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Posted Date: 18 February 2025

doi: 10.20944/preprints202502.1359.v1

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Resolving the Cosmological Constant Problem: Black Hole Entropy and Finite Geometry

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Abstract: The cosmological constant problem remains one of the most profound mysteries in physics. By effective field theory and dimensional analysis, Λ, the cosmological constant, is expected to be of the order of the Planck mass squared. Accordingly, the product of Λ and the Planck length squared must be nearly 1. By contrast, cosmological observations indicate that this product is close to the square of the ratio of the Planck length to the Hubble length ℓ_H, meaning that Λ is of the order ℓ_H^{-2}. Thus, the cosmological constant problem can be framed as the question of how ℓ_H gets involved in the prediction for Λ. The present paper demonstrates that such an involvement is a result of a straightforward combination of the black hole entropy bound with the assumption of a finite number of hodons, the fundamental elements of physical space.

Keywords: cosmological constant problem; vacuum catastrophe; dimensional analysis; Hubble length; black hole entropy; finite geometry; hodons

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1. Introduction

The cosmological constant problem is one of the most significant unsolved problems in physics. Originally introduced as Λ by Albert Einstein in his field equations of General Relativity (GR) to allow for a static universe, the cosmological constant (CC) corresponds to a constant energy density present everywhere, both in regions of empty space (the vacuum) and in regions that contain matter and radiation. Additionally, quantum field theory (QFT) posits that the vacuum is permeated with the zero-point energy from quantum fluctuations. This suggests that the zero-point energy contributes to the CC [1–3].

Now, recall that in effective field theory (EFT), it is common to estimate the contributions to physical quantities based on the energy scale of the theory. The Planck mass m_P represents the natural energy scale where quantum gravitational effects become significant. On the other hand, the CC has the dimensions of an inverse area (inverse square length). In natural units ($\hbar = c = 1$), the Planck mass squared, m_P^2, has dimensions of inverse length squared, which matches the dimensions of Λ. So, by EFT and dimensional analysis, Λ_{vac}, the contribution from zero-point energy to Λ, is expected to be of the order m_P^2 [4]. Accordingly, the product of Λ_{vac} and the Planck length squared, ℓ_P^2, must be nearly 1:

$$\Lambda_{\text{vac}} \cdot \ell_P^2 \sim 1 \quad . \tag{1}$$

This means that the CC is expected to be of order $\ell_P^{-2} \sim 10^{70}\,\text{m}^{-2}$. By contrast, cosmological observations deliver the number

$$\Lambda_{\text{vac}} \cdot \ell_P^2 \sim 10^{-122} \quad , \tag{2}$$

which indicates that the value of the CC must be $\Lambda \sim \Lambda_{\text{vac}} \sim 10^{-52}\,\text{m}^{-2}$, i.e., 122 orders of magnitude smaller than predicted by EFT and dimensional analysis [5,6].

This significant discrepancy embodies the CC problem, underscoring a knowledge gap in our understanding of the interplay between GR and quantum mechanics. To bridge this divide, various

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hypotheses and research methods are being explored, including modifications to GR, the introduction of new particles or fields, and a better understanding of quantum field behavior in curved spacetime.

The CC problem can be stated in an alternative manner. Notably, as highlighted in works such as [7,8], the value 10^{-122} is surprisingly close to the square of the ratio of the Planck length to the radius of the observable universe. This radius is approximately equivalent to the Hubble length, defined as $\ell_H \equiv c \cdot H_0^{-1}$ (where H_0 is the Hubble constant, i.e., the present value of the Hubble parameter H) [9]. Consequently, the product of Λ_{vac} and ℓ_P^2 can be expressed as:

$$\Lambda_{\text{vac}} \cdot \ell_P^2 \sim \left(\frac{\ell_P}{\ell_H}\right)^2 \quad . \tag{3}$$

Hence, the reason this product is nearly zero rather than close to one is that $\ell_P \sim 10^{-35}$ m is very much less than $\ell_H \sim 10^{27}$ m. The challenge, therefore, lies in explaining how the Hubble length gets involved in the prediction for Λ_{vac}.

2. Zel'dovich's Ansatz

To address this challenge, the paper [8] proposes utilizing Zel'dovich's Ansatz. This hypothesis, proposed by Yakov Zel'dovich, asserts that the observable vacuum energy density ρ_{vac} originates from the gravitational energy of virtual particle-antiparticle pairs generated and annihilated in the vacuum state. In keeping with this ansatz, the paper suggests that

$$\rho_{\text{vac}}(\ell) \sim \frac{G \cdot m^2(\ell)}{\ell} \cdot \frac{1}{\ell^2} \quad , \tag{4}$$

where $\rho_{\text{vac}}(\ell)$ and $m(\ell)$ represent the vacuum energy density and the mass of the particles, respectively, as perceived at a particular length scale ℓ. Defining $m(\ell)$ and $\Lambda_{\text{vac}}(\ell)$ as

$$m(\ell) \sim \frac{\hbar}{c \cdot \ell} \quad , \tag{5}$$

$$\Lambda_{\text{vac}}(\ell) \sim \frac{G}{c^4} \cdot \rho_{\text{vac}}(\ell) \quad , \tag{6}$$

one easily gets

$$\Lambda_{\text{vac}}(\ell) \cdot \ell_P^2 \sim \left(\frac{\ell_P}{\ell}\right)^6 \quad . \tag{7}$$

According to the studies [10,11], the minimum observable length scale is not the Planck length itself but a much larger one derived from the Bekenstein-Hawking entropy (black hole entropy) bound, namely,

$$\ell_{\min} = \sqrt[3]{\ell \cdot \ell_P^2} \quad . \tag{8}$$

By substituting ℓ by $\ell_{\min}$ in Eq. (7) and assuming that the CC is taken at the radius of the observable universe, and therefore

$$\Lambda_{\text{vac}} \equiv \Lambda_{\text{vac}}(\ell_H) \quad , \tag{9}$$

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the result (3) is obtained:

$$\Lambda_{\text{vac}} \cdot \ell_P^2 \sim \left(\frac{\ell_P}{\sqrt[3]{\ell \cdot \ell_P^2}} \right)^6 = \left(\frac{\ell_P}{\ell_H} \right)^2 \quad . \tag{10}$$

To critically evaluate this inference, it is essential to consider the established critiques of Zel'dovich's Ansatz. A primary criticism suggests that the ansatz may oversimplify the intricate interactions inherent in QFT, neglecting to account for all variables that affect vacuum energy density [12]. As a result, more comprehensive models may be necessary to accurately capture this phenomenon.

Additionally, some critics argue that Eq. (8) captures an unconditional uncertainty associated with a single geodesic [13]. They assert that relying on the uncertainty of just one geodesic length to assess the intrinsic uncertainty of the space-time metric is misguided. To accurately determine this intrinsic uncertainty, one must account for the simultaneous uncertainties of all geodesics or, at the very least, a pertinent subset of them. Ignoring the correlations among these uncertainties could lead to a substantial overestimation of the uncertainties in spacetime curvature.

Notwithstanding this criticism, the black hole bound entropy seems promising for understanding Eq. (3).

3. Causal Set Theory

In the paper [14], Eq. (3) is derived using the concept of a fluctuating cosmological constant based on the causal set hypothesis. Specifically, the CC is proposed to oscillate around zero as:

$$\Lambda = \overline{\Lambda} + \Delta\Lambda \quad , \tag{11}$$

where $\overline{\Lambda} = 0$ is the mean value of Λ and $\Delta\Lambda$ is the magnitude of its fluctuations stemming from the underlying discreteness of spacetime. This discreteness results in a finite number N of spacetime elements, making spacetime volume $\mathcal{V}$ directly reflect this count. Assuming fluctuations in N are of Poisson-type, the typical magnitude of fluctuations in the spacetime volume $\mathcal{V}$ is expected to be $\Delta\mathcal{V} = \pm\sqrt{\mathcal{V}}$.

Suppose that Λ can be determined to a certain extent, with fluctuations $\Delta\Lambda$ decreasing as $\mathcal{V}$ increases. Consequently,

$$\Delta\Lambda \sim \frac{1}{\sqrt{\mathcal{V}}} \quad . \tag{12}$$

Since a volume in spacetime (which has 4 dimensions) scales with the fourth power of the Hubble length ℓ_H, one finds

$$\Lambda = \Delta\Lambda \sim \frac{1}{\ell_P^2} \quad , \tag{13}$$

which results in Eq. (3).

It is important to recognize that the assumption of $\overline{\Lambda} = 0$ may not be consistent with contemporary understanding [15]. This assumption suggests that, on average, the contribution from zero-point energy offset other contributions, which appears to be quite improbable. Alternatively, assuming $\overline{\Lambda} = 0$ would indicate the existence of a mechanism or principle that, on average, suppresses the impact of zero-point energy on the cosmological constant to zero – a concept that lacks support from established physics.

One significant criticism pertains to the discrete nature of spacetime in causal set theory [16]. This inherent discreteness may conflict with the continuous spacetime observed at macroscopic scales. Consequently, this raises important questions regarding how the discrete structure of causal sets can lead to the familiar, continuous spacetime characterized by GR [17].

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Despite the criticism, the concept of a finite number of spacetime elements seems to have promise in clarifying Eq. (3).

In the present paper, it will be demonstrated that Eq. (3) is precisely what one obtains by a straightforward combination of the black hole entropy bound with the assumption of a finite number of the fundamental elements of physical space.

4. Finite Geometry

We will begin by defining a discrete space as one in which the number of the fundamental elements within any reasonably shaped (e.g., convex) region of finite volume is finite. As is customary, these fundamental elements will be referred to as *hodons* [18,19]. In the context of discrete geometry, hodons are regarded as the smallest, indivisible units constituting the fabric of the universe.

For any finite geometry proposal to be taken seriously, it must address three critical issues: *defining a distance function*, *managing anisotropy*, and *identifying hodons*.

Firstly, ***defining a distance function*** needs to resolve the "Pythagoras trouble", an objection raised by Weyl [20]. This objection argues that any regular (non-overlapping) tiling of space with randomly shaped hodons would not approximate Euclidean geometry because Pythagoras' Theorem would not hold, even approximately.

Second, ***managing anisotropy*** involves handling direction-dependent properties in finite geometry. The isotropy problem exists because any regular hodon shape will have preferred directions, making a discrete space inherently anisotropic.

Third, ***identifying hodons*** involves determining their measurable attributes. Unlike continuous geometry points, which lack size, volume or any other quantifiable characteristic, hodons in finite geometry must have measurable properties. The problem is determining which attributes to identify with hodons.

Various ways to define a distance function and tackle anisotropy have been suggested (see for example [21]). However, instead of adopting any of these methods, the paper [22] suggests representing a finite geometry using classical (continuous) space. This representation involves modelling or interpreting a discrete, or finite, geometric structure within the familiar framework of continuous classical geometry, such as Euclidean space. Specifically, this entails mapping or translating the discrete elements and properties of a finite geometry into the continuous framework of classical geometry.

Such an approach appears especially sensible as it closely mirrors the measurement process in quantum mechanics. This analogy can be understood through several key points:

1. Interpretation through Classical Physics: In quantum mechanics, a classical measuring apparatus is essential for interpreting and understanding quantum results in terms of classical physics. The act of measurement collapses the quantum wave function, producing a definite outcome that can be comprehended within the framework of classical physics.

2. Connecting Quantum Phenomena with Classical Experiences: Without a classical measuring apparatus, it would be problematic to connect quantum phenomena with our everyday classical experiences. The apparatus acts as a bridge, translating the abstract, probabilistic nature of quantum mechanics into tangible, observable results that align with our classical intuition.

3. Representation of Finite Geometry: Similarly, in the context of finite geometry, a classical measuring apparatus – by being part of our everyday continuous domain – provides a means to represent a finite geometry of quantum scale using classical (continuous) space. This allows us to apply our classical understanding of lengths, angles, and distances to discrete space-time.

4. Connecting Finite Geometry with Continuous Experiences: Without classical geometry, it would be problematic to connect the finite geometric nature of space with our everyday continuous experiences. Classical geometry serves as an interpretive framework that enables us to make sense of a discrete space within the context of a continuous geometrical model.

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Thus, just as a classical measuring apparatus is crucial for translating quantum phenomena into classical terms, classical geometry is essential for bridging the gap between finite geometry and our continuous spatial experiences. This analogy underscores the importance of classical frameworks in making sense of complex, abstract domains and highlights the interconnectedness of different areas of physics.

To address the problem of identifying hodons, it seems reasonable to apply Wheeler's idea that the fundamental building blocks of reality are informational rather than material, known as the "It from Bit" concept [23]. In line with this concept, each hodon, as a fundamental component of the universe, should be identified with *a bit of information*.

Since a hodon is essentially a unit of information, it does not require a definite shape when represented in classical space. Consequently, questions about aspects of a hodon, such as its boundary, are not applicable.

This implies that in classical space, the representation of exactly n hodons within a region can only be estimated probabilistically. Thus, one can only assess the likelihood that a particular region of classical space contains n hodons (n bits of information).

To be consistent with the causal set hypothesis [24], the correspondence between the underlying finite geometric structure of physical space and the classical space that represents it can be defined via a Poisson process of "sprinkling". This process uses a Poisson distribution to determine the number of hodons (units of information) mapped into a given region of classical space. This ensures that hodons are distributed randomly and independently.
In particular, the probability of sprinkling n hodons into a classical region of measure $\mathcal{M}$, where $\mathcal{M}$ represents a general measure of a region, which could be specified as area (A) or volume (V) depending on the context, is

$$P(n) = \frac{(\rho_{\mathcal{M}} \cdot \mathcal{M})^n \cdot e^{-\rho_{\mathcal{M}} \cdot \mathcal{M}}}{n!} \quad , \tag{14}$$

where $\rho_{\mathcal{M}}$ is the density of the sprinkling, so that the product $\rho_{\mathcal{M}} \cdot \mathcal{M}$ is the expected number of hodons mapped into the classical region.

5. Black Hole Entropy Bound

Consider a Riemannian manifold $\mathfrak{R}$ (i.e., a smooth manifold equipped with a Riemannian metric, which allows for the measurement of lengths, angles, and distances). Let $\mathcal{R}$ be a region in $\mathfrak{R}$ and $\delta\mathcal{R}$ its boundary, with measures $V(\mathcal{R})$ and $A(\delta\mathcal{R})$ respectively. Then, provided that $\rho_{V(\mathcal{R})} \cdot V(\mathcal{R})$ and $\rho_{A(\delta\mathcal{R})} \cdot A(\delta\mathcal{R})$ are the expected numbers of hodons (i.e., the expected amount of information) mapped into the region $\mathcal{R}$ and its boundary $\delta\mathcal{R}$, the expected entropy in $\mathcal{R}$ and $\delta\mathcal{R}$ can be defined as:

$$H(\mathcal{R}) = k_B \cdot \rho_{V(\mathcal{R})} \cdot V(\mathcal{R}) \quad , \tag{15}$$

$$H(\delta\mathcal{R}) = k_B \cdot \rho_{A(\delta\mathcal{R})} \cdot A(\delta\mathcal{R}) \quad . \tag{16}$$

If $\mathcal{R}$ is the region of a black hole and $\delta\mathcal{R}$ is the event horizon (the boundary of the region of a black hole), then the entropy assigned to the black hole to comply with the laws of thermodynamics as interpreted by external observers (known as the Bekenstein-Hawking entropy or black hole entropy [25,26]) is given by

$$S_{\mathrm{BH}} = k_B \cdot \frac{1}{4\ell_P^2} \cdot A(\delta\mathcal{R}) \quad . \tag{17}$$

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Assuming that $\frac{1}{4\ell_P^2} \cdot A(\delta\mathcal{R})$ is the expected number of hodons mapped onto the boundary $\delta\mathcal{R}$ and that a black hole is the most entropic object one can put inside the boundary $\delta\mathcal{R}$, we find

$$N\big(V(\mathcal{R})\big) \le \frac{1}{4\ell_P^2} \cdot A(\delta\mathcal{R}) \quad , \tag{18}$$

where $N\big(V(\mathcal{R})\big) = \rho_{V(\mathcal{R})} \cdot V(\mathcal{R})$ is the expected number of hodons mapped inside the boundary $\delta\mathcal{R}$. The above relation represents the black hole entropy bound in the context of finite geometry.

6. Cosmological Constant

Recall that according to QFT, each point in space is associated with the zero-point energy given by $E_0 = \frac{\hbar\omega}{2}$, where ω is the angular frequency. If we assume that space is discrete, this implies that each unit of information (i.e., hodon) must also be identified with the zero-point energy E_0 in accordance with QFT.

Allowing that all angular frequencies ω are alike, the vacuum energy present in the volume $V(\mathcal{R})$ can be expressed as the sum of all hodons mapped into $V(\mathcal{R})$:

$$E_{\text{vac}}\big(V(\mathcal{R})\big) = \sum_k \frac{\hbar\omega_k}{2} = \frac{\hbar\omega}{2} \cdot N\big(V(\mathcal{R})\big) \quad . \tag{19}$$

As acknowledged, on large scales, the space in which the universe exists is well approximated as three-dimensional and flat [27]. Given this, the Riemannian manifold $\mathfrak{R}$ representing the universe can be considered to have a flat geometry, meaning its Riemannian metric corresponds to the standard Euclidean space. Similarly, the region $\mathcal{R}_U$ of $\mathfrak{R}$ representing the space of the observable universe can be viewed as a 3-dimensional Euclidean ball with the radius $R_U \approx 3\ell_H$.

Naturally, the vacuum energy present in the observable universe can be expressed as the product of the vacuum energy density ρ_{vac} and the volume of the observable universe $V(\mathcal{R}_U)$:

$$E_{\text{vac}}\big(V(\mathcal{R}_U)\big) = \rho_{\text{vac}} \cdot V(\mathcal{R}_U) \quad , \tag{20}$$

where

$$\rho_{\text{vac}} = \frac{c^4\Lambda_{\text{vac}}}{8\pi G} \quad . \tag{21}$$

Conceding that the characteristic length $L(\mathcal{R})$ defining the linear scale of the region $\mathcal{R}$ is the ratio of the region's volume $V(\mathcal{R})$ to the area $A(\delta\mathcal{R})$ of the region boundary, i.e.,

$$L(\mathcal{R}) = \frac{V(\mathcal{R})}{A(\delta\mathcal{R})} \quad , \tag{22}$$

and assuming that the angular frequency ω is given by the expression

$$\omega = \frac{2\pi c}{L(\mathcal{R})} \quad , \tag{23}$$

the expected number of hodons mapped into the observable universe can be evaluated as

$$N\big(V(\mathcal{R}_U)\big) = \frac{2}{\hbar\omega} \cdot E_{\text{vac}}\big(V(\mathcal{R}_U)\big) = \frac{\Lambda_{\text{vac}}}{\ell_P^2} \cdot \frac{V^2(\mathcal{R}_U)}{8\pi^2 \cdot A(\delta\mathcal{R}_U)} \quad . \tag{24}$$

Substituting this evaluation into the black hole entropy bound (18), we find

$$\Lambda_{\mathrm{vac}} \leq 2\pi^2 \cdot \frac{1}{L^2(\mathcal{R}_U)} \quad . \tag{25}$$

Since the order of magnitude of $2\pi^2$ is 10^1 and the linear scale of a 3-dimensional Euclidean ball with the radius R_U is defined as

$$L(\mathcal{R}_U) = \frac{R_U}{3} \approx \ell_H \quad , \tag{26}$$

we finally get

$$\Lambda_{\mathrm{vac}} \lesssim 10^1 \cdot \frac{1}{\ell_H^2} \quad . \tag{27}$$

This indicates that the predicted upper limit of Λ_{vac} is reasonably close to the observed limit, $\Lambda_{\mathrm{vac}} \sim \ell_H^{-2}$. To be precise, the predicted and observed values that Λ_{vac} can reach are within an order of magnitude of each other.

7. Concluding Remarks

It is instructive to consider what classical region models (represents) a single hodon.

From the assumption that $\frac{1}{4\ell_P^2} \cdot A(\delta\mathcal{R})$ is the expected number of hodons mapped onto the boundary $\delta\mathcal{R}$ follows that a region of area $4\ell_P^2$ represents a single hodon on a surface. In symbols,

$$A(\delta\mathcal{R}_{\mathrm{hodon}}) = 4\ell_P^2 \quad . \tag{28}$$

Supposing that this region is a sphere, its radius must be

$$R_{\mathrm{hodon}} = \frac{\ell_P}{\sqrt{\pi}} \quad . \tag{29}$$

To find what region represents a single hodon in space, we can use the black hole entropy bound. Indeed, since

$$V(\mathcal{R}_{\mathrm{hodon}}) = \frac{V(\mathcal{R}_U)}{N\big(V(\mathcal{R}_U)\big)} \quad , \tag{30}$$

from Eq. (18) it follows that

$$V(\mathcal{R}_{\mathrm{hodon}}) \geq \frac{V(\mathcal{R}_U) \cdot 4\ell_P^2}{A(\delta\mathcal{R}_U)} \quad . \tag{31}$$

Thus, a region representing a single hodon in the space of the observable universe has the volume:

$$V(\mathcal{R}_{\mathrm{hodon}}) \gtrsim \ell_H \cdot 4\ell_P^2 \quad . \tag{32}$$

Again, supposing that this region is a sphere, its radius ought to be:

$$R_{\text{hodon}} \gtrsim \sqrt[3]{\frac{3\ell_H \cdot \ell_P^2}{\pi}} \quad . \tag{33}$$

Comparing this with Eq. (29) reveals that the radius of $A(\delta\mathcal{R}_{\text{hodon}})$, the surface area of the sphere representing a single hodon, differs from the radius of $V(\mathcal{R}_{\text{hodon}})$, the volume of the same sphere. This nonsensical situation shows that a true discrete geometry cannot be represented by classical space. In particular, a hodon cannot be represented by a regularly shaped region in Euclidean geometry.

Using the prediction(27) and the volume of a single hodon $V(\mathcal{R}_{\text{hodon}})$, the upper limit on the product of Λ_{vac} and ℓ_P^2 can be estimated by dimensional analysis as

$$\Lambda_{\text{vac}} \cdot \ell_P^2 \sim \frac{\ell_P^6}{V^2(\mathcal{R}_{\text{hodon}})} \quad . \tag{34}$$

Where a hodon to be represented in Euclidean space by a sphere of the Planck volume $V(\mathcal{R}_{\text{hodon}}) \sim \ell_P^3$, the product $\Lambda_{\text{vac}} \cdot \ell_P^2$ would be of order 1 making it "the worst theoretical prediction in the history of physics" [28]. However, due to the irrepresentability of hodon in continuous space, such a prediction is not applicable.

In conclusion, the theoretical framework of finite geometry, along with the black hole entropy bound, appears to capture key elements needed to solve the cosmological constant problem. Nonetheless, further refinement may enhance the precision of Λ_{vac} prediction.

Acknowledgments: This work was developed with the assistance of AI tools including Microsoft Copilot, ChatGPT for Word, and Autopilot. These tools were primarily utilized to verify the accuracy of statements in the paper, manage reference collections, and revise the text.

Conflicts of Interest: The author states that there is no conflict of interest.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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