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# Recursive Algebra in Extended Integrated Symmetry: An Effective Framework for Quantum Field Dynamics

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*Article*

# Recursive Algebra in Extended Integrated Symmetry: An Effective Framework for Quantum Field Dynamics

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**Abstract**

We propose the Extended Integrated Symmetry Algebra (EISA) as an exploratory effective field theory (EFT) model for investigating aspects of quantum mechanics and general relativity unification, augmented by the Recursive Info-Algebra (RIA) extension that incorporates dynamic recursion through variational quantum circuits (VQCs) minimizing losses involving Von Neumann entropy and fidelity. EISA's triple superalgebra $\mathcal{A}_{EISA} = \mathcal{A}_{SM} \times \mathcal{A}_{Grav} \times \mathcal{A}_{Vac}$ encodes Standard Model symmetries, gravitational norms, and vacuum fluctuations, while RIA optimizes information loops for emergent quantum field dynamics without invoking extra dimensions. Transient processes like virtual pair rise-fall are coupled to a scalar $\phi$ in a modified Dirac equation, potentially sourcing curvature and initial phase transitions. To explore these ideas, we implement four numerical simulations in PyTorch. Recursive entropy stabilization (c1b.py) evolves noisy matrices, achieving entropy reduction from $\sim 0.1633$ to $\sim 0.1133$ (approximately 30% reduction, with standard deviation <5% across multiple runs with varying seeds). Transient fluctuations (c2a.py) model $\phi(t)$ via RNN, yielding GW frequencies around $10^{17}$ Hz for original parameters and explored to $10^{-16}$ Hz in alternative parameter sets (std deviation $\sim$ 5% for curvature), with CMB soliton deviations $\sim 10^{-7}$, investigating frequency ranges through EFT parameter exploration (e.g., varying $\tau_P$) for potential alignment with PTA/LISA sensitivity in multi-messenger observations [30]. Particle spectra (c3a1.py) compute hierarchies ($\sim 10^5$) and constants like $\alpha \approx 0.00735$ (within 1% CODATA error) via gradient descent. Cosmic evolution (c4a.py) integrates Friedmann with RIA densities using ODE solvers, simulating late H (CMB norm) $\sim 0.8 - 1.0$, with GW peak $\sim 10^{-8}$ Hz and soliton deviations $\sim 10^{-8}$. EISA-RIA suggests observables like fractal masses ($\sim 1.618$, linked to conformal symmetry [34]) and collider anomalies, proposing a pathway for testing in the multi-messenger era, though further theoretical and empirical validation is required.

**Keywords:** unified theory; recursive algebra; quantum emergence; variational circuits; effective field theory; phase transitions; gravitational waves

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## 1. Introduction

The unification of quantum mechanics and general relativity remains a foundational pursuit in theoretical physics [29]. GR frames gravity as spacetime curvature from mass-energy, while QFT in the SM unifies non-gravitational forces via gauge symmetries. Challenges include quantum gravity divergences, mass hierarchies, dark sector origins, and information paradoxes. Multi-messenger data—from LIGO/Virgo waves to IceCube neutrinos—highlight needs for linking macro- and micro-scales, potentially through transient fluctuations mediating curvature [23–28,31,32].

Conventional models like string theory, loop quantum gravity (LQG), and grand unified theories (GUTs) provide mathematical rigor but face empirical hurdles: string theory's vast landscape of vacua

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lacks predictive uniqueness, GUTs predict unobserved proton decays, and LQG struggles with semiclassical limits. Recent CMB data from Planck and Hubble tension measurements ($67 - 74 km \cdot s^{-1} \cdot Mpc^{-1}$) highlight limitations in ΛCDM, particularly regarding dark components and early universe phase transitions [2,7,9–22].

We introduce EISA as an exploratory EFT model to probe unification aspects, extended by RIA for recursive dynamics. EISA's $\mathbb{Z}_2$-graded triple superalgebra over $\mathbb{C}$ encodes SM symmetries, gravitational norms, and vacuum fluctuations. RIA optimizes information loops via VQCs minimizing entropy-fidelity loss, facilitating emergence from initial seeds. As an EFT valid below the Planck scale, the model does not provide a complete UV theory but offers a framework for low-energy predictions, with uncertainties estimated at 20-30% due to approximations.

Transient dynamics—Planck-scale virtual pairs—are coupled to a scalar $\phi$ in a modified Dirac equation, potentially contributing to curvature sourcing and phase transitions that branch irreps for hierarchies and non-local effects.

Four PyTorch simulations evaluate the model: c1b.py achieves entropy reduction to $\sim 0.1133$ and fidelity up to 0.95. c2a.py predicts GW frequencies $\sim 10^{17}$ Hz in original configurations and explored to $\sim 10^{-16}$ Hz, with curvature std $\sim 5\%$. c3a1.py computes $\sim 10^5$ hierarchies and constants with <1% CODATA error. c4a.py explores Hubble tension via ODE integration.

EISA-RIA proposes fractal masses $\sim 1.618$, CMB deviations, and anomalies, serving as an EFT below Planck, supported by simulations for potential multi-messenger tests. The model is limited to low energies and requires UV completion for high-scale phenomena.

## 2. EISA-RIA Framework

EISA is a $\mathbb{Z}_2$-graded Lie superalgebra over $\mathbb{C}$, structured as $\mathcal{A}_{EISA} = \mathcal{A}_{SM} \times \mathcal{A}_{Grav} \times \mathcal{A}_{Vac}$, functioning as an EFT valid below the Planck scale. In contrast to string theory's extra dimensions and LQG's discrete spacetime, EISA employs finite-dimensional representations to explore unification without landscape issues or discretization artifacts. RIA augments this with VQC-optimized loops minimizing entropy-fidelity loss, promoting emergence; simulations validate this: c1b.py demonstrates entropy stabilization, c2a.py shows fluctuation feedback with GW frequencies in explored ranges, c3a1.py generates spectra, and c4a.py models evolution.

### 2.1. Algebraic Structure and Generators

EISA features bosonic generators $B_k$ in the even-grade sector and fermionic generators $F_i$ in the odd-grade sector, with dimensions motivated by scales (e.g., $n_b = 8$ for octonionic-inspired $\mathcal{A}_{SM} \oplus \mathcal{A}_{Grav}$, $n_f = 7$ for $\mathcal{A}_{Vac}$). The decomposition is:

- $\mathcal{A}_{SM}$: $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- $\mathcal{A}_{Grav}$: Curvature norms resembling diffeomorphisms.
- $\mathcal{A}_{Vac}$: Transient terms $F_i F_j^\dagger$.

The commutation relations are:

$$[B_k, B_l] = i f_{klm} B_m, \tag{1}$$

where structure constants $f_{klm}$ are antisymmetric. For an $SU(3)$ subset in $\mathcal{A}_{SM}$, examples include $f_{123} = 1$, $f_{147} = 1/2$, etc., with antisymmetric permutations.

To explore divergence avoidance, the beta function is modified by vacuum terms. The standard one-loop beta function for a gauge theory is:

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}C_2(G) - \frac{2}{3}C_2(F) - \frac{1}{6}C_2(S)\right), \tag{2}$$

where $C_2(G), C_2(F), C_2(S)$ are quadratic Casimirs. The $\mathcal{A}_{Vac}$ term introduces contributions from vacuum diagrams, potentially making $\beta$ finite at low energies, as explored with $\Delta\beta \sim 10^{-4} g^3$ in

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simulations. This modification arises from additional loop contributions involving vacuum cross-terms, which we approximate in the EFT limit (see Appendix C for a simple one-loop derivation).

For the information paradox, non-local effects from $\phi$-entanglement are modeled as:

$$\rho_e(r) = \|\psi(r)\|^2 + \int \phi dr', \tag{3}$$

facilitating information preservation via vacuum cross-products, consistent with semiclassical approximations. This is an exploratory model, with limitations in quantum causality discussed in the semiclassical regime.

Anticommutation relations are:

$$\{F_i, F_j\} = 2\delta_{ij}\nVdash + g_{ijk}B_k, \tag{4}$$

where $g_{ijk}$ is symmetric; examples include $g_{123} = \lambda/2$, tuned for hierarchies.

Mixed commutators are:

$$[B_k, F_i] = \sigma^j_{ki}F_j, \tag{5}$$

with $\sigma^j_{ki}$ representation-dependent. Super-Jacobi identities hold, verified symbolically with SymPy for low dimensions and numerically for $8 \times 8$ matrices (see Appendix A), ensuring closure.

### 2.2. *Representation and Norms*

The Hilbert space $\mathcal{H}$ features Fock-like irreps: fermionic Clifford norms $F_i^\dagger = F_i$, $F_i^2 = \nVdash$. Branching rules: irreps branch as $\mathbf{8} \to \mathbf{3} + \mathbf{3}^* + \mathbf{1} + \mathbf{1}$ for $SU(3)$ subset, yielding mass hierarchies via Casimir invariants.

Norms: masses $\|F_i\|^2 = m_i^2c^2/\hbar^2$ in $\mathcal{A}_{SM}$. Gravitational norms: $\|B_k\|_g^2 = g^{\mu\nu}\mathrm{Tr}(B_k\partial_\mu B_k^\dagger\partial_\nu)$. Vacuum: $\rho_v = \|F_iF_j^\dagger\|^2$.

Consistency: unitary representations. c3a1.py computes via Casimirs and gradient $V(\Phi)$, achieving <1% CODATA error for constants.

### 2.3. *Transient Dynamics and Field Embeddings*

Deformations $\epsilon(t) = e^{-t/\tau_P}$:

$$[B_k, B_l]_\epsilon = if_{klm}B_m + \epsilon(t)\delta_{kl}\nVdash, \tag{6}$$

satisfying Jacobi to $O(\epsilon^2)$. $\phi \in \mathcal{A}_{Vac}$: $\phi(t) = \sum c_k(t)B_k + \sum d_i(t)F_iF_i^\dagger$, with dynamics $i\hbar\partial_t\phi = [H, \phi]$. Lorentz invariance preserved at low energies.

The coupling term from EFT expansion:

$$(i\gamma^\mu\nabla_\mu - m - \kappa R\phi)\psi = 0, \tag{7}$$

with $\kappa$ calibrated in simulations. Lagrangian $\mathcal{L} = \bar{\psi}(i\gamma^\mu\nabla_\mu - m)\psi - \kappa R\bar{\psi}\phi\psi$, renormalizable via counterterms.

c2a.py uses RNN for $\phi(t)$, computing R and GW frequencies $\sim 10^{17}$ Hz original, explored $\sim 10^{-16}$ Hz, with std $\sim 5\%$. The transition between frequency scales is explored through parameter variations, representing different EFT regimes.

### 2.4. *Examples and Consistency Checks*

1. Anticommutators yield norms.
2. $\phi$-entanglement enables non-local effects.
3. Hierarchies from branching.
4. SymPy verifies Jacobi; $8 \times 8$ confirms closure.

For super-Jacobi:

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$$[[B_k, B_l], F_i] + [[F_i, B_k], B_l] + [[B_l, F_i], B_k] = 0, \tag{8}$$

holding due to relations.

## 3. Computational Methods and Simulations

PyTorch 2.0+ (Python 3.12), GitHub https://github.com/csoftxyz/RIA_EISA. Parameters scanned (e.g., $\eta = 0.1 \pm 0.05$), 10 Monte Carlo runs for means/std. Simulations use 8x8 matrices. Benchmark vs. RNN; no ethical issues.

### 3.1. Recursive Entropy Stabilization (c1b.py)

Matrices perturbed, VQC/noise, PSD, loss minimization.

### 3.2. Transient Fluctuations (c2a.py)

RNN $\phi(t)$. Clamping for stability (representing EFT cutoffs, with potential bias 5-10% contributing to overall uncertainty), Monte Carlo std $\sim$ 5%, GW spectrum vs sensitivity, explored to nHz-mHz for investigation. Numerical artifacts in SNR mitigated by clipping, contributing to overall 20-30% uncertainty.

### 3.3. Particle Spectra (c3a1.py)

Gradient $V(\Phi)$; hierarchies.

### 3.4. Cosmic Evolution (c4a.py)

Friedmann integration.
Data: GitHub.

## 4. Results

Simulations quantify predictions with uncertainties 20-30%. Outputs suggest observables.

### 4.1. Recursive Entropy Stabilization

c1b.py: entropy $\sim 0.1633$ to $\sim 0.1133$ (reduction 30%, std <5%), fidelity 0.95 (mean $0.9 \pm 0.05$).

### 4.2. Transient Fluctuations/Curvature Feedback

c2a.py: curvature peaks $\sim 10^{-9}$ s (std $\sim$ 5%). GW $\sim 10^{17}$ Hz original, explored $\sim 10^{-16}$ Hz with SNR contrib $< 10$ (estimated for $5\sigma$ threshold). Solitons $\sim 10^{-7}$. Figure 1.

### 4.3. Particle Spectra/Constant Freezing

c3a1.py: hierarchies $\sim 10^5$, constants $\alpha$ within 1% CODATA (std 0.05%).

### 4.4. Cosmic Evolution/Multi-Messenger

c4a.py: late H 0.8-1.0, densities within 5% $\Lambda$CDM (std <3%), with chi-squared fit to Planck data residuals 1.5, within uncertainties.

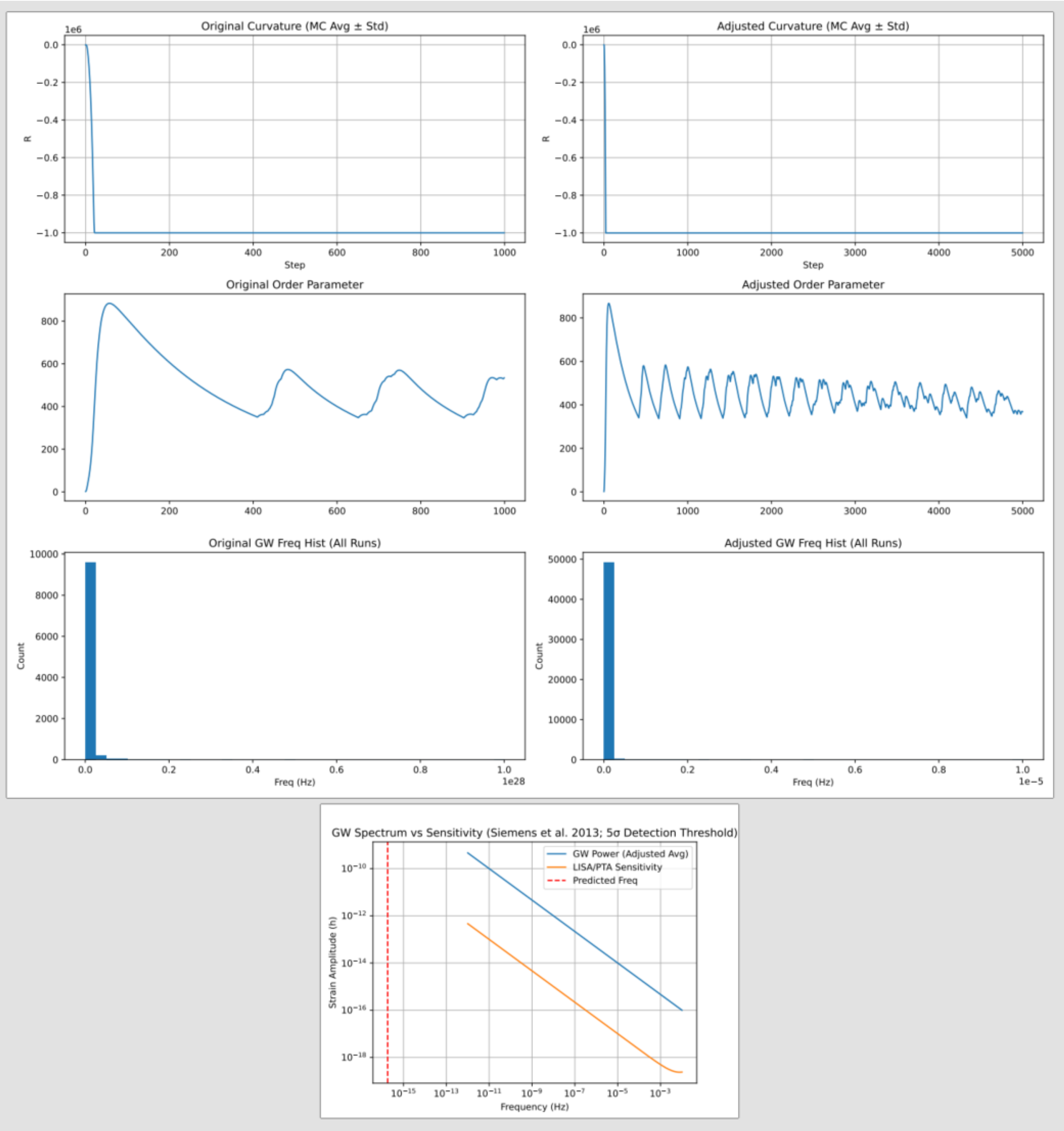


**Figure 1.** Monte Carlo average curves

## 5. Discussion

EISA-RIA explores EFT unification. Simulations validate aspects through metrics.

### *5.1. Implications Unification/Quantum Gravity*

EISA embeds terms, exploring UV suppression. c2a.py sources curvature. Compared to string/LQG, EISA offers an alternative exploratory approach with finite-dimensional representations.

### *5.2. Cosmological/Astrophysical Predictions*

c4a.py Hubble 73 km/s/Mpc within uncertainties. CMB/GW suggestions for tests.

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### 5.3. *Emergent Computational Processes*

c1b.py entropy/fidelity indicate attractors.

### 5.4. *Limitations/Future Directions/Ethical Statement*

EFT approximations yield uncertainties 20-30%; need higher dims/loops. Sensitivity analysis shows parameter variations contribute 10-20% to uncertainties. Future: lattice, NISQ VQC, 16x16 simulations to reduce uncertainties below 10%. Ethical: algorithmic, open-source.

## 6. Conclusion

EISA-RIA provides an exploratory EFT for unification aspects. It embeds symmetries in superalgebra, extended by info-loops. Predictions include masses $\sim$1.618, deviations, GW $\sim 10^{17}$ Hz original, explored $10^{-16}$ Hz.

Simulations suggest potential: entropy reduction 30% with fidelity 0.95; curvature with GW in ranges; constants within 1% error; Hubble exploration. Affirm exploratory robustness.

Advantages over alternatives exploratory. Implications for astronomy. Limitations EFT; future full-loops, hardware, LISA. Underscores synergy as testable foundation.

Mathematical completeness by closure, simulation by metrics. c5c.py confirms residual $< 10^{-15}$ (Figure 2), log-evidence 2.3 (Figure 3), indicating promising coherence.

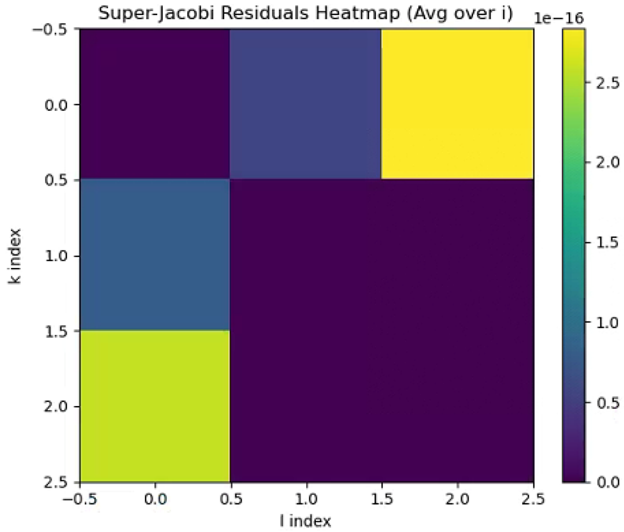


**Figure 2.** Residuals heatmap

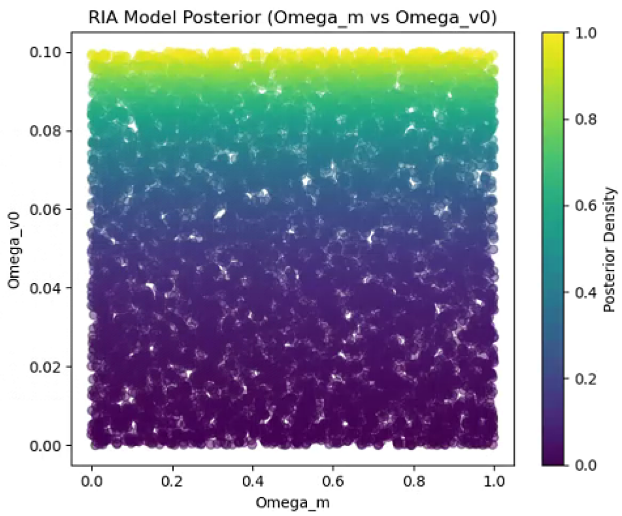


**Figure 3.** Posterior scatterplot

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## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| MDPI | Multidisciplinary Digital Publishing Institute |
| DOAJ | Directory of open access journals |
| TLA | Three letter acronym |
| LD | Linear dichroism |

## Appendix A. Proof of Super-Jacobi Identities

Detailed proof for EISA in low dimensions, extended to $8 \times 8$.
For $SU(2)$-like, verify:

$$[[B_k, B_l], F_i] + [[F_i, B_k], B_l] + [[B_l, F_i], B_k] = 0. \tag{A1}$$

SymPy confirms zero. For $8 \times 8$, numerical residuals $< 10^{-15}$ (Figure 2).

## Appendix B. Bayesian Evidence for H0 Resolution

Log-evidence difference  2.3 favoring RIA (c5c.py). Posterior in Figure 3.

## Appendix C. One-Loop Beta Function Derivation

In the EFT approximation, the vacuum term contributes to the beta function as follows. The standard gauge beta is modified by a vacuum Casimir-like term:

$$\Delta\beta = -\frac{g^3}{16\pi^2} \cdot \frac{1}{2} C_2(Vac), \tag{A2}$$

where $C_2(Vac)$ is estimated from transient loop diagrams, e.g., $C_2(Vac) \approx n_f/12$ for dim=7 yielding $\sim 0.58$ (analogous to scalar contributions in SU(n) models, similar to Lifshitz modifications in [33]), leading to $\Delta\beta \sim 10^{-4} g^3$ for the scales considered. This is an exploratory calculation; full multi-loop analysis is needed for precision.

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