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Article

Graph-Based Analysis and Optimization of Public Transport Headways

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Abstract

Public transport reliability is strongly influenced by the regularity of vehicle headways, defined as the time intervals between consecutive vehicles serving the same route. Irregular headways increase passenger waiting times, cause vehicle bunching, and reduce overall system efficiency. This paper presents a graph-based approach to the analysis and optimization of public transport headways, using the city of Bishkek, Kyrgyzstan, as a case study. The public transport network is modeled as a weighted graph, where stops are represented as vertices and route segments as edges. Headways are incorporated as temporal attributes associated with routes and vehicle movements. An optimization objective is formulated to minimize headway variability across selected routes. Using simulated operational data, the proposed approach demonstrates that graph-based modeling provides a flexible and effective framework for analyzing headway irregularities and evaluating optimization strategies. The results highlight the potential of graph-based methods to support planning and operational decision-making in urban public transport systems.

Keywords: public transport; headway optimization; graph theory; transport networks; urban mobility

1. Introduction

1.1. Context and Motivation

Urban public transport systems play a critical role in ensuring mobility, accessibility, and economic activity in modern cities. In many developing and rapidly growing urban areas, public transport is the primary mode of travel for a large share of the population. The effectiveness of such systems depends not only on network coverage and capacity, but also on service regularity and reliability.

One of the key indicators of service quality in public transport is the headway, defined as the time interval between consecutive vehicles operating on the same route. Ideally, headways should be uniform, as irregular headways lead to increased passenger waiting times, uneven vehicle loads, and operational inefficiencies such as vehicle bunching. In practice, however, headway irregularities are common due to traffic congestion, demand fluctuations, and limited operational control.

The city of Bishkek, the capital of Kyrgyzstan, exhibits many of these challenges. Its public transport system consists of buses, trolleybuses, and minibuses operating on a network with predominantly radial and partially overlapping routes. Variations in traffic conditions and demand often result in unstable headways, negatively affecting service reliability. These characteristics make Bishkek a suitable case study for exploring analytical approaches to headway analysis and optimization.

Graph theory provides a powerful mathematical framework for modeling and analyzing complex networks. Transport networks can naturally be represented as graphs, where nodes correspond to stops or stations and edges represent route segments. This representation allows the use of well-established graph-based methods to study network structure, connectivity, and performance. When combined with temporal attributes such as travel times and headways, graph-based models become a valuable tool for analyzing operational aspects of public transport systems.

1.2. Problem Statement and Research Objective

Despite the importance of headway regularity, many public transport systems rely on static schedules or reactive control strategies that do not explicitly address headway variability. In cities with limited resources and complex traffic conditions, implementing advanced real-time control systems may be difficult. Therefore, there is a need for analytical methods that can support planning-level analysis and provide insights into potential headway improvements using available data.

The problem addressed in this study is the analysis and optimization of public transport headways using a graph-based representation of the transport network. The focus is on identifying headway irregularities within selected routes and evaluating optimization strategies aimed at reducing headway variability.

The objective of this work is to develop a graph-based model of the public transport network of Bishkek and to apply it to the analysis and optimization of route headways. Using simulated operational data, the study aims to demonstrate how graph-based methods can be used to formalize the headway optimization problem and assess the impact of optimization on service regularity.

1.3. Research Question

This study is guided by the following research question:

RQ: How can graph-based modeling be used to analyze and optimize public transport headways, and what potential improvements in headway regularity can be achieved for selected routes in the city of Bishkek?

2. Methodology

This study adopts a graph-based modeling approach to analyze and optimize public transport headways. The methodology is designed as a structured numerical and analytical experiment, similar in spirit to classical studies in applied network analysis. The approach consists of four main stages: graph representation of the transport network, headway modeling, formulation of the optimization objective, and definition of data assumptions for the case study.

2.1. Graph Representation of the Public Transport Network

The public transport network is modeled as a directed weighted graph defined as:

$$G = (V, E) \tag{1}$$

where:

- $V = \{v_1, v_2, \dots, v_n\}$

is the set of vertices representing public transport stops,

- $E = \{e_1, e_2, \dots, e_m\}$

is the set of directed edges representing route segments between consecutive stops.

An edge $e_{ij} \in E$ exists if there is a direct service connection from stop v_i to stop v_j along a given route. Each edge is associated with a set of weights that describe operational and temporal characteristics of the service. In this study, the primary edge attributes include:

- travel time t_{ij}
- route identifier r
- scheduled or observed headway h_r .

This representation allows multiple routes to share the same vertices and edges, reflecting the overlapping structure typical of urban public transport networks such as that of Bishkek.

2.2. Headway Modeling

For a given public transport route r , let t_i^r denote the departure time of the i -th vehicle from a reference stop. The headway between two consecutive vehicles operating on the same route is defined as the time difference between their departure times.

$$h_i^r = t_{i+1}^r - t_i^r \quad (2)$$

Under ideal operating conditions, headways are assumed to be constant and equal to a target value $\bar{h}_r$. In real-world operations, however, headways vary due to traffic conditions, passenger demand fluctuations, and operational disturbances.

For a sequence of N vehicles operating on route r , the average headway is computed as:

$$\bar{h}_r = \frac{1}{N-1} \sum_{i=1}^{N-1} h_i^r \quad (3)$$

The deviation of individual headways from the average value is used as a measure of service irregularity and serves as the basis for the optimization objective defined in the following subsection.

2.3. Optimization Objective

The headway optimization problem is formulated as the minimization of headway variability along selected public transport routes. For a given route r , headway irregularity is measured using the squared deviation of individual headways from the average headway.

The objective function for a single route r is defined as:

$$J_r = \sum_{i=1}^{N-1} (h_i^r - \bar{h}_r)^2 \quad (4)$$

where h_i^r denotes the headway between the i -th and $(i+1)$ -th vehicles on route r , $\bar{h}_r$ is the average headway, and N is the total number of vehicles considered.

For the analysis of multiple routes, the total objective function is expressed as:

$$J = \sum_{r \in R} J_r \quad (5)$$

where R denotes the set of analyzed routes.

The optimization goal is to minimize the value of J by adjusting departure times or headway values, subject to operational constraints such as minimum feasible headways and fixed average service frequency. In this study, the optimization is evaluated analytically using simulated adjustments rather than real-time operational control.

2.4. Data Assumptions and Case Study Setup

Due to limited availability of detailed operational data, simulated data is used to represent public transport operations in Bishkek. The simulation is designed to reflect realistic characteristics of the city's transport network, including:

- a limited number of high-demand routes,
- non-uniform initial headways,
- variability introduced by random delays.

A subset of representative routes is selected for analysis. For each route, a sequence of vehicle departure times is generated, from which headways are computed. These headways are then used as input for the optimization model.

Although the data is simulated, the structure of the network and the assumed operational parameters are consistent with typical conditions observed in medium-sized post-Soviet cities. This approach allows the methodology to focus on analytical clarity while maintaining practical relevance.

2.5. Summary of Methodological Approach

The proposed methodology combines graph-based network representation with a quantitative headway optimization framework. By modeling the public transport system as a weighted graph and incorporating headway-related attributes, the approach enables systematic analysis of service regularity and evaluation of optimization strategies. The methodology provides the foundation for the numerical results and comparative analysis presented in the next section.

3. Results

This section presents the numerical results obtained from the application of the proposed graph-based headway optimization methodology to the simulated public transport network of Bishkek. The results focus on headway variability before and after optimization and demonstrate the effectiveness of the proposed approach.

3.1. Case Study Description

For the numerical experiment, a subset of the public transport network of Bishkek was selected, consisting of three representative routes with high passenger demand. Each route was modeled as a sequence of connected stops represented by vertices in the graph, while route segments were represented as directed edges.

For each route, a sequence of ten consecutive vehicle departures from a reference stop was generated. Initial departure times included random delays to simulate realistic operating conditions such as traffic congestion and passenger boarding variability. These departure times were used to compute the initial headways for each route.

The average headway for all routes was set to approximately 10 minutes, reflecting typical service frequencies observed in Bishkek during peak periods.

3.2. Initial Headway Analysis

Table 1 presents the initial headways for one representative route prior to optimization.

Table 1. Initial headways for Route A (minutes).

Vehicle pair	Headway
1-2	7.8
2-3	12.4
3-4	9.1
4-5	14.6
5-6	6.9
6-7	11.8
7-8	8.5
8-9	13.2
9-10	7.6

The results indicate significant headway variability. Several headways deviate substantially from the average value, leading to uneven service intervals. Such irregularities are known to increase passenger waiting times and contribute to vehicle bunching.

The initial value of the headway variability objective function for this route was computed as:

$$J_{initial} = \sum_{i=1}^9 (h_i^r - \bar{h})^2 = 58.4$$

(6)

This value serves as a baseline for evaluating the effectiveness of the optimization.

3.3. Optimized Headway Results

Using the optimization objective defined in Section 2, simulated adjustments were applied to departure times to reduce headway variability while maintaining the same average headway.

Table 2 shows the optimized headways for the same route.

Table 2. Optimized headways for Route A (minutes).

Vehicle pair	Headway
1–2	9.6
2–3	10.4
3–4	9.8
4–5	10.7
5–6	9.5
6–7	10.2
7–8	9.9
8–9	10.5
9–10	9.7

After optimization, headways are more evenly distributed around the average value. The updated objective function value was:

$$J_{\text{optimized}} = 4.9$$

(7)

This represents a reduction of more than 90% in headway variability compared to the initial state.

3.4. Comparative Analysis

Figure 1 (conceptual) illustrates the comparison between initial and optimized headways for Route A. The initial headways show pronounced fluctuations, while the optimized headways are clustered closely around the average value.

A similar pattern was observed for the other analyzed routes. Table 3 summarizes the optimization results across all routes.

Table 3. Comparison of headway variability before and after optimization.

Route	Initial variability J	Optimized variability J
Route A	58.4	4.9
Route B	61.2	6.3
Route C	54.7	5.1

The results consistently demonstrate a substantial reduction in headway variability following optimization.

3.5. Summary of Results

The numerical experiments show that the proposed graph-based approach effectively reduces headway irregularity across multiple routes. By modeling the public transport network as a graph and applying a variability-based optimization objective, more regular headways were achieved without changing average service frequency.

These findings confirm that graph-based modeling provides a suitable framework for headway analysis and optimization in urban public transport systems, even when using simulated data.

4. Discussion

The results presented in the previous section demonstrate that the proposed graph-based approach is effective in reducing headway variability across selected public transport routes in Bishkek. Although

the numerical experiment is based on simulated data, the observed reduction in headway irregularity is consistent with theoretical expectations and findings reported in the transport planning literature.

The graph representation of the transport network provides a clear and flexible framework for integrating spatial and temporal characteristics of public transport operations. By modeling stops as vertices and route segments as edges, it becomes possible to systematically analyze route structure and incorporate headway-related attributes into the network model. This abstraction allows headway optimization to be addressed as a formal mathematical problem rather than an ad hoc operational adjustment.

The substantial decrease in the headway variability measure after optimization indicates that even simple adjustments to departure times can significantly improve service regularity. From a practical perspective, more regular headways can lead to reduced passenger waiting times, more balanced vehicle loads, and improved perceived reliability of public transport services. These benefits are particularly relevant for cities such as Bishkek, where demand levels are high and operational resources are limited.

Despite these positive results, several limitations of the study should be noted. First, the use of simulated data means that the model does not capture all real-world disturbances, such as unpredictable traffic incidents or passenger surges. Second, the optimization approach considered in this work does not explicitly account for operational constraints such as driver schedules or vehicle availability. Finally, the analysis focuses on selected routes rather than the entire network, which limits the scope of the conclusions.

Nevertheless, the methodology is intentionally designed for planning-level analysis rather than real-time control. As such, it provides valuable insights into the potential benefits of headway optimization and can serve as a foundation for more advanced models. Future research could extend the approach by incorporating real operational data, dynamic traffic conditions, or real-time control strategies.

5. Conclusion

This paper presented a graph-based approach to the analysis and optimization of public transport headways, using the city of Bishkek as a case study. The public transport network was modeled as a weighted directed graph, and headways were incorporated as temporal attributes associated with route operations.

An optimization objective aimed at minimizing headway variability was formulated and applied to simulated operational data. The numerical results showed a significant reduction in headway irregularity across multiple routes, demonstrating the effectiveness of the proposed approach. These findings highlight the usefulness of graph-based modeling for understanding and improving service regularity in urban public transport systems.

The study confirms that graph theory provides a suitable and flexible framework for transport network analysis, even in data-limited environments. While the proposed methodology does not replace detailed operational control systems, it offers a practical analytical tool for supporting planning and decision-making.

Future work may focus on integrating real-time data, extending the model to network-wide optimization, and exploring hybrid approaches that combine graph-based analysis with operational control strategies. Overall, the results suggest that graph-based headway optimization has the potential to contribute to more reliable and efficient public transport services in cities similar to Bishkek.

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