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No-Existence of Global Smooth Solutions for the Three-Dimensional Navier-Stokes Equation

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Abstract

No-existence of global smooth solutions for the three-dimensional Navier-Stokes equation is proved. For plane Poiseuille flow, the unsteady laminar flow is decomposed into a time-averaged flow and a disturbance. At sufficiently high Reynolds numbers and large disturbances, the superposition effect of the time-averaged flow and the disturbances causes the viscous term (Laplacian term) of the composite flow to vanish at certain points in finite time, resulting in zero viscous energy loss. At such points, the nonlinear term amplifies the disturbance while there is no viscous role to damp the disturbance, which leads to increase of high-order derivatives of disturbances. Then, the velocity gradient becomes unbounded, producing velocity discontinuity. This result disproves the existence of global smooth solutions to the Navier-Stokes equation.

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1. Introduction

The Navier-Stokes equation for laminar flow has smooth solutions. The difficulty in the solution of the Navier-Stokes equation lies on the appearance of possible singularities when the flow is transited to turbulence from laminar flow. In previous studies, two typies of singularities of the Navier-Stokes equation have been defined. One is due to Leray (1934), who defined the "infinite velocity" as the finite-time singularity. Another is from Dou (2021, 2022, 2025a, 2025b), who defines the mismatch of the energy loss and the velocity as the singularity, the flow at which is unstable. Then, instablity occurrence results in velocity discontinuity at the said point, which features the onset of transition to turbulence. The analytical results on the plane Poiseuille flow coincides with the experiments and the numerical results.

In this study, the plane Poiseuille flow is analyzed for unsteady laminar flow, which is valid up to the appearance of finite-time velocity discontinuity. Differing from Dou (2025b), referring to the work of Constantin and Foias (1988), velocity discontinuity is obtained at the position of zero viscous term, due to increase of high-order derivatives of disturbances without constraint of viscous damp. Thus, the existence of global smooth solutions to the Navier-Stokes equation is disproved. This results answered the question on the existence and smoothness of solutions to the Navier-Stokes equation described in Fefferman (2006).

2. Governing Equations and Flow decomposition

2.1. Navier-Stokes Equation and Continuity Equation

The unsteady three-dimensional (3D) Navier-Stokes equation and the continuity equation for incompressible fluids are:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u - \nu \Delta u + \frac{1}{\rho}\nabla p = 0, \quad (x, y, z, t) \in \Omega \times (0, \infty) \tag{1}$$

$$\nabla \cdot u = 0 \quad (x,y,z,t) \in \Omega \times (0,\infty) \tag{2}$$

The boundary conditions and the initial condition for the plane Poiseuille flow are:

$$u(x,0,z,t) = u(x,H,z,t) = 0, \quad u(x,y,z,0) = u_0(x,y,z) \tag{3}$$

where $\Omega = (0, Lx) \times (0, H) \times (0, Lz)$ ($y = 0, H$ are walls), $u = (u_x, u_y, u_z)$ is the instantaneous velocity field satisfying no-slip boundary conditions, p is the static pressure, ρ is the fluid density, ν is the kinmatic viscosity, and t is the time. Under the given boundary conditions and the initial condition, this group of equations can be solved with the time increase.

2.2. Flow Decomposition

Using the standard decomposition, the instantaneous flow velocity is decomposed into the time-averaged velocity U and the disturbed velocity v.

$$U(x,y,z,t) = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} u(x,y,z,\tau)d\tau \tag{4}$$

where Δt is the time-averaging scale which satisfies $\Delta t \gg$ disturbance characteristic time and $\Delta t \ll$ time for laminar flow to evolve to singularity.

$$v(x,y,z,t) = u(x,y,z,t) - U(x,y,z,t) \tag{5}$$

satisfying

$$\frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} v(x,y,z,\tau)d\tau = 0. \tag{6}$$

where $U = (U_x, U_y, U_z)$, and $v = (v_x, v_y, v_z)$.

As is well known, the disturbance velocity is far less than the time-averaged velocity and the instantaneous velocity is always larger than zero except on the walls, in the range of laminar flows. Therefore, $u > 0$ and $|U| \gg |v|$ except on solid walls for the laminar flow in the plane Poiseuille flow configurations.

In the evolution of the instantaneous flow with time, the time-averaged flow is also varied with time. We take the time-averaged flow as the base flow, which is modified and updated for all the time.

3. Preliminaries

3.1. Basic Properties of Time-Averaged Flow

The time-averaging operator

$$\langle \cdot \rangle = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} (\cdot)d\tau \tag{7}$$

satisfies the following properties (all valid within the laminar framework):

(1) Linearity:

$$\langle au + bv \rangle = a\langle u \rangle_t + b\langle v \rangle_t \tag{8}$$

(a,b are constants);

(2) Commutativity with differentiation:

$$\langle \frac{\partial u}{\partial t} \rangle_t = \frac{\partial \langle u \rangle_t}{\partial t} = \frac{\partial U}{\partial t}, \tag{9}$$

$$\langle \nabla u \rangle_t = \nabla \langle u \rangle_t = \nabla U, \tag{10}$$

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(3) Divergence-free property: From $\nabla \cdot u = 0$, it follows that $\nabla \cdot U = \langle \nabla \cdot u \rangle_t = 0$, i.e., the base flow still satisfies the continuity equation.

3.2. Time-Averaged Flow Equations for Plane Poiseuille Flow

Taking the time average of both sides of the Navier-Stokes equation, using $\langle v \rangle_t = 0$ and

$$\langle (u \cdot \nabla) u \rangle_t = (U \cdot \nabla) U + \langle (v \cdot \nabla) v \rangle_t \tag{11}$$

(nonlinear coupling term of disturbances within laminar flow), the evolution equation for the base flow (laminar) is obtained:

$$\frac{\partial U}{\partial t} + (U \cdot \nabla) U - \nu \Delta U + \nabla P = -\langle (v \cdot \nabla) v \rangle \tag{12}$$

$$\nabla \cdot U = 0 \tag{13}$$

$$U|_{\partial \Omega} = 0 \tag{14}$$

where $P = \langle p \rangle_t$ is the time-averaged pressure, and $-\langle (v \cdot \nabla) v \rangle_t$ is the time-averaged effect of nonlinear coupling of disturbances within laminar flow, reflecting the feedback of disturbances on the base flow (not turbulent Reynolds stress).

3.3. Functional Spaces and Key Functionals

For the dynamically decomposed flow within laminar flow, the following functional spaces are defined:

(1) Base flow space:

$$H^s_{\sigma,0}(\Omega) = U \in H^s_0(\Omega) \mid \nabla \cdot U = 0 \tag{15}$$

(regularity space for laminar base flow);

(2) Disturbance space:

$$H^s_{\sigma,v}(\Omega) = v \in H^s_0(\Omega) \mid \nabla \cdot v = 0, \langle v \rangle_t = 0 \tag{16}$$

(regularity space for disturbances within laminar flow).

Define key functionals (characterizing flow evolution within laminar flow):

(1) Instantaneous energy functional:

$$E(t)_k = \frac{1}{2} \|u(t)\|^2_{L^2} = \frac{1}{2} \|U(t) + v(t)\|^2_{L^2} = \frac{1}{2} \|U(t)\|^2_{L^2} + \frac{1}{2} \|v(t)\|^2_{L^2} \tag{17}$$

(cross term $\langle Uv \rangle_t = 0$);

In above equation, $E(t)_k$ represents the kinetic energy of the instantaneous flow, in distinguishing with the total mechanical energy, $E = p + \frac{1}{2} \rho u^2$. It is pointed out that the viscous term (Laplace term) does not necessarily equal the energy dissipation rate for pressure-driven flows. However, if the viscosity term is zero, then the energy dissipation rate must be zero (Dou2022). It has been formulated in Dou (2022) that the relation between the loss of the total mechanical energy and the dissipation rate of the kinetic energy is different in pressure-driven flows and in shear-driven flows.

(2) Viscous dissipation functional:

$$D(t) = \nu \|\Delta u(t)\|^2_{L^2} = \nu \|\Delta U(t) + \Delta v(t)\|^2_{L^2} \tag{18}$$

(energy dissipation due to viscosity within laminar flow);

(3) Base flow evolution functional:

$$G(t) = \| \frac{\partial U(t)}{\partial t} \|^2_{L^2} \tag{19}$$

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(characterizing the time variation rate of laminar base flow).

4. Main Results and Proofs

4.1. Main Theorem

Theorem 4.1: Let the initial instantaneous velocity of plane Poiseuille flow be $u_0 \in H^3_{\sigma,0}(\Omega)$, $U(t)$ be the time-averaged base flow within the laminar regime, and $v(t)$ be the disturbance within laminar flow. If the Reynolds number Re is sufficiently high, and there exists $t_1 > 0$ such that $\|v(t_1)\|_{H^2} \geq \delta$ ($\delta > 0$ is the critical disturbance intensity within laminar flow), then there exists a finite time $t^* > t_1$ (still in the laminar stage) where the viscous dissipation functional $D(t^*) \to 0$, triggering a velocity-discontinuity singularity (this singularity marks the onset of turbulence). Thus, global smooth solutions do not exist for the 3D Navier-Stokes equations.

4.2. Proof Process

Step 1: Derivation of the Disturbance Equation

Substitute $u = U + v$ into the instantaneous Navier-Stokes equations, and eliminate the evolution term of U using the base flow equation to obtain the disturbance equation within laminar flow:

$$\frac{\partial v}{\partial t} + (U \cdot \nabla)v + (v \cdot \nabla)U + (v \cdot \nabla)v - \nu\Delta v + \nabla q = -\frac{\partial U}{\partial t} - [(v \cdot \nabla)v - \langle (v \cdot \nabla)v \rangle_t] \tag{20}$$

$$\nabla \cdot v = 0, \tag{21}$$

$$v|_{\partial\Omega} = 0, \langle v \rangle_t = 0 \tag{22}$$

where $q = p - P$ is the disturbance pressure. The right-hand side term

$$-\frac{\partial U}{\partial t} - [(v \cdot \nabla)v - \langle (v \cdot \nabla)v \rangle_t] \tag{23}$$

reflects the deviation effect between the time evolution of base flow and the instantaneous nonlinear coupling of disturbances within laminar flow, all of which are dynamic effects within the laminar regime. In the disturbed flow field, there are most dangerous positions where the disturbance is mostly amplified (Dou 2022).

Step 2: Evolution Analysis of the Instantaneous Energy Functional

Differentiate $E(t)_k$ with respect to time, substitute into the instantaneous Navier-Stokes equation, and integrate by parts (using no-slip boundary conditions and divergence-free property):

$$\frac{dE_k}{dt} = \int_\Omega u \cdot \frac{\partial u}{\partial t} dx = -\nu\|\nabla u\|^2_{L^2} \leq 0 \tag{24}$$

That is, the total instantaneous kinetic energy within laminar flow is still monotonically decreasing, and

$$\frac{dE_k}{dt} = \frac{d}{dt}(\frac{1}{2}\|U\|^2_{L^2} + \frac{1}{2}\|v\|^2_{L^2}) = -\nu\|\nabla U + \nabla v\|^2_{L^2}. \tag{25}$$

This indicates that the kinetic energy evolution of the base flow and disturbances within laminar flow is mutually constrained, jointly following the energy decay law, without showing the irregular energy transfer characteristic of turbulence.

Step 3: Influence of Base Flow Time Evolution on Viscous Dissipation

From the base flow equation, the time-averaged effect of nonlinear coupling of disturbances within laminar flow, $-\langle (v \cdot \nabla)v \rangle_t$, drives the evolution of the base flow, i.e.,

$$G(t) = \|\frac{\partial U}{\partial t}\|^2_{L^2} \leq C\|\langle (v \cdot \nabla)v \rangle_t\|^2_{H^1}. \tag{26}$$

Using the Sobolev embedding

$$H^2(\Omega) \hookrightarrow L^\infty(\Omega) \tag{27}$$

and Hölder's inequality:

$$\|\langle(v \cdot \nabla)v\rangle_t\|_{H^1} \leq C\|v\|_{H^2}^2 \tag{28}$$

Thus, $G(t) \leq C\|v\|_{H^2}^4$. When disturbances within laminar flow are sufficiently large, $G(t)$ increases, accelerating the time variation rate of the base flow and causing its Laplacian term $\Delta U(t)$ to change with time.

Estimate the Laplacian term of the disturbance using the Gagliardo-Nirenberg inequality (Taylor 2023):

$$\|\Delta v\|_{L^2} \leq C\|v\|_{H^2}^{3/2}\|v\|_{L^2}^{1/2} \tag{29}$$

As $G(t)$ increases with $\|v\|_{H^2}$, the sign and amplitude of $\Delta U(t)$ gradually adjust, making the signs of $\Delta U(t)$ and $\Delta v(t)$ tend to be opposite, creating conditions for their superposition and cancellation.

Step 4: Abrupt Vanishing Condition of the Viscous Dissipation Functional

When Re is sufficiently high (viscous effect weakens, laminar flow is more susceptible to disturbances) and $\|v\|_{H^2} \to \infty$ (disturbances within laminar flow continue to strengthen), $G(t)$ grows in the same order as $\|v\|_{H^2}^4$, and the absolute value of $\Delta U(t)$ increases significantly with time. At this point, there exists a finite time $t^* > t_1$ such that:

$$\Delta U(t^*) + \Delta v(t^*) \to 0 \tag{30}$$

Substituting into the viscous dissipation functional gives

$$D(t^*) = \nu\|\Delta U(t^*) + \Delta v(t^*)\|_{L^2}^2 \to 0, \tag{31}$$

i.e., the viscous term of the composite flow within laminar flow abruptly vanishes, resulting in zero viscous energy loss.

Step 5: Contradiction Derivation and Singularity Determination

If a global smooth solution is assumed to exist, then for any $t > 0$, $D(t) > 0$ and $\|v(t)\|_{H^2}$ is bounded (laminar flow maintains regularity). However, as $t \to t^*$ (still in the laminar stage):

(1) $D(t^*) \to 0$, the viscous term cannot dissipate energy, breaking the energy dissipation balance within laminar flow;

(2) The base flow evolution term $\frac{\partial U}{\partial t}$ and the disturbance nonlinear term $(v \cdot \nabla)v$ still transfer energy to the disturbances. However, because there is no viscous energy dissipation to stablize the flow, this will results in rapid increase of the high-order derivatives of the disturbances (Constantin and Foias 1988), causing the local velocity gradient to fail to change smoothly according to the monotonic decay law of $E(t)_k$, but instead increase sharply;

(3) From the Sobolev embedding

$$H^2(\Omega) \hookrightarrow C^1(\Omega), \tag{32}$$

$$\|\nabla u\|_{L^\infty} \leq C\|\Delta u\|_{L^2} \to \infty, \tag{33}$$

which contradicts the regularity requirement of smooth solutions. Thus, t^* is the time of the velocity-discontinuity singularity—this singularity marks the onset of turbulence, but the analysis of the singularity induction process in this paper is strictly limited to the laminar stage $t < t^*$.

In conclusion, the local smooth solution of the 3D Navier-Stokes equations cannot be extended to global time, and global smooth solutions do not exist.

4.3. Judging from the BKM Criterion for Solution Breakdown

The transport equayion of vorticity is very useful in understanding the vorticity variation in the process of turbulent transition (Lugt 1983; Dou 2025a), which shows the relationship between the velocity field and the vorticity field. In the study of regularity of the Euler equation for inviscid fluid, the BKM (Beale-Kato-Majda) criterion was proposed in Beale et al. (1984). Later, this criterion has also been proved to be valid for the Navier-Stokes equation (Kozono and Taniucj 2000; Zhao 2017; Gibbon et al. 2018).

Accoring to Beale et al. (1984), the unbounded velocity gradient will cause the vorticity blow up, violating the criterion of smooth solution,

$$\lim_{t \to t^*} \|\omega(t)\|_{L^\infty} = \infty \tag{34}$$

and

$$\int_0^{t^*} \|\omega(t)\|_{L^\infty} dt = \infty \tag{35}$$

where t^* is the first time beyond which the solution cannot be continued.

5. Discussions

5.1. Laminar Nature and Physical Significance of the Dynamic Decomposition

The “time-averaged base flow-disturbance" decomposition proposed in this paper strictly maintains laminar properties:

(1) The base flow U(t) is the time-averaged state of laminar flow; although it evolves with time, it does not show turbulence irregularity;

(2) The disturbance v(t) is a fluctuation within laminar flow, whose time average is zero and does not develop into a turbulent pulsating structure;

(3) The coupling effect between them is realized through the nonlinear term of disturbances, rather than turbulent Reynolds stress, and remains within the laminar framework.

Physical essence of viscous term abrupt vanishing: Within high-Reynolds-number laminar flow, the enhanced nonlinearity of disturbances drives the evolution of the base flow, causing the Laplacian terms of the two to superimpose and cancel in opposite directions. After viscous dissipation disappears, the flow loses regularization constraints, inevitably triggering a velocity-discontinuity singularity within the laminar stage, which is the starting point of turbulent transition.

5.2. Laminar Constraint Conditions for Key Parameters

1. Reynolds number Re: Must be in the range where “laminar flow can exist but is susceptible to disturbances" ($Re > Re_{critical\ laminar}$ but $Re < Re_{natural\ transition}$). Too high a value directly leads to turbulence, while too low a value makes it difficult for disturbances to drive significant evolution of the base flow;

2. Time-averaging scale Δt: Must satisfy $\Delta t \gg$ disturbance period (ensuring zero time average of disturbances) and $\Delta t \ll t^*$ (ensuring base flow evolution remains laminar). This paper takes $\Delta t \approx 10\tau_v$ (τ_v is the disturbance characteristic time);

3. Critical disturbance intensity δ: Must exceed the “laminar stability threshold", i.e., $\delta > \delta_0$ (δ_0 is the maximum disturbance intensity for laminar linear stability) to drive base flow evolution and trigger abrupt vanishing of the viscous term.

5.3. Limitations and Prospects

In this study, it is assumed that disturbances strengthen uniformly over the entire domain. Future research can investigate the influence of local disturbances (e.g., wall disturbances, inlet disturbances) on the evolution of base flow within laminar flow; meanwhile, combined with linear stability theory, the quantitative relationship between critical disturbance intensity δ and Reynolds number Re can be established like in Dou (2022). In addition, reproducing the process from viscous term abrupt vanishing to singularity appearance in laminar flow through numerical simulation will provide more intuitive verification for the theoretical results. A few of works in this area has been carried out in our group for the wake flow behind a sphere and the flow in Taylor-Couette configurations (Niu et al. 2024; Niu et al. 2025; Zhou et al. 2025a; Zhou et al. 2025b).

5.4. Velocity Discontinuity rather than Velocity Infinity to Leads to Solution Breakdown

In this study, the breakdown of the solutions to the Navier-Stokes equation is the blow up of the norm of the velocity gradient or the blow up of maximum of the vorticity magnitude, rather than the velocity infinity.

Although the conjecture of “velocity infinity" of the Navier-Stokes equation was proposed in Leray (1934), this idea has not been proved, nor disproved (Foias et al. 2004). The singularity at which zero viscous energy loss leads to “velocity discontinuity" has obtained agreement with the results from numerical simulations with direct numerical simulation (DNS) and large eddy simulation (LES) as well as experiments (Dou 2022; Dou 2025a; Niu et al. 2024; Niu et al. 2025; Zhou et al. 2025a; Zhou et al. 2025b).

6. Conclusions

In this study, it is proved that there is non-existence of global smooth solutions for the 3D Navier-Stokes equations based on the “time-averaged flow-disturbance” dynamic decomposition framework for the plane Poiseuille flow. The “time-averaged flow” is taken as the base flow and thus it is updated with the time increase and varies gradually at every points in the flow field. The key findings are that there is point where the viscous energy dissipation is zero, and at this point, “the high-order derivatives of the disturbances” increase rapidly, which leads to the velocity gradient unbounded. The core conclusions are as follows:

1. The base flow within laminar flow evolves with time, and its driving force comes from the time-averaged effect of nonlinear coupling of disturbances.

2. Within high-Reynolds-number and large-disturbance laminar flow, the Laplacian terms of the base flow and disturbances superimpose and cancel in opposite directions at some points, causing the viscous dissipation functional to abruptly vanish in finite time.

3. At the points where the viscous energy loss of the composite flow tends to zero, there is no viscous energy dissipation to stablize the flow. The unsteady term of the base flow and the disturbance nonlinear term still transfer energy to the disturbances, resulting in rapid increase of the high-order derivatives of the disturbances. This leads to the local velocity gradient to increase sharply, which causes velocity discontinuity.

4. This study is mutually evidenced and supported with the work in Dou (2025b). In Dou (2025b), the singularity is due to the mismatch of the energy loss and the velocity magnitude, where the energy loss is zero while the velocity is not zero. In this study, the singularity is characterized by the rapid increase of the high-order derivatives of the disturbances while the there is no viscous damp due to zero energy dissipation.

5. This research provides a new perspective for understanding the laminar-induced singularity mechanism of the Navier-Stokes equation and lays a theoretical foundation for the analysis of the initial stage of turbulent transition.

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