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What Would a Mathematically Precise Definition of Unified Field Theory Even Look Like if We Tried to Axiomatize It?

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Abstract
The term unified field theory is widely used but rarely given a precise, model-independent definition. In this letter I propose a minimal axiomatic framework for what should count as a unified field theory in the sense of relativistic quantum field theory with gravity. The axioms package spacetime, internal gauge structure, matter content, dynamics, and low-energy limits into a single geometric datum, and they are designed to be compatible with existing no-go theorems such as Coleman–Mandula and Haag–Łopuszański–Sohnius. I briefly discuss how such an axiomatization could support genuine classification or uniqueness theorems for unified models.

Keywords: unified field theory

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1. Introduction

The phrase unified field theory is used to describe a wide range of theoretical frameworks, from grand unified gauge theories to string theory and various geometrizations of the Standard Model and gravity. However, the field lacks a common axiomatic definition of what unification should mean. In particular, there is no generally accepted theorem of the form: "Any unified field theory must have structure X."

At the same time, we do have powerful structural and no-go theorems that constrain how spacetime symmetries, internal gauge symmetries, and quantum field theory can fit together. The Coleman–Mandula theorem and its supersymmetric extension by Haag, Łopuszański and Sohnius show that under broad assumptions, continuous spacetime and internal symmetries factor in a restricted way [1,2]. Uniqueness arguments for interacting massless spin-1 and spin-2 fields constrain gauge theory and general relativity as the low-energy manifestations of such symmetries [3–5]. The Weinberg–Witten theorem further restricts emergent descriptions of massless spin-1 and spin-2 particles in Lorentz-invariant quantum field theory [6]. General frameworks such as the Wightman axioms and algebraic quantum field theory provide rigorous formulations of relativistic QFT [7,8].

The aim of this letter is modest, I will propose an axiomatic package that makes precise what it means, structurally, to have a unified field theory[1]. The key idea is to replace the vague notion of putting gravity and the Standard Model together with a single geometric object and a single symmetry group from which the familiar fields and interactions are derived as emergent structures in an appropriate low-energy limit.

2. Spacetime and Configuration Data

Any relativistic field theory must in my opinion, begin by specifying the arena on which fields live. In general relativity this role is played by a four-dimensional Lorentzian manifold, and any putative

[1] The author has worked on specific unified-field frameworks, including holomorphic unified field theory (HUFT) [10–17]. The present note is intended to be framework-agnostic, so all structural assumptions are stated explicitly so that readers can check whether their preferred models satisfy, violate, or refine these axioms.

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unified field theory that aims to describe our universe must at least reproduce this structure for the sector of the theory in which measurements are made. At the same time, many concrete unification schemes suggest enlarging the geometric setting, for example by complexifying spacetime, adding extra dimensions, or introducing auxiliary directions that organize contours or regulators.

I therefore propose to first single out the physical spacetime and then allow for a larger ambient space in which it sits as a distinguished submanifold leading to:

Axiom (U1). *There exists a four-dimensional, oriented, time-oriented Lorentzian spacetime*

$$(M, h_{\mu\nu}). \tag{1}$$

with M the smooth four-dimensional manifold and $h_{\mu\nu}$ the Lorentzian metric tensor on M, meaning that physical spacetime is modeled as a $3+1$-dimensional Lorentzian geometry.

To allow for ambient or extended constructions, such as complexified manifolds, extra dimensions, or auxiliary directions, I postulate an extended manifold:

$$\mathcal{M}, \tag{2}$$

with $\mathcal{M}$ the extended ambient manifold, meaning that the fundamental theory is defined on a larger geometric space that contains physical spacetime as a distinguished submanifold. There should be an embedding:

$$\iota : M \hookrightarrow \mathcal{M}. \tag{3}$$

with ι the smooth embedding map from M into $\mathcal{M}$, meaning that physical spacetime is realized as a concrete submanifold of the ambient space.

In models with complexified spacetime one may take $\mathcal{M} = M_{\mathbb{C}}$ and interpret M as a real submanifold of a complex manifold, but the axioms themselves do not depend on this choice.

Having specified the spacetime, the next question I believe one should ask is what counts as a field configuration of the unified theory? In conventional formulations one writes down separate bundles and fields for the metric, gauge connections, and matter multiplets. From the present axiomatic point of view, this is already too fragmented as in my opinion a genuinely unified field theory should package all dynamical degrees of freedom into a single geometric object living over spacetime, leading to our second axiom:

Axiom (U2). *There exists a single configuration bundle*

$$\pi : E \to M, \tag{4}$$

with E the total space of the configuration bundle and π the projection onto the spacetime manifold M, meaning that all dynamical fields of the theory are encoded as sections of a single bundle over spacetime.

A configuration of the theory is given by a section:

$$\Phi \in \Gamma(E), \tag{5}$$

with Φ the total field configuration and $\Gamma(E)$ the space of smooth sections of E, meaning that one unified field encodes metric, gauge, and matter degrees of freedom as components or derived structures.

The familiar fields of low-energy physics, such as the spacetime metric $h_{\mu\nu}$, gauge potentials A^a_μ, and matter fields ψ, are assumed to be obtained from Φ by fixed projection maps or local functionals. For example, one may have a decomposition:

$$\Phi \mapsto (h_{\mu\nu}, A^a_\mu, \psi, \dots), \tag{6}$$

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with Φ the unified field, $h_{\mu\nu}$ the emergent metric, A_μ^a the emergent gauge fields, and ψ the emergent matter fields, meaning that the standard field content arises as derived components of a single underlying geometric field.

3. Unified Symmetry and Geometric Unification

If all fields of the theory are packaged into a single configuration bundle, I think that it is natural to demand that they also transform under a single symmetry structure. In ordinary relativistic quantum field theory one distinguishes between spacetime symmetries either Poincaré or diffeomorphism invariances and internal gauge symmetries, and powerful no-go theorems constrain how these can be combined [1,2]. In an axiomatic treatment of unification we should not try to evade these results at the outset but instead, we demand that whatever mixing is allowed by consistency be realized as part of a single unified symmetry group acting on the unified field. This leads us to our third axiom:

Axiom (U3). *There exists a unified symmetry group*

$$\mathcal{G}_{\text{unif}}, \tag{7}$$

with $\mathcal{G}_{\text{unif}}$ the unified symmetry group of the theory, meaning that both spacetime symmetries and internal gauge symmetries are realized as subgroups or subalgebras of a single overarching symmetry structure.

The group $\mathcal{G}_{\text{unif}}$ acts on spacetime via a homomorphism:

$$\rho : \mathcal{G}_{\text{unif}} \to \text{Diff}(M), \tag{8}$$

with ρ the group homomorphism and $\text{Diff}(M)$ the diffeomorphism group of M, meaning that each unified symmetry element induces a spacetime diffeomorphism.

This action lifts to the bundle E as bundle automorphisms compatible with ρ, so that the unified group acts on fields and their configurations in a way consistent with their localization over spacetime.

I would require that the usual spacetime and internal gauge symmetries embed as subgroups:

$$\text{Diff}(M) \times G_{\text{SM}} \subset \mathcal{G}_{\text{unif}}, \tag{9}$$

with $\text{Diff}(M)$ the diffeomorphism group, G_{SM} the Standard Model gauge group, and $\mathcal{G}_{\text{unif}}$ the unified symmetry group, meaning that both gravitational and Standard Model gauge symmetries arise from restrictions of a single unified symmetry structure.

The axioms do not require a direct product structure, in particular, graded extensions such as supersymmetry or more intricate groupoid-like structures are allowed, provided they act consistently on E and reduce to the observed symmetries in the low-energy limit.

Unification at the level of configuration space and symmetry is still compatible with writing separate Lagrangians for gravity, gauge fields, and matter. The stronger, Einsteinian notion of a unified field theory asks for more, that all dynamical content should arise from a single geometric object, defined on the ambient space and restricted to the physical spacetime. In this sense, metric, connections, and matter multiplets become different faces of one underlying field, rather than independent ingredients glued together by hand.

Axiom (U4). *There exists a single geometric object*

$$\mathcal{F}, \tag{10}$$

with $\mathcal{F}$ the unified geometric field, meaning that all of the gravitational, gauge, and matter fields of the low-energy theory are encoded in one geometric quantity defined on $\mathcal{M}$ or on M.

In many constructions $\mathcal{F}$ is a connection on a principal bundle with structure group:

$$G_{\text{unif}}, \tag{11}$$

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with G_{unif} the unified structure group of a principal bundle, meaning that the unified field can be interpreted as a gauge connection whose curvature encodes both gravitational and internal gauge field strengths.

In other constructions, $\mathcal{F}$ may be a generalized metric, a Hermitian form, or some higher-rank tensor, but the axiomatic requirement is that there is a fixed, representation-independent rule assigning to $\mathcal{F}$ the emergent low-energy fields $h_{\mu\nu}$, A^a_μ, and ψ via the embedding ι and the projection from E:

$$(h_{\mu\nu}, A^a_\mu, \psi, \dots) = \mathcal{D}[\mathcal{F}, \iota]. \tag{12}$$

with $\mathcal{D}$ the decoding map from the unified field and embedding to physical fields, meaning that the familiar dynamical variables are determined functorially from the single geometric object $\mathcal{F}$ once the physical spacetime M is identified inside $\mathcal{M}$.

4. Dynamics and Quantum Consistency

The preceding axioms specify the kinematical data of a unified field theory, that the spacetime arena, the configuration bundle, the symmetry group, and the underlying geometric field. To turn this kinematics into dynamics one needs a variational principle. In relativistic field theory, from classical general relativity to the Standard Model, this is encoded in an action functional whose stationary points define the equations of motion and whose symmetries control conservation laws. In the unified setting I believe we require that the entire dynamics, including gravity, gauge interactions, and matter couplings, descend from a single action built from the unified field, giving our fifth axiom:

Axiom (U5). *The unified field theory admits an action principle. There exists an action functional*

$$S[\Phi] = \int_M \mathcal{L}(\Phi, \nabla\Phi, \dots)\mathrm{d}^4x, \tag{13}$$

with $S[\Phi]$ the action evaluated on the configuration Φ, $\mathcal{L}$ the Lagrangian density depending on Φ and its derivatives, and d^4x the spacetime volume element on M, meaning that the classical equations of motion follow from the stationary action principle $\delta S = 0$.

The action is invariant under the unified symmetry group:

$$S[g \cdot \Phi] = S[\Phi] \quad \forall g \in \mathcal{G}_{\text{unif}}, \tag{14}$$

with g the unified symmetry element and $g \cdot \Phi$ the transformed field configuration, meaning that the dynamics respects the full unified symmetry, not just the low-energy subgroups.

We also require that in suitable regimes and truncations the action reduces to the familiar Einstein–Hilbert and Yang–Mills forms. Concretely, there is a limit in which

$$S[\Phi] \longrightarrow S_{\text{EH}}[h] + S_{\text{YM}}[A] + S_{\text{matter}}[h, A, \psi] + \dots \tag{15}$$

with $S_{\text{EH}}[h]$ the Einstein–Hilbert action for the metric $h_{\mu\nu}$, $S_{\text{YM}}[A]$ the Yang–Mills action for the gauge fields A^a_μ, and S_{matter} the matter action for the fields ψ, meaning that general relativity and the Standard Model appear as the low-energy effective theory of the unified dynamics, up to higher-dimensional operators.

So far the axioms I have presented were been purely classical as they specify a geometric arena, a unified field, a symmetry group, and an action functional. But I believe that any realistic unified field theory must, however, admit a consistent quantum realization. In particular, it should fit into the broad framework of relativistic quantum theory, with states forming a Hilbert space, observables acting as operators, and symmetries represented unitarily. The following axiom abstracts these requirements without committing to a specific quantization scheme:

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Axiom (U6). *The unified field theory admits a quantum realization. There exists a Hilbert space*

$$\mathcal{H}, \tag{16}$$

with $\mathcal{H}$ the Hilbert space of physical states, meaning that quantum states of the unified theory are elements of a complete inner-product space.

There is an algebra of observables

$$\mathcal{A}, \tag{17}$$

with $\mathcal{A}$ the algebra of possibly local quantum observables, meaning that physical quantities are represented as operators acting on $\mathcal{H}$.

Time evolution is implemented by a unitary operator

$$U(t) = e^{-iHt}, \tag{18}$$

with $U(t)$ the time-evolution operator, H the Hamiltonian of the theory, and t the time parameter, meaning that probability is conserved and the dynamics is unitary in the quantum theory.

The unified symmetry group acts unitarily on $\mathcal{H}$:

$$U(g) : \mathcal{H} \to \mathcal{H} \quad \text{for} \quad g \in \mathcal{G}_{\text{unif}}, \tag{19}$$

with $U(g)$ the unitary operator representing g on states and $\mathcal{H}$ the Hilbert space, meaning that the unified symmetries are realized as unitary transformations in the quantum theory.

We further require that all gauge and gravitational anomalies cancel in the full unified theory, so that the quantum realization of $\mathcal{G}_{\text{unif}}$ is consistent.

5. Low-Energy Limit and Phenomenology

An abstract unified field theory is only physically relevant if it reproduces the observed low-energy world. In practice this means that, below some characteristic scale, the theory must reduce to general relativity coupled to the Standard Model, up to higher-dimensional operators that are suppressed at accessible energies. Moreover, in a genuinely unified framework the plethora of low-energy parameters should not be arbitrary input data in my opinion, but descend in a structured way from the underlying geometry and representation theory, as shown my our seventh and final axiom:

Axiom (U7). *The unified field theory possesses a well-defined low-energy effective description. There exists an energy scale*

$$\Lambda_{\text{unif}}, \tag{20}$$

with Λ_{unif} the characteristic unification scale, meaning that physics at energies $E \ll \Lambda_{\text{unif}}$ is captured by an effective field theory obtained by integrating out heavy and auxiliary degrees of freedom.

The low-energy effective action can be written as

$$S_{\text{eff}}[h, A, \psi, \dots] = S_{\text{EH}}[h] + S_{\text{SM}}[h, A, \psi, \dots] + \Delta S_{\text{higher}}, \tag{21}$$

with S_{eff} the effective action for the light fields, S_{SM} the Standard Model action coupled to gravity, and ΔS_{higher} the collection of higher-dimensional operators suppressed by powers of Λ_{unif}, meaning that the unified theory reproduces known low-energy physics up to controlled, subleading corrections.

The parameters of the Standard Model, such as the masses, couplings, and mixing angles are not to be arbitrary but must arise from the geometry of $\mathcal{M}$, the structure of E, and the representation theory of $\mathcal{G}_{\text{unif}}$ applied to the unified field $\mathcal{F}$. In particular, I believe there should exist relations:

$$\{m_i, g_j, \theta_k, \dots\} = F_{\text{UV}}, \tag{22}$$

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with with F_{UV} dependent on geometry, topology, and representation, $\{m_i, g_j, \theta_k, \dots\}$ as the collection of low-energy parameters and F_{UV} the mapping determined by the unified theory, meaning that observable parameters are constrained or predicted by the unified structure rather than freely chosen.

6. A Mini-Theorem and a Conjecture

The axioms above allow one to make at least one simple but robust statement about the predictive content of any unified field theory and to formulate a natural conjecture about the tension between unification and standard locality assumptions.

Given a unified field theory in the sense of Axioms (U1)–(U7), let us collect all truly independent ultraviolet data into a finite-dimensional space:

$$\mathcal{D}_{\mathrm{UV}}, \tag{23}$$

with $\mathcal{D}_{\mathrm{UV}}$ the UV data space meaning that it parametrizes the geometric, topological and representation-theoretic choices of the unified theory (such as the topology of E, characteristic classes on M, and a finite set of continuous coupling parameters at the scale Λ_{unif}). Let:

$$\mathcal{P}_{\mathrm{IR}}, \tag{24}$$

denote the space of low-energy parameters, with $\mathcal{P}_{\mathrm{IR}}$ the IR parameter space meaning that its coordinates can be taken to be the Standard Model couplings, masses and mixing angles, and possibly a small number of gravitational couplings appearing in S_{SM} and S_{EH}.

By Axiom (U7), there is a map:

$$F_{\mathrm{UV}} : \mathcal{D}_{\mathrm{UV}} \longrightarrow \mathcal{P}_{\mathrm{IR}}, \tag{25}$$

with F_{UV} the UV-to-IR map meaning that it sends each choice of UV data such as geometry, topology, representations and high-scale couplings to the corresponding point in IR parameter space obtained after integrating out heavy modes and running to low energies.

Proposition: A Parameter-Counting Constraint). *Suppose $\mathcal{D}_{\mathrm{UV}}$ and $\mathcal{P}_{\mathrm{IR}}$ are smooth manifolds of dimensions*

$$N_{\mathrm{UV}} := \dim \mathcal{D}_{\mathrm{UV}}, \qquad N_{\mathrm{IR}} := \dim \mathcal{P}_{\mathrm{IR}}, \tag{26}$$

with N_{UV} the number of independent UV parameters and N_{IR} the number of independent low-energy parameters, meaning that N_{UV} counts the dimension of the UV data space and N_{IR} counts the dimension of the space of possible SM+GR parameter choices. Assume F_{UV} is a smooth map. If:

$$N_{\mathrm{UV}} < N_{\mathrm{IR}}, \tag{27}$$

with the inequality $N_{\mathrm{UV}} < N_{\mathrm{IR}}$ meaning that the unified theory has fewer UV input parameters than IR observables, then there exist at least:

$$N_{\mathrm{IR}} - N_{\mathrm{UV}} > 0 \tag{28}$$

functionally independent relations among the low-energy parameters in the image of F_{UV}, meaning that the IR parameters are constrained to lie on a submanifold of $\mathcal{P}_{\mathrm{IR}}$ of dimension at most N_{UV}.

This can be justified as the rank of the differential $\mathrm{d}F_{\mathrm{UV}}$ at any point is at most N_{UV}, so the image of F_{UV} has dimension at most N_{UV}. If $N_{\mathrm{UV}} < N_{\mathrm{IR}}$, the image is a proper submanifold of $\mathcal{P}_{\mathrm{IR}}$ of codimension at least $N_{\mathrm{IR}} - N_{\mathrm{UV}}$, which can be described locally as the common zero set of $N_{\mathrm{IR}} - N_{\mathrm{UV}}$ independent constraint functions on $\mathcal{P}_{\mathrm{IR}}$.

This simple proposition formalizes the intuitive statement that any genuinely unified model in the sense of the axioms, that is, any model in which a small amount of UV data determines all low-energy

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parameters must predict nontrivial relations among Standard Model and gravitational parameters. It also suggests a natural measure of degree of unification via the difference:

$$N_{\text{predict}} := N_{\text{IR}} - N_{\text{UV}}, \tag{29}$$

with N_{predict} the prediction index meaning that it counts the minimal number of independent relations among IR parameters implied by the unified structure.

The axioms also allow us to rephrase the tension between geometric unification and standard locality assumptions in a conjectural form. We may add to Axioms (U1)–(U7) the usual S-matrix hypotheses underlying the Coleman–Mandula theorem; an existence of an analytic S-matrix with a mass gap, nontrivial scattering, and strict locality on M.

Conjecture: Unification to no-go Tradeoff. *Consider a unified field theory in the sense of Axioms (U1)–(U7) in which the unified symmetry group*:

$$\mathcal{G}_{\text{unif}}, \tag{30}$$

acts irreducibly on the unified field:

$$\mathcal{F}, \tag{31}$$

with $\mathcal{G}_{\text{unif}}$ the unified symmetry group and $\mathcal{F}$ the unified geometric field, meaning that spacetime and internal quantum numbers are genuinely mixed in a single representation at high energies. Assume further that the theory admits a nontrivial analytic S-matrix satisfying the usual locality and mass-gap conditions on M. Then at least one of the following must fail:

1. *the irreducibility of the unified action of $\mathcal{G}_{\text{unif}}$ on $\mathcal{F}$, the unified field effectively decomposes into separate spacetime and internal pieces; or*
2. *the strict S-matrix hypotheses, for example, strict micro-locality or analyticity must be relaxed at or above the unification scale.*

In words, the conjecture states that one cannot have, simultaneously a genuinely irreducible geometric unification of spacetime and internal symmetries in the sense of Axiom (U4), and cannot have at the same time a completely orthodox local, analytic S-matrix satisfying the hypotheses of the Coleman–Mandula theorem. At a technical level this conjecture is a repackaging of the Coleman–Mandula obstruction into the language of a single unified field $\mathcal{F}$ and unified symmetry group $\mathcal{G}_{\text{unif}}$. What is new is the explicit formulation of this tension as a tradeoff within an axiomatic definition of unified field theory that any successful unified model must either relax some of the S-matrix assumptions for instance by introducing controlled nonlocality in the ultraviolet or abandon irreducible geometric mixing of spacetime and internal structures.

7. A No-Go Theorem for Strictly Local 4D Geometric Unification

The axioms (U1)–(U7) are deliberately broad enough to encompass a wide variety of candidate unified field theories. Now I feel we should ask that if one supplements the axioms with a set of additional strict locality and S-matrix assumptions in four dimensions, then no nontrivial geometrically unified model exists. The result is a conditional no-go theorem that any genuinely unified model in the sense of Axiom (U4) must relax at least one of these extra assumptions.

We consider a unified field theory satisfying Axioms (U1)–(U7) and impose the following additional conditions:

(L1) Strict 4D locality and polynomiality. The action can be written as:

$$S[\Phi] = \int_M \mathcal{L}(\Phi, \partial\Phi, \ldots, \partial^k\Phi)\, d^4x, \tag{32}$$

with $S[\Phi]$ the action for the unified field configuration Φ, $\mathcal{L}$ the local Lagrangian density depending only on the fields and a finite number of their derivatives at a single spacetime point, and d^4x the

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volume element on the four-dimensional spacetime manifold M, meaning that the theory is strictly local on M and described by a finite-derivative polynomial or analytic Lagrangian density.

(L2) Finite-component low-spin field content. The unified field Φ transforms in a finite-dimensional representation of the local Lorentz group and decomposes at low energies into a finite number of fields of spin:

$$s \in \{0, \tfrac{1}{2}, 1, 2\}, \tag{33}$$

with s the spin of each particle species in the low-energy spectrum, meaning that the only massless particles are scalars, spin-$\frac{1}{2}$ fermions, spin-1 gauge bosons and a single spin-2 graviton, as in the Standard Model plus general relativity.

(L3) Standard S-matrix hypotheses. The theory admits an S-matrix S in asymptotically flat regions of M with the usual properties:

$$S = \mathbb{1} + iT, \tag{34}$$

with S the scattering matrix relating in- and out-states, $\mathbb{1}$ the identity operator, and T the transition operator, meaning that there is a well-defined, nontrivial, analytic S-matrix describing scattering of asymptotic particle states with a mass gap above the vacuum.

(L4) Symmetry embedding at the S-matrix level. The connected Poincaré group $\mathcal{P}$ and a compact internal symmetry group G_{int} act as symmetries of the S-matrix such that:

$$\mathcal{P} \times G_{\text{int}} \subset \mathcal{G}_{\text{unif}}, \tag{35}$$

with $\mathcal{P}$ the Poincaré group, G_{int} the internal symmetry group that contains the Standard Model gauge group as a subgroup, and $\mathcal{G}_{\text{unif}}$ the unified symmetry group, meaning that at the level of scattering amplitudes the usual assumptions of relativistic QFT with internal symmetries are satisfied.

Now I will formulate a no-go theorem that links these additional assumptions to the geometric unification axiom (U4):

Theorem: no-go for strict 4D local geometric unification. *Consider a unified field theory satisfying Axioms (U1)–(U7) together with conditions (L1)–(L4). Suppose further that the unified symmetry group*

$$\mathcal{G}_{\text{unif}}, \tag{36}$$

acts irreducibly on the unified geometric field

$$\mathcal{F}, \tag{37}$$

with $\mathcal{G}_{\text{unif}}$ the unified symmetry group and $\mathcal{F}$ the unified geometric field introduced in Axiom (U4), meaning that spacetime and internal quantum numbers are genuinely mixed in a single representation at high energies. Then there exists no nontrivial theory satisfying all these conditions: either

$$\mathcal{G}_{\text{unif}} \simeq \mathcal{P} \times G_{\text{int}}, \tag{38}$$

with $\mathcal{G}_{\text{unif}}$ the unified symmetry group, $\mathcal{P}$ the Poincaré group and G_{int} the internal symmetry group, meaning that the symmetry algebra factorizes into a direct product and the unified geometric action is reducible, or at least one of the locality or S-matrix assumptions (L1)–(L4) must be violated.

The proof is essentially a corollary of the Coleman–Mandula theorem and the uniqueness of massless spin-1 and spin-2 couplings, reformulated in the language of the unified field axioms.

First, by (L1)–(L3) the theory falls within the class of local, analytic, relativistic quantum field theories with a mass gap and a nontrivial S-matrix. Under these conditions, the Coleman–Mandula theorem implies that the Lie algebra of continuous symmetries of the S-matrix must be a direct sum:

$$\mathfrak{g}_{\text{sym}} \simeq \mathfrak{p} \oplus \mathfrak{g}_{\text{int}}, \tag{39}$$

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with $\mathfrak{g}_{\text{sym}}$ the Lie algebra of all continuous S-matrix symmetries, $\mathfrak{p}$ the Poincaré Lie algebra and $\mathfrak{g}_{\text{int}}$ the Lie algebra of internal symmetry transformations, meaning that spacetime and internal symmetries cannot mix nontrivially in a simple Lie algebra acting on single-particle states.

On the other hand, Axiom (U4) and the irreducibility assumption state that the unified geometric field $\mathcal{F}$ transforms in a single representation:

$$R_{\text{unif}}, \tag{40}$$

with R_{unif} the unified representation of $\mathcal{G}_{\text{unif}}$ on $\mathcal{F}$, meaning that after restricting to the low-energy symmetry algebra the representation R_{unif} must carry both spacetime and internal indices in a genuinely mixed way.

However, the factorization (39) forces any finite-dimensional representation of $\mathfrak{g}_{\text{sym}}$ to decompose as a tensor product:

$$R_{\text{unif}} \simeq R_{\text{grav}} \otimes R_{\text{int}}, \tag{41}$$

with R_{grav} the representation of the Poincaré algebra capturing the spin content of the fields and R_{int} the representation of the internal algebra encoding gauge charges, meaning that the spacetime and internal parts of the representation separate and do not form a single irreducible multiplet mixing the two.

Furthermore, the uniqueness arguments for massless spin-1 and spin-2 couplings in local QFT show that the low-energy Lagrangian must reduce to a sum:

$$\mathcal{L}_{\text{IR}} = \mathcal{L}_{\text{EH}}(h) + \mathcal{L}_{\text{YM}}(A) + \mathcal{L}_{\text{matter}}(h, A, \psi) + \dots \tag{42}$$

with $\mathcal{L}_{\text{IR}}$ the low-energy Lagrangian density, $\mathcal{L}_{\text{EH}}(h)$ the Einstein–Hilbert Lagrangian for the metric $h_{\mu\nu}$, $\mathcal{L}_{\text{YM}}(A)$ the Yang–Mills Lagrangian for the gauge fields A^a_μ, and $\mathcal{L}_{\text{matter}}$ the matter Lagrangian for the fermions and scalars ψ, meaning that gravity and gauge interactions appear as separate, universal structures fixed by their spin and symmetry content.

Taken together, the factorization of the symmetry algebra (39), the representation factorization (41), and the universal low-energy form (42) imply that the unified geometric field $\mathcal{F}$ cannot remain irreducible in the sense of Axiom (U4) while satisfying (L1)–(L4); it necessarily decomposes into separate gravitational, gauge and matter pieces associated with a product symmetry group as in (38). This contradicts the assumption of a nontrivial, irreducible geometric unification. Therefore, any genuinely unified field theory in the sense of Axiom (U4) must relax at least one of the additional assumptions (L1)–(L4): for example, by introducing nonlocality or extended structure beyond strict 4D locality, by allowing an infinite tower of fields, or by giving up some of the standard S-matrix hypotheses in favor of a different notion of quantum consistency.

At its most basic form this new therom states that if you have a strictly local 4D QFT with an ordinary analytic S-matrix, a finite number of low-spin fields, and you try to make a genuinely irreducible geometric unification of spacetime and internal symmetries into one field $\mathcal{F}$ and group $\mathcal{G}_{\text{unif}}$, then the symmetry algebra factorizes and $\mathcal{F}$ decomposes. So any real geometric unification must relax locality or S-matrix assumptions.

8. Towards Theorems About Unification

Now once the notion of a unified field theory is axiomatized in terms of data:

$$(M, \mathcal{M}, E, \mathcal{G}_{\text{unif}}, \mathcal{F}, S, \mathcal{H}, \mathcal{A}), \tag{43}$$

with M the physical spacetime, $\mathcal{M}$ the ambient manifold, E the configuration bundle, $\mathcal{G}_{\text{unif}}$ the unified symmetry group, $\mathcal{F}$ the unified geometric field, S the action functional, $\mathcal{H}$ the Hilbert space, and $\mathcal{A}$ the observable algebra, meaning that the full content of the unified theory is packaged into a finite list of geometric and quantum structures, one can formulate precise questions and potential theorems.

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Examples include classification problems such as classifying all theories satisfying the axioms under additional regularity conditions, uniqueness questions like showing that a given choice of $\mathcal{G}_{\text{unif}}$ and E leads to essentially a unique action S, and no-go theorems meaning showing that under certain locality and analyticity assumptions no nontrivial mixing of spacetime and internal symmetries is possible beyond known constructions such as supersymmetry [1,2,9].

The present axioms are deliberately flexible enough to accommodate conventional grand unified models, extended geometries, and more exotic constructions, while still demanding that gravity, gauge fields, and matter arise from a single geometric object and a single symmetry structure and that the Standard Model plus general relativity emerge in the appropriate limit.

9. Conclusions

A precise axiomatic definition of unified field theory is a first step towards turning unification from a slogan into a mathematical framework. Even if no single, completely general classification theorem is within reach, partial results restricting the space of possible unified theories should be attainable under these axioms. Conversely, concrete models can be evaluated against the axioms to clarify in what precise sense they achieve unification, what assumptions they relax, and how they evade or realize known no-go theorems.

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