvature-Residual Localization on an Orbital Domain

This section presents a minimal reproducible numerical workflow illustrating the spectral-selection mechanism described in Section 3.2.

3.3.1. Objective

This workflow demonstrates the following steps:

1. Definition of an orbital domain Σ_{orb}.
2. Construction of a coherence deviation field ΔC(x) using Laplacian eigenmodes.
3. Computation of the curvature trace $\nabla^2(\Delta C)$.
4. Evaluation of the curvature-residual functional

$$R_1(x) = (\nabla^2(\Delta C)(x) - \langle \nabla^2(\Delta C) \rangle)_+, \qquad (40)$$

Visualize the predicted localization pattern.

This workflow is meant to support independent replication and to facilitate further exploration by other researchers.

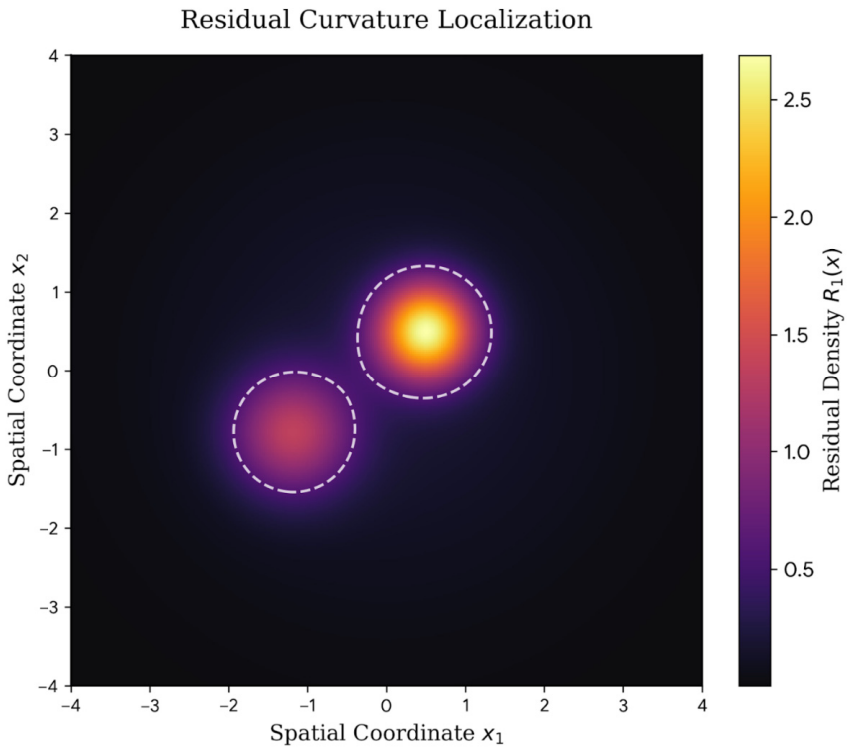

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Figure 4. Residual curvature density $R_1(x)$. Regions where the curvature trace exceeds its domain average generate localized residual contributions. Within the VTT interpretation, these regions correspond to electron-mass localization.

3.3.2. Minimal Domain Choices

Two baseline domain geometries are convenient for numerical exploration of the spectral-selection mechanism.

(A) 1D interval (debug / validation)

$$\Sigma_{orb} = [0, L] \tag{41}$$

Eigenmodes (Dirichlet):

$$\varphi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \lambda_n = \left(\frac{n\pi}{L}\right)^2. \tag{42}$$

(B) 2D disk / 3D ball (orbital-like geometry)

- 2D disk: eigenmodes in Bessel functions
- 3D ball: spherical Bessel harmonics

For numerical prototyping, a regular Cartesian grid with Dirichlet boundary conditions provides a convenient and computationally efficient approximation.

3.3.3. Construction of $\boldsymbol{\Delta C}$: Low-Mode Basin + High-Mode Residual

We construct the coherence deviation field as a spectral decomposition:

$$\Delta C(x) = \underbrace{\sum_{n=1}^{N_L} a_n \varphi_n(x)}_{\text{stable basin (low modes)}} + \underbrace{\sum_{n=N_L+1}^{N} a_n \varphi_n(x)}_{\text{curvature residue (high modes)}} . \tag{43}$$

where the first term represents the stabilized basin geometry (low-frequency modes) and the second term represents curvature-residual fluctuations (high-frequency modes).

A convenient spectral envelope is a decaying power law:

$$a_n = \frac{A}{n^p}, p \in [1,3], \tag{44}$$

with an optional "residual boost" for high modes:

$$a_n \leftarrow \alpha a_n \text{for } n > N_L, \alpha > 1. \tag{45}$$

This explicitly tests the mechanism of this section.

3.3.4. Numerical Implementation Procedure

The following numerical workflow is intended solely as an illustrative implementation of the theoretical framework and does not represent a unique computational method.

Input: domain Σ_{orb}, Spectral truncation N, Low-mode cutoff N_L, coefficients a_n, and optional coupling κ.

Output: fields $\Delta C(x)$, curvature trace $\nabla^2(\Delta C)$, residual density $R_1(x)$, and the integrated residual M_1.

Procedure:

1. Compute (or approximate numerically) the Laplacian eigenpairs eigenpairs $(\lambda_n, \varphi_n)$of $-\nabla^2$on Σ_{orb}.
2. Construct the coherence deviation field $\Delta C(x) = \sum_{n=1}^{N} a_n \varphi_n(x)$.
3. Evaluate the curvature trace using the spectral expansion:

$$\nabla^2 \Delta C(x) = -\sum_{n=1}^{N} \lambda_n a_n \varphi_n(x). \tag{46}$$

4. Compute the domain average:

$$\langle \nabla^2 \Delta C \rangle = \frac{1}{|\Sigma_{orb}|} \int_{\Sigma_{orb}} \nabla^2 \Delta C \, d\Sigma. \tag{47}$$

5. Evaluate the residual density:

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$$R_1(x) = \max\left(0, \nabla^2 \Delta C(x) - \langle \nabla^2 \Delta C \rangle\right). \quad (48)$$

6. Compute integrated residual mass:

$$\mathcal{M}_1 = \kappa \int_{\Sigma_{orb}} R_1(x)\, d\Sigma. \quad (49)$$

7. Visualize the filed: $\Delta C(x)$ (basin geometry), $\nabla^2 \Delta C(x)$ (curvature density), $R_1(x)$ (residual localization).

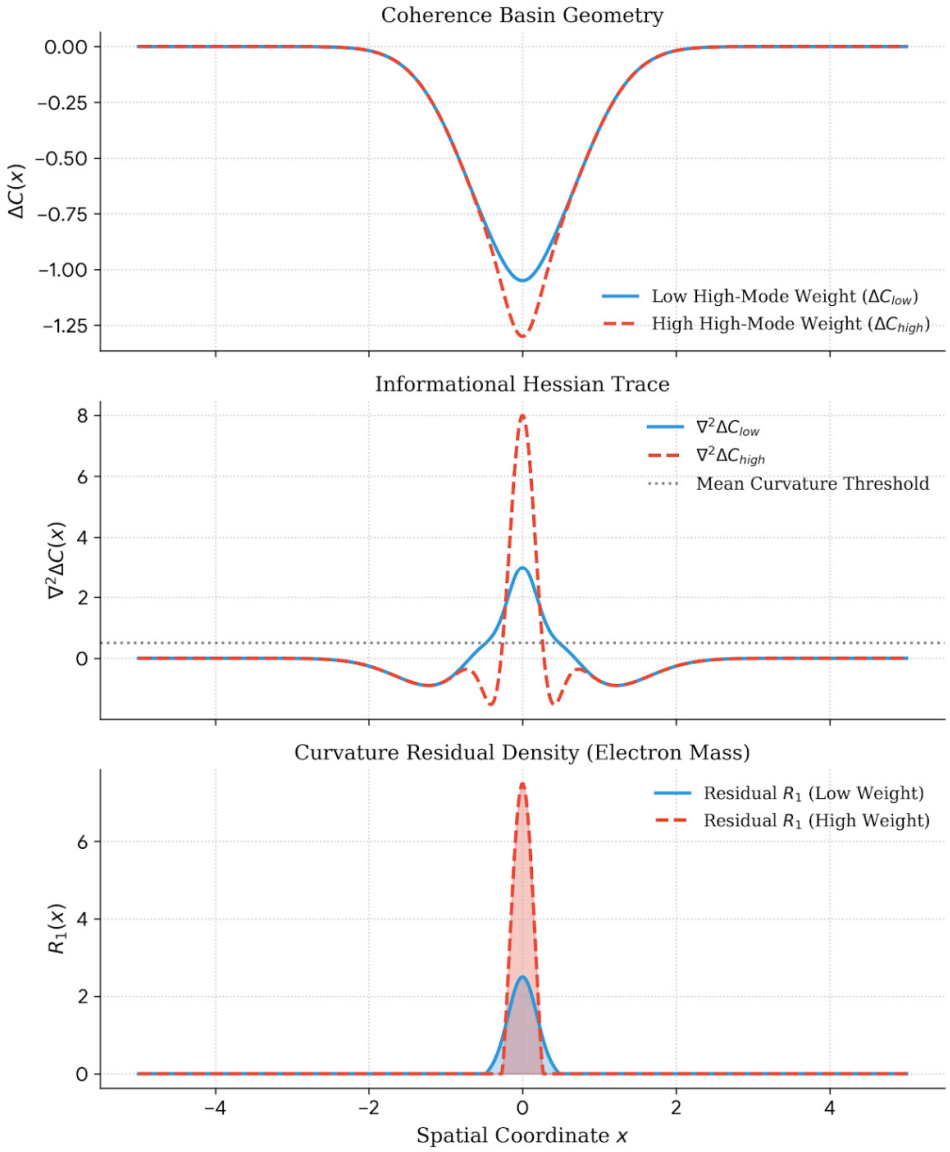

Figure 5. Numerical illustration of the spectral-selection mechanism. Increasing the spectral weight of higher-frequency modes enhances curvature-residual localization while leaving the large-scale basin geometry largely unchanged.

A minimal Python implementation illustrating this workflow is provided in Appendix B.

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4. Discussion

Compatibility with Schrödinger Dynamics

The spectral and curvature-based structures developed in Sections 2–3 admit a natural comparison with the stationary Schrödinger equation [15]. The goal of this section is not to modify quantum mechanics but to highlight a structural correspondence between the curvature operator governing the informational deviation field and the Laplacian operator appearing in quantum dynamics.

The stationary Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \quad (50)$$

can be rewritten as:

$$\nabla^2\psi = k^2(x)\psi, \quad (51)$$

where:

$$k^2(x) = \frac{2m}{\hbar^2}(V - E). \quad (52)$$

In VTT language:

$$\mathrm{Tr}(\mathcal{H}) = \nabla^2(\Delta C). \quad (53)$$

Thus, both the quantum wavefunction ψ and the informational deviation field ΔC are governed by Laplacian-type operators acting on spatial structure.

If one considers a proportional correspondence of the form

$$\Delta C \sim \psi, \quad (54)$$

then the Laplacian of coherence deviation satisfies the same structural operator as the quantum eigenvalue equation.

This correspondence should not be interpreted as an identification of ΔC with the quantum wavefunction. Rather, it indicates that both descriptions rely on Laplacian spectral structure governing spatial organization.

Under this interpretation:

- Schrödinger stationary solutions correspond to eigenmodes of the Laplacian operator.
- Stabilized orbital basins correspond to eigenmodes of the informational curvature operator.
- Orbital quantization may therefore be viewed as the discretization of stable curvature modes.
- The electron mass parameter appears as a coupling constant in the curvature-residual functional introduced earlier.

Importantly, this reinterpretation does not modify the predictive structure of quantum mechanics. Rather, it provides a geometric framework in which quantum stationary states may be viewed as manifestations of stabilized informational curvature modes.

5. Conclusions

In this work we introduced a curvature-based interpretation of orbital formation within the Viscous Time Theory (VTT) framework. Orbital structures were modeled as stabilized basins of informational curvature arising from the geometry of the coherence deviation field $\Delta C(x)$. Within this setting, the informational Hessian characterizes local curvature structure, while spectral properties of its trace operator $\nabla^2(\Delta C)$determine basin stability and curvature localization.

A residual curvature functional was then introduced to quantify curvature excess relative to the basin average. This residual term naturally generates localized contributions whenever curvature stabilization is spatially non-uniform. Under this interpretation, electron mass may be viewed as a residual curvature density that remains when curvature transport cannot fully smooth local curvature variations within the orbital basin.

Spectral analysis further showed that curvature-residual functionals preferentially weight higher-curvature eigenmodes of the Laplacian operator. This mechanism provides a natural explanation for how localized residual structure may emerge on top of a stabilized low-frequency

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geometric basin. A minimal numerical workflow illustrated this spectral-selection effect and demonstrated how localized curvature residuals arise in simple orbital domains.

Importantly, the framework remains structurally compatible with conventional quantum mechanics. Schrödinger stationary states correspond to eigenmodes of the Laplacian operator, while VTT orbital basins correspond to eigenmodes of the informational curvature operator. Under this correspondence, orbital quantization may be interpreted as the discretization of stable curvature modes. No modification of the predictive structure of quantum theory is required; rather, the present approach offers a geometric reinterpretation of orbital stability and mass localization.

Within this perspective, localized mass may be viewed as a residual geometric effect of curvature stabilization processes. If this interpretation is further developed, atomic orbitals, molecular bonding, and potentially larger-scale structure formation may be understood as manifestations of curvature hierarchy across scales.

Several directions for further investigation remain open. Future work may explore whether:

- informational curvature tensors admit scale-invariant structure,
- residual curvature mechanisms extend to nuclear or cosmological regimes, and
- electron mass generation connects to broader coherence-transport principles.

The present work therefore provides a mathematical foundation for exploring curvature-residual interpretations of quantum structure while remaining fully compatible with established physical theory. In this sense, the framework proposed here suggests that mass localization within atomic structure may emerge as a geometric consequence of curvature stabilization in informational fields, providing a new perspective on orbital structure that complements the standard quantum mechanical description.

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Conflicts of Interest: The author declares no conflict of interest.

Appendix A. Mathematical Properties of the Residual Functional

A.1. Lemma (Non-Negativity)

Lemma 1.

If $\Delta C \in C^2(\Sigma_{orb})$, then

$$\mathcal{M}[\Delta C] \geq 0. \tag{A1}$$

Proof.

By definition:)

$$R(x) = \max\{0, \mathrm{Tr}(\mathcal{H})(x) - \langle \mathrm{Tr}(\mathcal{H}) \rangle\}. \tag{A2}$$

Thus:

$$R(x) \geq 0 \forall x \in \Sigma_{orb}. \tag{A3}$$

Since $\kappa > 0$,

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$$\mathcal{M}[\Delta C] \geq 0. \quad (A4)$$

A.2. Theorem (Existence of Residual Under Non-Uniform Curvature)

Theorem 1.

If $\mathrm{Tr}(\mathcal{H})$ is not constant over Σ_{orb}, then: $\mathcal{M}[\Delta C] > 0$.

Proof.

If $\mathrm{Tr}(\mathcal{H})$is not constant, there exists a measurable subset $A \subset \Sigma_{orb}$ such that:

$$\mathrm{Tr}(\mathcal{H})(x) > \langle \mathrm{Tr}(\mathcal{H}) \rangle \text{for } x \in A. \quad (A5)$$

Since Σ_{orb} is compact and $\mathrm{Tr}(\mathcal{H}) \in C^0$,

- Ahas nonzero measure,
- $R(x) > 0$ on A.

Therefore:

$$\int_{\Sigma_{orb}} R(x) d\Sigma > 0. \quad (A6)$$

Hence: $\mathcal{M}[\Delta C] > 0$.

A.3. Physical Interpretation

This theorem implies:

- Perfectly uniform curvature produces no residual mass.
- Any non-uniform curvature basin necessarily generates residual density.
- Electron mass emerges whenever orbital curvature is spatially non-homogeneous.

This is crucial: Residual mass is not imposed.

It is mathematically unavoidable under curvature variability.

A.4. Stability Under Small Perturbations

Let:

$$\Delta C_\epsilon = \Delta C + \epsilon f(x) \quad (A7)$$

with $f \in C^2(\Sigma_{orb})$and $\epsilon \ll 1$.

Then:

$$\mathcal{H}_\epsilon = \mathcal{H} + \epsilon \nabla^2 f. \quad (A8)$$

Thus:

$$\mathcal{M}[\Delta C_\epsilon] = \mathcal{M}[\Delta C] + O(\epsilon). \quad (A9)$$

Therefore:

The residual functional is stable under small perturbations. This ensures robustness of the mass interpretation.

A.5. Boundary Condition Case

If:

$$\mathrm{Tr}(\mathcal{H}) = \text{constantover } \Sigma_{orb}, \quad (A10)$$

then: $\mathcal{M}[\Delta C] = 0$.

This corresponds to idealized perfect harmonic curvature — a theoretical limiting case not observed in atomic solutions

Appendix B

B.1. Python Skeleton (Replicable Minimal Code)

```
import numpy as np


def build_1d_dirichlet_basis(L, x, N):
```

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```
    # eigenmodes on [0, L] with Dirichlet boundary conditions
    # phi_n(x) = sqrt(2/L) sin(n pi x / L), lambda_n = (n pi / L)^2
    phi = np.zeros((N, len(x)))
    lam = np.zeros(N)
    for n in range(1, N+1):
        lam[n-1] = (n*np.pi/L)**2
        phi[n-1, :] = np.sqrt(2.0/L) * np.sin(n*np.pi*x/L)
    return lam, phi

def synthesize_deltaC(phi, a):
    # deltaC(x) = sum_n a_n phi_n(x)
    return (a[:, None] * phi).sum(axis=0)

def laplacian_deltaC_from_eigens(lam, phi, a):
    # ∇² deltaC = - sum_n lambda_n a_n phi_n
    return (-lam[:, None] * a[:, None] * phi).sum(axis=0)

def positive_part(z):
    return np.maximum(0.0, z)

def compute_residual(L, N=80, NL=10, p=2.0, alpha=3.0, kappa=1.0, grid=2000):
    x = np.linspace(0.0, L, grid)
    lam, phi = build_1d_dirichlet_basis(L, x, N)

    # coefficients a_n = A / n^p, with high-mode boost
    n = np.arange(1, N+1, dtype=float)
    A = 1.0
    a = A / (n**p)
    a[NL:] *= alpha

    deltaC = synthesize_deltaC(phi, a)
    lapC = laplacian_deltaC_from_eigens(lam, phi, a)

    mean_lapC = lapC.mean()
    R1 = positive_part(lapC - mean_lapC)

    # approximate integral on uniform grid
    M1 = kappa * np.trapz(R1, x)

    return x, deltaC, lapC, R1, M1

# Example run:
```

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```
x, deltaC, lapC, R1, M1 = compute_residual(L=1.0, N=120, NL=12, p=2.0, alpha=4.0, kappa=1.0)
print("Integrated residual M1 =", M1)
```

Expected qualitative outcome: Increasing α (high-mode weight) increases curvature fluctuations and produces sharper localized residual regions in R_1, consistent with Appendix C.

B.2. Minimal Numerical Experiments (Suggested)

1. **High-mode amplification sweep**
   Fix N, N_L, p. Sweep $\alpha \in \{1,2,4,8\}$.
   Observe monotonic growth of $\mathcal{M}_1$ and increased localization of R_1.
2. **Spectral decay sweep**
   Fix α. Sweep $p \in \{1,2,3\}$.
   Observe that slower decay (smaller p) yields stronger residual localization.
3. **Invariant comparison (optional)**
   Compute R_2using curvature energy $I_2 = \mathrm{Tr}(\mathcal{H}^2)$ in discretized form.
   Check whether localization patterns agree with R_1.

B.3. Notes for Extensions

- Replace 1D basis with 2D/3D eigenmodes (finite differences or FEM).
- Use spherical domains to resemble orbital geometry.
- Introduce a viscosity-weighted residual operator if desired:

$$R_{1,\eta}(x) = (\nabla^2 \Delta C(x) - \langle \nabla^2 \Delta C \rangle)_+ \cdot w(\eta(x)). \qquad \text{(B1)}$$

Appendix C. Open Questions and Future Research Directions

The present work proposes a conservative geometric reinterpretation of orbital structure and electron mass within the VTT framework. While internally consistent and compatible with standard quantum mechanics, it opens several concrete research directions.

The following points outline several directions for further investigation within the present framework.

C.1. Spectral Calibration of the Residual Functional

The residual mass functional was defined using curvature invariants of the Informational Hessian. An immediate problem is:

Determine which invariant (trace, quadratic energy, determinant, or combination) best correlates with physical electron mass when applied to realistic orbital solutions.

This requires:

- Expanding hydrogenic orbitals in a Laplacian eigenbasis,
- Evaluating candidate residual functionals,
- Testing scaling behavior against known mass parameters.

C.2. Boundary Condition Sensitivity

Orbital manifolds were treated abstractly as compact domains. Future work should investigate:

- Dependence of the residual functional on boundary conditions (Dirichlet vs Neumann),
- Stability under domain deformation,
- Behavior under spherical vs non-spherical geometries.

This analysis may clarify whether residual localization is a geometric invariant or partially boundary-driven.

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C.3. Compatibility with Relativistic Frameworks

The present treatment is non-relativistic and Schrödinger-compatible. A natural extension is:

- Replace Laplacian eigenbasis with Klein–Gordon or Dirac operator eigenstructure,
- Examine whether residual curvature functionals remain well-defined under Lorentz-covariant operators.

This would test whether the residual mechanism is structural or purely non-relativistic.

C.4. Viscosity-Coupled Residual Dynamics

The metastability condition introduced viscosity η_I. An open problem is:
Can residual generation be dynamically derived from a viscous transport equation for ΔC?
Specifically, consider evolution equations of the form:

$$\frac{D}{D\tau}\Delta C = \eta_I \nabla^2 \Delta C - \mathcal{N}(\Delta C), \qquad \text{(C1)}$$

and analyze whether steady-state curvature residual arises generically.
This would transform the residual interpretation from static to dynamical.

C.5. Multi-Scale Curvature Hierarchy

If residual mass is a curvature-excess phenomenon, one may ask:

- Does an analogous residual mechanism operate at nuclear or composite scales?
- Is there a curvature hierarchy across atomic, molecular, and possibly larger domains?

This question is speculative but structurally natural.
We do not assert such scaling — we propose it as a testable geometric hypothesis.

C.6. Numerical Validation on Real Orbital Solutions

A concrete computational program would:

1. Use known hydrogenic wavefunctions,
2. Construct candidate ΔC mappings,
3. Compute Hessian invariants numerically,
4. Evaluate residual functionals,
5. Compare relative scaling across orbitals.

This is a tractable finite-element or spectral computation problem.

C.7. Conceptual Clarification: Geometry vs Ontology

The present paper does not claim that electrons are not fundamental particles.
Rather, it proposes:
A geometric reinterpretation in which localized mass may be understood as residual curvature of stabilized informational basins.
Whether this reinterpretation is merely mathematical, or reflects deeper ontological structure, remains an open question.
We explicitly invite independent mathematical and physical scrutiny.

C.8. Outlook

If validated, the framework suggests a general principle:
Localized physical quantities may emerge as residuals of geometric stabilization processes rather than as primitive inputs.
Such a principle would unify curvature, localization, and quantization under a single structural mechanism.
This remains a research direction — not a conclusion.

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Appendix D. Symbol Table

D.1. Core Informational Quantities

Symbol	Meaning	Description
$\Delta C(x)$	Coherence deviation field	Local deviation of informational coherence from equilibrium baseline
$\Delta C_0(x)$	Stabilized coherence basin	Stationary curvature configuration
$\epsilon(x)$	Oscillatory amplitude	Small perturbative modulation around stationary basin
τ	Viscous time parameter	Informational time coordinate in VTT
η_I	Informational viscosity	Resistance to coherence transport
Φ_α	Admissibility function	Binary/continuous gate for coherence collapse
ΔI	Informational inertia	Measure of accumulated coherence transport

D.2. Differential Geometry Operators

Symbol	Meaning	Description
$\nabla\Delta C$	Gradient of coherence field	First spatial derivative of ΔC
$\nabla^2\Delta C$	Laplacian of coherence field	Trace of Hessian, total curvature density
$\mathcal{H}_{ij}$	Informational Hessian	Second derivative tensor of coherence field
$\lambda_k(\mathcal{H})$	Eigenvalues of Hessian	Local curvature eigenvalues
$\mathrm{Tr}(\mathcal{H})$	Trace of Hessian	Sum of principal curvatures

$$\mathcal{H}_{ij} = \frac{\partial^2(\Delta C)}{\partial x_i \partial x_j} \qquad (D1)$$

D.3. Orbital and Residual Quantities

Symbol	Meaning	Description
Σ_{orb}	Orbital manifold	Spatial region defining stabilized curvature basin
m_e	Electron mass	Residual curvature functional
κ	Coupling constant	Dimensional proportionality constant
$\langle\cdot\rangle$	Basin average	Mean value over orbital manifold
$(\ \cdot)_+$	Positive part operator	Selects curvature excess above equilibrium

Residual mass functional:

$$m_e = \kappa \int_{\Sigma_{orb}} (\mathrm{Tr}(\mathcal{H}) - \langle \mathrm{Tr}(\mathcal{H}) \rangle)_+ \, d\Sigma \qquad (D2)$$

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D.4. Quantum Mechanical Correspondence

Symbol	Meaning	Description
$\psi(x,t)$	Wavefunction	Quantum probability amplitude
$\phi(x)$	Spatial eigenfunction	Stationary solution of Schrödinger equation
E	Energy eigenvalue	Quantized energy level
$\hbar$	Reduced Planck constant	Quantum action constant
$V(x)$	Potential	External potential in Schrödinger formalism
$k^2(x)$	Spectral parameter	Effective curvature coefficient

Schrödinger stationary equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \qquad \text{(D3)}$$

Structural correspondence:

$$\mathrm{Tr}(\mathcal{H}) \sim \nabla^2(\Delta C) \qquad \textit{(D4)}$$

D.5. Stability Conditions

Condition	Meaning
$\nabla\Delta C = 0$	Stationary curvature point
$\lambda_k(\mathcal{H}) > 0$	Stable curvature basin
$\lambda_{min}(\mathcal{H}) > -\eta_I$	Metastable curvature attractor
$\frac{D}{D\tau}(\Delta C) = 0$	Time-averaged stability

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