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Article

CKM Matrix Values Found from Fibonacci Numbers

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Abstract: The author shows that it is possible to obtain the CKM transition probabilities from the values of quark masses. For example in order to obtain the probability for the transition from the quark d to the quark u the author shows that this value can be found from the value of the mass of the quark u and the values of the masses of quarks that can transform into the u quark. Alternatively, every value of the CKM matrix can be found from few Fibonacci numbers. The author shows how these two findings are related.

Keywords: CKM matrix; quark masses; quantum physics; fibonacci numbers; golden ratio

1. Introduction

The Cabibbo–Kobayashi–Maskawa matrix or CKM matrix (or quark mixing matrix) is a unitary matrix which describes the transition probabilities of the flavour-changing weak interactions. In 1963, Nicola Cabibbo introduced the Cabibbo angle (θ_c) that is related to the relative probability that down and strange quarks decay into up quarks (V_{ud}^2 and V_{us}^2 , respectively). When the charm quark was discovered in 1974, it was noticed that the down and strange quark could decay into either the up or charm quark, leading to the introduction of the Cabibbo angle and the “Cabibbo matrix”:

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \quad (1)$$

The Cabibbo matrix was subsequently expanded to the 3×3 CKM matrix by Kobayashi and Maskawa. The generalized Cabibbo matrix is the Cabibbo–Kobayashi–Maskawa matrix (or CKM matrix) that describes the probabilities of the weak decays of the three generations of quarks:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (2)$$

where V_{ij} represents the probability that the j quark decay into the i quark. Nowadays the values of this matrix are considered physical constants by the field and the matrix assume values, reported in Eq. 3, that are found experimentally. In this paper we want to propose that the values of the CKM matrix depend from other physical constants and that they can be found mathematically from few known numbers, the Fibonacci numbers. In particular, the author discovers that the values of this matrix can be found solely from quark masses or, alternatively, from Fibonacci numbers.

2. Methods

The experimental values for the CKM matrix from the Particle Group 2022 review [2] is reported in Eq. 3 together with the standard error for each value. To explore how these values were found see [1].

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.973730 & 0.22430 & 0.00382 \\ 0.2210 & 0.9750 & 0.0408 \\ 0.00860 & 0.0415 & 0.014 \end{pmatrix} \pm \begin{pmatrix} 0.000310 & 0.00080 & 0.00020 \\ 0.0040 & 0.0060 & 0.0014 \\ 0.00020 & 0.00090 & 0.029 \end{pmatrix} \quad (3)$$

In Eq. 3 V_{ij} represents the probability that the quark of flavor j decays into a quark of flavor i .

The author finds that these probabilities can be estimated using the ratio of the mass of the produced quark and the mass of the original quark mass divided by the sum of ratios of the produced quark divided by any mass of quark that can decay in the produced quark (see Eq. 4).

We found the following equation for computing the value of the transition probability V_{ij} from the heavier quark j to the lighter quark i :

$$V_{ij}^2 = \frac{\left(\frac{m_i}{m_j}\right)}{\left(\frac{m_i}{m_j}\right) + \left(\frac{m_i}{m_k}\right) + \left(\frac{m_i}{m_l}\right)} \quad (4)$$

where quark j , quark k and quark l are the possible quarks that transform into quark i (can be two, three or more). In the case there exist other (unknown) particles transforming into quark i , this equation provides an upper bound.

Alternatively, we can find that the transition probabilities can be found using the Fibonacci numbers. The author will use the following finding [first paper]:

$$\left(\frac{m_i}{m_j}\right) = \left(\frac{m_t/F_i}{m_t/F_j}\right) = \left(\frac{F_i}{F_j}\right)^{-1} \quad (5)$$

where m_t is the mass of the top quark and F_i are Fibonacci numbers. The equation that put in relation masses and Fibonacci numbers (Eq. 5) is introduced in [first paper].

We can rewrite Eq. (4) as a function of Fibonacci numbers, and only them. This results in an equation for V_{ij} that is all numerical (no physical variables are involved).

$$V_{ij}^2 = \frac{\left(\frac{F_j}{F_i}\right)}{\left(\frac{F_j}{F_i}\right) + \left(\frac{F_k}{F_i}\right) + \left(\frac{F_l}{F_i}\right)} \quad (6)$$

In the next section the author will use Eq. 4 and Eq. 6 to find every element of the CKM matrix.

3. Results

Let's begin considering that you can obtain a quark u via weak interaction starting from either a quark d or quark s or quark b . The transition probability of the transition from quark d to quark u can be found from the masses involved in the described three transitions. Applying Eq. 4, the author obtains the following result:

$$V_{ud}^2 = \frac{\left(\frac{m_u}{m_d}\right)}{\left(\frac{m_u}{m_d}\right) + \left(\frac{m_u}{m_s}\right) + \left(\frac{m_u}{m_b}\right)} = \frac{\frac{2.16}{4.67}}{\frac{2.16}{4.67} + \frac{2.16}{93.4} + \frac{2.16}{4180}} = 0.9514 \quad (7)$$

where Eq. 4 has been applied to the transition $d \rightarrow u$. Taking the squared root of equation 7, we obtain:

$$V_{ud} = 0.9754 \quad (8)$$

to be compared to the experimental value:

$$V_{ud} = 0.97373 \pm 0.00031 \quad (9)$$

that is one of the value of the CKM matrix in Eq. 3.

The value for V_{ud} that the author found in Eq. 8 is compatible with the experimental value in Eq. 9 within 1 standard deviation.

Using Eq. 6, instead, we can rewrite Eq. (7) as a function of Fibonacci numbers, and only them (Eq. 10). This results in an equation for V_{ud} that is all numerical (Eq. 11).

$$V_{ud}^2 = \frac{\left(\frac{F_d}{F_u}\right)}{\left(\frac{F_d}{F_u}\right) + \left(\frac{F_s}{F_u}\right) + \left(\frac{F_b}{F_u}\right)} \quad (10)$$

$$V_{ud}^2 = \frac{\left(\frac{37512.5}{75025}\right)}{\left(\frac{37512.5}{75025}\right) + \left(\frac{2090.5}{75025}\right) + \left(\frac{1}{75025}\right)} = 0.94615 \quad (11)$$

The author considered the transitions described in the first row of the CKM matrix, that describes the three transitions of possible quarks able to decay into the u quark.

Taking the root of Eq. 11, we can obtain the transition probability V_{ud} :

$$V_{ud} = 0.9727 \quad (12)$$

The value in Eq. 12 is even a better estimation of the experimental value for V_{ud} (Eq. 9). The value of V_{ud} computed in Eq. 12 is compatible within 1 standard deviation from the experimental value (see Eq. 9 for the experimental error for V_{ud}) with a relative error of 4.8%. In the following, the theoretical estimations are obtained for the transition probabilities V_{us} and V_{ub} :

$$V_{us}^2 = \frac{\left(\frac{F_s}{F_u}\right)}{\left(\frac{F_s}{F_u}\right) + \left(\frac{F_d}{F_u}\right) + \left(\frac{F_b}{F_u}\right)} \quad (13)$$

$$V_{us}^2 = \frac{\left(\frac{2090.5}{75025}\right)}{\left(\frac{37512.5}{75025}\right) + \left(\frac{2090.5}{75025}\right) + \left(\frac{44.5}{75025}\right)} = 0.0527 \quad (14)$$

$$V_{us} = 0.2293 \quad (15)$$

The experimental value for the transition quark_s → quark_u is

$$V_{us}^{experimental} = 0.2243 \pm 0.0008 \quad (16)$$

The value that we found with this framework (Eq. 15) is out of 3 standard deviation from the experimental value, however the relative error is very small (err = 2.2%). One might also realized this result might be subject to corrections, for example in the case of an unknown particle or another quark that is able to transform to the quark u , even in the case that these transitions might be very rare, attesting that Eq. 4 or Eq. 6 (Eq. 15 in this case) might be an upper bound on the transition probability. Let's consider now the transition between quark b and quark u . A similar framework brings to the following estimation for the element V_{ub} :

$$V_{ub}^2 = \frac{\left(\frac{F_b}{F_u}\right)}{\left(\frac{F_b}{F_u}\right) + \left(\frac{F_b}{F_c}\right) + \left(\frac{F_b}{F_t}\right)} = 1.3237E - 05 \quad (19)$$

$$V_{ub} = 0.003525$$

In these case we considered the transitions described in the third column in the CKM matrix. The author is unaware of the reason behind using a row or a column from the CKM matrix. We rewrite for convenience the experimental value found in the CKM matrix (Eq. 3):

$$V_{ub}^{experimental} = 0.00382 \pm 0.00020 \quad (20)$$

The value that we found within this framework (Eq. 19) is compatible with the experimental value within 2 standard deviations.

In the same way we obtain in Eq. 21-23 the other elements of the CKM matrix.

$$V_{cd}^2 = \frac{\left(\frac{F_c}{F_d}\right)}{\left(\frac{F_c}{F_d}\right) + \left(\frac{F_c}{F_s}\right)} = \frac{\frac{144}{37512.5}}{\frac{144}{37512.5} + \frac{144}{2090.5}} = 0.05279 \quad (21)$$

$$V_{cd} = 0.230$$

Eq. 21 has to be compared to the experimental value $V_{cd}^{experimental} = 0.221 \pm 0.004$. Note in this case we considered $\left(\frac{F_c}{F_d}\right)$ in place of $\left(\frac{F_d}{F_c}\right)$. The author is not aware of the reason, possibly it is because it is the quark c (heavier) that decay into the quark d (lighter).

$$V_{cs}^2 = \frac{\left(\frac{F_c}{F_s}\right)}{\left(\frac{F_c}{F_s}\right) + \left(\frac{F_c}{F_d}\right)} = \frac{\frac{144}{2090.5}}{\frac{144}{2090.5} + \frac{144}{37512.5}} = 0.9472 \quad (22)$$

$$V_{cs} = 0.973V_{cs}^{experimental} = 0.975 \pm 0.006$$

$$V_{cb}^2 = \frac{\left(\frac{F_c}{F_b}\right)}{\left(\frac{F_c}{F_b}\right) + \left(\frac{F_c}{F_d}\right) + \left(\frac{F_c}{F_s}\right)} = \frac{\frac{144}{44.5}}{\frac{144}{44.5} + \frac{144}{37512.5} + \frac{144}{2090.5}} = 0.00116 \quad (23)$$

$$V_{cb} = 0.0432V_{cb}^{experimental} = 0.0408 \pm 0.0014$$

The estimations of the three values in Eq. 21, 22 and 23 are off from the experimental error in the 3rd digit after comma, exactly where the experimental error is. The computed three values in Eq. 21, 22 and 23 are within 3, 1 and 2 standard deviations, respectively. In Eq. 23 one would expect to find the ratio $\left(\frac{F_b}{F_c}\right)$ rather than its reverse since it is the quark b (heavier) that decay into the quark c (lighter). The reason is unknown to the author.

Using the same formalism we obtain the last three values of the CKM matrix (last row, Eq. 3).

$$V_{td}^2 = \frac{\left(\frac{F_t}{F_d}\right)}{\left(\frac{F_t}{F_d}\right) + \left(\frac{F_u}{F_d}\right) + \left(\frac{F_c}{F_d}\right)} = \frac{\frac{1}{37512.5}}{\frac{1}{37512.5} + \frac{1}{75025} + \frac{144}{37512.5}} = 5.291E - 05 \quad (24)$$

$$V_{td} = 0.0073V_{td}^{experimental} = 0.0086 \pm 0.0002$$

The considered quarks in Eq. 24 are the quarks involved in the transitions described by the first column in the CKM matrix (quark u , c and t). One should have expected the usage of the quarks involved in the transitions the quark t is involved that are described by the last row of the CKM matrix. The reason is unknown to the author. There is, though, a symmetry respect to the CKM diagonal between V_{ub} and V_{td} and a symmetry in the computation of these two variables in this framework, since in calculating both of them we have considered the transitions of the column (they belong to) in the CKM matrix.

Let's compute now the matrix element V_{ts} :

$$V_{ts}^2 = \frac{\left(\frac{F_s}{F_t}\right)}{\left(\frac{F_s}{F_t}\right) + \left(\frac{F_b}{F_t}\right) + \left(\frac{F_u}{F_t}\right)} = \frac{\left(\frac{1}{2090.5}\right)}{\left(\frac{1}{2090.5}\right) + \frac{44.5}{144} + \frac{2090.5}{75025}} = 0.001418 \quad (25)$$

$$V_{ts} = 0.0377V_{ts}^{experimental} = 0.0415 \pm 0.0009$$

This estimated value is outside 3 standard deviations compared to the experimental value. The relative error is 10.2%.

The value of the element V_{tb} is computed in the following:

$$V_{tb}^2 = \frac{\left(\frac{F_t}{F_b}\right)}{\left(\frac{F_t}{F_b}\right) + \left(\frac{F_t}{F_s}\right) + \left(\frac{F_t}{F_d}\right)} = \frac{\frac{1}{44.5}}{\frac{1}{44.5} + \frac{1}{2090.5} + \frac{1}{37512.5}} = 0.999989 \quad (27)$$

$$V_{tb} = 0.999994V_{ts}^{experimental} = 1.014 \pm 0.029$$

Since it seems highly luckily from experiments that the value for V_{tb} is close to 1, if not exactly 1, the computed value is an accurate estimate for V_{tb} . The value found for V_{tb} in Eq. 27 is compatible with the experimental value within 1 standard deviation. In case other transitions have to be considered in the denominator of Eq. 27, the value in Eq. 27 has to be considered an upper bound.

We can write a general formula for the probabilities V_{ij} for the transition from the heavier quark j to the lighter quark i

$$V_{ij}^2 = \frac{\left(\frac{m_i}{m_j}\right)}{\left(\frac{m_i}{m_j}\right) + \sum \left(\frac{m_i}{m_k}\right)} \quad (28)$$

$$V_{ij}^2 = \frac{\left(\frac{F_j}{F_i}\right)}{\left(\frac{F_j}{F_i}\right) + \sum \left(\frac{F_j}{F_k}\right)} \quad (29)$$

where n might be 1, 2, ...

Using the theoretical framework just presented one can check the unitary of the CKM matrix, using the theoretical values for the elements of the CKM matrix. In particular, the first-row matrix elements give:

$$V_{ub}^2 + V_{us}^2 + V_{ub}^2 = 0.946 + 0.0527 + 1.323 \cdot 10^{-5} = 0.9989 \quad (30)$$

to be compared to the experimental value:

$$(V_{ub}^2 + V_{us}^2 + V_{ub}^2)^{experimental} = 0.9985 \pm 0.0007 \quad (31)$$

The value in Eq. 30 calculated with the author's framework is compatible with the experimental value within 1 standard deviation. A theoretical value different than 1, like in Eq. 30, indicates possible theories of physics beyond the Standard Model.

4. Discussion

The values of the CKM matrix have been computed experimentally. By using the mathematical formalism presented in this work, the author is able to compute the values for the CKM transition probabilities. The author shows that the values of this matrix can be found solely from the values of the quark masses or, alternatively, from Fibonacci numbers. The physical values of the CKM matrix found in this paper have relative errors as little as 0.008% with average relative errors of 2.8%. The values compatible with the experimental values within 3 standard deviations are 6 out of 9, with the other three theoretical values assuming values close to the 3 standard deviations limit. Some limitations in the formalism developed here has to be found in a formula that is not unique but has been modified for some transitions: this is the case of Eq. 21 and 22 where only two quarks are considered in the denominator and Eq. 23, where the author is not aware of the reason behind considering the ratio $\left(\frac{F_b}{F_c}\right)$ rather than $\left(\frac{F_c}{F_b}\right)$. However, this work proves that it is possible to explain the probabilities of quark transitions starting from the values of the quark masses or, alternatively, from Fibonacci numbers. This work, together with the study in [first paper], might give insights on how masses are generated during particle transitions, that is relevant to the Mass Generation problem (open problem in Physics). The CKM matrix values are used in computing different quantities in particle physics and they are involved in different hypotheses, including the weak universality and the CP violating phase. Therefore this theory might have important consequences and give important insights to the field. Applying a similar formalism to the Pontecorvo–Maki–Nakagawa–Sakata matrix, one might discover some relationship between the masses of neutrinos. Several major experimental efforts are underway to help establish the masses of neutrinos. This work might help in finding them theoretically. Also, we compute the first-row matrix element parameter in Eq. 30, finding a value different than 1, that indicate possible physics beyond the Standard Model. Therefore, this work might inspire some possible theory of physics beyond the Standard Model. To conclude, this work might be relevant for future research in different fields.

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