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Horizon and Regime Dependent Performance of GARCH Type Models: Evidence from Volatility Forecasting in a Frontier Market

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Abstract

This study examined the comparative performance of GARCH family models including GARCH, EGARCH, GJR GARCH, APARCH, and FIGARCH within a horizon and regime aware framework to assess forecasting accuracy. Using daily prices of equity and foreign exchange markets in Kenya covering 1997–2024, volatility was modelled and validated through Value at Risk and Expected Shortfall back tests to establish economic relevance. The results reveal strong horizon and regime dependence: EGARCH performs well in capturing short run volatility in the equity market and turbulent foreign exchange regimes, while FIGARCH dominates in calm equity markets and at medium term horizons. Risk validation confirms that FIGARCH delivers reliable tail risk forecasts for equities, whereas EGARCH excels in turbulent foreign exchange markets. Unlike prior comparative studies that focus on efficient markets with stable volatility structures, this study applied GARCH family models to a frontier market, comparing forecasting accuracy across varying horizons and regimes. The study advanced beyond best fit evaluations by linking forecasting performance to horizon length, asset type, and regime shifts, thereby contributing new evidence on modelling volatility in African frontier markets and offering insights relevant for regulators, institutional investors, and policymakers concerned with financial stability.

Keywords: GARCH; forecasting volatility; frontier markets; FIGARCH; EGARCH; regime shifts; horizons; Value at Risk; Expected Shortfall

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1. Introduction

Modelling Volatility has remained central to financial econometrics since the seminal contributions of Engle (1982) and Bollerslev (1986), which introduced the ARCH and GARCH frameworks. These models became the dominant empirical approach for capturing time-varying volatility and the stylized facts of clustering and persistence (Hansen & Lunde, 2005; Poon & Granger, 2003). Over time, extensions such as EGARCH (Nelson, 1991a), GJR-GARCH (Glosten et al., 1993), APARCH (Ding et al., 1993), and FIGARCH (Baillie et al., 1996) were developed to incorporate leverage effects, power transformations, and long-memory dynamics. Despite these advances, comparative studies show that no single specification dominates across assets, horizons, or regimes (Patton & Sheppard, 2015). This unresolved debate has motivated research into horizon- and regime-specific performance of GARCH-type models.

Forecasting volatility is regime-dependent and horizon-specific. Standard GARCH models produce smooth forecasts that fail to distinguish between calm and turbulent regimes (Hamilton & Susmel, 1994), while regime-switching extensions demonstrate superior performance during crises (Marcucci, 2005). Asymmetric models tend to outperform in high-volatility regimes, whereas long-memory specifications provide gains at longer horizons (Conrad & Haag, 2006). Recent contributions emphasize integrating macroeconomic volatility components and adopting non-parametric approaches to improve robustness against structural breaks (Engle & Rangel, 2008; Kamronnaher et al., 2024; Makatjane & Mmelesi, 2024). These findings highlight ongoing controversies: whether volatility is best explained by structural regimes, asymmetric responses, or long-memory persistence, and whether model rankings remain stable across contexts.

African frontier markets present a challenging environment for modelling volatility. Volatility dynamics are shaped by recurrent structural breaks, thin market depth, and sensitivity to external shocks (Balcilar et al., 2015; Salisu et al., 2020). Evidence from Nigeria, South Africa, and North African economies shows that crises and regime shifts disrupt volatility patterns severely than in developed markets, with asymmetric and long-memory models outperforming standard GARCH specifications during turbulent periods (Salisu et al., 2020). Yet systematic evaluations of horizon- and regime-dependent performance remain scarce in Africa, leaving unresolved questions about how volatility models behave under conditions of thin liquidity, political uncertainty, and recurrent macroeconomic shocks. This gap is critical given the reliance of regulators and institutional investors on volatility forecasts for risk management and financial stability.

Another unresolved debate concerns the economic relevance of volatility forecasts. While many studies rank GARCH models using statistical loss functions, fewer examine their usefulness for risk management through Value-at-Risk (VaR) and Expected Shortfall (ES). Regulatory bodies rely on these tail-risk measures, yet evidence on whether GARCH-family models generate reliable inputs for such applications in frontier markets remains limited (Patton et al., 2019). Addressing this gap requires a horizon- and regime-aware evaluation framework that compares statistical forecasting accuracy and assesses the economic value of volatility forecasts for risk management.

Against this background, this study examined how the performance and ranking of GARCH-family models vary across asset classes, forecast horizons, and market regimes in Kenya. In Kenya thin trading, exchange rate volatility, and exposure to global commodity shocks amplify volatility persistence and asymmetry. Unlike prior studies that focus on statistical loss functions, this study examined the economic relevance of volatility forecasts by assessing their performance in VaR and ES backtests. Therefore, this study contributes to the debate on horizon- and regime-dependent model performance, extends comparative volatility modelling to African frontier markets, and provides insights relevant for regulators, institutional investors, and policymakers concerned with financial stability.

1.1. Theoretical Framework

Modelling volatility is anchored in several theories that explain market behaviour, risk transmission, and investor psychology. The Efficient Market Hypothesis (EMH) (Fama, 1970) posits that asset prices reflect available information, with its weak form aligning to the random walk theory (Kendall & Hill, 1953). While EMH suggests that volatility arises from new information, empirical evidence from frontier markets validates persistent inefficiencies, thin liquidity, and behavioural biases that amplify volatility (Shiller, 2003). The tension between theoretical efficiency and observed inefficiency underscores the need for models that capture clustering, persistence, and asymmetry.

Modern Portfolio Theory (MPT) (Markowitz, 1952)and the Capital Asset Pricing Model (CAPM) (Sharpe, 1964) conceptualize volatility as a measure of risk, emphasizing diversification and systematic exposure. Both frameworks assume normal return distributions and rational investor behaviour which is violated in African markets where recurrent shocks, political instability, and thin trading produce fat-tailed distributions and extreme volatility (Mandelbrot & Hudson, 2010). The Arbitrage Pricing Theory (APT) (Ross, 1976) attempts to incorporate multiple sources of risk, however its empirical application in frontier markets is limited(Faruque, 2011).

Stochastic process theory provides the mathematical foundation for modelling volatility. The random walk hypothesis (Bachelier, 1900)and Brownian motion underpin modern time-series analysis, while mean-reverting processes capture the tendency of volatility to revert to long-run levels. Building on these principles, ARCH and GARCH models (Bollerslev, 1986; Engle, 1982) introduced conditional heteroskedasticity, enabling researchers to capture volatility clustering and persistence.

Behavioural finance challenges rational frameworks by incorporating cognitive biases and emotional influences. Prospect theory (Kahneman & Tversky, 2013) demonstrates that investors exhibit loss aversion, leading to asymmetric responses to gains and losses. Herding behaviour, overconfidence, and anchoring distort price discovery, producing excess volatility and speculative bubbles (Shleifer, 1986). These behavioural tendencies are relevant in markets, where limited information circulation and weak institutional structures amplify the impact of investor sentiment on volatility dynamics (Balcilar et al., 2015; Salisu et al., 2020). Behavioural insights justify the use of asymmetric GARCH variants such as EGARCH and GJR-GARCH, which model leverage effects and asymmetric responses to shocks (Dinga et al., 2023; Watard et al., 2024).

The financialization of commodities has restructured volatility transmission by increasing cross-market linkages. Commodities, once driven by supply and demand fundamentals, are now influenced by institutional investors, hedge funds, and algorithmic trading (Cheng & Xiong, 2014). This integration has increased co-movement between commodity and equity markets, reducing diversification benefits and increasing systemic risk(Domanski & Heath, 2007; Tang & Xiong, 2012). For Kenya's economy, which is exposed to commodity price shocks and exchange rate volatility, financialization magnifies vulnerability to global shocks and underscores the need for volatility models that incorporate both domestic and international drivers.

1.2. Empirical Literature Review

Empirical research on modelling volatility has evolved since the introduction of ARCH and GARCH frameworks (Bollerslev, 1986; Engle, 1982). While these models captured short-run persistence, they failed to account for the slow hyperbolic decay in volatility autocorrelations (Baillie et al., 1996; Ding et al., 1993). This limitation motivated the development of long-memory models such as FIGARCH and HYGARCH, which outperform standard GARCH specifications in environments characterized by persistent information flows and heterogeneous agents (Andersen & Bollerslev, 1998; Davidson, 2004)(Andersen & Bollerslev, 1998; Davidson, 2004). Parallel advances introduced asymmetric models that incorporate leverage and size effects, improving estimation of volatility during periods of market stress (Orakcioglu, 2015). Evidence suggests that asymmetries and persistence effects are noticeable in emerging and frontier markets, reflecting structural differences in liquidity, market depth, and information circulation (Lim & Sek, 2013; Othman et al., 2019).

Recent studies emphasize the regime-dependent and horizon-specific nature of volatility forecasts. Standard GARCH models produce smooth forecasts that fail to distinguish between calm and turbulent regimes (Hamilton & Susmel, 1994), while regime-switching extensions demonstrate superior performance at short horizons and during crisis periods (Dueker, 1997; Marcucci, 2005). Asymmetric specifications tend to dominate under stress conditions, whereas long-memory models provide gains at longer horizons (Brownlees et al., 2011; Conrad & Haag, 2006). Recent contributions highlight the importance of integrating macroeconomic volatility components and adopting non-parametric approaches to improve robustness against structural breaks (Engle & Rangel, 2008; Kamronnaher et al., 2024; Makatjane & Mmelesi, 2024). These findings underscore the continuing debate: whether volatility is best explained by structural regimes, asymmetric responses, or long-memory persistence, and whether model rankings remain stable across horizons and asset classes.

Studies in Risk management have shifted from variance-based measures to tail-risk metrics such as Value-at-Risk (VaR) and Expected Shortfall (ES). While VaR is widely used, its non-coherent nature has prompted regulators to recommend ES as a reliable measure of extreme losses (Tian et al., 2019). Empirical evidence shows that models incorporating long memory and asymmetry, with skewed Student's t distributions, provide superior estimates of both VaR and ES (Aloui & Ben Hamida, 2015).

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However, the superiority of regime-switching models remains horizon-dependent, with single-regime models excelling in short-term forecasts and regime-switching models outperforming over medium and long horizons (Hoang & Luu, 2024). This debate is relevant in frontier markets, where volatility dynamics are shaped by thin liquidity, political uncertainty, and exposure to commodity price shocks.

Despite these advances, comparative studies have focused on developed and emerging markets, leaving frontier markets underexplored. Existing evidence confirms that although volatility persistence, asymmetry, and regime dependence are stronger in less mature markets, there is limited evaluations of horizon-specific and regime-dependent performance. This gap is relevant in Africa, where integrated financial markets are vulnerable to macroeconomic instability and political shocks. Equity and foreign exchange markets in Kenya exhibit stylized facts of volatility clustering and leverage effects, yet remain underrepresented in global volatility forecasting research. By focusing on Kenya, this study provides new insights into the horizon- and regime-dependent performance of GARCH-family models in a frontier market setting, thereby contributing to both academic debates and practical risk management applications.

2. Materials and Methods

2.1. Research Design

This study adopted a quantitative research design to investigate the horizon- and regime-dependent performance of GARCH-family models in a frontier market context. The study used daily equity and foreign exchange return data. Research objectives focused on model estimation, forecast evaluation, and comparative performance analysis. Financial return series are stochastic and exhibit stylized facts. As such, econometric models provide an appropriate framework for capturing these dynamics (Bollerslev, 1986; Engle, 1982). The research design is guided by the theoretical framework of conditional heteroskedasticity (Bollerslev, 1986; Engle, 1982). Extensions of this framework justify the inclusion of asymmetric models to capture leverage effects (Glosten et al., 1993; Nelson, 1991b), as well as long-memory models to account for persistent volatility behaviour (Baillie et al., 1996). The study employed multiple GARCH-type specifications to address the first objective by conducting both in-sample assessment and out-of-sample evaluation. The study incorporated a multi-horizon and regime-dependent framework to address the second objective by evaluating forecast performance across different horizons and regimes in order to capture structural changes in volatility dynamics. The third objective was addressed by integrating an economic evaluation component into the research design. Model performance was assessed based on the ability to capture downside risk using Value-at-Risk (VaR) and Expected Shortfall (ES) measures.

2.2. Data Description

Raw data includes Daily NSE 20 Share index and USD/KES exchange rate from November 1997–December 2024.

Daily returns are derived as continuously compounded logarithmic differences of prices, expressed as

$$r_t = 100 * \ln\left(\frac{P_t}{P_{t-1}}\right), \quad (1)$$

where P_t represents the closing value of the asset exchange rate on day t.

2.3. Model Specifications

Five GARCH specifications are estimated using Student-t distributed errors.

Let:

$$r_t = \mu + \varepsilon_t, \qquad \varepsilon_t = \sigma_t z_t, \qquad z_t \sim i.i.d.\ (0,1). \quad (2)$$

GARCH(1,1) Model

The GARCH specification is used because of its ability to capture volatility clustering in financial time series.

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$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \qquad \omega > 0,\ \alpha, \beta \geq 0,\ \alpha + \beta < 1. \tag{3}$$

The GARCH (1,1) model assumes symmetric response to shocks.

EGARCH(1,1) Model

EGARCH model, introduced by Nelson (1991) addresses limitations of the standard GARCH model.

$$\ln(\sigma_t^2) = \omega + \alpha\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\ln(\sigma_{t-1}^2). \tag{4}$$

A negative value of γ signals the presence of a leverage effect.

GJR-GARCH(1,1) Model

The GJR-GARCH model, introduced by Glosten, Jagannathan, and Runkle (1993), extends the standard GARCH framework by incorporating asymmetric responses to positive and negative shocks.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{5}$$

where

$$I_{t-1} = \begin{cases} 1, & \text{if } \varepsilon_{t-1} < 0, \\ 0, & \text{otherwise.} \end{cases}$$

Here $\gamma > 0$ reflects the extra sensitivity of volatility to negative shocks, creating asymmetry in the conditional variance.

APARCH(1,1) Model

APARCH model, introduced by Ding, Granger, and Engle (1993), accommodates asymmetry, power transformations of volatility, and leverage effects.

$$\sigma_t^\delta = \omega + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta, \tag{6}$$

where the power parameter $\delta > 0$ allows flexible scaling of volatility magnitude, and γ captures leverage.

FIGARCH(1,d,1) Model

The Fractionally Integrated GARCH (FIGARCH) model, introduced by Baillie, Bollerslev, and Mikkelsen (1996), allows for fractional integration in the conditional variance process.

In compact lag-polynomial notation,

$$\phi(L)(1-L)^d\left(\varepsilon_t^2 - \sigma^2\right) = \omega + [1-\beta(L)]v_t, \qquad 0 \leq d < 1, \tag{7}$$

where , L denotes the lag operator, while $(1-L)^d$ represents the fractional differencing operator, with d capturing the degree of fractional integration. When $d > 0$, volatility shocks diminish hyperbolically rather than exponentially.

2.4. Estimation Technique

Model estimation is carried out using quasi-maximum likelihood with the Bollerslev–Wooldridge robust covariance approach. The suitability of each specification is assessed using information criteria alongside residual-based diagnostics.

2.5. Forecast Design

Out-of-sample volatility forecasts are obtained through a rolling window framework. Models are initially fitted on a starting sample, then they are re-estimated sequentially to generate conditional variance forecasts at horizons.

$$h \in \{1,5,20\} \text{ (days)}. \tag{8}$$

Forecast accuracy is assessed from statistical and economic perspectives. Statistical evaluation uses RMSE, MAE, QLIKE, and Diebold–Mariano tests. Economic performance is assessed via VaR and ES with back-testing. Models are ranked across assets, horizons, and regimes.

2.6. Testing Horizon and Regime Dependence

Three-stage analysis evaluates 1-, 5-, and 20-day forecasts using loss metrics and Diebold–Mariano pairwise comparisons.

To evaluate regime dependence, out-of-sample period is divided into calm and crisis segments, identified through historical events or smoothed regime probabilities.

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In GARCH-type specifications, volatility persistence is summarized by

$$\phi = \alpha + \beta, \tag{9}$$

Corresponding half-life of shocks is derived as

$$H = \frac{\ln(0.5)}{\ln(\phi)}, \tag{10}$$

provided that $\phi < 1$.

2.7. Model Selection and Interpretation

Model selection is based on a dominance framework using multiple criteria. Preference is given to specifications that pass diagnostic checks, ensuring residuals show no remaining serial correlation or ARCH effects.

3. Results

This section presents the empirical findings from the comparative evaluation of GARCH models applied to Kenyan equity and foreign exchange market.

3.1. Descriptive Statistics

The descriptive statistics in table 1 confirm the presence of stylized facts. Both series show near-zero mean returns and excess kurtosis. The Augmented Dickey–Fuller (ADF) tests confirm stationarity for both return series (p < .01) in line with evidence that financial return series are stationary processes (Engle, 1982). High kurtosis calls for for long memory specifications while mild skewness favours asymmetric models (Orakcioglu, 2015). Differences in the magnitude of volatility between equity and forex highlight regime dependence consistent with (Marcucci, 2005) and support risk based evaluation using VaR and ES where FIGARCH and APARCH are superior consistent with Aloui and Ben Hamida (2015).

Table 1. *Descriptive Statistics and Diagnostics.*

Asset	count	mean	std	skew	kurtosis	ADF(p-value)	LB_resid (p)	LB_sq (p)	JB (p-value)
NSE 20	6703	-0.006	0.782	0.213	9.35	0	0	0.933	0
USD/KES	6703	0.011	0.432	0.061	23.99	0	0	1	0

Source(s): Author's own work.

Figures 1 and 2 confirm volatility persistence and regime shifts in equity and forex markets. Volatility persistence is dominant in frontier economies where shocks spread gradually due to limited market depth and slower information circulation (Alberg *et al.*, 2008; Caporale and Zekokh, 2019). Volatility processes are characterized by shifts between high- and low-variance states (Ardia *et al.*, 2018; Conrad and Kleen, 2020). In frontier markets regime transitions are high due to exposure to external shocks and concentrated investor bases (Caporale and Zekokh, 2019; Mensi *et al.*, 2021).

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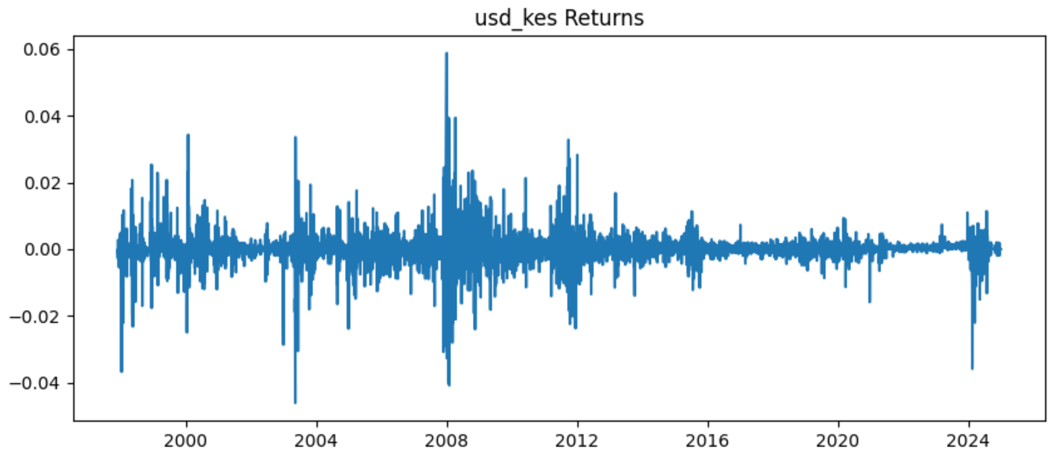

Figure 1. *USD / KES daily returns time series plot, Source: Author's own creation.*

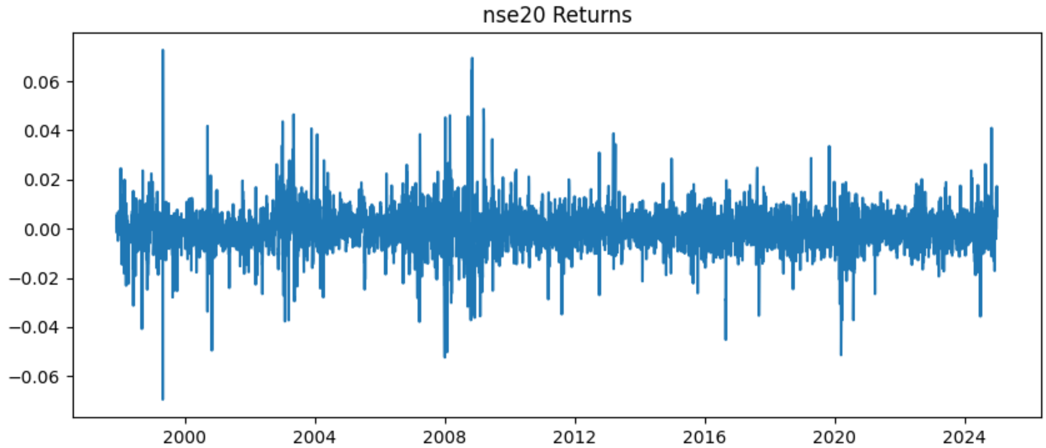

Figure 2. *NSE 20 Share Index Daily Returns time series plot, Source: Author's own creation.*

3.2. Model Parameters Estimates

Parameters in table 2 provide evidence that volatility dynamics in Kenyan financial markets are heterogeneous. For equity, the fractional integration parameter ($d = 0.228$, $p < .01$) confirms presence of long-range dependence in conditional volatility. Long-memory behaviour is prevalent in equity markets exposed to structural uncertainty and occasional macroeconomic shocks in frontier contexts (Baillie and Morana, 2009; Conrad and Kleen, 2020; Mensi *et al*., 2021). The magnitude of the fractional parameter ($d < 0.5$) indicates covariance stationarity with slow mean reversion, a property observed in markets characterised by thin trading and gradual information circulation (Caporale and Zekokh, 2019). Forex dynamics are dominated by asymmetric responses to market innovations. The asymmetry parameters in EGARCH ($\gamma = 0.084$, $p < .01$) and APARCH ($\gamma = -0.061$, $p = .016$) confirm leverage effects. Exchange rates in emerging economies exhibit asymmetric volatility responses, reflecting risk repricing, capital flow reversals, and policy uncertainty (Ardia *et al*., 2018; Caporale and Zekokh, 2019; Mensi *et al*., 2021). The high persistence parameter in the EGARCH specification ($\beta = 0.977$) implies that once volatility spikes occur, they disintegrate slowly consistent with findings that FX markets experience prolonged volatility following negative external shocks (Conrad and Kleen, 2020).

Table 2. *Model Parameter Estimates and Preliminary diagnostics.*

Asset	Model	Parameter	Estimate	Std. Error	t-Statistic	p-value
NSE 20	FIGARCH	d	0.228	0.044	5.134	0.000
NSE 20	FIGARCH	omega	0.069	0.013	5.408	0.000
NSE 20	FIGARCH	beta	0.444	0.096	4.618	0.000
USD/KES	EGARCH	alpha	0.450	0.034	13.330	0.000
USD/KES	EGARCH	beta	0.977	0.004	256.410	0.000
USD/KES	EGARCH	gamma	0.084	0.013	6.728	0.000
USD/KES	APARCH	delta	2.375	0.507	4.683	0.000
USD/KES	APARCH	gamma	-0.061	0.025	-2.395	0.016

Source(s): Author's own work.

3.3. In-Sample Estimation

Table 3 presents in-sample model selection criteria. For equity, FIGARCH achieved the lowest AIC and BIC values consistent with Baillie and Morana (2009) and Conrad and Kleen (2020). For the FX market, EGARCH yields the lowest AIC and BIC values, indicating superior in-sample fit consistent with Ardia *et al*. (2018) and Caporale and Zekokh ((2019).

Table 3. *In Sample Model Selection Criteria.*

Asset	Model	Log-Likelihood	AIC	BIC
NSE 20	FIGARCH	-6535.81	13083.63	13124.49
	APARCH	-6547.02	13108.04	13155.71
	GARCH	-6550.60	13111.20	13145.30
	EGARCH	-6561.31	13133.00	13166.70
USD/KES	EGARCH	368.60	-727.31	-693.26
	FIGARCH	347.83	-683.66	-642.80
	APARCH	331.01	-648.01	-600.34
	GARCH	323.01	-636.01	-601.96

Source(s): Author's own work.

3.4. Forecast Performance Across Horizons

Results in table 4 confirm that model performance is horizon-dependent and regime-sensitive. No single GARCH specification dominates across forecast horizons (Conrad & Kleen, 2020; Hansen & Lunde, 2005; Patton, 2011). For Equity FIGARCH was dominant at the 20-day horizon. Long-memory models outperform short-memory alternatives when forecasting medium- to long-term volatility (Baillie & Morana, 2009; Conrad & Kleen, 2020). This finding supports Caporale and Zekokh (2019) and Mensi *et al*. (2021). The efficiency of GARCH (1,1) at the 5-day horizon indicates that short-term volatility exhibits localized mean-reversion consistent with weekly trading cycles. Similar horizon-specific reversals were documented by Hansen and Lunde (2005), and Patton (2011).

In the forex market, the shifting dominance across horizons reinforces the asset-specific and regime-dependent nature of modelling volatility. The short-term and crisis-period performance of FIGARCH suggests that during turbulent phases, volatility shocks are persistent consistent with Conrad and Kleen (2020); Mensi *et al*. (2021) . The superior long-horizon performance of EGARCH indicates that asymmetric leverage effects drive forecast accuracy over long periods. Exchange rates in developing economies respond asymmetrically to depreciation (Ardia *et al*., 2018; Caporale and Zekokh, 2019)

Table 4. *Forecast Performance Across Horizons.*

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Asset	Model	MSE (1)	MSE (5)	MSE (20)	QLIKE (1)	QLIKE (5)	QLIKE (20)	QLIKE (Calm)	QLIKE (Turb)
NSE 20	FIGARCH	4.267	52.672	467.333	0.532	4.093	18.64	-0.209	1.274
	APARCH	4.399	52.246	468.576	0.615	3.69	20.289	0.049	1.18
	EGARCH	4.382	52.132	468.911	0.606	3.598	20.786	0.028	1.184
	GARCH	4.396	51.9	468.42	0.613	3.428	20.067	0.046	1.181
USD/KES	EGARCH	0.936	10.584	92.735	20.054	49.265	5.68	-4.465	44.574
	FIGARCH	0.935	10.582	96.68	13.644	46.113	124.205	-4.267	31.555
	APARCH	0.936	10.595	96.676	19.168	76.162	122.202	-4.443	42.78
	GARCH	0.936	10.597	96.763	19.65	86.542	203.504	-4.455	43.755

Source(s): Author's own work.

3.5. Diebold-Mariano (DM) Test for Forecast Significance by Regime

The DM test results in table 5 confirm that forecast accuracy is regime-dependent. Model dominance is conditional on market states rather than universal across environments (Diebold *et al*., 2005; Patton, 2011). Ranking GARCH models reverses between calm and turbulent periods in markets characterized by occasional shocks (Conrad and Kleen, 2020; Mensi *et al*., 2021). During calm regimes, FIGARCH was the best forecasting model for equity market across horizons. DM statistics and QLIKE values indicate that incorporating fractional integration improves forecast performance compared to short-memory GARCH specifications consistent with Billie and Morana (2009) and Caporale and Zekokh (2019). For the FX market under calm conditions, the dominance of EGARCH suggests that asymmetric responses are relevant even when volatility levels are subdued. Exchange rates in emerging economies exhibit persistent leverage effects due to asymmetric reactions to depreciation, policy signals and capital flows (Ardia *et al*., 2018; Mensi *et al*., 2021).

Table 5. *DM Forecast Results by Regime.*

Asset	Horizon	Market State	Loss Metric	Best Model	DM Statistic	p-value
NSE 20	1-Day	Turbulent	QLIKE	FIGARCH	-6.31	0.000
NSE 20	1-Day	Calm	QLIKE	FIGARCH	1223.08	0.000
NSE 20	20-Day	Calm	QLIKE	FIGARCH	44.80	0.000
NSE 20	20-Day	Turbulent	QLIKE	FIGARCH	12.47	0.000
USD/KES	1-Day	Turbulent	QLIKE	FIGARCH	15.85	0.000
USD/KES	1-Day	Calm	QLIKE	EGARCH	69.60	0.000
USD/KES	5-Day	Turbulent	MSE	FIGARCH	11.99	0.000
USD/KES	20-Day	Calm	QLIKE	EGARCH	17.10	0.000

Source(s): Author's own work.

3.6. Parameter Estimates and Distributional Diagnostics

The fractional integration parameter of 0.228 for the equity FIGARCH model in table 6 provides evidence of long-memory dynamics in the Kenyan equity market, confirming that volatility shocks decay slowly over time, consistent with Baillie *et al*. (1996) and Bollerslev and Mikkelsen (1996). The Student-t shape parameter of 5.187 supports the presence of leptokurtosis, consistent with Brownlees *et al*. (2011).

In the FX market, APARCH has a significant asymmetry parameter of -0.0607, providing evidence of leverage effects. This finding is consistent with Orakcioglu (2015. The significant power parameter of 2.375 justifies the use of flexible power transformations, consistent with Bali *et al*. (2012) and Brownlees *et al*. (2011). EGARCH diagnostics, with a Student-t shape parameter of 2.8, confirm the heavy-tailed nature of currency returns, in line with Liu and Hung (2010) and Gupta (2023).

Table 6. *Parameter Estimates and Distributional Diagnostics.*

Asset	Model	Parameter	Estimate	Std. Error	t-Stat	p-value
NSE 20	FIGARCH	d (Long Memory)	0.228	0.044	5.134	0.000
NSE 20	FIGARCH	Shape (ν)	5.187	0.355	14.616	0.000
USD/KES	EGARCH	ω (Constant)	0.031	0.018	1.743	0.081
USD/KES	EGARCH	Shape (ν)	2.800	0.115	24.288	0.000
USD/KES	APARCH	γ (Asymmetry)	-0.061	0.025	-2.395	0.016
USD/KES	APARCH	δ (Power)	2.375	0.507	4.683	0.000

Source(s): Author's own work.

3.7. Value-at-Risk and Expected Shortfall

Table 7 present the back testing results for Value-at-Risk and Expected Shortfall. For equity, all models are robust at the 1% VaR level, with one violation out of 500 observations, supporting earlier evidence that GARCH models can provide reliable risk estimates in equity markets. The Kupiec LR statistic of 4.81 ($p = 0.028$) indicates that these models overestimate risk, thereby offering higher capital buffers. This is consistent with Hansen and Lunde (2006) and Patton (2011). The Christoffersen test p-values of 0.949 confirm that violations are independent.

The FX results show systemic failure across all models at the 1% level, with violations exceeding expectations and Kupiec and Christoffersen p-values of 0.000. This outcome reflects the limitations of GARCH models in capturing extreme volatility and clustering of shocks inherent in frontier currency markets, consistent with Ardia *et al*. (2018) and Othman *et al*. (2019). The EGARCH model provides conservative Expected Shortfall (-1.2) and the lowest number of violations (24). Liu and Hung (2010) and Gupta (2023) emphasized the ability of EGARCH to capture leverage effects and extreme shocks in currency markets.

Table 7. *Back testing Results.*

Asset	Model	VaR (1%)	ES (1%)	Violations	Kupiec LR	p-value	Christoffersen LR	Christ. p-value
NSE 20	GARCH	-3.387	-4.477	1	4.81	0.028	0.004	0.949
NSE 20	EGARCH	-3.355	-4.435	1	4.81	0.028	0.004	0.949
NSE 20	APARCH	-3.393	-4.485	1	4.81	0.028	0.004	0.949
NSE 20	FIGARCH	-2.965	-3.919	1	4.81	0.028	0.004	0.949

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USD/KES	GARCH	-0.378	-0.593	34	74.08	0.000	48.923	0.000
USD/KES	EGARCH	-0.585	-1.200	24	38.03	0.000	33.621	0.000
USD/KES	APARCH	-0.382	-0.599	31	62.51	0.000	43.529	0.000
USD/KES	FIGARCH	-0.440	-0.689	28	51.56	0.000	37.775	0.000

Source(s): Author's own work.

4. Discussion

4.1. Horizon Dependence

The Diebold-Mariano (DM) test results reveal sensitivity of model performance to the forecast horizon. At the one-day horizon, the negative and significant DM statistics for pairs involving the FIGARCH model against GARCH and EGARCH specifications suggest that capturing long-memory dynamics is essential for short-term volatility forecasts. This finding is consistent with Maqsood *et al.* (2017) who argue that the Nairobi Securities Exchange (NSE) lacks weak-form efficiency, causing information shocks to persist rather than dissipate rapidly. As the forecast window extends to five and twenty days, this persistence becomes critical for the equity market during turbulent regimes where the FIGARCH model maintains statistical superiority. Such results corroborate the assertions of Ahmed and Suliman (2011) regarding the explosive nature of conditional variance in African frontier markets, where the structural characteristics of volatility require models that account for fractional integration over longer durations (Ahmed and Suliman, 2011; Maqsood *et al.*, 2017).

While asymmetric models like EGARCH and APARCH demonstrate superior predictive power at shorter horizons their relative advantage shifts during prolonged calm periods. At the twenty-day horizon for USD/KES in a calm regime, the DM statistics turn positive, indicating that parsimonious models may match or exceed the performance of complex long-memory specifications. The marginal utility of modelling long-range dependence diminishes when the currency reaches a state of relative equilibrium or is subject to stabilizing central bank interventions consistent with Lim and Sek (2013).

4.2. Regime Dependence

The Diebold-Mariano (DM) tests highlight regime dependence in the predictive accuracy of the GARCH models, suggesting that the structural dynamics of the Kenyan market undergo shifts between periods of stability and stress. For the Equity market the regime-specific results demonstrate a contrast between market conditions. In calm regimes, the standard GARCH yields positive and significant DM statistics against FIGARCH and EGARCH across horizons. At the five-day horizon in a calm state, the DM statistic for the GARCH versus FIGARCH pair stands at 5.51, indicating that the parsimonious symmetric specification is sufficient and superior when market volatility is low. However, this relationship reverses during turbulent regimes, where the DM statistics for GARCH against FIGARCH and EGARCH become negative (-14.76 for GARCH-FIGARCH at h=5). This reversal emphasises the inadequacy of symmetric models in capturing asymmetric shocks and long-memory persistence that characterize Kenyan equity market crises, necessitating the use of non-linear frameworks during periods of economic instability. In the forex market the optimal model specification is sensitive to the prevailing market environment. During turbulent periods the symmetric GARCH model is outperformed by the EGARCH and APARCH models, which account for the leverage effect and the aggressive market reactions to currency depreciation. This is reflected in the negative DM statistics during crises, which confirm that ignoring asymmetry leads to a significant loss in forecast precision. The USDKES data reveals that while FIGARCH shows short-term gains in calm periods at the one-day horizon, the standard GARCH model regains its statistical dominance as the horizon extends to five and twenty days.

Regime-dependent performance validates the theoretical underpinnings of Markov regime-switching frameworks popularized by Hamilton (1989). The results are consistent with Ardia *et al*. (2018) and Kambouroudis *et al*. (2021), who argue that a single-regime GARCH model frequently misprices risk because it averages volatility parameters across distinct market states (Ardia *et al*., 2018; Kambouroudis and McMillan, 2016). This is consistent with Kirui *et al*. (2014), who noted that the response of Kenyan markets to news is contingent on the existing macro-economic climate.

4.3. Economic Significance

The back testing results for the equity and forex market reveal divergence in risk model performance. For the FX market all the GARCH models failed the Kupiec and Christoffersen test.

Results for the equity market were conservative. At the 5% level, the models recorded a single violation despite an expected count of twenty-five, indicating that the risk parameters are inflated. By overstating potential losses, these models force institutional investors to hold excessive reserves against equity portfolios, thereby reducing the capital available for active investment and potentially dampening overall market liquidity. While the currency market is exposed to hidden risks, the equity market is hampered by a cautious risk framework that fails to reflect the true distribution of daily returns on the NSE.

The failure of symmetric models in the currency market validates the leverage effect. The performance gains of EGARCH and FIGARCH over the standard GARCH in the USD/KES tests support the necessity of accounting for leptokurtosis and long-memory persistence in emerging market data sets. The conservative bias observed in the NSE 20 Share index results is consistent with Kambouroudis *et al*. (2021), who argued that traditional GARCH models can over-react to historical shocks in markets with low trading frequency . The rejection of the conditional coverage tests for the USD/KES indicates that the underlying return distribution is non-normal.

5. Conclusions

This study examined the performance of GARCH models in a frontier market , focusing on equity and FX across multiple horizons and regimes. Forecasting volatility depends on the asset class, forecast horizon, and market. FIGARCH is the best for equity while EGARCH is ideal for the FX.

Policy Implications

The Central Bank of Kenya and the Capital Markets Authority should advocate for state-aware risk management practices. Risk management models for the equity market are over-conservative. Regulators can unlock dead capital by recalibrating risk management models to reflect the actual loss distribution of the Nairobi Securities Exchange. The underestimation of tail risk in the FX market is a red flag for financial stability. CBK should allow the use of ES as a primary risk metric for FX exposure. Risk management strategies for portfolio management should be duration-specific.

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Abbreviations

The following abbreviations are used in this manuscript:

ARCH	Autoregressive Conditional Heteroskedasticity
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
GJR-GARCH	Glosten–Jagannathan–Runkle Generalized Autoregressive Conditional Heteroskedasticity
APARCH	Asymmetric Power Autoregressive Conditional Heteroskedasticity
FIGARCH	Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity
VaR	Value-at-Risk
ES	Expected Shortfall
MAE	Mean Absolute Error
QLIKE	Quasi-Likelihood Loss Function
DM Test	Diebold–Mariano Test
NSE	Nairobi Securities Exchange
USD/KES	United States Dollar / Kenyan Shilling Exchange Rate
EMH	Efficient Market Hypothesis
MPT	Modern Portfolio Theory
CAPM	Capital Asset Pricing Model
APT	Arbitrage Pricing Theory
MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	Linear dichroism

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