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A Perspective Discovery of Hidden Fundamental Principle: GENERIC Stochastic Formalism, with Gauge Integration for Unitary Evolution: A Unified QED Framework

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Article

A Perspective Discovery of Hidden Fundamental Principle: GENERIC Stochastic Formalism, with Gauge Integration for Unitary Evolution: A Unified QED Framework

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Abstract

We leverage and clarify a unified nonequilibrium framework in which stochastic GENERIC dynamics, entropy production, gauge evolution, and relativistic structure are treated within a single thermodynamically admissible formalism. Entropy production is identified as a geometric driver capable of generating effective spacetime structure, observer acceleration, and horizon formation. Under entropy-balance conditions, the GENERIC system undergoes Hamiltonian closure, yielding unitary evolution and forcing gauge unification at a dynamically selected scale. The framework introduces a stochastic entropy-based controller whose deterministic limit reproduces renormalization-group (RG) flow under a logarithmic entropy clock $\sigma = \ln(\mu/\mu_c)$. Within this mapping, thermodynamic entropy flux corresponds directly to gauge-theoretic β -functions, and the standard one- and two-loop structures—including threshold effects—emerge as physically valid limits of the controller dynamics. This demonstrates that loop-level assumptions in gauge unification arise from entropy production under GENERIC self-consistency, rather than being imposed empirically. A corrective entropy channel is shown to be required for global entropy balance in cosmology. Incorporating this channel enforces gauge-level unification (“forced unification”) while preserving thermodynamic admissibility and field-theoretic consistency. The resulting three-channel entropy structure—matter, geometric (horizon), and dissipative—provides an operator-level mechanism for unification across quantum, gauge, and cosmological scales, identifying the additive entropy contribution as a potential source of previously unaccounted physical structure. At entropy balance, dissipation vanishes and the GENERIC dynamics reduce to purely Hamiltonian flow, yielding unitary evolution and fixing the unification scale without external normalization. In this sense, renormalization is not an auxiliary prescription but follows internally from the gauge framework itself, defining a *no-normalization condition*. The construction integrates emergent gravity, gauge equivalence, and thermodynamic closure within a single dynamical system, demonstrating that conventional unification appears as a static subset of a more general entropy-driven dynamical unification. As a consequences we realized that gravity is of discrete quanta, therefore, clarifying the physical meaning of geometric structure: as quantum in nature.

Keywords: a unified scientific framework; GENERIC; fundamentals; high energy physics

1. Introduction

Nonequilibrium phenomena arise across physics whenever irreversible processes, dissipation, and multiscale dynamics play a central role, from complex fluids and soft matter to relativistic and quantum systems. Among the most general and structurally consistent approaches to nonequilibrium dynamics is the GENERIC formalism (General Equation for Non-Equilibrium Reversible–Irreversible Coupling), developed by Grmela and Öttinger, which provides a unified framework for constructing evolution equations that simultaneously conserve energy, ensure nonnegative entropy production, and remain invariant under coarse-graining [1–3]. GENERIC has been successfully applied across a wide range of systems, including complex fluids, interfaces, multiphase flows, relativistic media, and open

quantum systems [4,5]. Importantly, while the formalism guarantees thermodynamic admissibility and structural consistency, it remains agnostic to the physical origin and geometric interpretation of entropy production, treating spacetime and kinematics as externally prescribed structures. In particular, although compatibility with special relativity and gravitational settings has been demonstrated [6–9], the emergence of spacetime structure itself is not addressed within the GENERIC framework. The present work builds on the structural foundations of GENERIC while providing a concrete physical and geometric interpretation of entropy-driven dynamics, showing how effective relativistic kinematics, observer-dependent horizons, and spacetime structure can arise from the underlying thermodynamic state-space evolution.

The main contributions of this work are both a total synthesis, stochastic interpretations, and identification of entropy production as a geometric driver that generates effective spacetime structure, observer acceleration, and horizon formation, rather than treating entropy as a scalar bookkeeping variable [Reference Table 1 Results Section]. We therefore provide an explicit unified QED framework for use in gauge integration.

2. Results Generalized

Table 1. Results: Extended Einstein framework incorporating the dissipative tensor $Q_{\mu\nu}$ and its connection to information-theoretic entropy production. This formulation reflects the GENERIC principle that geometric evolution encodes real physical processes—irreversible dissipation and entropy flow—rather than purely symbolic structures. In the zero-diffusion, deterministic limit, the framework reduces to inverse optimal control formulations, consistent with GENERIC projections and Hamiltonian structure.

Domain	What GR + Dissipative Tensor $Q_{\mu\nu}$ Adds	Why It's Valuable For Future Applications
Cosmology	Models entropy production, horizon formation, and irreversible structure formation without violating General Relativity, consistent with GENERIC-based causal dissipative cosmology [7,8].	Provides a thermodynamically admissible description of entropy flow across cosmological horizons and gravitational collapse, extending GENERIC treatments of nonequilibrium gravity [9].
Plasma Physics & Magnetohydrodynamics	The tensor $Q_{\mu\nu}$ acts as a covariant dissipative channel analogous to GENERIC friction operators used in relativistic transport and plasma models [1,6].	Enables thermodynamically consistent relativistic plasma modeling for stellar interiors, accretion disks, and fusion plasmas, extending Hamiltonian plasma formulations with entropy production [10,11].
Atmospheric & Ocean Dynamics	Encodes energy and entropy fluxes geometrically, consistent with GENERIC-based formulations of fluid dynamics and turbulence [1,5].	Provides a multiscale, entropy-consistent framework for geophysical flows without ad hoc dissipation parameters, aligning with Hamiltonian-GENERIC approaches to climate and turbulence modeling [12,13].
Geophysics / Solid Earth	Stress, strain, and thermal dissipation are represented as curvature and entropy-production effects within a local spacetime manifold, consistent with GENERIC continuum mechanics [5].	Offers a unified, thermodynamically admissible framework for mantle convection, fault dynamics, and crustal deformation grounded in GENERIC and Hamiltonian continuum theory [14,15].
Biological Systems / Neuroscience	Connects energy flow, entropy regulation, and information processing through $Q_{\mu\nu}$, consistent with GENERIC's treatment of open, nonequilibrium systems [5].	Provides a geometric-thermodynamic foundation for cognition and self-organization compatible with entropy minimization and the free-energy principle [16?].
Computational Modeling	Transforms complex temporal dynamics into geometrically constrained evolution equations obeying conservation and entropy laws, as required by GENERIC [1,4].	Enables stable, entropy-consistent numerical solvers for high-dimensional systems such as climate models, relativistic plasmas, and biological networks.

3. Gauge-Based Construction and Entropic Unification

We construct a cosmological framework in which gauge unification emerges as a structural consequence of renormalization-group (RG) flow subject to entropy constraints; under mild assumptions, the unification scale is fixed by the particle spectrum and corresponds to a vanishing-dissipation point in a GENERIC system.

Lemma 1 (Vanishing-Threshold Crossing). *If threshold corrections vanish ($\Delta_i = 0$), the one-loop pairwise crossing condition reduces to*

$$\sigma_{ij} = \frac{2\pi}{b_i - b_j} (\alpha_i^{-1}(\mu_0) - \alpha_j^{-1}(\mu_0)),$$

where $\sigma = \ln(\mu / \mu_0)$; hence the crossing scale σ_c is uniquely fixed by $\{b_i\}$ and the couplings at μ_0 .

Proof. At one loop

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \Delta_i - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0},$$

so subtracting i, j and setting $\Delta_i = \Delta_j = 0$ gives

$$0 = (\alpha_i^{-1}(\mu_0) - \alpha_j^{-1}(\mu_0)) - \frac{b_i - b_j}{2\pi} \ln \frac{\mu_c}{\mu_0},$$

i.e.

$$\sigma_{ij} = \ln(\mu_c / \mu_0) = \frac{2\pi}{b_i - b_j} (\alpha_i^{-1}(\mu_0) - \alpha_j^{-1}(\mu_0)).$$

□

The double loop proof is provided in the Appendix at the readers freedom.

Lemma 2 (GENERIC Dissipation). *Let $\dot{y} = L\nabla H + M\nabla S$ with $L^\top = -L$ and $M \succeq 0$; then $\dot{H} = 0$ and $\dot{S} \geq 0$.*

Lemma 3 (Cosmological Entropy Channels). *In an expanding cosmology, the total entropy production decomposes as*

$$\dot{S}_{\text{tot}} = \dot{S}_{\text{m}} + \dot{S}_{\text{h}},$$

where \dot{S}_{m} is bulk (matter) entropy production and \dot{S}_{h} is geometric (horizon) entropy production.

Lemma 4 (GENERIC Closure). *A finite or countable sum of GENERIC subsystems may be represented as a single GENERIC system:*

$$\sum_n (L_n \nabla H_n + M_n \nabla S_n) = L \nabla H + M \nabla S$$

with $L = \sum_n L_n$ and $M = \sum_n M_n$.

Lemma 5 (Global GENERIC Structure). *If global H and S exist over cosmological time, then the universe admits a GENERIC description*

$$\dot{y} = L(t) \nabla H(t) + M(t) \nabla S(t)$$

for $t \in [0, \infty)$.

Lemma 6 (Emergent Gravity). *Near local equilibrium, applying $\delta Q = T dS$ to local horizons reproduces Einstein's equations (Jacobson), so gravitational dynamics are consistent with and contained within the GENERIC entropy structure.*

Lemma 7 (Hamiltonian Closure at Vanishing Dissipation). *If there exists a scale at which $\dot{S}_{\text{tot}} = 0$, then $M\nabla S = 0$ and the GENERIC evolution reduces to a purely Hamiltonian flow*

$$\dot{y} = L\nabla H.$$

Lemma 8 (Entropy Balance Scale). *Assume \dot{S}_{m} and \dot{S}_{h} dominate in different regimes and are continuous in the RG/cosmological clock; then there exists at least one scale where $\dot{S}_{\text{m}} = \dot{S}_{\text{h}}$, implying (by Lemma 7) Hamiltonian closure at that scale.*

Lemma 9 (Forced Gauge Unification in GENERIC). *At an entropy-balanced scale satisfying Lemma 8, the GENERIC closure $M\nabla S = 0$ holds; combined with the RG crossing structure of Lemma 1, this identifies a unification scale where gauge couplings coincide (up to higher-loop corrections).*

4. Failure of the One-Loop Unification Diagnostic and Entropy-Based Replacement

4.1. Limitation of the One-Loop Unification Lemma

At one loop with vanishing thresholds, the conventional unification condition reads

$$\Delta\alpha_{ij}^{-1}(\mu_0) = \frac{(b_i - b_j)}{2\pi} \ln \frac{\mu_U}{\mu_0}, \quad (b_i - b_j) = \frac{2\pi \Delta\alpha_{ij}^{-1}(\mu_0)}{\ln(\mu_U/\mu_0)}. \quad (1)$$

For $\mu_0 \neq \mu_U$, this finite-difference form is well defined. However, in the unification limit $\mu_0 \rightarrow \mu_U$, two pathologies arise:

- If $\Delta\alpha_{ij}^{-1} \neq 0$, the expression diverges.
- If $\Delta\alpha_{ij}^{-1} = 0$, the expression collapses to an indeterminate 0/0 form.

In the latter case, the correct limiting expression is the differential form

$$(b_i - b_j) = -2\pi \left. \frac{d}{d \ln \mu} \Delta\alpha_{ij}^{-1}(\mu) \right|_{\mu=\mu_U}, \quad (2)$$

which is simply the renormalization-group equation itself.

This demonstrates that the one-loop unification lemma ceases to be diagnostic at unification: RG slopes alone cannot extract ultraviolet microphysics at μ_U . The apparent singularity is not physical but instead signals missing information in the effective description.

In particle-physics language, the 0/0 limit reflects an underdetermined UV completion. In cosmological language, it signals a missing entropy-production channel. Unification remains preserved in form, but its scale cannot be fixed using RG data alone.

4.2. Entropy-Based Reformulation of the Unification Condition

To restore a meaningful diagnostic, we replace the one-loop unification lemma with an entropy-balance condition formulated in a cosmological setting.

We define two canonical entropy-production channels in an FRW cosmology:

Matter (bulk-viscous channel).

$$\dot{S}_m(T) = C_m \left(\frac{\zeta}{s} \right) (T) T, \quad C_m = \text{constant}. \quad (3)$$

Horizon (Bekenstein–Hawking channel).

$$\dot{S}_h(T) = C_h T^{-2}, \quad C_h = \text{constant}. \quad (4)$$

At high temperature, these channels vanish with different scalings,

$$\dot{S}_m(T) \sim T^{-3}, \quad \dot{S}_h(T) \sim T^{-2}. \quad (5)$$

4.3. Controller Formulation

Introduce the RG-like controller variable

$$\mu = \mu_0 e^\sigma, \quad \sigma = \ln \left(\frac{\mu}{\mu_0} \right). \quad (6)$$

Define the entropy controller

$$\dot{\sigma} = -\kappa G(\sigma), \quad G(\sigma) = \dot{S}_m(\mu_0 e^\sigma) - \dot{S}_h(\mu_0 e^\sigma). \quad (7)$$

Equilibrium requires

$$G(\sigma^*) = 0 \iff \dot{S}_m(\sigma^*) = \dot{S}_h(\sigma^*). \quad (8)$$

4.4. Failure of the Two-Channel Entropy Model

Assuming $(\zeta/s)(T) \sim T^{-4}$ at high temperature, one finds

$$\dot{S}_m(T) \sim \frac{C_m}{T^3}, \quad \dot{S}_h(T) \sim \frac{C_h}{T^2}. \quad (9)$$

Therefore,

$$\lim_{T \rightarrow \infty} \frac{\dot{S}_m(T)}{\dot{S}_h(T)} \sim \frac{1}{T} \rightarrow 0. \quad (10)$$

The matter and horizon channels vanish at different rates and cannot balance except at a fine-tuned point. Thus, a two-channel entropy controller fails to enforce unification generically.

4.5. Resolution via a Missing Entropy Channel

To restore entropy balance, we impose

$$\dot{S}_m(T) + \dot{S}_3(T) = \dot{S}_h(T), \quad (11)$$

where the additional channel is defined by

$$\dot{S}_3(T) = \dot{S}_h(T) - \dot{S}_m(T). \quad (12)$$

This term is not arbitrary; it is required by entropy consistency. It satisfies:

- $\dot{S}_3 > 0$ at unification,
- $\dot{S}_3 \sim T^{-2}$ at high temperature,
- $\dot{S}_3 \simeq -\dot{S}_m$ at low temperature.

The correction channel carries no conservative energy and must therefore reside in the dissipative sector of the GENERIC framework,

$$\dot{S}_3 \geq 0. \quad (13)$$

The total entropy production is

$$\dot{S}_{\text{tot}} = \dot{S}_m + \dot{S}_h + \dot{S}_3 \geq 0. \quad (14)$$

4.6. Three-Channel Entropy Diagnostic

Define

$$\dot{S}_{\text{tot}} = \dot{S}_m + \dot{S}_h + \dot{S}_3, \quad (15)$$

and introduce the diagnostic ratio

$$R(\sigma) = \frac{\dot{S}_m + \dot{S}_3}{\dot{S}_h}. \quad (16)$$

Unification is defined by

$$R(\sigma_c) = 1. \quad (17)$$

Near σ_c , applying L'Hôpital's rule yields

$$\lim_{\sigma \rightarrow \sigma_c} R(\sigma) = \frac{S''_m(\sigma_c) + S''_3(\sigma_c)}{S''_h(\sigma_c)} < \infty. \quad (18)$$

Thus, the apparent divergence encountered in the one-loop unification lemma is parametrization-dependent rather than physical. The entropy-based diagnostic remains smooth and well defined at unification, marking the boundary between dissipative and Hamiltonian regimes.

Structural Insight Box. The GENERIC framework developed here admits both stochastic and deterministic interpretations. Expressing the Gauge component as a controller demonstrates that entropy balance and gauge coupling encode the same physical structure, enforcing a forced cosmological requirement. As a result, GUT entropy is intrinsically coupled to the dynamics, and additive entropy analysis aligns with microscopic and RG-based descriptions.

□

5. How Controllers Represent Fundamental Principles (Discretely)

As a primer to fundamental control theory, see Refs. [17–20].

The most general form of a stochastic controller may be written as

$$\dot{\sigma} = -\kappa G(\sigma, t) + \eta u(t) + 2D \zeta(t), \quad (19)$$

where $\zeta(t)$ is zero-mean noise and D is the diffusion (noise intensity). In a physical setting, diffusion corresponds to additive heating, while noise represents quantum or microscopic corrections. A purely deterministic controller contains no diffusion term.

Using entropy balance, we have shown that the cosmological Universe admits a representation in which the controller error is

$$G(\sigma, t) \equiv \sum_i \alpha_i \dot{S}_i(\sigma, t), \quad (20)$$

with α_i fixed by the physical role of each entropy channel.

When external forcing and noise vanish, $u(t) \rightarrow 0$ and $D \rightarrow 0$, Eq. (19) reduces to the deterministic feedback law

$$\dot{\sigma} = -\kappa G(\sigma, t).$$

At steady state,

$$\dot{\sigma} = 0 \iff G(\sigma^*) = 0 \iff \sum_i \alpha_i \dot{S}_i(\sigma^*) = 0. \quad (21)$$

Linearizing about a balance point σ_c yields

$$\delta\dot{\sigma} \approx -\kappa (\partial_\sigma G)|_{\sigma_c} \delta\sigma + \eta u(t) + 2D \zeta(t). \quad (22)$$

For $\kappa > 0$ and $(\partial_\sigma G)|_{\sigma_c} > 0$, the feedback is negative and the equilibrium is stable, with perturbations decaying in time. Here κ sets the relaxation rate and η determines how strongly external disturbances couple to σ .

5.1. Control Notation in the Gauge Framework

Setup and notation.

We introduce the logarithmic RG/entropy clock as the controller variable,

$$\sigma \equiv \ln\left(\frac{\mu}{\mu_0}\right), \quad (23)$$

assumed monotone along the RG flow. Entropy channels are functions of σ ,

$$\dot{S}_i = \dot{S}_i(\sigma, t), \quad i \in \{m, 3, h\},$$

corresponding to matter (m), an additive/dissipative channel (3), and the horizon (h). The sign vector is fixed as

$$\alpha_m = \alpha_3 = +1, \quad \alpha_h = -1.$$

Reparameterization by σ .

Assuming

$$S_i = S_i(\sigma), \quad \sigma = \ln(\mu/\mu_0) \text{ monotone}, \quad (24)$$

the chain rule gives

$$\dot{S}_i = \frac{dS_i}{d\sigma} \dot{\sigma} \equiv S'_i(\sigma) \dot{\sigma}. \quad (25)$$

Thus the dynamics may be expressed in terms of σ rather than t without loss of information. Monotonicity preserves ordering, and the GENERIC degeneracy condition $\nabla H \cdot M \nabla S = 0$ is maintained under this reparameterization. In the Hamiltonian limit $M \nabla S = 0$, dissipation vanishes and only the reversible flow $L \nabla H$ remains.

Compact controller form.

Define

$$\dot{\mathbf{S}}(\sigma, t) = (\dot{S}_m, \dot{S}_3, \dot{S}_h)^\top, \quad \boldsymbol{\alpha} = (1, 1, -1)^\top.$$

Neglecting noise, the controller equation becomes

$$\dot{\sigma} = -\kappa \boldsymbol{\alpha}^\top \dot{\mathbf{S}}(\sigma, t) + \eta u(t). \quad (26)$$

Connection to the ratio diagnostic.

Introduce the balance ratio

$$R(\sigma, t) \equiv \frac{\dot{S}_m(\sigma, t) + \dot{S}_3(\sigma, t)}{\dot{S}_h(\sigma, t)}. \quad (27)$$

Using Eq. (20),

$$G(\sigma, t) = \dot{S}_m + \dot{S}_3 - \dot{S}_h = \dot{S}_h(\sigma, t) [R(\sigma, t) - 1]. \quad (28)$$

Hence $G = 0$ if and only if $R = 1$, and the controller drives the system toward the balance (unification) condition.

5.2. Summary

Replacing renormalization time by the entropy clock

$$\sigma = \ln\left(\frac{\mu}{\mu_0}\right)$$

renders the energy scale an entropy-driven variable. All couplings and entropy rates evolve as functions of σ ,

$$\dot{S}_i(\sigma), \quad i \in \{m, 3, h\}.$$

The control law

$$\dot{\sigma} = -\kappa G(\sigma), \quad G(\sigma) = \dot{S}_m + \dot{S}_3 - \dot{S}_h, \quad (29)$$

with $\kappa > 0$, replaces the multi-operator RG system with a single scalar equation. Unification is thereby recast as a dynamical entropy-balancing process, fully consistent with the Gauge framework.

5.3. Stochastic Regime and Balance Failure

Far from unification, entropy channels fluctuate stochastically, as expected in a GENERIC system. The controller must then be interpreted in the mean,

$$\langle \dot{\sigma} \rangle = -\kappa \langle G(\sigma) \rangle. \quad (30)$$

In this regime, $G(\sigma)$ acts as thermodynamic noise: it does not vanish instantaneously, but its mean trends toward equilibrium. Consequently, $R(\sigma)$ oscillates about unity, reflecting incomplete balance between microscopic and macroscopic entropy production.

On removing explicit noise and heating terms.

Taking the limits $u(t) \rightarrow 0$, $D \rightarrow 0$, and $\dot{S}_3 \rightarrow 0$ does not remove quantum or thermal physics. By embedding the additive entropy channel S_3 within the total entropy functional, all admissible stochastic, quantum, and thermal effects are already encoded in the GENERIC structure. These limits therefore describe the global equilibrium state in which entropy channels balance and fluctuations are thermodynamically absorbed.

Final interpretation.

The final controller equation requires no explicit noise or diffusion terms because entropy itself carries stochastic, quantum, and thermal information through its gradient structure. Entropy is thus an active driver of dynamics, not a passive bookkeeping quantity. The resulting closure is complete and self-consistent across all scales. Recent work by Beyen and Maes [21] provides a complementary foundation for the reversible–irreversible decomposition used here, while the present framework extends these ideas to scale evolution and renormalization-group behavior.

6. Resolving the Controller and Its Coupling to Thresholds

Solution Terms and Fundamental Control Reference.

We define the entropy–channel ratio in σ -space as

$$R(\sigma) = \frac{S'_m(\sigma) + S'_3(\sigma)}{S'_h(\sigma)}. \quad (31)$$

The corresponding imbalance in σ -space is

$$\Delta_\sigma(\sigma) \equiv S'_h(\sigma) - (S'_m(\sigma) + S'_3(\sigma)) = S'_h(\sigma) [1 - R(\sigma)]. \quad (32)$$

The analogous ratio in the time domain is

$$R_t(\sigma) = \frac{\dot{S}_m(\sigma) + \dot{S}_3(\sigma)}{\dot{S}_h(\sigma)}. \quad (33)$$

We define the controller error in the time domain as

$$G(\sigma) \equiv \dot{S}_m(\sigma) + \dot{S}_3(\sigma) - \dot{S}_h(\sigma) = \dot{S}_h(\sigma) [R_t(\sigma) - 1]. \quad (34)$$

The time–domain imbalance (preferred for control) is

$$\Delta_t(\sigma) \equiv \dot{S}_h(\sigma) - (\dot{S}_m(\sigma) + \dot{S}_3(\sigma)), \quad \text{so that} \quad G(\sigma) = -\Delta_t(\sigma). \quad (35)$$

The σ -space imbalance (preferred for diagnostics) is therefore

$$\Delta_\sigma(\sigma) = S'_h(\sigma) - (S'_m(\sigma) + S'_3(\sigma)) = S'_h(\sigma) [1 - R(\sigma)]. \quad (36)$$

The chain rule links the two domains,

$$\begin{aligned}\dot{X}(\sigma) &= X'(\sigma) \dot{\sigma}, \\ \Delta_t(\sigma) &= \dot{\sigma} \Delta_\sigma(\sigma), \\ G(\sigma) &= -\dot{\sigma} \Delta_\sigma(\sigma).\end{aligned}\tag{37}$$

The control law (standard negative feedback) then takes the form

$$\dot{\sigma} = -\kappa G(\sigma) \iff \dot{\sigma} = \kappa \Delta_t(\sigma),\tag{38}$$

with $\kappa > 0$ a relaxation coefficient.

The balance (unification) condition is satisfied when

$$R = 1 \iff \Delta_\sigma = 0 \iff \Delta_t = 0 \iff G = 0.\tag{39}$$

Stochastic Controller Evolution.

In the stochastic formulation, the entropy–balance controller evolves according to

$$\dot{\sigma} = -\kappa G(\sigma),\tag{40}$$

where

$$G(\sigma) \equiv \dot{S}_m(\sigma) + \dot{S}_3(\sigma) - \dot{S}_h(\sigma)$$

is the instantaneous imbalance between entropy channels.

Approximate (Threshold–Free) Controller.

When no explicit threshold is imposed, the system remains stochastic and the controller evolves cumulatively. Integrating from a reference point σ_0 yields

$$\sigma - \sigma_0 = -\kappa \int_{t_0}^t G(\sigma(t')) dt'.\tag{41}$$

Since $G(\sigma)$ is the imbalance $\Delta(\sigma)$,

$$\sigma - \sigma_0 = -\kappa \int_{t_0}^t \Delta(\sigma(t')) dt'.\tag{42}$$

In this approximate regime, σ records the integrated imbalance stochastically. No boundary or normalization is imposed, and balance is realized only in the mean,

$$\langle \dot{\sigma} \rangle = -\kappa \langle \Delta(\sigma) \rangle.$$

Exact (Threshold–Defined) Controller.

To obtain an exact deterministic controller, we impose a threshold condition defining the balance manifold,

$$\Delta(\sigma_c) = 0, \quad \sigma_c = 0.$$

This fixes the origin of the σ coordinate system. Repeating the integration gives

$$\sigma - \sigma_c = -\kappa \int_{t_c}^t \Delta(\sigma(t')) dt'.\tag{43}$$

Since $\sigma_c = 0$, we write

$$\sigma = -\kappa \int_{t_c}^t \Delta(\sigma(t')) dt' \equiv \Delta\sigma.\tag{44}$$

Thus, σ is inherently a *difference variable*, measuring deviation from the balanced (unified) state. Imposing the threshold converts the stochastic approximation into a deterministic law, anchoring the entropy dynamics to a fixed reference—analogue to gauge fixing at unification. By definition, $\sigma = 0$ at unification.

In the zero-noise limit of stochastic control, the system becomes self-consistent. With the entropy modification, we therefore obtain cosmological entropy unification and microscopic unification through forced gauge unification.

Interpretation and Sign Consistency.

Defining $G(\sigma) = -\Delta(\sigma)$ ensures that

$$\dot{\sigma} = -\kappa G(\sigma) = \kappa \Delta(\sigma) \quad (45)$$

implements negative feedback. If $\Delta > 0$ (geometry excess), then $\dot{\sigma} > 0$; if $\Delta < 0$ (matter excess), then $\dot{\sigma} < 0$. In both cases, the system is driven toward the equilibrium manifold $\Delta(\sigma_c) = 0$. This guarantees dynamic stability and convergence to the balanced fixed point.

Concept	Before normalization	After threshold $\sigma_c = 0$
σ definition	$\ln(\mu/\mu_0)$	$\ln(\mu/\mu_c)$
Free constant	μ_0 arbitrary	$\mu_0 = \mu_c$
Balance point	$\sigma = \sigma_c$	$\sigma = 0$
External normalization	Yes	No
Controller target	$\sigma \rightarrow \sigma_c$	$\sigma \rightarrow 0$

6.1. Relation Between Controller and Gauge Thresholds

Mathematical Consistency.

The scalar imbalance is a projection of the full tensorial structure. For state variables x_i , the total entropy production is

$$\dot{S}_{\text{tot}} = \frac{\partial S}{\partial x_i} M_{ij} \frac{\partial S}{\partial x_j}, \quad (46)$$

with M_{ij} symmetric and positive semidefinite. The imbalance tensor is

$$\Delta_{ij} = M_{ij} (\partial_i S_h - \partial_i (S_m + S_3)) (\partial_j S_h - \partial_j (S_m + S_3)). \quad (47)$$

In one dimension this reduces to

$$\Delta = M (\partial_\sigma S_h - \partial_\sigma S_m - \partial_\sigma S_3)^2. \quad (48)$$

At unification,

$$\Delta_{ij}(0) = 0 \quad \forall i, j, \quad (49)$$

enforcing directional balance,

$$\partial_i S_h = \partial_i (S_m + S_3). \quad (50)$$

Thus the controller threshold and gauge threshold coincide exactly.

Physical Equivalence of Thresholds.

Embedding the RG clock into the controller enforces identical physical structure. Consequently, the unification point of the RG gauge framework is equivalent to the vanishing-threshold condition in the stochastic GENERIC analysis.

6.2. Section Summary

At unification, stochastic terms vanish by definition,

$$G(\sigma_c) = 0 \quad \Rightarrow \quad \dot{S}_m + \dot{S}_3 = \dot{S}_h, \quad (51)$$

and $R = 1$ exactly. The GENERIC dynamics reduce to their Hamiltonian form,

$$\dot{y} = L\nabla H, \quad (52)$$

since $M\nabla S = 0$. Entropy production saturates and dissipation ceases: the deterministic limit of the stochastic controller corresponds precisely to thermodynamic unification.

Because

$$\dot{y} = L\nabla H + M\nabla S, \quad (53)$$

contains both reversible and irreversible operators, the σ -controller allows coherent evolution of microscopic and macroscopic physics. The balance condition $\Delta = 0$ arises internally through gauge fixing of σ , with no external normalization.

Structural Insight Box. The key structural observation underlying this framework is that the parameters of particle physics (such as gauge couplings, masses, and scalar interactions) may be treated as the state variables of a GENERIC dynamical manifold. The reversible and irreversible operators ($L\nabla H$, $M\nabla S$) therefore induce the minimal geometric and entropic structure required for their consistent evolution.

In the reversible limit, this induced structure reproduces gravitational dynamics, while in the irreversible limit it yields the monotonic, scale-defining behaviour traditionally described by renormalization-group flow. Unification arises as a consequence of entropy-balance constraints within this operator-generated state geometry. A detailed particle-physics analysis of this underlying principle is left for future work.

□

7. Layered Jacobian Diagnostics for the Gauge Spectrum

This section replaces a purely spectral interpretation of gauge unification with a layered Jacobian diagnostic derived from the GENERIC formulation. The goal is not to infer microphysics from eigenvalues alone, but to identify how Hamiltonian and dissipative structures organize the gauge spectrum near unification.

7.1. RG-Clock Gauge Dynamics

We define the RG-clock gauge dynamics by

$$f(y, \sigma) = L(y, S) \nabla_y H(y; g(\sigma)) + M(y, S) \nabla_y S(y), \quad (54)$$

where y denotes the slow gauge degrees of freedom and σ is the RG-entropy clock.

Linearization refers to the first-order Taylor expansion of the drift around the unification point (y^*, σ_c) :

$$f(y^* + \delta y, \sigma_c) = f(y^*, \sigma_c) + J \delta y + O(\|\delta y\|^2), \quad J = \left. \frac{\partial f}{\partial y} \right|_{(y^*, \sigma_c)}. \quad (55)$$

Throughout this section, the GENERIC operators L and M are frozen locally at (y^*, σ_c) .

7.2. Frozen Jacobian Structure

Taking the derivative with respect to y at y^* yields

$$J \approx \underbrace{L^* \text{Hess}_y[H(\cdot; g(\sigma_c))]}_{\text{skew} \Rightarrow \text{Im } \lambda} \Big|_{y^*} + \underbrace{M^* \text{Hess}_y[\mathcal{S}]}_{\text{symmetric} \Rightarrow \text{Re } \lambda \leq 0} \Big|_{y^*}, \quad (56)$$

with $L^* := L(y^*, S^*)$ and $M^* := M(y^*, S^*)$. The Hamiltonian curvature generates reversible oscillations, while the entropy curvature supplies genuine dissipation.

7.3. Projection onto the Slow Manifold

Because only the slow variables are physically observable at the RG scale of interest, we project onto the tangent space $T\mathcal{M}_\varepsilon$ of the slow manifold at y^* .

Let $B \in \mathbb{R}^{n \times n_s}$ have columns spanning $T\mathcal{M}_\varepsilon$. The orthogonal projector is

$$\Pi := B(B^\top B)^{-1}B^\top, \quad \Pi^2 = \Pi, \quad \Pi^\top = \Pi. \quad (57)$$

Intuitively, Π retains slow coordinates and suppresses fast modes.

The projected Jacobian is therefore

$$\hat{J} = \Pi J \Pi^\top = \Pi \left(L^* \text{Hess}_y H + M^* \text{Hess}_y \mathcal{S} \right) \Pi^\top. \quad (58)$$

7.4. Minimal Mode Count and Gauge Structure

The symmetry properties of GENERIC enforce a four-dimensional Hamiltonian subspace (effectively \mathbb{R}^4) together with a single RG–entropy clock direction. Thus, the slow sector has a minimal dimensionality

$$N = 4 + 1. \quad (59)$$

In gauge-theory language, supporting three interacting fundamental forces requires at least two independent gauge-difference pairs. In control-theory language, dissipation is essential; without it, no stability or alignment analysis would be required. Both perspectives therefore select the same minimal mode structure.

7.5. Linearized Spectrum at Unification

Linearizing explicitly at the unification point σ_c , the projected Jacobian \hat{J} admits five eigenvalues:

$$\lambda_{1,2} = -\gamma_1 \pm i\omega_1, \quad (60)$$

$$\lambda_{3,4} = -\gamma_2 \pm i\omega_2, \quad (61)$$

$$\lambda_5 = -\kappa \Delta'(\sigma_c). \quad (62)$$

The damping rates $\gamma_{1,2} > 0$ arise from the dissipative block $M^* \text{Hess}_y \mathcal{S}$ (Assumption B3), while $\kappa \Delta'(\sigma_c) > 0$ is supplied by feedback along the RG clock.

The complex-conjugate pairs correspond to damped gauge oscillations generated by the Hamiltonian block $L^* \nabla_y^2 H^*$, whereas the single real eigenvalue represents slow dissipative alignment along the entropy-clock direction.

7.6. Entropy Clock and Feedback Mismatch

The clock mismatch driving the slow mode is defined as

$$\Delta(\sigma) := S'_h(\sigma) - \left(S'_m(\sigma) + S'_3(\sigma) \right), \quad (63)$$

where S_h is the horizon entropy, S_m the matter/radiation entropy in a Hubble patch, and S_3 an auxiliary entropy channel (e.g. coarse-graining or particle creation). Primes denote derivatives with respect to the clock variable σ .

The clock dynamics follows

$$J_{\text{ctrl}}(\sigma) = \frac{d}{d\sigma}[-\kappa \Delta(\sigma)] = -\kappa \Delta'(\sigma), \quad (64)$$

which supplies the only dynamics along an otherwise neutral Casimir direction. Dissipation therefore appears both in the plant (via $M^* \nabla_y^2 S^*$) and along the clock (via $-\kappa \Delta'(\sigma_c)$).

7.7. Why Linearize at Unification

At σ_c the three gauge couplings coincide. The Hamiltonian block is maximally symmetric, and dissipation opens a clean spectral gap, minimizing noise and separating modes most effectively.

Because heat and diffusion are intrinsic to GENERIC, additional hydrodynamic modes are expected. These do not appear in the QED analysis of Medvedev [11], and while they arise naturally in our framework, a full classification is deferred to future work. This layered diagnostic is consistent with the more abstract proof presented earlier.

8. Results Generalized

We state the main results in their cleanest form; broader generalizations (e.g. additional entropy channels, $k \neq 0$ FRW, and explicit field-theoretic realizations of $Q_{\mu\nu}$) are deferred to future work.

- One-loop unification diagnostics become non-informative at μ_U (a 0/0 limit), motivating an entropy-based replacement.
- An entropy-balance controller defines a physical RG–entropy clock σ and yields a smooth unification diagnostic via $R(\sigma)$.
- At entropy balance ($\dot{S}_{\text{tot}} = 0$), the GENERIC flow undergoes Hamiltonian closure ($M\nabla S = 0$).
- The unification scale is identified structurally by combining entropy balance with RG crossing (up to higher-loop corrections).
- Linearization near σ_c produces a projected Jacobian with damped oscillatory gauge modes plus a slow entropy-clock alignment mode.

9. Conclusions

We have developed a unified nonequilibrium framework in which stochastic GENERIC dynamics, entropy production, gauge evolution, and relativistic structure are treated within a single, thermodynamically admissible formalism. The central result is that gauge unification, renormalization-group flow, and spacetime structure are not independent inputs, but emerge as coupled consequences of entropy balance within the GENERIC operator framework.

A key technical outcome of this work is the demonstration that conventional one-loop unification diagnostics cease to be informative at the unification point itself, where they reduce to an indeterminate 0/0 form. We have shown that this pathology is not physical, but instead reflects a missing thermodynamic constraint. Replacing the purely spectral diagnostic with an entropy-based controller yields a smooth, well-defined criterion for unification that remains valid precisely where the one-loop method fails.

By analyzing the entropy structure to high order—explicitly including additive and cross-channel entropy contributions—we show that the controller governing the RG–entropy clock is internally stable at gauge equivalence. This stability implies that high-energy unification is dynamically selected, not imposed, and that global scaling to a universal time parameter arises automatically from the gauge framework itself. Higher-order entropy terms beyond those explicitly retained here may be analyzed systematically within the same formalism, as demonstrated in earlier preprints, but are not required to establish the existence, stability, or physical meaning of the unification point.

A central conceptual result is what we term the *no-normalization condition*: the unification scale does not require external normalization, boundary fixing, or tuning. Renormalization is not an auxiliary assumption but is enforced internally by the gauge structure once entropy balance is imposed. All assumptions used in the construction are independently validated by thermodynamic admissibility, GENERIC degeneracy conditions, and control-theoretic stability, ensuring logical closure of the framework.

At entropy balance, the irreversible sector of the GENERIC dynamics closes ($M\dot{V}S = 0$), and the evolution reduces to a purely Hamiltonian flow. In this limit, Einstein gravity emerges as the nondissipative, time-symmetric sector of the underlying thermodynamic geometry. Away from balance, dissipative corrections naturally enter through entropy production, allowing general relativity to be consistently embedded within a broader material, stochastic, and gauge-equivalent setting.

Taken together, these results establish full thermodynamic unification across gauge, gravitational, and nonequilibrium domains. Importantly, we show that conventional (static) unification is not a fundamental endpoint, but rather a special, measure-zero subset of a more general *dynamical unification* process governed by entropy flow. Unification is therefore not merely a coincidence of coupling constants, but a structural feature of the universe's irreversible dynamics.

Future work will focus on explicit field-theoretic realizations of the dissipative tensor, higher-loop and nonperturbative extensions, and quantitative applications to cosmology, relativistic plasmas, and complex material systems. The framework presented here provides a self-consistent foundation for these developments, grounded in first principles and free of external normalization assumptions.

Appendix A. Two-Loop Spectral-Gap Derivations (NLO Closed Form)

This Appendix provides a compact derivation of the next-to-leading order (NLO) two-loop solution for the running couplings. The construction mirrors the one-loop “no-normalization” technique used in the main text, but retains the first nontrivial two-loop correction.

Appendix A.1. Two-Loop RG Equation and Inverse-Coupling Form

Start from the two-loop RG equation

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi} \alpha_i^2 + \frac{1}{8\pi^2} \sum_j b_{ij} \alpha_i^2 \alpha_j + \mathcal{O}(\alpha^4), \quad (\text{A1})$$

where $t \equiv \ln(\mu/\mu_0)$.

Define the inverse couplings $a_i \equiv \alpha_i^{-1}$. Then

$$\frac{da_i}{dt} = \frac{d}{dt} \left(\frac{1}{\alpha_i} \right) = -\frac{1}{\alpha_i^2} \frac{d\alpha_i}{dt}.$$

Using Eq. (A1) gives

$$\boxed{\frac{da_i}{dt} = -\frac{b_i}{2\pi} - \frac{1}{8\pi^2} \sum_j b_{ij} \alpha_j + \mathcal{O}(\alpha^2)}. \quad (\text{A2})$$

Appendix A.2. Single Integration and NLO Evaluation

Let

$$A_i \equiv \alpha_i^{-1}(\mu_0) - \Delta_i, \quad \beta_i \equiv \frac{b_i}{2\pi}, \quad t \equiv \ln(\mu/\mu_0).$$

Integrating Eq. (A2) from $t' = 0$ to $t' = t$ yields

$$a_i(t) = A_i - \beta_i t - \frac{1}{8\pi^2} \sum_j b_{ij} \int_0^t \alpha_j(t') dt' + \mathcal{O}(\alpha^2). \quad (\text{A3})$$

To evaluate the integral at NLO, substitute the one-loop solution for α_j :

$$a_j^{(1)}(t) = A_j - \beta_j t \quad \Rightarrow \quad \alpha_j^{(1)}(t) = \frac{1}{A_j - \beta_j t}.$$

Then

$$\int_0^t \alpha_j^{(1)}(t') dt' = \int_0^t \frac{dt'}{A_j - \beta_j t'} = -\frac{1}{\beta_j} \ln\left(\frac{A_j - \beta_j t}{A_j}\right).$$

Appendix A.3. Validated NLO Closed Form

Inserting the NLO integral into Eq. (A3) gives

$$a_i(t) = A_i - \beta_i t + \sum_j \frac{b_{ij}}{8\pi^2 \beta_j} \ln\left(\frac{A_j - \beta_j t}{A_j}\right) + \mathcal{O}(\alpha^2), \quad \beta_i \equiv \frac{b_i}{2\pi}. \quad (\text{A4})$$

Equivalently, in terms of μ ,

$$\alpha_i^{-1}(\mu) = A_i - \beta_i \ln \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{8\pi^2 \beta_j} \ln\left(\frac{A_j - \beta_j \ln(\mu/\mu_0)}{A_j}\right) + \mathcal{O}(\alpha^2). \quad (\text{A5})$$

This expression is the standard NLO (two-loop) correction written in inverse-coupling form, matching the one-loop structure while retaining the first logarithmic correction sourced by operator mixing.

Appendix B. Cosmological Entropy Balance Primer and Additive-Channel Assumptions

This Appendix collects standard FRW thermodynamic identities and horizon-entropy formulas used in the main text. We include the derivations explicitly for clarity and consistent notation.

References used in this Appendix.

Gibbs relation, FRW thermodynamics, and Friedmann/acceleration equations [22]; bulk viscous entropy production in FRW [23]; black-hole/causal-horizon entropy and the thermodynamic route to Einstein equations [24–26]; and apparent-horizon formulas in FRW (including $S = A/4G$, $r_A = (H^2 + k/a^2)^{-1/2}$, and $T = 1/2\pi r_A$). We work in units $c = \hbar = k_B = 1$ and assume spatial flatness ($k = 0$) unless noted.

Appendix B.1. Total Entropy Decomposition

We decompose the total entropy into matter and horizon contributions,

$$S(t) = S_{\text{matter}}(t) + S_{\text{horizon}}(t), \quad (\text{A6})$$

consistent with the GENERIC degeneracy conditions $L\nabla S = 0$ and $M\nabla E = 0$. The entropy production rate splits as

$$\dot{S} = \dot{S}_{\text{matter}} + \dot{S}_{\text{horizon}} \geq 0. \quad (\text{A7})$$

Appendix B.2. Matter Entropy Production in FRW

For a perfect fluid, the Gibbs relation gives

$$T dS_{\text{matter}} = dE_{\text{matter}} + p dV, \quad (\text{A8})$$

with $E_{\text{matter}} = \rho V$ and a comoving patch whose physical volume scales as

$$V = V_0 a^3, \quad (\text{A9})$$

where V_0 is an arbitrary constant (e.g. $4\pi/3$).

Using the FRW continuity equation,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (\text{A10})$$

we obtain

$$\dot{S}_{\text{matter}} = \frac{1}{T} [\dot{\rho} V + (\rho + p) \dot{V}] = \frac{1}{T} [\dot{\rho} V + 3H(\rho + p)V] = 0, \quad (\text{A11})$$

i.e. adiabatic expansion for the perfect-fluid case [22].

Bulk-viscous correction.

If dissipative bulk viscosity Π is present (effective pressure $p_{\text{eff}} = p + \Pi$), the continuity equation becomes $\dot{\rho} + 3H(\rho + p + \Pi) = 0$, and the Gibbs relation yields the standard FRW result

$$\dot{S}_{\text{matter}} = -\frac{3H\Pi V}{T} \geq 0 \quad (\text{for } H > 0 \text{ and } \Pi < 0), \quad (\text{A12})$$

in agreement with causal thermodynamics in cosmology [23].

Appendix B.3. Horizon Entropy Production in FRW

For the (apparent) horizon of a spatially flat FRW Universe, $r_H = r_A = H^{-1}$ and $A = 4\pi r_H^2$, so the Bekenstein–Hawking relation gives

$$S_{\text{horizon}} = \frac{A}{4G} = \frac{\pi}{GH^2}. \quad (\text{A13})$$

Differentiating,

$$\dot{S}_{\text{horizon}} = -\frac{2\pi}{GH^3} \dot{H}. \quad (\text{A14})$$

Using the acceleration equation for $k = 0$,

$$\dot{H} = -4\pi G(\rho + p), \quad (\text{A15})$$

we find

$$\dot{S}_{\text{horizon}} = \frac{8\pi^2}{H^3} (\rho + p) \geq 0 \quad (\text{A16})$$

for matter satisfying $\rho + p \geq 0$ [24–26].

Appendix C. Reading Notes on Bekenstein’s Generalized Second Law

For convenience, we summarize key definitions from Bekenstein’s formulation of black-hole entropy and the generalized second law (GSL).

Bekenstein defines the black-hole entropy in terms of horizon area A (with a dimensionless proportionality constant later fixed by Hawking), and expresses the first-law form for stationary black holes,

$$dM = T_{\text{BH}} dS_{\text{BH}} + \Phi dQ + \Omega dJ,$$

establishing that S_{BH} behaves as a thermodynamic entropy. Matter entropy outside the horizon is written as an integral of the entropy density s over the exterior region, and the generalized total entropy is

$$S_{\text{tot}} = S_{\text{BH}} + S_{\text{matter}}. \quad (\text{A17})$$

The GSL asserts monotonicity,

$$\frac{d}{dt}(S_{\text{BH}} + S_{\text{matter}}) \geq 0. \quad (\text{A18})$$

In the cosmological extension used here, we identify $S_h \equiv S_{\text{BH}}$ and $S_m \equiv S_{\text{matter}}$, and include an additional dissipative channel S_3 to account for nonequilibrium entropy production:

$$\dot{S}_{\text{tot}} = \dot{S}_m + \dot{S}_h + \dot{S}_3 \geq 0, \quad (\text{A19})$$

preserving the GSL structure within a cosmological and stochastic setting.

Appendix D. Interpretive Extensions and Dissipative Tensor Ansatz

This work does not attempt a full variational derivation of modified Einstein equations incorporating an explicit dissipative tensor $Q_{\mu\nu}$. The objective is to provide a physically interpretable framework in which $Q_{\mu\nu}$ captures entropy production, dissipation, and information flow in a geometric language. A rigorous derivation (including explicit coupling between $Q_{\mu\nu}$ and $T_{\mu\nu}$) is left for future work.

In this viewpoint, General Relativity corresponds to the nondissipative, time-symmetric limit of the thermodynamic geometry: at entropy balance (unification) the irreversible sector closes and $Q_{\mu\nu} \rightarrow 0$.

A minimal phenomenological ansatz is

$$G_{\mu\nu} + Q_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (\text{A20})$$

with $Q_{\mu\nu}$ defined in terms of an entropy-flux four-vector s_μ by

$$Q_{\mu\nu} = \lambda \nabla_{(\mu} s_{\nu)} + \kappa g_{\mu\nu} (\nabla_\alpha s^\alpha), \quad (\text{A21})$$

where λ and κ are transport coefficients associated with irreversible entropy flow. In the thermodynamic equilibrium limit (entropy balance), $\dot{S} \rightarrow 0$ and $Q_{\mu\nu} \rightarrow 0$.

Appendix E. Planck Scaling as a Consistency Lock

Combining microbalanced GENERIC dynamics with KMS/FDT relations yields a single action quantum per exchange,

$$\Delta I = \hbar = k_B T_* \tau_*, \quad (\text{A22})$$

which enforces the \hbar -scaled commutator structure on canonical variables in the Hamiltonian limit. When the irreversible sector is balanced, the macroscopic evolution reduces to Hamiltonian flow,

$$\dot{y} = L \nabla H_{\text{GR}}, \quad (\text{A23})$$

corresponding to Einstein gravity in the nondissipative limit.

At the Planck point,

$$k_B T_P t_P = \hbar \quad (\text{A24})$$

provides kinematic consistency with G and c , confirming that gravity is intrinsically quantum at this scale, while GR appears as a late-time fixed point.

This correspondence applies across epochs via the thermal-time rule

$$\tau_* = \frac{\hbar}{k_B T_*}, \quad (\text{A25})$$

with numerical conversion

$$\frac{\hbar}{k_B} = 7.638 \times 10^{-12} \text{ K s}. \quad (\text{A26})$$

Examples:

- **Planck epoch:** $T_* = 1.4168 \times 10^{32} \text{ K} \Rightarrow \tau_* = 5.39 \times 10^{-44} \text{ s}$;
- **GUT epoch:** $T_* \sim 10^{29} \text{ K} \Rightarrow \tau_* \approx 7.64 \times 10^{-41} \text{ s}$;
- **Electroweak epoch:** $T_* \sim 10^{15} \text{ K} \Rightarrow \tau_* \approx 7.64 \times 10^{-27} \text{ s}$.

For reference,

$$t_P = \sqrt{\frac{\hbar G}{c^5}}, \quad T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}}, \quad k_B T_P t_P = \hbar. \quad (\text{A27})$$

Appendix F. Validation of Entropy Selection Orders and Mixed-Scaling Rules

Appendix F.1. Controller Structure and Additive Channel

We begin with the entropy controller,

$$\dot{\sigma} = -\kappa G(\sigma), \quad G(\sigma) = \dot{S}_m(\sigma) - \dot{S}_h(\sigma), \quad (\text{A28})$$

where \dot{S}_m and \dot{S}_h denote the matter (viscous) and horizon (geometric) entropy-production rates. In the high-energy limit ($T = \mu_0 e^\sigma$), take

$$\dot{S}_m = a_m T^{-p}, \quad \dot{S}_h = a_h T^{-q}, \quad p > q > 0. \quad (\text{A29})$$

For $p > q$, $G(\sigma)$ changes sign at most once, yielding a unique fixed point under standard regularity assumptions.

To preserve global thermodynamic consistency, we introduce an additive entropy channel,

$$\dot{S}_3 = -(\dot{S}_m + \dot{S}_h), \quad (\text{A30})$$

so that the total entropy production satisfies

$$\dot{S}_{\text{tot}} = \dot{S}_m + \dot{S}_h + \dot{S}_3 = 0. \quad (\text{A31})$$

This S_3 represents residual nonequilibrium entropy production required for boundedness and continuity of the total entropy functional.

Appendix F.2. Scaling of the Additive Term

With the above scalings,

$$\dot{S}_3 = -(a_m T^{-p} + a_h T^{-q}), \quad (\text{A32})$$

so the ultraviolet contribution is dominated by the slowest-decaying term, $\dot{S}_3 \sim -a_h T^{-q}$.

Under radiation domination, using $\dot{T} \sim -HT$ and the usual scaling $H \propto T^2$ (up to constants), one obtains the schematic integrated form

$$S_3(T) \propto a_m T^{-(p-2)} + a_h T^{-(q-2)}. \quad (\text{A33})$$

For the canonical ($\dot{S}_h \sim T^{-2}$) and ($\dot{S}_m \sim T^{-3}$) case,

$$S_3(T) \approx A_0 + A_1 T^{-1} + \mathcal{O}(T^{-2}), \quad (\text{A34})$$

where A_0 is a scale-invariant offset and $A_1 T^{-1}$ is the leading UV correction.

Appendix F.3. Geometric-Mean Mixed Scaling

If S_3 couples dynamically into the controller as

$$G_{\text{eff}}(\sigma) = \dot{S}_m - \dot{S}_h + \epsilon \dot{S}_3, \quad (\text{A35})$$

then minimizing curvature (schematically $dG_{\text{eff}}/dT = 0$) yields

$$r = \frac{p+q}{2}, \quad (\text{A36})$$

so that

$$\dot{S}_3 \propto T^{-(p+q)/2}, \quad S_3 \propto T^{-(p+q)/2+1}. \quad (\text{A37})$$

For $(p, q) = (3, 2)$ this gives $\dot{S}_3 \sim T^{-2.5}$ and $S_3 \sim T^{-1.5}$ (up to model-dependent prefactors).

Appendix F.4. Correspondence with Mixed Scaling in RG and Nonequilibrium Structure

The geometric-mean scaling resembles mixed-scaling behavior in RG operator mixing and cross-dissipative entropy production in nonequilibrium thermodynamics. In RG language, interacting scaling operators with exponents Δ_1 and Δ_2 can acquire composite scaling $\Delta_{12} \approx (\Delta_1 + \Delta_2)/2$ [? ?].

In GENERIC/EIT language, cross-dissipative production can scale as a geometric mean,

$$\dot{S}_{12} \sim \sqrt{\dot{S}_1 \dot{S}_2} \Rightarrow \dot{S}_{12} \sim T^{-(p+q)/2}, \quad (\text{A38})$$

consistent with the mixed-scaling structure used above [1,5?].

Appendix F.5. A Compact Estimate for the Balance Temperature

Decompose the horizon prefactor as $C_h^{\text{tot}} = C_h^{\text{Hub}} + C_h^{\text{BH}}$ and assume a small black-hole share,

$$\varepsilon \equiv \frac{C_h^{\text{BH}}}{C_h^{\text{tot}}} \ll 1 \quad \Longrightarrow \quad C_h^{\text{tot}} = (1 - \varepsilon)^{-1} C_h^{\text{Hub}}.$$

Near the balance point use the power-law ansatz

$$S_h(T) = C_h^{\text{tot}} T^{-p}, \quad S_m(T) = C_m T^{-q}, \quad p > q > 0,$$

and impose $S_3 \rightarrow S_m$ at balance so that

$$\left. \frac{dS_h}{dT} \right|_{T_c} = 2 \left. \frac{dS_m}{dT} \right|_{T_c}.$$

Differentiating gives $dS_h/dT = -pC_h^{\text{tot}}T^{-p-1}$ and $dS_m/dT = -qC_mT^{-q-1}$, hence

$$T_c \simeq \left(\frac{p C_h^{\text{Hub}}}{2q C_m} \right)^{\frac{1}{p-q}}, \quad \frac{\Delta T_c}{T_c} \approx \frac{\varepsilon}{p-q}. \quad (\text{A39})$$

For $(p, q) = (4, 3)$ this reduces to $T_c \simeq \frac{2 C_h^{\text{Hub}}}{3 C_m}$, and a 5% black-hole fraction shifts T_c by $\sim 5\%$.

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