Preprints.org

Article

Not peer-reviewed version

# Photon Entanglement, Bell Inequality Violation, and Energy Interpretation of the Born Rule in Maxwell–Schwartz Field Theory

David Carfì *

Posted Date: 20 March 2026

doi: 10.20944/preprints202603.1612.v1

Keywords: Maxwell fields; Bell correlations; CHSH inequality; tempered distributions; polarization states; electromagnetic fields; energy norm; Born rule

*Article*

# Photon Entanglement, Bell Inequality Violation, and Energy Interpretation of the Born Rule in Maxwell–Schwartz Field Theory

**David Carfì** [1,2]

[1] Section of Theoretical Physics, Department of Mathematics, Computer Sciences, Physical Sciences and Earth Sciences, University of Messina, 98166 Messina, Italy; dcarfi@unime.it

[2] Fractal Analysis, Dynamical Systems and Mathematical Physics Group, Department of Mathematics, University of California Riverside, Riverside, CA 92521, USA

**Abstract**

In this paper we study photon entanglement in the framework of Maxwell–Schwartz field theory. The ambient state space is the complex Maxwellian distribution space $W = \mathcal{S}'(M_4, \mathbb{C}^3)$, whose elements are fields of the form $F = E + icB$. Polarization is realized as a two-dimensional complex subspace of $W$, generated by suitable linearly polarized Maxwellian solutions associated with opposite propagation directions. This yields canonical polarization sectors $P_A$ and $P_B$, each naturally isomorphic to $\mathbb{C}^2$. Within this setting, the Bell singlet state is represented by a non-factorizable tensorial Maxwellian field in $P_A \otimes P_B \subset W \otimes W$. By means of the induced rotated polarization bases, the standard joint probabilities of the photon polarization experiment are recovered exactly, and the correlation law $E(a, b) = -\cos(2(a - b))$ is obtained. Consequently, the usual CHSH value $2\sqrt{2}$ is reproduced in the Maxwell–Schwartz framework. To clarify the meaning of this violation, we first formulate the CHSH inequality in a purely measure-theoretic form, as a theorem about four correlators represented on a single probability space by bounded measurable functions. We then show that the correlators produced by the intrinsic Maxwellian Bell state do not admit such a common representation. The obstruction is structural: the ontic state is a global non-product field configuration, and the four correlations arise from different polarization resolutions of the same tensorial Maxwellian state. A second main result concerns the Born rule. For $L^2$ quantum states in the domain of the Maxwellian correspondence, we prove that the squared Hilbert norm coincides with the electromagnetic energy of the associated field. This leads to an energy interpretation of the Born rule: the Born probability measure is identified with the normalized electromagnetic energy density, and polarization probabilities become energy contributions of the corresponding field components. These results show that photon entanglement, Bell inequality violation, and the Born rule admit a coherent interpretation within Maxwell–Schwartz field theory, where the basic ontological objects are electromagnetic-like fields rather than abstract state vectors.

**Keywords:** Maxwell fields; Bell correlations; CHSH inequality; tempered distributions; polarization states; electromagnetic fields; energy norm; Born rule

**MSC:** 81P15; 81Q65; 35Q61; 46F12

---

## 1. Introduction

Bell's theorem is one of the central results in the foundations of quantum mechanics. It shows that the correlations predicted by quantum theory violate inequalities that hold for a large class of probabilistic models.

The usual interpretation is that no local realistic description of quantum phenomena is possible. However, Bell's theorem is not a statement about physics in general, but about a specific mathematical structure. In particular, it applies to models in which all correlations are defined on a single probability space and are generated by bounded measurable functions.

The aim of this work is to analyze Bell correlations within a different framework, based on Maxwell's electromagnetic theory formulated in the space of tempered distributions.

In this setting, the fundamental objects are electromagnetic fields

$$F = E + icB$$

belonging to the space

$$W = \mathcal{S}'(M_4, \mathbb{C}^3).$$

Instead of representing polarization by an external doubling of the field space, we introduce an intrinsic two-dimensional subspace

$$\mathcal{P} \subset W$$

generated by linearly polarized Maxwellian waves. This subspace plays the role of the polarization space and is naturally isomorphic to $\mathbb{C}^2$.

Within this intrinsic construction, the polarization states of two photons are represented in the tensor product space

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

and the Bell state appears as a non-factorizable Maxwellian field configuration.

A central point of this work is that the Hilbert structure of the polarization space is not imposed externally, but emerges from the physical structure of the electromagnetic field. In particular, the square norm of a quantum state coincides with the electromagnetic energy of the associated field. More precisely, for $L^2$ states one has

$$||\psi||^2 = ||F(\psi)||^2,$$

where the right-hand side represents the total electromagnetic energy divided by $\varepsilon_0$.

This identification allows one to interpret the Born probability measure as a normalized electromagnetic energy density.

In order to analyze Bell correlations, we use a purely measure-theoretic formulation of the CHSH inequality. In this formulation, the theorem is a statement about the existence of a single probability space supporting four correlation functions defined by bounded measurable functions.

We show that the Maxwellian model reproduces the usual quantum correlations, but does not admit such a probabilistic representation. The reason is structural: the ontic states of the theory are global electromagnetic field configurations, which are not decomposable into independent local variables.

From this perspective, the violation of Bell inequalities does not require superluminal influences. Rather, it reflects the nonseparable structure of electromagnetic field states.

The paper is organized as follows.

In Section 2 we present the CHSH inequality in a purely measure-theoretic form. In Section 3 we recall the quantum polarization model. In Section 4 we construct the intrinsic Maxwellian polarization sector. In Section 5 we represent Bell states in the Maxwellian framework. In Section 6 we establish the energy–norm correspondence. In Section 7 we analyze the failure of the CHSH representation. Finally, Section 8 contains a discussion and concluding remarks.

## 2. Historical and Conceptual Background

The problem of understanding quantum correlations has a long and well-known history. Its modern formulation begins with the work of Einstein, Podolsky, and Rosen, who questioned whether the quantum-mechanical description of physical reality could be considered complete. Their argument, originally formulated for position and momentum, made explicit the tension between the completeness

 doi:10.20944/preprints202603.1612.v1



of the wave function and the possibility of a local realist description of distant systems. This question later became central to the study of entanglement and nonclassical correlations. [1]

A decisive step was taken by Bell, who transformed the original philosophical difficulty into a precise mathematical problem. Bell showed that a broad class of probabilistic models, based on a single hidden-variable space and local response functions, must satisfy specific inequalities. The later Clauser–Horne–Shimony–Holt formulation made this structure directly testable in the laboratory and established the now standard CHSH inequality. From that point onward, the discussion of quantum nonclassicality became inseparable from the problem of representing several correlators on a single probability space. [2,3]

The experimental situation quickly confirmed the theoretical predictions of quantum mechanics. Early tests by Freedman and Clauser, and later the optical experiments of Aspect, Grangier, and Roger, showed that polarization correlations of photon pairs violate Bell-type inequalities in the way predicted by the quantum formalism. These experiments made polarization-entangled electromagnetic systems the privileged setting in which Bell's theorem is both mathematically transparent and physically concrete. [4–6]

From the mathematical point of view, the standard quantum description of the Bell experiment is formulated in Hilbert spaces, using polarization vectors in $\mathbb{C}^2$ and tensor products of such spaces for bipartite systems. At the same time, the experimental object under study is not abstract: it is electromagnetic radiation. This suggests that the polarization formalism of quantum mechanics should admit a direct realization inside a suitable field-theoretic space. The present work develops exactly such a realization, by constructing polarization sectors directly inside a Maxwellian field space and representing Bell states as tensorial electromagnetic field configurations.

A second historical strand enters through the theory of generalized functions. The theory of tempered distributions, introduced by Laurent Schwartz in the late 1940s and early 1950s, provides the natural setting for wave propagation, singular sources, and localized detection events. In modern mathematical physics it is standard to regard distributional methods as indispensable whenever one wants to pass from smooth wave models to singular or concentrated physical phenomena. This makes the distributional setting particularly appropriate for a Maxwellian formulation of quantum states and measurement processes. [7–9]

The present paper lies at the intersection of these two lines of development. On one side stands the Bell–CHSH problem, understood in its precise measure-theoretic form; on the other stands the electromagnetic and distributional structure of Maxwellian fields. Our aim is to show that the polarization Bell experiment can be realized intrinsically inside the Maxwellian field space $W = \mathcal{S}'(M_4, \mathbb{C}^3)$, with polarization represented by distinguished two-dimensional subspaces and Bell states represented by non-factorizable tensorial field configurations. In this way, the usual quantum correlations are recovered while the underlying ontological objects are no longer abstract vectors but electromagnetic fields.

## 3. Theoretical Background I: CHSH Inequality in Measure-Theoretic Form

In this section we present the CHSH inequality in a purely measure-theoretic form. The aim is to isolate the analytic structure of the theorem, without introducing any physical interpretation.

### 3.1. *The Probabilistic Framework*

Let $(\Lambda, \mathcal{F}, \mu)$ be a probability space, that is, $\mu$ is a positive measure on $(\Lambda, \mathcal{F})$ such that $\mu(\Lambda) = 1$.
Let $\mathcal{A} = \{a, a'\}$ and $\mathcal{B} = \{b, b'\}$.
We consider two maps

$$A : \mathcal{A} \times \Lambda \to \mathbb{R}, \qquad B : \mathcal{B} \times \Lambda \to \mathbb{R},$$

such that, for every $x \in \mathcal{A}$, the section $A(x, \cdot) : \Lambda \to \mathbb{R}$ is $\mathcal{F}$-measurable, and for every $y \in \mathcal{B}$, the section $B(y, \cdot) : \Lambda \to \mathbb{R}$ is $\mathcal{F}$-measurable.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



We assume the uniform bounds

$$|A(x,\cdot)| \le 1, \qquad |B(y,\cdot)| \le 1,$$

pointwise on $\Lambda$, for all $x \in \mathcal{A}$ and $y \in \mathcal{B}$.

### 3.2. Correlation Functionals

For every $(x,y) \in \mathcal{A} \times \mathcal{B}$ we define

$$E(x,y) := \int_\Lambda A(x,\cdot)\, B(y,\cdot)\, (\mu).$$

These four numbers,

$$E(a,b), \quad E(a,b'), \quad E(a',b), \quad E(a',b'),$$

are the correlation functionals associated with the pair $(A,B)$.

### 3.3. Statement of the Theorem

**Theorem 1** (CHSH inequality, measure-theoretic form)**.** *Let $(\Lambda, \mathcal{F}, \mu)$ be a probability space. Let $\mathcal{A} = \{a,a'\}$ and $\mathcal{B} = \{b,b'\}$.*

*Let*

$$A : \mathcal{A} \times \Lambda \to \mathbb{R}, \qquad B : \mathcal{B} \times \Lambda \to \mathbb{R},$$

*be measurable in the sense described above, and assume*

$$|A(x,\cdot)| \le 1, \qquad |B(y,\cdot)| \le 1.$$

*Define*

$$E(x,y) := \int_\Lambda A(x,\cdot)\, B(y,\cdot)\, (\mu).$$

*Then*

$$\big|E(a,b) + E(a,b') + E(a',b) - E(a',b')\big| \le 2.$$

### 3.4. Proof

**Proof.** Define the measurable function $C : \Lambda \to \mathbb{R}$ by

$$C := A(a,\cdot)B(b,\cdot) + A(a,\cdot)B(b',\cdot) + A(a',\cdot)B(b,\cdot) - A(a',\cdot)B(b',\cdot).$$

Grouping terms, we obtain

$$C = A(a,\cdot)\big(B(b,\cdot) + B(b',\cdot)\big) + A(a',\cdot)\big(B(b,\cdot) - B(b',\cdot)\big).$$

Using $|A| \le 1$ and the triangle inequality, we get

$$|C| \le |B(b,\cdot) + B(b',\cdot)| + |B(b,\cdot) - B(b',\cdot)|.$$

We now use the elementary fact that for all real numbers $u, v$ with $|u| \le 1$ and $|v| \le 1$,

$$|u+v| + |u-v| \le 2.$$

Applying this pointwise with $u = B(b,\cdot)$ and $v = B(b',\cdot)$, we obtain

$$|C| \le 2 \quad \text{on } \Lambda.$$

Integrating, we get

$$\left| \int_{\Lambda} C\,(\mu) \right| \leq \int_{\Lambda} |C|\,(\mu) \leq \int_{\Lambda} 2\,(\mu) = 2.$$

By linearity of the integral,

$$\int_{\Lambda} C\,(\mu) = E(a,b) + E(a,b') + E(a',b) - E(a',b').$$

This proves the inequality.

□

### 3.5. Deterministic Case

If

$$A(x,\cdot),\ B(y,\cdot) \in \{-1,+1\}$$

for all $x, y$, then the estimate is pointwise sharp, and $|C| = 2$ on $\Lambda$.

### 3.6. Interpretation

The theorem is purely measure-theoretic. It states that if four correlators are represented on a single probability space by bounded measurable functions, then the CHSH bound holds.

### 3.7. Quantum Case

In quantum mechanics, the correlators are of the form

$$E(x,y) = \langle \psi, \hat{A}_x \otimes \hat{B}_y\, \psi \rangle,$$

with non-commuting operators in general. In this case, the four correlators do not arise from a single classical probability space of the above type, and the CHSH bound does not apply.

## 4. Theoretical Background II: Quantum Model of the Polarization Bell Experiment

In this section we recall the standard quantum-mechanical description of the polarization Bell experiment. The purpose is to fix the mathematical structure of the four correlators that will later be compared with the measure-theoretic framework of Section 2.

### 4.1. Single-Photon Polarization Space

Consider a photon propagating in a fixed spatial direction. Its polarization degrees of freedom are described by the complex vector space

$$H = \mathbb{C}^2.$$

We fix the orthonormal basis

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

representing horizontal and vertical polarization.

These vectors satisfy

$$\langle H|H\rangle = 1, \qquad \langle V|V\rangle = 1, \qquad \langle H|V\rangle = 0.$$

For every angle $\theta \in \mathbb{R}$, we define the linear polarization state

$$|\theta\rangle = \cos\theta\,|H\rangle + \sin\theta\,|V\rangle,$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



and the orthogonal state

$$|\theta^{\perp}\rangle = -\sin\theta\,|H\rangle + \cos\theta\,|V\rangle.$$

*4.2. Two-Photon System*

For a pair of photons, the state space is the tensor product

$$H_A \otimes H_B \simeq \mathbb{C}^2 \otimes \mathbb{C}^2.$$

We use the standard notation

$$|HH\rangle, \quad |HV\rangle, \quad |VH\rangle, \quad |VV\rangle$$

for the canonical basis.

The Bell state considered in polarization experiments is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\big(|HV\rangle - |VH\rangle\big).$$

This state is normalized and not factorizable.

*4.3. Measurement Operators*

Let $a$ and $b$ be the angles of the polarizers used by the two observers.

The measurement at angle $a$ is described by the orthogonal projections

$$P_a = |\theta = a\rangle\langle\theta = a|, \qquad P_a^{\perp} = |\theta^{\perp} = a\rangle\langle\theta^{\perp} = a|.$$

Similarly for $b$:

$$P_b = |\theta = b\rangle\langle\theta = b|, \qquad P_b^{\perp} = |\theta^{\perp} = b\rangle\langle\theta^{\perp} = b|.$$

Each measurement has two possible outcomes, which we encode as

$$+1 \quad \text{for } P, \qquad -1 \quad \text{for } P^{\perp}.$$

*4.4. Correlation Function*

The correlation function is defined by

$$E(a,b) := \langle\Psi, (\hat{A}_a \otimes \hat{B}_b)\Psi\rangle,$$

where the observables are given by

$$\hat{A}_a = P_a - P_a^{\perp}, \qquad \hat{B}_b = P_b - P_b^{\perp}.$$

A direct computation gives

$$E(a,b) = -\cos\big(2(a-b)\big).$$

*4.5. Violation of the CHSH Inequality*

Consider the standard choice of angles

$$a = 0, \qquad a' = \frac{\pi}{4}, \qquad b = \frac{\pi}{8}, \qquad b' = -\frac{\pi}{8}.$$

Then one obtains

$$|E(a,b) + E(a,b') + E(a',b) - E(a',b')| = 2\sqrt{2}.$$

This exceeds the bound established in Section 2.

 doi:10.20944/preprints202603.1612.v1



### 4.6. Conceptual Remark

The four quantum correlators

$$E(a,b), \quad E(a,b'), \quad E(a',b), \quad E(a',b')$$

are defined through non-commuting operators.

For this reason, they do not arise from a single probability space supporting four bounded measurable functions as in Section 2.

This is the precise analytic reason why the CHSH inequality does not apply to the quantum model.

## 5. Results I: Intrinsic Maxwellian Polarization Sectors

In this section we construct the intrinsic Maxwellian realization of photon polarization. The basic idea is that the two-dimensional polarization space is not introduced by an external Cartesian doubling of the Maxwellian field space, but appears as a distinguished two-dimensional complex subspace of the Maxwellian space itself.

### 5.1. The Ambient Maxwellian Field Space

Let $M_4$ denote Minkowski spacetime, with coordinates $x = (ct, \vec{x})$, where $\vec{x} = (x_1, x_2, x_3)$.

We consider the complex Maxwellian field space

$$W = \mathcal{S}'(M_4, \mathbb{C}^3).$$

An element $F \in W$ is interpreted as a complex electromagnetic field of the form

$$F = E + i\,cB,$$

where $E$ is the electric field and $B$ is the magnetic field.

In vacuum, the complex Maxwell equations take the form

$$i\,\partial_t F = c\,\nabla \times F,$$

together with the transversality condition

$$\nabla \cdot F = 0.$$

The purpose of this section is to exhibit, inside the space $W$, distinguished two-dimensional complex subspaces carrying the usual polarization structure of quantum mechanics.

### 5.2. Alice'S Polarization Basis

We first construct the polarization sector associated with Alice.

Let $(e_1, e_2, e_3)$ denote the canonical basis of $\mathbb{R}^3$. We assume that Alice's wave propagates in the positive $e_3$ direction. Accordingly, we fix

$$\vec{k}_A = k_3 e_3, \qquad k_3 > 0,$$

and define

$$\omega = c\,k_3.$$

We then introduce the phase function

$$\theta_A = k_3 x_3 - \omega t.$$

 doi:10.20944/preprints202603.1612.v1



Since $x_3 = z$, this can also be written as

$$\theta_A = k_3 z - \omega t.$$

Now define the complex transverse vector

$$f_A = e_1 + ie_2.$$

We introduce the two fields

$$H_A = \cos(\theta_A)\,(e_1 + ie_2),$$

and

$$V_A = -\sin(\theta_A)\,(e_2 - ie_1).$$

Since

$$e_2 - ie_1 = -i(e_1 + ie_2),$$

the second field can be rewritten more transparently as

$$V_A = i\sin(\theta_A)\,(e_1 + ie_2) = i\sin(\theta_A)\,f_A.$$

Thus Alice's basis waves may be written in the compact form

$$H_A = \cos(\theta_A)\,f_A, \qquad V_A = i\sin(\theta_A)\,f_A.$$

Their sum is

$$H_A + V_A = \big(\cos(\theta_A) + i\sin(\theta_A)\big) f_A = e^{i\theta_A} f_A.$$

Hence, if we define

$$w_A = e^{i\theta_A}(e_1 + ie_2),$$

we obtain

$$w_A = H_A + V_A.$$

This shows that the usual circularly polarized Maxwellian mode splits naturally into two linearly polarized Maxwellian fields.

### *5.3. Verification of Maxwell's Equations for Alice's Basis Waves*

We now verify directly that $H_A$ and $V_A$ both satisfy the complex Maxwell equations.

Time derivative of $H_A$

Since

$$H_A = \cos(\theta_A)\,f_A, \qquad \theta_A = k_3 z - \omega t,$$

we have

$$\partial_t \theta_A = -\omega.$$

Therefore

$$\partial_t H_A = \partial_t(\cos\theta_A)\,f_A = (-\sin\theta_A)(-\omega)\,f_A = \omega\sin\theta_A\,f_A.$$

Multiplying by $i$, we obtain

$$i\,\partial_t H_A = i\omega\sin\theta_A\,f_A.$$

Since

$$V_A = i\sin\theta_A\,f_A,$$

it follows that

$$i\,\partial_t H_A = \omega V_A.$$

 doi:10.20944/preprints202603.1612.v1



Curl of $H_A$

Since $f_A$ is constant in space,

$$\nabla \times H_A = \nabla(\cos\theta_A) \times f_A.$$

Now

$$\nabla\theta_A = k_3 e_3,$$

hence

$$\nabla(\cos\theta_A) = -\sin\theta_A\, \nabla\theta_A = -k_3 \sin\theta_A\, e_3.$$

Thus

$$\nabla \times H_A = -k_3 \sin\theta_A\, (e_3 \times f_A).$$

Using

$$e_3 \times e_1 = e_2, \qquad e_3 \times e_2 = -e_1,$$

we get

$$e_3 \times f_A = e_3 \times (e_1 + ie_2) = e_2 - ie_1.$$

Therefore

$$\nabla \times H_A = -k_3 \sin\theta_A\, (e_2 - ie_1) = k_3 V_A.$$

Multiplying by $c$ and using $\omega = c\, k_3$, we obtain

$$c\, \nabla \times H_A = \omega V_A.$$

Hence

$$i\, \partial_t H_A = \omega V_A = c\, \nabla \times H_A.$$

Time derivative of $V_A$

Since

$$V_A = i \sin\theta_A\, f_A,$$

we have

$$\partial_t V_A = i\, \partial_t(\sin\theta_A)\, f_A = i(\cos\theta_A)(-\omega)\, f_A = -\, i\omega \cos\theta_A\, f_A.$$

Therefore

$$i\, \partial_t V_A = \omega \cos\theta_A\, f_A = \omega H_A.$$

Curl of $V_A$

Again, since $f_A$ is constant,

$$\nabla \times V_A = \nabla(i \sin\theta_A) \times f_A.$$

Now

$$\nabla(i \sin\theta_A) = i \cos\theta_A\, \nabla\theta_A = ik_3 \cos\theta_A\, e_3.$$

Thus

$$\nabla \times V_A = ik_3 \cos\theta_A\, (e_3 \times f_A).$$

Since

$$e_3 \times f_A = e_2 - ie_1,$$

and

$$i(e_2 - ie_1) = e_1 + ie_2 = f_A,$$

 doi:10.20944/preprints202603.1612.v1



we obtain
$$\nabla \times V_A = k_3 \cos\theta_A\, f_A = k_3 H_A.$$

Hence
$$c\,\nabla \times V_A = \omega H_A.$$

Therefore
$$i\,\partial_t V_A = \omega H_A = c\,\nabla \times V_A.$$

Divergence condition

Both $H_A$ and $V_A$ are proportional to $f_A = e_1 + ie_2$, which has no $e_3$ component, while the phase depends only on $z$ and $t$. Therefore

$$\nabla \cdot H_A = 0, \qquad \nabla \cdot V_A = 0.$$

Thus both fields satisfy the full complex Maxwell system.

*5.4. Bob'S Polarization Basis*

We now construct the polarization basis associated with Bob.

The propagation direction is the negative $e_3$ direction. We fix

$$\vec{k}_B = -k_3 e_3, \qquad k_3 > 0, \qquad \omega = c\,k_3.$$

We define the phase
$$\theta_B = -k_3 z - \omega t.$$

We choose the orthonormal triple
$$(e_2, e_1, -e_3),$$
which is right-handed, since
$$e_2 \times e_1 = -e_3.$$

We define the complex transverse vector

$$f_B = e_2 + ie_1.$$

We now introduce the two fields

$$H_B = i\sin(\theta_B)\, f_B, \qquad V_B = \cos(\theta_B)\, f_B.$$

*Physical interpretation*

We first verify that the naming of the two fields agrees with the standard physical convention, according to which polarization is determined by the direction of the electric field, that is, by the real part of the complex Maxwellian field.

Expanding $H_B$, we obtain

$$H_B = i\sin(\theta_B)(e_2 + ie_1) = i\sin(\theta_B)\, e_2 - \sin(\theta_B)\, e_1.$$

Therefore
$$\mathrm{Re}(H_B) = -\sin(\theta_B)\, e_1,$$
so the electric field of $H_B$ is directed along the horizontal axis $e_1$.

Similarly,
$$V_B = \cos(\theta_B)(e_2 + ie_1) = \cos(\theta_B)\, e_2 + i\cos(\theta_B)\, e_1,$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



hence

$$\mathrm{Re}(V_B) = \cos(\theta_B)\, e_2,$$

so the electric field of $V_B$ is directed along the vertical axis $e_2$.

Thus:

- $H_B$ is horizontally polarized;
- $V_B$ is vertically polarized.

*Elementary geometric identity*

Before computing the derivatives, we record the key identity involving the vector $f_B$. Using

$$e_3 \times e_2 = -e_1, \qquad e_3 \times e_1 = e_2,$$

we obtain

$$e_3 \times f_B = e_3 \times (e_2 + ie_1) = -e_1 + ie_2.$$

On the other hand,

$$if_B = i(e_2 + ie_1) = ie_2 - e_1 = -e_1 + ie_2.$$

Hence

$$e_3 \times f_B = if_B.$$

This is the identity that will enter the curl computations.

*Time and space derivatives of the phase*

Since

$$\theta_B = -k_3 z - \omega t,$$

we have

$$\partial_t \theta_B = -\omega, \qquad \nabla\theta_B = -k_3 e_3.$$

*Time derivative of $H_B$*

We compute

$$\partial_t H_B = i\,\partial_t(\sin\theta_B)\, f_B = i\cos(\theta_B)\,\partial_t\theta_B\, f_B.$$

Since $\partial_t\theta_B = -\omega$, this gives

$$\partial_t H_B = -i\omega\cos(\theta_B)\, f_B.$$

Multiplying by $i$, we obtain

$$i\,\partial_t H_B = \omega\cos(\theta_B)\, f_B.$$

But

$$V_B = \cos(\theta_B)\, f_B,$$

therefore

$$i\,\partial_t H_B = \omega V_B.$$

*Curl of $H_B$*

Since $f_B$ is constant, we have

$$\nabla \times H_B = \nabla\big(i\sin(\theta_B)\big) \times f_B.$$

Now

$$\nabla\big(i\sin(\theta_B)\big) = i\cos(\theta_B)\,\nabla\theta_B = -ik_3\cos(\theta_B)\, e_3.$$

doi:10.20944/preprints202603.1612.v1



Therefore
$$\nabla \times H_B = -ik_3 \cos(\theta_B)\,(e_3 \times f_B).$$

Using the identity
$$e_3 \times f_B = i f_B,$$

we get
$$\nabla \times H_B = -ik_3 \cos(\theta_B)\,(i f_B) = k_3 \cos(\theta_B)\, f_B.$$

Hence
$$\nabla \times H_B = k_3 V_B.$$

Multiplying by $c$ and using $\omega = c\,k_3$, we conclude that
$$c\,\nabla \times H_B = \omega V_B.$$

Combining this with the previous computation gives
$$i\,\partial_t H_B = c\,\nabla \times H_B.$$

*Time derivative of $V_B$*

We now compute
$$\partial_t V_B = \partial_t(\cos\theta_B)\, f_B = -\sin(\theta_B)\,\partial_t\theta_B\, f_B.$$

Since $\partial_t\theta_B = -\omega$, we obtain
$$\partial_t V_B = \omega \sin(\theta_B)\, f_B.$$

Multiplying by $i$, we get
$$i\,\partial_t V_B = i\omega \sin(\theta_B)\, f_B.$$

But
$$H_B = i \sin(\theta_B)\, f_B,$$

therefore
$$i\,\partial_t V_B = \omega H_B.$$

*Curl of $V_B$*

Again, since $f_B$ is constant,
$$\nabla \times V_B = \nabla(\cos\theta_B) \times f_B.$$

Now
$$\nabla(\cos\theta_B) = -\sin(\theta_B)\,\nabla\theta_B = k_3 \sin(\theta_B)\, e_3.$$

Thus
$$\nabla \times V_B = k_3 \sin(\theta_B)\,(e_3 \times f_B).$$

Using
$$e_3 \times f_B = i f_B,$$

we obtain
$$\nabla \times V_B = ik_3 \sin(\theta_B)\, f_B.$$

Since
$$H_B = i \sin(\theta_B)\, f_B,$$

it follows that
$$\nabla \times V_B = k_3 H_B.$$

 doi:10.20944/preprints202603.1612.v1



Therefore

$$c\,\nabla \times V_B = \omega H_B.$$

Combining this with the computation of the time derivative, we obtain

$$i\,\partial_t V_B = c\,\nabla \times V_B.$$

*Divergence condition*

Finally, we verify the transversality condition.

Since $f_B = e_2 + ie_1$ has no $e_3$ component, and since the phase depends only on $z$ and $t$, one has

$$\nabla \cdot H_B = 0, \qquad \nabla \cdot V_B = 0.$$

Thus both fields satisfy the divergence-free condition.

*Conclusion*

We have constructed two exact solutions of the complex Maxwell equations,

$$H_B = i\sin(\theta_B)\,f_B, \qquad V_B = \cos(\theta_B)\,f_B, \qquad f_B = e_2 + ie_1,$$

adapted to propagation in the negative $e_3$ direction.

These fields form Bob's polarization basis and satisfy the following properties:

- they are compatible with the right-handed frame $(e_2, e_1, -e_3)$;
- their electric fields have the correct horizontal and vertical directions;
- they satisfy the complex Maxwell equations

$$i\,\partial_t H_B = c\,\nabla \times H_B, \qquad i\,\partial_t V_B = c\,\nabla \times V_B;$$

- they satisfy the transversality conditions

$$\nabla \cdot H_B = 0, \qquad \nabla \cdot V_B = 0.$$

### *5.5. Intrinsic Polarization Subspaces*

We now define the polarization sectors associated with Alice and Bob as intrinsic subspaces of the Maxwellian field space

$$W = \mathcal{S}'(M_4, \mathbb{C}^3).$$

For Alice, we set

$$\mathcal{P}_A = \mathrm{Span}_{\mathbb{C}}\{H_A, V_A\} \subset W.$$

For Bob, we set

$$\mathcal{P}_B = \mathrm{Span}_{\mathbb{C}}\{H_B, V_B\} \subset W.$$

Since the pairs $\{H_A, V_A\}$ and $\{H_B, V_B\}$ consist of linearly independent Maxwellian fields, each of the spaces $\mathcal{P}_A$ and $\mathcal{P}_B$ is two-dimensional over $\mathbb{C}$.

These spaces represent the intrinsic polarization sectors associated with Alice and Bob, respectively.

In particular:

- $\mathcal{P}_A$ contains all complex linear combinations of the fields $H_A$ and $V_A$, corresponding to all possible polarization states propagating in the $+e_3$ direction;
- $\mathcal{P}_B$ contains all complex linear combinations of the fields $H_B$ and $V_B$, corresponding to all possible polarization states propagating in the $-e_3$ direction.

 doi:10.20944/preprints202603.1612.v1



Thus polarization is realized as an internal degree of freedom of the Maxwellian field space, rather than as an external tensor factor.

*5.6. Canonical Identification with $\mathbb{C}^2$*

We now establish a canonical identification between the intrinsic polarization subspaces and the standard quantum polarization space $\mathbb{C}^2$.

For Alice, we define the linear map

$$\Phi_A : \mathcal{P}_A \to \mathbb{C}^2,$$

by setting

$$\Phi_A(H_A) = (1,0), \qquad \Phi_A(V_A) = (0,1),$$

and extending $\mathbb{C}$-linearly to all of $\mathcal{P}_A$.

Similarly, for Bob, we define the linear map

$$\Phi_B : \mathcal{P}_B \to \mathbb{C}^2,$$

by setting

$$\Phi_B(H_B) = (1,0), \qquad \Phi_B(V_B) = (0,1),$$

and extending $\mathbb{C}$-linearly.

Since the families $\{H_A, V_A\}$ and $\{H_B, V_B\}$ are linearly independent, the maps $\Phi_A$ and $\Phi_B$ are linear isomorphisms.

Therefore, we obtain canonical identifications

$$\mathcal{P}_A \simeq \mathbb{C}^2, \qquad \mathcal{P}_B \simeq \mathbb{C}^2.$$

In this way, the usual quantum polarization space $\mathbb{C}^2$ is realized intrinsically as a subspace of the Maxwellian field space.

Under these identifications:

- quantum polarization states correspond to Maxwellian fields;
- linear combinations of quantum states correspond to superpositions of electromagnetic fields;
- the tensor product $\mathbb{C}^2 \otimes \mathbb{C}^2$ corresponds to the tensorial Maxwellian space $\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W$.

Thus the standard quantum polarization formalism is embedded directly into the Maxwellian framework. We shall use these identifications in the next subsection to construct the intrinsic Maxwellian Bell state.

*Maxwell equations*

The verification that $H_B$ and $V_B$ satisfy the complex Maxwell equations is completely analogous to the case of Alice.

First, we have

$$\partial_t \theta_B = -\omega, \qquad \nabla \theta_B = -k_3 e_3.$$

From this one obtains

$$i\,\partial_t H_B = \omega V_B, \qquad i\,\partial_t V_B = \omega H_B.$$

Moreover, using

$$e_3 \times (e_1 - ie_2) = e_2 + ie_1 = i f_B,$$

one finds

$$\nabla \times H_B = k_3 V_B, \qquad \nabla \times V_B = k_3 H_B.$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



Therefore

$$i\,\partial_t H_B = c\,\nabla \times H_B, \qquad i\,\partial_t V_B = c\,\nabla \times V_B.$$

Finally, since the fields are transverse,

$$\nabla \cdot H_B = 0, \qquad \nabla \cdot V_B = 0.$$

Thus both $H_B$ and $V_B$ are exact solutions of Maxwell's equations.

**Theorem 2** (Intrinsic Maxwellian realization of polarization)**.** *Let*

$$W = \mathcal{S}'(M_4, \mathbb{C}^3)$$

*be the complex Maxwellian field space.*

*Let $H_A, V_A \in W$ and $H_B, V_B \in W$ be the fields constructed in Sections 5.2 and 5.4.*

*Define*

$$P_A = \mathrm{Span}_{\mathbb{C}}\{H_A, V_A\}, \quad P_B = \mathrm{Span}_{\mathbb{C}}\{H_B, V_B\}.$$

*Then:*

1. *The fields $H_A, V_A, H_B, V_B$ are solutions of the complex Maxwell system*

$$i\partial_t F = c\nabla \times F, \quad \nabla \cdot F = 0.$$

2. *The spaces $P_A$ and $P_B$ are two-dimensional complex subspaces of $W$.*
3. *The maps*

$$\Phi_A : P_A \to \mathbb{C}^2, \quad \Phi_B : P_B \to \mathbb{C}^2,$$

*defined by*

$$\Phi_A(H_A) = (1,0),\ \Phi_A(V_A) = (0,1), \quad \Phi_B(H_B) = (1,0),\ \Phi_B(V_B) = (0,1),$$

*are linear isomorphisms.*

*Therefore, the quantum polarization space $\mathbb{C}^2$ is realized intrinsically as a subspace of the Maxwellian field space $W$.*

**Theorem 3** (Decomposition of circular Maxwellian modes)**.** *Let*

$$w_A = e^{i\theta_A}(e_1 + ie_2)$$

*be the circularly polarized Maxwellian plane wave.*

*Then there exist real-valued functions $\alpha, \beta$ such that*

$$w_A = \alpha H_A + \beta V_A,$$

*and explicitly*

$$w_A = H_A + V_A.$$

*In particular, circular polarization decomposes into two linearly polarized Maxwellian fields belonging to $P_A$.*

*5.7. Meaning of the Construction*

The importance of this construction is that photon polarization is now represented directly inside the Maxwellian ontology.

No external Cartesian doubling is needed. Horizontal and vertical polarization are realized as two linearly independent Maxwellian waves inside the same field space.

This makes the present formulation more intrinsic, more geometric, and closer to the usual physical interpretation of polarization in electromagnetism.

In the next section we shall use the tensor product $\mathcal{P}_A \otimes \mathcal{P}_B$ to construct Maxwellian Bell states and recover the usual Bell correlations.

## 6. Results II: Bell States in the Intrinsic Maxwellian Framework

In this section we construct the Bell state associated with the intrinsic Maxwellian polarization sectors introduced in the previous section.

The key point is that the standard polarization Bell experiment can be realized directly inside the tensor product

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

where $\mathcal{P}_A$ and $\mathcal{P}_B$ are the two-dimensional Maxwellian polarization spaces associated with Alice and Bob.

### 6.1. The Tensor Product Polarization Space

Recall that

$$\mathcal{P}_A = \mathrm{Span}_{\mathbb{C}}\{H_A, V_A\} \subset W,$$

and

$$\mathcal{P}_B = \mathrm{Span}_{\mathbb{C}}\{H_B, V_B\} \subset W.$$

Since both spaces are two-dimensional over $\mathbb{C}$, their tensor product

$$\mathcal{P}_A \otimes \mathcal{P}_B$$

is four-dimensional over $\mathbb{C}$.

A natural basis is given by

$$H_A \otimes H_B, \qquad H_A \otimes V_B, \qquad V_A \otimes H_B, \qquad V_A \otimes V_B.$$

Under the canonical identifications

$$\Phi_A : \mathcal{P}_A \to \mathbb{C}^2, \qquad \Phi_A(H_A) = (1,0), \qquad \Phi_A(V_A) = (0,1),$$

and

$$\Phi_B : \mathcal{P}_B \to \mathbb{C}^2, \qquad \Phi_B(H_B) = (1,0), \qquad \Phi_B(V_B) = (0,1),$$

the tensor product space $\mathcal{P}_A \otimes \mathcal{P}_B$ is canonically isomorphic to

$$\mathbb{C}^2 \otimes \mathbb{C}^2.$$

Thus the usual two-photon polarization space is realized as an intrinsic tensorial sector of the Maxwellian field space.

### 6.2. The Intrinsic Maxwellian Bell State

We now define the intrinsic Maxwellian Bell state by

$$\Psi_{\mathrm{Bell}} = \frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B).$$

This element belongs to

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W.$$

 doi:10.20944/preprints202603.1612.v1



Under the isomorphism

$$\Phi_A \otimes \Phi_B : \mathcal{P}_A \otimes \mathcal{P}_B \to \mathbb{C}^2 \otimes \mathbb{C}^2,$$

the state $\Psi_{\text{Bell}}$ is sent to

$$\frac{1}{\sqrt{2}}(|H\rangle_A \otimes |V\rangle_B - |V\rangle_A \otimes |H\rangle_B),$$

which is exactly the standard Bell singlet state of the polarization experiment.

Therefore the intrinsic Maxwellian Bell state is not an analogue of the usual quantum Bell state, but its direct realization inside the Maxwellian framework.

We can summarize some results.

**Theorem 4** (Intrinsic Maxwellian Bell state)**.** *Let $P_A$ and $P_B$ be the intrinsic polarization subspaces defined in Section 5.*

*Define*

$$\Psi_{\text{Bell}} = \frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B) \in P_A \otimes P_B.$$

*Then:*

1. *$\Psi_{\text{Bell}}$ is not factorizable, i.e., there do not exist*

$$F_A \in P_A, \quad F_B \in P_B$$

*such that*

$$\Psi_{\text{Bell}} = F_A \otimes F_B.$$

2. *Under the canonical isomorphism*

$$\Phi_A \otimes \Phi_B : P_A \otimes P_B \to \mathbb{C}^2 \otimes \mathbb{C}^2,$$

*one has*

$$(\Phi_A \otimes \Phi_B)(\Psi_{\text{Bell}}) = \frac{1}{\sqrt{2}}(|H\rangle \otimes |V\rangle - |V\rangle \otimes |H\rangle).$$

*Therefore, the standard Bell singlet state is realized intrinsically as a Maxwellian field configuration.*

**Theorem 5** (Maxwellian Bell correlation law)**.** *Let $\Psi_{\text{Bell}} \in P_A \otimes P_B$ be the intrinsic Maxwellian Bell state.*

*For angles $a, b \in \mathbb{R}$, let the rotated polarization bases be defined as in Section 6.4.*

*Then the associated correlation function is*

$$E(a, b) = -\cos(2(a - b)).$$

*In particular, for the standard CHSH angles*

$$a = 0, \quad a' = \frac{\pi}{4}, \quad b = \frac{\pi}{8}, \quad b' = -\frac{\pi}{8},$$

*one has*

$$|E(a, b) + E(a, b') + E(a', b) - E(a', b')| = 2\sqrt{2}.$$

*6.3. Non-Factorizability of the Bell State*

The state $\Psi_{\text{Bell}}$ is not factorizable. That is, there do not exist

$$F_A \in \mathcal{P}_A, \qquad F_B \in \mathcal{P}_B,$$

such that

$$\Psi_{\text{Bell}} = F_A \otimes F_B.$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



Indeed, if

$$F_A = aH_A + bV_A, \qquad F_B = cH_B + dV_B,$$

then

$$F_A \otimes F_B = ac\, H_A \otimes H_B + ad\, H_A \otimes V_B + bc\, V_A \otimes H_B + bd\, V_A \otimes V_B.$$

In order for this to coincide with

$$\frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B),$$

one would need

$$ac = 0, \qquad bd = 0, \qquad ad = \frac{1}{\sqrt{2}}, \qquad bc = -\frac{1}{\sqrt{2}}.$$

But the last two identities imply that $a, d, b, c$ are all nonzero, which contradicts the first two equalities. Hence such a factorization is impossible.

Thus the Maxwellian Bell state is intrinsically entangled.

### 6.4. Rotated Polarization Bases

In order to describe polarization measurements, we introduce rotated bases in the two polarization sectors.

For Alice, at angle $a$, define

$$H_A^{(a)} = \cos a\, H_A + \sin a\, V_A,$$

and

$$V_A^{(a)} = -\sin a\, H_A + \cos a\, V_A.$$

Similarly, for Bob, at angle $b$, define

$$H_B^{(b)} = \cos b\, H_B + \sin b\, V_B,$$

and

$$V_B^{(b)} = -\sin b\, H_B + \cos b\, V_B.$$

These are exactly the Maxwellian counterparts of the standard rotated linear polarization states.

### 6.5. Joint Probabilities

Because the identifications $\Phi_A$ and $\Phi_B$ are linear isomorphisms preserving the polarization structure, the Born probabilities in the intrinsic Maxwellian model coincide with the usual quantum probabilities.

Thus, for the Bell state $\Psi_{\text{Bell}}$, one has

$$P(+,+ \mid a,b) = \frac{1}{2}\sin^2(a-b),$$

$$P(-,- \mid a,b) = \frac{1}{2}\sin^2(a-b),$$

$$P(+,- \mid a,b) = \frac{1}{2}\cos^2(a-b),$$

and

$$P(-,+ \mid a,b) = \frac{1}{2}\cos^2(a-b).$$

Here the outcome $+1$ corresponds to the projection onto the rotated horizontal direction, while the outcome $-1$ corresponds to the orthogonal rotated vertical direction.

 doi:10.20944/preprints202603.1612.v1



### 6.6. Correlation Function

The correlation function is therefore

$$E(a,b) = P(+,+ \mid a,b) + P(-,- \mid a,b) - P(+,- \mid a,b) - P(-,+ \mid a,b).$$

Substituting the above probabilities, we obtain

$$E(a,b) = \frac{1}{2}\sin^2(a-b) + \frac{1}{2}\sin^2(a-b) - \frac{1}{2}\cos^2(a-b) - \frac{1}{2}\cos^2(a-b).$$

Hence

$$E(a,b) = \sin^2(a-b) - \cos^2(a-b),$$

that is,

$$E(a,b) = -\cos(2(a-b)).$$

This is exactly the standard correlation law of the polarization Bell experiment.

### 6.7. Violation of the CHSH Inequality

Choose the standard four analyzer settings

$$a = 0, \qquad a' = \frac{\pi}{4}, \qquad b = \frac{\pi}{8}, \qquad b' = -\frac{\pi}{8}.$$

Then

$$E(a,b) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},$$

$$E(a,b') = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},$$

$$E(a',b) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},$$

and

$$E(a',b') = -\cos\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}.$$

Therefore

$$E(a,b) + E(a,b') + E(a',b) - E(a',b') = -2\sqrt{2},$$

and so

$$\left|E(a,b) + E(a,b') + E(a',b) - E(a',b')\right| = 2\sqrt{2}.$$

Thus the CHSH bound is violated.

### 6.8. Conclusion of the Construction

The intrinsic Maxwellian model reproduces exactly the standard Bell correlations inside the tensor product of two Maxwellian polarization sectors.

The Bell state is realized as an intrinsically non-factorizable electromagnetic field configuration in

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W.$$

This shows that the Bell experiment admits a fully Maxwellian realization in which the polarization qubit is encoded directly inside the ambient field space.

In the next section we shall show that, although the CHSH violation is reproduced exactly, the corresponding correlators do not admit a single measure-theoretic representation of the type described in Section 2.

 doi:10.20944/preprints202603.1612.v1



## 7. Results III: Failure of a Single Measure-Theoretic CHSH Representation in the Intrinsic Maxwellian Model

In Section 2 we proved the CHSH inequality in a purely measure-theoretic form. The theorem states that if the four correlators

$$E(a,b), \qquad E(a,b'), \qquad E(a',b), \qquad E(a',b')$$

are represented on a single probability space by bounded measurable functions, then the CHSH combination is bounded by 2.

In Section 5 we constructed the intrinsic Maxwellian Bell state

$$\Psi_{\text{Bell}} = \frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B) \in \mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

and we showed that the corresponding correlation function is

$$E(a,b) = -\cos(2(a-b)).$$

Consequently, for suitable analyzer settings, one has

$$\left|E(a,b) + E(a,b') + E(a',b) - E(a',b')\right| = 2\sqrt{2}.$$

The purpose of the present section is to explain, in the precise measure-theoretic language of Section 2, why this does not contradict the CHSH theorem. The reason is that the intrinsic Maxwellian Bell model does not admit a single measure-theoretic representation of the required type.

### 7.1. The Hypothesis of the CHSH Theorem

We first recall the exact hypothesis of the theorem proved in Section 2.

The CHSH bound holds if there exist:

- a probability space $(\Lambda, \mathcal{F}, \mu)$;
- two maps
$$A : \{a, a'\} \times \Lambda \to \mathbb{R}, \qquad B : \{b, b'\} \times \Lambda \to \mathbb{R},$$
such that, for each fixed analyzer angle, the corresponding sections
$$A(a,\cdot), \quad A(a',\cdot), \quad B(b,\cdot), \quad B(b',\cdot)$$
are measurable and bounded by 1 in absolute value;
- and such that the four correlators are simultaneously represented by
$$E(x,y) = \int_\Lambda A(x,\cdot)\, B(y,\cdot)\, (\mu), \qquad x \in \{a,a'\},\ y \in \{b,b'\}.$$

Thus the essential analytic assumption is the existence of a single probability space supporting all four correlators at once.

### 7.2. The Intrinsic Maxwellian Correlators

In the intrinsic Maxwellian model, the Bell state belongs to

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

where

$$\mathcal{P}_A = \operatorname{Span}_{\mathbb{C}}\{H_A, V_A\}, \qquad \mathcal{P}_B = \operatorname{Span}_{\mathbb{C}}\{H_B, V_B\}.$$

 doi:10.20944/preprints202603.1612.v1



For every analyzer angle $a$, Alice's measurement is described by the rotated polarization basis

$$H_A^{(a)} = \cos a \, H_A + \sin a \, V_A, \qquad V_A^{(a)} = -\sin a \, H_A + \cos a \, V_A,$$

and for every analyzer angle $b$, Bob's measurement is described by

$$H_B^{(b)} = \cos b \, H_B + \sin b \, V_B, \qquad V_B^{(b)} = -\sin b \, H_B + \cos b \, V_B.$$

For each pair of angles $(a, b)$, the four outcome probabilities are computed from the Bell state $\Psi_{\text{Bell}}$ by using the corresponding polarization decomposition. This yields the correlation

$$E(a, b) = -\cos(2(a - b)).$$

Therefore the intrinsic Maxwellian model reproduces exactly the same family of correlators as the usual quantum polarization model.

### *7.3. Why a Single Classical Representation Is Not Available*

The crucial point is that, although the four correlators are all well-defined, they are not realized by means of a single classical probability space carrying four bounded measurable functions of the type required in Section 2.

Indeed, the Maxwellian Bell state is a single global tensorial field configuration, but the polarization decompositions associated with the four analyzer settings

$$a, \qquad a', \qquad b, \qquad b'$$

are not simultaneously realized as one fixed family of classical random variables on a common sample space.

For each pair of analyzer settings $(x, y)$, with

$$x \in \{a, a'\}, \qquad y \in \{b, b'\},$$

the correlation $E(x, y)$ is computed by resolving the same Bell field state with respect to the pair of rotated Maxwellian bases determined by $x$ and $y$. Thus each correlator is attached to a different pair of polarization resolutions.

The four correlators are therefore not obtained by evaluating four pre-existing measurable functions on a single underlying field of classical events. They arise from four distinct measurement contexts applied to one and the same global Maxwellian state.

### *7.4. Context Dependence of the Polarization Resolutions*

This point may be formulated more explicitly.

For each analyzer angle $x$ on Alice's side, the pair

$$\{H_A^{(x)}, V_A^{(x)}\}$$

is a basis of the same two-dimensional space $\mathcal{P}_A$. Likewise, for each analyzer angle $y$ on Bob's side, the pair

$$\{H_B^{(y)}, V_B^{(y)}\}$$

is a basis of the same two-dimensional space $\mathcal{P}_B$.

However, the bases corresponding to different angles are not identical. In particular, the basis associated with $a$ is not the same as the basis associated with $a'$, and similarly for $b$ and $b'$.

Therefore the outcome decomposition used to define the correlation $E(a, b)$ is not the same as the outcome decomposition used to define $E(a', b)$, nor the same as that used for $E(a, b')$, nor for $E(a', b')$.

 doi:10.20944/preprints202603.1612.v1



This means that the four correlators are not jointly represented by one fixed family of bounded measurable functions on a common probability space. They are obtained from one global state by means of four distinct measurement resolutions.

**Remark 1** (On the absence of a joint measurable structure)**.** *The obstruction exhibited above may be formulated in a stronger and more precise way.*

*In the measure-theoretic framework of Section 2, the CHSH inequality holds under the assumption that there exists a single probability space $(\Lambda, \mathcal{F}, \mu)$ supporting four bounded measurable functions*

$$A(a,\cdot),\ A(a',\cdot),\ B(b,\cdot),\ B(b',\cdot),$$

*defined simultaneously on $\Lambda$.*

*In the intrinsic Maxwellian model, such a jointly measurable family does not exist. Indeed, the four correlators are not obtained by evaluating pre-existing functions on a common space of events. Rather, each correlator $E(x,y)$ arises from a distinct polarization resolution of a single global field*

$$\Psi_{\mathrm{Bell}} \in P_A \otimes P_B \subset W \otimes W.$$

*Therefore, the quantities $A(a,\cdot)$ and $A(a',\cdot)$ (and similarly on Bob's side) cannot be interpreted as jointly defined random variables on a common probability space.*

*The failure of the CHSH representation is thus not a consequence of nonlocal dynamical effects, but of the non-coexistence of the corresponding measurement contexts as measurable functions on a single sample space. It reflects the fact that polarization observables are realized as context-dependent decompositions of a global Maxwellian field configuration, rather than as evaluations of pre-existing local variables.*

### *7.5. The Structural Source of the Obstruction*

The obstruction is not merely formal. It comes from the structure of the intrinsic Maxwellian Bell state itself.

The state

$$\Psi_{\mathrm{Bell}} = \frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B)$$

is not factorizable. It does not have the form

$$F_A \otimes F_B$$

with

$$F_A \in \mathcal{P}_A, \qquad F_B \in \mathcal{P}_B.$$

Thus the ontic object represented by $\Psi_{\mathrm{Bell}}$ is not a pair of independent local polarization states. It is a single global electromagnetic configuration in a tensor product space.

If one had a single probability space

$$(\Lambda, \mathcal{F}, \mu)$$

supporting four bounded measurable functions

$$A(a,\cdot), \qquad A(a',\cdot), \qquad B(b,\cdot), \qquad B(b',\cdot),$$

then the four correlators would be encoded by one common classical probabilistic structure.

But this would amount to replacing the global tensorial Maxwellian state by a classical joint representation of all four contexts. That is exactly what the intrinsic Bell state does not permit.

### *7.6. Formal Contradiction with the CHSH Hypothesis*

We may now state the conclusion in theorem form.

 doi:10.20944/preprints202603.1612.v1



**Theorem 6** (No single probability representation)**.** *The four correlators produced by the intrinsic Maxwellian Bell state do not admit a simultaneous representation on a single probability space of the form*

$$E(x,y) = \int_\Lambda A(x,\cdot)\, B(y,\cdot)\, (\mu), \qquad x \in \{a,a'\},\ y \in \{b,b'\},$$

*with*

$$A : \{a,a'\} \times \Lambda \to \mathbb{R}, \qquad B : \{b,b'\} \times \Lambda \to \mathbb{R},$$

*measurable in the sense of Section 2 and satisfying*

$$|A(x,\cdot)| \le 1, \qquad |B(y,\cdot)| \le 1.$$

**Proof.** Assume, by contradiction, that such a probability space and such functions exist.

Then all hypotheses of the measure-theoretic CHSH theorem of Section 2 are satisfied. Therefore one would have

$$\left|E(a,b) + E(a,b') + E(a',b) - E(a',b')\right| \le 2.$$

But for the intrinsic Maxwellian Bell state we have already proved in Section 5 that

$$E(a,b) = -\cos(2(a-b)),$$

and therefore, for the standard analyzer settings,

$$\left|E(a,b) + E(a,b') + E(a',b) - E(a',b')\right| = 2\sqrt{2}.$$

This contradicts the CHSH bound. Therefore no such common measure-theoretic representation exists.

□

### *7.7. Conceptual Conclusion*

The significance of the previous theorem is that the intrinsic Maxwellian Bell model does not contradict the CHSH inequality. Rather, it lies outside its hypotheses.

The CHSH theorem is a statement about the existence of a single classical probability space supporting four bounded measurable correlators. The intrinsic Maxwellian model reproduces the same four numerical correlations, but not by means of such a representation.

The reason is that the ontic state is a global non-factorizable field configuration in

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

and the four correlations arise from different polarization resolutions of that same state.

Thus the violation of the CHSH bound is not evidence of any inconsistency. It is simply the indication that the intrinsic Maxwellian Bell model is not representable within the single-space measure-theoretic scheme of Section 2.

### *7.8. From Abstract Quantum Structures to Electromagnetic Realizations*

One of the central features of the intrinsic Maxwellian model is that the usual quantum structures are not merely reproduced, but are identified with concrete electromagnetic objects.

In the standard formalism, polarization states belong to the abstract space $\mathbb{C}^2$, and observables are represented by self-adjoint operators acting on that space. In the present framework, the polarization space is realized as a two-dimensional subspace

$$\mathcal{P} \subset W = \mathcal{S}'(M_4, \mathbb{C}^3),$$

 doi:10.20944/preprints202603.1612.v1



whose elements are genuine electromagnetic fields.

Under the canonical identification

$$\mathcal{P} \simeq \mathbb{C}^2,$$

quantum states correspond to Maxwellian fields, and linear combinations of states correspond to superpositions of electromagnetic fields.

Observables are likewise realized concretely. A polarization measurement at angle $a$ is not described abstractly by a projection operator on $\mathbb{C}^2$, but by a rotation of the transverse electromagnetic field followed by a decomposition along the rotated directions.

Thus the usual quantum projections are interpreted as physical operations on electromagnetic fields.

At the level of correlations, the standard quantum expression

$$E(a,b) = \langle \psi, A_a \otimes B_b \psi \rangle$$

is realized as a mean value associated with the decomposition of a tensorial Maxwellian field configuration in

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W.$$

In this sense, the intrinsic Maxwellian model provides a direct realization of the quantum formalism: abstract Hilbert space structures are identified with geometric and analytic structures of electromagnetic fields.

This correspondence extends also to dynamical operators. For instance, the abstract momentum operator corresponds, in the Maxwellian setting, to differential operators such as the curl, which act directly on the physical fields.

Thus the passage from quantum theory to the intrinsic Maxwellian model may be viewed not as a reinterpretation, but as a realization of the quantum formalism within the theory of electromagnetic fields.

## 8. Results IV: Energy–Norm Correspondence and Probability as Electromagnetic Energy Density

One of the central results of the intrinsic Maxwellian framework is that the Hilbert norm of a quantum state is naturally identified with the electromagnetic energy of the associated field.

This establishes a direct correspondence between quantum probability and classical electromagnetic energy density.

### *8.1. Maxwellian Norm and Electromagnetic Energy*

Let

$$W = \mathcal{S}'(M_4, \mathbb{C}^3)$$

be the space of complex Maxwellian fields

$$F = E + icB,$$

where $E$ and $B$ are real vector-valued distributions representing the electric and magnetic fields.

We define the (formal) Maxwellian norm of a field $F$ by

$$\|F\|_W^2 = \int_{\mathbb{R}^3} \frac{|E|^2 + |cB|^2}{2\varepsilon_0}.$$

This quantity coincides with the total electromagnetic energy of the field.

 doi:10.20944/preprints202603.1612.v1



### 8.2. Energy Measure Associated with a Field

To each field $F \in W$ with finite energy we associate a measure

$$\mu_F : \mathcal{B}(\mathbb{R}^3) \to \mathbb{R},$$

defined by

$$\mu_F(M) = \|F\|_W^{-2} \int_M \frac{|E|^2 + |cB|^2}{2\varepsilon_0}.$$

Here $M$ is any Borel subset of $\mathbb{R}^3$.

By construction, $\mu_F$ is a probability measure:

$$\mu_F(\mathbb{R}^3) = 1.$$

Thus the electromagnetic energy density determines a normalized probability distribution on physical space.

### 8.3. Quantum States and Maxwellian Fields

Let

$$V_e \subset L^2(M_4)$$

be the quantum state space generated by the de Broglie basis, and let

$$F_e : V_e \to W_e \subset W$$

be the Schwartz-linear isomorphism associating to each quantum state $\psi$ its corresponding Maxwellian field $F_e(\psi)$.

We assume that $\psi$ belongs to $L^2$ and that the corresponding field has finite electromagnetic energy.

### 8.4. Energy–Norm Theorem

**Theorem 7** (Energy–norm equivalence)**.** *Let $\psi \in V_e \cap L^2(M_4)$ and let*

$$F = F_e(\psi) \in W_e.$$

*Then*

$$\|\psi\|_{L^2}^2 = \|F\|_W^2.$$

**Proof.** By construction, the map $F_e$ is defined by transporting the spectral representation of $\psi$ with respect to the de Broglie basis into the corresponding Maxwellian basis.

The $L^2$ norm of $\psi$ is given by

$$\|\psi\|_{L^2}^2 = \int |\psi|^2,$$

which represents the total probability weight.

On the Maxwellian side, the same spectral coefficients determine the field $F$, whose energy is given by

$$\|F\|_W^2 = \int_{\mathbb{R}^3} \frac{|E|^2 + |cB|^2}{2\varepsilon_0}.$$

Since the mapping $F_e$ preserves the spectral amplitudes and the normalization of the basis elements is chosen so that each mode carries energy proportional to its squared amplitude, the two quantities coincide.

Therefore

$$\|\psi\|_{L^2}^2 = \|F\|_W^2.$$

□

 doi:10.20944/preprints202603.1612.v1



### 8.5. Probability as Normalized Energy Density

As a direct consequence, the Born probability distribution associated with $\psi$ admits a natural physical interpretation.

Let $\psi \in L^2$ and $F = F_e(\psi)$. The Born probability density $|\psi|^2$ corresponds to the normalized energy density of the electromagnetic field.

More precisely, for every Borel set $M \subset \mathbb{R}^3$,

$$\int_M |\psi|^2 = \mu_F(M) = \|F\|_W^{-2} \int_M \frac{|E|^2 + |cB|^2}{2\varepsilon_0}.$$

Thus the Born rule is identified with the normalized distribution of electromagnetic energy in space.

### 8.6. Extension to Polarization Sectors

In the intrinsic polarization model, a state

$$F = F_1 H + F_2 V \in \mathcal{P} \subset W$$

corresponds to a pair of quantum amplitudes

$$\psi = (\psi_1, \psi_2) \in \mathbb{C}^2.$$

The total energy is given by

$$\|F\|_W^2 = \|F_1\|_W^2 + \|F_2\|_W^2,$$

which corresponds to

$$|\psi_1|^2 + |\psi_2|^2.$$

Thus the probabilities of the two polarization outcomes are directly given by the energy contributions of the corresponding components.

### 8.7. Interpretation

The previous results show that quantum probability is not an abstract statistical structure, but a direct manifestation of the distribution of electromagnetic energy.

In particular:

- the Born norm corresponds to total electromagnetic energy;
- the Born probability measure corresponds to normalized energy density;
- polarization probabilities correspond to energy splitting between field components.

Therefore, within the intrinsic Maxwellian framework, probability is not introduced as an independent postulate, but emerges from the physical structure of the electromagnetic field.

## 9. Discussion

In this section we discuss the conceptual and structural implications of the intrinsic Maxwellian realization of Bell correlations developed in the previous sections.

### 9.1. Non-Product Structure of Maxwellian Bell States

A central feature of the intrinsic Maxwellian model is the existence of non-factorizable field configurations in the tensor product space

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W.$$

 doi:10.20944/preprints202603.1612.v1



The Bell state

$$\Psi_{\text{Bell}} = \frac{1}{\sqrt{2}}(H_A \otimes V_B - V_A \otimes H_B)$$

is a paradigmatic example of such a configuration.

This state is not representable in the form

$$F_A \otimes F_B, \qquad F_A \in \mathcal{P}_A, \quad F_B \in \mathcal{P}_B.$$

Thus the ontic content of the model is not given by pairs of independent local fields, but by a single global electromagnetic configuration.

This non-product structure is the precise mathematical origin of the Bell correlations observed in the model.

### *9.2. Global Field Ontology Versus Local Random Variables*

The measure-theoretic formulation of the CHSH theorem assumes that the correlations arise from bounded measurable functions defined on a single probability space.

In such a framework, one attempts to represent the outcomes of all measurement settings as functions of a common hidden parameter.

In the intrinsic Maxwellian model, no such representation exists. The ontic object is a global field in a tensor product space, and the correlations are obtained by resolving this field with respect to different polarization bases.

Therefore, the model does not assign pre-existing values to all measurement settings simultaneously. Instead, each pair of settings defines a specific decomposition of the same underlying field.

This shows that the failure of Bell factorization is not due to any lack of locality in the dynamical sense, but to the structural difference between global field configurations and classical random variables.

### *9.3. Realization of Quantum Projections as Field Operations*

Another important aspect of the model is that quantum projections are realized as concrete operations on electromagnetic fields.

In the standard formalism, a polarization measurement at angle $a$ is represented by a projection operator on $\mathbb{C}^2$. In the intrinsic Maxwellian framework, the same operation is realized as:

- a rotation of the transverse electromagnetic field;
- followed by a decomposition along the rotated directions.

Thus abstract quantum observables are identified with geometric operations on physical fields.

In particular, the Bloch sphere of polarization states is interpreted as the space of physical orientations of the transverse electromagnetic field.

### *9.4. Correlation Functions as Field-Theoretic Mean Values*

In the standard quantum formalism, the correlation function is written as

$$E(a,b) = \langle \psi, A_a \otimes B_b \psi \rangle.$$

In the intrinsic Maxwellian model, the same quantity is obtained from the decomposition of a tensorial electromagnetic field configuration.

Thus the correlation function is interpreted as a mean value associated with the structure of the field, rather than as an expectation value of abstract operators.

This provides a concrete realization of the quantum formalism in terms of electromagnetic fields.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 20 March 2026 doi:10.20944/preprints202603.1612.v1



**Remark 2** (Correspondence between quantum and Maxwell–Schwartz structures)**.** *The intrinsic Maxwellian framework establishes a systematic correspondence between the standard quantum formalism and the theory of Maxwellian fields in the space*

$$W = \mathcal{S}'(M_4, \mathbb{C}^3).$$

*More precisely, let*

$$\mathcal{V}_e \subset \mathcal{S}'(M_4, \mathbb{C})$$

*be the quantum state space generated by the de Broglie basis, and let*

$$\mathcal{F}_e : \mathcal{V}_e \to \mathcal{W}_e \subset W$$

*be the associated Schwartz-linear correspondence.*

*Then the following identifications hold:*

| *Quantum structure* | *Maxwell–Schwartz realization* |
|---|---|
| $\psi \in \mathcal{S}'(M_4, \mathbb{C})$ | $F = \mathcal{F}_e(\psi) \in W$ |
| *Polarization space* $\mathbb{C}^2$ | *Subspace* $P = \mathrm{Span}_{\mathbb{C}}\{H, V\} \subset W$ |
| *Bipartite space* $\mathbb{C}^2 \otimes \mathbb{C}^2$ | $P_A \otimes P_B \subset W \otimes W$ |
| *State vector* | *Maxwellian field configuration* |
| *Superposition* | *Linear combination of fields in* $W$ |
| *Projection* $P_a$ | *Rotation and decomposition of the transverse field* |
| *Observable* $A_a$ | *Field-theoretic polarization operation* |
| *Expectation* $\langle \psi, A\psi \rangle$ | *Mean value induced by field decomposition* |
| *Norm* $\|\psi\|^2$ | *Electromagnetic energy* $\|F\|_W^2$ |
| *Born probability* $|\psi|^2$ | *Normalized energy density of F* |

*This correspondence shows that the abstract Hilbert space structures of quantum mechanics admit a concrete realization in terms of electromagnetic fields. In particular, quantum states, observables, and probabilities are interpreted as geometric and energetic properties of Maxwellian field configurations.*

### 9.5. Probability and Energy

The energy–norm correspondence established in Section 6 shows that quantum probability is directly related to electromagnetic energy.

The Born norm corresponds to total energy, and the Born probability measure corresponds to normalized energy density.

Thus the probabilistic structure of quantum mechanics is not introduced as an independent axiom, but emerges from the physical structure of the field.

This gives a unified interpretation of quantum amplitudes, probabilities, and electromagnetic energy.

### 9.6. On Locality and Bell Correlations

The intrinsic Maxwellian model reproduces the standard Bell correlations while remaining entirely within the framework of Maxwell's equations.

In particular, the dynamical evolution of the fields is local and compatible with special relativity.

The violation of the CHSH inequality does not arise from any form of superluminal signal or interaction. Rather, it reflects the fact that the correlations are produced by a global field configuration that is not decomposable into independent local components.

Thus the apparent tension between quantum mechanics and locality is resolved at the level of ontology: the relevant objects are not point-like particles or local random variables, but extended electromagnetic fields.

 doi:10.20944/preprints202603.1612.v1



### 9.7. Conceptual Summary

The intrinsic Maxwellian framework leads to the following picture:

- quantum states are electromagnetic fields;
- polarization spaces are subspaces of the Maxwellian field space;
- Bell states are non-factorizable tensorial field configurations;
- quantum observables are realized as field operations;
- probabilities are normalized energy densities;
- Bell correlations arise from the global structure of fields.

In this sense, the intrinsic Maxwellian model provides a direct realization of quantum mechanics within classical field theory, without modifying either the mathematical structure of quantum correlations or the local dynamics of electromagnetic fields.

## 10. Conclusion

In this work we have developed an intrinsic Maxwellian framework for the description of polarization states and Bell correlations.

The starting point is the identification of the polarization space with a two-dimensional subspace

$$\mathcal{P} \subset W = \mathcal{S}'(M_4, \mathbb{C}^3),$$

generated by linearly polarized solutions of Maxwell's equations. This allows one to realize the usual quantum space $\mathbb{C}^2$ directly inside the electromagnetic field space.

Within this framework, bipartite systems are represented in the tensor product space

$$\mathcal{P}_A \otimes \mathcal{P}_B \subset W \otimes W,$$

and Bell states are realized as non-factorizable Maxwellian field configurations.

We have shown that:

- the intrinsic Maxwellian Bell state reproduces exactly the standard polarization correlations;
- the CHSH inequality is violated with the usual maximal value $2\sqrt{2}$;
- nevertheless, the hypotheses of the measure-theoretic CHSH theorem are not satisfied, since the correlators do not admit a representation on a single probability space;
- the obstruction arises from the non-product structure of the underlying electromagnetic field configuration.

A central result of the paper is the energy–norm correspondence, which establishes that the Hilbert norm of a quantum state coincides with the electromagnetic energy of the associated field. As a consequence, the Born probability measure is identified with the normalized energy density.

Thus the probabilistic structure of quantum mechanics is not introduced as an independent postulate, but emerges from the physical structure of electromagnetic fields.

From a conceptual point of view, the intrinsic Maxwellian framework provides a direct realization of quantum mechanics:

- quantum states are electromagnetic fields;
- quantum observables correspond to operations on fields;
- quantum probabilities correspond to energy distributions;
- Bell correlations arise from global field configurations.

In this perspective, the apparent conflict between Bell inequalities and locality is resolved by recognizing that the relevant ontic objects are not collections of local random variables, but global non-factorizable field configurations.

 doi:10.20944/preprints202603.1612.v1



The results of this work suggest that a large part of the quantum formalism may admit a natural interpretation within the theory of electromagnetic fields, opening the way to further developments in the study of quantum phenomena in distributional and field-theoretic settings.

Future work will address extensions of the present framework to more general classes of quantum systems, as well as a deeper analysis of the relation between electromagnetic potentials, gauge structures, and quantum observables.

**Author Contributions:** "Conceptualization, D.C.; methodology, D.C.; validation, D.C.; formal analysis, D.C.; writing—original draft preparation, D.C.; writing—review and editing, D.C.; visualization, D.C.; supervision, D.C.; project administration, D.C. All authors have read and agreed to the published version of the manuscript."

**Funding:** This research received no external funding

**Informed Consent Statement:** Not applicable

**Conflicts of Interest:** The authors declare no conflicts of interest.

## References

1. A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete?, *Physical Review* **47** (1935), 777–780.
2. J. S. Bell, On the Einstein Podolsky Rosen paradox, *Physics* **1** (1964), 195–200.
3. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, *Physical Review Letters* **23** (1969), 880–884.
4. S. J. Freedman and J. F. Clauser, Experimental test of local hidden-variable theories, *Physical Review Letters* **28** (1972), 938–941.
5. A. Aspect, P. Grangier, and G. Roger, Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A new violation of Bell's inequalities, *Physical Review Letters* **49** (1982), 91–94.
6. B. S. Cirel'son, Quantum generalizations of Bell's inequality, *Letters in Mathematical Physics* **4** (1980), 93–100.
7. L. Schwartz, *Théorie des distributions, Tome I*, Hermann, Paris, 1950.
8. L. Schwartz, *Théorie des distributions*, Nouvelle édition, Hermann, Paris, 1966.
9. E. Zeidler, *Quantum Field Theory I: Basics in Mathematics and Physics*, Springer, Berlin, 2006.