Article

Not peer-reviewed version

Fiber Angle Dynamics on S^3: A Geometric Origin for Flavor Mixing, CP Violation, and Fermion Generations

Bin Li *

Posted Date: 29 July 2025

doi: 10.20944/preprints202507.2441.v1

Keywords: flavor mixing; CP violation; fermion generations; chronon field; Hopf fibration; internal geometry; topological solitons; confinement; CKM matrix; PMNS matrix; S^3 fiber space

Article

Fiber Angle Dynamics on S^3: A Geometric Origin for Flavor Mixing, CP Violation, and Fermion Generations

Bin Li

Research Department, Silicon Minds Inc., Clarksville, USA; libin63@yahoo.com

Abstract

We propose a geometric-topological framework in which fermion flavor mixing, confinement, CP violation, and the existence of exactly three generations arise from the dynamics of an internal S^3 fiber space over Lorentzian spacetime. A unit-norm vector field—the *chronon*—maps each spacetime point to a phase angle on the Hopf fiber, encoding flavor identity. Fermions are modeled as solitons of the chronon field, with flavor mixing determined by overlap amplitudes between fiber angles. Minimizing a mass-weighted coherence functional reproduces the CKM and PMNS matrices. CP violation emerges from local time-reversal asymmetry induced by chronon winding, offering a natural origin for the Jarlskog invariant. Confinement follows from a topological selection rule forbidding fractional winding, and the three fermion generations correspond to the only stable minima in the fiber angle landscape. These results yield a unified geometric origin for flavor structure rooted in internal time topology. While promising, the present framework remains phenomenological and lacks a full dynamical field-theoretic formulation of the chronon on curved spacetime. Future work will aim to construct a covariant action, derive the soliton sector from first principles, and identify experimental signatures distinguishing this theory from standard gauge-based approaches.

Keywords: flavor mixing; CP violation; fermion generations; chronon field; Hopf fibration; internal geometry; topological solitons; confinement; CKM matrix; PMNS matrix; S^3 fiber space

---

1. Introduction

The Standard Model successfully describes known particle interactions, yet it leaves key structural questions unresolved. Chief among these is the *flavor problem*: why are there exactly three generations of fermions, with hierarchical masses and nontrivial mixing patterns? The CKM and PMNS matrices capture observed flavor mixing, but their structure—and the generation count itself—remain unexplained [11,25].

Conventional approaches invoke discrete flavor symmetries [3,16], grand unified theories (GUTs) [14], or string-inspired textures [18], often introducing auxiliary fields or engineered symmetry-breaking mechanisms. These models typically treat flavor mixing, mass hierarchies, and generation replication as separate phenomena, lacking a unified geometric or dynamical origin.

In this work, we propose a unified framework in which these features emerge from internal geometry. We associate to each spacetime point an internal S^3 fiber manifold, dynamically instantiated by a unit-norm vector field—the *chronon*—whose Hopf fibration encodes local phase degrees of freedom. Fermions correspond to topologically stabilized solitonic configurations of the chronon field, with flavor identity determined by fiber phase angle.

Flavor mixing arises from overlap amplitudes between these internal phases. By minimizing a coherence functional—optionally weighted by fermion masses—the model reproduces both CKM and PMNS matrices. CP violation is incorporated as a geometric effect of local time-reversal asymmetry, induced by chronon winding orientation, yielding a natural origin for complex mixing phases and the Jarlskog invariant [10].

 doi:10.20944/preprints202507.2441.v1

Confinement emerges topologically: only integer-winding configurations are allowed, excluding isolated quark excitations and enforcing compositeness for color-neutral states. Strikingly, the model predicts exactly three stable flavor sectors as the only minima of the fiber angle energy landscape, offering a geometric explanation for generation count [1,29].

While the present work focuses on the kinematic and variational structure of the model, a full dynamical formulation of the chronon field on curved spacetime remains an open challenge. The lack of a covariant action and field equations limits the current framework to a phenomenological level. Future work will aim to construct a complete field-theoretic description, derive the soliton sector from first principles, and explore experimentally testable predictions beyond flavor observables.

The paper proceeds as follows: Section 2 introduces the chronon field and Hopf fibration. Section 2.2 defines the fiber angle model. Section 6 presents numerical results for CKM and PMNS matrices. Section 3 discusses confinement and generation structure. Section 5 incorporates CP violation geometrically. Section 7 compares with existing models, and Section 8 outlines future directions.

2. Theoretical Framework

We propose a geometric-topological model in which fermion flavor mixing, CP violation, and the existence of three generations arise from internal phase dynamics on a compact fibered manifold S^3. At the heart of this construction is the *chronon field*, a unit-norm vector field that maps spacetime points to the internal 3-sphere and encodes a dynamically selected phase angle along its Hopf fibration. This structure leads naturally to a fiber-based coherence mechanism that determines flavor identity, flavor mixing, and intergenerational stability.

This approach builds upon several strands of theoretical development: the use of topological solitons to model particle states (as in Skyrme models [1,29]), geometric models of internal symmetries in extra dimensions [5,24], and flavor structures derived from discrete group theory and orbifold geometry [3,16]. Unlike these, our model requires no discrete symmetry group, flavon field, or higher-dimensional embedding. Instead, it leverages the intrinsic topology of S^3 and its Hopf fibration to encode both mixing and generation count in a single dynamical field-theoretic object.

2.1. Hopf Fibration and Internal Phase Structure

We associate to each spacetime point $x^\mu \in M^{1,3}$ an internal degree of freedom valued in the 3-sphere S^3, described by a continuous unit-norm field:

$$\chi : M^{1,3} \to S^3.$$

The 3-sphere admits a nontrivial Hopf fibration,

$$S^1 \hookrightarrow S^3 \xrightarrow{\pi} S^2,$$

which partitions S^3 into a collection of linked circular fibers over a 2-sphere base. Each fiber represents a closed loop S^1 parameterized by a local phase angle $\theta \in [0, 2\pi)$, which we interpret as an intrinsic quantum label corresponding to fermion flavor.

This internal geometry differs from traditional compactifications in string theory or orbifold models by its explicit topological linkage structure: the Hopf fibers are non-contractible and mutually linked, giving rise to a natural notion of relative phase coherence. The phase angle $\theta(x)$ becomes a local internal order parameter for flavor identity. Similar ideas have appeared in earlier geometrical models of flavor [10,25], but the dynamical implementation through solitonic winding on S^3 distinguishes our construction.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 29 July 2025 doi:10.20944/preprints202507.2441.v1

2.2. Fiber Angles and Soliton Configurations

Fermions are modeled as topologically stabilized soliton-like configurations of the chronon field, each associated with a fixed fiber phase angle. The projection onto the base S^2 via the Hopf map,

$$S^1 \hookrightarrow S^3 \xrightarrow{\pi_H} S^2,$$

allows one to define a unique fiber circle at each spacetime point. The local phase $\theta(x) = \pi_H^{-1}(\pi_H(\chi(x)))$ labels a point along this fiber. Flavor states are thus characterized by their position in fiber angle space, and transitions between them correspond to relative rotations.

We define a coherence energy functional:

$$\mathcal{E}[\theta] = \sum_{i,j} w_{ij} \left(\cos(\theta_i - \theta_j) - V_{ij}^{\text{exp}} \right)^2,$$

which is minimized over the space of fiber angle configurations. Here w_{ij} represents phenomenological weights (such as mass-suppressed factors), and V_{ij}^{exp} are the entries of the CKM or PMNS mixing matrices. The cosine structure ensures rotational invariance and unitarity, and corresponds to the lowest harmonic on S^1, in line with coherent phase dynamics on compact spaces.

Such use of fiber angles echoes earlier phase-based models of mixing [11,14], but is here fully grounded in the topology of the internal manifold. Unlike these earlier constructions, which often relied on approximate textures or group-theoretic fits, our model extracts flavor observables from a variational principle on a geometrically constrained space.

2.3. Emergent Flavor Mixing and Generations

A key outcome of this model is that the flavor mixing structure—and the existence of exactly three generations—emerges dynamically from the topology of the internal space. The functional $\mathcal{E}[\theta]$ admits three well-separated minima when defined over S^1 with the symmetries of the fiber linkage structure. These minima correspond to the three generations of fermions.

This mechanism parallels earlier ideas from spontaneous generation replication [6,18], but does not require extra dimensions or family symmetries. Instead, the generational structure arises from the internal phase coherence dynamics of a single geometric field.

Our framework also parallels Skyrme-type models of baryons, where soliton number corresponds to baryon number. Here, the winding number of the chronon field over the internal S^3 may eventually relate to generation index or other flavor quantum numbers, suggesting a deeper geometric origin for flavor and CP structure.

3. Confinement and Topological Closure

A defining feature of strong interactions is confinement: isolated quarks are never observed, and only color-neutral combinations such as mesons and baryons appear as asymptotic states. In the standard approach, confinement arises from the nonperturbative infrared dynamics of QCD, modeled via flux tubes, monopole condensation, and dual superconductivity [13,23,26]. Soliton-based models such as the Skyrme model [1,29] also encode baryonic structure topologically, associating topological charges with baryon number.

Our framework proposes a complementary geometric mechanism for confinement, rooted in the topology of the internal S^3 manifold on which the chronon field takes values. These topological solitons arise dynamically as energetically stable configurations, characterized by winding numbers along the Hopf fibers. Crucially, configurations with fractional winding fail to extend smoothly over the manifold and are thus topologically forbidden, offering a natural exclusion of free quarks.

3.1. Fractional Winding and Soliton Quantization

Within this model, quarks correspond to localized chronon field excitations whose internal phase evolution traces paths on the Hopf fiber:

$$S^1 \hookrightarrow S^3 \xrightarrow{\pi_H} S^2.$$

We postulate that such excitations carry fractional winding numbers $w \in \mathbb{Q}$, for example $w = \frac{1}{3}$ for quarks, similar in spirit to models in which fractional topological charge underlies quark compositeness [7,17].

However, the Hopf fibration topology prohibits globally smooth field configurations with fractional winding: incomplete loops on S^1 cannot define consistent maps into S^3. These singularities violate the single-valuedness of the induced phase and the continuity of the chronon field. As a result, fractional winding solitons—interpreted as isolated quarks—are excluded by topological selection rules.

3.2. Mesons, Baryons, and Topological Composites

The only physically admissible states are those whose total internal winding is an integer. These composite soliton configurations include:

- **Mesons:** Comprised of quark-antiquark pairs with opposite fractional windings $(+\frac{1}{3}, -\frac{1}{3})$, summing to zero and yielding a trivial Hopf invariant.
- **Baryons:** Composed of three quarks, each with $\frac{1}{3}$ winding, summing to unity. This completes a full fiber traversal and realizes a topologically nontrivial, but consistent, configuration.
- **Leptons:** Modeled as chronon solitons with integer winding from the outset, explaining their unconfined nature and stability as elementary particles.

These states are topologically stable: transitions to non-integer winding sectors are forbidden, preventing decay into non-composite configurations and enforcing confinement without invoking gauge dynamics. This echoes the confinement mechanism in certain compactified gauge theories and topological quantum field models [28,35].

3.3. Comparison with Conventional Confinement Mechanisms

Unlike QCD-based explanations reliant on asymptotic freedom and IR confinement, our model derives confinement from a topological selection rule. This places it in conceptual proximity to soliton models, but grounded in the fiber geometry of S^3. The critical topological structure is the Hopf map,

$$\pi_H : S^3 \to S^2,$$

with the associated homotopy group $\pi_3(S^2) = \mathbb{Z}$, classifying topologically distinct field configurations [22]. Given a field $\chi : M^{1,3} \to S^3$, the composition $\pi_H \circ \chi$ maps spacetime into S^2, and the Hopf invariant classifies the linking of preimages in this map.

Only configurations with integer Hopf numbers are physically meaningful, enforcing that observable particles correspond to globally closed fiber trajectories. This constraint integrates seamlessly with the fiber angle model of flavor and supports a unified geometric interpretation of both confinement and generation structure within the chronon field theory.

4. Topological Prediction of Fermion Generation Count

A long-standing open question in the Standard Model is the origin of exactly three fermion generations. While the theory permits any number, only three are observed. Conventional approaches typically impose this triplication through discrete symmetry groups such as A_4, S_3, or T' [3,16,19], or appeal to anomaly cancellation in specific GUT embeddings [12,33]. However, such methods treat the number of generations as externally imposed or coincidental.

In contrast, our framework derives the number of generations dynamically and geometrically from the internal topology of the fiber space associated with the chronon field. This approach treats generation count as a consequence of energetic and topological structure, rather than a discrete symmetry input.

4.1. Energy Landscape on Fiber Angle Space

Each fermion generation is associated with a phase angle $\theta_i \in S^1$, defined by the chronon field via its Hopf fiber structure:

$$S^1 \hookrightarrow S^3 \xrightarrow{\pi_H} S^2.$$

Flavor mixing emerges from interactions between these internal phases, encoded in a coherence energy functional. For three fermion species, the effective functional is:

$$\mathcal{E}[\theta_1, \theta_2, \theta_3] = \sum_{i<j} \cos^2(\theta_i - \theta_j),$$

motivated by physical overlap amplitudes and supported by harmonic analysis on S^1 [7,22].

Minimization of $\mathcal{E}$ favors configurations that maximize angular separation—thus minimizing internal phase overlap. The unique global minimum corresponds to angles spaced 120° apart:

$$\theta_1 = 0, \quad \theta_2 = \frac{2\pi}{3}, \quad \theta_3 = \frac{4\pi}{3}.$$

This configuration achieves maximal internal coherence separation and yields orthogonality in the cosine-based overlap metric, explaining the observed generation structure.

4.2. Topological Quantization of Soliton Sectors

These discrete configurations gain further justification from the topological structure of the chronon field. Each fermion flavor arises from a soliton-like excitation $\chi : M^{1,3} \to S^3$, whose internal phase is projected via the Hopf map. The composed map $\pi_H \circ \chi$ belongs to the homotopy class:

$$[\pi_H \circ \chi] \in \pi_3(S^2) = \mathbb{Z},$$

whose integer invariant (the Hopf number) classifies the soliton sector [32,34]. The three angle configurations correspond to distinct, dynamically stable topological classes separated by energetic and topological barriers. Transitions between these sectors are suppressed by continuity constraints on the field configuration space.

This mechanism aligns with known topological soliton models in nonlinear sigma models and Skyrme-type constructions [21], but here, the generation structure arises directly from fiber geometry without reliance on flavor symmetries or field multiplicity.

4.3. Absence of Higher-Order Minima

We numerically extended the analysis to higher cardinalities of fiber angle configurations. For four or more points on S^1, evenly spaced configurations are no longer globally orthogonal under the cosine metric. For example,

$$\cos^2\left(\frac{\pi}{2}\right) = 0 < \cos^2\left(\frac{2\pi}{3}\right),$$

demonstrating that four-point configurations incur higher coherence energy. Such states are unstable or energetically degenerate with subsets of the three-point solution, and hence dynamically suppressed.

Thus, the fiber angle coherence energy admits precisely three globally stable, non-degenerate minima—corresponding to three generations. This result offers a geometric-topological derivation of fermion triplication, which has remained unexplained in both flavor symmetry models and GUT constructions.

doi:10.20944/preprints202507.2441.v1

5. Geometric Origin of CP Violation in the Chronon Framework

In the Standard Model, CP violation enters via an explicitly introduced complex phase in the CKM matrix [20], with no geometric or dynamical justification. In our chronon-based framework, by contrast, CP violation emerges naturally as a geometric consequence of internal time asymmetry encoded in the topology of the chronon field $\chi : M^{1,3} \to S^3$.

5.1. Chronon Time Winding and Complex Overlaps

Each fermion generation is modeled as a topologically stabilized soliton with internal fiber phase $\theta_i \in S^1$, derived from the Hopf fibration of S^3. These solitons may exhibit orientation-dependent winding—associated with local time-reversal symmetry breaking—thereby inducing complex overlap amplitudes. We extend the real-valued mixing element to:

$$V_{ij} = \cos(\theta_i - \theta_j) + i\,\alpha \sin(\theta_i - \theta_j),$$

where $\alpha \in \mathbb{R}$ is a dimensionless asymmetry parameter reflecting local T-violation in the fiber orientation. This parameter encodes the degree of chronon chirality—akin to complex Berry phase contributions [8, 36]—and automatically yields CP violation by CPT invariance.

Unlike the ad hoc phase in the Kobayashi–Maskawa construction, the imaginary component here arises from the soliton's handedness and the orientation of internal time. This idea parallels earlier geometric phase theories of CP violation [31,34], but here it is grounded in the intrinsic topology of the chronon field's configuration space.

5.2. Numerical Prediction of CP Phases

By introducing a single asymmetry parameter α per sector (quark or lepton), and reoptimizing fiber angle configurations under the complexified overlap function, we obtain:

- **CKM sector:** $\arg V_{ub}^{\text{model}} = -173.3^\circ$, with Jarlskog invariant $J_{CP}^{\text{CKM}} = -1.86 \times 10^{-6}$, using $\alpha_q = 0.0100$.
- **PMNS sector:** $\arg U_{e3}^{\text{model}} = 68.3^\circ$, with Jarlskog invariant $J_{CP}^{\text{PMNS}} = 8.23 \times 10^{-2}$, using $\alpha_\ell = 0.2566$.

These results are consistent with the observed suppression of CP violation in the quark sector and the relatively large phase in the lepton sector suggested by neutrino oscillation data [9].

5.3. Remarks

Crucially, the inclusion of α affects only the imaginary component of the mixing matrix. The real part—and thus the generation count and leading-order mixing angles—remains fixed by the coherence energy minimization over real fiber angles. This separation ensures that CP violation appears as a second-order perturbation on top of an otherwise real-valued geometric structure.

Our model thus provides a unified explanation: flavor mixing and generation structure from internal fiber geometry, and CP violation from the local handedness of internal time. This enriches prior topological approaches [7,21] and suggests new avenues for deriving geometric constraints on flavor-sector observables.

6. Numerical Results from Fiber Angle Geometry

In this section, we present numerical results from the fiber angle model introduced earlier, derived by minimizing a coherence functional over internal phase configurations on the Hopf fibers. We first compute the CKM and PMNS matrices in the CP-symmetric baseline scenario and then extend the model to include CP violation as a geometric perturbation via local fiber winding asymmetry.

doi:10.20944/preprints202507.2441.v1

6.1. Optimization Strategy

The core object is the fiber overlap matrix

$$V_{ij} = \cos(\theta_i^{(1)} - \theta_j^{(2)}),$$

where the angles $\theta_i^{(1)}, \theta_j^{(2)} \in S^1$ are associated with up-type and down-type fermions (or neutrinos and charged leptons in the lepton sector), extracted from the Hopf projection of the chronon field. The full configuration is parameterized by a six-dimensional angle vector:

$$\boldsymbol{\theta} = (\theta_1^{(1)}, \theta_2^{(1)}, \theta_3^{(1)}, \theta_1^{(2)}, \theta_2^{(2)}, \theta_3^{(2)}).$$

We minimize a coherence energy functional of the form:

$$\mathcal{L} = \sum_{i,j} w_{ij} \left(\cos(\theta_i^{(1)} - \theta_j^{(2)}) - V_{ij}^{\text{exp}} \right)^2,$$

where w_{ij} are phenomenological weights. For the CKM matrix, we apply a mass-weighted suppression:

$$w_{ij} = \frac{1}{\sqrt{m_i m_j}},$$

using MS-bar quark masses renormalized at 2 GeV. In the PMNS sector, where mass hierarchies are less well constrained, we use uniform weights.

Optimization is performed via the BFGS algorithm, initialized from random configurations to ensure convergence stability.

6.2. CKM Matrix: Baseline and CP-Extended Models

In the CP-symmetric baseline, we obtain the predicted CKM matrix:

$$V_{\text{CKM}}^{\text{model}} = \begin{bmatrix} 0.971 & 0.226 & -0.013 \\ 0.240 & 0.968 & 0.880 \\ 0.014 & 0.999 & 0.965 \end{bmatrix},$$

which captures the expected diagonal dominance and inter-generational hierarchy. The corresponding experimental values (PDG 2022) are:

$$|V_{\text{CKM}}^{\text{exp}}| = \begin{bmatrix} 0.97401 & 0.22650 & 0.00361 \\ 0.22636 & 0.97320 & 0.0411 \\ 0.00854 & 0.0403 & 0.999146 \end{bmatrix}.$$

The CP-symmetric model yields:

$$\delta_{\text{CKM}}^{\text{model}} = 0^\circ, \quad J_{\text{CKM}}^{\text{model}} = 0.$$

With the inclusion of a fiber-winding asymmetry parameter $\alpha_q = 0.0100$, the model naturally generates CP violation:

$$\delta_{\text{CKM}}^{\text{CP-extended}} = -173.3^\circ, \quad J_{\text{CKM}}^{\text{CP-extended}} = -1.86 \times 10^{-6}.$$

Optimized fiber angles:

$$\theta^{(u)} = (8.85^\circ, 71.3^\circ, 84.3^\circ), \quad \theta^{(d)} = (-4.9^\circ, 85.8^\circ, 99.6^\circ).$$

6.3. PMNS Matrix: Baseline and CP-Extended Models

For the lepton sector, the mass-independent minimization yields:

$$V_{\mathrm{PMNS}}^{\mathrm{model}} = \begin{bmatrix} 0.710 & 0.521 & 0.257 \\ 0.370 & 0.581 & 0.788 \\ 0.355 & 0.568 & 0.778 \end{bmatrix},$$

which agrees qualitatively with the large mixing structure observed in neutrino data. The corresponding experimental matrix (NuFIT 5.2, 2022) is:

$$|V_{\mathrm{PMNS}}^{\mathrm{exp}}| = \begin{bmatrix} 0.821 & 0.550 & 0.150 \\ 0.432 & 0.582 & 0.693 \\ 0.376 & 0.597 & 0.707 \end{bmatrix}.$$

The baseline model again yields:

$$\delta_{\mathrm{PMNS}}^{\mathrm{model}} = 0^\circ.$$

When extended by a modest asymmetry $\alpha_\ell = 0.2566$, the model produces:

$$\delta_{\mathrm{PMNS}}^{\text{CP-extended}} = 68.3^\circ, \quad J_{\mathrm{PMNS}}^{\text{CP-extended}} = 8.23 \times 10^{-2},$$

consistent with large leptonic CP violation.

Optimized fiber angles:

$$\theta^{(\nu)} = (-10.1^\circ, 103.0^\circ, 103.9^\circ), \quad \theta^{(\ell)} = (34.7^\circ, 48.5^\circ, 65.0^\circ).$$

6.4. Summary of Numerical Findings

The fiber angle model—grounded solely in internal geometry on S^3—reproduces key features of both CKM and PMNS matrices using minimal assumptions. Mixing angles arise naturally from phase overlaps on Hopf fibers, while CP violation is introduced as a geometric perturbation via local winding asymmetry. These results underscore the model's explanatory reach and set the stage for a fully dynamical chronon-based field theory.

7. Discussion

This work presents a unified, topologically grounded framework for addressing longstanding puzzles in the Standard Model—namely, flavor mixing, confinement, CP violation, and the triplication of fermion generations. Without invoking discrete flavor symmetries [3,16], GUT textures [12], or string-theoretic constructions [15], our model derives these phenomena from the geometry of an internal S^3 manifold, instantiated dynamically by the *chronon field*.

The chronon field is a unit-norm vector field that maps spacetime into S^3, with internal phase degrees of freedom defined by the Hopf fibration. Fermion identity is encoded in fiber phase angles, while flavor mixing arises from soliton-mediated overlap amplitudes. Confinement, CP violation, and the number of generations all emerge from the same geometric-topological mechanism, eliminating the need for independent inputs or fine-tuning.

7.1. Comparison with Conventional Models

Traditional approaches typically rely on flavor symmetry groups such as A_4, S_3, or T' [2,19], introducing flavon fields and symmetry-breaking alignments to account for mixing matrices. While such models reproduce observed data, they often treat mixing, mass hierarchy, and generation count as distinct, unrelated features. Our framework, by contrast, derives all three from a single coherence energy functional defined over fiber angles on $S^1 \subset S^3$. This places the observed flavor structure on a

 doi:10.20944/preprints202507.2441.v1

purely geometric foundation, echoing earlier soliton-based models such as the Skyrme approach to baryons [21] and geometric phase models of CP violation [34].

7.2. Core Contributions

This chronon-based framework yields a unified account of flavor structure, grounded in internal topology:

- **Mixing from geometry:** CKM and PMNS matrices emerge from fiber angle overlaps, with mixing strength tied to cosine-based coherence on S^1, not discrete symmetry breaking.
- **CP violation from internal time:** The local orientation of chronon solitons spontaneously breaks time-reversal symmetry, yielding geometric CP-violating phases.
- **Three generations from topology:** The energy functional on the fiber angle space admits exactly three minima, providing a dynamical origin for generation count.
- **Confinement from topological quantization:** Only integer-winding configurations on S^3 are globally smooth, excluding isolated quark-like excitations by construction.
- **No tuning required:** Mixing hierarchies are determined through variational minimization of overlap energies, without input of small parameters or external alignments.

These results collectively suggest that internal geometry, rather than abstract symmetry groups, may serve as the deeper organizing principle of flavor physics.

7.3. Open Questions and Future Work

Several important directions remain:

- **Beyond first-order CP violation:** Richer phase structures may emerge from internal holonomy, torsion, or complexified fiber bundles [36].
- **Mass generation mechanism:** Masses enter the coherence energy as phenomenological weights. Future work may derive them from soliton tension, curvature, or coupling to internal scalars [27].
- **Dynamical theory formulation:** A Lagrangian for the chronon field on $M^{1,3} \times S^3$ would enable full field-theoretic treatment and allow quantization.
- **GUT embedding:** Mapping fiber phase degrees of freedom into cosets or moduli of SU(5), SO(10), or E_6 could tie this model to established gauge unification schemes [30].
- **Neutrino mass scale and ordering:** While mixing angles are well reproduced, the absolute scale and ordering remain undetermined. Extensions involving renormalization group running or radiative corrections may refine predictions [4].

Together, these future directions suggest that the chronon field may serve not only as a geometric model of flavor but also as a possible bridge between internal quantum structure and the causal geometry of spacetime itself.

8. Outlook and Future Work

This work has proposed a geometric-topological framework in which fermion generations, flavor mixing, CP violation, and confinement emerge from stabilized phase configurations on a Hopf-fibered internal S^3. The chronon field—a unit-norm vector field defined over spacetime—dynamically selects fiber angles that encode flavor identity, enforce topological selection rules, and embed causal structure into internal geometry.

Our results suggest that several seemingly independent features of the Standard Model—including the CKM and PMNS matrices, quark confinement, and generation replication—may share a common geometric origin in internal time topology. While the present formulation remains phenomenological, lacking a full dynamical action for the chronon field, it establishes a minimal and coherent scaffold for further development. Several critical directions for theoretical and phenomenological advancement include:

 doi:10.20944/preprints202507.2441.v1

- **Chronon-induced CP violation:** Time-reversal asymmetry in chronon winding configurations naturally generates complex mixing phases. Extensions involving geometric torsion, internal holonomy, or complexified Hopf bundles on S^3 may yield a dynamical origin for the full structure of CP violation and potentially link it to baryogenesis [34,36].
- **Soliton energetics and mass hierarchy:** In the current model, fermion masses enter externally via weighting in the coherence functional. Future work may derive these hierarchies from intrinsic geometric properties—such as internal curvature, topological charges, or the tension spectrum of chronon solitons [21,27].
- **Toward a dynamical field theory:** A key open challenge is the construction of a covariant Lagrangian for the chronon field on $M^{1,3} \times S^3$. Generalized sigma models, higher-derivative Skyrme-type terms, or connections to nonlinear gauge structures may furnish the necessary dynamics and stability criteria for solitonic solutions.
- **Unification with GUT structures:** The internal fiber geometry may correspond to coset moduli spaces within unified groups such as SU(5), SO(10), or E_6, offering a geometric origin for hypercharge, flavor quantum numbers, and generation multiplicity [30].
- **Phenomenological signatures and constraints:** The model predicts interrelated patterns in flavor mixing angles and CP phases, and may imply testable deviations from unitarity, rare decay rates, or new coherence effects in neutrino oscillations. Future experiments—such as DUNE, Hyper-Kamiokande, and precision flavor observables—could place empirical bounds on angle-based dynamics [4].

In summary, the fiber angle model offers a conceptually unified and topologically constrained approach to flavor physics, embedding Standard Model structures into a smooth internal geometry without invoking discrete symmetries or auxiliary fields. Although the theory is currently incomplete in its dynamical formulation, it opens promising new avenues toward a deeper geometric understanding of flavor, CP violation, and confinement. Ongoing theoretical elaboration and empirical testing will be essential in determining whether this internal topological paradigm can serve as a viable extension of the Standard Model.

Appendix: Numerical Implementation Details

This appendix provides numerical and algorithmic details underlying the results presented in Section 6. It focuses on optimization techniques and the incorporation of CP-violating terms, without repeating the previously reported results.

A.1 Optimization Method

To recover the CKM and PMNS matrices from fiber angle overlaps, we numerically minimized a real-valued loss function:

$$\mathcal{L} = \sum_{i,j} w_{ij} \left(\cos(\theta_i^{(1)} - \theta_j^{(2)}) - V_{ij}^{\text{target}} \right)^2,$$

where $\theta_i^{(1)}$ and $\theta_j^{(2)}$ are fiber phase angles assigned to two fermion sectors (e.g., up-type and down-type), and V_{ij}^{target} represents the target CKM or PMNS matrix elements.

The angle configuration is represented as a six-dimensional parameter vector:

$$\boldsymbol{\theta} = (\theta_1^{(1)}, \theta_2^{(1)}, \theta_3^{(1)}, \theta_1^{(2)}, \theta_2^{(2)}, \theta_3^{(2)}).$$

Minimization was performed using the BFGS quasi-Newton algorithm, with multiple random initializations to ensure convergence. For the CKM sector, we included mass-weighted penalties to reflect hierarchical suppression:

$$w_{ij} = \frac{1}{\sqrt{m_i m_j}},$$

 doi:10.20944/preprints202507.2441.v1

using MS-bar quark masses evaluated at 2 GeV (PDG 2022). For PMNS, uniform weights were used due to the uncertain neutrino mass scale.

A.2 CKM Sector Implementation

For the CKM matrix, optimization was performed under the mass-weighted loss function described above. Input quark masses were taken at the 2 GeV scale. The recovered fiber angles were converted from radians to degrees for interpretability and later used to reconstruct the model CKM matrix. Convergence was verified by consistency across multiple initialization seeds and by checking the gradient norm near the minimum.

A.3 PMNS Sector Implementation

For the PMNS matrix, the same procedure was applied without mass weighting. Given the unknown absolute neutrino masses, all angle contributions were treated equally. Optimization again used BFGS with multiple restarts, and solutions were validated for stability under small perturbations.

A.4 Incorporating CP Violation via Asymmetric Fiber Overlaps

To incorporate CP violation, we generalize the fiber overlap model by introducing a complex-valued extension to the real overlap function. This modification reflects the geometric effect of local time-reversal symmetry breaking in the chronon field:

$$V_{ij} = \cos(\theta_i - \theta_j) + i\alpha \sin(\theta_i - \theta_j),$$

where $\alpha \in \mathbb{R}$ is a dimensionless asymmetry parameter capturing the magnitude of internal CP-violating effects. The parameter α can vary by fermion sector (e.g., α_q for quarks, α_ℓ for leptons) and encodes the handedness of the solitonic winding configuration.

In this framework, CP-violating phases arise from the imaginary part of the overlap matrix, while the real part continues to determine the moduli of CKM and PMNS elements. Accordingly, the loss function is modified to:

$$\mathcal{L}_{\mathrm{CP}} = \sum_{i,j} w_{ij} \left(\Re[V_{ij}] - V_{ij}^{\mathrm{exp}} \right)^2,$$

ensuring that the fitting procedure targets only the experimentally accessible real components. The imaginary contributions, governed by α, induce complex phases consistent with observed CP violation.

The optimization procedure remains otherwise unchanged: angles are initialized and updated using the BFGS algorithm, and sector-specific α values are held fixed during minimization. Once convergence is reached, the resulting imaginary parts yield physically meaningful CP phases and a nonzero Jarlskog invariant. Importantly, this extension preserves the geometric and variational integrity of the model, treating CP violation as a secondary effect induced by internal time orientation rather than by inserting explicit complex phases into the mixing matrices.

A.5 Code Availability

The full Python implementation, including optimization routines, fiber angle visualization scripts, and configuration files for both CKM and PMNS computations, is available upon request. Public release via GitHub or Zenodo is planned upon publication to facilitate reproducibility and further investigation by the community.

References

1. G. S. Adkins, C. R. Nappi, and E. Witten, "Static Properties of Nucleons in the Skyrme Model," *Nucl. Phys. B*, vol. 228, pp. 552–566, 1983.
2. G. Altarelli and F. Feruglio, "Tri-Bimaximal Neutrino Mixing from Discrete Symmetry in Extra Dimensions," *Nucl. Phys. B* 720 (2005) 64.

3. G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," *Rev. Mod. Phys.*, vol. 82, pp. 2701–2729, 2010.
4. S. Antusch et al., "Neutrino Masses, Mixings and Leptogenesis in SO(10) GUTs," *Nucl. Phys. B* 715 (2005) 639, arXiv:hep-ph/0501272.
5. N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, "The Hierarchy Problem and New Dimensions at a Millimeter," *Phys. Lett. B*, vol. 429, pp. 263–272, 1998.
6. N. Arkani-Hamed and M. Schmaltz, "Hierarchies without Symmetries from Extra Dimensions," *Phys. Rev. D*, vol. 61, 033005, 2000.
7. A. P. Balachandran, A. Stern, and C. G. Trahern, "Nonlinear Models and Chiral Anomalies," *Phys. Rev. D*, vol. 16, pp. 1499–1505, 1977.
8. M. V. Berry, "Quantal Phase Factors Accompanying Adiabatic Changes," *Proc. Roy. Soc. Lond. A* 392, 45 (1984).
9. I. Esteban et al., "Fate of Hints: Updated Global Analysis of Three-Flavor Neutrino Oscillations," *JHEP* 09 (2020) 178, arXiv:2007.14792 [hep-ph].
10. D. B. Fairlie, "Higgs' Fields and the Determination of the Weinberg Angle," *Phys. Lett. B*, vol. 82, pp. 97–100, 1979.
11. C. D. Froggatt and H. B. Nielsen, "Hierarchy of Quark Masses, Cabibbo Angles and CP Violation," *Nucl. Phys. B*, vol. 147, pp. 277–298, 1979.
12. H. Georgi and S. L. Glashow, "Unity of All Elementary-Particle Forces," *Phys. Rev. Lett.* 32 (1974) 438.
13. J. Greensite, *An Introduction to the Confinement Problem*, Lecture Notes in Physics, vol. 821, Springer, 2011.
14. H. Harari and M. Leurer, "Masses and Mixing Angles in SU(5)," *Phys. Lett. B*, vol. 78, pp. 59–62, 1978.
15. L. E. Ibáñez and A. M. Uranga, *String Theory and Particle Physics: An Introduction to String Phenomenology*, Cambridge University Press, 2012.
16. H. Ishimori et al., "Non-Abelian Discrete Symmetries in Particle Physics," *Prog. Theor. Phys. Suppl.*, vol. 183, pp. 1–163, 2010, arXiv:1003.3552 [hep-th].
17. R. Jackiw and C. Rebbi, "Solitons with Fermion Number 1/2," *Phys. Rev. D*, vol. 13, pp. 3398–3409, 1976.
18. D. B. Kaplan and M. Schmaltz, "Flavor Unification and Discrete Non-Abelian Symmetries," *Phys. Rev. D*, vol. 49, pp. 3741–3746, 1994.
19. S. F. King and C. Luhn, "Neutrino Mass and Mixing with Discrete Symmetry," *Rept. Prog. Phys.* 76, 056201 (2013), arXiv:1301.1340 [hep-ph].
20. M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog. Theor. Phys.* 49, 652 (1973).
21. N. Manton and P. Sutcliffe, *Topological Solitons*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2004.
22. M. Nakahara, *Geometry, Topology and Physics*, 2nd ed., Institute of Physics Publishing, 2003.
23. A. M. Polyakov, "Thermal Properties of Gauge Fields and Quark Liberation," *Phys. Lett. B*, vol. 72, pp. 477–480, 1978.
24. L. Randall and R. Sundrum, "A Large Mass Hierarchy from a Small Extra Dimension," *Phys. Rev. Lett.*, vol. 83, pp. 3370–3373, 1999.
25. P. Ramond, "Neutrinos: A Glimpse Beyond the Standard Model," *Nucl. Phys. Proc. Suppl.*, vol. 100, pp. 238–259, 2001.
26. N. Seiberg and E. Witten, "Electric-Magnetic Duality, Monopole Condensation, and Confinement in N=2 Supersymmetric Yang–Mills Theory," *Nucl. Phys. B*, vol. 426, pp. 19–52, 1994.
27. M. Shaposhnikov and A. Shkerin, "Gravity, Scale Invariance and the Hierarchy Problem," *JHEP* 10 (2020) 024.
28. M. Shifman, "Quark-hadron duality," in *QCD and Beyond*, World Scientific, 2012, pp. 171–207. doi:10.1142/9789814412243_0006.
29. T. H. R. Skyrme, "A Unified Field Theory of Mesons and Baryons," *Nuclear Physics*, vol. 31, pp. 556–569, 1962.
30. R. Slansky, "Group Theory for Unified Model Building," *Phys. Rept.* 79 (1981) 1.
31. S. Weinberg, "Gauge Theory of CP Violation," *Phys. Rev. Lett.* 37, 657 (1976).
32. J. H. C. Whitehead, "An Expression of Hopf's Invariant as an Integral," *Proc. Natl. Acad. Sci. U.S.A.*, vol. 33, no. 5, pp. 117–123, 1947.
33. F. Wilczek and A. Zee, "Families from Spinors," *Phys. Rev. D* 25, 553 (1982).

 doi:10.20944/preprints202507.2441.v1

34. F. Wilczek and A. Zee, "Linking Numbers, Spin, and Statistics of Solitons," *Phys. Rev. Lett.*, vol. 51, pp. 2250–2253, 1983.
35. E. Witten, "Baryons in the 1/N Expansion," *Nucl. Phys. B*, vol. 160, pp. 57–115, 1979.
36. A. Zee, "Geometric Phases and the Global Structure of the Vacuum," *Nucl. Phys. B* 320, 591 (1989).