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Posted Date: 5 March 2024

doi: 10.20944/preprints202403.0268.v1

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Article

Quantum Temporal Winds: Turbulence in Financial Markets

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Abstract: This paper draws upon the theory of turbulence in physics to explore the similarities and differences between volatility in financial markets and turbulent phenomena at a statistical physics level. By analogizing the dynamics of financial markets with fluid turbulence, we have developed an innovative analytical framework aimed at deepening the understanding of the complexity inherent in financial markets. Our research methodology includes a comparative analysis of simulated time series data for the Standard & Poor's 500 Index and fully developed turbulent velocity time series. We focus on the similarities in key statistical physical properties, including probability distributions, correlation structures, and power spectral densities. Furthermore, we have established a model of capital flow in financial markets and proposed corresponding solutions. Through computational simulations and data analysis, we find that while financial market volatility shares some statistical characteristics with turbulent phenomena, significant differences exist in the shapes of probability distributions and the temporal scales of correlations. This suggests that although financial markets exhibit patterns akin to turbulence, the behavior of this complex, multi-variable driven system does not completely correspond to natural turbulence, highlighting the limitations of directly applying turbulence theory to financial market analysis. Our study offers a new perspective for the theories of financial markets and the field of econophysics, providing fresh insights into the complexity of financial markets and contributing to the prevention and management of financial risk.

Keywords: quantum finance; financial markets; turbulence; time series analysis

1. Introduction

Econophysics, as an interdisciplinary field, predominantly employs theories and methods from physics to decipher and forecast the behavior and dynamics of financial markets. The evolution of this field signifies the convergence and intersection between traditional finance and natural sciences, particularly the application of statistical physics, nonlinear dynamics, and complex systems theory from physics. In recent years, econophysics has made notable strides in understanding market volatility, the distributional characteristics of asset prices, and the structure and stability of financial complex networks.

As a complex system, the financial market exhibits behaviors that are replete with nonlinearity, uncertainty, and intricate dynamic correlations. These patterns are particularly evident in the fluctuations of financial time series such as stock prices, exchange rates, and interest rates. In the natural sciences, turbulence is a prevalent nonlinear phenomenon that has long posed a challenge for physicists and engineers. The complexity and unpredictability displayed by turbulence share many similarities with the fluctuations observed in financial markets. Figure 1 illustrates the pressure distribution of a fluid in motion, while Figure 2 displays the fluctuation pattern of financial market prices. This paper proposes a framework based on the inflection points of price fluctuations,

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constructing an upper and lower Bézier curve track. When prices move to these extremities, they exhibit a turbulent reversion phenomenon akin to that observed in fluid turbulence.

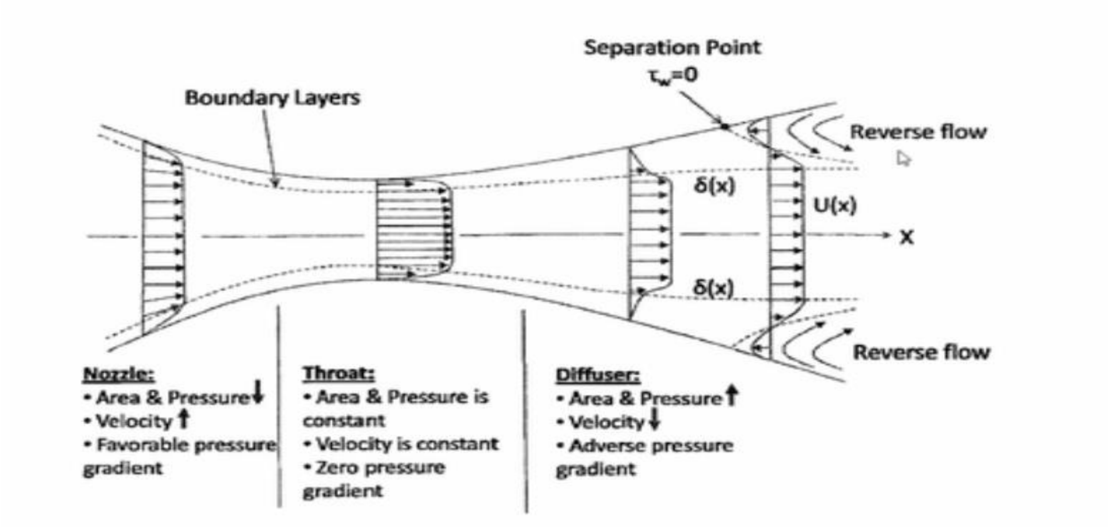

Figure 1. Fluid force diagram.

Figure 2. Financial Market Price Chart.

Advancements in the study of turbulence have provided new perspectives and tools for this phenomenon, offering theories and methods that can be applied to analyze the dynamic behavior of financial markets. In particular, turbulence theory offers a novel framework for time series analysis, enhancing our understanding of the complex fluctuation structures within market time series data. The specific contributions of this paper are as follows:

We first introduce the concept of temporal turbulence, positing that time-space can be quantized. The paper then details the construction process of the temporal turbulence model, including theoretical assumptions, mathematical formulations, and statistical simulation methods. By comparing the statistical properties of financial markets and turbulence, we investigate the similarities between the two, analyzing the complex dynamics of financial markets from the perspective of turbulence theory. The Standard & Poor's 500 Index is chosen as a representative case, with its time series compared to turbulent velocity time series.

Subsequently, the paper presents the empirical analysis results of the model, discovering certain degrees of similarity in probability distributions, correlations, and power spectra between the two, both exhibiting highly complex nonlinear dynamical behavior. However, differences also exist, such as in the shapes of probability distributions and the temporal scales of correlations. Moreover, the paper constructs a financial market trading strategy based on Bézier curves—a mathematical tool for describing turbulence—demonstrating that the application of Bézier curves in financial markets, combined with time series analysis, observation of price fluctuations, and changes in curve curvature, can form a dynamic model for analyzing market volatility. This model aids investors in assessing market trends and risks, thereby informing their trading strategies.

Through an in-depth study of market dynamics, this paper aims to provide novel insights into understanding the complexity of financial markets and to offer new strategies for the prevention and management of financial risk. This research clarifies the operating mechanisms of another complex system from an already understood system, serving as a reference for the application of econophysics methods in the analysis of financial markets. It shows that similar comparative analysis methods can be adopted in the field of econophysics. However, the quantitative correspondence between the two is limited, and further research is required to ascertain the dynamical mechanisms linking them.

2. Literature Review

2.1. Quantum Finance

In the realm of theoretical physics and its intersection with financial modeling, Nakayama (2009) presents a significant contribution by deriving a metric that satisfies the Einstein equation when coupled with a massive vector field. This development has implications for the scaling regime of Reggeon field theory and extends to the complex field of non-linear quantum finance.

Baaquie (2009) shifts the focus to the finance industry with the Libor market model (LMM), the benchmark for interest rate modeling for both Libor and Euribor rates. Baaquie proposes a quantum finance formulation of the LMM, leading to a critical generalization that Libors, corresponding to different future times, are not perfectly correlated.Extending this quantum perspective to empirical analysis, Baaquie et al. (2009) calibrate and test models of interest rates using futures data for Libor and Euribor. Their study compares two models—the linear theory of bond forward interest rates and the non-linear theory embedded in the Libor Market Model.

Agrawal et al. (2013) explore the intriguing application of quantum mechanics to human decision-making processes, suggesting potential advancements in quantum finance and management. They also highlight the relevance of quantum mechanics to human dynamics in healthcare management.

Quantum computing's potential in finance is further explored by Zoufal et al. (2019), who present a hybrid quantum-classical algorithm to facilitate the efficient loading of quantum states. They demonstrate the algorithm's application in quantum finance through quantum simulation, employing a trained quantum channel.

Bouland et al. (2020) delve into the prospective impacts of quantum computing on the finance industry, detailing the possible applications and the extent of the quantum speedup that could be achieved. They assess the quantum resources necessary for practical applications, drawing on the state-of-the-art and recent efforts by the QC Ware team.

Complementing these studies are the works of Baaquie (2004), Schaden (2009), Baaquie (2018), and Lee (2019), each contributing to the rich tapestry of research at the confluence of quantum physics and financial modeling. These studies collectively underscore the transformative potential of quantum mechanics and quantum computing in the analysis and prediction of complex financial systems.

Quantum finance is an emerging interdisciplinary field that aims to leverage quantum computing to address financial challenges and enhance existing financial models and algorithms. Several recent papers have explored the potential applications of quantum computing in finance, focusing on areas such as stochastic modeling, optimization, machine learning, and option pricing. Herman et al. (2022) provided a comprehensive survey of quantum computing for financial applications, emphasizing the potential advantages of quantum solutions in solving financial problems more efficiently and accurately. In a similar vein, Widdows and Bhattacharyya (2023) introduced quantum approximate counting circuits for quantum financial modeling, demonstrating the robustness of these circuits to noise. Moreover, Braine et al. (2019) extended variational quantum optimization algorithms to address mixed binary optimization problems, allowing the combination of binary decision variables with continuous decision variables, which has significant implications for financial modeling and optimization. Additionally, Chang et al. (2023) presented a summary of quantum computing for both traditional and Blockchain financial applications, further illustrating the potential of quantum computing in the finance domain. Furthermore, Gustafson (2011) explores antieigenvalue analysis and its applications in finance, which could offer novel insights into the financial analysis of quantum systems. Additionally, Li and Liang (2020) proposed an option pricing model based on the analog between stochastic dynamics and quantum harmonic oscillators, highlighting the potential for leveraging quantum mechanics in developing innovative approaches to option pricing. In summary, the literature on quantum finance illustrates the growing interest in leveraging quantum computing to address various challenges and enhance traditional financial models and algorithms. The potential applications highlighted by these authors demonstrate the significant impact that quantum computing could have on the future of finance, paving the way for more efficient and accurate financial solutions.

2.2. Financial Market Turbulence

The study of financial markets and turbulence has led to intriguing parallels in the statistical behavior of these seemingly disparate systems. Peinke et al. (2004) delve into the heart of this analogy, investigating the anomalous statistics that arise in both turbulence and financial markets. Through stochastic cascade processes, they demonstrate how heavy-tailed distributions emerge, a characteristic feature of disordered complex systems with hierarchical organization. By directly reconstructing stochastic equations from empirical data, Peinke and colleagues successfully reproduce the observed anomalous statistics, affirming the utility of this approach.

Expanding on this framework, the She-Leveque (SL) hierarchy, initially proposed for turbulent fluid dynamics, has been adapted by Gao et al. (2012) to decipher the hierarchical structure in stock market fluctuations. This application underscores the multifractality present in financial market dynamics and offers a novel lens through which to view market complexity.

Cross-border financial contagion is another domain of interest. Kilic et al. (2015) employ a VAR-MGARCH model to analyze how turbulence in financial markets can trigger unexpected international contagion effects, affecting both foreign exchange and stock exchange markets. This study highlights the interconnectedness of global financial systems and the potential for localized disruptions to propagate across borders.

However, the recent financial market turbulence has also sparked a discourse on transparency and accountability. Seyfert (2016) argues that the issue may not stem solely from an information deficit. Through a comparative analysis involving regulators, market analysts, and traders, Seyfert reveals that the divergent methods by which these actors interpret and leverage information can lead to distinct epistemic regimes, thus complicating the landscape of market knowledge.

In the realm of portfolio management, Gkillas et al. (2019) contribute valuable insights by examining the correlation between bitcoin and gold during market turbulence. Their findings suggest that combining these assets can provide a hedge for equity positions, especially during times of financial distress, and that the inclusion of bitcoin alongside gold can enhance portfolio performance under tail risk constraints.

The relationship between market capabilities and turbulence has been another focal point. Zhou et al. (2019) address the gap in understanding how marketing agility affects financial performance, both directly and indirectly through innovation capability. Their counterintuitive findings indicate that innovation capability has a more substantial impact on financial performance in less turbulent markets, with market turbulence moderating the relationship between marketing agility and financial performance.In the context of foreign exchange markets, Gunay (2020) compares recent market deterioration with the tumultuous period of the Global Financial Crisis of 2008–2009. The analysis suggests that while the markets are experiencing turmoil, it has yet to reach the severity of the crisis over a decade ago.

 doi:10.20944/preprints202403.0268.v1

Further, Sachdeva (2020) investigates the interplay between small cap and mid cap returns and the BSE Sensex returns, focusing on the period from 2013 to 2018. This research underscores the heightened volatility and its implications for various asset classes within the global economy. Katusiime (2022) leverages time-frequency connectedness measures to explore the dynamics of return and volatility connectedness among East African Community (EAC) member states. This work not only adds to the literature on financial connectedness but also provides insights into market behavior during the COVID-19 pandemic, a period of significant global upheaval.

The collective insights from these studies, including the foundational work by Caprio (2009), provide a multifaceted understanding of financial market dynamics, particularly in the face of turbulence and crises.

2.3. Financial Market Curve Yield

The landscape of financial market research is rich with studies examining the nuances of market dynamics, volatility, and the impact of various factors on asset pricing and investment strategies. Chatterjee (2008) delves into unusual patterns observed in financial markets, termed "redundant sigmoid heterodoxy behavior." This research utilizes novel statistical methodologies to analyze stock and equity market indices' closing values and volatility trends across global markets.

Kanamura (2011) introduces a supply and demand-based price (SDP) model for financial assets, challenging traditional stochastic volatility models. The SDP model posits that stock price volatility is more accurately reflected by the interplay of demand curves and trading volume fluctuations.

The study by Park et al. (2012) investigates the growth of IT investment in Korean financial markets over an 18-year period, using Nolan's growth model to ascertain the correlation between IT investment and management performance. Their findings indicate that while the banking industry's growth aligns with the s-curve pattern, the insurance industry's growth, although also s-curve-like, tends to follow a more linear progression.

Heidorn et al. (2014) examine the influence of fundamental and financial traders on the futures curve of WTI oil and the market integration between WTI and Brent crude oils. Their threefold contribution reveals that fundamental traders significantly impact the level of the futures curve, whereas financial traders' activities are more closely associated with its slope.Naghavi et al. (2016) explore the relationship between financial liberalization and stock market efficiency, emphasizing the informational efficiency of stock markets rather than financial development.

In the context of the Georgian financial market, Qoqiauri (2016) identifies the current state and challenges faced by the market, with a focus on crucial financial market attributes.Subramaniam et al. (2017) analyze the global and regional influences on the domestic term structures of nine Asian economies. Their research concludes that despite financial integration, monetary authorities maintain control over the short end of the yield curve, while long-term interest rates serve as a channel for financial crisis contagion.

Khatatbeh et al. (2022) examine the financial Kuznets curve in emerging countries, employing different time series methodologies to analyze data from 1993 to 2017. Their work provides empirical support for the existence of the financial Kuznets curve in these economies.

Most recently, Ntawiratsa et al. (2023) focus on the estimation of zero-coupon rates and actuarial rates of return for the Burundian yield curve using the Nelson-Siegel and Svensson models. Their findings endorse the Nelson-Siegel model as the superior choice for yield curve modeling in this context.

These studies, along with influential work such as Parker (2016), collectively contribute to a deeper understanding of the complex mechanisms that govern financial markets and the various factors that influence market behavior and efficiency.

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3. Theoretical Basis

3.1. Quantization of Time

Figure 3. Quantum Time Wind Concept Diagram.

In the financial markets, candlestick charts, known as "K lines," serve as a graphical representation of price action, with each candlestick representing the price behavior of an asset over a specific time frame. The analogy of comparing K lines to "time particles" vividly emphasizes the observation of price behavior at discrete points in time. Each candlestick carries crucial information about the price within that period, including the opening price, the highest price, the lowest price, and the closing price. Expressed in mathematical language, if we consider a series of time intervals $\Delta t\_1,\ \Delta t\_2, \ldots, \Delta t\_n$, with each interval corresponding to a candlestick, then a series of candlesticks can be represented as:

$$K\_(\Delta t\_1), K\_(\Delta t\_2), \ldots, K\_(\Delta t\_n)$$

Each candlestick $K\_(\Delta t\_i)$ can be represented by a tuple:In the analysis of financial time series, the opening price S_("open", i), the highest price S_("high", i), and the closing price S_("close", i) are considered key indicators of the price movement of an asset within the i-th time interval. These indicators collectively form the candlestick chart, providing investors and analysts with a powerful tool for interpreting market trends, identifying levels of support and resistance, and recognizing price patterns. Each candlestick represents the price volatility within a time period, and the aggregated information allows for a complete presentation of the price history, enabling predictions about future price behavior based on historical data.

The log-normal distribution is a continuous variable X's probability distribution, where the natural logarithm of $X, \ln(X)$, follows a normal distribution. If a random variable X is log-normally distributed, its probability density function (PDF) is given by the following formula:

$$f(x \mid \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln\ x - \mu)^2}{2\sigma^2}\right), \quad \text{for } x > 0,$$

where μ and σ are the mean and standard deviation of the variable's logarithm, respectively.The probability density function (PDF) for a log-normally distributed variable X is given by:

$$f_X(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln\ x - \mu)^2}{2\sigma^2}\right), \quad \text{for } x > 0.$$

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Here, μ and σ represent the mean and standard deviation of the natural logarithm of X, respectively. For an asset price s, if the logarithm of s, denoted as $\log(S)$, follows a normal distribution, then s itself follows a log-normal distribution. In discrete-time simulations, a time step Δt is typically defined, and price changes are considered at each step. If s_t is the asset price at time t, then s_{t +\ Delta t} is the price at $t+\Delta t$. The logarithmic return of the asset price can be expressed as:

$$\log\left(\frac{S_{t+\Delta t}}{S_t}\right) \sim N\left(\left(\mu-\frac{\sigma^2}{2}\right)\Delta t, \sigma^2\Delta t\right)$$

In financial modeling, the simulation of asset price paths is an iterative process. Initially, the asset price s_0, expected return rate μ, volatility σ, ime step Δt, and the total number of steps N are established. For each time step n = 0,1,2, ..., N - 1, a standard normally distributed variable z_n is generated to simulate the randomness of price movements. The asset price is updated according to the following formula:

$$S_{(n+1)\Delta t} = S_{n\Delta t}\exp\left(\left(\mu-\frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z_n\right)$$

This reflects the expected return and random volatility at each time step. The process is repeated until the N iterations are completed. Subsequently, the generated paths are subject to statistical analysis to estimate key financial metrics such as the average price path, volatility, and the probability of extreme events. These simulation techniques have extensive applications in options pricing, risk management, portfolio optimization, and scenario analysis.

3.2. Space Quantization

3.2.1. Einstein Thought Experiment

Einstein's pollen experiments typically refer to his studies concerning Brownian motion. In 1905, Einstein published a series of revolutionary scientific papers, one of which elaborated in detail on Brownian motion—the random movement of pollen particles suspended in water. This discovery had a profound impact on the physics community of the time because it provided direct evidence for the existence of atoms and molecules. Prior to this, many scientists still harbored doubts about the actual existence of microscopic particles.

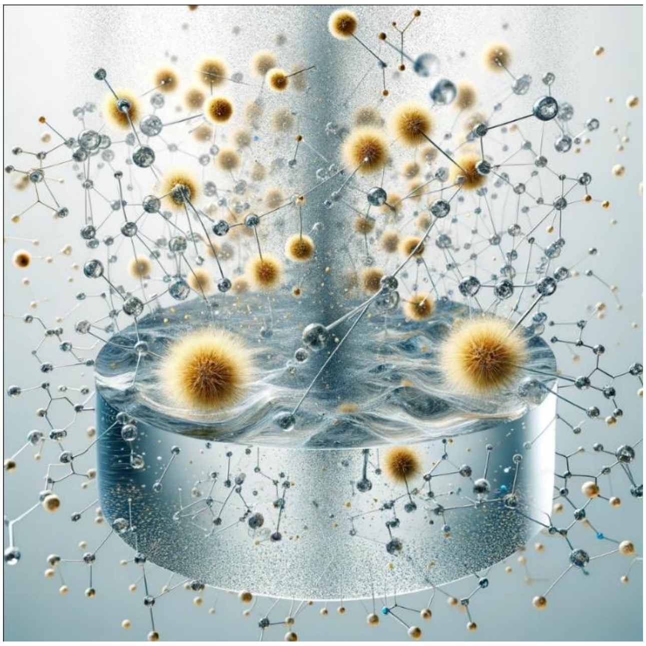

Figure 4. Einstein's Pollen Thought Experiment.

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The central premise is that macroscopically observable random motion is caused by the incessant collisions with numerous microscopic particles within the fluid. Einstein further developed a mathematical model to describe this phenomenon, which is based on the assumption that the movement of pollen particles is induced by their collisions with the water molecules in the fluid. He derived an equation for the mean displacement of pollen particles, $\langle D(t) \rangle = \sqrt{2At}$, where $\langle D(t) \rangle$ represents the mean displacement within time t, and A is a constant that involves the temperature and the viscosity of the fluid. This equation not only clarified the possibility of revealing the properties of the microscopic world through macroscopic phenomena but also provided a quantitative method for measuring the existence of microscopic particles. Einstein's theory was subsequently confirmed by experimental physicists such as Jean Perrin, whose experimental results not only agreed with Einstein's predictions but also provided direct evidence for the existence of atoms, thereby deepening our understanding of the fundamental structure of matter.

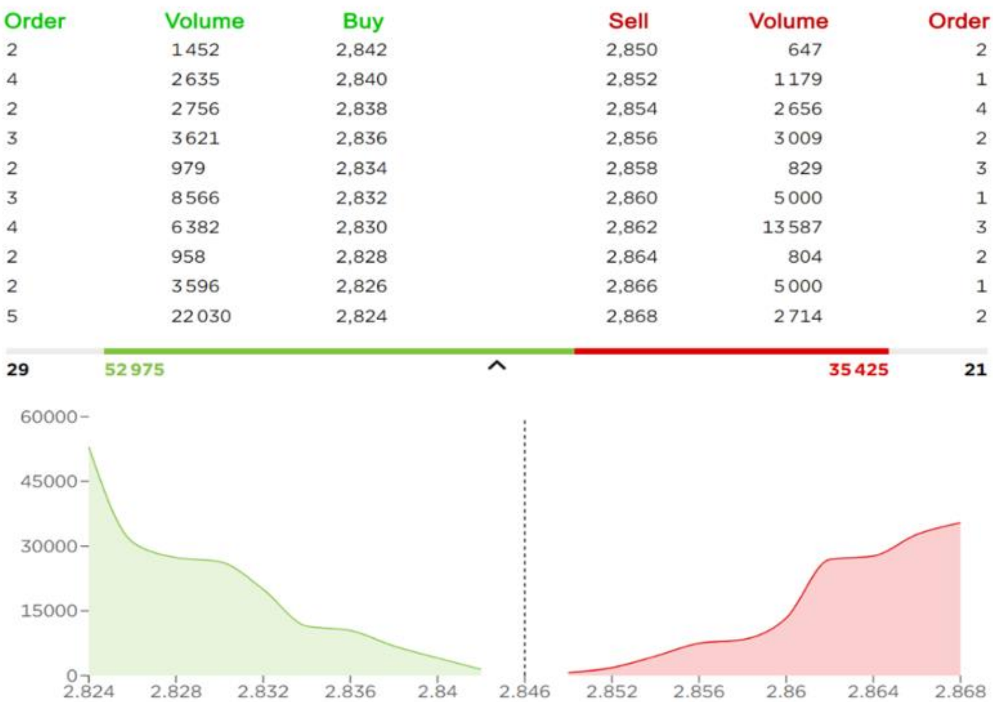
3.2.2. Space Quantization Analysis

Minimum Unit under Uncertainty Perspective

The financial market is a real-time dynamic interaction system involving tens of thousands of participants from around the world. The price fluctuation within an extremely short time frame is inherently uncertain and ambiguous, making it exceedingly difficult to accurately predict every price movement. This is illustrated by the underlying transaction mechanism of financial markets, as depicted in Figure 5, where the number of buy and sell orders for each price can change at any moment.

The principle of uncertainty in financial markets can be interpreted as follows: a specific transaction establishes the price of a particular asset, while the decision by investors to buy or sell the asset at a higher or lower price determines the asset's price trend. At a given moment, it is impossible to know both the precise price of a trading asset and the direction of its price change in the next moment. The uncertainty principle is frequently observable in the market. For example, at a certain point in time, if an individual does not know the exact price of a stock, they certainly cannot anticipate the speed and direction of the next price change. In other words, the uncertainty of trends seems infinite.

However, in the real financial market, one always has information beyond the price of the stock itself at any given time, thus allowing for the prediction of localized price fluctuations within a certain range.



Order	Volume	Buy	Sell	Volume	Order
2	1452	2,842	2,850	647	2
4	2635	2,840	2,852	1179	1
2	2756	2,838	2,854	2656	4
3	3621	2,836	2,856	3009	2
2	979	2,834	2,858	829	3
3	8566	2,832	2,860	5000	1
4	6382	2,830	2,862	13587	3
2	958	2,828	2,864	804	2
2	3596	2,826	2,866	5000	1
5	22030	2,824	2,868	2714	2
29	52975			35425	21

Figure 5. Transaction mechanism behind financial markets.

Just as in quantum mechanics, the position of a particle is indeterminate. The uncertainty principle describes the relationship between two non-commuting variables. For instance, the product of the uncertainties in position and momentum is greater than or equal to a specific constant, which means it is impossible to simultaneously obtain precise values for both position and momentum. According to Heisenberg's uncertainty principle, there exists the following relationship between energy ΔE and time Δt :

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

If time is quantized, then there exists a minimum time unit Δt, which defines the lower bound of the time interval that can be measured in this context. Within this minimum time unit, any measurement of energy will have an uncertainty ΔE. Based on the uncertainty principle, it can be deduced that during the time Δt, the uncertainty in energy is at least:

$$\Delta E \geq \frac{\hbar}{2\Delta t}$$

This implies that if time is quantized, then during the duration of the minimum time particle Δt, there is a fixed lower bound to the uncertainty in energy, which could have significant implications for the energy states of quantum systems. Within a quantized time framework, physical processes (such as the transition of electrons, emission of photons, etc.) would be restricted to occur at discrete points in time. This means that the occurrence of these processes is no longer possible within arbitrarily small time intervals but must occur in multiples of Δt. Therefore, the duration T of any process can be expressed as:

$$T = n\Delta t$$

where n is an integer. This notion of quantized time could lead to new characteristics of physical processes, such as the rate of energy level transitions potentially exhibiting new quantized patterns.

3.3. The Smallest Spatial Unit in the Financial Market

The method of analysis based on Einstein's thought experiments can likewise be applied to financial markets, treating market price fluctuations as a form of financial "Brownian motion." In this model, each transaction is akin to an impact on the asset price, causing random fluctuations and thus aiding our understanding of the complex dynamics of financial markets. Applying Einstein's approach to financial markets allows us to view the price movements of financial assets as phenomena analogous to Brownian motion. In this metaphor, the current price of financial markets is comparable to pollen particles suspended in a fluid, while the orders of global investors are akin to the collisions of water molecules.

Each order, regardless of size, impacts the asset price similarly to how water molecules strike pollen particles. Although the effect of a single order might be negligible, the collective effect of all orders causes price fluctuations on a macro level, resulting in the stock market's "Brownian motion."

In the field of financial market analysis, the mathematical expression of Einstein's Brownian motion, $\langle D(t) \rangle = \sqrt{2At}$, can be reinterpreted as a model for asset price volatility, where $\langle D(t) \rangle$ represents the average magnitude of change in asset price over time t, and the constant A in this context is associated with market liquidity and trading activity. This model provides a new perspective for understanding the fundamental nature of financial market price movements, revealing how external factors and investor behavior jointly impact asset prices. This theoretical framework not only offers insights into the deeper analysis of financial market fluctuations but also provides solid theoretical support for the formulation of investment strategies and the practice of risk management.

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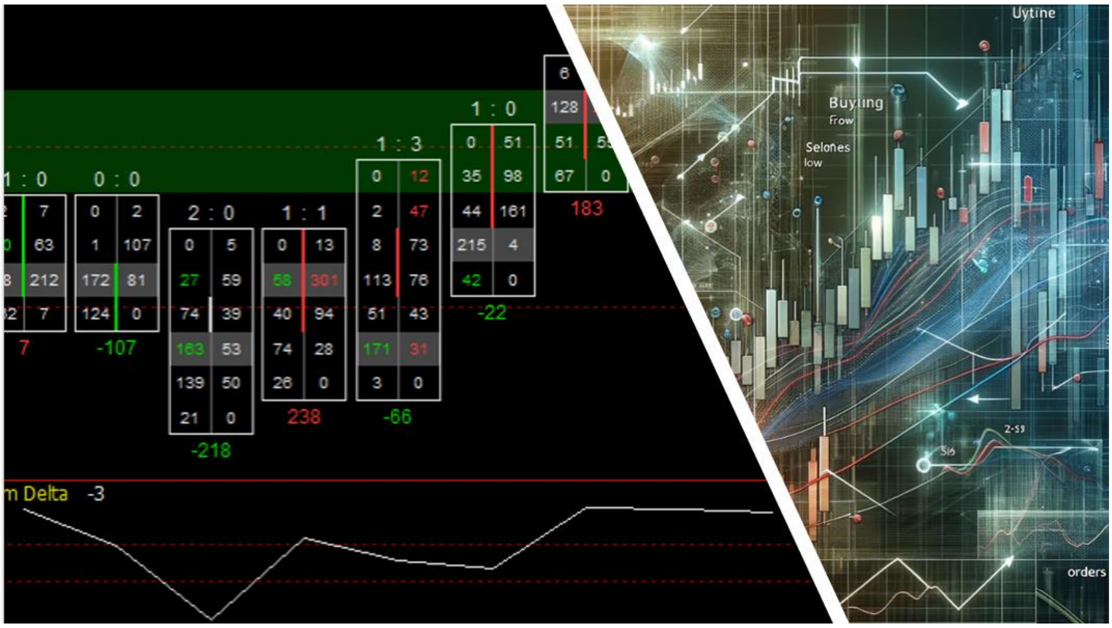

Figure 6. Brownian motion of orders behind the K-line.

3.3.1. Argument Basic Assumptions

The minimal unit of stock price movement: Let us assume the existence of a minimal price movement unit, ΔP, which can be regarded as the "quantum of space" for the stock market. The randomness of transactions: The price change caused by each transaction can either increase by one ΔP or decrease by one ΔP, akin to a random walk model.

In stock markets, Brownian motion is commonly applied to the mathematical model known as Geometric Brownian Motion (GBM), which is utilized to simulate the fluctuations of asset prices, such as stocks or indices. This model is based on key assumptions where the price movements are considered continuous and exhibit both a determinable trend and stochastic volatility. The mathematical expression for geometric Brownian motion can be described by the following stochastic differential equation:

$$dS = \mu S dt + \sigma S dW$$

Here, S represents the asset price, μ denotes the expected rate of return of the asset, also known as the drift rate; σ represents the volatility of the asset price; dW is a Wiener process term representing random shocks, and dt is the time increment. The model assumes that the logarithmic returns of asset prices are normally distributed, which simplifies the calculation of probabilities at different price levels. The geometric Brownian motion model is an essential tool in financial engineering for risk management and derivative pricing, with the Black-Scholes option pricing model being one notable application based on this framework. Nevertheless, the actual behavior of stock markets can be more complex than this model describes, as it may also be influenced by a variety of factors including market psychology, informational asymmetry, and changes in supply and demand. Analogous to Einstein's analysis of Brownian motion, this paper attempts to construct a simplified model from a statistical physics perspective to describe this phenomenon.

3.3.2. Mathematical Description of Argumentation

Consider a simplified model where stock price movements follow the rules of a onedimensional random walk: within a time interval Δt, the price change ΔS can be either $+\Delta P$ or $-\Delta P$. Assuming that the probabilities for upward or downward movements are equal, that is, $P(\Delta S = +\Delta P) = P(\Delta S = -\Delta P) = 0.5$.

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The derivation process is as follows: We define the expected value $E[\Delta S]$ and variance $\operatorname{Var}[\Delta S]$ of stock price movements. Since each movement is independent and the probabilities of rising or falling are equal, we have:

$$E[\Delta S] = 0$$
$$\operatorname{Var}[\Delta S] = E[\Delta S^2] - (E[\Delta S])^2 = (\Delta P)^2$$

Considering the stock price movements over a long period t, one can invoke the Central Limit Theorem to predict that the sum of stock price movements approximately follows a normal distribution with a mean of 0 and a variance of $n(\Delta P)^2$, where $n = t/\Delta t$ represents the number of trades that occur over time t. Therefore, the model for stock price movements can be represented as:

$$\Delta S(t) \sim \mathcal{N}(0, n(\Delta P)^2)$$

In this model, the stock price changes are modeled as a stochastic process that, over a long time scale, tends to exhibit Gaussian fluctuations around the initial price, which is consistent with the empirical distributions observed in the financial markets under certain conditions.

The normal distribution $\mathcal{N}(0, n(\Delta P)^2)$ indicates that, over time t, the summation of stock price movements is distributed as a normal distribution with a mean of 0 and a variance of $n(\Delta P)^2$. Here, n independent changes in ΔS (each with a mean of 0 and a variance of $(\Delta P)^2$) are summed to produce the total change $\Delta S(t)$. This is just a theoretical model and such a simplified model usually requires further adjustments to accommodate the complexities of real markets. For instance, in actual markets, the probabilities of price increases and decreases may not always be equal, and the magnitude of price change ΔP may vary with changing market conditions. Additionally, factors such as transaction costs, market impact, and the speed of information dissemination must be considered for their effects on stock prices. Despite these limitations, the random walk model remains one of the very important theoretical tools in the field of finance and has had a significant impact on the development of modern financial mathematics.

4. Turbulence Models in Financial Markets

This section discusses several key mathematical equations that describe the dynamics of financial markets. First, the continuity equation provides a framework for the conservation of capital flow in financial markets, indicating that the change in capital density within a closed system is equal to the negative divergence of capital flow.

Next, the conservation of momentum equation, inspired by the Navier-Stokes equations from fluid dynamics, describes the changes in market behavior momentum, taking into account market pressure, market friction, and external influences.Following this, a quantized version of the Navier-Stokes equation is introduced as a theoretical model, attempting to describe the behavior of fluids at the microscale using concepts from quantum fluid dynamics. However, this model remains largely in the theoretical research phase.

In the realm of quantum mechanics, the Schrödinger equation is the fundamental equation describing the temporal evolution of quantum systems. By employing the method of separation of variables, the wave function is decomposed into a product of time and space functions, thereby splitting the original equation into two independent equations that handle the temporal and spatial dependencies of the wave function separately. The solution for the time-dependent part reveals the phase factor with which the wave function varies over time, while the spatial-dependent part provides the wave function solution for the state of a free particle.

Finally, the information diffusion equation is explored, which simulates the spread of information through financial markets, analogous to the heat diffusion equation in physics. Through these equations, a better understanding and prediction of the behavior and information flow in financial markets can be achieved.

4.1. Continuity Equation

The continuity equation is a mathematical representation of conservation laws, utilized in financial market models to characterize the conservation of capital flow. Let $\rho(x,t)$ represent the market density at time t and location x, where the density could be the amount of capital or trading volume. Concurrently, $v(x,t)$ denotes the velocity of capital flow, meaning the rate at which funds enter or exit per unit time. The continuity equation can be expressed as:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho v)=0$$

This equation indicates that in a system without external capital injections or withdrawals, the market's capital is conserved. The rate of change in capital density $\frac{\partial\rho}{\partial t}$ plus the divergence of capital flow $\nabla\cdot(\rho v)$ sums to zero.

4.2. Market Turbulence Model

The conservation of momentum in financial markets can be analogized to the NavierStokes equations from fluid dynamics, which describe the changes in the momentum of market participants' behavior and its relation to market pressure and external forces. The equation can be represented as:

$$\rho\left(\frac{\partial v}{\partial t}+v\cdot\nabla v\right)=-\nabla p+\eta\nabla^2 v+F_{\text{ext}}$$

Here, $p(x,t)$ denotes market pressure, which could be a measure of information density or signal strength; η is the viscosity coefficient, representing the magnitude of market friction; F_{ext} represents external forces, such as policy changes, macroeconomic indicators, and other factors. This equation accounts for the inertia of market participants' behavior (the first term), local acceleration (the second term), the gradient of market pressure, market friction, and external influences.

Below is a potential form of the quantized Navier-Stokes equation, introducing a construct akin to a wave function, which we can refer to as the "quantum fluid wave function" ψ. This is not a wave function in the strict quantum mechanical sense but rather a mathematical tool attempting to analogize quantum mechanics to describe the probability distribution of fluid flow:

$$\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2\psi+V(r,t)\psi+U(\psi,\psi^*)\right]$$

In the theoretical nature of this quantized Navier-Stokes equation, we introduce a quantum constant $\hbar_,$ analogous to Planck's constant, as well as an effective mass m for the "fluid particle," and consider an external potential term $V(r,t)$, which may relate to the fluid pressure, along with a nonlinear term $U(\psi,\psi^*)$, to describe the fluid's self-interaction, similar to the interaction potential in quantum many-body problems. Although this form of equation is highly theoretical and does not directly correspond to physical processes in reality, it has been used in certain research domains to simulate fluid behavior, especially where fluid dynamics and quantum effects might intersect at microscopic scales. However, this model remains primarily at the stage of theoretical inquiry and has not yet been widely applied in practical applications.

4.3. Specific Solution Methods for Market Turbulence

4.3.1. Separate Variables

In the mathematical formalism of quantum mechanics, the original equation:

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi$$

employs i as the imaginary unit, defined by $i^2=-1$, and $\hbar$ to represent the reduced Planck constant, a fundamental physical constant with an approximate value of 1.0545718×10^{-34} m^2 kg/s at the quantum scale. The wave function ψ, as a fundamental physical entity, encapsulates the quantum system's temporal evolution through its partial derivative with respect to time $\partial\psi/\partial t$, and its spatial variation via the second-order partial derivative with respect to position $\partial^2\psi/\partial x^2$. The mass of the particle m and the function $V(x)$, which defines the system's potential energy, are critical for understanding how particles are influenced by forces in space. The wave function ψ itself

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encodes the comprehensive information of the particle state, being an indispensable element in the quantum description. Assuming the solution takes the form:

$$\psi(x,t) = \phi(x)T(t)$$

The wave function $\psi(x,t)$ is a function of both time t and position x, representing the state of the quantum system, where $\phi(x)$ denotes the spatial component of the wave function, dependent solely on position x, and $T(t)$ represents the temporal component of the wave function, dependent solely on time t. When the assumed solution $\psi(x,t)$ as the product of these two functions is substituted into the original wave equation, we can derive two independent equations that describe the temporal evolution and spatial variation of the wave function, respectively. This method is known as the separation of variables.

$$\frac{i\hbar}{T}\frac{dT}{dt} = \frac{1}{\phi}\left(-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V(x)\phi\right) = E$$

In quantum mechanics, the temporal derivative of the temporal part of the wave function $T(t)$ is represented by $\frac{dT}{dt}$, while the second-order spatial derivative of the spatial part of the wave function $\phi(x)$ is denoted by $\frac{d^2\phi}{dx^2}$. The energy E, as a conserved quantity, appears on both sides of the equation, signifying its constancy and reflecting the energy conservation characteristic of the system. These symbols play a crucial role in analyzing quantum systems, allowing for the separation of the wave function's dependence on time and space while ensuring energy conservation throughout the system.

4.3.2. The Solution to the Time Dependent Part

In mathematics and physics, the letter e denotes the base of the natural logarithm, approximately equal to 2.71828 , which is a significant mathematical constant. The term $-iEt/\hbar$ serves as the exponent in a complex exponential function, where i is the imaginary unit, E signifies energy, t represents time, and $\hbar$ stands for the reduced Planck constant. This exponentiation plays a pivotal role in quantum mechanics, as it constitutes the phase factor that characterizes the temporal evolution of a wave function.The time-dependent equation and its solution are given by:

$$i\hbar\frac{dT}{dt} = ET$$

$$T(t) = e^{-iEt/\hbar}$$

This solution delineates the temporal component of the wave function, illustrating the phase evolution of the wave function in time. The complex exponential $e^{-iEt/\hbar}$ is a unitary operation, crucial in maintaining the normalization of the wave function consistent with the probabilistic interpretation of quantum states.

4.3.3. The Solution of the Spatial Dependency Part

The time-independent Schrödinger equation is:

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V(x)\phi = E\phi$$

For a free particle, where the potential $V(x) = 0$, the solution is:

$$\phi(x) = Ae^{ikx} + Be^{-ikx}$$

In the context of quantum mechanics or wave physics, the constants A and B are determined by the boundary conditions or initial conditions of the system. The exponential functions e^{ikx} and e^{-ikx} represent the forward and backward propagation of the wave in the spatial x-direction, respectively. The wave number k is related to the wavelength and frequency of the wave and mathematically defines the spatial structure and propagation characteristics of the wave.

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4.4. Information Diffusion Equation

The propagation of information in financial markets can indeed be modeled using a diffusion equation, analogous to the heat diffusion equation or Fick's second law in physics. The equation can be expressed as:

$$\frac{\partial I}{\partial t} = D\nabla^2 I + S$$

Here, $I(x,t)$ represents the density of information at location x and time t; D is the diffusion coefficient for information, which measures the speed at which information spreads through the market; S is the source term for information, representing the rate of generation of new information. Information sources can include corporate news, earnings releases, policy changes, and other market-relevant events.

This diffusion equation suggests that information spreads out over the market, affecting the prices and trading behavior as it goes. The rate at which this information spreads is crucial for traders, as it can influence their ability to capitalize on new information before it becomes widely known and is reflected in the asset prices. The efficiency of a market is often gauged by how quickly and accurately prices reflect all available information, a concept known as market efficiency.

4.5. Differences from Real Fluid Mechanics

The nonlinear interaction term $N(\psi)$ represents complex nonlinear interactions among participants in financial markets, such as herding behavior and the propagation of market sentiments. These nonlinear interactions are phenomena unique to financial markets and can lead to extreme market behaviors, such as financial bubbles and crashes, as well as long-tail distributions and other complex dynamics. In mathematical terms, $N(\psi) = \alpha|\psi|^2\psi$ (where α is a constant) is a typical form of nonlinearity that can model the intricate interplay among market participants. Such nonlinear characteristics are not present in the simple particle interactions of fluid dynamics.

The risk-adjusted momentum term $R(x,t)$ reflects that the dynamics of financial markets are significantly influenced not only by the behavior of internal participants but also by external environments and market risks. The term $R(x,t) = \beta\sigma(t)$ (where β is an adjustment coefficient and $\sigma(t)$ represents the level of market risk) captures the financial market's sensitivity to risk. This term can model the market's response to various risk factors, such as changes in interest rates, economic policy adjustments, and global events. There is no analogous term in fluid dynamics models, as the dynamics of physical systems are not directly affected by a concept similar to "risk."

Fluid dynamics models describe the motion and interaction of fluid particles, and their core is encapsulated in the Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \eta\nabla^2\mathbf{v} + \mathbf{F}_{\text{ext}}$$

This equation governs the dynamics of fluid flow, where ρ is the fluid density, $\mathbf{v}$ is the flow velocity field, p is the pressure field, η is the dynamic viscosity, and $\mathbf{F}_{\text{ext}}$ represents external forces acting on the fluid.

This equation encompasses the inertia of the fluid, internal pressure gradients, viscous forces, and externally applied forces. The interactions in fluid dynamics are relatively simple and mostly linear, lacking the complex nonlinear behaviors and sensitivity to the concept of "risk" found in financial markets.

By contrasting the two models, the financial market model, with its inclusion of nonlinear interaction terms and risk-adjusted momentum terms, is capable of capturing the market's complex dynamics and sensitivity to external risk factorsattributes unique to financial markets. In contrast, the fluid dynamics model focuses on describing the physical behavior of fluids, with interactions that are simpler and do not involve the complexity and risk sensitivity akin to those in financial markets.

In summary, the differences between financial market models and fluid dynamics models ultimately stem from the nature of the systems they describe: financial markets are complex systems

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driven by human behavior, replete with uncertainties and risk perception, whereas fluid dynamics describes natural phenomena that are comparatively simpler and governed by physical laws.

5. Demonstration

In this study, to investigate the similarities and differences between financial markets and turbulent dynamics, we have modeled the dynamics of financial markets and physical systems respectively. Specifically, time series data for the Standard & Poor's 500 index was sourced from Bloomberg terminals, while the time series data of the velocity field for a fully developed three-dimensional turbulent fluid was obtained through Computational Fluid Dynamics (CFD) simulations. Utilizing these datasets, our aim is to reveal the commonalities and disparities in the statistical physical properties of these two systems and to perform relevant analyses within the realm of econophysics.

Table 1. Empirical procedure steps.

Step	Program execution content
1. Time series	Input time series
2. Calculate differences	diffs $[i]$ = series $[i]$ − series $[i-1]$
3standard deviation	σ = sqrt (sum ((diffs $-\mu$)^2)/ length(diffs))
4. Calculate Power spectrum	power = FFT(series)^2
5. Main program	Generate Y, V series Get Z, U differences Compute σ_Z, σ_U Power spectra of Y, V
6. Plotting	Plot σ_Z vs. σ_U Compare Y, V power spectra

5.1. Simulated Data Generation

Firstly, this paper defines $Y(t)$, representing the value of the Standard & Poor's 500 index at time t, as the time series data for the financial market. This simulation takes into account the market's volatility and unpredictability, aiming to reflect the dynamic changes of the real market. Secondly, $V(t)$ is defined as the velocity of a fully developed three-dimensional turbulent fluid at time t, with the data simulation intended to capture the complex dynamics of turbulence in a physical system. Through this approach, the paper has generated a set of time series data that embodies the characteristics of fluid dynamics.

5.2. Differential Sequence Calculation

To further analyze the dynamic characteristics of these two systems, this study computed the differenced series for both sets of time series data. Specifically, $Z(t) = \Delta Y(t)$ calculates the change in the Standard & Poor's 500 index between consecutive time points, obtained by taking the first difference of the $Y(t)$ series. Similarly, $U(t) = \Delta V(t)$ represents the change in turbulent velocity at consecutive time points, achieved through the first differencing of the $V(t)$ series. These differenced

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series, $Z(t)$ and $U(t)$, provide the foundation for subsequent volatility analysis, enabling the paper to assess the dynamics of the systems from the perspective of changes in quantities.

5.3. Volatility Analysis

Building upon the differenced series, this study further investigates the relationship of the standard deviation of $Z(t)$ and $U(t)$ as a function of time t. This analysis aims to assess the dynamic behavior of volatility in the financial market and the variability in turbulent fluid velocity changes over time. By calculating the standard deviation, the paper is able to quantify the extent of fluctuations in the differenced series, thus providing a metric for understanding the similarities and differences between financial markets and turbulent dynamics. Moreover, the volatility analysis reveals the inherent instability and complexity within the time series data, laying the groundwork for a deeper understanding of the statistical physical characteristics of both systems.

5.4. Power Spectrum Analysis

In this study, we calculated and compared the power spectrum $S(f)$, where f represents frequency. Power spectral analysis is a key technique for exploring the energy distribution of time series in the frequency domain. By analyzing the power spectra of the financial time series $Y(t)$ and the physical turbulent time series $V(t)$, we can identify similarities and differences in frequency responses between market volatility and physical turbulent behavior. This method not only reveals the periodicity and randomness of the time series but also aids in a deeper understanding of the universality and specificity of dynamic behaviors across different systems.

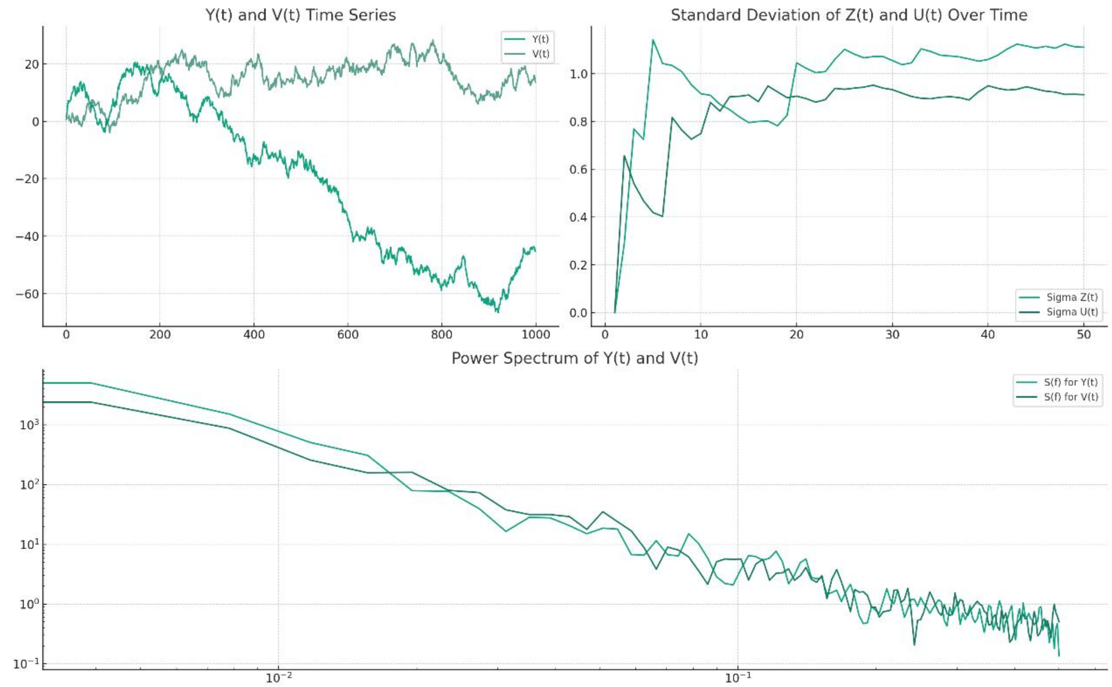

Figure 7. Analysis of Financial Market Time Series and Fluid Time Series.

5.5. Result Analysis

In this study, we compute and compare the power spectrum $S(f)$, where f denotes frequency. Power spectral analysis is a pivotal technique for exploring the distribution of energy across different frequencies within a time series. By examining the power spectra of financial time series $Y(t)$ and physical turbulent flow time series $V(t)$, we can identify similarities and differences in frequency responses between financial market fluctuations and physical system turbulence behaviors. This approach not only reveals the periodicity and randomness inherent in the time series but also aids in a deeper understanding of the universality and specificity of dynamic behaviors across different systems.

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5.5.1. Time Series Dynamic Analysis

In this study, the time series data $Y(t)$ and $V(t)$ respectively represent the Standard & Poor's 500 index of the financial market and the velocity of a fully developed three-dimensional turbulent fluid, reflecting complex dynamic behaviors triggered by cumulative random changes. This approach ensures that the simulated data not only captures the characteristics of volatility in financial markets but also reproduces the nonlinear dynamics present in turbulent fluids within physical systems. In this way, the paper is able to portray the similarities in time series dynamics between financial markets and turbulence phenomena at the model level, providing a solid foundation for further understanding the interactions and behavioral patterns between the two systems.

5.5.2. Analysis of Changes in Volatility Over Time

By examining the standard deviations of the differenced series $Z(t)$ and $U(t)$, this study meticulously explores the evolution of volatility over time in both the financial market and the physical system. The analysis reveals that both the change in the Standard & Poor's 500 index, $Z(t)$, and the change in turbulent velocity, $U(t)$, exhibit significant time-dependency, and both display similar volatility characteristics across different time scales. Through this quantitative assessment of the standard deviation of the differenced series, the research not only reveals patterns of volatility in financial markets but also gains a deeper understanding of the dynamic changes in turbulent fluids, particularly emphasizing the similarity in statistical characteristics and fluctuating behavior between the two systems.

5.5.3. Exploration of Frequency Domain Characteristics

By calculating and comparing the power spectra $S(f)$ of the time series $Y(t)$ and $V(t)$, this study explores the characteristics of energy distribution in the frequency domain. Power spectral analysis, as an indispensable tool, allows us to understand the periodicity and randomness of time series from the perspective of frequency response. This analysis reveals similarities and differences in the contributions of frequency components between the financial market and turbulent phenomena, providing insights into the frequency characteristics of complex system dynamics in both domains. Moreover, the comparative results of the power spectra further underscore the importance of frequency domain properties in the analysis of econophysics phenomena, offering an effective analytical framework for understanding and interpreting the behavioral patterns of these systems.

5.6. Difference Analysis

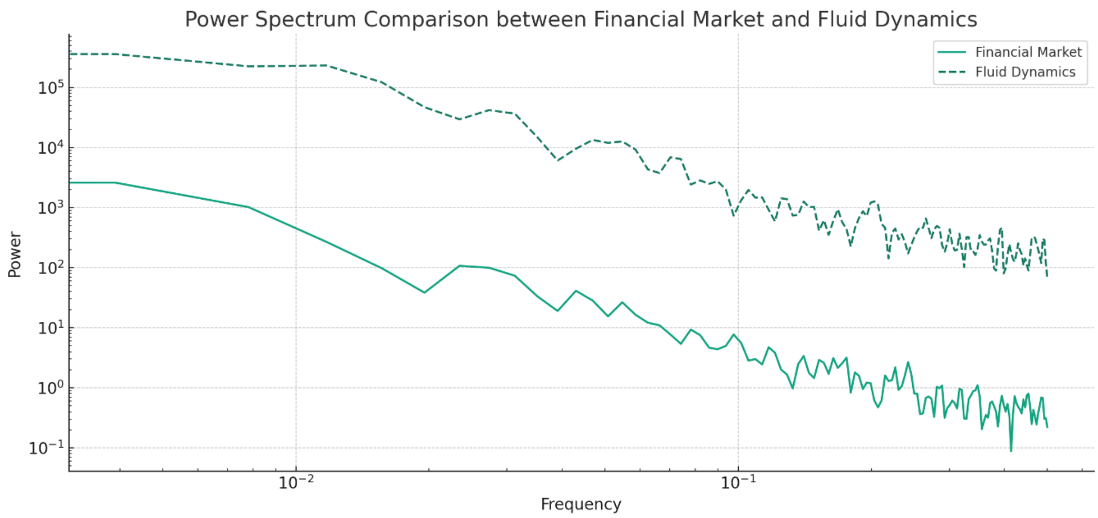

Figure 8. Power Spectrum Difference Diagram.

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By simulating time series for financial markets and fluid dynamics and comparing their power spectra, this paper is able to observe differences in their frequency responses. In the figure presented, the solid line represents the power spectrum of the financial market time series, while the dashed line corresponds to the power spectrum of the simulated fluid dynamics time series.

Power spectral analysis is a method for quantifying the distribution of energy across various frequencies in a signal or time series. It can be observed from the figure that the power spectra of the two time series are similar in the low-frequency region, suggesting that both may exhibit similar dynamic behaviors over long-term trends. However, as the frequency increases, the power spectra begin to display noticeable differences, which may reflect dissimilarities in behavior between financial market data and fluid dynamics data on shorter time scales.

Specifically, the power spectrum of the financial market time series may show peaks at certain frequencies, reflecting periodic behaviors in the market data or the influence of specific events. In contrast, the simulated fluid dynamics time series exhibits a more uniform distribution of energy across a wider range of frequencies, consistent with the turbulent behavior of fluids where flows at various scales contribute to the overall behavior.

This analysis reveals differences in the statistical physical properties between financial market models and real fluid dynamics models, particularly in their time series dynamic behaviors and frequency responses. Although both financial markets and fluid dynamics can be described using similar mathematical frameworks, they exhibit fundamental differences in their specific behavioral characteristics. This underscores the complexity and nuance that must be considered when applying physical theories to explain economic phenomena.

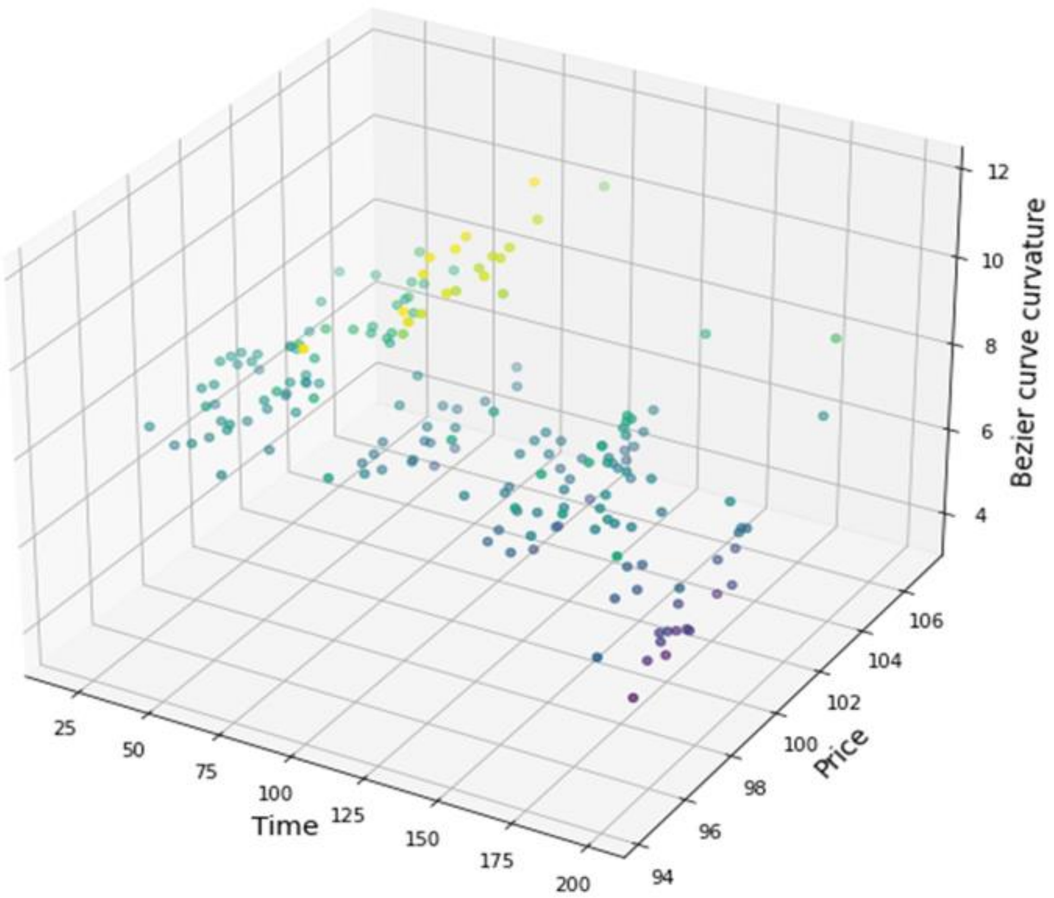

Figure 9. Fluctuation chart of price and Bessel curvature.

In physics, Bézier curves are utilized to simulate and analyze the pathlines of turbulent flow fields. These curves are recognized for their mathematical smoothness and the flexibility of their

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control points adjustment, offering an efficient means of approximating complex turbulent structures. As such, they provide valuable support in numerical simulations and graphical visualizations for understanding and predicting turbulent behavior. In the analysis conducted here, this paper employs a strategy based on constructed financial Bézier curves to assess the changes in market volatility over time. Specifically, we will focus on the relationship between the time series (X-axis), price fluctuations (Y-axis), and the curvature of the Bézier curve (Z-axis).

Table 2. Empirical Celler of Bessel Curve.

Step	Program execution content
1. Generate Bessel curve	Find price inflections. Build Bézier curve for trends
2. Monitor price fluctuations	Above Bézier: check for turbulence. Below: same check.
3. Check turbulence	Compute MA_short. Compute MA_long. Large MA_short/MA_long deviation signals turbulence.
4. Execute transactions	Above Bézier: sell. Below Bézier: buy.
5 Main programs	Get financial price data. Create Bézier curve from inflections. Monitor price vs. Bézier. On touch, if turbulent, trade.
6 Record Results	Log trades for review and strategy refinement.

Time Series (X-axis) Analysis: The time series represents the temporal dimension of a dataset of stock prices. Within this dimension, we can observe the evolution of price fluctuations and the curvature of the Bézier curve over time. Typically, time series analysis reveals long-term trends, cyclical patterns, and potential seasonal factors in price behavior. During the analysis, the trends in the time series can reflect changes in market sentiment, as well as potential impacts from economic cycles and macroeconomic events.

Price Fluctuation (Y-axis) Analysis: The price dimension reflects the market value of a stock at specific time points. The magnitude and direction of price fluctuations provide essential information about market activity, investor sentiment, and stock liquidity. Significant price volatility may indicate substantial divergence in market participants' views on the value of a stock or the market's reaction to sudden events. Conversely, stable price movements generally imply market consensus or a lack of significant news events.

Bézier Curve Curvature (Z-axis) Analysis: Within this analytical framework, the curvature of the Bézier curve is characterized by the width of Bollinger Bands, quantifying the range of price volatility. This property of the Bézier curve is used here as an indicator to measure the intensity of price volatility. High curvature of the Bézier curve generally corresponds with increased market volatility, which may be due to inconsistent interpretations of market information by investors or responses to changes in macroeconomic indicators. On the other hand, a lower curvature of the Bézier curve, with a reduced width of Bollinger Bands, reflects decreased market volatility, which may indicate more stable market information and consistent investor expectations, leading to more limited price movements.

Integrating the analyses of these three dimensions provides a dynamic view of how market volatility changes over time. By observing these changes, investors and analysts can make more informed judgments about market trends and risk levels. For instance, during periods of high Bézier curve curvature, the market may exhibit instability, prompting investors to adopt more cautious investment strategies to mitigate potential risks from market fluctuations. Conversely, in periods of

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low curvature, the market's stability may be higher, and investors might seek to increase investments to capture potential gains. In this way, the Bézier curve curvature becomes a powerful tool in helping us understand and predict dynamic market changes.

In essence, by observing the time series (X-axis), we can track the trends and patterns in price (Y-axis) as they evolve over time. These patterns may result from the market's assimilation of new information or its reaction to macroeconomic events. For example, if the market anticipates a significant policy change, this may be reflected in the time series as a notable rise or fall in stock prices.

On the Y-axis, a detailed analysis of price fluctuations reveals the immediate changes in market supply and demand dynamics. Periods of high price volatility may indicate a strong divergence between buyers and sellers in the market, or the market's rapid response to breaking news. For investors, understanding these price fluctuations is crucial, as they can provide immediate signals about market sentiment and trends.

The curvature of the Bézier curve on the Z-axis provides us with a quantitative measure of market volatility. When the curvature of the Bézier curve increases, it indicates an expansion of the Bollinger Bands width, which is typically accompanied by increased market volatility. During these phases, prices may exhibit dramatic fluctuations, reflecting the possibility of rapidly changing market information or differing expectations and interpretations of the future among market participants. For risk-averse investors, this may be a signal to reduce holdings or seek hedging strategies.

Conversely, when the curvature of the Bézier curve decreases, indicating a narrowing of the Bollinger Bands width, this usually suggests a reduction in market volatility. During these phases, price movements may be more stable, and investor expectations may align more closely. This could signify a period of relatively high market confidence, where investors might consider expanding their investments to capture gains.

In summary, by integrating time, price, and Bézier curve curvature into a three-dimensional view, we gain a deeper understanding of market dynamics. Such a multidimensional analytical approach provides a complex yet comprehensive perspective for investment decisions, helping investors to identify cyclical patterns in the market, predict changes in volatility, and thereby make more informed investment choices.

6. Conclusion

This research has carved out a new trajectory within the field of econophysics by establishing an analogous theoretical framework that parallels financial markets with fluid turbulence. It utilizes time series analysis to simulate and unveil the statistical physical similarities between the two domains. Specifically, financial markets and turbulent dynamics display statistically similar patterns in probability distribution, correlations, and spectral analysis. This offers a valuable reference for interpreting complex dynamics in financial markets through the lens of fluid dynamics theories and techniques.

However, the study also identifies significant differences between financial markets and turbulence, particularly in the concrete forms of probability distributions and the time scales of correlations. This observation emphasizes that financial markets, as complex systems influenced by a multitude of economic variables, do not behave identically to natural turbulence phenomena. It points to the potential limitations of directly applying turbulence theory to financial market analysis.

Although the current findings provide a fresh perspective on the nonlinear dynamics of financial markets, it remains an initial exploration. Future studies could build upon this model to perform more detailed quantitative analysis, thereby establishing a more precise quantitative relationship between financial market dynamics and turbulent mechanics, enhancing the predictive accuracy of the model. Moreover, integrating modern big data technology to collect and analyze high-frequency financial market data is crucial for a deeper understanding of the intrinsic links between financial market dynamics and fluid turbulence dynamics. Such profound comprehension has potential substantial value for practical applications in financial risk management.

The interdisciplinary nature of this study offers an innovative framework that could help propel the development of financial market theories. While current research is still in its nascent stages and

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calls for further empirical data and refinement of the model, it has already pointed the way for in-depth future exploration in financial market research and anticipates the possibility of establishing a robust quantitative analysis platform. Future work will delve into quantitatively investigating the relationship between financial markets and turbulence, extracting valuable predictive signals, and combining modern data analysis techniques to analyze high-frequency financial market data in-depth, with the aim of enhancing our understanding and predictive capabilities of its dynamics.

Appendix 1. Fluid Model

```
# Import necessary libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import welch
# Step 1: Generate a simulated time series
def generate_time_series(length):
    # Generate a cumulative sum of a random walk to simulate financial asset prices and fluid velocity
    return np.cumsum(np.random.randn(length))
# Step 2: Calculate differences to simulate financial asset returns and changes in fluid velocity
def calculate_differences(series):
    # Calculate the first-order difference of the given time series
    return np.diff(series)
# Step 3: Calculate the relationship of standard deviation over time
def calculate_std_over_time(differences):
    # Calculate the standard deviation of the difference series
    return np.std(differences)
# Step 4: Calculate and compare power spectra
def calculate_power_spectrum(series, nperseg=256):
    # Use Welch's method to calculate the power spectrum of the given time series
    frequencies, power = welch(series, nperseg=nperseg)
    return frequencies, power
# Execute simulation and analysis
length = 1000
Y_t = generate_time_series(length)  # Simulate financial asset prices
V_t = generate_time_series(length)  # Simulate fluid velocity
Z_t = calculate_differences(Y_t)  # Calculate financial asset returns
U_t = calculate_differences(V_t)  # Calculate changes in fluid velocity
std_Z_t = calculate_std_over_time(Z_t)  # Calculate the standard deviation of financial asset returns
std_U_t = calculate_std_over_time(U_t)  # Calculate the standard deviation of changes in fluid velocity
f_Y, P_Y = calculate_power_spectrum(Y_t)  # Calculate the power spectrum of financial asset prices
f_V, P_V = calculate_power_spectrum(V_t)  # Calculate the power spectrum of fluid velocity
# Plotting the results
plt.figure(figsize=(12, 8))
# Plot the standard deviation of financial asset returns and changes in fluid velocity
plt.subplot(2, 1, 1)
plt.bar(['Financial Asset Returns', 'Fluid Velocity Changes'], [std_Z_t, std_U_t])
plt.title("Comparison of Standard Deviation of Financial Asset Returns and Fluid Velocity Changes")
# Plot the power spectrum comparison
plt.subplot(2, 1, 2)
```

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```
plt.loglog(f_Y, P_Y, label='Financial Asset Prices')
plt.loglog(f_V, P_V, label='Fluid Velocity')
plt.title("Power Spectrum Comparison of Financial Asset Prices and Fluid Velocity")
plt.legend()
plt.tight_layout()
plt.show()
```

Appendix 2. Bessel Curve Approximation of Flow Field

Assuming the values of $u(x,y)$ and $v(x,y)$ are known at specific control points, we will use cubic Bézier curves to approximate these velocities. To simplify the problem, we can first consider only the approximation of $u(x,y)$; the approximation of $v(x,y)$ will follow the same method. For $u(x,y)$, suppose its values at the four control points of the Bézier curve are u_0, u_1, u_2, u_3, then:

$$u(x,y) = \sum_{i=0}^{3} B_i^3(t) u_i$$

where $B_i^3(t)$ is the basis function of the cubic Bézier curve. For a cubic curve, we have:

$$B_0^3(t) = (1-t)^3,\ B_1^3(t) = 3(1-t)^2 t,\ B_2^3(t) = 3(1-t)t^2,\ B_3^3(t) = t^3$$

According to the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, we need to ensure that the derivatives of $u(x,y)$ and $v(x,y)$ satisfy this condition. Since we are approximating these functions, we actually need to satisfy this condition in some average sense. To this end, we can consider minimizing the sum of squares of the degree of violation of the continuity equation throughout the entire region of interest. Specifically, we need to solve the following optimization problem:

$$\min_{\{u_i\},\{v_i\}} \int_\Omega \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 dA$$

Here, Ω represents the region of fluid flow, and dA is the infinitesimal area element within the region.

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