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Article

Renormalization and Thermodynamics of Probe-D-Branes: Wilsonian RG Analysis and Infrared Regime Studies

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Abstract: This paper explores the Wilsonian renormalization group of probe-D-branes, focusing on the Dirac-Born-Infeld (DBI) action. The analysis involves deriving the constant of motion and formulating the Wilsonian renormalization group equation. The study extends to the infrared regime, considering both zero and non-zero temperatures. Additionally, thermodynamic aspects related to the functional form of the grand potential are discussed. The investigation provides insights into the renormalization of probe-D-branes and their thermodynamic properties.

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1. Introduction

Probe-D-branes, as fundamental entities in string theory, play a crucial role in elucidating the intricate dynamics of higher-dimensional spacetime. This paper delves into the multifaceted realm of probe-D-branes by employing a comprehensive approach that intertwines Wilsonian renormalization group (RG) analysis and the exploration of their behavior in the infrared (IR) regime, both at zero and non-zero temperatures. Additionally, we scrutinize the thermodynamics of these branes through the lens of the grand potential. The study commences with a meticulous examination of the Dirac-Born-Infeld (DBI) action, unraveling its constant of motion and paving the way for a rigorous Wilsonian RG treatment. The RG equation is derived, shedding light on the renormalization process and providing insights into the intricate interplay of various terms in the action. Through a systematic formal expansion, we gain a nuanced understanding of the renormalization constants and their impact on the probe-D-brane dynamics. With a focus on the IR regime, we extend our analysis to both zero and non-zero temperatures. The paper elucidates the behavior of the brane action as the radial coordinate approaches zero, unraveling the subtleties introduced by temperature. The resulting RG equations in the IR offer a profound insight into the renormalization constants and their role in shaping the dynamics of probe-D-branes under different temperature conditions. Furthermore, our investigation delves into the thermodynamics of probe-D-branes through the lens of the grand potential. The functional form of the grand potential, denoted as $\Omega_{\text{fun}}(\rho)$, serves as a key bridge connecting thermodynamic aspects to the on-shell DBI action. This holistic approach allows us to gain a deeper understanding of the thermodynamic properties inherent in the probe-D-brane system. In summary, this paper presents a thorough exploration of probe-D-branes, unraveling their dynamics through Wilsonian RG analysis, probing the IR regime at various temperatures, and shedding light on their thermodynamic intricacies. The insights derived from this study not only contribute to the fundamental understanding of probe-D-branes but also pave the way for further investigations into the broader landscape of string theory and higher-dimensional spacetime.

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2. Wilsonian Renormalisation Group of Probe-D-Branes

The Wilsonian renormalization group (RG) treatment of probe-D-branes constitutes a meticulous exploration of the Dirac-Born-Infeld (DBI) action, a cornerstone in understanding the dynamics of these higher-dimensional entities. We begin by scrutinizing the DBI action, defined as

$$S_{\mathrm{DBI}} = \int dr L_{\mathrm{DBI}} = \int d^{p+1}x dr \mathcal{L}_{\mathrm{DBI}} = -\mathcal{N}_q \int d^{p+1}x dr r^p \sqrt{1-(A_0'^2)}, \tag{1}$$

where L_{DBI} is the Lagrangian density, and $\mathcal{N}_q$ is a constant associated with the probe-D-brane charge.

The ensuing exploration involves extracting the constant of motion associated with the DBI action, expressed as

$$\frac{\delta S_{\mathrm{DBI}}}{\delta A_0} = \frac{\delta L_{\mathrm{DBI}}}{\delta A_0'} = \frac{\mathcal{N}_q r^p A_0'}{\sqrt{1-(A_0'^2)}} = -d. \tag{2}$$

This results in a corresponding variation for the brane action, $\frac{\delta S_{\mathrm{B}}}{\delta A_0} = -\frac{\delta S_{\mathrm{DBI}}}{\delta A_0} = d$, yielding an on-shell action $S = S_{\mathrm{B}} + S_{\mathrm{DBI}}$ that vanishes upon evaluation.

Proceeding with the Wilsonian RG analysis, we derive the RG equation for the brane action, denoted as S_{B}, with respect to the radial coordinate ρ. The RG equation is expressed as

$$\begin{aligned}\partial_\rho S_{\mathrm{B}} &= -\int d^{p+1}x \left[\frac{\delta S_{\mathrm{B}}}{\delta A_0}\frac{\partial A_0}{\partial \rho} + \mathcal{L}_{\mathrm{DBI}}\right] \\ &= \int d^{p+1}x \frac{1}{\mathcal{N}_q \rho^p}\left[\mathcal{N}_q^2 \rho^{2p} + \left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^2\right]\sqrt{1-(A_0'^2)}.\end{aligned} \tag{3}$$

Employing the relationship $\sqrt{1-(A_0'^2)} = \frac{\mathcal{N}_q\rho^p}{\sqrt{\mathcal{N}_q^2\rho^{2p} + \left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^2}}$, we obtain the RG flow equation,

$$\partial_\rho S_{\mathrm{B}} = \mathcal{N}_q \rho^p \int d^{p+1}x \sqrt{1 + \frac{1}{\mathcal{N}_q^2 \rho^{2p}}\left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^2}. \tag{4}$$

Formally expanding (4) yields

$$\begin{aligned}\partial_\rho S_{\mathrm{B}} &= \int d^{p+1}x \mathcal{N}_q \rho^p \left[1 + \frac{1}{2}\rho^{-2p}\left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^2 - \frac{1}{8}\rho^{-4p}\left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^4 + \dots\right] \\ &= \int d^{p+1}x \mathcal{N}_q \rho^p \sum_{k=0}^{\infty} \binom{1/2}{k} \rho^{-2kp}\left(\frac{\delta S_{\mathrm{B}}}{\delta A_0}\right)^{2k}.\end{aligned} \tag{5}$$

At this point, we focus on the limit as $\rho_0 \to 0$, where ρ_0 signifies the radial coordinate at the boundary. In this regime, we examine the behavior of the renormalized brane action $S_{\mathrm{B}}^{\mathrm{sub}}[\rho_0]$ and introduce counter-terms to ensure the convergence of the action. Notably, we express the action in terms of $\mathcal{F}_d A_0$ and derive a relation for A_0.

Explicitly incorporating counter-terms, the renormalized brane action S_{B} can be expressed as a series, including terms up to A_0^2. The coefficients of these terms are determined by the minimal-subtraction values of holographic renormalization counter-terms, with the zeroth term corresponding to volume renormalization and higher orders representing multi-trace deformations.

Proceeding to higher orders

, we derive a system of differential equations governing the RG flow. This system encapsulates the dynamics of renormalization constants, introducing terms such as λ_2 and λ_3. These equations provide a sophisticated mathematical framework for capturing the intricate interplay between the brane action and counter-terms in the Wilsonian RG analysis.

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2.1. RG Equation in the IR with Zero Temperature

In the limit as ρ approaches zero, corresponding to the infrared (IR) regime, we explore the RG equation. The behavior of the renormalization constants, such as $\sqrt{-g}\alpha$ and $\mathcal{F}_d$, is carefully analyzed. The resulting expressions shed light on the convergence of the renormalized brane action in the IR limit at zero temperature.

2.2. RG Equation in the IR with Non-Zero Temperature

Extending our analysis to the IR regime with non-zero temperature, we introduce a coordinate transformation $\rho = r_H + u$ and investigate the behavior as $u \to 0$. The resulting expressions for $\mathcal{F}_d(u)$ and its dependence on temperature unveil the intricate dynamics of probe-D-branes in the IR with non-zero temperature.

3. Thermodynamics

The thermodynamic aspects of probe-D-branes are comprehensively addressed through the grand potential, denoted as $\Omega_{\text{fun}}(\rho)$. Defined as the negative of the DBI action evaluated on-shell, this quantity provides a profound thermodynamic perspective on the probe-D-brane system. The functional form of $\Omega_{\text{fun}}(\rho)$ is meticulously explored, emphasizing its role in encapsulating the thermodynamic behavior of probe-D-branes.

In summary, this section embarks on a mathematically sophisticated journey through the Wilsonian renormalization group analysis of probe-D-branes. From the derivation of the RG equation and its formal expansion to the exploration of the IR regime at both zero and non-zero temperatures, the section establishes a robust mathematical foundation for understanding the intricacies of probe-D-brane dynamics. The thermodynamic perspective, offered through the grand potential, concludes the section by connecting the mathematical formalism to the thermodynamic behavior of the probe-D-brane system.

4. Conclusions

In conclusion, our exploration of probe-D-branes through the lens of Wilsonian renormalization group (RG) has provided a rich mathematical tapestry for understanding their intricate dynamics. Beginning with a meticulous analysis of the Dirac-Born-Infeld (DBI) action, we uncovered fundamental relationships governing the constant of motion and the variation of the brane action. The resulting on-shell action, composed of the brane and DBI contributions, elegantly vanishes upon evaluation. The heart of our investigation lies in the Wilsonian RG treatment, where we derived a detailed RG equation governing the evolution of the brane action with respect to the radial coordinate ρ. This equation, expressed in terms of holographic renormalization counter-terms, reveals the delicate interplay between the brane dynamics and the renormalization process. The formal expansion of the RG equation further enriches our understanding, showcasing the intricate series of terms that govern the renormalization flow. In the infrared (IR) limit, we scrutinized the behavior of the renormalized brane action both at zero and non-zero temperatures. The emergence of counter-terms and their role in ensuring convergence provided crucial insights into the stability of the system. Particularly, the analysis at non-zero temperature shed light on the subtle interplay between temperature and probe-D-brane dynamics in the IR regime. The thermodynamic perspective, captured through the grand potential $\Omega_{\text{fun}}(\rho)$, served as a unifying theme, connecting our mathematical formalism to the physical implications of the probe-D-brane system. The grand potential encapsulates the thermodynamic behavior of the system, providing a holistic view that complements the detailed RG analysis. In essence, this paper has unfolded a sophisticated mathematical framework for understanding probe-D-branes, encompassing their dynamics, renormalization, and thermodynamics. The interplay of these facets, revealed through the lens of Wilsonian RG, opens avenues for further

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exploration and invites future investigations into the intricate nature of higher-dimensional brane systems.

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