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Article

p-Adic Heisenberg-Robertson-Schrodinger and p-Adic Maccone-Pati Uncertainty Principles

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Abstract

Let $\mathcal{X}$ be a p-adic Hilbert space over a conjugated non-Archimedean valued field $\mathbb{K}$ with $2 \neq 0$. Let $A : \mathcal{D}(A) \subseteq \mathcal{X} \to \mathcal{X}$ and $B : \mathcal{D}(B) \subseteq \mathcal{X} \to \mathcal{X}$ be possibly unbounded self-adjoint linear operators. For $x \in \mathcal{D}(A)$ with $\langle x, x\rangle = 1$, define $\Delta_x(A) := \|Ax - \langle Ax, x\rangle x\|$. Then for all $x \in \mathcal{D}(AB) \cap \mathcal{D}(BA)$ with $\langle x, x\rangle = 1$, we show that (1) $\Delta_x(A) + \Delta_x(B) \geq \max\{\Delta_x(A), \Delta_x(B)\} \geq \frac{\sqrt{\left|\left\langle [A,B]x, x\right\rangle^2 + \left(\langle \{A,B\}x, x\rangle - 2\langle Ax, x\rangle\langle Bx, x\rangle\right)^2\right|}}{\sqrt{|2|}}$ and (2) $\Delta_x(A) + \Delta_x(B) \geq \max\{\Delta_x(A), \Delta_x(B)\} \geq |\langle (A + B)x, y\rangle|$, $\forall y \in \mathcal{X}$ satisfying $\|y\| \leq 1, \langle x, y\rangle = 0$. We call Inequality (1) as p-adic Heisenberg-Robertson-Schrodinger uncertainty principle and Inequality (2) as p-adic Maccone-Pati uncertainty principle.

Keywords: uncertainty principle; p-adic Hilbert space

MSC: 46S10; 12J25; 32P05

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1. Introduction

Let $\mathcal{H}$ be a complex Hilbert space and A be a possibly unbounded self-adjoint linear operator defined on the domain $\mathcal{D}(A) \subseteq \mathcal{H}$. For $h \in \mathcal{D}(A)$ with $\|h\| = 1$, define the uncertainty of A at the point h as

$$\Delta_h(A) := \|Ah - \langle Ah, h\rangle h\| = \sqrt{\|Ah\|^2 - \langle Ah, h\rangle^2}.$$

In 1929, Robertson [1] derived the following mathematical form of the uncertainty principle of Heisenberg derived in 1927 [2]. Recall that for two operators $A : \mathcal{D}(A) \subseteq \mathcal{H} \to \mathcal{H}$ and $B : \mathcal{D}(B) \subseteq \mathcal{H} \to \mathcal{H}$, we define the commutator $[A, B] := AB - BA$ and anti-commutator $\{A, B\} := AB + BA$.

Theorem 1.1. *[1–4] (**Heisenberg-Robertson Uncertainty Principle**) Let $A : \mathcal{D}(A) \subseteq \mathcal{H} \to \mathcal{H}$ and $B : \mathcal{D}(B) \subseteq \mathcal{H} \to \mathcal{H}$ be self-adjoint linear operators. Then for all $h \in \mathcal{D}(AB) \cap \mathcal{D}(BA)$ with $\|h\| = 1$, we have*

$$\frac{1}{2}\left(\Delta_h(A)^2 + \Delta_h(B)^2\right) \geq \left(\frac{\Delta_h(A) + \Delta_h(B)}{2}\right)^2 \geq \Delta_h(A)\Delta_h(B) \geq \frac{1}{2}|\langle [A, B]h, h\rangle|. \tag{1}$$

In 1930, Schrodinger made the following improvement of Inequality (1).

Theorem 1.2. *[5,6] (**Heisenberg-Robertson-Schrodinger Uncertainty Principle**) Let $A : \mathcal{D}(A) \subseteq \mathcal{H} \to \mathcal{H}$ and $B : \mathcal{D}(B) \subseteq \mathcal{H} \to \mathcal{H}$ be self-adjoint linear operators. Then for all $h \in \mathcal{D}(AB) \cap \mathcal{D}(BA)$ with $\|h\| = 1$, we have*

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$$\Delta_h(A)\Delta_h(B) \geq |\langle Ah, Bh\rangle - \langle Ah, h\rangle\langle Bh, h\rangle| = \frac{\sqrt{|\langle [A,B]h, h\rangle|^2 + |\langle \{A,B\}h, h\rangle - 2\langle Ah, h\rangle\langle Bh, h\rangle|^2}}{2}$$
$$= \frac{\sqrt{(\langle \{A,B\}h, h\rangle - 2\langle Ah, h\rangle\langle Bh, h\rangle)^2 - \langle [A,B]h, h\rangle^2}}{2}.$$

Surprisingly, in 2014, Maccone and Pati derived the following uncertainty principle which works for any unit vector which is orthogonal to given unit vector [7].

Theorem 1.3. *[7] (**Maccone-Pati Uncertainty Principle**) Let $A : \mathcal{D}(A) \subseteq \mathcal{H} \to \mathcal{H}$ and $B : \mathcal{D}(B) \subseteq \mathcal{H} \to \mathcal{H}$ be self-adjoint linear operators. Then for all $h \in \mathcal{D}(A) \cap \mathcal{D}(B)$ with $\|h\| = 1$, we have*

$$\Delta_h(A)^2 + \Delta_h(B)^2 \geq \frac{1}{2}\Big(|\langle (A+B)h, k\rangle|^2 + |\langle (A-B)h, k\rangle|^2\Big), \quad \forall k \in \mathcal{H} \text{ satisfying } \|k\| = 1, \langle h, k\rangle = 0.$$

As the study of p-adic Hilbert spaces is equally important as the study of Hilbert spaces, we naturally ask the following question.

Question 1.4. *What are p-adic versions of Theorems 1.2 and 1.3?*

In this paper, we answer Question 1.4.

2. p-Adic Heisenberg-Robertson-Schrodinger Uncertainty Principle and p-Adic Maccone-Pati Uncertainty Principle

We are going to consider the following notion of p-adic Hilbert space which was introduced by Kalisch [8] in 1947.

Definition 2.1. *Let $\mathbb{K}$ be a non-Archimedean valued field (with valuation $|\cdot|$). A map $\sigma : \mathbb{K} \to \mathbb{K}$ is said to be a conjugation if following conditions hold.*

(i) $\sigma(\alpha + \beta) = \sigma(\alpha) + \sigma(\beta)$ *for all* $\alpha, \beta \in \mathbb{K}$.
(ii) $\sigma(\alpha\beta) = \sigma(\alpha)\sigma(\beta)$ *for all* $\alpha, \beta \in \mathbb{K}$.
(iii) $\sigma(\sigma(\alpha)) = \alpha$ *for all* $\alpha \in \mathbb{K}$.
(iv) $|\sigma(\alpha)| = |\alpha|$ *for all* $\alpha \in \mathbb{K}$.

In this case, we say $\mathbb{K}$ is σ-conjugated.

Definition 2.2. *Let $\mathbb{K}$ be a σ-conjugated non-Archimedean valued field (with valuation $|\cdot|$) and $\mathcal{X}$ be a non-Archimedean Banach space (with norm $\|\cdot\|$) over $\mathbb{K}$. We say that $\mathcal{X}$ is a **p-adic Hilbert space** if there is a map (called as p-adic inner product) $\langle\cdot,\cdot\rangle : \mathcal{X} \times \mathcal{X} \to \mathbb{K}$ satisfying following.*

(i) *If $x \in \mathcal{X}$ is such that $\langle x, y\rangle = 0$ for all $y \in \mathcal{X}$, then $x = 0$.*
(ii) $\langle x, y\rangle = \sigma(\langle y, x\rangle)$ *for all* $x, y \in \mathcal{X}$.
(iii) $\langle x, y + z\rangle = \langle x, y\rangle + \langle x, z\rangle$ *for all* $x, y, z \in \mathcal{X}$.
(iv) $\langle \alpha x, y\rangle = \alpha\langle x, y\rangle$ *for all* $x, y \in \mathcal{X}$, *for all* $\alpha \in \mathbb{K}$.
(v) $|\langle x, y\rangle| \leq \|x\|\|y\|$ *for all* $x, y \in \mathcal{X}$.

Following are standard examples.

Example 2.3. *Let $d \in \mathbb{N}$ and $\mathbb{K}$ be a non-Archimedean valued field. Then $\mathbb{K}^d$ is a p-adic Hilbert space w.r.t. ultranorm*

$$\|(x_j)_{j=1}^d\| := \max_{1\leq j\leq d} |x_j|, \quad \forall (x_j)_{j=1}^d \in \mathbb{K}^d$$

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and p-adic inner product

$$\langle (x_j)_{j=1}^d, (y_j)_{j=1}^d \rangle := \sum_{j=1}^d x_j y_j, \quad \forall (x_j)_{j=1}^d, (y_j)_{j=1}^d \in \mathbb{K}^d.$$

Example 2.4. *Let $\mathbb{K}$ be a non-Archimedean valued field. Define*

$$c_0(\mathbb{N}, \mathbb{K}) := \{(x_n)_{n=1}^\infty : x_n \in \mathbb{K}, \forall n \in \mathbb{N}, \lim_{n\to\infty} x_n = 0\}.$$

Then $c_0(\mathbb{N}, \mathbb{K})$ is a p-adic Hilbert space w.r.t. ultranorm

$$\|(x_n)_{n=1}^\infty\| := \sup_{n\in\mathbb{N}} |x_n|, \quad \forall (x_n)_{n=1}^\infty \in c_0(\mathbb{N}, \mathbb{K})$$

and p-adic inner product

$$\langle (x_n)_{n=1}^\infty, (y_n)_{n=1}^\infty \rangle := \sum_{n=1}^\infty x_n y_n, \quad \forall (x_n)_{n=1}^\infty, (y_n)_{n=1}^\infty \in c_0(\mathbb{N}, \mathbb{K}).$$

Let $\mathcal{X}, \mathcal{Y}$ be p-adic Hilbert spaces and $T : \mathcal{X} \to \mathcal{Y}$ be a linear operator. We say that T is adjointable if there is a linear operator, denoted by $T^* : \mathcal{Y} \to \mathcal{X}$ such that $\langle Tx, y\rangle = \langle x, T^* y\rangle, \forall x \in \mathcal{X}, \forall y \in \mathcal{Y}$. Note that (i) in Definition 2.2 says that adjoint, if exists, is unique. An adjointable linear operator $T : \mathcal{X} \to \mathcal{X}$ is said to be self-adjoint if $T^* = T$.
Let A be a possibly unbounded linear operator (need not be self-adjoint) defined on domain $\mathcal{D}(A) \subseteq \mathcal{X}$. For $x \in \mathcal{D}(A)$ with $\langle x, x\rangle = 1$, define the uncertainty of A at the point x as

$$\Delta_x(A) := \|Ax - \langle Ax, x\rangle x\|.$$

We now have the p-adic version of Theorem 1.2.

Theorem 2.5. ***(p-adic Heisenberg-Robertson-Schrodinger Uncertainty Principles)*** *Let $\mathcal{X}$ be a p-adic Hilbert space. Let $A : \mathcal{D}(A) \subseteq \mathcal{X} \to \mathcal{X}$ and $B : \mathcal{D}(B) \subseteq \mathcal{X} \to \mathcal{X}$ be linear operators. Then for all $x \in \mathcal{D}(AB) \cap \mathcal{D}(BA)$ with $\langle x, x\rangle = 1$, we have*

(i)

$$\Delta_x(A)\Delta_x(B) \geq |\langle Ax, Bx\rangle - \langle Ax, x\rangle\langle x, Bx\rangle| = |\langle Bx, Ax\rangle - \langle x, Ax\rangle\langle Bx, x\rangle|.$$

In particular, if A and B are self-adjoint, then

$$\Delta_x(A)\Delta_x(B) \geq |\langle BAx, x\rangle - \langle Ax, x\rangle\langle Bx, x\rangle| = |\langle ABx, x\rangle - \langle Ax, x\rangle\langle Bx, x\rangle|.$$

(ii) If A and B are self-adjoint and $2 \neq 0$, then

$$\Delta_x(A) + \Delta_x(B) \geq \max\{\Delta_x(A), \Delta_x(B)\} \geq \frac{\sqrt{\left|\langle [A, B]x, x\rangle^2 + \left(\langle \{A, B\}x, x\rangle - 2\langle Ax, x\rangle\langle Bx, x\rangle\right)^2\right|}}{\sqrt{|2|}}.$$

(iii) If A and B are adjointable and $2 \neq 0$, then

$$\max\{\Delta_x(A), \Delta_x(B)\} \geq \sqrt{\left|\langle (A^*A + B^*B)x, x\rangle - \frac{\langle (A+B)x, x\rangle\langle x, (A+B)x\rangle + \langle (A-B)x, x\rangle\langle x, (A-B)x\rangle}{2}\right|}.$$

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In particular, if A and B are self-adjoint, then

$$\max\{\Delta_x(A),\Delta_x(B)\} \geq \sqrt{\left|\langle(A^2+B^2)x,x\rangle - \frac{\langle(A+B)x,x\rangle^2+\langle(A-B)x,x\rangle^2}{2}\right|}$$
$$= \sqrt{\left|\frac{\langle(A+B)^2x,x\rangle+\langle(A-B)^2x,x\rangle-\langle(A+B)x,x\rangle^2-\langle(A-B)x,x\rangle^2}{2}\right|}.$$

(iv) If A and B are adjointable, then

$$\Delta_x(A)+\Delta_x(B) \geq \sqrt{|\langle(A^*A-B^*B)x,x\rangle - (\langle Ax,x\rangle\langle x,Ax\rangle - \langle Bx,x\rangle\langle x,Bx\rangle)|}.$$

In particular, if A and B are self-adjoint, then

$$\max\{\Delta_x(A),\Delta_x(B)\} \geq \sqrt{|\langle(A^2-B^2)x,x\rangle - \langle(A+B)x,x\rangle\langle(A-B)x,x\rangle|}.$$

(v)

$$\Delta_x(A)+\Delta_x(B) \geq \max\{\Delta_x(A),\Delta_x(B)\} \geq \sqrt{|\langle(A+B)x,(A+B)x\rangle - \langle(A+B)x,x\rangle\langle x,(A+B)x\rangle|}.$$

(vi)

$$\Delta_x(A)+\Delta_x(B) \geq \max\{\Delta_x(A),\Delta_x(B)\} \geq \sqrt{|\langle(A-B)x,(A-B)x\rangle - \langle(A-B)x,x\rangle\langle x,(A-B)x\rangle|}.$$

Proof. Let $x \in \mathcal{D}(AB)\cap\mathcal{D}(BA)$ be such that $\langle x,x\rangle = 1$.

(i) By using the definition of p-adic inner product,

$$\begin{aligned}\Delta_x(A)\Delta_x(B) &= \|Ax-\langle Ax,x\rangle x\|\|Bx-\langle Bx,x\rangle x\| \\ &\geq |\langle Ax-\langle Ax,x\rangle x, Bx-\langle Bx,x\rangle x\rangle| \\ &= |\langle Ax,Bx\rangle - \langle Ax,x\rangle\langle x,Bx\rangle|.\end{aligned}$$

(ii) By making a direct expansion and simplification, we see that

$$\begin{aligned}&\langle[A,B]x,x\rangle^2 + (\langle\{A,B\}x,x\rangle - 2\langle Ax,x\rangle\langle Bx,x\rangle)^2 \\ &= (\langle ABx,x\rangle - \langle BAx,x\rangle)^2 + \langle\{A,B\}x,x\rangle^2 + 4\langle Ax,x\rangle^2\langle Bx,x\rangle^2 - 4\langle\{A,B\}x,x\rangle\langle Ax,x\rangle\langle Bx,x\rangle \\ &= (\langle ABx,x\rangle - \langle BAx,x\rangle)^2 + (\langle ABx,x\rangle + \langle BAx,x\rangle)^2 + 4\langle Ax,x\rangle^2\langle Bx,x\rangle^2 \\ &\quad - 4\langle ABx,x\rangle\langle Ax,x\rangle\langle Bx,x\rangle - 4\langle BAx,x\rangle\langle Ax,x\rangle\langle Bx,x\rangle \\ &= 2\langle ABx,x\rangle^2 + 2\langle BAx,x\rangle^2 + 4\langle Ax,x\rangle^2\langle Bx,x\rangle^2 - 4\langle ABx,x\rangle\langle Ax,x\rangle\langle Bx,x\rangle - 4\langle BAx,x\rangle\langle Ax,x\rangle\langle Bx,x\rangle \\ &= 2(\langle ABx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2 + 2(\langle BAx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2.\end{aligned}$$

Therefore (by using previous equation and self-adjointness of A and B)

$$\begin{aligned}&\left|\langle[A,B]x,x\rangle^2 + (\langle\{A,B\}x,x\rangle - 2\langle Ax,x\rangle\langle Bx,x\rangle)^2\right| \\ &= |2|\left|(\langle ABx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2 + (\langle BAx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2\right| \\ &\leq |2|\max\left\{\left|(\langle ABx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2\right|, \left|(\langle BAx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle)^2\right|\right\} \\ &= |2|\max\left\{|\langle ABx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle|^2, |\langle BAx,x\rangle - \langle Ax,x\rangle\langle Bx,x\rangle|^2\right\} \\ &\leq |2|\max\left\{\max\{\Delta_x(A)^2,\Delta_x(B)^2\}, \max\{\Delta_x(B)^2,\Delta_x(A)^2\}\right\} = |2|\max\{\Delta_x(A)^2,\Delta_x(B)^2\}.\end{aligned}$$

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(iii) By using the non-Archimedean triangle inequality and the definition of p-adic inner product,

$$\begin{aligned}
\max\{\Delta_x(A), \Delta_x(B)\} &= \max\{\|Ax - \langle Ax, x\rangle x\|, \|Bx - \langle Bx, x\rangle x\|\} \\
&\geq \max\{\sqrt{|\langle Ax - \langle Ax, x\rangle x, Ax - \langle Ax, x\rangle x\rangle|}, \sqrt{|\langle Bx - \langle Bx, x\rangle x, Bx - \langle Bx, x\rangle x\rangle|}\} \\
&= \max\{\sqrt{|\langle Ax, Ax\rangle - \langle Ax, x\rangle\langle x, Ax\rangle|}, \sqrt{|\langle Bx, Bx\rangle - \langle Bx, x\rangle\langle x, Bx\rangle|}\} \\
&= \sqrt{\max\{|\langle Ax, Ax\rangle - \langle Ax, x\rangle\langle x, Ax\rangle|, |\langle Bx, Bx\rangle - \langle Bx, x\rangle\langle x, Bx\rangle|\}} \\
&\geq \sqrt{|\langle Ax, Ax\rangle - \langle Ax, x\rangle\langle x, Ax\rangle + \langle Bx, Bx\rangle - \langle Bx, x\rangle\langle x, Bx\rangle|} \\
&= \sqrt{|\langle A^*Ax, x\rangle + \langle B^*Bx, x\rangle - (\langle Ax, x\rangle\langle x, Ax\rangle + \langle Bx, x\rangle\langle x, Bx\rangle)|} \\
&= \sqrt{\left|\langle (A^*A + B^*B)x, x\rangle - \frac{\langle (A+B)x, x\rangle\langle x, (A+B)x\rangle + \langle (A-B)x, x\rangle\langle x, (A-B)x\rangle}{2}\right|}.
\end{aligned}$$

(iv) Using initial calculations in (iii),

$$\begin{aligned}
\max\{\Delta_x(A), \Delta_x(B)\} &\geq \sqrt{\max\{|\langle Ax, Ax\rangle - \langle Ax, x\rangle\langle x, Ax\rangle|, |\langle Bx, Bx\rangle - \langle Bx, x\rangle\langle x, Bx\rangle|\}} \\
&\geq \sqrt{|\langle Ax, Ax\rangle - \langle Ax, x\rangle\langle x, Ax\rangle - \langle Bx, Bx\rangle + \langle Bx, x\rangle\langle x, Bx\rangle|} \\
&= \sqrt{|\langle A^*Ax, x\rangle - \langle B^*Bx, x\rangle - (\langle Ax, x\rangle\langle x, Ax\rangle - \langle Bx, x\rangle\langle x, Bx\rangle)|} \\
&= \sqrt{|\langle (A^*A - B^*B)x, x\rangle - (\langle Ax, x\rangle\langle x, Ax\rangle - \langle Bx, x\rangle\langle x, Bx\rangle)|}
\end{aligned}$$

(v) Using ultrametric inequality first and then using p-adic inner product we get

$$\begin{aligned}
&\max\{\Delta_x(A), \Delta_x(B)\} = \max\{\|Ax - \langle Ax, x\rangle x\|, \|Bx - \langle Bx, x\rangle x\|\} \\
&\geq \|Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x\| \\
&\geq \sqrt{|\langle Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x, Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x\rangle|} \\
&= \sqrt{|\langle Ax, Ax\rangle + \langle Bx, Bx\rangle + \langle Ax, Bx\rangle + \langle Bx, Ax\rangle - \langle Ax, x\rangle\langle x, Bx\rangle - \langle x, Ax\rangle\langle Bx, x\rangle - \langle Ax, x\rangle\langle x, Ax\rangle - \langle Bx, x\rangle\langle x, Bx\rangle|} \\
&= \sqrt{|\langle (A+B)x, (A+B)x\rangle - \langle (A+B)x, x\rangle\langle x, (A+B)x\rangle|}.
\end{aligned}$$

(vi) Using initial calculations in (v),

$$\begin{aligned}
\max\{\Delta_x(A), \Delta_x(B)\} &= \max\{\|Ax - \langle Ax, x\rangle x\|, \|Bx - \langle Bx, x\rangle x\|\} \\
&\geq \|Ax - \langle Ax, x\rangle x - Bx + \langle Bx, x\rangle x\| \\
&\geq \sqrt{|\langle Ax - \langle Ax, x\rangle x - Bx + \langle Bx, x\rangle x, Ax - \langle Ax, x\rangle x - Bx + \langle Bx, x\rangle x\rangle|} \\
&= \sqrt{|\langle (A-B)x, (A-B)x\rangle - \langle (A-B)x, x\rangle\langle x, (A-B)x\rangle|}.
\end{aligned}$$

□

We next derive p-adic version of Theorem 1.3.

Theorem 2.6. *(**p-adic Maccone-Pati Uncertainty Principle**) Let $\mathcal{X}$ be a p-adic Hilbert space. Let $A : \mathcal{D}(A) \subseteq \mathcal{X} \to \mathcal{X}$ and $B : \mathcal{D}(B) \subseteq \mathcal{X} \to \mathcal{X}$ be linear operators. Then for all $x \in \mathcal{D}(A) \cap \mathcal{D}(B)$ with $\langle x, x\rangle = 1$, we have*

$$\Delta_x(A) + \Delta_x(B) \geq \max\{\Delta_x(A), \Delta_x(B)\} \geq |\langle (A+B)x, y\rangle|, \quad \forall y \in \mathcal{X} \text{ satisfying } \|y\| \leq 1, \langle x, y\rangle = 0$$

and

$$\Delta_x(A) + \Delta_x(B) \geq \max\{\Delta_x(A), \Delta_x(B)\} \geq |\langle (A-B)x, y\rangle|, \quad \forall y \in \mathcal{X} \text{ satisfying } \|y\| \leq 1, \langle x, y\rangle = 0.$$

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Proof. Let $x \in \mathcal{D}(A) \cap \mathcal{D}(B)$ be such that $\langle x, x\rangle = 1$. Let $y \in \mathcal{X}$ satisfies $\|y\| \leq 1$ and $\langle x, y\rangle = 0$. Then

$$\begin{aligned}
|\langle (A+B)x, y\rangle| &= |\langle Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x, y\rangle| \\
&\leq \|Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x\| \|y\| \\
&\leq \|Ax - \langle Ax, x\rangle x + Bx - \langle Bx, x\rangle x\| \\
&\leq \max\{\|Ax - \langle Ax, x\rangle x\|, \|Bx - \langle Bx, x\rangle x\|\} \\
&= \max\{\Delta_x(A), \Delta_x(B)\}.
\end{aligned}$$

□

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