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Imaginary dimensions in physics require an imaginary set of base Planck units and some negative parameter c_n corresponding to the speed of light in vacuum c. Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene introduce the second, negative fine-structure constant $\alpha_2^{-1} \approx -140.178$ as a fundamental constant of nature and this constant introduces these imaginary base Planck units along with this negative parameter $c_n \approx -3.06 \times 10^8$ [m/s]. Neutron stars and white dwarfs, *objects* emitting perfect black-body radiation, are conjectured to possess energy exceeding their mass-energy equivalence ratios, The imaginary parts of complex energies, inaccessible for direct observation, make storing excess of these energies possible. It is shown that black holes are fundamentally uncharged; charged micro neutron stars and white dwarfs with masses lower than 5.7275×10^{-10} [kg] cannot be observed; and the radii of white dwarfs' cores are limited to $R_{WD} < 6.7933\, GM_{WD}/c^2$. It is conjectured that the maximum atomic number $Z = 238$. A black-body *object* is in the equilibrium of complex energies of masses, charges, and wavelengths if its radius $R_{eq} \approx 2.7665\, GM_{BBO}/c^2$. Thus, R_{eq} corrects the value of the photon sphere radius $R = 3GM/c^2$, as it takes into account the value(s) of the fine-structure constant(s), which is otherwise neglected in general relativity.

Keywords: emergent dimensionality; imaginary dimensions; Planck units; fine-structure constant; black holes; neutron stars; white dwarfs; complex energy; complex force; extended periodic table; Buchdahl's theorem; photon sphere; general relativity; holographic principle; mathematical physics

I. INTRODUCTION

The universe began with the Big Bang, which is a current prevailing scientific opinion. But this Big Bang was not an explosion of 4-dimensional spacetime, which also is a current prevailing scientific opinion, but an explosion of dimensions. More precisely, in the −1-dimensional void, a 0-dimensional point appeared implying the appearance of countably infinitely other points indistinguishable from the first one. The breach made by the first operation of the *dimensional successor function* of the Peano axioms inevitably continued leading to the formation of 1-dimensional, real and imaginary lines allowing for an ordering of points using multiples of real units (ones) or imaginary units ($a \in \mathbb{R} \Leftrightarrow a = 1b$[1], $a \in \mathbb{I} \Leftrightarrow a = ib, b \in \mathbb{R}$). Then out of two lines of each kind, crossing each other only at one initial point $(0, 0)$, the dimensional successor function formed 2-dimensional $\mathbb{R}^2$, $\mathbb{I}^2$, and $\mathbb{R} \times \mathbb{I}$ Euclidean planes, with $\mathbb{I}^2$ being a mirror reflection of $\mathbb{R}^2$. And so on, forming n-dimensional Euclidean spaces $\mathbb{R}^a \times \mathbb{I}^b$ with $a \in \mathbb{N}$ real and $b \in \mathbb{N}$ imaginary lines, $n := a + b$, and the scalar product defined by

$$\mathbf{x} \cdot \mathbf{y} = \left(x_1, \ldots, x_a, ix'_1, \ldots, ix'_b\right)\left(y_1, \ldots, y_a, iy'_1, \ldots, iy'_b\right) := \\ := \sum_{k=1}^{a} x_k y_k + \sum_{l=1}^{b} x'_l \overline{y'_l}, \tag{1}$$

where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^a \times \mathbb{I}^b$.

With the onset of the first 0-dimensional point, information began to evolve [1–6].

However, dimensional properties are not uniform. Concerning regular convex n-polytopes in natural dimensions, for example, there are countably infinitely many regular convex polygons, five regular convex polyhedra (Platonic solids), six regular convex 4-polytopes, and only three regular convex n-polytopes if $n > 3$ [7]. In particular, 4-dimensional euclidean space is endowed with a peculiar property known as exotic $\mathbb{R}^4$ [8]. This property allowed for variation of phenotypic traits within populations of individuals [9] perceiving emergent Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space of three real and one imaginary (time) dimension observer-dependently [10] and at present [11], when $i0 = 0$ is *real*. The evolution of information extended into biological evolution.

Each dimension requires certain units of measure. In real dimensions, the *natural units of measure*, were introduced by Max Planck in 1899 as "independent of special *bodies* or *substances*, thereby necessarily retaining their meaning for all times and for all civilizations, including extraterrestrial and non-human ones" [12].

This study introduces the complementary set of Planck units applicable for imaginary dimensions, including the imaginary base units, and outlines certain prospects for their research. As the speed of electromagnetic radiation is the product of its wavelength and frequency (which are real and positive), and both these quantities are imaginary in imaginary dimensions, some real but negative parameter $c_n = \nu_i \lambda_i$ corresponding to the speed of light in vacuum c (i.e., the Planck speed) is also necessary as $i^2 = -1$. It turns out that the imaginary Planck energy $E_{\mathrm{P}i}$ and temperature $T_{\mathrm{P}i}$ are larger in moduli than the Planck energy E_{P} and temperature T_{P}. Thus, the minimum energy principle sets more favorable conditions for biological evolution to emerge in $\mathbb{R}^3 \times \mathbb{I}$ Euclidean space than in $\mathbb{I}^3 \times \mathbb{R}$ Euclidean one.

The study shows that the energies of neutron stars and white dwarfs exceed their mass–energy equivalences. Therefore, the excess of these energies must be stored in imaginary dimensions and is inaccessible to direct observations. This results in the upper bound on the slope of the radius of their cores as a function of their masses.

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[1] This is, of course, a circular definition, but it is given for clarity.

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The paper is structured as follows. Section II shows that Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene introduce the second, negative fine-structure constant α_2 as a fundamental constant of nature. Section III shows that nature endows us with the imaginary base Planck units by this second fine-structure constant. Section IV introduces the concept of a black-body *object* in thermodynamic equilibrium emitting black-body radiation and discusses its necessary properties. Section V introduces two complex energies of masses and charges and applies them to black-body *objects*. Section VI introduces four additional complex energies of masses, charges, and wavelengths to derive the black-body *object* equilibrium, correcting the photon sphere radius of general relativity. Section VII summarizes the findings of this study.

II. THE SECOND FINE-STRUCTURE CONSTANT

Numerous publications provide Fresnel coefficients for the normal incidence of electromagnetic radiation (EMR) on monolayer graphene (MLG), which are remarkably defined only by π and the fine-structure constant α

$$\alpha^{-1} = \left(\frac{q_P}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036. \tag{2}$$

Transmittance (T) of MLG

$$T = \frac{1}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.9775, \tag{3}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [13] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [14, 15] for photon energies between about 0.5 and 2.5 [eV]. T was related to reflectance (R) [16] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 1.2843\times 10^{-4}, \tag{4}$$

The above equations for T and R, as well as the equation for the absorptance

$$A = \frac{\pi\alpha}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.0224, \tag{5}$$

were also derived [17] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate).

The sum of transmittance (3) and the reflectance (4) at normal EMR incidence on MLG was also derived [18] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} \\ &= \frac{1+\frac{1}{4}\pi^2\alpha^2}{\left(1+\frac{\pi\alpha}{2}\right)^2} \approx 0.9776, \end{aligned} \tag{6}$$

where η is the vacuum impedance

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \approx 376.73 \ [\Omega], \tag{7}$$

ϵ_0 is vacuum permittivity, $\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [19], and $\chi = 0$ is the electric susceptibility of vacuum.

These coefficients are thus well-established theoretically and experimentally confirmed [13–15, 18, 20, 21].

As a consequence of the conservation of energy

$$(T + A) + R = 1. \tag{8}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The reflectance $R = 0.013\%$ (4) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}(R-1)\pi^2\alpha^2 + R\pi\alpha + R = 0, \tag{9}$$

having two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \quad \text{and} \tag{10}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{11}$$

Therefore, the quadratic equation (9) introduces the second, negative fine-structure constant α_2.

The sum of the reciprocals of these fine-structure constants (10) and (11)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{12}$$

is remarkably independent of the reflectance R (The same result can be obtained for the sum of T and A, as shown in Appendix A).

Furthermore, this result is intriguing in the context of a peculiar algebraic expression for the fine-structure constant [22]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036303776 \tag{13}$$

that contains a *free* π term and is very close to the physical definition (2) of α^{-1}, which according to the CODATA 2018 value is 137.035999084. Notably, the value of the fine-structure constant is not *constant* but increases with time [23–27]. Thus, the algebraic value given by (13) can be interpreted as the asymptote of the α increase.

Using relations (12) and (13), we can express the negative reciprocal of the 2$^{\text{nd}}$ fine-structure constant α_2^{-1} that emerged in the quadratic equation (9) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.177896429, \tag{14}$$

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and this value can also be interpreted as the asymptote of the α_2 decrease with the current value ammounting to $\alpha_2^{-1} \approx -140.177591737$, assuming the rate of change is the same for α and α_2.

Using relations (13) and (14), transmittance T (3), reflectance R (4), and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π. Furthermore, equation (9) introduces two π-like constants for two surfaces with positive and negative Gaussian curvatures (cf. Appendix B).

III. α_2-SET OF PLANCK UNITS

Planck units can be derived from numerous starting points [5, 28] (cf. Appendix C). The definition of the Planck charge $q_P = \sqrt{4\pi\epsilon_0\hbar c}$ can be solved for the speed of light yielding $c = q_P^2/(4\pi\epsilon_0\hbar)$. Furthermore, the ratio of charges definition of the fine-structure constant $\alpha = e^2/q_P^2$ (2) applied for the negative α_2, requires an introduction of some imaginary Planck charge q_{Pi} so that its square would yield a negative value of α_2

$$\frac{q_{Pi}^2}{e^2} = \alpha_2^{-1} \approx -140.178 < 0. \tag{15}$$

Since the elementary charge e is real

$$q_{Pi} = \pm\sqrt{\frac{e^2}{\alpha_2}} = \pm\sqrt{4\pi\epsilon_0\hbar c_n}, \tag{16}$$

and almost all physical constants of the $\sqrt{4\pi\epsilon_0\hbar c_n}$ term are positive[2], only the $c_n = \nu_i\lambda_i$ parameter, corresponding to the speed of light, must be negative as both frequency ν_i and wavelength λ_i are imaginary in imaginary dimensions. Therefore, equation (16) can be solved for c_n yielding

$$c_n = q_{Pi}^2/(4\pi\epsilon_0\hbar) \approx -3.066653 \times 10^8 \text{ [m/s]}, \tag{17}$$

which is greater than the speed of light in vacuum c in modulus[3].

The negative parameter c_n (17) introduces the imaginary set of base Planck units q_{Pi}, ℓ_{Pi}, m_{Pi}, t_{Pi}, and T_{Pi} that redefined by square roots containing c_n raised to an odd (1, 3, 5) power become imaginary and bivalued

$$q_{Pi} = \pm\sqrt{4\pi\epsilon_0\hbar c_n} = \pm q_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \approx \pm i1.8969 \times 10^{-18} \text{ [C]} \quad (|q_{Pi}| > |q_P|), \tag{18}$$

$$\ell_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^3}} = \pm\ell_P\sqrt{\frac{\alpha_2^3}{\alpha^3}} \approx \approx \pm i1.5622 \times 10^{-35} \text{ [m]} \quad (|\ell_{Pi}| < |\ell_P|), \tag{19}$$

$$m_{Pi} = \pm\sqrt{\frac{\hbar c_n}{G}} = \pm m_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \approx \pm i2.2012 \times 10^{-8} \text{ [kg]} \quad (|m_{Pi}| > |m_P|), \tag{20}$$

$$t_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^5}} = \pm t_P\sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \approx \pm i5.0942 \times 10^{-44} \text{ [s]} \quad (|t_{Pi}| < |t_P|), \tag{21}$$

$$T_{Pi} = \pm\sqrt{\frac{\hbar c_n^5}{Gk_B^2}} = \pm T_P\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \approx \pm i1.4994 \times 10^{32} \text{ [K]} \quad (|T_{Pi}| > |T_P|), \tag{22}$$

and can be expressed, using the relation (31), in terms of base Planck units q_P, ℓ_P, m_P, t_P, and T_P.

Planck units derived from the imaginary base units (19)-(21) are generally not imaginary. The α_2 Planck volume

$$\ell_{Pi}^3 = \pm\left(\frac{\hbar G}{c_n^3}\right)^{3/2} = \pm\ell_P^3\sqrt{\frac{\alpha_2^9}{\alpha^9}} \approx \approx \pm i3.8127 \times 10^{-105} \text{ [m}^3\text{]} \quad \left(< |\ell_P^3|\right), \tag{23}$$

the α_2 Planck momentum

$$p_{Pi} = \pm m_{Pi}c_n = \pm\sqrt{\frac{\hbar c_n^3}{G}} = \pm m_P c\sqrt{\frac{\alpha^3}{\alpha_2^3}} \approx \approx \pm i6.7504 \text{ [kg m/s]} \quad (|m_{Pi}c_n| > |m_P c|), \tag{24}$$

the α_2 Planck energy

$$E_{Pi} = \pm m_{Pi}c_n^2 = \pm\sqrt{\frac{\hbar c_n^5}{G}} = \pm E_P\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \approx \pm i2.0701 \times 10^9 \text{ [J]} \quad (|E_{Pi}| > |E_P|), \tag{25}$$

and the α_2 Planck acceleration

$$a_{Pi} = \pm\frac{c_n}{t_{Pi}} = \pm\sqrt{\frac{c_n^7}{\hbar G}} = \pm a_P\sqrt{\frac{\alpha^7}{\alpha_2^7}} \approx \approx \pm i6.0198 \times 10^{51} \text{ [m/s}^2\text{]} \quad (|a_{Pi}| > |a_P|), \tag{26}$$

are imaginary and bivalued. However, the α_2 Planck force

$$F_{P2} = \pm\frac{E_{Pi}}{\ell_{Pi}} = \pm\frac{c_n^4}{G} = \pm F_P\frac{\alpha^4}{\alpha_2^4} \approx \approx \pm 1.3251 \times 10^{44} \text{ [N]} \quad (|F_{P2}| > |F_P|), \tag{27}$$

[2] Vacuum permittivity ϵ_0 is the value of the absolute dielectric permittivity of classical vacuum. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative; negative probabilities do not seem to withstand Occam's razor.

[3] Their average $(c + c_n)/2 \approx -3.436417 \times 10^6$ [m/s] is in the range of the Fermi velocity.

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and the α_2 Planck density

$$\rho_{P2} = \pm\frac{m_{Pi}}{\ell_{Pi}^3} = \pm\frac{c_n^5}{\hbar G^2} = \pm\rho_P\frac{\alpha^5}{\alpha_2^5} \approx \approx \pm 5.7735 \times 10^{96}\ [\mathrm{kg/m^3}] \quad (|\rho_{P2}| > |\rho_P|)\,, \tag{28}$$

are real and bivalued. On the other hand, the α_2 Planck area

$$\ell_{Pi}^2 = \frac{\hbar G}{c_n^3} = \ell_P^2\frac{\alpha_2^3}{\alpha^3} \approx \approx -2.4406 \times 10^{-70}\ [\mathrm{m^2}] \quad \left(|\ell_{Pi}^2| < |\ell_P^2|\right), \tag{29}$$

is strictly negative, while the Planck area ℓ_P^2 is strictly positive.

Both α_2 and c_n introduce the second, negative vacuum impedance

$$\eta_2 = \frac{4\pi\alpha_2\hbar}{e^2} = \frac{1}{\epsilon_0 c_n} \approx -368.29\ [\Omega] \quad (|\eta_2| < |\eta|)\,. \tag{30}$$

Solving both impedances (7) and (30) for $4\pi\hbar\epsilon_0/e^2$ and comparing with each other yields the following important relation between the speed of light in vacuum c, negative parameter c_n, and the fine-structure constants α, α_2

$$c\alpha = c_n\alpha_2 \quad (= v_e)\,, \tag{31}$$

where, notably, v_e is the electron's velocity at the first circular orbit in the Bohr model of the hydrogen atom. This is not the only α to α_2 relation. Along with the two π-like constants relations (B8) and (B10) (cf. Appendix B)

$$\frac{\alpha_2}{\alpha} = \frac{c}{c_n} = \frac{\pi_1}{\pi} = \frac{\pi}{\pi_2} \approx -0.9776. \tag{32}$$

The relations between time (21) and temperature (22) α_2 Planck units are inverted, $\alpha^5 t_{Pi}^2 = \alpha_2^5 t_P^2$, $\alpha_2^5 T_{Pi}^2 = \alpha^5 T_P^2$, and saturate Heisenberg's (energy-time version) uncertainty principle taking energy from the equipartition theorem for one degree of freedom (or one bit of information [5, 29])

$$\frac{1}{2}k_B T_P t_P = \frac{1}{2}k_B T_{Pi} t_{Pi} = \frac{\hbar}{2}. \tag{33}$$

Furthermore, eliminating α and α_2 from (18)-(20), yield

$$\frac{q_P^2}{m_P^2} = \frac{q_{Pi}^2}{m_{Pi}^2} = 4\pi\epsilon_0 G, \tag{34}$$

and

$$\ell_P m_P^3 = \ell_{Pi} m_{Pi}^3 \quad \text{and} \quad \ell_P q_P^3 = \ell_{Pi} q_{Pi}^3. \tag{35}$$

Base Planck units themselves admit negative values as negative square roots. By choosing complex analysis, within the framework of emergent dimensionality [5, 9, 11, 30, 31], we enter into bivalence by the very nature of this analysis. All geometric *objects* have both positive and negative volumes and surfaces [31] equal in moduli. On the other hand, imaginary and negative physical quantities are the subject of research. In particular, the subject of scientific research is thermodynamics in the complex plane. Lee–Yang zeros, for example, have been experimentally observed [32, 33].

We note here that the imaginary Planck Units are not imaginary due to being multiplied by the imaginary unit i. They are imaginary numbers $\mathbb{I}$ due to the negativity of odd powers of c_n being the square root argument; thus, they define imaginary physical quantities inaccessible to direct measurements[4]. The complementary Planck units do not apply only to the time dimension but to any imaginary dimension. However, in our four-dimensional Euclidean $\mathbb{R}^3\times\mathbb{I}$ space-time, Planck units apply in general to the spatial dimensions, while the imaginary ones in general to the imaginary temporal dimension, wherein the seemingly interchangeable meaning of the Planck imaginary length ℓ_{Pi} and time t_{Pi} requires further research. All the complementary Planck units have physical meanings. However, some of them are elusive, like the negative area or imaginary volume, which require two or three orthogonal imaginary dimensions.

IV. BLACK BODY OBJECTS

There seem to be only three observable *objects* in nature that emit perfect black-body radiation: unsupported black holes (*BH*, the densest), neutron stars (*NS*) supported, as it is accepted, by neutron degeneracy pressure, and white dwarfs (*WD*), supported by electron degeneracy pressure (the least dense). We shall collectively call them black-body *objects* (*BBO*). It has recently been experimentally confirmed that the so-called *accretion instability* is a fundamental physical process [34] common for all *BBO*s.

As black-body radiation is radiation emitted by a body in global thermodynamic equilibrium, it is patternless (thermal noise) radiation and depends only on the temperature of this body. In the case of *BH*s, this is known as Hawking radiation, wherein the *BH* temperature $T_{BH} = T_P/(2\pi d_{BH})$, where T_P is the Planck temperature, is a function of the *BH* diameter [5] $D_{BH} = d_{BH}\ell_P$, where $d_{BH} \in \mathbb{R}$ (in the following d is also called a diameter) and ℓ_P is the Planck length.

As Hawking radiation depends only on the diameter of a *BH*, it must be the same for a given *BH*, even though it is momentary as it fluctuates (cf. Appendix D). As the interiors of the *BBO*s are inaccessible to an exterior observer [35], *BBO*s do not *enclose* interiors and can only be defined by their diameters (cf. [5] Fig. 2(b)). The term *object* as a collection of *matter* is a misnomer in general, as it neglects quantum nonlocality. But it is a particularly staring misnomer if applied to *BBO*s. Thus we use emphasis for (indistinguishable) *particle* and (distinguishable) *object*, as well as for *matter* and *distance*, as these terms have no absolute meaning in emergent dimensionality. In particular, given the recent observation of *quasiparticles* in classical systems [36].

[4] Quantum measurement outcomes are *real* eigenvalues of hermitian operators.

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But not only *BBO*s are perfectly spherical. Also, the early epochs of their collisions must be perfectly spherical, as it has been recently, experimentally confirmed [37] for *NS* s based on the AT2017gfo kilonova data. One can hardly expect a collision of two perfectly spherical, patternless thermal noises to produce some aspherical pattern instead of another perfectly spherical patternless noise. Where would the information about this pattern come from at the moment of the collision? From the point of impact? No point of impact is distinct on a patternless surface.

As black-body radiation is patternless, the triangulated [5] *BBO*s, as well as their early epoch collisions, must contain a balanced number of Planck area triangles, each carrying binary potential $\delta\varphi_k = -c^2 \cdot \{0, 1\}$, as it has been shown for *BH*s [5], based on Bekenstein-Hawking entropy

$$S_{BH} = \frac{1}{4} k_{\mathrm{B}} N_{BH}, \tag{36}$$

where $N_{BH} := 4\pi R_{BH}^2/\ell_{\mathrm{P}}^2 = \pi d_{BH}^2$ is the *BH* information capacity (i.e., the number of the triangular Planck areas at the *BH* horizon, corresponding to bits of information [29, 35, 38] and the fractional part triangle $\{\pi d_{BH}^2\}$ to small to carry a single bit of information [*sic*!]) and $R_{BH} = 2GM_{BH}/c^2$ is the *BH* (Schwarzschild) radius. The *BH* entropy (36) can be derived from the Bekenstein bound

$$S \leq \frac{2\pi k_{\mathrm{B}} R E}{\hbar c}, \tag{37}$$

an upper limit on the thermodynamic entropy S that can be contained within a sphere of radius R having energy E, where k_{B} is the Boltzmann constant and $\hbar$ is the reduced Planck constant, after plugging the *BH* radius R_{BH} and energy $E_{BH} = M_{BH}c^2$ taken from mass-energy equivalence into the bound (37) .

Since the patternless nature of the perfect black-body radiation was derived [5] by comparing *BH* entropy (36) with the binary entropy variation $\delta S = k_{\mathrm{B}} N_1/2$ ([5] Eq. (55)), which is valid for any holographic sphere, where $N_1 \in \mathbb{N}$ denotes the number of active Planck areas with binary potential $\delta\varphi_k = -c^2$, the BH entropy (36) must be valid also for *NS* s and *WD*s. Thus, defining the generalized radius of a holographic sphere of mass M as a function of GM/c^2 multiplier k [5]

$$R := k\frac{GM}{c^2}, \tag{38}$$

and the generalized energy E of this sphere as a function of Mc^2 multiplier a

$$E := aMc^2, \tag{39}$$

with $k, a \in \mathbb{R} : k \geq 2$, the generalized Bekenstein bound (37) becomes

$$S \leq \frac{1}{2} k_{\mathrm{B}} \frac{a}{k} N, \tag{40}$$

where $N := 4\pi R^2/\ell_{\mathrm{P}}^2$ is the information capacity of this sphere, the surface of which contains $\lfloor N \rfloor$ Planck triangles, where "$\lfloor x \rfloor$" is the floor function that yields the greatest integer less than or equal to its argument x.

The generalized Bekenstein bound (40) equals the *BH* entropy (36) if $\frac{a}{2k} = \frac{1}{4} \Rightarrow a = \frac{k}{2}$. Thus, the energy of all *BBO*s having a radius (38) is

$$E_{BBO} = \frac{k}{2} M_{BBO} c^2, \tag{41}$$

with $k = 2$ in the case of *BH*s and $k > 2$ for *NS* s and *WD*s.

Schwarzschild *BH*s are fundamentally uncharged, contrary to *NS* s and *WD*s, since the entropy (36) of any *BH* is equal to that of the uncharged Schwarzschild *BH* with the same area by the Penrose process. It is accepted that in the case of *NS* s, electrons combine with protons to form neutrons, but it is never the case that all electrons and all protons become neutrons; *WD*s are charged by definition as they are composed mostly of electron-degenerate *matter*.

As the entropy of independent systems is additive, a collision of two *BBO*s, BBO_1 and BBO_2, having entropies $S_{BBO_1} = \frac{1}{4} k_{\mathrm{B}} N_{BBO_1} = \frac{1}{4} k_{\mathrm{B}} \pi d_{BBO_1}^2$ and $S_{BBO_2} = \frac{1}{4} k_{\mathrm{B}} \pi d_{BBO_2}^2$, produces another BBO_C having entropy

$$S_{BBO_C} = S_{BBO_1} + S_{BBO_2} \quad \Rightarrow d_{BBO_C}^2 = d_{BBO_1}^2 + d_{BBO_1}^2. \tag{42}$$

This shows that a collision of two primordial *BH*s, each having the Planck length diameter, the reduced Planck temperature $\frac{T_{\mathrm{P}}}{2\pi}$ (which is the largest physically significant temperature [11]), and no tangential acceleration a_{LL} [5, 11], produces a *BH* having $d_{BH} = \pm\sqrt{2}$ which represents the minimum *BH* diameter allowing for the notion of time [11], while a collision of the latter two *BH*s produces a *BH* having $d_{BH} = \pm 2$ having the triangulation defining only one precise diameter between its poles (cf. [5] Fig. 3(b)). Diameter $d_{BH} = \pm 2$ is also recovered [5] from Heisenberg's Uncertainty Principle (cf. Appendix C).

The hitherto considerations may be unsettling for the reader, as the energy (41) of *BBO*s other than *BH*s exceeds mass-energy equivalence $E = Mc^2$ for $k > 2$, which is the limit of the maximum *real* energy. Thus, a part of the energy of *NS* s and *WD*s must be imaginary and thus unmeasurable. We shall consider this question in the subsequent section.

V. COMPLEX ENERGIES

A complex energy formula

$$E_R := E_{M_R} + iE_{Q_R} = (1 + i\beta_R) M_R c^2, \tag{43}$$

where $E_{M_R} = M_R c^2$ and iE_{Q_R} represent respectively real and imaginary energy of an *object* having mass M_R and charge Q_R[5], and

$$\beta_R := \frac{E_{Q_R}}{E_{M_R}} = \frac{Q_R}{2M_R\sqrt{\pi\epsilon_0 G}}, \tag{44}$$

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[5] Charges in the cited study are defined in CGS units; here we adopt SI.

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is the imaginary-real energy ratio[6], was proposed in [39] (Eqs. (1), (3), and (4)). Equations (43) and (44) consider real (physically measurable) masses M_R and charges Q_R.

Planck charge relations (2) and (16) imply that the elementary charge e is the same both in real and imaginary dimensions since

$$e^2 = \alpha q_{\mathrm{P}}^2 = \alpha_2 q_{\mathrm{P}i}^2. \quad (45)$$

On the other hand, there is no physically meaningful *elementary mass* $m_e = \pm 3.4566 \times 10^{-18}$ [kg] that would satisfy the analogous relation (20)

$$m_e^2 = \alpha m_{\mathrm{P}}^2 = \alpha_2 m_{\mathrm{P}i}^2. \quad (46)$$

Thus, as to the modulus, charges are the same in real and imaginary dimensions, while masses are different. We note that the same form of the relations (45) and (46) reflect the same form of Coulomb's law and Newton's law of gravity, which are inverse-square laws.

In the following, where deemed appropriate, dimensional quantities were discretized using Planck units as

$$\begin{aligned} &Q := qe, \quad Q_i := iQ = iqe, \quad q \in \mathbb{Z} \\ &M := mm_{\mathrm{P}}, \quad M_i := m_i m_{\mathrm{P}i}, \quad m, m_i \in \mathbb{R}, \\ &\lambda := l\ell_{\mathrm{P}}, \quad \lambda_i := l_i \ell_{\mathrm{P}i}, \quad l, l_i \in \mathbb{R}, \\ &R := r\ell_{\mathrm{P}}, \quad R_i := r_i \ell_{\mathrm{P}i}, \quad r, r_i \in \mathbb{R}, \end{aligned} \quad (47)$$

although the discretization of charges by integer multiples q of the elementary charge e is far-fetched, considering the fractional charge of *quasiparticles*.

We shall now modify the equation (43) to a form involving imaginary masses M_i and charges Q_i by defining the following two complex energies, the complex energy of real mass M and imaginary charge Q_i

$$\begin{aligned} E_{MQ_i} &:= E_M + E_{Q_i} = \\ &= (1 + \beta_{Q_i}) Mc^2 = \left(M + iq\sqrt{\alpha} m_{\mathrm{P}}\right) c^2, \end{aligned} \quad (48)$$

and the complex energy of real charge Q and imaginary mass M_i

$$\begin{aligned} E_{QM_i} &:= E_Q + E_{M_i} = \\ &= (\beta_Q + 1) M_i c_n^2 = \left(q\sqrt{\alpha_2} m_{\mathrm{P}i} + M_i\right) c_n^2, \end{aligned} \quad (49)$$

where

$$\beta_{Q_i} := \frac{Q_i}{2M\sqrt{\pi\epsilon_0 G}} = \frac{iq\sqrt{\alpha} m_{\mathrm{P}}}{M} \in \mathbb{I}, \quad (50)$$

$$\beta_Q := \frac{Q}{2M_i\sqrt{\pi\epsilon_0 G}} = \frac{q\sqrt{\alpha_2} m_{\mathrm{P}i}}{M_i} \in \mathbb{I}. \quad (51)$$

We note in passing that using the different speed of light parameters in energies E_{MQ_i} (49) and E_{QM_i} (48) yields a contradiction (cf. Appendix F).

Equations (48)-(51) yield two different quanta of the charge-dependent energies corresponding to the elementary charge, the imaginary quantum

$$E_{Q_i}(q = \pm 1) = \pm i\sqrt{\alpha} E_{\mathrm{P}} \approx \pm i 1.67 \times 10^8 \text{ [J]}, \quad (52)$$

and the - larger in modulus - real quantum

$$E_Q(q = \pm 1) = \pm\sqrt{\alpha_2} E_{\mathrm{P}i} \approx \pm 1.75 \times 10^8 \text{ [J]}. \quad (53)$$

Furthermore, $\forall q, \alpha^2 E_{Qi} = i\alpha_2^2 E_Q$.

The squared moduli of the energies (48) and (49) can be expressed as

$$|E_{MQ_i}|^2 = M^2 c^4 \left(1 - \beta_{Qi}^2\right) = \left(M^2 + q^2 \alpha m_{\mathrm{P}}^2\right) c^4, \quad (54)$$

and (using relations (31) and (20))

$$\begin{aligned} |E_{QM_i}|^2 &= M_i^2 c_n^4 \left(\beta_{Mi}^2 - 1\right) = \left(q^2 \alpha_2 m_{\mathrm{P}i}^2 - M_i^2\right) c_n^4 = \\ &= \frac{\alpha^4}{\alpha_2^4} \left(q^2 \alpha m_{\mathrm{P}}^2 - M_i^2\right) c^4. \end{aligned} \quad (55)$$

If we assume these moduli are equal, we shall obtain the value of the imaginary mass M_i corresponding to an *object* having mass M and charge Q

$$\begin{aligned} &\alpha_2^4 \left(M^2 + q^2 \alpha m_{\mathrm{P}}^2\right) = \alpha^4 \left(q^2 \alpha m_{\mathrm{P}}^2 - M_i^2\right), \\ &M_i = \pm\sqrt{q^2 \alpha m_{\mathrm{P}}^2 \left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - \frac{\alpha_2^4}{\alpha^4} M^2}, \end{aligned} \quad (56)$$

as an equation with three unknowns M, M_i, and q.

In particular for an uncharged mass M ($q = 0$) this yields

$$M_i \alpha^2 = \pm i M \alpha_2^2 \quad \text{or} \quad M_i = \pm i \frac{\alpha_2^2}{\alpha^2} M \approx \pm 0.9557 i M. \quad (57)$$

Since mass M_i is imaginary by definition, the argument of the square root in the relation (56) must be negative

$$|M| > |q| m_{\mathrm{P}} \sqrt{\alpha \left(\frac{\alpha^4}{\alpha_2^4} - 1\right)} \approx |q| 5.7275 \times 10^{-10} \text{ [kg]}, \quad (58)$$

which means that masses of uncharged micro BHs ($q = 0$) can be arbitrary but micro NSs and micro WDs cannot be observed, as achieving a net charge $Q = 0$ is impossible in this case. Even a single elementary charge renders the mass $M = 5.7275 \times 10^{-10}$ [kg] comparable to the mass of a grain of sand.

We conjecture that the threshold (58) pertains solely to charged distinguishable *objects* having masses larger than the threshold of distinguishability [5] of $M < 2\pi m_{\mathrm{P}} \approx 1.3675 \times 10^{-7}$ [kg] (i.e., masses having Compton wavelengths smaller than the Planck length). A hypothetical (indistinguishable) element at this threshold would have an atomic number $Z = \left\lfloor 2\pi / \sqrt{\alpha(\alpha^4/\alpha_2^4 - 1)} \right\rfloor = 238$, corresponding to q, which - as

[6] In the cited study it is called α, so we shall call it β to avoid confusion with the fine-structure constant α.

we conjecture - sets the limit on an extended periodic table, and is lower that the accepted limit of 300 [40].

We can interpret the modulus of the generalized energy of *BBO*s (41) as the modulus of the complex energy of real mass (54), taking the observable real energy $E_{BBO} = M_{BBO}c^2$ of the *BBO* as the real part of this energy. Thus

$$\left(\frac{k}{2}M_{BBO}c^2\right)^2 = \left(M_{BBO}^2 + q_{BBO}^2 \alpha m_{\mathrm{P}}^2\right)c^4,$$
$$q_{BBO} = \pm\frac{M_{BBO}}{m_{\mathrm{P}}}\sqrt{\frac{1}{\alpha}\left(\frac{k^2}{4} - 1\right)}, \tag{59}$$

which is real for $k \geq 2$ and for $k = 2$ confirms vanishing net charge of *BH*s. Similarly, we can interpret the modulus of the generalized energy of *BBO*s (41) as the modulus of the complex energy of real charge (55). Thus

$$\frac{k^2}{4}M_{BBO}^2 = \frac{\alpha^4}{\alpha_2^4}\left(q_{BBO}^2 \alpha m_{\mathrm{P}}^2 - M_{iBBO}^2\right),$$
$$M_{iBBO}^2 = q_{BBO}^2 \alpha m_{\mathrm{P}}^2 - \frac{\alpha_2^4}{\alpha^4}\frac{k^2}{4}M_{BBO}^2. \tag{60}$$

Substituting q_{BBO}^2 from the relation (59) into the relation (60) yields

$$M_{iBBO}^2 = \left[\frac{k^2}{4}\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - 1\right]M_{BBO}^2,$$
$$M_{iBBO} = \pm M_{BBO}\sqrt{\frac{k^2}{4}\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - 1}, \tag{61}$$

which for *BH*s ($k = 2$) also corresponds to the relation (57) between uncharged masses M and M_i, where no assumptions concerning the *BBO* energy were made.

Furthermore, the argument of the square root in the relation (61) must be negative, as mass M_i is imaginary by definition. This leads to the maximum GM/c^2 multiplier

$$|k_{max}| < \frac{2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx 6.7933. \tag{62}$$

Relations (59) and (61) are shown in Fig 1.

The relation (62) sets the upper bound on the *BBO* radius (38) and energy (39)

$$R_{BBO} < \frac{R_{BH}}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \quad \text{and} \quad E_{BBO} < \frac{M_{BBO}c^2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}}, \tag{63}$$

where R_{BH} is the radius of the *BH* having a mass of the *BBO*. As *WD*s are the least dense *BBO*s, this bound defines the maximum radius of a *WD* core.

Furthermore, combining the minimum mass threshold (58) with the maximum mass multiplier threshold (62) of the charged *BBO* radius yields the minimum charge required for a given *BBO* diameter

$$q_{BBO} > \frac{1}{4}\sqrt{\frac{\alpha_2^4}{\alpha^5}}d_{BBO}. \tag{64}$$

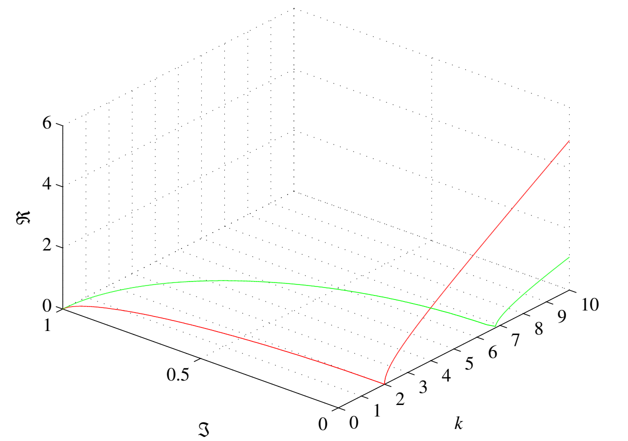

Figure 1. Ratios of imaginary mass M_{iBBO} to real mass M_{BBO} (green) and real charge $q_{BBO}m_{\mathrm{P}}\sqrt{\alpha}$ to M_{BBO} (red) of a *BBO* as a function of GM/c^2 multiplier $0 \leq k \leq 10$. Mass M_{iBBO} is imaginary up to $k_{max} < 6.79$. Charge q_{BBO} is real for $k \geq 2$.

Thus for example 1-bit *BBO* ($d_{BBO} = 1/\sqrt{\pi}$) requires $q_{BBO} > 1.5780$, while π-bit *BBO* ($d_{BBO} = 1$) requires $q_{BBO} > 2.7969$.

These results show that the radius of charged *BBO*s (i.e., *BBO*s other than *BH*s) is a continuous function of $k \in \mathbb{R} : 2 < k < k_{max}$; the largest k satisfying the *BBO* entropy relation (36), a necessary condition of patternless perfect black body radiation [5]. We shall consider this question in the subsequent section.

VI. MASS, CHARGE, EMR - PHOTON SPHERE RADIUS

Besides complex energies of masses and charges (48), (49)

$$E_{MQ_i} := Mc^2 + \frac{Q_i c^2}{2\sqrt{\pi\epsilon_0 G}} = \left(m + iq\sqrt{\alpha}\right)E_{\mathrm{P}}, \tag{65}$$

$$E_{QM_i} := \frac{Qc_n^2}{2\sqrt{\pi\epsilon_0 G}} + M_i c_n^2 = \frac{\sqrt{\alpha}\alpha^2}{\alpha_2^2}\left(q + \frac{m_i}{\sqrt{\alpha_2}}\right)E_{\mathrm{P}}, \tag{66}$$

we can also define the complex energies of real wavelength and imaginary mass/charge E_{RM_i}, E_{RQ_i}

$$E_{RM_i} := \frac{hc}{\lambda} + M_i c_n^2 = \frac{2\pi E_{\mathrm{P}}}{l} + m_i E_{\mathrm{P}i} =$$
$$= \left(\frac{2\pi}{l} + \sqrt{\frac{\alpha^5}{\alpha_2^5}}m_i\right)E_{\mathrm{P}}, \tag{67}$$

$$E_{RQ_i} := \frac{hc}{\lambda} + \frac{Q_i c^2}{2\sqrt{\pi\epsilon_0 G}} = \left(\frac{2\pi}{l} + iq\sqrt{\alpha}\right)E_{\mathrm{P}}, \tag{68}$$

and of imaginary wavelength and real mass/charge E_{MR_i}, E_{QR_i},

$$E_{MR_i} := Mc^2 + \frac{hc_n}{\lambda_i} = mE_{\mathrm{P}} + \frac{2\pi E_{\mathrm{P}i}}{l_i} = = \left(m + \sqrt{\frac{\alpha^5}{\alpha_2^5}}\frac{2\pi}{l_i}\right)E_{\mathrm{P}}, \tag{69}$$

$$E_{QR_i} := \frac{Qc_n^2}{2\sqrt{\pi\epsilon_0 G}} + \frac{hc_n}{\lambda_i} = \frac{\alpha^2}{\alpha_2^2}q\sqrt{\alpha}E_{\mathrm{P}} + \frac{2\pi}{l_i}E_{\mathrm{P}i} = = \frac{\alpha^2\sqrt{\alpha}}{\alpha_2^2}\left(q + \frac{2\pi}{l_i\sqrt{\alpha_2}}\right)E_{\mathrm{P}}, \tag{70}$$

where we applied discretizations (47). The energies (67)-(70) link the electromagnetic radiation with gravity/inertia and electrodynamics within the framework of emergent dimensionality.

Complex energies (65)-(70) are real-to-imaginary *balanced* if moduli of their real and imaginary parts are equal. This holds for

$$\begin{aligned} &|M| = i\,|M_i| = \left|m_{\mathrm{P}} q\sqrt{\alpha}\right|, \\ &|M_i| = i\left|\frac{\alpha_2^2}{\alpha^2}\frac{h}{c\lambda}\right|, \quad |M| = i\left|\frac{\alpha}{\alpha_2}\frac{h}{c\lambda_i}\right|, \\ &|\lambda| = \left|\frac{2\pi l_{\mathrm{P}}}{q\sqrt{\alpha}}\right|, \quad |\lambda_i| = i\left|\frac{2\pi l_{\mathrm{P}}\alpha_2}{q\sqrt{\alpha^3}}\right|. \end{aligned} \tag{71}$$

Furthermore, complex energies (65)-(70) define complex forces between two *objects* acting over real and imaginary *distances* R, R_i (cf. Appendix E).

This time, comparing the squared moduli of the energies (65)-(70) with the squared *BBO* energy (41) yield a solvable system of six equations with six unknowns k, q, m, m_i, l, l_i

$$\begin{aligned} &|E_{MQ_i}|^2 \Rightarrow q^2\alpha = m^2\left(\frac{k^2}{4} - 1\right), \\ &|E_{QM_i}|^2 \Rightarrow \frac{\alpha^4}{\alpha_2^4}q^2\alpha - \frac{\alpha^5}{\alpha_2^5}m_i^2 = \frac{k^2}{4}m^2, \\ &|E_{RM_i}|^2 \Rightarrow \frac{4\pi^2}{l^2} - \frac{\alpha^5}{\alpha_2^5}m_i^2 = \frac{k^2}{4}m^2, \\ &|E_{RQ_i}|^2 \Rightarrow \frac{4\pi^2}{l^2} + q^2\alpha = \frac{k^2}{4}m^2, \\ &|E_{MR_i}|^2 \Rightarrow m^2\left(1 - \frac{k^2}{4}\right) = \frac{\alpha^5}{\alpha_2^5}\frac{4\pi^2}{l_i^2}, \\ &|E_{QR_i}|^2 \Rightarrow \frac{\alpha^4}{\alpha_2^4}q^2\alpha - \frac{4\pi^2}{l_i^2}\frac{\alpha^5}{\alpha_2^5} = \frac{k^2}{4}m^2. \end{aligned} \tag{72}$$

Substituting $q^2\alpha = m^2\left(\frac{k^2}{4} - 1\right)$ from $|E_{MQ_i}|^2$ to $|E_{RQ_i}|^2$ recovers the Compton wavelength of the *BBO*, $\lambda_{BBO} = \frac{h}{M_{BBO}c}$, in its discrete form $l^2 = \frac{4\pi^2}{m^2}$.

Furthermore, by substituting $q^2\alpha$ and the Compton mass $m^2 = \frac{4\pi^2}{l^2}$ into $|E_{QM_i}|^2$, and comparing the LHSs of $|E_{QM_i}|^2$ and $|E_{RM_i}|^2$ we obtain the *BBO* equilibrium multiplier

$$\frac{k_{eq}^2}{4} = \frac{\alpha_2^4}{\alpha^4} + 1 \Rightarrow |k_{eq}| = 2\sqrt{\frac{\alpha_2^4}{\alpha^4} + 1} \approx 2.7665, \tag{73}$$

at which all the moduli (72) are equal. The equilibrium multiplier k_{eq} (73) is related to the bound k_{max} (62) by $k_{eq}^2 + 16/k_{max}^2 = 8$. Also, the following relations can be derived from the relations (72) for the *BBO* in the equilibrium k_{eq} (73)

$$m_i^2 = -\frac{\alpha_2^9}{\alpha^9}m^2 \quad \Leftrightarrow \quad M_{iBBO_{eq}} = \pm i\frac{\alpha_2^4}{\alpha^4}M_{BBO_{eq}}, \tag{74}$$

$$l_i^2 = -\frac{\alpha^9}{\alpha_2^9}l^2 \quad \Leftrightarrow \quad \lambda_{iBBO_{eq}} = \pm i\frac{\alpha^3}{\alpha_2^3}\lambda_{BBO_{eq}}, \tag{75}$$

$$l^2 = \frac{4\pi^2}{m^2} \quad \Leftrightarrow \quad \lambda_{BBO_{eq}} = \frac{h}{M_{BBO_{eq}}c} \tag{76}$$

$$q^2\alpha = \frac{\alpha_2^4}{\alpha^4}m^2 \quad \Leftrightarrow \quad \frac{\alpha_2^4}{\alpha^4} = -\beta_{QiBBO_{eq}}^2 = \pm i\frac{M_{iBBO_{eq}}}{M_{BBO_{eq}}}, \tag{77}$$

where in the last relation, we used the definition (50) and applied the relation (74).

Notably, $2.25 < k_{eq} < 3$, where $9/4$ is the multiplier of a radius of the maximal sustainable density for gravitating spherical *matter* given by Buchdahl's theorem, and 3 is the multiplier of a *BH* photon sphere radius.

This shows that $k \approx 2.766$ is a true photon sphere radius, where *BBO* gravity, charge, and photon energies remain at equilibrium. Aside from the Schwarzschild radius (derivable from escape velocity $v_{esc}^2 = 2GM/R$ of mass M by setting $v_{esc}^2 = c^2$), all the remaining thresholds of general relativity, such as Buchdahl's threshold ($k = 9/4$) or a photon sphere radius ($k = 3$), are only crude approximations. It must be so, since general relativity neglects the value of the fine-structure constants α and α_2, which, similarly as π or the base of the natural logarithm, are the fundamental constants of nature.

VII. DISCUSSION

The reflectance of graphene under the normal incidence of electromagnetic radiation expressed as the quadratic equation for the fine-structure constant α introduces the 2$^{\text{nd}}$ negative fine-structure constant α_2. The sum of the reciprocal of this 2$^{\text{nd}}$ fine-structure constant α_2 with the reciprocal of the fine-structure constant α (2) is independent of the reflectance value R and remarkably equals simply $-\pi$. Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, containing the free π term, can be interpreted as the asymptote of the CODATA value α^{-1}, the value of which increases with time. The negative fine-structure constant α_2 introduces the complementary set of Planck units applicable to imaginary dimensions, including imaginary Planck units (18)-(26). Real and imaginary mass and charge units (34), length and mass units (35) units, and temperature and time units (33) are directly related to each other. Also, the elementary charge e is common for real and imaginary dimensions (45).

Applying the complementary Planck units to a complex energy formula [39] yields complex energies (48), (49) setting the atomic number $Z = 238$ as the limit on an extended periodic table.

The generalized energy (41) of all perfect black-body *objects* (black holes, neutron stars, and white dwarfs) having the generalized radius $R_{BBO} = kGM/c^2$ exceed mass-energy equivalence if $k > 2$. Complex energies (48), (49) allow for storing the excess of this energy in their imaginary parts, inaccessible for direct observation. The results show that the perfect black-body *objects* other than black holes cannot have masses lower than 5.7275×10^{-10} [kg] and that the maximum slope of the radius of their cores as a function of mass is defined, as $k_{max} \approx 6.7933$, by the relation (62). It is further shown that a black-body *object* is in the equilibrium of complex energies if its radius $R_{eq} \approx 2.7665\, GM_{BBO}/c^2$ (73). It is conjectured that this is the correct value of the photon sphere radius.

The findings of this study inquire further research in the context of information-theoretic approach [1–6] and emergent dimensionality [9, 30, 31].

In the context of the results of this study, monolayer graphene, a truly 2-dimensional material with no thickness[7], is a *keyhole* to other, unperceivable [5], dimensionalities. Graphene history is also instructive. Discovered in 1947 [42], graphene was long considered an *academic material* until it was eventually pulled from graphite in 2004 [43] by means of ordinary Scotch tape[8]. These fifty-seven years, along with twenty-nine years (1935-1964) between the condemnation of quantum theory as *incomplete* [44] and Bell's mathematical theorem [45] asserting that it is not true, and the fifty-eight years (1964-2022) between the formulation of this theorem and 2022 Nobel prize in physics for its experimental *loophole-free* confirmation, should remind us that Max Planck, the genius who discovered Planck units, has also discovered Planck's principle.

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[7] Thickness of MLG is reported [41] as 0.37 [nm] with other reported values up to 1.7 [nm]. However, considering that 0.335 [nm] is the established inter-layer *distance* and consequently the thickness of bilayer graphene, these results do not seem credible: the thickness of bilayer graphene is not $2 \times 0.37 + 0.335 = 1.075$ [nm].

[8] Introduced into the market in 1932.

Appendix A: Other quadratic equations

The quadratic equation for the sum of transmittance (3) and absorptance (5), putting $C_{TA} \coloneqq T + A$, is

$$\frac{1}{4} C_{TA}\pi^2\alpha^2 + (C_{TA} - 1)\,\pi\alpha + (C_{TA} - 1) = 0, \tag{A1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is, similarly as the relation (12), also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (6) expressed as the quadratic equation and putting $C_{TR} \coloneqq T + R$, is

$$\frac{1}{4}\,(C_{TR} - 1)\,\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A6}$$

whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A7}$$

is dependent on T and R.

Appendix B: Two π-like constants

With algebraic definitions of α (13) and α_2 (14), transmittance T (3), reflectance R (4) and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π. For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (13) they become

$$T(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)^2}{(8\pi^2 + 2\pi + 3)^2} \approx 0.9775, \tag{B1}$$

$$A(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)}{(8\pi^2 + 2\pi + 3)^2} \approx 0.0224, \tag{B2}$$

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while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (14) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)^2}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 1.0228, \tag{B3}$$

$$A(\alpha_2) = -\frac{4\left(4\pi^2 + \pi + 2\right)}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx -0.0229, \tag{B4}$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 1.2843 \times 10^{-4}. \tag{B5}$$

$(T(\alpha)+A(\alpha))+R(\alpha) = (T(\alpha_2)+A(\alpha_2))+R(\alpha_2) = 1$ as required by the law of conservation of energy (8), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. Nonetheless, physical interpretation of $T(\alpha_2) > 1$ and $A(\alpha_2) < 0$ invites further research. $A(\alpha) > 0$ implies a *sink*, whereas $A(\alpha_2) < 0$ implies a *source*, whereas the opposite holds true for the transmittance T, as illustrated schematically in Fig 2. Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (11) or (14) could be explained in terms of graphene spontaneous emission.

The quadratic equation (9) describing the reflectance R of MLG under normal incidence of EMR (or alternatively (A1)) can also be solved for π yielding two roots

$$\pi(R, \alpha_*)_1 = \frac{2\sqrt{R}}{\alpha_*(1 - \sqrt{R})}, \quad \text{and} \tag{B6}$$

$$\pi(R, \alpha_*)_2 = \frac{-2\sqrt{R}}{\alpha_*(1 + \sqrt{R})}, \tag{B7}$$

dependent on R and α_*, where α_* indicates α or α_2. This can be further evaluated using the MLG reflectance R (4) or (B5) (which is the same for both α and α_2), yielding four, yet only three distinct, possibilities

$$\pi_1 = \pi(\alpha)_1 = -\pi\frac{4\pi^2 + \pi + 1}{4\pi^2 + \pi + 2} = \pi\frac{\alpha_2}{\alpha} \approx -3.0712, \tag{B8}$$

$$\pi(\alpha)_2 = \pi(\alpha_2)_1 = \pi \approx 3.1416, \quad \text{and} \tag{B9}$$

$$\pi_2 = \pi(\alpha_2)_2 = -\pi\frac{4\pi^2 + \pi + 2}{4\pi^2 + \pi + 1} = \pi\frac{\alpha}{\alpha_2} \approx -3.2136. \tag{B10}$$

The modulus of π_1 (B8) corresponds to a convex surface having a positive Gaussian curvature, whereas the modulus of π_2 (B10) - to a negative Gaussian curvature. Their product $\pi_1\pi_2 = \pi^2$ is independent of α_*, and their quotient $\pi_1/\pi_2 = \alpha_2^2/\alpha^2$ is independent of π. It remains to be found, whether each of them describes the ratio of circumference of a circle drawn on the respective surface to its diameter (π_c) or the ratio of the area of this circle to the square of its radius (π_a). These definitions produce different results on curved surfaces, whereas $\pi_a > \pi_c$ on convex surfaces, while $\pi_a < \pi_c$ on saddle surfaces [48].

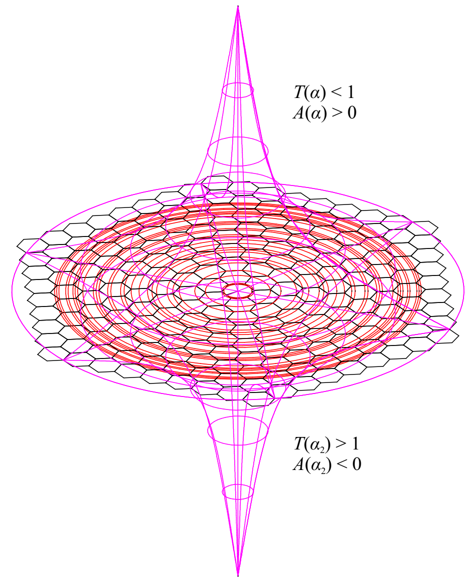

Figure 2. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

Appendix C: Planck units and HUP

Perhaps the simplest derivation of the squared Planck length is based on Heisenberg's uncertainty principle

$$\delta P_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2} \quad \text{or} \quad \delta E_{\text{HUP}}\delta t_{\text{HUP}} \geq \frac{\hbar}{2}, \tag{C1}$$

where δP_{HUP}, δR_{HUP}, δE_{HUP}, and δt_{HUP} denote momentum, position, energy, and time uncertainties, by replacing energy uncertainty $\delta E_{\text{HUP}} = \delta M_{\text{HUP}}c^2$ with mass uncertainty and time uncertainty with position uncertainty, using mass-energy equivalence and $\delta t_{\text{HUP}} = \delta R/c_{\text{HUP}}$ [28], which yields

$$\delta M_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2c}. \tag{C2}$$

Plugging $\delta M_{\text{HUP}} = \delta R_{\text{HUP}}c^2/(2G)$ for *BH* mass into (C2) we arrive at $\delta R_{\text{HUP}}^2 = \ell_{\text{P}}^2 \Rightarrow \delta D_{\text{HUP}} = \pm 2\ell_{\text{P}}$ and recover *BH* diameter $d_{BH} = \pm 2$.

However, using the same procedure but inserting the *BH* radius, instead of the *BH* mass, into the uncertainty principle (C2) leads to $\delta M_{\text{HUP}}^2 = \frac{1}{4}\hbar c/G = \frac{1}{4}m_{\text{P}}^2$. In general, using the generalized radius (38) in both procedures, one obtains

$$\delta M_{\text{HUP}}^2 = \frac{1}{2k}m_{\text{P}}^2 \quad \text{and} \quad \delta R_{\text{HUP}}^2 = \frac{k}{2}\ell_{\text{P}}^2. \tag{C3}$$

Thus, if k increases mass δM_{HUP} decreases, and δR_{HUP} increases and the factor is the same for $k = 1$ i.e., for *orbital speed radius* $\delta R = G\delta M/c^2$ or the *orbital speed mass* $\delta M = \delta Rc^2/G$.

Appendix D: Fluctuations of the holographic spheres

A simple relation describing the *BH* information capacity after absorption ($+16\pi^2 d_{BH}/l$) or emission ($-16\pi^2 d_{BH}/l$) of a *particle* having the wavelength λ multiplier $l = \lambda/\ell_P$ has been derived in [5] (Eq. (18)) as

$$N_{BH}^{A/E}(d_{BH}, l) = 64\pi^3 \frac{1}{l^2} \pm 16\pi^2 \frac{d_{BH}}{l} + \pi d_{BH}^2. \tag{D1}$$

Using the generalized radius (38), this equation can be generalized to all holographic spheres, including *BBO*s

$$N^{A/E}(d, l) = 16k^2\pi^3 \frac{1}{l^2} \pm 8k\pi^2 \frac{d}{l} + \pi d^2. \tag{D2}$$

Appendix E: Complex forces

Complex energies (65)-(70) define complex forces between two *objects*, real R or imaginary R_i *distance* apart. We exclude mixed forces (of real and imaginary masses/charges) as real and imaginary dimensions are orthogonal. Thus, using discretizations (47), we obtain the following products

$$\begin{aligned} &E_{2mq_i}E_{2mq_i} := E_{1MQ_i}E_{2MQ_i}/E_P^2 = \\ &= m_1m_2 - q_1q_2\alpha + i\sqrt{\alpha}(m_1q_2 + m_2q_1), \\ &E_{1qm_i}E_{2qm_i} := E_{1QM_i}E_{2QM_i}/E_P^2 = \\ &= \frac{\alpha^5}{\alpha_2^4}\left(q_1q_2 + \frac{1}{\alpha_2}m_{1i}m_{2i} + \frac{1}{\sqrt{\alpha_2}}(q_1m_{2i} + q_2m_{1i})\right), \end{aligned} \tag{E1}$$

$$\begin{aligned} &E_{1rm_i}E_{2rm_i} := E_{1RM_i}E_{2RM_i}/E_P^2 = \\ &= \frac{4\pi^2}{l_1l_2} + \frac{\alpha^5}{\alpha_2^5}m_{1i}m_{2i} + 2\pi\sqrt{\frac{\alpha^5}{\alpha_2^5}}\left(\frac{m_{2i}}{l_1} + \frac{m_{1i}}{l_2}\right), \\ &E_{1mr_i}E_{2mr_i} := E_{1MR_i}E_{2MR_i}/E_P^2 = \\ &= m_1m_2 + \frac{\alpha^5}{\alpha_2^5}\frac{4\pi^2}{l_{1i}l_{2i}} + 2\pi\sqrt{\frac{\alpha^5}{\alpha_2^5}}\left(\frac{m_1}{l_{2i}} + \frac{m_2}{l_{1i}}\right), \end{aligned} \tag{E2}$$

$$\begin{aligned} &E_{1qr_i}E_{2qr_i} := E_{1QR_i}E_{2QR_i}/E_P^2 = \\ &= \frac{\alpha^4}{\alpha_2^4}q_1q_2\alpha + \frac{4\pi^2}{l_1l_2}\frac{\alpha^5}{\alpha_2^5} + 2\pi\frac{\alpha^5}{\sqrt{\alpha_2^9}}\left(\frac{q_1}{l_2} + \frac{q_2}{l_1}\right), \\ &E_{1rq_i}E_{2qr_i} := E_{1RQ_i}E_{2RQ_i}/E_P^2 = \\ &= \frac{4\pi^2}{l_1l_2} + i2\pi\sqrt{\alpha}\left(\frac{q_2}{l_1} + \frac{q_1}{l_2}\right) - q_1q_2\alpha, \end{aligned} \tag{E3}$$

defining six complex forces; between two *particles* or *objects* acting over a real *distance* R

$$F_{AB_i} = \frac{G}{c^4R^2}E_{1AB_i}E_{2AB_i} = \frac{F_P}{r^2}E_{1ab_i}E_{2ab_i}, \tag{E4}$$

and six complex forces; between two *particles* or *objects* acting over an imaginary *distance* R_i

$$\tilde{F}_{AB_i} = \frac{G}{c_n^4R_i^2}E_{1AB_i}E_{2AB_i} = \frac{\alpha_2}{\alpha}\frac{F_P}{r_i^2}E_{1ab_i}E_{2ab_i}, \tag{E5}$$

where $A, B \in \{M, Q, R\}$ and $a, b \in \{m, q, r\}$, and

$$\alpha_2r^2F_{AB_i} = \alpha r_i^2\tilde{F}_{AB_i}. \tag{E6}$$

In the case of masses and charges (65) and (66), the complex forces (E1) unify Coulomb's law and Newton's law of gravity (similarly to the complex energy of real masses and charges (43), [39] Eq. (7)).

Furthermore, under a simplifying assumption of $r = r_i$, the forces acting over a real *distance* R are stronger and opposite to the corresponding forces acting over an imaginary *distance* R_i (if these *distances* are equal in modulus) even though the Planck force is lower in modulus than the complementary (and, in this case, real) Planck force (27). This issue requires further research.

Appendix F: A mixed speeds hypothesis

Let us define the mass/charge energies with different speeds of light, i.e., the charge part of the energy E_{MQ_i} with c_n and the charge part of the energy E_{QM_i} with c

$$\begin{aligned} \hat{E}_{MQ_i} &:= Mc^2 + \frac{Q_ic_n^2}{2\sqrt{\pi\epsilon_0G}} = Mc^2 \pm iq\sqrt{\alpha}m_P\frac{\alpha^2}{\alpha_2^2}c^2, \\ \hat{E}_{QM_i} &:= \frac{Qc^2}{2\sqrt{\pi\epsilon_0G}} + M_ic_n^2 = \pm q\sqrt{\alpha}m_Pc^2 + M_i\frac{\alpha^2}{\alpha_2^2}c^2, \end{aligned} \tag{F1}$$

If their moduli are equal, then

$$\begin{aligned} &M^2 + q^2\alpha m_P^2\frac{\alpha^4}{\alpha_2^4} = q^2\alpha m_P^2 - M_i^2\frac{\alpha^4}{\alpha_2^4}, \\ &M_i = \pm\sqrt{q^2\alpha m_P^2\left(\frac{\alpha_2^4}{\alpha^4} - 1\right) - \frac{\alpha_2^4}{\alpha^4}M^2}. \end{aligned} \tag{F2}$$

For an uncharged mass M, this relation corresponds to (57). However, since mass M_i is imaginary, the argument of the square root in the relation (F2) must be negative, i.e.,

$$|M| \not> |q|m_P\sqrt{\alpha\left(1 - \frac{\alpha^4}{\alpha_2^4}\right)}. \tag{F3}$$

But $\alpha^4 > \alpha_2^4$, yielding imaginary M, while M is real by definition. Therefore, complex energies E_{MQ_i} (48) and E_{QM_i} (49) must be parametrized respectively by c and c_n.

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