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Spacetime and Internal Symmetry from Split Bioctonions and the Two Extra SU(3)'s of $E_8 \times \omega E_8$

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Abstract

Over the last few years, we have attempted to develop an $E_8 \times E_8$ theory of unification to combine the standard model with general relativity. In the present new work, we give a self-contained construction in which the two extra SU(3) factors that appear in the maximal subgroup chain $E_8 \supset E_6 \times \mathrm{SU}(3)$ on each side of $E_8 \times \omega E_8$ generate: (i) a six-dimensional base (M_6, g) of signature $(3,3)$; (ii) two embedded Lorentzian 4D spacetimes; and (iii) per side, a canonical *real* 4-dimensional internal fibre naturally identified with the tangent of $\mathbb{C}P^2 = \mathrm{SU}(3)/S(U(2) \times U(1))$. The key algebraic ingredient is the octonionic split $\mathbb{O} = \mathbb{H} \oplus \mathbb{H}\varepsilon$ with $\varepsilon \perp \mathbb{H}$, by which the branch $\mathrm{Adj}\,\mathrm{SU}(3) \to \mathbf{3}_0 \oplus \mathbf{2}_{+1} \oplus \overline{\mathbf{2}}_{-1} \oplus \mathbf{1}_0$ is realised as $\Im\mathbb{H} \oplus (\mathbb{H}\varepsilon)_{\mathbb{R}} \oplus \mathbb{R}$. The two $U(1)$ factors play the role of Spin^c connections on the $\mathbb{C}P^2$ fibres. The role of AI in this work is explicitly acknowledged, and highlighted in an appendix.

Keywords: E8 x E8 unification; octonions; trinification; spacetime; internal symmetry; adjoint SU(3)

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1. Introduction

Over the last few years we have proposed and developed an $E_8 \times \omega E_8$ theory of unification which aims to unify the standard model with gravitation described by the general theory of relativity [1–3]. Here, ω is the split complex number. It is assumed that each of the two E_8 branches as $SU(3) \times E_6$ and each of the two resulting E_6 undergoes a trinification $E_6 \to SU(3) \times SU(3) \times SU(3)$. The split complex number plays a crucial role, enabling the emergence of a Lorentzian signature for spacetime, and enabling the emergence of chiral fermions. Its origin in our theory can be traced to (left acting) octonionic chains made from the algebra $\mathbb{O} \oplus \omega\mathbb{O}$ of split bioctonions. Complex split bioctonions generate the Clifford algebra $Cl(7) \cong Cl(6) \oplus \omega Cl(6)$ which is used to obtain one generation of standard model chiral quarks and leptons [4].

The trinification provides the following interpretation of the branching of the two E_8, as discussed in detail in [3]:

$$\begin{aligned} E_{8L} &\to SU(3)_L^{geom} \times E_{6L} \xrightarrow{E_{6L}} SU(3)_c \times SU(3)_{F,L} \times SU(3)_L \xrightarrow{SU(3)_L} SU(2)_L \times U(1)_Y \\ E_{8R} &\to SU(3)_R^{geom} \times E_{6R} \xrightarrow{E_{6R}} SU(3)_{c'} \times SU(3)_{F,R} \times SU(3)_R \xrightarrow{SU(3)_R} SU(2)_R \times U(1)_{Ydem} \end{aligned} \tag{1}$$

Of the three $SU(3)$s arising from the branching of E_{6L}, the $SU(3)_c$ implements the color gauge symmetry of QCD. Furthermore, $SU(3)_{F,L}$ is the non-gauged global flavor symmetry which is responsible for three left-handed fermion generations [3] described by the exceptional Jordan algebra $J_3(\mathbb{O}_c)$. $SU(3)_L$ branches as $SU(2)_L \times U(1)_Y$ giving rise to the electroweak sector, and the $SU(2)_L$ acts only on left-handed particles.

As regards the branching of the second E_6, which we label E_{6R}, the $SU(3)_{c'}$ symmetry is preferably not gauged (so as to be consistent with phenomenology) - it is global and explicitly broken at the electroweak scale. Its role is to rotate the so-called Jordan frame which arises in the Peirce decomposition of the exceptional Jordan algebra matrices [3]. The $\omega SU(3)_{F,R}$ is the global flavor symmetry which describes three generations of right-handed fermions. The $SU(3)_R$ branches as $SU(2)_R \times U(1)_{Ydem}$.

Here, the spontaneously broken $SU(2)_R$ gives rise to general relativity, and the unbroken $U(1)_{dem}$ arising from $U(1)_{Ydem} \to U(1)_{dem}$ is a new force, dubbed dark electromagnetism. It is sourced by the square-root of mass and can be made cosmologically and phenomenologically safe. The emergence of general relativity in this manner, and the preceding gravi-weak unification, was briefly discussed in [1] and is analysed in detail in a forthcoming publication [5].

The focus of the present article is the two extra $SU(3)$s which we label $SU(3)^{geom}_L$ and $SU(3)^{geom}_R$, which arise in the decomposition of the two E_8, and which are apart from the six $SU(3)s$ arising in the trinification of the two E_6. As was already proposed by us in [1], the role of $SU(3)^{geom}_L$ and $\omega SU(3)^{geom}_R$ is to describe spacetime and internal symmetry space. The split complex number is crucial in enabling a Lorentzian signature to emerge from combination of two compact groups, as we rigorously demonstrate below. The split bioctonions also play an important role here: their split quaternionic subalgebra here gives rise to a 6D spacetime with signature (3,3), which upon electroweak symmetry breaking gives rise to two overlapping 4D spacetimes with signature (3,1) and (1,3) respectively. One of these is our familiar 4D spacetime, curved by gravitation. The other 4D spacetime has flipped signature and a (1,1) intersection with our spacetime - its directions are interpreted as being curved by the weak force [5]. The complementary remaining quaternionic subalgebras describe internal symmetry space for holding the unbroken symmetries $SU(3)_c$ and $SU(3)_{c'}$. A decomposition of the adjoint **8** of $SU(3)^{geom}_{L,R}$ shows an elegant mapping to the octonions as described below.

Thus the goal of the present short note is to rigorously demonstrate how $SU(3)^{geom}_L$ and $\omega SU(3)^{geom}_R$ provide the scaffolding on which the fields contained in $E_{6L} \times E_{6R}$ live. This construction realises the all-encompassing role of $E_8 \times \omega E_8$, making the unified symmetry a source of space-time as well as of matter. It is in this spirit that we call the primitive entities possessing $E_8 \times \omega E_8$ symmetry 'atoms of space-time-matter' [2].

2. Roadmap

Our mass-ratio programme [3] places matter in $E_{6L} \times E_{6R}$ and uses the exceptional Jordan algebra $J_3(\mathbb{O}_\mathbb{C})$ to derive charged-fermion square-root mass ratios and mixings. The present paper supplies the geometric scaffolding promised [see Sec. XII.H of [3]] in the uplift to $E_8 \times \omega E_8$: the *two extra* SU(3)'s are taken as *geometric* structure groups that yield precisely a $(3,3)$ base, two 4D spacetimes, and two real 4D internal fibres. The $E_6^{L,R}$ fields then live on this scaffold. A complementary 6D gravi–weak reduction shows how two 4D leaves arise dynamically from a 6D BF theory [5].

What is new here.

(i) A detailed octonionic identification of the 4D fibres with the realification of $\mathbf{2} \oplus \bar{\mathbf{2}}$, using $\mathbb{O} = \mathbb{H} \oplus \mathbb{H}\varepsilon$ and an intrinsic complex structure on $\mathbb{H}\varepsilon$. (ii) A clean role for each extra $U(1)$ as the Spin^c line on the $\mathbb{C}P^2$ fibre. (iii) A unified presentation tying the split-bioctonion base, the E_8 branching, and the 6D gravi–weak localisation.

Plan.

Secs. 3–4 build M_6 from split bioctonions and embed the two 4D spacetimes. Secs. 5–6 relate the $\mathrm{SU}(3)^{geom}_{L,R}$ branching to $\Im\mathbb{H}$ and $\mathbb{H}\varepsilon$ and identify the 4D fibres with $T\mathbb{C}P^2$. Sec. 7 clarifies the two $U(1)$'s. Section 8 explains how the $E_{6L} \times E_{6R}$ fields live on the proposed scaffold, and Section 9 gives the big picture and interpretation.

3. Split-Bioctonionic Base (M_6, g)

Definition 1 (Split bioctonions $\mathbb{O} \oplus \omega\mathbb{O}$ [4] and metric)**.** *Let $\mathbb{C}_s$ denote split-complex numbers with generator $\omega^2 = +1$ and let $\mathbb{O}$ be the octonions. Choose quaternionic subalgebras $\mathbb{H}_L \subset \mathbb{O}$ and $\mathbb{H}_R \subset \mathbb{O}$. Define*

$$M_6 := \Im\mathbb{H}_L \oplus \omega\, \Im\mathbb{H}_R, \qquad g(x_L \oplus \omega x_R, y_L \oplus \omega y_R) := \langle x_L, y_L\rangle - \langle x_R, y_R\rangle, \tag{2}$$

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with $\langle\cdot,\cdot\rangle$ *the Euclidean inner product induced by the octonion norm.*

Proposition 1 (Signature and isometries). (M_6, g) *has signature* $(3,3)$*. The action of unit quaternions by conjugation on each* $\Im\mathbb{H}$ *yields an isometry action of* $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$*, which sits inside the maximal compact* $\mathrm{SO}(4)$ *of* $\mathrm{Spin}(3,3) \cong \mathrm{SL}(4,\mathbb{R})$.

4. Two Embedded 4D Spacetimes

Pick unit vectors $t_L \in \Im\mathbb{H}_L$ and $t_R \in \Im\mathbb{H}_R$. Define

$$\mathcal{M}_4^{(R)} := \Im\mathbb{H}_R \oplus \mathrm{span}\{t_L\} \quad (\text{signature } (3,1)), \tag{3}$$

$$\mathcal{M}_4^{(L)} := \Im\mathbb{H}_L \oplus \mathrm{span}\{\omega t_R\} \quad (\text{signature } (1,3)). \tag{4}$$

These two Lorentzian 4-planes intersect in a neutral $(1,1)$ 2-plane $\mathrm{span}\{t_L, \omega t_R\}$. This is the kinematic version of the two-leaf picture; a dynamical realisation from a 6D BF theory is given in [5].

5. The Two Extra SU(3)'s and Their Branching

On each side of $E_8 \times \omega E_8$ one has the maximal chain

$$E_8 \supset E_6 \times \mathrm{SU}(3), \qquad \mathbf{248} = (\mathbf{78},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8}) \oplus (\mathbf{27},\mathbf{3}) \oplus (\overline{\mathbf{27}},\overline{\mathbf{3}}). \tag{5}$$

We use the two extra SU(3)'s as *geometry*. Pick the standard embedding $\mathrm{SU}(2) \subset \mathrm{SU}(3)$ and take the complementary $U(1)$ generator proportional to $\mathrm{diag}(1,1,-2)$. With the convenient normalisation where the doublet has unit charge, the adjoint branches as

$$\mathbf{8} \longrightarrow \mathbf{3}_0 \oplus \mathbf{2}_{+1} \oplus \overline{\mathbf{2}}_{-1} \oplus \mathbf{1}_0. \tag{6}$$

The $\mathbf{3}_0$ from $SU(3)_L^{geom}$ will supply the three directions of $\Im\mathbb{H}_L$ and the $\mathbf{3}_0$ from $SU(3)_R^{geom}$ will supply the three directions of $\Im\mathbb{H}_R$; together forming M_6. The $\mathbf{2} \oplus \bar{\mathbf{2}}$ will furnish a *real* 4D internal fibre, one each from the two $SU(3)^{geom}$.

6. Octonionic Realisation: $\mathbb{O} = \mathbb{H} \oplus \mathbb{H}\varepsilon$ and the 4D Fibre

Fix a quaternionic subalgebra $\mathbb{H} = \langle 1, u, v, uv\rangle \subset \mathbb{O}$ and choose $\varepsilon \in \Im\mathbb{O}$ orthogonal to $\mathbb{H}$ with $\varepsilon^2 = -1$. Then

$$\mathbb{O} = \mathbb{H} \oplus \mathbb{H}\,\varepsilon, \qquad \Im\mathbb{O} = \Im\mathbb{H} \oplus \mathbb{H}\varepsilon. \tag{7}$$

Define on the real 4-space $\mathbb{H}\varepsilon$ the complex structure J by left multiplication with a fixed unit $u \in \Im\mathbb{H}$:

$$J(a\,\varepsilon) := (u\,a)\,\varepsilon, \qquad J^2 = -\mathrm{Id}_{\mathbb{H}\varepsilon}. \tag{8}$$

Let SU(2) act on $\mathbb{H}$ by unit-quaternion conjugation and trivially on ε; let $U(1)$ act as phases $e^{\theta J}$ on $(\mathbb{H}\varepsilon, J)$. Then:

Proposition 2 (Identifications). *(a)* $\Im\mathbb{H} \cong \mathrm{Adj}\,\mathrm{SU}(2) \cong \mathbf{3}_0$ *as real representations.*
(b) $(\mathbb{H}\varepsilon, J) \cong \mathbf{2}_{+1}$ *as a complex* $\mathrm{SU}(2) \times U(1)$ *representation. Forgetting the complex structure, the underlying real representation is*

$$\mathbb{H}\varepsilon \cong (\mathbf{2}_{+1} \oplus \overline{\mathbf{2}}_{-1})_{\mathbb{R}}, \tag{9}$$

a real 4*-vector space. This is the internal fibre* F_4.

Thus the octonionic split realises the branching (6) concretely: $\Im\mathbb{H}$ gives the 3's for spacetime directions; $\mathbb{H}\varepsilon$ gives the 4 internal directions.

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6.1. Relation to $\mathbb{C}P^2$ and Kaluza–Klein Intuition

At a point $[n] \in \mathbb{C}P^2 = \mathrm{SU}(3)/S(U(2) \times U(1))$ one has

$$T_{[n]}\mathbb{C}P^2 \cong \mathrm{Hom}(\mathbb{C}n,\ \mathbb{C}^3/\mathbb{C}n) \cong \mathbb{C}^2\ \cong\ \mathbf{2}_{+1}, \tag{10}$$

so the *real* tangent is 4-dimensional. The octonionic identification above gives a canonical isomorphism

$$F_4\ \simeq\ T\mathbb{C}P^2 \quad \text{(real rank 4 at each point)}. \tag{11}$$

This matches the minimal Kaluza–Klein choice for an SU(3) internal, explaining why four internal real directions are "right" for a QCD-like sector at the level of geometry.

7. The Two $U(1)$'s

The $U(1)$ in (6) is the isotropy phase in $S(U(2) \times U(1))$. On $(\mathbb{H}\varepsilon, J)$ it acts by $e^{\theta J}$. For each extra SU(3) we therefore obtain a natural *line bundle* whose connection is the Spin^c twist needed on $\mathbb{C}P^2$ (which is non-spin). We *do not* identify the octonion real line $\mathbb{R} \cdot 1 \subset \mathbb{O}$ with this $U(1)$; rather, the $U(1)$ from $SU(3)^{geom}_{L,R} \to SU(2) \times U(1)$ is a Lie-algebra direction acting on the $\mathbb{C}^2$ tangent. It is the Spin^c line on $T\mathbb{C}P^2$. In model-building one may consider mixing these geometric $U(1)$'s with abelian factors inside E_6, but the Spin^c role is canonical and model-independent.

8. How the $E_6^L \times E_6^R$ Fields Sit on the Scaffold

We treat the two extra SU(3)'s as *structure* only. Over M_6 take principal bundles for E_6^L and E_6^R; matter sits in associated **27** vector bundles and gauge fields in adjoint bundles. Tangent/internal decomposition uses $TM_6 = (\Im\mathbb{H}_L) \oplus (\Im\mathbb{H}_R)$ and the two fibres $F_4^{L,R} \simeq \mathbb{H}_{L,R}\varepsilon_{L,R}$ from above. The visible interactions come from $E_6^{L,R}$; the extra SU(3)'s supply geometry and Spin^c lines on the internal fibres.

9. Big Picture and Interpretation

Three layers.

1. **Geometric $\mathbf{SU(3)^{geom}_{L,R}}$** from $E_8 \supset E_6 \times \mathrm{SU}(3)$ on each side: not gauged. Purpose: carve the base and fibre geometry. After $\mathrm{SU}(3) \to \mathrm{SU}(2) \times U(1)$,

$$TM_6\ \cong\ (\Im\mathbb{H}_L)\ \oplus\ (\Im\mathbb{H}_R), \qquad F_4^X\ \cong\ (\mathbf{2}_{+1} \oplus \bar{\mathbf{2}}_{-1})_{\mathbb{R}}\ \simeq\ T\mathbb{C}P^2,\ X = L, R,$$

   with $\Im\mathbb{H}$ realizing $\mathbf{3}_0$ and $\mathbb{H}\varepsilon$ realizing $(\mathbf{2} \oplus \bar{\mathbf{2}})_{\mathbb{R}}$. The two $U(1)$'s act as Spin^c line connections on the $\mathbb{C}P^2$ fibres.
2. **Gauge $\mathbf{SU(3)}$'s** inside each E_6 (trinification): these are dynamical. On the left: $E_6^L \to \mathrm{SU}(3)_c \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_{F,L}$ with $\mathrm{SU}(3)_L \to \mathrm{SU}(2)_L \times U(1)_Y$. On the right: $E_6^R \to \mathrm{SU}(3)_{c'} \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_{F,R}$ with $\mathrm{SU}(3)_R \to \mathrm{SU}(2)_R \times U(1)_{\mathrm{Ydem}}$.
3. **Localization and Lorentz breaking** in 6D: two Higgs order parameters define localized 4D leaves $\Sigma_R, \Sigma_L \subset M_6$ via a covariant two-form density ρ_X. Normal 2-frames U_X implement $SO(3,3) \to SO(3,1)$ on Σ_R and $SO(3,3) \to SO(1,3)$ on Σ_L, eating 9 Lorentz coset modes per leaf and leaving the tangent spin connections massless [5].

Where Do the Unbroken Gauge Groups Live?

Because every sector action is wedged with ρ_X, dynamics is *localized* on the leaves. Hence unbroken $\mathrm{SU}(3)_c$ (and $\mathrm{SU}(3)_{c'}$ if retained) are 4D gauge symmetries on the relevant Σ's. The 6D base M_6 persists as the ambient bundle base, but low-energy fields do not propagate in the bulk.

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What are the fibres relative to spacetime?

$F_4^{L,R}$ are rank-4 *internal* vector bundles over M_6, canonically $F_4^X \simeq \mathbb{H}_X \varepsilon_X \cong (\mathbf{2}_{+1} \oplus \overline{\mathbf{2}}_{-1})_{\mathbb{R}} \simeq T\mathbb{C}P^2$. They are not extra spacetime directions. Restriction to a leaf gives internal fibres $F_4^X|_{\Sigma_X}$ on which internal interactions act. The two 4D spacetimes come from the six tangent directions $(\Im\mathbb{H}_L) \oplus (\Im\mathbb{H}_R)$ plus one opposite-side normal on each leaf.

Two Consistent Options for $\mathbf{SU(3)}_{c'}$.

- *Decoupled/hidden:* break or confine $\mathrm{SU}(3)_{c'}$ above the localization scale; only visible $\mathrm{SU}(3)_c$ remains on Σ_R.
- *Gauged on a leaf:* keep $\mathrm{SU}(3)_{c'}$ dynamical on one leaf (typically Σ_R or Σ_L). Portal terms can live on $X_3 = \Sigma_R \cap \Sigma_L$ under the BF matching conditions.

Dictionary (one line).

$$\boxed{\text{structural } \mathrm{SU}(3)^{\mathrm{geom}}_{L,R} \Rightarrow (M_6, F_4^{L,R}) \quad \text{vs} \quad \text{gauge } \mathrm{SU}(3) \subset E_6^{L,R} \Rightarrow \text{4D forces on } \Sigma_{L,R}\ .}$$

10. Summary

- The $(3,3)$ base M_6 is $\Im\mathbb{H}_L \oplus \omega\, \Im\mathbb{H}_R$; the two 4D spacetimes are 4-planes obtained by adding a single normal from the opposite side.
- The extra $\mathrm{SU}(3)$'s branch as $\mathbf{8} \to \mathbf{3}_0 \oplus \mathbf{2}_{+1} \oplus \overline{\mathbf{2}}_{-1} \oplus \mathbf{1}_0$.
- $\Im\mathbb{H}$ realises $\mathbf{3}_0$; $\mathbb{H}\varepsilon$ realises $(\mathbf{2} \oplus \bar{\mathbf{2}})_{\mathbb{R}}$ and is the 4D fibre $F_4 \simeq T\mathbb{C}P^2$.
- Each geometric $U(1)$ is the Spin^c line on $\mathbb{C}P^2$; we do not confuse it with the octonion real line.

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The author gratefully acknowledges the support received from Open AI's ChatGPT-5 Pro in the analysis described in this article. In particular, the physical ideas conceived by the author were input to the AI as one prompt, and a mathematical formulation of the said ideas was requested. The highly satisfactory and original response from Pro is shown in the Appendix, alongside the prompt. Pro also assisted in the writing and preparation of this manuscript. All the results presented in the article above have been worked out, verified and confirmed by the author, and the author takes sole responsibility for their correctness. I thank Jose Isidro and P Samuel Wesley for useful discussions.

Appendix A. Appendix : Role of AI in the Present Analysis

The figures below show the single prompt, and the detailed reply from ChatGPT-5 Pro which helped in an important way in this work.

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pls see the discussion in Section XII.H of our mass ratios paper. We discuss there the uplift from E_6 to E_8. The uplift gives us additional SU(3) x \omega SU(3), apart from the earlier E_6 x \omega E_6. We want to firstly associate the additional SU(3) x \omega E_6 with split bioctonions. I suppose that can be done using the adjoint 8D rep of SU(3)? Next, consider that each of these two additional SU(3) branch as SU(2) x U(1), giving us an SU(2) x \omega SU(2) pair, and a U(1) x \omega U(1) pair. Right now I don't know what to make of these two U(1)s so I will set them aside for now. I want to use the two SU(2)s and map them, one each, to the three imaginary directions of a quaternion..giving me a 6D entity from which I want to generate a 6D spacetime with signature (3,3) and in this 6D spacetime I want to embed two overlapping 4D spacetimes. Can this be done? Also, when each of these two SU(3) branches this way, can each one of them leave an internal 4D purely imaginary fibre space on which internal forces lie [such as QCD)? My big picture is that these two additional SU(3) coming from E_8 ---> SU(3) x E_6 should provide the scaffolding [spacetime plus internal space] on which the fields provided by E_6 x E_6 lie. The leptons and the electroweak and gravi-dem sector lie in 6D spacetime, and the internal symmetries lie on the internal imaginary octonion space left behind after segregation of spacetime. Please see if you can rigorously implement this big picture for the additional two SU(3)s coming from E_8 x E_8.

Figure A1. Prompt to ChatGPT-5 Pro

Yes. Here is a concrete construction that realizes your picture with the two extra $SU(3)$'s from the $E_8 \times E_8$ uplift.

1) Uplift and role of the extra $SU(3)$'s.
Use the standard maximal chain on each side

$$E_8 \supset E_6 \times SU(3), \qquad 248 = (78,1) \oplus (1,8) \oplus (27,3) \oplus (\overline{27},\overline{3}).$$

At the product level:

$$E_8^L \times E_8^R \;\to\; (E_6^L \times SU(3)_L) \times (E_6^R \times SU(3)_R).$$

These two additional $SU(3)$'s are taken as **structure groups** for a 6D sector that scaffolds the $E_6 \times E_6$ fields. This is exactly the setup sketched in Sec. XII.H (and the magic-star rationale in XII.I). Archive (14) Archive (14)

Figure A2. Response from ChatGPT-5 Pro

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2) Split-bioctonion model of the 6D base.
Let $\mathbb{B}_s := \mathbb{C}_s \otimes \mathbb{O}$ with $\omega^2 = +1$ (split complex). Choose two quaternionic subalgebras $\mathbb{H}_L, \mathbb{H}_R \subset \mathbb{O}$. Define the 6D real vector space

$$M_6 := \operatorname{Im}\mathbb{H}_L \;\oplus\; \omega\, \operatorname{Im}\mathbb{H}_R$$

with metric

$$g(x_L \oplus \omega x_R,\; y_L \oplus \omega y_R) = \langle x_L, y_L\rangle \;-\; \langle x_R, y_R\rangle,$$

so M_6 has signature $(3,3)$. The split tag ω implements the sign flip for the right copy, matching the "$\omega SU(3)$" language and the p(3,3) sector envisaged in XII.H. Archive (14)

3) Branch $SU(3) \to SU(2) \times U(1)$ and obtain the 6D "quaternionic" directions.
For each extra $SU(3)$, fix an $SU(2) \times U(1) \subset SU(3)$. Use the **adjoint triplet** of each $SU(2)$ (not the doublet) and identify it with $\operatorname{Im}\mathbb{H} \simeq \mathbb{R}^3$ (unit quaternions act by conjugation). Then

$$\underbrace{\operatorname{Im}\mathbb{H}_L}_{\text{from } SU(2)_L} \;\oplus\; \underbrace{\operatorname{Im}\mathbb{H}_R}_{\text{from } SU(2)_R}$$

provides the $3+3$ real directions of M_6. The two $SU(2)$'s embed as the maximal compact $SU(2) \times SU(2) \subset \mathrm{Spin}(3,3)$, so they act isometrically on (M_6, g). (The $U(1)$'s are kept in reserve as you suggested.)

Figure A3. Response (contd. from above)

4) Two overlapping 4D spacetimes inside $(3,3)$.
Pick unit vectors $t_R \in \operatorname{Im}\mathbb{H}_R$ and $t_L \in \operatorname{Im}\mathbb{H}_L$. Define two Lorentzian 4-planes

$$\mathcal{M}_4^{(L)} := \operatorname{Im}\mathbb{H}_L \;\oplus\; \operatorname{span}\{\omega t_R\} \quad (\text{signature } (3,1)),$$

$$\mathcal{M}_4^{(R)} := \operatorname{Im}\mathbb{H}_R \;\oplus\; \operatorname{span}\{t_L\} \quad (\text{signature } (1,3)).$$

They overlap on $\operatorname{span}\{t_L, \omega t_R\}$ (signature $(1,1)$). This realizes two embedded, overlapping 4D spacetimes inside the 6D split sector envisaged in XII.H. Ghost control is handled by restricting dynamics to these slices, as flagged there. Archive (14)

5) Internal 4D fibres per extra $SU(3)$.
Under $SU(3) \to SU(2) \times U(1)$, the adjoint 8 decomposes as

$$8 \;\to\; 3_0 \;\oplus\; 2_{+3} \;\oplus\; \overline{2}_{-3} \;\oplus\; 1_0.$$

Interpret:

- 3_0: the three **spacetime** directions $\operatorname{Im}\mathbb{H}$ picked above.
- $2 \oplus \overline{2}$: a **real 4D internal fibre** F_4 attached to that $SU(3)$.
  Thus each extra $SU(3)$ leaves a canonical 4D fibre F_4^L and F_4^R (the "purely imaginary" complement to $\operatorname{Im}\mathbb{H}_{L/R}$ inside the realified fundamental), on which internal forces can live. This cleanly matches your request that QCD-like internal symmetries ide on the internal imaginary space after segregating spacetime

Figure A4. Response (contd. from above)

6) Place the $E_6 \times E_6$ fields on this scaffolding.

- Base: M_6 with two Lorentzian 4D slices $\mathcal{M}_4^{(L)}, \mathcal{M}_4^{(R)}$.
- Structure bundles: principal $SU(3)_L \times SU(3)_R$ over M_6.
- Associated internal bundles: F_4^L and F_4^R from the $2 \oplus \overline{2}$ of each $SU(3)$.
- Gauge–matter: the $E_6^L \times E_6^R$ connections and 27-plets live as fields on M_6, with components along the two 4D slices and along the internal fibres. This is precisely the "extra $SU(3)$ supplies the scaffolding, $E_6 \times E_6$ supplies the fields" picture in XII.H/I. Archive (14) Archive (14)

7) Real-form choice and ω bookkeeping.
Pick real forms so the left structure group is compact $SU(3)_L$ and the right copy is marked by the split tag ω to implement the sign flip (equivalently choose $SU(3)_R$ acting on the negative 3-plane). This mirrors the ω prescription and p(3,3) sector described in XII.H, and isolates the open items (real-form choice, chirality projection, ghost-free slicing) that were listed there. Archive (14)

Figure A5. Response (contd. from above)

8) Summary of mappings.

- $E_8^{L,R} \to E_6^{L,R} \times SU(3)_{L,R}$ (structure + fields). Archive (14)
- $SU(3)_{L,R} \to SU(2)_{L,R} \times U(1)_{L,R}$.
- $\mathrm{Adj}\ SU(2)_{L,R} \cong \mathrm{Im}\,\mathbb{H}_{L,R}$ gives $3+3$ real directions $\Rightarrow M_6$ with $(3,3)$.
- Complement $2 \oplus \overline{2}$ in each $SU(3)$ gives an internal real $4D$ fibre F_4^L, F_4^R.
- Two embedded 4D spacetimes: $\mathcal{M}_4^{(L)}$ and $\mathcal{M}_4^{(R)} \subset M_6$.
- $E_6 \times E_6$ fields propagate on these slices and fibres (your leptons/EW/gravi-dem in 6D; internal symmetries on the fibres). Archive (14)

9) Consistency checks / open knobs.

- Matches the magic-star motivation for the $E_8 \to SU(3) \times E_6$ chain. Archive (14)
- The three-generation story and $J_3(\mathbb{O}_{\mathbb{C}})$ dynamics remain intact; the uplift only adds the geometric sector. Archive (14)
- Items to fix (all already noted in XII.H): precise real forms, the chirality projection removing mirrors, and ensuring ghosts are projected out by restricting dynamics to the 4D slices. Archive (14)

This implements the two additional $SU(3)$'s as the **spacetime + internal-fibre** scaffolding for your $E_6 \times E_6$ fields, with a concrete $(3,3)$ split-bioctonion realization and two overlapping 4D spacetimes as requested. Archive (14)

Figure A6. Response (contd. from above)

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