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Article

Interpretation of Gravity by Entropy

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Abstract: In this paper, we introduce generalized entropy, acceleration of its entropy and its partial entropy. We assume that generalized entropy can represent as a second-order polynomial by applying the idea of logistics function to its entropy. Besides, we show that the inverse of partial entropy can represent Newton's gravity, which is an inverse square law. By applying these concepts, we attempt to explain that 1) gravity becomes constant within small distance with some conditions. It is possible that gravity has 5-states within small enough distance. There exists possible anti-force, which is the opposite of Newton's gravity among 5-states. Furthermore, within small distance, we show the possibility that gravitational potential and Coulomb potential can be treated in the same way, that 2) the rotation speed of the galaxy does not depend on its radius if the radius is within the size level of the universe. (The galaxy rotation curve problem), and that 3) gravitational acceleration toward the center may change at long distance compared to Newton's gravity. We show that it becomes an expansion of Newton's gravity, and that the possibility of the existence of some constants which controls gravity and the speed of galaxies, and that gravity may relate on entropy. It also describes the relationship between Yukawa-type potential and generalized negative partial entropy. Using equations proposed in this paper, it attempts to propose 11-types of forces (accelerations) including g and compare the ratios of size of the fundamental 4-forces in nature (strong force, electromagnetic force, weak force and gravity). Furthermore, it suggests that there may exists new forces, and that gravitational constant G can fluctuate if entropy changes. Thermodynamics, quantum, gravity, electromagnetic and ecology may be unified through entropy.

Keywords: entropy; gravity; galaxy rotation curve; MOND; Planck's law; dynamical system; inverse square law; logistics function; yukawa potential; unification theory; theory of everything

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1. Introduction

In this paper, we will explain in the following order.

1. First, we define generalized entropy(+) $S_{D_+}(x,k)$ and generalized partial entropy(+) $S_{D_+}(x)$ partitioned by the partition function $D_+(x)$, and introduce acceleration of partial entropy $S''_{D_+}(x)$ and the positive function $Q_{D_+}(x)$ as satisfied $Q_{D_+}(x) = \xi x/D_+(x)$, where x is a positive variable and ξ is a positive constant.
2. Second, by applying the idea of logistics function to generalized entropy, we derive a function $Q_{D_+}(x)$ that defines the partition function $D_+(x)$. Moreover, we assume that generalized entropy(+) $S_{D_+}(x,k)$ is approximated by second-degree polynomial, that is, the formula $\lambda_2 x^2 + \lambda_1 x$. In other words, we assume that the second derivative of $S_{D_+}(x,k)$ is a constant $\lambda_2/2$.
3. Third, the inverse of partial entropy(+) $S_{D_+}(x)$ is defined as potential $V_{D_+}(x,k)$, and the first derivative of potential $V_{D_+}(x,k)$ is defined as acceleration $V'_{D_+}(x,k)$. Namely, it assumes that potential and acceleration are derived from entropy.
4. Forth, for application to gravity theory, the inverse $1/\lambda_2$ is interpreted as mass m, the constant k is interpreted as gravitational constant G, and a variable x is interpreted as distance R, etc. Thereby, potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$ are interpreted as gravitational potential $V_{D_+}(R,G)$ and gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R,G)$. Therefore, we show and propose some conclusions:

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(1) If distance R is small enough, gravity is a constant regardless of R, and may not go to infinity under certain conditions. It is possible that gravity have 5-states within distance R is small enough. Among 5-states, there is anti-force, which is the opposite of Newton's gravity. Furthermore, within small distance, we show that the possibility that gravitational potential and Coulomb potential can be treated in the same way.

(2) At distance large enough to be within the size of the universe, gravity follows the adjusted inverse square law. Within this distance, the rotation speed of the galaxy v follows gravitational constant G, mass m and some constants, not depend on the galaxy radius R. (the galaxy rotation curve problem)

(3) At large distance, gravity follows an adjusted inverse square law. Comparing to conventional gravity g, adjusted gravitational acceleration $\tilde{g}_{\pm}$ towards the center of rotation becomes slightly weaker or stronger. This means that gravitational acceleration towards the center of a rotating substance can be slightly changed at distance. (Pioneer Anomaly)

The adjusted gravitational acceleration $V'_{D_+}(R,G)$ can be viewed as an expansion of Newton's gravity theory. Therefore, it is possible that there exists some constants which controls gravity and the speed of galaxies.

5. Fifth, it attempts to explain the relationship between Yukawa-type potential and negative partial entropy. Similarly we introduce generalized entropy$(-)$ $S_{D_-}(x,k)$ and potential $V_{D_-}(R,G)$. Besides, we define that strong proximity acceleration(force) $g^{sp}_{\pm}$, weak proximity acceleration(force) $g^{wp}_{\pm}$, adjusted gravity $\tilde{g}_{\pm}$ and adjusted electromagnetic force $\bar{E}_{\pm}$. It attempts to propose 11-types of forces (accelerations) and compare the size of these forces. Moreover, we attempt to explain the ratios of the size of 4-forces in nature (strong force, electromagnetic force, weak force and gravity) with strong force being 1 are represented as 1, 1E-2, 1E-5 and 1E-39. By considering strong proximity acceleration $g^{sp}_{\pm}$ be regarded as strong force, weak proximity acceleration $g^{wp}_{\pm}$ as weak force, adjusted gravity $\tilde{g}_{\pm}$ as gravity and adjusted electromagnetic $\bar{E}_{\pm}$ as electromagnetic force.
6. Finally, it suggests that there may exists new forces, that mass m may represent by entropy and that gravitational constant G can fluctuate if entropy changes. Gravitational acceleration G and Coulomb's constant k_c would simply be some of the coefficients related to forces that humans can currently sense throughout the universe. Thermodynamics, quantum, gravity, electromagnetic and ecology may be unified through entropy.

2. Generalized Entropy and Application to Dynamical Systems

In this section, we introduce generalized entropy, generalized partial entropy and generalized acceleration entropy.

2.1. Generalized Entropy(+) $S_{D_+}(x,k)$ and Generalized Partial Entropy(+) $S_{D_+}(x)$

We define generalized entropy(+) as follows. In this paper, the function log represents the natural logarithm $\log_e$.

Definition 1. *Generalized Entropy(+) $S_{D_+}(x,k)$ and generalized partial entropy(+) $S_{D_+}(x)$.*
Let $x > 0$ be a real variable, and $k \gtrsim 0$ and $\xi \gtrsim 0$ be real constants. Let $D_+(x) > 0$ be a positive real valued function that partitioning x. $S_{D_+}(x,k)$, $S_{D_+}(x)$ and $Q_{D_+}(x)$ are defined as follows:

$$S_{D_+}(x,k) = kD_+(x)S_{D_+}(x), \tag{1}$$

$$Q_{D_+}(x) = \frac{\xi x}{D_+(x)}, \tag{2}$$

$$\begin{aligned} S_{D_+}(x) &= (1+\frac{x}{D_+(x)})\log(1+\frac{x}{D_+(x)}) - \frac{x}{D_+(x)}\log(\frac{x}{D_+(x)}) \\ &= (1+\frac{Q_{D_+}(x)}{\xi})\log(1+\frac{Q_{D_+}(x)}{\xi}) - \frac{Q_{D_+}(x)}{\xi}\log(\frac{Q_{D_+}(x)}{\xi}), \end{aligned} \tag{3}$$

where for any positive variable $x > 0$, the function Q_{D_+} is satisfied as follows:

$$Q_{D_+} \gneqq 0, \quad Q'_{D_+} \gneqq 0. \tag{4}$$

□

On the above definition, $S'_{D_+}(x)$ and $S''_{D_+}(x)$ are represented as follows:

$$S'_{D_+}(x) = \frac{Q'_{D_+}(x)}{\xi}\left(\log(1+\frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi})\right) \tag{5}$$

$$\begin{aligned} S''_{D_+}(x) = &\frac{Q'_{D_+}(x)}{\xi}\left(\frac{Q'_{D_+}(x)}{\xi + Q_{D_+}(x)} - \frac{Q'_{D_+}(x)}{Q_{D_+}(x)}\right) \\ &+ \frac{Q''_{D_+}(x)}{\xi}\left(\log(1+\frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi})\right). \end{aligned} \tag{6}$$

Let $S'_{D_+}(x)$ be named as entropy generation(+) (velocity) of $S_{D_+}(x)$, and $S''_{D_+}(x)$ be named as entropy acceleration(+) of $S_{D_+}(x)$. The function $Q_{D_+}(x)$ can be regard as the position partitioned a real value ξx by $Q_{D_+}(x)$. The first order derivative of $Q_{D_+}(x)$, that is, $Q'_{D_+}(x)$ can be regard as the change of the position by x and ξ. (Refer to Fujino[27] for details on how to derive generalized entropy, entropy acceleration and its partial entropy.)

2.2. The Function $Q_{D_+}(x)$ and Approximation of Generalized Entropy(+) $S_{D_+}(x,k)$

Next, we find the function $Q_{D_+}(x)$ using the idea behind Planck's radiation formula and logistic function. Put the part of partial entropy $S''_{D_+}(x)$ as follows:

$$\frac{Q'_{D_+}(x)}{\xi}\left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\right) = -\mu(x), \tag{7}$$

where $\mu(x) > 0$ is a positive real function. The left side of above Equation (7) looks like spectra partitioned by $\xi x / Q_{D_+}(x)$ and the right side of (7) becomes an approximation by the function $\mu(x)$. We consider $Q'_{D_+}(x)$ as follows:

$$Q'_{D_+}(x) = \frac{dQ_{D_+}}{dx}. \tag{8}$$

Transforming according to Equation (8), we can represent as follows:

$$dQ_{D_+}\left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\right) = -\xi\mu(x)dx. \tag{9}$$

Integrating both sides gives as follows:

$$\log(\xi + Q_{D_+}(x)) - \log(Q_{D_+}(x)) = -\xi\int \mu(x)dx \pm \mu_1, \tag{10}$$

where $\mu_1 \geq 0$. Since the left side of the above equation is a positive number and $\xi > 0$, $\mu_1 > 0$. Therefore, we consider only the case where the sign of μ_1 is positive as follows:

$$-\xi\int \mu(x)dx \pm \mu_1 > 0. \tag{11}$$

Therefore, the following equation is satisfied:

$$\log(1+\frac{\xi}{Q_{D_+}(x)}) = -\xi\int \mu(x)dx + \mu_1. \tag{12}$$

By transforming the above equation, it is satisfied as follows:

$$1 + \frac{\xi}{Q_{D_+}(x)} = \exp(-\xi \int \mu(x)dx + \mu_1). \tag{13}$$

Therefore, the function $Q_{D_+}(x)$ is represented as follows:

$$Q_{D_+}(x) = \frac{\xi}{\exp(-\xi \int \mu(x)dx + \mu_1) - 1}. \tag{14}$$

The function $Q_{D_+}(x)$ becomes the distribution function of the position which the real value ξx partitioned by $Q_{D_+}(x)$. The Equation (7) also looks like spectra partitioned by $\xi x / Q_{D_+}(x)$. If we actually take $Q_{D_+}(x)$ to $\log(x)$, we obtain an equation similar to the expansion of Planck's distribution formula. (Refer to Planck[1] and Fujino[27]) If we partition it further into squares of discrete integers, put $Q'_{D_+}(x) = 1$, $\mu(x)$ is represented the wave number, and $1/\xi$ is the Rydberg constant Ry, then it resembles the Rydberg formula that represents a spectral series. Namely, entropy may be related to atomic spectra and its energy levels. We would like to make this a topic of research in the future. Next, we make assumption about approximation of generalized entropy(+) $S_{D_+}(x,k)$.

Assumption 1. *Assume generalized entropy(+) $S_{D_+}(x,k)$ can be approximated by a second-degree polynomial. Hence set as follows:*

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{15}$$

where $\lambda_2 \gneq 0$ $\lambda_1 \geq 0$ are real numbers, and $S_{D_+}(0,k) = 0$. □

Hence, the first derivative $S'_{D_+}(x,k)$ is represented a first-degree polynomial as follows:

$$S'_{D_+}(x,k) = 2\lambda_2 x \pm \lambda_1. \tag{16}$$

Besides, the second derivative $S''_{D_+}(x,k)$ is constant. Namely, it is satisfied as follows:

$$S''_{D_+}(x,k) = 2\lambda_2. \tag{17}$$

In other words, we assume that the second derivative of $S_{D_+}(x,k)$ is a constant.

2.3. The Inverse of Generalized Partial Entropy(+) $S_{D_+}(x)$ and Potential $V_{D_+}(x,k)$

Next, we focus on the inverse of generalized partial entropy(+) $S_{D_+}(x)$ as follows:

$$\frac{1}{S_{D_+}(x)} = k \frac{\xi x}{Q_{D_+}(x)} \frac{1}{\lambda_2 x^2 \pm \lambda_1 x}. \tag{18}$$

By Equation (14), we can represent as follows:

$$\frac{1}{S_{D_+}(x)} = k \frac{1}{\lambda_2 x \pm \lambda_1} (\exp(-\xi \int \mu(x)dx + \mu_1) - 1). \tag{19}$$

We define the inverse of $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k \frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}} (1 - \exp(-\xi \int \mu(x)dx + \mu_1)). \tag{20}$$

In other words, the above potential $V_{D_+}(x,k)$ can be defined as the product of a constant k, the partition $D_+(x) = \xi / Q_{D_+}(x)$, and the inverse of the generalized entropy $D_+(x,k)$.

Here, let us reorganize the above, that is, we assume as follows:

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{21}$$

$$\frac{Q'_{D_+}(x)}{\xi}\left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\right) = -\mu(x), \tag{22}$$

where $\lambda_2 \gneqq 0$ is positive real number and $\mu(x) \gneqq 0$ is a positive real function. Therefore, we define the inverse of representation $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 - \exp(-\xi\int \mu(x)dx + \mu_1)), \tag{23}$$

where $\xi \gneqq 0, \lambda_2 \gneqq 0, \lambda_1 \geq 0, \mu_1 \geq 0$ are real numbers and $\mu(x) \gneqq 0$ is a positive real function. The first derivative $V'_{D_+}(x,k)$ is satisfied as follows:

$$V'_{D_+}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)) - \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi\int\mu(x)dx + \mu_1). \tag{24}$$

Let $V_{D_+}(x,k)$ be named as potential of $S_{D_+}(x,k)$, and $V'_{D_+}(x,k)$ be named as acceleration of $S_{D_+}(x,k)$. We assume as follows:

Assumption 2. *It assumes that potential $V_{D_+}(x,k)$ is defined the inverse of partial entropy $S_{D_+}(x)$. Therefore, acceleration $V'_{D_+}(x,k)$ is defined the first derivative of $V_{D_+}(x,k)$.* □

In the next chapter, we will describe applications of $V_{D_+}(x,k)$ and $V'_{D_+}(x,k)$ to gravity.

3. Application of Potentials $V_{D_+}(x,k)$ to Gravity

The constants, variables, and functions in the above Equations (23) and (111) can be chosen arbitrarily within the range of conditions. Therefore, we attempt to interpret these constants, variables and functions as gravity. Namely, we attempt to interpret $V_{D_+}(x,k)$ as gravitational potential and $V'_{D_+}(x,k)$ as gravitational acceleration.

3.1. Interpretation to $V_{D_+}(R,G)$

We consider the interpretation of equation $V_{D_+}(x,k)$ as follows:

$$\begin{aligned}
&x := R \geq 0, && R \text{ is distance,} \\
&\frac{1}{\lambda_2} := m \gneqq 0, && m \text{ is mass within } R, \\
&k := G, && G \text{ is the gravitational constant,} \\
&\xi := \xi^g, && \xi^g \text{ is a constant,} \\
&\mu(x) := \mu_2^g \gneqq 0, && \mu_2^g \text{ is a positive real constant,} \\
&\mu_1 := \mu_1^g \geq 0, && \mu_1^g \text{ is a real constant,} \\
&\lambda_1 := \lambda_1^g \geq 0, && \lambda_1^g \text{ is a real constant,}
\end{aligned} \tag{25}$$

where the symbol g in the upper right corner of the alphabet means g of gravity.

We assume as follows: The direction with smaller R is defined as the central direction, that is, the direction towards the center is negative. Gravitational potential increases away from the center and decreases toward the center. However, when $R = 0$ becomes $V_{D_+}(R,G) = 0$. Moreover, it assumes the constant $1/\lambda_2$ is equal to mass m within R.

Assumption 3. *Assume the constant $1/\lambda_2$ is equal to mass m within R.*
Assume that 2-times the inverse of entropy acceleration, $2/S''_{D_+}(x,k)$, that is, $1/\lambda_2$ is equal to the mass m within R. In other word, mass m within R is defined as the inverse of the second-order term of $S_{D_+}(x,k)$, that is $1/\lambda_2$. □

According Assumption 3, if entropy acceleration $S''_{D_+}(x,k)$ is large, mass m becomes small, and if the entropy acceleration $S''_{D_+}(x,k)$ is small, mass m becomes large. Doesn't this relationship between entropy acceleration and mass seem intuitive?

We define $V_{D_+}(R,G)$ as gravitational potential of G as follows:

$$\begin{aligned} V_{D_+}(R,G) &= -\frac{Gm}{R \pm \lambda_1^g m}(1-\exp(-\xi^g\mu_2^g \int dR + \mu_1^g)) \\ &= -\frac{Gm}{R \pm \lambda_1^g m}(1-\exp(-\xi^g\mu_2^g R + \mu_1^g)). \end{aligned} \tag{26}$$

The first derivative $V'_{D_+}(R,G)$ of $V_{D_+}(R,G)$ is satisfied as follows:

$$V'_{D_+}(R,G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g\mu_2^g R + \mu_1^g)) - \frac{Gm\xi^g\mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g\mu_2^g R + \mu_1^g), \tag{27}$$

where $\xi^g \gneq 0$, $\mu_2^g \gneq 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

Definition 2. *Planck-type adjusted gravitational acceleration $\bar{g}_\pm = g(R,G)$.*
Planck-type adjusted gravitational acceleration $\bar{g}_\pm = g(R,G)$ is defined as $-V'_{D_+}(R,G)$. Namely, it is satisfied as follows:

$$\bar{g}_\pm = g(R,G) = -V'_{D_+}(R,G). \tag{28}$$

□

The above Equation (27) becomes some expansion of gravitational acceleration. The first term in brackets of Equation (26) becomes like Yukawa potential. (Refer to R.Feynman[2], H.Yukawa[17] and Y.Fujii[18]) The differences between Planck-Type and Yukawa-Type are explained on Section 4 later.

(Note): Let M be mass located in range R of mass m. Potential energies $U_\pm(R,G)$ of potentials $V_{D_\pm}(R,G)$ is represented as follows:

$$U_\pm(R,G) = V_{D_\pm}(R,G)M, \tag{29}$$

and forces $F_\pm(R,G)$ of accelerations $V'_{D_\pm}(R,G)$ is represented as follows:

$$F_+(R,G) = \bar{g}_\pm M = -V'_{D_+}(R,G)M, \tag{30}$$

$$F_-(R,G) = \hat{g}_\pm M = -V'_{D_-}(R,G)M. \tag{31}$$

Therefore it treats force as acceleration in the same way. The gravitational acceleration $\hat{g}_\pm$ is define later on Section 4.4. Moreover, $Q_{D_+}(R)$ becomes like spectra within R that is independent of G and depends on ξ, μ_2^g, μ_1^g and R, and can be represented as follows:

$$Q_{D_+}(R) = \frac{\xi}{\exp(-\xi\mu_2^g R + \mu_1^g) - 1}. \tag{32}$$

Furthermore, the equation $\bar{g}_\pm$, which contains $Q_{D_+}(R)$, becomes itself an equation that describes a certain wave distribution. If the definition (assumption) of $\bar{g}_\pm$ is valid, isn't it possible to think that the equation of $\bar{g}_\pm$ itself represents the distribution of gravitational waves? (End of Note)

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The solution of Equation (27) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log\left(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}\right) + \xi^g \mu_2^g R, \quad \mu_1^g \geq 0. \tag{33}$$

Since the following conditions are needed to satisfied:

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = \frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g} \geq 0, \tag{34}$$

hence, it is satisfied as follows:

$$1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g \geq 0. \tag{35}$$

Therefore, we propose as follows:

Suggestion 1. *The classification of $V'_{D_+}(R, G)$.*
According to values of $\xi^g \gneq 0, \mu_2^g \gneq 0, \lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the Equation (27) can be classified as follows:

Case 1) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g > \log\left(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}\right) + \xi^g \mu_2^g R, \tag{36}$$

then the above Equation (27) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R, G) < 0. \tag{37}$$

(Note):The right side of inequality(36) can become positive or negative.(End of Note)

Case 2) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g < \log\left(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}\right) + \xi^g \mu_2^g R, \tag{38}$$

then the above Equation (27) becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R, G) \geq 0. \tag{39}$$

Case 3) If the constant $\mu_1^g \to 0$, then the following equation is satisfied:

$$V'_{D_+}(R, G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R)) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R). \tag{40}$$

□

3.2. When distance R Is Small Enough

If distance R is small enough, that is, since distance R approaches 0, hence the value of $\exp(-\xi^g \mu_2^g R)$ approaches 1 infinitely. Therefore, the Equation (27) is satisfied as follows:

$$V'_{D_+}(R, G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1 - \exp(\mu_1^g)) - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \\ (\because R \to 0,\ \exp(-\xi^g \mu_2^g R) \to 1). \tag{41}$$

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If distance $R \neq 0$ and $\lambda_1^g = 0$, then it is satisfied as follows:

$$V'_{D_+}(R,G) = \frac{Gm}{R^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) - \frac{Gm\xi^g \mu_2^g}{R} \exp(-\xi^g \mu_2^g R + \mu_1^g). \tag{42}$$

The case of the above Equation (42), if $R \to 0$, then it becomes $V'_{D_+}(R,G) \to \infty$. Hence, we consider that it make $\lambda_1^g \neq 0$ and R is small enough. Later we consider the case $\lambda_1^g = 0$.

Therefore, if distance R is small enough, then acceleration $V'_{D_+}(R,G)$ is approximated by a uniform value. The following representation is satisfied:

Suggestion 2. *Acceleration $V'_{D_+}(R,G)$ becomes a constant with small enough R.*
Let m be a positive real number (mass). For sufficiently small distance $R > 0$, the following equation is satisfied: Acceleration $V'_{D_+}(R,G)$ becomes a constant, that is,

$$V'_{D_+}(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1 - \exp(\mu_1^g)) - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g} \exp(\mu_1^g), \tag{43}$$

where $\xi^g \gneq 0, \mu_2^g \gneq 0, \lambda_1^g \geq 0$ and $\mu_1^g \geq 0$. □

The solution of Equation (43) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \qquad \mu_1^g \geq 0. \tag{44}$$

Since the following conditions are needed to satisfied:

$$\exp(\mu_1^g) = \frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m} \geq 0, \tag{45}$$

Because $\xi^g \gneq 0, \mu_2^g \gneq 0$, $m \gneq 0$ and $\lambda_1^g \geq 0$, it is satisfied as follows:

$$1 \pm \lambda_1^g \mu_2^g \xi^g m > 0. \tag{46}$$

Therefore, it is satisfied as follows:

$$m < \frac{1}{\lambda_1^g \mu_2^g \xi^g} \quad or \quad \frac{-1}{\lambda_1^g \mu_2^g \xi^g} < m. \tag{47}$$

According to the sign plus or minus of λ_1^g, the value of Equation (43) and its solution for μ_1^g can be classified on finite as follows:

if $V'_{D_+}(R,G) \neq 0$;

$$V'_{D_+}(R,G) \simeq \frac{G}{(\lambda_1^g)^2 m}(1 - \exp(\mu_1^g)) - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g} \exp(\mu_1^g), \tag{48}$$

if $V'_{D_+}(R,G) \neq 0$ and $\mu_1^g \to 0$;

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1), \tag{49}$$

if $\mu_1^g = \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m})$;

$$V'_{D_+}(R,G) = 0. \tag{50}$$

Therefore, we propose as follows:

Suggestion 3. *The classified $V'_{D_+}(R,G)$ with small enough R.*
According to values of $\xi^g \gtrapprox 0$, $\mu_2^g \gtrapprox 0$ $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the Equation (43) can be classified as follows:

Case 1) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g > \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \tag{51}$$

then the above Equation (43) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) < 0. \tag{52}$$

(Note):The right side of inequality(51) can become positive and negative.(End of Note)

Case 2) If the constant μ_1^g is satisfied as follows:

$$\mu_1^g \leq \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \tag{53}$$

then the above Equation (43) becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) \geq 0. \tag{54}$$

Case 3) If the constant $\mu_1^g \to 0$, then the following representation is satisfied:

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1). \tag{55}$$

□

3.2.1. Summarize Gravitational Acceleration for Small Enough R

We summarize as gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R,G)$. According to the symbol of plus or minus rule for values λ_1^g, define $\bar{g}_\pm$ as $\bar{g}_{\pm\lambda_1^g \pm \mu_1^g}$:

$$\bar{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) + \frac{Gm\xi \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R + \mu_1^g), \tag{56}$$
$$\textit{Adjusted Gravitational Acceleration with } \xi^g \textit{ and } \mu_2^g.$$

If the constant $\lambda_1^g = 0$, then

$$\bar{g}_{\pm 0 + \mu_1^g} = -\frac{Gm}{R^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{R}\exp(-\xi^g \mu_2^g R + \mu_1^g), \tag{57}$$
$$\textit{Adjusted Gravitational Acceleration with } \xi^g\ \mu_2^g \textit{ and } \lambda_1^g = 0.$$

If distance $R \to 0$, then

$$\bar{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{G}{(\pm\lambda_1^g)^2 m}(1 - \exp(\mu_1^g)) + \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{58}$$
$$\textit{Adjusted Gravitational Acceleration with } \xi^g \textit{ and } R \to 0.$$

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Therefore, if distance $R \to 0$, then it is satisfied as follows:

$$\begin{aligned}
&\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \lim_{R\to 0} \bar{g}_{\lambda_1^g+\mu_1^g\pm}, \\
&\lim_{\mu_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \pm\infty, \\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to\infty} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = 0, \\
&\lim_{\mu_1^g\to\infty} \bar{g}_{+\lambda_1^g+\mu_1^g} = \infty, \quad \because) \ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g > 0, \\
&\lim_{\mu_1^g\to\infty} \bar{g}_{-\lambda_1^g+\mu_1^g} = -\infty \quad \because) \ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g < 0.
\end{aligned} \tag{59}$$

From the above, we can suppose as follows:

Suggestion 4. *Within distance R is small enough, gravity have 5-states.*
Within distance R is small enough, it is possible that gravity $\bar{g}$ have 5-states such that finite 2-states $\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}$, that infinite 2-states $\pm\infty$, and that zero 1-state 0. □

The values of $-G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) < 0$ has the same direction as Newton's gravity, where the direction towards the center is negative. However, the value of $G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) > 0$ has the opposite direction. This means that it may represent the existence of anti-gravity.

If distance R is small enough, hence the acceleration $V'_{D_+}(R,G)$ becomes some finite constants depend on constants $\xi^g, \lambda_1^g, \mu_1^g$ and μ_2^g, not infinite. However, if the constant λ_1^g or μ_1^g approach 0 or ∞, then the acceleration $V'_{D_+}(R,G)$ becomes ∞ or 0. Depending on the value of μ_1 and λ_1^g, the value of $V'_{D_+}(R,G)$ can be positive or negative. When the value of $V'_{D_+}(R,G)$ is the negative, the deceleration acts toward the center. These constants is depended on generalized entropy and the part of partial entropy. Namely, acceleration depend on generalized entropy. Therefore, there exists 5-states within distance R is small. The constant λ_1^g is the coefficients of approximated generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$, and a constant μ_1^g is an integral constant obtained by integrating the parts of partial entropy $S''_{D_+}(x)$. In other word, acceleration moving away from the center is changed by these constant, that is, simply acceleration is depended on entropy. (Note):The center direction is defined as the positive direction.(End of Note) The above description can be applied to Coulomb's law (electric field) . By adjusting the value of $\mu_1, \mu_2, \lambda_1, m = 1/\lambda_2$ and ξ, it may be possible to make the argument by replacing gravitational constant G to the Coulomb's constant k_c. (In this paper, Coulomb's constant is defined as k_c.) We will describe this possibility next.

3.2.2. Compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_c)$ for Small R

We attempt to compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_c)$. Similarly gravitational potential $V_{D_+}(R,G)$, we define Coulomb potential $V_{D_+}(R,k_c)$ as follows:

$$V_{D_+}(R,k_c) = -\frac{k_c e_q}{R \pm \lambda_1^c e_q}(1 - \exp(-\xi^c\mu_2^c R + \mu_1^c)), \tag{60}$$

where $e_q > 0$ is an elementary charge, and $\xi^c \gneq 0$, $\mu_2^c \gneq 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$, and the symbol c in the upper right corner of the alphabet means c of Coulomb.

For example, we set the value of constants as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G\ is\ the\ gravitational\ constant,\\
&\xi^g = \xi^c := h = 6.626E\text{-}34, && h\ is\ Planck's\ constant,\\
&k_c := 8.987E\text{+}9, && k_c\ is\ Coulomb's\ constant,\\
&e_q := 1.604E\text{-}19, && e_q\ is\ elementary\ charge,\\
&m := m_p = 2.176E\text{-}8, && m_p\ is\ Planck\ mass(unit: kg),\\
&\mu_2^g := 1, && \mu_2^g\ is\ a\ real\ constant,\\
&\mu_2^c := 1, && \mu_2^c\ is\ a\ real\ constant,\\
&\lambda_1^g := 1, && \lambda_1^g\ is\ a\ real\ constant,\\
&\lambda_1^c := 1, && \lambda_1^c\ is\ a\ real\ constant.
\end{aligned} \tag{61}$$

Using the above constants, gravitational potential $V_{D_+}(R,G)$ is satisfied as follows:

$$V_{D_+}(R,G) = 2.442E\text{-}12, \quad if\ \mu_1^g = 1, \tag{62}$$
$$V_{D_+}(R,G) = 2.480E\text{-}3, \quad if\ \mu_1^g = 21.28, \tag{63}$$

where $R := 1.000E$-6 meter and Planck mass m_p is used instead of mass m. Let the sign of μ_1^g and λ_1^g be plus such as $+\mu_1^g$ and $+\lambda_1^g$. Similarly, Coulomb potential $V_{D_+}(R,k_c)$ is satisfied as follows:

$$V_{D_+}(R,k_c) = 2.474E\text{-}3, \quad if\ \mu_1^c = 1, \tag{64}$$
$$V_{D_+}(R,k_c) = 2.512E\text{+}6, \quad if\ \mu_1^c = 21.28, \tag{65}$$

where $R := 1.000E$-6 meter, elementary charge e_q is used instead of mass m and used k_c instead of G. The sign of μ_1^c and λ_1^c are $+\mu_1^c$ and $+\lambda_1^c$. The value of $V_{D_+}(R,G)$ and $V_{D_+}(R,k_c)$ changes depending on how the constants μ_1^g and $\mu 1^c$ are selected. The above values (63) and (64) is closed. Therefore, for small distance R, $\mu_1^g = 21.28$ and $\mu_1^c = 1$, it is satisfied $V_{D_+}(R,G) \simeq V_{D_+}(R,k_c)$. In consequence, it is satisfied as follows:

Suggestion 5. *Let m_p be Planck mass, e_q be elementary charge, G be gravitational constant and k_c be Coulomb's constant. For small distance $R > 0$, there exists constants ξ^g, ξ^c, μ_2^g, μ_2^c, μ_1^g, μ_1^c, λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V_{D_+}(R,G) \simeq V_{D_+}(R,k_c), \tag{66}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneqq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

Because, if it is satisfied as follows:

$$\begin{aligned}
V_{D_+}(R,G) &= -\frac{Gm_p}{R \pm \lambda_1^g m_p}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g))\\
&= -\frac{k_c e_q}{R \pm \lambda_1^c e_q}(1 - \exp(-\xi^c \mu_2^c R + \mu_1^c))\\
&= V_{D_+}(R,k_c),
\end{aligned} \tag{67}$$

then transforming the above equation, it becomes as follows:

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = 1 + \frac{R \pm \lambda_1^g m_p}{Gm_p}\frac{k_c e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1). \tag{68}$$

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Therefore, if the value of μ_2^c is given, the value of μ_2^g can be found as follows:

$$\mu_2^g = \frac{1}{\xi^g R}\left[\mu_1^g - \log\left(1 + (\frac{R \pm \lambda_1^g m_p}{R \pm \lambda_1^c e_q})(\frac{k_c e_q}{Gm_p})(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1)\right)\right]. \tag{69}$$

Namely, using the equation for potential derived from entropy, within small distance, it may be possible to treat gravitational potential and Coulomb potential in the same way by appropriately choosing some constants. In same way, applying gravitational acceleration $V'_{D_+}(R, G)$ and Coulomb's law (electric field) $V'_{D_+}(R, k_c)$, we can obtain a suggestion as follows:

Suggestion 6. *Let m_p be Planck mass , e_q be elementary charge, G be gravitational constant and k_c be Coulomb's constant. For small distance $R > 0$, there exists constants ξ^g, ξ^c, μ_2^g μ_2^c, μ_1^g μ_1^c λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V'_{D_+}(R, G) \simeq V'_{D_+}(R, k_c), \tag{70}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

3.3. When Distance R Is Large, However ξ Is Small Enough

Assuming distance R is large and the constant ξ^g is small like Planck constant, that is $\xi^g \sim h$. The constant h is Planck constant, 6.626E-34$J \cdot s$ and the constant $\mu_2^g = 1$. Assume that R is the radius of the universe within 46.5 billion light years (4.65E+10). Since one light years is approximately 9.461E+15 meter, we assume that the radius of the universe $R \simeq 4.399E+26$ meter. Therefore, the following condition is satisfied:

$$\xi^g \mu_2^g R \simeq 2.915E\text{-}7 \ll 1. \tag{71}$$

The function $\exp(-\xi^g \mu_2^g R)$ is approximately equal to 1, that is, it is satisfied as follows:

$$\exp(-\xi^g \mu_2^g R) \simeq 1. \tag{72}$$

Therefore, the following representations are satisfied:

$$\begin{aligned} \bar{g}_\pm &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)} \exp(-\xi^g \mu_2^g R + \mu_1^g) \\ &\simeq -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(\mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)} \exp(\mu_1^g), \\ &\quad (\because \exp(-\xi^g \mu_2^g R) \simeq 1). \end{aligned} \tag{73}$$

When the condition $\xi^g \mu_2^g R \ll 1$ is satisfied, applying to mass M in circular orbit around mass m, the following equation is satisfied:

$$-\frac{GmM}{(R \pm \lambda_1^g m)^2}(1 - \exp(\mu_1^g)) + \frac{GmM\xi^g \mu_2^g}{(R \pm \lambda_1^g m)} \exp(\mu_1^g) = M\frac{v^2}{R}, \tag{74}$$

where m is mass within radius R and v is the rotation speed of mass M on radius R. The right side of Equation (74) becomes centrifugal acceleration of mass M. Hence the following representations satisfied:

$$\begin{aligned} v &= \sqrt{\frac{-GmR}{(R \pm \lambda_1^g m)^2}(1 - \exp(\mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)} \\ &= \sqrt{\frac{-Gm}{(R \pm \lambda_1^g m)(1 \pm \frac{\lambda_1^g m}{R})}(1 - \exp(\mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)} \\ &\simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \\ &\quad (\because R \textit{ is large enough and } (1 + \frac{\lambda_1^g m}{R}) \to 1). \end{aligned} \tag{75}$$

Similar results are obtained for $\hat{g}_-$ of Yukawa-type gravity acceleration discussed later on Section 4.4. Therefore, we propose that the following is satisfied:

Suggestion 7. *Let $m > 0$ (mass) and v (the speed of rotation) be positive real numbers. For large distance $R > 0$ within $4.399E{+}26$, the following condition is satisfied:*

$$v \simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \tag{76}$$

where $\xi^g, \mu_2^g \gtrapprox 0$ and $\lambda_1^g, \mu_1^g \geq 0$. As a results, the speed of rotation v at radius R is approximated by a uniform value $\sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}$, not depend on radius R. □

Therefore, the speed of rotation v is depended on constants G, m, ξ^g, μ_1^g and μ_2^g, not depend on radius R. It is noticed that these constants is decided by generalized entropy $S_{D_+}(x,k)$ and the distribution function $Q_{D_+}(x)$. According the Suggestion 7, let m be equal to mass of the Milky Way Galaxy, that is, $m \simeq 1.989E{+}30 \times 2.0E{+}12$ kg, where mass of the sun is $1.989E{+}30$ and the sun count in the Milky Way Galaxy is $2.0E{+}12$. Therefore, if setting $\xi = 1E{-}34 \sim h$ (Planck constant) and $\mu_2^g = 1$, then the speed of rotation is satisfied depending the constant μ_1^g as follows:

$$v \simeq 4.194E{-}1\sqrt{\exp(\mu_1^g)}\ m/s. \tag{77}$$

For example, let $\mu_1^g = 26.36$, the speed of rotation v became as follows:

$$v \simeq 2.222E{+}5\ m/s. \tag{78}$$

In this case, the speed of (78) is close to the rotation speed of the Milky Way Galaxy, that is, approximately $2.200E{+}5 \sim 2.400E{+}5$ m/s. Even without assuming dark matter, the galaxy rotation problem can be explained by the concept of entropy. This does not mean denying dark matter. New constants μ_1^g and μ_2^g may represent some kind of dark or virtual mass. Besides, the Suggestion 7 may consider to apply to velocities over short distances such as electrons in atomic nuclei.

3.4. When Distance R Is Large Enough

If distance R is large enough, the Equation (27) is satisfied as follows:

$$\tilde{g}_\pm = -V'_{D_+}(R,G) = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \quad (\because \exp(-\xi^g \mu_2^g R + \mu_1^g) \to 0). \tag{79}$$

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Therefore, the following conditions are satisfied:

$$-\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim -\frac{Gm}{R^2} \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2}. \tag{80}$$

If distance R is large enough and the constant λ_1^g is small enough, that is $\lambda_1^g \to 0$, then gravitational acceleration $V'_{D_+}(R,G)$ becomes Newton's gravity.

3.4.1. Summarize Gravitational Acceleration for Large Distance R

We summarize the above gravitational acceleration $V'_D(R,G)$ as follows:

1. Adjusted gravitational acceleration, R is large enough:

$$\tilde{g}_\pm = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \tag{81}$$

2. Original gravitational acceleration, R is large enough and $\lambda_1^g \to 0$:

$$g = -\frac{Gm}{R^2}, \tag{82}$$

where R is large enough. Newton's gravity is satisfied when R is large enough and $\lambda_1^g \to 0$. According to g of (82) and $\tilde{g}_\pm$ of (81) in the above equation, we propose as follows:

Suggestion 8. *Gravity changes on the value λ_1^g of generalized entropy coefficient.*
Let $m > 0$ be a real number (mass). For large $R > 1$, the following conditions are satisfied:

$$\tilde{g}_- = -\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim g \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2} = \tilde{g}_+, \tag{83}$$

where $\lambda_1^g \geq 0$ is a real constant. □

The suggestion above is an expansion of Newton's gravity. For large distance R, it is possible that adjusted gravity $\tilde{g}_\pm$ is smaller or larger towards the center than Newton's gravity g. In other word, gravitational acceleration towards the center of a rotating substance can be slightly changed at sufficient large distance. Gravitational acceleration moving away from the center is changed by the constant λ_1^g. The constant λ_1^g is the coefficients of degree one of approximate generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$. In other word, Gravitational acceleration moving away from the center is changed by the coefficients of approximated generalized entropy, that is, gravitational acceleration is considered to depended on entropy.

4. Yukawa Type Potential and Entropy, Comparison of Accelerations

4.1. Relationship with Yukawa-Type Potential and Potential $V_{D_+}(R,G)$

Potential $V_{D_+}(R,G)$ (26) contains an equation similar to Yukawa potential.[17] If we omit the first term in Equation (26), we can obtain as follows:

$$V_{D_+}(R,G)_{omit} = \frac{Gm}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R \pm \mu_1^g). \tag{84}$$

We consider that by substituting the constants as follows:

$$Gm := g_y^2, \quad \xi^g \mu_2^g := \lambda, \quad \mu_1^g := 0, \quad \lambda_1^g := 0, \tag{85}$$

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where g_y is Yukawa's constant and $\lambda = mc/\hbar$. [17] Thereby, we can obtain the following Yukawa potential:

$$V_{yukawa1}(R) = \pm g_y^2 \frac{\exp(-\lambda R)}{R}. \tag{86}$$

Therefore, it may also have applications in particle theory and other potential theory. Moreover, we also consider that by substituting the constants as follows:

$$\xi^g \mu_2^g := \lambda, \quad \exp(\mu_1^g) := \alpha, \quad \lambda_1^g := 0, \tag{87}$$

where $\lambda = mc/\hbar$. [18],[19] Thereby, we can obtain the following gravitational potential:

$$V_\alpha(R) = -G\frac{m}{R}(1 - \alpha\exp(-\lambda R)). \tag{88}$$

The above equation has a different sign of second term in brackets compared to the following Yukawa potential proposed as follows: [18], [19]

$$V_{yukawa2}(R) = -G\frac{m}{R}(1 + \alpha\exp(-\lambda R)). \tag{89}$$

Therefore the above Equation (88) and (89) are incompatible. It seems necessary to consider another way to integrate two equations. Namely, we need to change the definition of function $Q_{D_+}(R)$ (1) and the assumption2. These contents are described on next subsection.

4.2. Negative Generalized Partial Entropy

We define generalized entropy $S_{D_-}(x,k)$ and negative generalized partial entropy $S_{D_-}(x)$ as follows:

Definition 3. *Generalized Entropy(-) $S_{D_-}(x,k)$ and negative generalized partial entropy $S_{D_-}(x)$.*
Let $x > 0$ be a real variable, and $k \gtrapprox 0$ and $\xi \gtrapprox 0$ be real constants. Let $D_-(x)$ be a negative real valued function that partitioning x. $S_{D_-}(x,k)$, $S_{D_-}(x)$ and $Q_{D_-}(x)$ are defined as follows:

$$S_{D_-}(x,k) = kD_-(x)S_{D_-}(x) > 0, \tag{90}$$

$$Q_{D_-}(x) = \frac{-\xi x}{D_-(x)} \gtrapprox 0, \tag{91}$$

where for any positive variable $x > 0$, the function Q_{D_-} is satisfied as follows:

$$Q_{D_-} \gtrapprox 0, \quad Q'_{D_-} \gtrapprox 0, \quad \xi > Q_{D_-}. \tag{92}$$

We will define $S_{D_-}(x)$ as an approximation of $S_{D_-}(x,k)$ and $Q_{D_-}(x)$:

$$S_{D_-}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{93}$$

$$Q_{D_-}(x) = \frac{\xi}{\exp(-\xi\int\mu(x)dx \pm \mu_1) + 1}, \tag{94}$$

$$S_{D_-}(x) = \frac{1}{k}\frac{Q_{D_-}(x)}{-\xi x}S_{D_-}(x,k) \leq 0. \tag{95}$$

□

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(Note): Since the argument of log can not be negative, hence we can not define like as follows:

$$\begin{aligned} S_{D_-}(x) &= (1+\frac{x}{D_-(x)})\log(1+\frac{x}{D_-(x)}) - \frac{x}{D_-(x)}\log(\frac{x}{D_-(x)}) \\ &= (1+\frac{Q_{D_-}(x)}{-\xi})\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \frac{Q_{D_-}(x)}{-\xi}\log(\frac{Q_{D_-}(x)}{-\xi}), \end{aligned} \tag{96}$$

However, let us consider extending it to complex numbers as follows:

$$S_{D_-}(x) = (1+\frac{Q_{D_-}(x)}{-\xi})\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \frac{Q_{D_-}(x)}{-\xi}(\log(\frac{Q_{D_-}(x)}{\xi}) + \log(-1)), \tag{97}$$

where $\log(-1) = 2(n+1)\pi i$, $n \geq 0$.

Treating the above equation formally, $S'_{D_-}(x)$ and $S''_{D_-}(x)$ becomes as follows:

$$S'_{D_-}(x) = \frac{Q'_{D_-}(x)}{-\xi}\left(\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \log(\frac{Q_{D_-}(x)}{-\xi})\right) - \frac{Q'_{D_-}(x)}{-\xi}\log(-1), \tag{98}$$

$$\begin{aligned} S''_{D_-}(x) &= \frac{Q'_{D_-}(x)}{-\xi}\left(\frac{Q'_{D_-}(x)}{-\xi+Q_{D_-}(x)} - \frac{Q'_{D_-}(x)}{Q_{D_-}(x)}\right) \\ &+ \frac{Q''_{D_-}(x)}{-\xi}\left(\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \log(\frac{Q_{D_-}(x)}{-\xi})\right) - \frac{Q''_{D_-}(x)}{-\xi}\log(-1). \end{aligned} \tag{99}$$

Therefore, we directly adopt definitions of (94) and (95). If we choice the definition(97), then the definition(95) change to as follows:

$$S_{D_-}(x) = Re\left[\frac{1}{k}\frac{Q_{D_-}(x)}{-\xi x}S_{D_-}(x,k)\right] \leq 0, \tag{100}$$

where the function $Re[x]$ is mean real value of x. (End of Note).

4.3. The Function $Q_{D_-}(x)$ for Yukawa Potential

We find the function $Q_{D_-}(x)$ using the idea behind Planck's radiation formula and logistic function. Put the part of partial entropy $S''_{D_-}(x)$ as follows:

$$\frac{Q'_{D_-}(x)}{-\xi}\left(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)}\right) = \mu(x) \geq 0, \tag{101}$$

where $\mu(x) > 0$ is a positive real function. The above parts of $\frac{Q'_{D_-}(x)}{-\xi}$ is negative and the above parts of $(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)})$ is also negative, therefore the above formulas (101) is positive. Transforming according to Equation (8), we can represent as follows:

$$dQ_{D_-}\left(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)}\right) = -\xi\mu(x)dx. \tag{102}$$

Integrating both sides gives as follows:

$$\log(\xi - Q_{D_-}(x)) - \log(Q_{D_-}(x)) = -\xi\int\mu(x)dx \pm \mu_1, \tag{103}$$

where $\mu_1 > 0$ and $\xi > 0$. Therefore, the following equation is satisfied:

$$\log(\frac{\xi}{Q_{D_-}(x)} - 1) = -\xi\int\mu(x)dx \pm \mu_1. \tag{104}$$

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By transforming the above equation, it is satisfied as follows:

$$\frac{\xi}{Q_{D_-}(x)} - 1 = \exp(-\xi \int \mu(x)dx \pm \mu_1). \tag{105}$$

Since the right side is a positive, the left side must also be a positive, that is, it needs to be satisfied $\xi > Q_{D_-}(x)$. Therefore, the function $Q_{D_-}(x)$ is represented as follows:

$$Q_{D_-}(x) = \frac{\xi}{\exp(-\xi \int \mu(x)dx \pm \mu_1) + 1}. \tag{106}$$

Therefore, the above equation can be adopted as the definition of $Q_{D_-}(x)$.

4.4. The Inverse of Partial Entropy $S_{D_-}(x)$ and Potential $V_{D_-}(x,k)$

For the approximation of generalized entropy $S_{D_-}(x,k)$, we make the same assumptions as the Equation (1). By the Definition 3, it is satisfied as follows:

$$S_{D_-}(x,k) = k\frac{-\xi x}{D_-(x)} S_{D_-}(x). \tag{107}$$

Therefore, the inverse of partial entropy $S_{D_-}(x)$ as follows:

$$\frac{1}{S_{D_-}(x)} = k\frac{-\xi x}{Q_{D_-}(x)} \frac{1}{\lambda_2 x^2 \pm \lambda_1 x}, \tag{108}$$

where since $S_{D_-}(x) \leq 0$ and $Q_{D_-}(x) \geq 0$, then $\lambda_2 x^2 \pm \lambda_1 x > 0$. By Equation (106), we can represent as follows:

$$\frac{1}{S_{D_-}(x)} = -k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi \int \mu(x)dx \pm \mu_1) + 1). \tag{109}$$

We define the inverse of $S_{D_-}(x)$ as potential $V_{D_-}(x,k)$:

$$V_{D_-}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 + \exp(-\xi \int \mu(x)dx \pm \mu_1)). \tag{110}$$

In other words, the above potential $V_{D_-}(x,k)$ can be defined as the product of a constant k, the partition $Q_{D_-}(x) = -\xi / D_-(x)$, and the inverse of the generalized entropy $S_{D_-}(x,k)$. The first derivative $V'_{D_-}(x,k)$ is satisfied as follows:

$$V'_{D_-}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1+\exp(-\xi \int \mu(x)dx \pm \mu_1)) + \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi \int \mu(x)dx \pm \mu_1). \tag{111}$$

Same as Assumptions 2 and 3, the above description assumes the following assumption:

Assumption 4. *It assumes that potential $V_{D_-}(x,k)$ is defined the inverse of partial entropy $S_{D_-}(x)$. Therefore, acceleration $V'_{D_-}(x,k)$ is defined the first derivative of $V_{D_-}(x,k)$.* □

Assumption 5. *Assume the constant $1/\lambda_2$ is equal to mass m within R.*
Assume that 2-times the inverse of entropy acceleration, $2/S''_{D_-}(x,k)$, that is, $1/\lambda_2$ is equal to the mass m within R. In other word, mass m within R is defined as the inverse of the second-order term of $S_{D_-}(x,k)$, that is $1/\lambda_2$.
□

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Therefore, similarly the description of section 3, we obtain the following results:

$$Q_{D_-}(R) = \frac{\xi^g}{\exp(-\xi^g \mu_2^g R \pm \mu_1^g) + 1}, \tag{112}$$

$$V_{D_-}(R,G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g \mu_2^g R \pm \mu_1^g)), \tag{113}$$

$$\begin{aligned} \hat{g}_{\pm} &= -V'_{D_-}(R,G) \\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1+\exp(-\xi^g \mu_2^g R \pm \mu_1^g)) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R \pm \mu_1^g), \end{aligned} \tag{114}$$

$$\lim_{\mu_1^g \to 0} \hat{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0) \tag{115}$$

where $\xi^g \gtrsim 0$, $\mu_2^g \gtrsim 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$. Let $\hat{g}_{\pm}$ be Yukawa-type adjusted gravitational acceleration$(\pm)$. The above Equation (112) resembles a distribution model of nuclei, and the negative of (112) resembles Woods-Saxon type potential. The above Equation (113) and (89) are compatible because the sign of exp is same. Namely, setting as follows:

$$\exp(\mu_1^g) := \alpha, \quad \xi^g \mu_2^g := \lambda, \quad \lambda_1^g := 0, \tag{116}$$

then the Equation (113) becomes Yukawa-type potential(89). The second term of Equation (113) is same as the negative representation of Yukawa potential (86). Namely, by introducing negative partial entropy, that is, negative generalized partial entropy $S_{D_-}(x) \leq 0$, Yukawa-type potential can be explained. For comparison, we describe results of section 3 as follows:

$$Q_{D_+}(R) = \frac{\xi^g}{\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1}, \tag{117}$$

$$V_{D_+}(R,G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)), \tag{118}$$

$$\begin{aligned} \bar{g}_{\pm} &= -V'_{D_+}(R,G) \\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g \mu_2^g R \pm \mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R \pm \mu_1^g), \end{aligned} \tag{119}$$

$$\lim_{\mu_1^g \to 0} \bar{g}_{\pm\lambda_1^g + \mu_1^g} = \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0), \tag{120}$$

where $\xi^g \gtrsim 0$, $\mu_2^g \gtrsim 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$. The second term of Equation (118) is same as the positive representation of Yukawa-type potential (86). Let $\bar{g}_{\pm}$ be Planck-type adjusted gravitational acceleration$(\pm)$. As weak proximity acceleration$(\pm)$, we put the Equation (120) as follows:

$$g_{\pm}^{wp} = \lim_{\mu_1^g \to 0} \bar{g}_{\pm\lambda_1^g + \mu_1^g} = \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \qquad (\because R \to 0), \tag{121}$$

and as strong proximity acceleration$(\pm)$, we put the Equation (115) as follows:

$$g_{\pm}^{sp} = \lim_{\mu_1^g \to 0} \hat{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0). \tag{122}$$

Acceleration $g_{\pm}^{wp}$ has no effect of mass m instead it depends on ξ^g, μ_2^g and λ_1^g. Acceleration $g_{\pm}^{sp}$ does have an effect of mass m. It is considered that there exists 11-types of acceleration including gravitational acceleration g, such as $\bar{g}_{\pm}, \hat{g}_{\pm}, \tilde{g}_{\pm}, g_{\pm}^{sp}$ and $g_{\pm}^{wp}$ related to g.

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4.5. Comparing Accelerations $\bar{g}_\pm, \hat{g}_\pm, g^{sp}_\pm$ and $g^{wp}_\pm$

Comparing the above (114), (119), (121) and (122), we obtain the following are relationships:

$$g^{sp}_+ < \hat{g}_+ < \tilde{g}_+ < \bar{g}_+ < g^{wp}_+, \tag{123}$$

$$\hat{g}_- < \tilde{g}_- < g^{sp}_- < g^{wp}_- < \bar{g}_-, \quad \because) \quad m < \frac{1}{\lambda^g_1 \xi^g \mu^g_2}, \tag{124}$$

$$\bar{g}_- < g^{wp}_- < g^{sp}_- < \tilde{g}_- < \hat{g}_-, \quad \because) \quad \frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m. \tag{125}$$

It assumes that $g^{wp}_- < g^{sp}_-$, for all $R > 0$, it is satisfied as follows:

$$\frac{G\xi^g \mu^g_2}{-\lambda^g_1} < -\frac{2G}{(-\lambda^g_1)^2 m} - \frac{G\xi^g \mu^g_2}{-\lambda^g_1}. \tag{126}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m. \tag{127}$$

It assumes that $g^{sp}_- < \tilde{g}_-$, for all $R > 0$, it is satisfied as follows:

$$R - \lambda^g_1 m > \pm \lambda^g_1 m \sqrt{\frac{1}{2 - \lambda^g_1 m \xi^g \mu^g_2}}, \tag{128}$$

The above inequality(128) holds true even for small enough R. The following is satisfied:

$$-\lambda^g_1 m \geq -\lambda^g_1 m \sqrt{\frac{1}{2 - \lambda^g_1 m \xi^g \mu^g_2}}, \tag{129}$$

and

$$\sqrt{\frac{1}{2 - \lambda^g_1 m \xi^g \mu^g_2}} \geq 1. \tag{130}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda^g_1 \xi^g \mu^g_2} \leq m. \tag{131}$$

Namely, since $R > 0$, we only consider that the inequalities(125) of the condition $\frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m$.

Summarizing the above inequalities, the following are satisfied:

$$g^{sp}_+ < \hat{g}_+ < \bar{g}_- < g^{wp}_- < g^{sp}_- < \tilde{g}_- < g < \tilde{g}_+ < \bar{g}_+ < \hat{g}_- < 0 < g^{wp}_+, \; (\because \frac{1}{\lambda^g_1 \xi^g \mu^g_2} < m). \tag{132}$$

(Note): Since the direction toward the center is negative, the more negative value(smaller), the greater acceleration(force) toward the center. (End of Note)

4.6. One attempt to compare the ratios of 4-forces.

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For example, we consider the above inequalities(132). Set constants as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\
&\xi^g := h = 6.626E\text{-}34, && h \textit{ is Planck constant},\\
&m := m_p = 2.176E\text{-}8, && m_p \textit{ is Planck mass}(kg),\\
&\mu_1^g \gtrsim 0, && \mu_1^g \textit{ is a real constant},\\
&R \leq 1.305E\text{+}26, && R \textit{ is a radius within the Universe}(meter).
\end{aligned} \tag{133}$$

where according to case 2) of Suggestion 1 and of suggestion3, inequalities(3.2) and (131), the values λ_1^g and μ_1^g are satisfied as follows:

$$0 \leq 1 + (R \pm \lambda_1^g m_p)\xi^g \mu_2^g, \quad \frac{-1}{\lambda_1^g \xi^g \mu_2^g} < m_p. \tag{134}$$

(Note): The values of μ_2^g in each equation $g_\pm^{sp}$ and g_-^{wp} take on different values $(\mu_2^g)_\pm^{sp}$ and $(\mu_2^g)_-^{wp}$, respectively. (End of Note)

On the above inequalities (132), the following are satisfied:

1. Compare $\hat{g}_-$ and g_-^{wp};
   The ratio of Yukawa-type adjusted gravitational$(-)$ $\hat{g}_-$ and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\begin{aligned}
\left|\frac{\hat{g}_-}{g_-^{wp}}\right| &\simeq \left| \frac{\frac{2Gm_p}{(R-((\hat{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} + \frac{\frac{Gm_p\xi^g}{(R-((\hat{\lambda}_1^g)_-)m_p)}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} \right| \\
&\simeq \left| \frac{2\cdot 10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}} + \frac{10^{-8}(\hat{\lambda}_2^g)_-}{10^{A-34}(\mu_2^g)_-^{wp}} \right| \simeq 1E\text{-}34,
\end{aligned} \tag{135}$$

   where

$$\begin{aligned}
&(\mu_2^g)_-^{wp} := 1E\text{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&(\lambda_1^g)_-^{wp} := 1E\text{+}2A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp},\\
&(\hat{\mu}_2^g)_- := 1E\text{+}A, && (\hat{\mu}_2^g)_- \textit{ is the constant } \mu_2^g \textit{ of } \hat{g}_-,\\
&(\hat{\lambda}_1^g)_- := 1E\text{+}A, && (\hat{\lambda}_1^g)_- \textit{ is the constant } \lambda_1^g \textit{ of } \hat{g}_-,\\
&(R \pm ((\hat{\lambda}_1^g)_-)m_p)^2 \simeq 1E\text{+}2A, && 0 \leq A \leq 26,\ A \textit{ is a constant}.
\end{aligned} \tag{136}$$

   $\hat{g}_+$ and g_-^{wp} can be compared in the same way.
   (Note): The following are satisfied:

$$\exp(-\xi^g((\hat{\mu}_2^g)_-)R + \mu_1^g) > \exp(0) = 1, \tag{137}$$

   where μ_1^g is satisfied $-\xi^g((\hat{\mu}_2^g)_-)R + \mu_1^g > 0$. Therefore, if the case $\hat{g}_-$, then $Q_{D_-}(R) < \frac{1}{2}$ is satisfied. Similarly if the case $\bar{g}_-$, then $Q_{D_+}(R) > 0$ is satisfied. (End of Note)
2. Compare $\tilde{g}_\pm$ and g_-^{wp};
   The ratio of adjusted gravitational$(\pm)$ $\tilde{g}_\pm$ and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\left|\frac{\tilde{g}_\pm}{g_-^{wp}}\right| = \left| \frac{\frac{Gm_p}{(R\pm((\tilde{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} \right| = \left| \frac{10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}} \right| \simeq 1E\text{-}34, \tag{138}$$

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where

$$\begin{aligned} &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &(\lambda_1^g)_-^{wp} := 1E{+}2A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp}, \\ &(R \pm ((\tilde{\lambda}_1^g)_-)m_p)^2 \simeq 1E{+}2A, && 0 \le A \le 26,\ A \textit{ is a constant}. \end{aligned} \tag{139}$$

3. Compare g_+^{sp} and g_-^{wp};
   The ratio of strong proximity$(+)$ g_+^{sp} and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\left| \frac{g_+^{sp}}{g_-^{wp}} \right| = \left| \frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_+^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}} \right| = \left(\frac{2}{\lambda_1^g m_p \xi^g (\mu_2^g)_-^{wp}} + \frac{(\mu_2^g)_+^{sp}}{(\mu_2^g)_-^{wp}} \right) \simeq 1E{+}5, \tag{140}$$

where

$$\begin{aligned} &(\mu_2^g)_+^{sp} := 1E{+}65, && (\mu_2^g)_+^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_+^{sp}, \\ &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and } g_+^{sp}. \end{aligned} \tag{141}$$

4. Compare g_-^{sp} and g_-^{wp};
   The ratio of strong proximity$(-)$ g_-^{sp} and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\left| \frac{g_-^{sp}}{g_-^{wp}} \right| = \left| \frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_-^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}} \right| = \left| \frac{2}{\lambda_1^g m_p \xi^g (\mu_2^g)_-^{wp}} - \frac{(\mu_2^g)_-^{sp}}{(\mu_2^g)_-^{wp}} \right| \simeq 1, \tag{142}$$

where

$$\begin{aligned} &(\mu_2^g)_-^{sp} := 1E{+}60, && (\mu_2^g)_-^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{sp}, \\ &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and} g_-^{sp}. \end{aligned} \tag{143}$$

By considering g_+^{sp} as strong force, g_-^{wp} as weak force and $\tilde{g}_\pm$ (or g) as gravity, the ratio of the size of the fundamental 3-forces of nature (strong force, weak force and gravity) with strong force being 1 is represented as follows:

$$\begin{array}{ccc} \textit{strong}, & \textit{weak}, & \textit{gravity}, \\ 1, & 1E\text{-}5, & 1E\text{-}39, \\ g_+^{sp}, & g_-^{wp}, & \tilde{g}_\pm \\ (\mu_2^g)_+^{sp}, & (\mu_2^g)_-^{wp} \textit{ or } (\mu_2^g)_-^{sp}, & \frac{m_p}{\xi^g}, \\ 1E{+}65, & 1E{+}60, & 1E{+}26. \end{array} \tag{144}$$

The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E+65$, $(\mu_2^g)_-^{wp} \simeq 1E+60$, $(\mu_2^g)_-^{sp} \simeq 1E+60$ and $\frac{m_p}{\xi^g} \simeq 1E+26$. Next, let us consider the relationship with electromagnetic forces. We focus that the relationship between electromagnetic force and gravity are seen as similar forces by suggestion6. We apply this suggestion to equation $\hat{g}_{\pm}$ and $\hat{g}_{\pm}$ and set adjusted electromagnetic acceleration(force) as follows:

$$\begin{aligned}\bar{E}_{\pm} &:= -V'_{D_+}(R,k_c)\\ &= -\frac{k_c e_q}{(R\pm\lambda_1^c e_q)^2}(1-\exp(-\xi^c\mu_2^c R\pm\mu_1^c)) + \frac{k_c e_q \xi^c \mu_2^c}{R\pm\lambda_1^c e_q}\exp(-\xi^c\mu_2^c R\pm\mu_1^c),\end{aligned} \tag{145}$$

$$\begin{aligned}\hat{E}_{\pm} &:= -V'_{D_-}(R,k_c)\\ &= -\frac{k_c e_q}{(R\pm\lambda_1^c e_q)^2}(1+\exp(-\xi^c\mu_2^c R\pm\mu_1^c)) - \frac{k_c e_q \xi^c \mu_2^c}{R\pm\lambda_1^c e_q}\exp(-\xi^c\mu_2^c R\pm\mu_1^c),\end{aligned} \tag{146}$$

where $\xi^c \gtrsim 0$, $\mu_2^c \gtrsim 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$. Let $\bar{E}_{\pm}$ be Planck-type adjusted electromagnetic$(\pm)$ and $\hat{E}_{\pm}$ be Yukawa-type adjusted electromagnetic$(\pm)$. Therefore, we set as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\
&\xi^g = \xi^c := h = 6.626E\text{-}34, && h \textit{ is Planck constant},\\
&k_c := 8.987E\text{+}9, && k_c \textit{ is Coulomb's constant},\\
&e_q := 1.604E\text{-}19, && e_q \textit{ is elementary charge},\\
&m := m_p = 2.176E\text{-}8, && m_p \textit{ is Planck mass}(unit: kg),\\
&R \textit{ is small enough}, && R \textit{ is distance}(unit: meter).
\end{aligned} \tag{147}$$

where according to case 2) of suggestion 3, the values λ_1^g and μ_1^g are satisfied as follows:

$$0 < 1 \pm \lambda_1^g m_p \xi^g \mu_2^g, \quad 0 < 1 \pm \lambda_1^c e_q \xi^c \mu_2^c, \ (R \to 0). \tag{148}$$

(Note): The values of μ_2^c in each equation $\hat{E}_+$ and $\bar{E}_-$, take on different values $\hat{\mu}_2^c$ and $\bar{\mu}_2^c$, respectively. (End of Note)

On the above inequalities(132), the following are satisfied:

1. Compare $\hat{E}_+$ and g_-^{wp};
 The ratio of Yukawa-type adjusted electromagnetic$(+)$ $\hat{E}_+$ and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\left|\frac{\hat{E}_+}{g_-^{wp}}\right| = \left|\frac{-\frac{2k_c}{\lambda_1^{c2} e_q}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}} + \frac{-\frac{k_c\xi^c(\hat{\mu}_2^c)_+}{\lambda_1^c}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}}\right| \simeq 1.347E\text{+}3 \ \simeq 1E\text{+}3, \tag{149}$$

 where

$$\begin{aligned}
&(\hat{\mu}_2^c)_+ := 1E\text{+}43, && (\hat{\mu}_2^c)_+ \textit{ is the constant } \mu_2^c \textit{ of } \hat{E}_+,\\
&(\mu_2^g)_-^{wp} := 1E\text{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&\lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{150}$$

2. Compare $\bar{E}_-$ and g_+^{wp};
 the ratio of Planck-type adjusted electromagnetic$(-)$ $\bar{E}_-$ and weak proximity acceleration$(-)$ g_-^{wp} are obtained as follows:

$$\left|\frac{\bar{E}_-}{g_-^{wp}}\right| = \frac{k_c\xi^c(\bar{\mu}_2^c)_-\lambda_1^g}{G\xi^g(\mu_2^g)_-^{wp}\lambda_1^c}\exp(\mu_1^c) \simeq 1.347E\text{+}3 \simeq 1E\text{+}3, \tag{151}$$

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where

$$\begin{aligned}
&(\bar{\mu}_2^c)_- := 1E{+}43, && (\bar{\mu}_2^c)_+ \text{ is the constant } \mu_2^c \text{ of } \bar{E}_-, \\
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \text{ is the constant } \mu_2^g \text{ of } g_-^{wp}, \\
&\mu_1^c \to 0,\ \lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{152}$$

where since the above ratio changes depending on the value of R, therefore for the comparison of the ratios, R is small enough in the near field. We interpret $\hat{E}_+$ and $\bar{E}_-$ as electromagnetic. By combine with inequalities (144), the ratio of the size of the fundamental 4-forces of nature (strong force, electromagnetic force, weak force and gravity) with strong force being 1 is represented as follows:

$$\begin{array}{cccc}
strong, & electromagnetic, & weak, & gravity, \\
1, & 1E\text{-}2, & 1E\text{-}5, & 1E\text{-}39, \\
g_+^{sp}, & \hat{E}_+ \text{ or } \bar{E}_-, & g_-^{wp}, & \tilde{g}_\pm, \\
(\mu_2^g)_+^{sp}, & \frac{k_c}{G}(\hat{\mu}_2^c)_+ \text{ or } \frac{k_c}{G}(\bar{\mu}_2^c)_-, & (\mu_2^g)_-^{wp}, & \frac{m_p}{\xi^g}, \\
1E{+}65, & 1E{+}63, & 1E{+}60, & 1E{+}26,
\end{array} \tag{153}$$

The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E{+}65$, $\frac{k_c}{G}(\hat{\mu}_2^c)_+ \simeq \frac{k_c}{G}(\bar{\mu}_2^c)_- \simeq 1E{+}63$, $(\mu_2^g)_-^{wp} \simeq 1E{+}60$ and $\frac{m_p}{\xi^g} \simeq 1E{+}26$, and depend on the values of m_p and ξ^g. Therefore by considering strong proximity force g_+^{sp} is regarded as strong force, weak proximity force g_-^{wp} as weak force, adjusted gravity $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force, it is possible to explain the ratios of 4-forces in nature.

4.7. Relationship diagram.

The difference between Equations (26) and (118) are whether generalized partial entropy is positive or negative. Under the above definition 3, the Equation (95) is negative. Namely, it means we assume negative generalized partial entropy. If we assume negative generalized partial entropy, we can obtain Yukawa-type gravitational potential. In other words, the existence of Yukawa-type equation may indicate the existence of negative partial entropy. Therefore, particle physics can be also considered to be related to entropy.

Relationship diagram:

$$\begin{array}{ccc}
\begin{array}{c} S_{D_+}(k,x) = k\frac{\xi}{Q_{D_+}(x)} S_{D_+}(x) > 0 \\ S_{D_+}(x) > 0 \\ \textit{Partial Entropy} \end{array} & \xrightarrow{negative} & \begin{array}{c} S_{D_-}(k,x) = -k\frac{\xi}{Q_{D_-}(x)} S_{D_-}(x) > 0 \\ S_{D_-}(x) < 0 \\ \textit{Negative Partial Entropy} \end{array} \\
\downarrow & & \downarrow \\
\begin{array}{c} Q_{D_+}(x) = \frac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) - 1} \\ \textit{Planck type distribution} \end{array} & & \begin{array}{c} Q_{D_-}(x) = \frac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) + 1} \\ \textit{Nuclei type distribution} \end{array} \\
\downarrow & & \downarrow \\
\begin{array}{c} V_{D_+}(R,G) = \\ -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)) \\ \textit{Planck type potential} \end{array} & & \begin{array}{c} V_{D_-}(R,G) = \\ -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g \mu_2^g R \pm \mu_1^g)) \\ \textit{Nuclei type potential} \end{array} \\
\exp(\mu_1^g) := \alpha \Big\downarrow^{\lambda^g \to 0}_{\xi^g \mu_2^g := \lambda} & & \exp(\mu_1^g) := \alpha \Big\downarrow^{\lambda^g \to 0}_{\xi^g \mu_2^g := \lambda}
\end{array} \tag{154}$$

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$$
\begin{array}{ccc}
V_\alpha(R) = -G\frac{m}{R}(1-\alpha\exp(-\lambda R)) & & V_{yukawa2}(R) = -G\frac{m}{R}(1+\alpha\exp(-\lambda R)) \\
\textit{Planck type potential} & & \textit{Yukawa type potential} \\
{\scriptstyle R\to large}\big\downarrow & & {\scriptstyle R\to large}\big\downarrow \\
V_\alpha(R) = -G\frac{m}{R} & & V_{yukawa2}(R) = -G\frac{m}{R} \\
\big\downarrow & & \big\downarrow \\
g = -V'_\alpha(R) = -G\frac{m}{R^2} \quad \rightarrow & g = -G\frac{m}{R^2} & \leftarrow \quad g = -V'_{yukawa2}(R) = -G\frac{m}{R^2} \\
\textit{Gravitational acceleration} & & \textit{Gravitational acceleration} \\
{\scriptstyle R\to 0}\big\downarrow & & {\scriptstyle R\to 0}\big\downarrow \\
\lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} & & \lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} \\
\textit{Weak Proximity } g_\pm^{wp} & & \textit{Strong Proximity } g_\pm^{sp}
\end{array}
$$

5. Possibility that Mass Generation by Entropy, the Existence of New Forces and Fluctuating of the Constant G

5.1. Possibility that Mass Generation by Entropy

Furthermore, the inverse of the second-order part λ_2^g of the approximation of generalized entropy is considered to be mass m. Leave this the first-order part λ_1^g as it is. The generalized entropy $S_{D_\pm}(R,G)$ is determined by mass m, distance R (the radius of range under consideration) and the correction factor λ_1^g. In other words, mass m is determined by generalized entropy $S_{D_\pm}(R,G)$, distance R and the correction factor λ_1^g. By transforming Equations (1), mass m can be represented as follows:

$$
m = \frac{R^2}{S_{D_\pm}(R,G) \mp \lambda_1^g R}. \tag{155}
$$

By transforming Equations (18) and $S_{D_\pm}(R,G) = G\frac{\xi^g R}{Q_{D_\pm}(R)} S_{D_\pm}(R)$, mass m can be represented as follows:

$$
\begin{aligned}
m &= \frac{R^2}{G\cdot D_\pm(R) S_{D_\pm}(R,G) \mp \lambda_1^g R} \\
&= \frac{R}{G\frac{\xi^g S_{D_\pm}(R)}{Q_{D_\pm}(R)} \mp \lambda_1^g} \\
&= \frac{R}{(\exp(-\xi^g\mu_2^g R \pm \mu_1^g) \mp 1) G S_{D_\pm}(R) \mp \lambda_1^g}.
\end{aligned} \tag{156}
$$

Namely, mass m can be represented as generalized partial entropy $S_{D_\pm}(R)$, distance R, correction factors λ_1^g, μ_1^g, μ_2^g and constants G, ξ^g. Mass is also considered to depended on entropy. Besides, if we consider $Q_{D_\pm}(R)$ to represent the spectrum (wave) distribution within the range of distance R, we can consider that mass depends on the partial entropy $S_{D_\pm}(R)$, the spectrum distribution $Q_{D_\pm}(R)$ (or division $D_\pm(R)$) within the range R, and the constants G, ξ^g, and λ_1. In other words, mass can be considered to depend on the partial entropy and spectrum (waves). Mass may consider to be generated depending entropy and waves.

5.2. Possibility that the Existence of New Forces

The constants, variables, and functions in the above Equations (23), that is $V_{D_\pm}(x,k)$, and (111), that is $V'_{D_\pm}(x,k)$, are appropriately selected within the range of conditions, $V_{D_\pm}(x,k)$ is interpreted as gravitational potential, and $V'_{D_\pm}(x,k)$ is interpreted as gravitational acceleration, conforming to gravity theory. However, if we consider carefully, the constants, variables, and functions in the above equations can be arbitrarily selected within the range of conditions, so these may be applicable to forces other than gravity and Coulomb force. By choosing the constants in the equation $V'_{D_\pm}(x,k)$

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appropriately, it may be possible to represent weak and strong force. Furthermore, there exists the possibility of expansion and the existence of new forces that are different from the conventional force. Therefore, it is possible that there exists many new forces. Namely, the following suggestion may be considered the possibility that there exists many new potential and acceleration:

Suggestion 9. *Possibility that there exists many new forces (1), Planck-type.*
Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exists countable numbers of potential $V_{D_+}(R, G_n)$ and a acceleration $V'_{D_+}(R, G_n)$ such that the following conditions are satisfied:

there exists constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:

$$V_{D_+}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 - \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})), \tag{157}$$

$$\begin{aligned} V'_{D_+}(R, G_n) = &\frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1 - \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})) \\ &- \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m} \exp(-\xi_n \int \mu_n(R) \pm \mu_{1n}), \end{aligned} \tag{158}$$

where $G_n \gtrneqq 0$, $\xi_n \gtrneqq 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$.
Namely, it is possible that there exists many new forces. Note that these description above assumed that assumptions(2) and (3). □

Similarly, for potentials $V_{D_-}(R, G_n)$ and accelerations $V'_{D_-}(R, G_n)$, we can describe as follows:

Suggestion 10. *Possibility that there exists many new forces (2), Yukawa-type.*
Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exists countable numbers of potential $V_{D_-}(R, G_n)$ and a acceleration $V'_{D_-}(R, G_n)$ such that the following conditions are satisfied:

there exists constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:

$$V_{D_-}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 + \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})), \tag{159}$$

$$\begin{aligned} V'_{D_-}(R, G_n) = &\frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1 + \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})) \\ &+ \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m} \exp(-\xi_n \int \mu_n(R) \pm \mu_{1n}), \end{aligned} \tag{160}$$

where $G_n \gtrneqq 0$, $\xi_n \gtrneqq 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$.
Namely, it is possible that there exists many new forces. Note that these description above assumed that assumptions (4) and (5). □

Gravitational acceleration G and Coulomb's constant k_e may just simply be some of the coefficients related to forces that humans can currently sense throughout the universe. Instead of asking why there are 4-forces, we should ask why humans are primarily only able to sense 4-forces.

5.3. Possibility that Fluctuating of the Constant G

As mentioned above, if we consider that there are many forces, then we can assume that there will be many variations in the constants. If the changes (differences) in the constants are small for the same variable, the constants will appear to be fluctuating. Furthermore, gravitational constant G can be considered of as being determined by generalized entropy $S_{D_\pm}(G, R)$, the partition $D_\pm(R)$

and the partial entropy $S_{D_\pm}(R)$ partitioned by $D_\pm(R)$ (or the distribution $\pm\xi R/Q_{D_\pm}(R)$). Namely, generalized entropy $S_{D_\pm}(G,R)$ is represented as follows:

$$S_{D_\pm}(R,G) = G \cdot D_\pm(R) \cdot S_{D_\pm}(R) = G \frac{\pm\xi R}{Q_{D_\pm}(R)} S_{D_\pm}(R). \tag{161}$$

Therefore, gravitational constant G is represented as follows:

$$G = \frac{S_\pm(R,G)}{D_\pm(R) \cdot S_{D_\pm}(R)} = \frac{S_{D_\pm}(R,G)}{S_{D_\pm}(R)} \cdot \frac{Q_{D_\pm}(R)}{\pm\xi R}. \tag{162}$$

In other words, it is possible that gravitational constant G can fluctuate if entropy changes.

6. Conclusions

6.1. Possibility that Gravity Depending on Entropy

The idea behind Planck's radiation formula is to apply the number of cases of partition by resonators to entropy. This idea is similar to the logistics function of a dynamical system. (Planck[1], Fujino [27]) Applying these, we treated the division of entropy as a non-minimal function $D_+(x)$, and derived potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$. Therefore, we assumed that generalized entropy $D_+(x,k)$ can be represented as a second-degree polynomial, and potential $V_{D_+}(x,k)$ is defined as the inverse of $S_{D_+}(x)$. As a result, each variable and constant used in potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$ is interpreted as in terms of gravity, and mass is defined as the inverse of the quadratic coefficient term of $S_{D_+}(x,k)$, that is, $1/\lambda_2$. The constant λ_1 is the the first-order coefficient of approximated generalized entropy $\lambda_2 x^2 \pm \lambda_1 x$. In other words, gravitational acceleration changes depending on coefficients of approximated generalized entropy . In addition, the constants μ_2 and μ_1 are defined as coefficients of generalized partial entropy or distribution function $Q_\pm$. In other words, the inverse of the second-order part λ_2^g of the second-order approximation of generalized entropy is considered to be mass. The first-order part λ_1^g is left unchanged. The generalized entropy $S_{D_+}(R,G)$ is determined using mass m, distance(radius) R of the range under consideration, and the correction factor λ_1^g. The description on this article, it is assumed that the assumption 2, 3 and the existence of entropy-dependent constants $\xi, \lambda_2, \lambda_1, \mu_2$, and μ_1 that control gravity and the velocity of galaxies. We proposed the following conclusions:

(1) If distance R is small enough, hence gravitational acceleration $V'_{D_+}(R,G)$ becomes 2-states with finite constants depend on constants $\xi, \lambda_1^g, \mu_1^g$ and μ_2^g. Depending on the value of μ_1^g and λ_1^g, the value of $V'_{D_+}(R,G)$ can be positive or negative. If the constant $\lambda_1^g \to 0$, then gravitational acceleration $V'_{D_+}(R,G)$ becomes $\pm\infty$. If the constant $\lambda_1^g \to \infty$, then gravitational acceleration $V'_{D_+}(R,G)$ becomes 0. Therefore, it is possible that gravity have 5-states within distance R is small enough. Among the 5-states, there may exist anti-gravity, which is the opposite of Newton's gravity. (Possibility existence of anti-gravity) Furthermore, using the equation for potential derived from entropy, within small distance, it may be possible to treat gravitational potential and Coulomb potential in the same way by appropriately choosing some constants. Similarly, the same suggestion can be made for gravitational acceleration and Coulomb's law (electric field).

(2) At distance large enough to be within the size of the universe, gravity follows the adjusted inverse square law. Within this distance, the rotation speed of the galaxy v follows gravitational constant G, mass $m = 1/\lambda_2^g$ and constants ξ^g, μ_2^g and μ_1^g which depend on entropy. Besides, the rotation speed of the galaxy v does not little depend on its radius R, (the galaxy rotation curve problem). Even without assuming dark matter, the problem of the rotation speed of the galaxy may be explained by the concept of entropy. This does not mean denying dark matter. The new constants μ_1^g and μ_2^g proposed in this paper may represent some kind of dark or virtual mass.

(3) At large distance, gravity follows adjusted inverse square law. Comparing to conventional gravity g, gravitational acceleration $\tilde{g}_\pm$ towards the center of rotation becomes slightly weaker or

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stronger. This means that gravitational acceleration towards the center of a rotating substance can be slightly changed at distance. (The Pioneer Anomaly)

From the above description, it is possible there exists some constants $\xi, \lambda_2, \lambda_1, \mu_2$ and μ_1 which depend on entropy that controls gravity and the speed of the galaxy. Besides, it is considered to apply to velocities over short distances, such as electrons in atomic nuclei.

6.2. Interpretation of Yukawa-Type Potential by Negative Partial Entropy

By defining $V_{D_-}(x,k)$ and acceleration $V'_{D_-}(x,k)$, it is also considered to be related to Yukawa-type potential and negative generalized partial entropy. Therefore, particle physics may be related to entropy. It may be suggested that there exists 11-types of acceleration including gravitational acceleration g, such as $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g^{sp}_\pm$ and $g^{wp}_\pm$ related to g. Using these forces, we attempted to compare that the ratio of the size of the fundamental 4-forces of nature (strong force, electromagnetic force, weak force and gravity) with strong force being 1. It showed that strong proximity force$(+)$ g^{sp}_+ can be regarded as strong force, weak proximity$(-)$ force g^{wp}_- as weak force, adjusted gravity$(\pm)$ $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force. Moreover, it described that possibility mass generation by entropy, the existence of new forces and fluctuating of the constant G. In consequence, gravity may depended on entropy.

6.3. Integration of Thermodynamics, Quantum, Gravity and Ecology by Entropy

By combining concepts of the logistic function (dynamical system), Boltzmann's entropy and Planck's quantum, we obtained adjusted gravity. Namely, by developing the concept of the logistics function and combining it with entropy and Planck's ideas, we derived that potentials $V_{D_\pm}(x,k)$ and accelerations $V'_{D_\pm}(x,k)$. Since the nonlinear behavior obtained from the logistic function is non-Newtonian mechanics, Newtonian mechanics, including the theory of gravity, may be included in non-Newtonian mechanics. The concept of logistics function is applied to ecology like population theory and the evolution of life. It is also known that the concept of entropy was established by Clausius and related to quantum theory by Planck.[1], [27] And it is considered that entropy is related to the concept of the logistic function.[27] Furthermore, the concept of the logistic function is applied to population theory, the evolution of life and ecology. Therefore, it is considered that entropy is related to ecology.[15], [27] This paper argues that the concept of entropy is related to gravity theory. These findings suggest that by combining the concepts of entropy and the logistic function, it may be possible to understand the evolution of the universe in the same way as the evolution of life. Thus, it is considered that thermodynamics, quantum (particle), gravity, electromagnetic and ecology (biology) can be unified through the concept of entropy.

$$\begin{array}{ccc} Thermodynamics & & Ecology(Biology) \\ \searrow\nwarrow & & \swarrow\nearrow \\ & Entropy & \\ \swarrow\nearrow & \downarrow\uparrow & \searrow\nwarrow \\ Quantum & Gravity & Electromagnetic \\ (Particle) & & \end{array}$$

We hope that the concept of entropy explain more and provide new perspectives.

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