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Jiazheng Liu *

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Article

Unique Bandlimited Green’s Function as Physical Field: Analytic Unification of Gauge Fields, Quantum Gravity, Fermions, Dissipative Dynamics, and Celestial Holography on the Light Cone

Jiazheng Liu

Independent Researcher; lib88419@gmail.com

Abstract

We prove that $\delta^{(4)}(x - y) \notin L^2(\mathbb{R}^4)$ is not a legitimate physical Green’s function under the quantum-mechanical postulate of finite energy (A1). A fourth postulate of closed sourcelessness (A4)—methodologically analogous to Einstein’s postulate of the constancy of the speed of light—is derived as a theorem from the quantum-gravity result $\dim \mathcal{H}_{\text{universe}} = 1$ [1–8]. Under three independent postulates A1–A3 together with this result, we derive the unique physical Green’s function $G = \sin(\Omega\sqrt{-\sigma^2 - i\epsilon}) / (\Omega\sqrt{-\sigma^2 - i\epsilon})$, $\Omega = \pi/t_p$. The bandlimited two-point function K of the resulting Paley–Wiener space PW_{π/t_p} admits the spherical Bessel decomposition $K(x, x') = \frac{\Omega^3}{2\pi^2} \sum_{l=0}^\infty (2l + 1) j_l(\Omega r) j_l(\Omega r') P_l(\cos \theta)$. We prove: (i) the $l = 0, 1, 2$ sectors are precisely the scalar, photon, and graviton propagators; (ii) gauge symmetry emerges as the zero-set geometry of j_l ; (iii) restriction to the light cone $\sigma^2 = 0$ yields the celestial sphere S^2 with 2D CFT two-point structure and conformal dimensions $\Delta_l = l + 1$, parameter-free; (iv) tensor structure Π_l follows from the $\text{SO}(4, 2)$ representation theory of massless fields on the six-dimensional light cone [9,10]; (v) fermions arise necessarily from the spinor representations of $\text{SO}(4, 2)$ via $\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{pos}} \otimes \mathcal{H}_{\text{int}}$. All four physical regimes (QFT, quantum gravity, gauge fields, dissipation) are restrictions of the single entire function $f(z) = \sin(z)/z$ to different domains of $\mathbb{C}$. Bandlimitedness is a theorem, not an assumption. Since all cosmological observables—CMB (TT, TE, EE), large-scale structure, and weak lensing—are recorded along null geodesics ($\sigma^2 = 0$ where $G = 1$ exactly, with no dimensional suppression), they jointly probe the same $\Delta_l = l + 1$ structure on the celestial sphere. Their combined Bayesian posterior $P(\theta|\text{data}) \propto \prod_i \mathcal{L}_i$ compresses the posterior width as $1/\sqrt{N_{\text{datasets}}}$, providing a simultaneous, parameter-free observational test.

Keywords: bandlimited Green’s function; Paley-Wiener space; quantum gravity; gauge field unification; celestial holography; 2D CFT; light cone; parameter-free prediction; COSMIC Microwave Background (CMB); spherical bessel functions; spin representation; finite energy postulate

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Introduction

Standard QFT uses $\delta^{(4)}(x - y)$ as the source in $\square G = \delta^{(4)}$. Since $\|\delta^{(4)}\|_{L^2}^2 = \delta^{(4)}(0) = \infty$, this distribution violates the quantum-mechanical postulate that physical fields carry finite energy. This is a category error; its correction is the subject of this Letter.

The correction yields a unique physical Green’s function $G \in \text{PW}_{\pi/t_p} \subset L^2$, whose spherical Bessel decomposition provides a mathematical unification of all spin fields within a single bandlimited two-point function. The light-cone value $G|_{\sigma^2=0} = 1$ (a removable singularity of $\sin(z)/z$) connects the Planck-scale propagator to the celestial sphere and to 2D conformal field theory, and—crucially—carries *no* dimensional suppression: all cosmological observables recorded along null geodesics sense the same $\Delta_l = l + 1$ conformal structure simultaneously.

Relation to prior work.

Kempf and collaborators [11–14] developed bandlimited QFT with an assumed UV cutoff. Our contribution is fivefold: (i) bandlimitedness is a theorem under A1–A4; (ii) A4 itself is a theorem from $\dim \mathcal{H} = 1$; (iii) the spherical Bessel decomposition gives a parameter-free unification of gauge fields and gravity on the light cone; (iv) fermions arise from the spinor representations of $SO(4, 2)$ without additional postulates; (v) the light-cone value $G = 1$ makes the $\Delta_l = l + 1$ prediction testable with every null-geodesic cosmological probe simultaneously.

Postulates and A4 as Theorem

A1 (Quantum mechanics). $\varphi \in L^2(\mathbb{R}^4)$.

A2 (Analytic completeness). Any analytic continuation of a physical field from $\mathbb{R}^4$ is uniquely determined by its real-domain values (identity theorem for analytic functions).

A3 (Singularity). Penrose–Hawking geodesic incompleteness under the strong energy condition [15,16].

Methodological remark on A4.

Einstein’s 1905 special relativity rests on the postulate of the constancy of the speed of light—a statement adopted because it is internally consistent and empirically fruitful. A4 occupies the same epistemological position. Rather than treating it as an independent axiom, we show it follows from established quantum-gravity results, but the logical force of the framework survives even if one adopts it directly as a postulate, just as special relativity survives independent of the derivation of c from Maxwell’s equations.

[A4 from $\dim \mathcal{H} = 1$].

The results of [1–8]—established respectively via quantum extremal surfaces, replica wormholes, topological models, two-dimensional gravity, Swampland conditions, and gravitational path integrals with observers—all imply $\dim \mathcal{H}_{\text{universe}} = 1$. Specifically:

1. Quantum extremal surface calculations show that the entanglement entropy of a closed universe with any putative external system is identically zero, implying the encoding map is a rank-one projection [1,2].
2. Gravitational path integrals with replica wormholes show that the inner-product matrix of distinct universe states has rank one: all apparently distinct states are the same state in the fundamental description [3].
3. Holographic CFT computations confirm the rank-one inner-product structure [4].
4. Swampland arguments show that gauge redundancies of quantum gravity force $\dim \mathcal{H}_{\text{BU}} = 1$ in $d > 3$ spacetime dimensions [5].
5. Gravitational path-integral analyses with an explicit internal observer confirm the one-dimensional structure in the observer-free limit [6–8].

A one-dimensional Hilbert space contains no external parameters: any state $|\Psi(\lambda)\rangle = c(\lambda)|\Psi_0\rangle$ has $c(\lambda) \in \mathbb{C}$ unobservable by the Born rule, so λ has no physical effect (A4).

[Planck time from dimensional uniqueness].

Under A4, the only time scale constructible from the universal constants $c, \hbar, G$ is $t_P = \sqrt{\hbar G/c^5}$ (Appendix A).

[Bandlimit theorem].

Under A1–A4, $\varphi \in \text{PW}_{\pi/t_P}$ with unique cardinal series (Eq. (2)) and Planck time t_P fixed by dimensional analysis. The unique bandlimited two-point function of PW_{π/t_P} is

$$K(t, s) = \frac{\sin(\pi(t - s)/t_P)}{\pi(t - s)/t_P}, \quad (1)$$

with cardinal series

$$\varphi(t) = \sum_{n \in \mathbb{Z}} \varphi(nt_P) K(t, nt_P). \quad (2)$$

Bandlimitedness is the *conclusion* of the theorem, not its hypothesis.

Unique Physical Green's Function

Principle (physical field status). $G \in \text{PW}_{\pi/t_p} \subset L^2$ is a physical field, not a distribution. The modified field equation $\square_x G(x, y) = K(\sigma(x, y)/t_p)$ is closed within PW_{π/t_p} . [Existence and uniqueness].

The unique field satisfying (C1) $G \in \text{PW}_{\pi/t_p}$, (C2) real-domain values determine its analytic continuation, (C3) Feynman boundary conditions, is

$$G(x, y) = \frac{\sin(\Omega \sqrt{-\sigma^2(x, y) - i\varepsilon})}{\Omega \sqrt{-\sigma^2(x, y) - i\varepsilon}}, \quad \Omega = \frac{\pi}{t_p}. \quad (3)$$

Three regions. (i) **Timelike** ($\sigma^2 < 0$, $\tau = \sqrt{-\sigma^2}$): $G = j_0(\Omega\tau) \in L^2$. (ii) **Light cone** ($\sigma^2 = 0$): $G|_{\sigma^2=0} = \lim_{z \rightarrow 0} \sin(z)/z = 1$ (removable singularity). (iii) **Spacelike** ($\sigma^2 > 0$, $s = \sqrt{\sigma^2}$): $G = (2/\pi)K_1(\Omega s)/(\Omega s) \sim e^{-\Omega s} \in L^2$, the unique L^2 branch selected by the Feynman $i\varepsilon$ prescription (Appendix B).

The standard QFT Green's function is recovered in the distributional limit $t_p \rightarrow 0$ with error $\mathcal{O}(t_p^2)$.

Gauge Field Unification on the Light Cone

Spherical Bessel decomposition.

In three spatial dimensions the bandlimited two-point function takes the form

$$K(x, x') = \frac{\Omega^3}{2\pi^2} \sum_{l=0}^{\infty} (2l+1) j_l(\Omega r) j_l(\Omega r') P_l(\cos \theta), \quad (4)$$

where $r = |\mathbf{x}|$, $r' = |\mathbf{x}'|$, θ is the angle between $\mathbf{x}$ and $\mathbf{x}'$, and j_l are spherical Bessel functions of the first kind. This expansion follows from the three-dimensional Fourier transform of $\hat{K}(\mathbf{k}) = (\pi/\Omega) \mathbf{1}_{|\mathbf{k}| \leq \Omega}$ combined with the Jacobi–Anger plane-wave expansion (Appendix C).

Tensor structure from $\text{SO}(4, 2)$ representation theory.

Massless fields in four-dimensional Minkowski space have asymptotic symmetry group $\text{SO}(4, 2)$ (Penrose [17]). Dirac [9] showed that the natural arena for $\text{SO}(4, 2)$ is the six-dimensional light cone $\eta_{AB} X^A X^B = 0$. Aldaya, Calixto, and Cerveró [10] proved that unitary realisation of $\text{SO}(4, 2)$ while maintaining $P^\mu P_\mu = 0$ requires the full double cover $\text{Spin}(4, 2) \cong \text{SU}(2, 2)$. Under the restriction $\text{SO}(4, 2) \downarrow \text{ISO}(3, 1)$, helicity $|\lambda| = l$ determines:

- $l = 0$: trivial representation $\Rightarrow$ scalar, $\Pi_0 = \mathbf{1}$.
- $l = 1$: vector representation, transverse projector $\Pi_{ij}^T = \delta_{ij} - \hat{r}_i \hat{r}_j$.
- $l = 2$: symmetric rank-2 tensor, transverse-traceless projector $\Pi_{\mu\nu\rho\sigma}^{TT}$ uniquely fixed by the $(2, 2)$ irreducible representation of the little group $\text{SO}(2)$.

[Gauge field unification].

The l -th sector of decomposition (4) is proportional to the radial part of the spin- l massless propagator:

$$l = 0 : G_{\text{scalar}} \propto j_0(\Omega r), \quad (5)$$

$$l = 1 : G_{\mu\nu}^T \propto \Pi_1 j_1(\Omega r), \quad (6)$$

$$l = 2 : G_{\mu\nu\rho\sigma}^{TT} \propto \Pi_2 j_2(\Omega r). \quad (7)$$

Gauge symmetry as zero-set geometry of j_l .

The zero-set of $j_0(x) = \sin(x)/x$ is $x = n\pi$ ($n \geq 1$), i.e.,

$$r_n = \frac{n\pi}{\Omega} = n t_P, \quad n \in \mathbb{Z}^+. \quad (8)$$

A gauge parameter $\chi \in \text{PW}_{\pi/t_P}$ in the l -th angular sector vanishes on the zero-set of j_l : $\chi^{(l)}(r_n^{(l)}) = 0$, $j_l(\Omega r_n^{(l)}) = 0$. For $l = 1$ the zero-set $\{r_n^{(1)}\}$ provides the geometric origin of $U(1)$ gauge invariance and the Aharonov–Bohm effect. For $l = 2$ the zero-set yields the BMS supertranslation boundary conditions at null infinity [18].

Fermions from $SO(4,2)$ Spinor Representations

The spherical Bessel expansion (4) involves integer l because spherical harmonics Y_l^m must be single-valued: $e^{2\pi i l} = 1$ forces $l \in \mathbb{Z}_{\geq 0}$. Hence fermions cannot arise from the orbital sector $\mathcal{H}_{\text{pos}}$ alone; they arise from representation theory.

The total state space is

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{pos}} \otimes \mathcal{H}_{\text{int}}, \quad (9)$$

where $\mathcal{H}_{\text{int}}$ is the $SO(4,2)$ representation space. The Aldaya–Calixto–Cerveró theorem [10] shows that realising $SO(4,2)$ unitarily on $\mathcal{H}_{\text{int}}$ requires the full double cover $\text{Spin}(4,2) \cong \text{SU}(2,2)$, which necessarily admits half-integer spin representations. Including only tensor (integer) representations would reduce the symmetry group from $\text{Spin}(4,2)$ to $SO(4,2)$, breaking the unitary implementation required by A1.

[Fermion necessity].

Under A1–A4, the full unitary representation of $\text{Spin}(4,2)$ on $\mathcal{H}_{\text{tot}}$ necessarily includes half-integer spin sectors. The Dirac spinor (spin $\frac{1}{2}$) and all higher half-integer spin fields appear as sectors of $\mathcal{H}_{\text{tot}}$ without additional postulates. Explicitly: $l = 0, s = \frac{1}{2}$: $j = \frac{1}{2}$ (s-wave Dirac fermion); $l = 1, s = \frac{1}{2}$: $j = \frac{1}{2}, \frac{3}{2}$ (including the gravitino at $j = \frac{3}{2}$).

Light Cone, Celestial Sphere, and 2D CFT

[Celestial 2D CFT from the light cone].

On the light cone $\sigma^2 = 0$ with fixed $r = |\mathbf{x} - \mathbf{x}'|$, the angular part of the bandlimited two-point function satisfies

$$\sum_{l=0}^{\infty} (2l+1) j_l(\Omega r)^2 P_l(\cos \theta) \xrightarrow{\Omega r \rightarrow \infty} \delta^{(2)}(\hat{n} - \hat{n}'), \quad (10)$$

which is the identity two-point function of a 2D CFT on S^2 .

Conformal dimensions.

Using $j_l(x) \sim \sin(x - l\pi/2)/x$ for $x \rightarrow \infty$, one reads off

$$\Delta_l = l + 1, \quad (11)$$

parameter-free: $\Delta_\gamma = 2$ for photons, $\Delta_h = 3$ for gravitons, consistent with and extending the Pasterski–Shao–Strominger conventions [19,20].

The peak value $\sum_l (2l+1) j_l(\Omega r)^2 P_l(1) = K(\Omega r, \Omega r) = 1$ holds exactly, consistent with $G|_{\sigma^2=0} = 1$. The gauge unification diagram reads:

$$K(x, x') \longrightarrow \begin{cases} l = 0 & \text{scalar propagator} \\ l = 1 & \text{photon propagator (U(1))} \\ l = 2 & \text{graviton propagator (Diff)} \end{cases} \quad (12)$$

$$\sigma^2 \rightarrow 0 : K|_{S^2} \rightarrow \text{2D CFT on celestial sphere, } \Delta_l = l + 1.$$

Multi-Probe Cosmological Testability

Light-cone physics is unsuppressed.

CMB photons travel null geodesics ($\sigma^2 = 0$), so they sense $G|_{\sigma^2=0} = 1$ exactly—no dimensional hierarchy of the form $(k/\Omega)^2 \sim 10^{-122}$ applies at Hubble scales. The prediction $\Delta_l = l + 1$ is therefore not a small correction to existing results; it is the *exact* two-point structure on the celestial sphere.

Physical origin of cosmic variance.

The cosmic variance per multipole

$$\frac{\Delta C_l}{C_l} \propto \frac{1}{\sqrt{l + \frac{1}{2}}}$$

reflects the $2l + 1$ independent modes on S^2 —the same geometric origin as $\Delta_l = l + 1$ in Eq. (11).

Null-geodesic universality.

All principal cosmological probe classes are recorded along null geodesics and therefore probe the same $\Delta_l = l + 1$ structure:

- CMB temperature TT : constrains acoustic peak positions.
- CMB polarisation EE : constrains reionisation optical depth.
- CMB cross-spectrum TE : constrains the cross-correlation of the above.
- Large-scale structure $P(k)$: constrains the matter growth rate.
- Weak gravitational lensing: constrains the integrated gravitational potential.

These datasets constrain the same prediction $\Delta_l = l + 1$ from approximately orthogonal directions in parameter space.

Joint Bayesian posterior.

The combined posterior is

$$P(\theta|\text{data}) \propto \mathcal{L}_{TT} \mathcal{L}_{TE} \mathcal{L}_{EE} \mathcal{L}_{LSS} \mathcal{L}_{\text{lens}} \cdots \times \pi(\theta). \quad (13)$$

Because each $\mathcal{L}_i$ constrains θ from a complementary direction, the product compresses the posterior width as $1/\sqrt{N_{\text{datasets}}}$ —validated in precision cosmology by the joint CMB + BAO + SNe determination of dark energy parameters.

Distinguishing signature.

Other quantum-gravity schemes predict timelike corrections with $(k/\Omega)^2$ suppression and no exact $\Delta_l = l + 1$ structure on the celestial sphere: string theory predicts a Gaussian e^{-k^2/M_s^2} ; loop quantum gravity predicts polynomial corrections [12]; minimum-length models predict exponential cutoffs [14]. The zero-suppression, parameter-free prediction $\Delta_l = l + 1$ simultaneously constrained by every null-geodesic probe is the observable signature distinguishing the present framework.

Analytic Unity Theorem

[Analytic unity].

The entire function $f(z) = \sin(z)/z$ ($f(0) = 1$) unifies all physical regimes as restrictions to domains of $\mathbb{C}$:

1. QFT ($z \in \mathbb{R}^+$): $f(\Omega\tau) = j_0(\Omega\tau) \in [-1, 1]$; $\hat{f}(\omega) = (\pi/\Omega)\mathbf{1}_{|\omega| \leq \Omega}$.
2. Quantum gravity ($i\epsilon$ branch): $f(\Omega s e^{i(\pi/2-\delta)}) \rightarrow (2/\pi)K_1(\Omega s)/(\Omega s)$, $\delta \rightarrow 0^+$, exponential decay.
3. Light cone: $f(0) = 1$ (removable singularity).
4. Gauge fields: l -projected sectors of (4), with tensor structure Π_l from $\text{SO}(4, 2)$.
5. Fermions: spinor sectors of $\mathcal{H}_{\text{tot}}$.
6. Dissipation (Laplace transform, $\text{Re}(s) > 0$): $\mathcal{L}[f(\Omega \cdot)](s) = \Omega^{-1} \arctan(\Omega/s)$.

All items are uniquely determined by A2: an entire function is fixed by its values on any set with an accumulation point.

Resolution of Fundamental Conflicts

UV divergences. $\tilde{G}(\mathbf{k}) = 0$ for $|\mathbf{k}| > \Omega$; all loop integrals are finite.

Light-cone singularity. $G|_{\sigma^2=0} = 1$, finite; the divergence $G_{\text{std}}|_{\sigma^2=0} \rightarrow \infty$ was an artifact of $\delta^{(4)} \notin L^2$.

Gauge/gravity separation. Photon and graviton propagators are here projections of the same bandlimited two-point function (4), with tensor structures uniquely determined by $\text{SO}(4,2)$.

Boson/fermion separation. Integer-spin and half-integer-spin sectors of the same $\text{Spin}(4,2)$ representation on $\mathcal{H}_{\text{tot}}$.

Celestial CFT basis. The celestial holography programme [19,20] identifies 4D scattering amplitudes with 2D CFT correlators on S^2 . The above provides the field-theoretic origin: the celestial sphere is the angular section of the light cone, and the 2D CFT is the restriction of the unique bandlimited two-point function to $\sigma^2 = 0$.

Rank-one correspondence. The Green's function matrix on the Planck lattice $G(mt_P, nt_P) = \delta_{m,0}\delta_{n,0}$ is rank-one, in precise correspondence with the rank-one inner-product matrix of closed-universe quantum gravity [1,2,6].

Observable Predictions

All corrections are fixed by t_P without free parameters; functional forms are theorems.

Casimir energy: $\Delta E_{\text{Cas}}/E_{\text{Cas}}^{\text{std}} = c_1(t_P c/d)^4 + \mathcal{O}(t_P^6)$, $c_1 = \mathcal{O}(1)$.

Lamb shift: $\delta E_{\text{Lamb}}/E_{\text{Lamb}}^{\text{std}} = c_2(\alpha/\pi)(m_e/m_P)^2 \sim 10^{-44}$.

Celestial conformal dimensions: $\Delta_{\text{scalar}} = 1$, $\Delta_\gamma = 2$, $\Delta_h = 3$ (Eq. (11)), consistent with and extending [19].

Multi-probe CMB/LSS/lensing test: The prediction $\Delta_l = l + 1$ is simultaneously constrained by all null-geodesic probes (CMB TT , TE , EE , $P(k)$, lensing) with no free parameters and no dimensional suppression.

Planck-shell zeros: The zero-set of the scalar propagator occurs at exactly $r_n = n t_P$ (Eq. (??)), in principle testable as a Planck-scale structure of vacuum fluctuation correlators.

Scope and Limitations

L1 (Flat spacetime). Curved-spacetime extension requires the Hadamard construction and Kempf's spectral-geometry approach [12]. The light-cone results have conformal universality and are not affected by bulk curvature corrections at leading order.

L2 (Free fields). Interaction corrections are UV-finite but require order-by-order computation.

L3 (Higher l). Spins $l \geq 3$ appear as $l \geq 3$ sectors of (4); their consistency with known no-go theorems for higher-spin fields is left for separate analysis.

L4 (Non-Abelian gauge groups). $l = 1$ gives the radial structure of the vector propagator; connecting colour indices to the bandlimited function space structure for $\text{SU}(2)$, $\text{SU}(3)$ is an open problem.

Conclusion

Under three independent postulates (A1–A3) plus the quantum-gravity theorem $\dim \mathcal{H}_{\text{universe}} = 1$ [1–8], we have derived:

1. The unique physical Green's function $G \in \text{PW}_{\pi/t_P}$, with bandlimitedness a theorem not a postulate.
2. All spin- l bosonic propagators as l -projected sectors of K , with tensor structure uniquely determined by $\text{SO}(4,2)$.
3. Fermions as necessarily arising from spinor representations of $\text{Spin}(4,2)$ on $\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{pos}} \otimes \mathcal{H}_{\text{int}}$.
4. Gauge symmetry as the zero-set geometry of j_l .
5. The celestial sphere 2D CFT with conformal dimensions $\Delta_l = l + 1$ as the light-cone restriction of K .

6. Analytic unity of QFT, quantum gravity, gauge fields, fermions, and dissipation as restrictions of $f(z) = \sin(z)/z$.
7. A simultaneous, parameter-free observational prediction: all null-geodesic cosmological probes (CMB $TT/TE/EE$, LSS, lensing) probe the same $\Delta_l = l + 1$ structure with no dimensional suppression, yielding a joint posterior that contracts as $1/\sqrt{N_{\text{datasets}}}$.

The key insight is that all spin fields, all spacetime regimes, and the coherent/dissipative distinction are encoded in a single entire function evaluated at different points of $\mathbb{C}$, together with the representation theory of $\text{Spin}(4, 2)$. Their apparent separation arose from the category error of treating $\delta^{(4)} \notin L^2$ as a physical Green's function. Correcting this error yields a parameter-free, self-consistent framework encompassing the full spin content of the Standard Model and gravity within one bandlimited two-point function—with its light-cone restriction simultaneously providing the foundation for celestial holography and a multi-probe cosmological test.

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Appendix A. Planck Time: Uniqueness from Dimensional Analysis

Lemma A1 (Dimensional uniqueness). *Let $c, \hbar, G$ be three mutually independent universal constants with dimensions $[c] = LT^{-1}$, $[\hbar] = ML^2T^{-1}$, $[G] = M^{-1}L^3T^{-2}$. The unique combination $c^a \hbar^b G^c$ with dimension $[T]$ is $t_P = \sqrt{\hbar G/c^5}$.*

Proof. Setting $[c^a \hbar^b G^c] = [T]$ and matching powers of M, L, T : $M : 0 = b - c$; $L : 0 = a + 2b + 3c$; $T : 1 = -a - b - 2c$. The unique solution is $a = -5/2, b = 1/2, c = 1/2$. $\square$

Theorem A1 (Planck time as the minimal physical interval). *A closed sourceless universe (A4) emerging from a singularity (A3) has, as the unique time scale consistent with A1–A4, the interval t_P . Consequently the physically observable event sequence forms an equally spaced lattice $\Lambda = \{nt_P\}_{n \in \mathbb{Z}}$.*

Appendix B. Uniqueness of the $i\epsilon$ Branch

Condition C1 in the spacelike region ($s > 0$) requires $G \in L^2$. The branch $z = i\Omega s$ gives $\sinh(\Omega s)/(\Omega s) \sim e^{\Omega s} \notin L^2$. The unique L^2 branch is $z = \Omega s e^{i(\pi/2 - \delta)}$ ($\delta \rightarrow 0^+$), yielding $(2/\pi)K_1(\Omega s)/(\Omega s) \sim e^{-\Omega s}$. This is identical to the Feynman $i\epsilon$ prescription, so C1 and C3 uniquely select the same branch simultaneously.

Appendix C. Spherical Bessel Decomposition and Jacobi–Anger

The three-dimensional bandlimited two-point function of PW_{π/t_P} is

$$K(\mathbf{x}, \mathbf{x}') = \int_{|\mathbf{k}| \leq \Omega} \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \quad (\text{A1})$$

Inserting the Jacobi–Anger expansion $e^{i\mathbf{k} \cdot \mathbf{r}} = 4\pi \sum_{l,m} i^l j_l(kr) Y_l^m(\hat{k}) Y_l^{m*}(\hat{r})$ and integrating over $\hat{k}$ using orthogonality of Y_l^m , the k -integral reduces to $\int_0^\Omega k^2 j_l(kr) j_l(kr') dk$, giving the normalisation in (4).

Appendix D. Mathematical Foundation of Bandlimitedness

Theorem A2 (Paley–Wiener, 1934 [21]). *Let $f \in L^2(\mathbb{R})$. The following are equivalent: (i) $\text{supp } \hat{f} \subseteq [-\sigma, \sigma]$; (ii) f is the restriction to $\mathbb{R}$ of an entire function F with $|F(z)| \leq Ce^{\sigma|\text{Im } z|}$; (iii) $f \in \text{PW}_\sigma$.*

Theorem A3 (Physical field bandlimit theorem). *Let φ be a physical field satisfying A1 and A2, and let the universe satisfy A3 and A4. Then $\varphi \in \text{PW}_{\pi/t_P}$, and φ has the unique representation*

$$\varphi(t) = \sum_{n \in \mathbb{Z}} \varphi(nt_P) \frac{\sin\left(\frac{\pi}{t_P}(t - nt_P)\right)}{\frac{\pi}{t_P}(t - nt_P)}.$$

Bandlimitedness is the conclusion of this theorem, not its hypothesis.

The proof follows from: (Step 1) Aliasing on the Planck lattice shows $\text{supp } \hat{\varphi} \subseteq [-\pi/t_P, \pi/t_P]$ by A4; (Step 2) Theorem D.1 then gives $\varphi \in \text{PW}_{\pi/t_P}$; (Step 3)–(Step 4) Completeness and orthogonality of the Whittaker–Shannon basis [22–24] give uniqueness of the expansion.

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