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An Accurate Empirical Determination of the Mass of the Electron from Properties of the W Boson Vector

Vincenzo Vinciguerra [1,2]

[1] Cultore in Fisica, Medicine and Surgery Department, Kore University of Enna, Cittadella Universitaria, 94100 Enna, Italy; vincenzo.vinciguerra@st.com

[2] Advanced R&D Senior Scientist, GTR&D, STMicroelectronics, Stradale Primosole 50, 95121 Catania, Italy

Abstract

An accurate and empirical relationship between the electron mass and the mass of the W boson vector is proposed. The electron mass, calculated in this way, differs from the experimental value by 25 ppm and can be improved up to about 7.3 ppm; the error arises from the uncertainty of the W boson mass. By calculating the vacuum expectation value of the Wilson loop (rectangle $R \times T$) for a $U(1)$ gauge field, it becomes necessary to compare the Bohr energy of a suitably chosen particle-antiparticle pair with the time scale determined by the electron mass, possibly corrected for the value of the electron's gyromagnetic ratio. It is demonstrated that by appropriately choosing an off-shell W^+W^- pair, a Bohr energy can be determined that agrees with the mass value of the electron-positron pair. From these considerations, a dependence on the square of the fine-structure constant α, the mass of the W boson, and the cosine of an angle compatible with the Weinberg angle is deduced. Once appropriately validated, the theory allows for the deduction of the electron's coupling constant with the Higgs field and reduces the number of free parameters in the Standard Model.

Keywords: electron; Wilson loop; magnetic flux quantization; charge; mass; electroweak theory; Bohr's energy; W boson vector; standard model

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1. Introduction

Of all the elementary particles [1] investigated in the standard model (SM) [2–5], electrons have been definitely central in the development of the semiconductor industry being their properties, either classical or quantal, widely exploited in the whole set of integrated circuits made of downscaled solid-state devices [6–8].

For technologists and experts involved in the investigation of matter, electrons are fundamental particles upon which devising explanations regarding atoms, molecules, condensed matter and general observed phenomena. Indeed, thanks also to the rise and reliability of computational methods tested over the years and established on fundamental principles, scientists can implement today a set of theoretical frameworks to further understand the structure of molecules and materials [9,10], also based on the density functional theory (DFT) [11–14]. Their use has had an impact on our understanding of quantum effects in solid state thin films (e.g. in the quantum integer and fractional Hall effect) [15] and also in the development of new device technologies [16].

To complete the picture, the physical foundations of the methods illustrated above rely upon the theory of quantum electrodynamics (QED) [17], and its further developments within the electroweak theory and quantum field theory (QFT) [18–21].

However, after more than a century from the discovery [22] of the electron as a fundamental building block of matter, the electron is still perceived mostly as an enigmatic particle in its essence [23]. Moreover, it is emblematic that Weinberg last paper was devoted in reviving the interest on the explanation of the mass patterns of quarks and leptons [24], and that, one of his first paper, a fundamental paper for the foundation of the standard model, was titled a "Model of leptons" [25]. However, despite the title, no hint on how to determine the mass of the electron was reported. Similar attempts to determine the electron mass have been pursued in the past [26,27] though with limited success. As a fact the determination of the mass of the electron is still an open problem. This paper seeks to solve this issue.

The paper is organized as follows. First, we derive and demonstrate a relationship existing between the U(1) Wilson loop of a particle-antiparticle pair and the electron mass. Then we search for an opportunely chosen particle-antiparticle pair that can fit the picture and by trial and errors (heuristically) we propose an empirical relationship between the mass of the W boson vector and the electron mass. Going further in the paper, we check the solution against the measurements of the mass of the W boson accomplished in recent years. Then we considered an extension to the case of heavier leptons, such as muons. Finally, a discussion and the conclusions are reported.

2. On the Relationship Between the U(1) Wilson Loop of a Particle-Antiparticle Pair and the Mass of the Electron

Whenever a particle-antiparticle pair (e.g. a quark-antiquark, an electron-positron pair or a W^+W^- pair) is created at a distance R, let to evolve for a time T and finally annihilated, it is possible to probe their interaction energy, determined by the gauge field of the exchanged bosons (photons for the QED case, gluons for the QCD case), by evaluating the vacuum expectation value (VEV) of the Wilson-loop related to the rectangular space-time path $C = R \times T$.

For the case of a free-photon field, the exponent of the VEV of the Wilson-loop is related to the Coulomb energy of the particle-antiparticle pair as (see Srednicki [20] and Figure 1):

$$\langle 0|W_C|0\rangle = exp\left[-\left(\frac{2\alpha}{\pi a}-\frac{\alpha}{R}\right)T\right] \quad (1)$$

where α is the fine structure constant ($\frac{e^2}{4\pi}$ in natural units), and a is a cut-off length, such that $a \ll R \ll T$.

In the case of the Coulomb potential, because of the virial theorem, the kinetic energy K of the pair, in natural units, is equal to $K = \frac{1}{2}\frac{\alpha}{R}$, whereas the total energy is $K + U = -\frac{1}{2}\frac{\alpha}{R}$. The total energy of the antiparticle-particle pair E_{pair} will be a function of R, that is also quantized according to $E_{pair} = -\frac{1}{2}\frac{\alpha}{n^2 a_0}$, where a_0 is the Bohr radius. Let us, hence, indicate with $E_B = -\frac{1}{2}\frac{\alpha}{a_0}$ the Bohr energy.

If we consider the function $g(T)$ defined as

$$g(T) = exp\left(\frac{1}{2}\frac{2\alpha}{\pi a}T\right)\sum_{n=1}^{\infty}\langle 0|W_C|0\rangle_n^{1/2} = \sum_{n=1}^{\infty} exp\left[-\left(-\frac{1}{2}\frac{\alpha}{n^2 a_0}\right)T\right] \quad (2)$$

and evaluate the limit $\lim_{T\to\infty}\frac{g(T+\Delta T)}{g(T)}$, where ΔT is an opportunely chosen time interval. Being $-\frac{1}{2}\frac{\alpha}{a_0} < -\frac{1}{2}\frac{\alpha}{n^2 a_0}$, it results that

$$\lim_{T\to\infty}\frac{g(T+\Delta T)}{g(T)} = exp[-E_B\Delta T] \quad (3)$$

By choosing the time interval ΔT, see Figure 1, related to the zitterbewegung frequency of the electron, that is $\Delta T = 2\pi\frac{\hbar}{2m_e c^2}$, we can write down, in cgs units

$$\lim_{T\to\infty}\frac{g(T+\Delta T)}{g(T)} = exp\left[-\frac{1}{\hbar c}\left(-\frac{1}{2}\frac{e^2}{a_0}\right)2\pi\frac{\hbar c}{2m_e c^2}\right] = exp\left[-2\pi\left(-\frac{1}{2}\frac{e^2}{a_0}\right)\frac{1}{2m_e c^2}\right] = exp\left[2\pi\frac{-E_B}{2m_e c^2}\right] \quad (4)$$

Moreover, after some manipulations, we can view the previous result as related to the Bohr magneton μ_B of the electron and to the magnetic flux $\Phi(\uparrow)$ determined by an electron having a well-defined spin state [28].

In fact,

$$-2\pi\frac{E_B}{2m_e c^2} = -2\pi\frac{1}{\hbar c}\left(-\frac{1}{2}\frac{e^2}{a_0}\right)\frac{\hbar c}{2m_e c^2} = -2\pi\frac{e}{\hbar c}\left(-\frac{1}{2}\frac{e}{a_0}\right)\frac{\hbar}{2m_e c} = -2\pi\frac{e}{\hbar c}\left(-\frac{1}{2a_0}\right)\frac{\hbar e}{2m_e c}2\pi\frac{e}{\hbar c}\left(\frac{2\pi}{2a_0}\right)\mu_B = 2\pi\frac{e}{\hbar c}\Phi(\uparrow). \quad (5)$$

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Because of the anomalous magnetic moment of the electron, the magnetic moment in the previous expression must be corrected by the electron's gyromagnetic ratio g_s, by substituting μ_B with $\frac{g_s}{2}\mu_B$.

In the event, by doing so and by restoring the real time $\Delta T \rightarrow -i\Delta T$, we can re-express our equation as:

$$\lim_{T\to\infty}\frac{g(T+\Delta T)}{g(T)} = exp[-E_B(-i\Delta T)] = exp\left[-2\pi i\frac{g_s}{2}\frac{-E_B}{2m_ec^2}\right] \tag{6}$$

Hence, if it is possible to determine a particle-antiparticle pair such that $-\frac{g_s}{2}\frac{E_B}{2m_ec^2} = k$, with $k \in \mathbb{N}$, the limit arising from the Wilson's loop will be equal to $\lim_{T\to\infty}\frac{g(T+\Delta T)}{g(T)} = exp[-2\pi ki] = 1$

In conclusion, by calculating the vacuum expectation value (VEV) of the Wilson loop (rectangle $R\times T$) for a $U(1)$ gauge field, it becomes reasonable to compare the Bohr energy of a suitably chosen particle-antiparticle pair with the time scale determined by the electron mass, possibly corrected for the value of the electron's gyromagnetic ratio g_s.

Hence, investigating the properties of the VEV of the Wilson loop in the $U(1)$ gauge field allows us to pose a question about the existence of a particle-antiparticle pair with a Bohr energy comparable to that of the electron-positron system. If such energy exists, the mass of the electron can be calculated as

$$m_ec^2 = -\frac{g_s}{2}\frac{E_B}{2k} \tag{7}$$

with $k \in \mathbb{N}$.

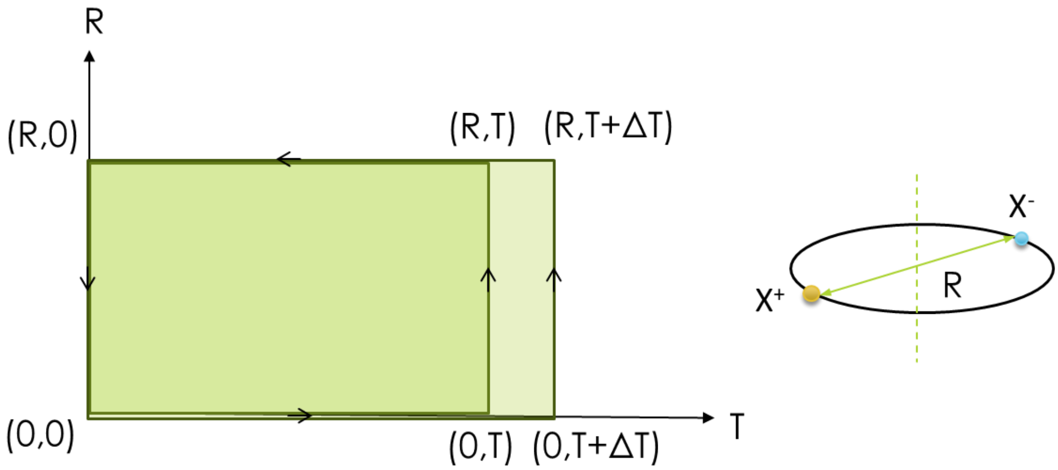

Figure 1. Rectangular contours $R\times T$, $R\times(T+\Delta T)$ exploited for the calculation of the VEV Wilson loop, corresponding to a particle-antiparticle X^+X^- pair, created at a certain distance R, let to survive for a time T, $(T+\Delta T)$ and then annihilated. The interaction energy can be probed from the phase factor resulting from the VEV of Wilson Loop. From the Wilson loop VEV in the $U(1)$ gauge field it is possible to determine the ratio of the Bohr energy of the particle-antiparticle pair with the energy at rest of the electron-positron pair.

3. Formulation of an Energy Scale Comparable to the Energy at Rest of the Electron

In the following, starting from eq. 7, by heuristic trials and errors attempts, we will formulate a Bohr energy which matches the energy of an electron-positron pair.

In the hypothesis that eq. 7 is valid, also the opposite of the Bohr energy will depend on the second power of the fine structure constant α. Indeed, if we indicate with M_X the mass of the unknown particle that intervenes in the particle-antiparticle pair, by considering that the reduced mass of pair will be equal to $\frac{M_X}{2}$, the opposite of the Bohr energy equals $-E_B = \frac{\alpha^2M_X}{4}$, and eq. 7 can be written as:

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$$m_e = \frac{g_s}{2}\frac{\alpha^2 M_X}{4k} \quad (8)$$

Patterns of the values of the masses for the elementary particles involving the fine structure constant were first proposed by Yōichirō Nambu [29]. Similarly, we may observe as in defining the classical radius r_o of the electron (also related to the Thomson elastic scattering), the Compton wavelength of the electron is proportional to the fine structure constant α, whereas the classical radius of the electron can be associated to the Compton wavelength of a more massive particle whose mass reads $\frac{m_e c^2}{\alpha} = \frac{\hbar c}{r_0}$. If we consider the Barut formula [30,31] $m_\mu = m_e + m_e \frac{3}{2\alpha}$ for the mass of a muon, being $\frac{m_e c^2}{\alpha} = \frac{\hbar c}{r_0} = \frac{2}{3}(m_\mu - m_e)c^2$, we can observe as the mass associated to the classical radius of the electron is of the same order of magnitude of the mass of a muon. In such a way we can provide a value of the mass of the electron in terms of that of a muon. Similarly, this has been reported in [27] though with different and more complex considerations. However, usually we express the mass of heavier leptons in terms of the lightest lepton mass, the electron. Its mass is considered as a fundamental constant and used as a reference to compare the values of the masses of heavier particles.

Moreover, the diversification of the masses of fermions, and hence also of leptons, can be explained by the Yukawa mechanism of spontaneous symmetry breaking of the gauge symmetry [18], coupling massless charged leptons with the Higgs field. If this is the case, we should be able to provide an evaluation of the electron mass by considering, in some way, the electroweak forces or the parameters involved in the electroweak theory, such as the VEV of the Higgs field. However, the Yukawa coupling constant is a parameter that cannot be determined in the theory [19] of the Standard Model and is introduced ad hoc from the experimental values of the masses.

On the other hand, if we conjecture a dependence of the mass of the electron on the fine structure constant α that might involve a higher power of α, such as α^2, the α^2 quantity must be multiplied by the mass value of a heavier particle opportunely chosen. We can also expect that the unknown mass M_X is, in a certain way, related to standardized expressions of use within the quantum theory among which we can make a choice. In particular, a mass-energy formula that we can consider stems from the atomic theory. In fact, the equation that relates the Bohr energy of the hydrogen energy levels to the mass of the electron has the same dependence on α^2. This is what is suggested by equation 7, a Bohr energy of an opportunely chosen mass of a particle-antiparticle pair. Having the objective of pursuing the determination of a formula for the mass of the electron within a scheme which includes quantities appearing in the electroweak theory, we have to consider the possibility that such heavier mass might be related to the masses of the vector bosons mediating the weak force. Since electrons and leptons originate in the β decay from W bosons, a W^+W^-pair is a natural candidate to test our conjecture.

Hence, let us consider that the mass M_X required to calculate the opposite value of the Bohr energy $-E_B$ is the energy determined by the electromagnetic interaction of a W^+W^- pair, where the short lifetime of such particles is neglected. By considering for the fine structure constant α the value recommended by the 2022 CODATA value [32] and for the mass of the W boson, the value of $M_X = M_W = 80.3692 \pm 0.0133\, GeV$ measured in 2024 [33], it results for $-E_B = \frac{1}{2}\alpha^2 \frac{M_W c^2}{2}$ a value of $1.07\, MeV$.

This energy value is very close to the energy of an e^+e^-pair, which amounts to $2m_e c^2 = 1.02 MeV$ if both particles are at rest. If this is the case, we can conjecture that eq. 7 is valid, with a value of $k = 2$.

And indeed, we can say that the energy scale of an electron-positron pair is comparable to the opposite of the Bohr energy of a W^+W^- pair: $2m_e c^2 \approx \frac{1}{4}\alpha^2 M_W c^2$.

Moreover, the difference $2\Delta E$ between the Bohr energy E_B and the electron-positron pair mass results of 48.15 keV:

$$2\Delta E = \frac{1}{4}\alpha^2 M_W c^2 - 2m_e c^2 = 48.15\, keV. \quad (9)$$

It is known that there are off-shell values of the mass of the W. Indicating with $\delta M_W = 2.085 \pm 0.042\ GeV$ [34] the decay width of a charged vector boson, assuming that $2\delta M_W$ is the uncertainty of the mass of the W^+W^- pair, it results that:

$$\frac{1}{4}\alpha^2(2\delta M_W)c^2 = 55.5 \pm 1.1\ keV. \quad (10)$$

In the hypothesis that such a difference turns out from an indetermination on the mass of the charged boson vector W, we can wright equation (9) as:

$$\Delta E = \frac{1}{8}\alpha^2 M_W c^2 - m_e c^2 \approx \frac{1}{8}\alpha^2 2\delta M_W c^2. \quad (11)$$

Therefore, the error associated to the mismatch of the two energy scales, the mass of the electron $m_e c^2$ and the corrected Bohr energy of a W^+W^- pair $\frac{1}{8}\alpha^2(M_W - 2\delta M_W)c^2$ is:

$$1 - \frac{\frac{1}{8}\alpha^2 M_W c^2 - \frac{1}{8}\alpha^2 2\delta M_W c^2}{m_e c^2} \approx 0.7\%, \quad (12)$$

and differs only by 0.7 % from the mass at rest of an electron.

It is also possible to go further in our evaluation. Indeed, we can introduce an angle θ such that to consider the deviation of the difference reported in eq. 11 with respect to the quantity calculated in eq. 10, that is:

$$\frac{1}{4}\alpha^2(2\delta M_W)c^2 cos\theta = \frac{1}{4}\alpha^2 M_W c^2 - 2m_e c^2. \quad (13)$$

There are at least two possible argumentations to consider such an angle. First, in the investigation of alternate currents (AC), the power factor $cos\phi$, measures the ratio between the real power and the apparent power. Moreover, today, in machine learning applications, $cos\theta$ is exploited to determine the similarity of two data sets considered as vectors. In our case, the $cos\theta$ can measure how close the energies reported in eq. 9 and eq. 10 are.

It results that a reckoning of $cos\theta$ provides a value of θ which amounts to $\theta = acos\left(\frac{48.15\ keV}{55.5 \pm 1.1\ keV}\right) = (29.8 \pm 2)°$. Such a value is comparable with the Weinberg angle $\theta_W = 28.1°$ [35].

In the hypothesis that the angle θ is the Weinberg angle θ_W, we can gain a value for the energy scale under evaluation, which is even closer to the energy at rest of the electron:

$$m_e c^2 = \frac{1}{8}\alpha^2 M_W c^2 - \frac{1}{8}\alpha^2(2\delta M_W)c^2 cos\theta_W = 0.5106(5)\ MeV\ (0.08\%), \quad (14)$$

and such a value differs only by 0.08% with respect to the mass of the electron. On the other hand, the Bohr energy is an electromagnetic form of energy; by considering eq. (7), we must multiply by half of the spin g-factor $\frac{g_s}{2}$ the second member of eq. 14. By making such a correction, exploiting the value of $g_s = 2.00231930436092(36)$ [36] we can obtain the value of an energy scale which differs by only 0.04% with respect to the mass of the electron:

$$m_e c^2 = \frac{g_s}{2}\left(\frac{1}{8}\alpha^2 M_W c^2 - \frac{1}{8}\alpha^2(2\delta M_W)c^2 cos\theta_W\right) = 0.5112(5)\ MeV\ (0.04\%). \quad (15)$$

4. Testing the Empirical Formula Against the Measured Values of the W-Boson Mass

The empirical formula reported in eq. 15 has been tested against a set of measured values of the mass of the W boson collected during recent years by different collaborations. Such measurements are summarized in the chart of Figure 2.

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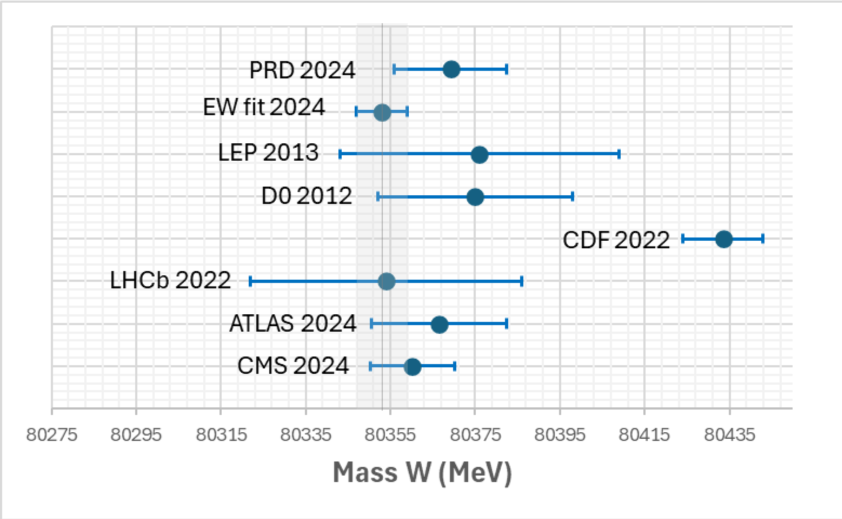

Figure 2. Comparison of the W boson mass measurements with the electroweak standard model predicted value. The values of the mass are the results of the CMS, ATLAS, LHCb, CDF, D0, LEP collaborations, compared with the CODATA 2022 value reported in PRD 2024 [33] and the Electroweak (EW) fit [44], resulting from theoretical calculation pursued within the standard model. PRD 2024 [33], LEP 2013 [43], D0 2012 [42], CDF 2022 [41], LHCb 2022 [40], ATLAS 2024 [39], CMS 2024 [38].

To calculate the mass of the electron with eq. 15, along with the mass of the W boson, we used the whole sets of quantities whose up-to-date measured values are reported in Table 1.

To make a comparison, we exploited the CODATA 2022 value for the mass of the electron [37], which amounts to $m_e c^2$ =0.510 998 950 69(16) MeV. As far as we know, it is the most accurate measurement of the mass of the electron available today.

Table 1. Values of the quantities exploited to calculate the electron mass according to eq. 15.

Quantity	Symbol	Value	Reference
Fine structure constant	α	$0.0072973525643(11)$	[32]
$sin^2(\theta_W)$ of the Weinberg angle θ_W	s_W^2	a) 0.22290(30) b) 0.22305(23)	[35]
W decay width (GeV)	δM_W	$2.085 \pm 0.042\ GeV$	[34]
Electron's gyromagnetic ratio	g_s	$2.00231930436092(36)$	[36]
Electron mass (MeV)	$m_e c^2$	0.510 998 950 69(16)	[37]

In Table 2 we reported, in the third column, the calculated value of the electron mass according to eq. 15 by using the 2018 CODATA [35] value of 0.22290(30) for the square of the sine of the Weinberg angle s_W^2, for the different measured values of the mass of the W. The 1st column reports the measured value of the mass of the W, the 2nd column the reference, the 3rd column the calculated value of the mass of the electron, the 4th column reports the experimental value of the mass of the electron. The last column reports the agreement of the calculated value with respect to the measured value in ppm. It is worth observing as the electroweak fit, that allows the calculation of the mass of the W boson according to the standard model, agrees with the measured value by 25 ppm. Moreover, by exploiting the measured value of the LHCb experiment we have an even better agreement which amounts to 12 ppm.

Table 2. Test of the agreement of the calculated mass of the electron mass according to eq. 15 for different experimental measurements of the W boson mass. In this case we used the value of 0.22290(30) for s_W^2, reported in the 2018 CODATA [35].

Measured mass of the W boson (MeV)	Reference	Mass of the electron (calculated value (MeV) eq. 15)	Mass of the electron (experimental value (MeV))	Agreement in ppm
80360.2 ± 9.9	[38] CMS collaboration https://arxiv.org/pdf/2412.13872	0.51103418077	0.51099895069	68.94
80366.5 ±15.9	[39] ATLAS collaboration arXiv:2403.15085	0.51107616484	0.51099895069	151.10
80354 ± 32	**[40] LHCb JHEP 01 (2022) 036, https://arxiv.org/abs/2109.01113**	**0.51099286311**	**0.51099895069**	**-11.91**
80433.5 ± 9.4	[41] CDF Science 376 (2022) 6589	0.51152266211	0.51099895069	1024.88
80375 ± 23	[42] D0 PRL 108 (2012) 151804	0.51113281002	0.51099895069	261.96
80376 ± 33	[43] LEP combination Phys. Rep. 532 (2013) 11	0.51113947415	0.51099895069	275.00
80353 ± 6	**[44] Electroweak fit PRD 110 (2024) 030001, https://arxiv.org/pdf/2311.08203**	**0.51098619897**	**0.51099895069**	**-24.95**
80369.2 ± 13.3	[34] S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024).	0.51109415801	0.51099895069	186.32

In Figure 3 we report a chart of the calculated value of the mass of the electron according to eq. 15 completed with an analysis of the errors (reported in the Appendix A) obtained by considering the sum of squares of the errors of the quantities involved in eq. 15. The major uncertainties come out from the uncertainty on the value of the decay width δM_W, the uncertainty on the value of the mass of the W boson and the uncertainty on the s_W^2 value, respectively. The uncertainties on the fine structure constant α and on the value of g_s are negligeable and do not contribute much.

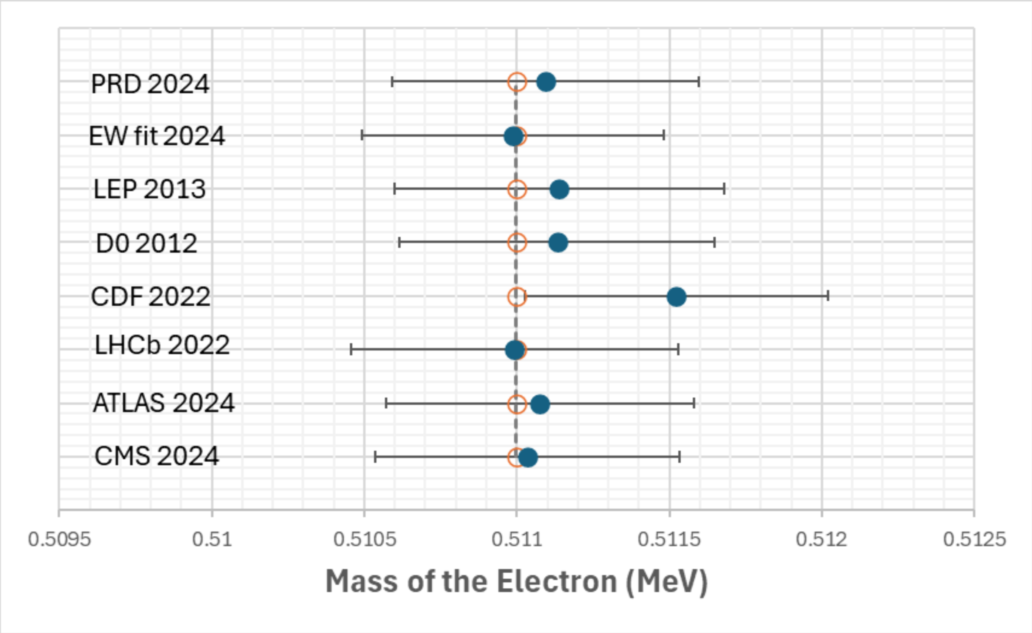

Figure 3. Chart of the test of the agreement of the calculated mass of the electron mass according to eq. 15 for different experimental measurements of the W boson mass. In this case we used a value of 0.22290(30) for s_W^2. The central dashed vertical line with red hollow dots ○ is the experimental value of the mass of the electron.

In Table 3 we repeated the same calculation, by considering the 2022 updated CODATA [32] value of the s_W^2. It is worth observing the further improvements of the agreement with the Electroweak fit, which reduces to about 20 ppm, and with the LHCb measurement which reduced to 7.3 ppm.

Table 3. Test of the agreement of the calculated mass of the electron mass according to eq. 15 according to different experimental measurements of the W boson mass. In this case we used a value of 0.22305(23) for s_W^2, which is the 2022 updated CODATA value [32].

Measured mass of the W boson (MeV)	Reference	Mass of the electron calculated value (MeV)	Mass of the electron experimental value (MeV)	Agreement in ppm
80360.2 ± 9.9	[38] CMS collaboration https://arxiv.org/pdf/2412.13872	0.51103654518	0.51099895069	73.57
80366.5 ±15.9	[39] ATLAS collaboration arXiv:2403.15085	0.51107852926	0.51099895069	155.73
80354 ± 32	**[40] LHCb JHEP 01 (2022) 036, https://arxiv.org/abs/2109.01113**	**0.51099522752**	**0.51099895069**	**-7.29**
80433.5 ± 9.4	[41] CDF Science 376 (2022) 6589	0.51152502653	0.51099895069	1029.50
80375 ± 23	[42] D0 PRL 108 (2012) 151804	0.51113517443	0.51099895069	266.58
80376 ± 33	[43] LEP combination Phys. Rep. 532 (2013) 11	0.51114183857	0.51099895069	279.62
80353 ± 6	**[44] Electroweak fit PRD 110 (2024) 030001, https://arxiv.org/pdf/2311.08203**	**0.51098856339**	**0.51099895069**	**-20.33**

80369.2 ± 13.3	[33,34] S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024).	0.51109652243	0.51099895069	190.94

In Figure 4 we reported the chart of the calculated values of the electron mass according to the updates reported in Table 3, by considering also the propagation of the uncertainties.

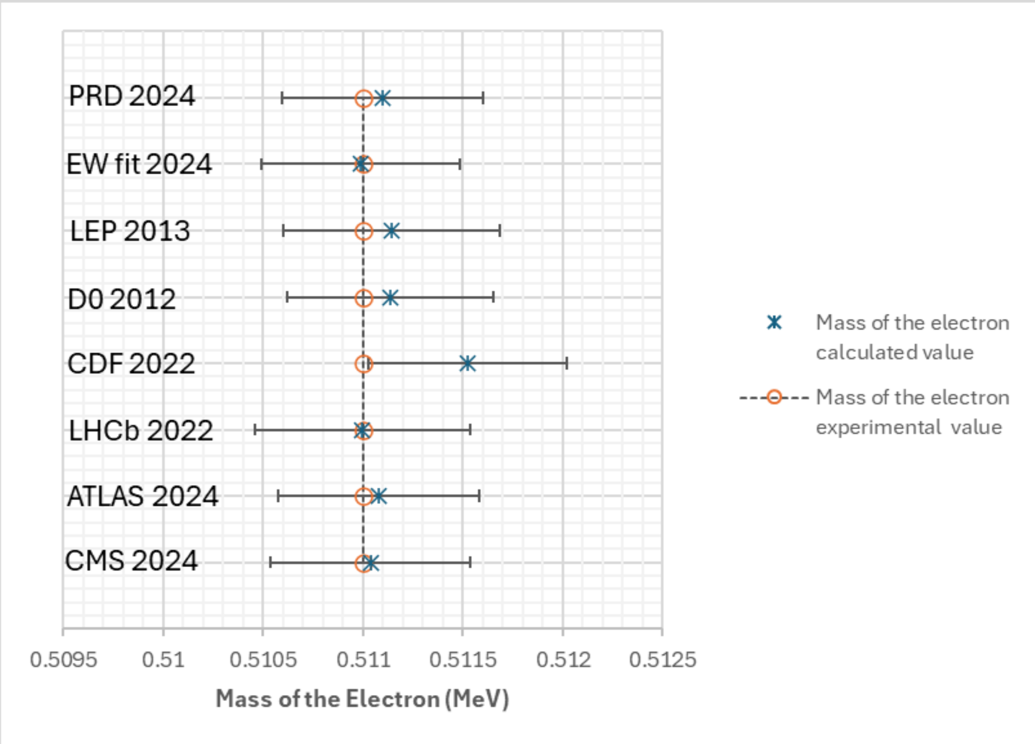

Figure 4. Plot of the Test of the agreement of the calculated mass of the electron mass according to eq. 15 for different experimental measurements of the W boson mass. In this case we used a value of 0.22305(23) for s_W^2. The central dashed vertical line with red hollow dots ○ is the experimental value of the mass of the electron.

Finally, in Table 4, we summarized the results on the calculated values of the mass of the electron according to eq. 15, by considering the significant digits resulting from the analysis of the errors.

Table 4. Calculated masses of the electron as resulting from eq. 15 reported by considering the significant digits.

Mass of the W (Reference)	**Mass of the electron (MeV)** s_W^2 =0.22290(30)	**Error (MeV)**	**Mass of the electron (MeV)** s_W^2 = 0.22305(23)	**Error (MeV)**
[38] CMS collaboration https://arxiv.org/pdf/2412.13872	0.51103	0.00050	0.511037	0.000498
[39] ATLAS collaboration arXiv:2403.15085	0.51108	0.00050	0.511079	0.000505
[40] LHCb JHEP 01 (2022) 036, https://arxiv.org/abs/2109.01113	0.51099	0.00054	0.510995	0.000538
[41] CDF Science 376 (2022) 6589	0.51152	0.00050	0.511525	0.000497
[42] D0 PRL 108 (2012) 151804	0.51113	0.00052	0.511135	0.000517

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[43] LEP combination Phys. Rep. 532 (2013) 11	0.51114	0.00054	0.511142	0.000540
[44] Electroweak fit PRD 110 (2024) 030001, https://arxiv.org/pdf/2311.08203	0.51099	0.00050	0.510989	0.000495
[33,34]S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024).	0.51109	0.00050	0.511097	0.000501

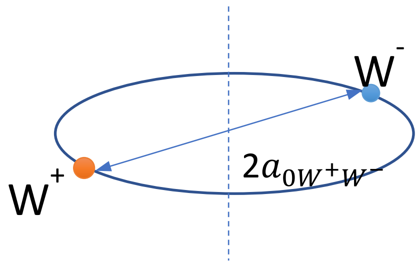
5. Extension to the Case of the Muon

It is worth observing that because of eq, 15, the classical radius of the electron can be expressed as the Bohr radius of the off shell W^+W^- pair system

$$r_0 = \alpha \frac{\hbar}{m_e c} = \frac{e^2}{\frac{g_s}{2}\frac{1}{8}\alpha^2 c^2 (M_W - (2\delta M_W)cos\theta_W)} = a_{0W^+W^-}. \quad (16)$$

As is well known, there is a relationship, involving α, between the classical radius of the electron, the Compton wavelength and the Bohr radius. The classical radius of the electron will depend on α and can be considered as the Compton wavelength of a heavier particle, whose mass is close to that of the muon:

$$r_0 = \alpha \frac{\hbar}{m_e c} = a_{0W^+W^-} = \frac{3}{2}\frac{\hbar}{(m_\mu - m_e)c} \approx \frac{\hbar}{m_\mu c} \; . \quad (17)$$



$$a_{0W^+W^-} = \frac{e^2}{m_e c^2} \approx \frac{\hbar}{m_\mu c}$$

Figure 5. A pair of W vector bosons (W^+W^-) is considered to survive long enough to result in a hydrogen-like system. The Bohr's radius $a_{0W^+W^-}$ of the system is comparable to the classical radius of the electron r_0 and to the Compton's wavelength of the muon.

Moreover, either by considering the Barut theory [30,31] or also the works of Barr and Zee [27] Eq, 15 can be easily extended to determine the mass of muons as a function of the W mass. Indeed, since the mass m_μ of the muon is approximately proportional to α^{-1} the mass of the electron, there will be a linear dependence on α, that is:

$$m_\mu \propto \frac{g_s}{2}\frac{1}{8}\alpha(M_W - 2\delta M_W cos\theta_W) \; .(18)$$

More precisely, according to Barut's formula, $m_\mu = m_e + m_e \frac{3}{2\alpha}$, or also $\frac{m_\mu}{m_e} = 1 + \frac{3}{2\alpha} = \frac{1}{\alpha}\left(\alpha + \frac{3}{2}\right)$. Hence the mass of the muon can be expressed approximately as:

$$m_\mu \approx \left(\alpha^2 + \frac{3}{2}\alpha\right)\frac{g_s}{2}\frac{1}{8}(M_W - 2\delta M_W cos\theta_W) \; (19)$$

Following still Barut, a similar equation for the mass m_τ of the tau particle can be written. Indeed, being $m_\tau \approx m_e\left(1 + \frac{3}{2\alpha}\sum_{k=0}^{2} k^4\right)$, we have that

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$$m_\tau \approx \left(\alpha^2 + \frac{3}{2}\alpha(1+2^4)\right)\frac{g_s}{2}\frac{1}{8}(M_W - 2\delta M_W cos\theta_W) \quad (20)$$

We can also consider the Koide mass formula [45]

$$\mathrm{Q} = \frac{\mathrm{m_e}+\mathrm{m_\mu}+\mathrm{m_\tau}}{\left(\sqrt{\mathrm{m_e}}+\sqrt{\mathrm{m_\mu}}+\sqrt{\mathrm{m_\tau}}\right)^2} = \frac{2}{3} \quad (21)$$

and gain either the mass of the tau lepton as a function of the mass of the mass of the electron and the muon

$$\sqrt{m_\tau} = 2\left(\sqrt{m_e}+\sqrt{m_\mu}\right) + \sqrt{3\left(m_e+m_\mu\right) + 4\cdot 3\sqrt{m_e}\sqrt{m_\mu}} \quad (22)$$

or also calculate the mass of the electron as a function of the masses of the muon and tau particles:

$$\sqrt{m_e} = 2\left(\sqrt{m_\tau}+\sqrt{m_\mu}\right) - \sqrt{3\left(m_\tau+m_\mu\right) + 4\cdot 3\sqrt{m_\tau}\sqrt{m_\mu}} \quad (23)$$

In this case it is worth observing that the resulting mass of the electron will be of 0.51064 MeV.

Going back to the results of previous sections, by considering eq. 7 we may reason in a similar way. Indeed, we can extend eq. 7 to the case of the muon m_μ

$$\frac{g_\mu}{2}\frac{-E_B\frac{m_\mu g_s}{m_e g_\mu}}{2m_\mu c^2} = k \quad (24)$$

In this case $-E_B\frac{m_\mu}{m_e}\frac{g_s}{g_\mu}$ is the opposite value of a higher binding energy that according to Barut model is determined by the contribution of the magnetic energy emerging at closer distances. On the other hand, by exploiting the same reasoning pursued by Barut in his paper [30] because of eq. 15 we can determine a mass associated to the muon according to the Barut formula. If this is the case, eq. 24 suggests correcting the Barut formula for the muon by multiplying the value of the mass by $\frac{g_\mu}{2}$, where g_μ is the gyromagnetic ratio of the muon. By doing so we obtain an increase of the calculated mass of the muon from 105.55 MeV, which is -0.1 % lower with respect to the experimental value 105.6583755(23) MeV) [32] to 105.67 MeV, which is 0.013% higher with respect to the experimental value.

In conclusion, by extending eq, 7, the mass of the muon can be calculated if it is possible to find a ground state energy of a particle-antiparticle pair, with the same energy at rest of the muon-antimuon pair. According to Barut this extra energy comes out from the contribution of a magnetic energy.

6. Conclusions

In conclusion, by calculating the expectation value of the Wilson loop (rectangle RxT) for a U(1) gauge field, it has been shown that the mass of the electron can be calculated as the opposite value of the Bohr energy of a suitably chosen particle-antiparticle pair, corrected by the contribution of the electron's gyromagnetic ratio. We have demonstrated that by appropriately choosing an off-shell W^+W^- pair, the opposite value of a Bohr energy, summarized by the empirical formula of eq.15, that matches the energy at rest of the electron can be determined. The empirical formula obtained has been tested against the experimental measured values of the W mass and agrees with the mass value of an electron-positron pair up to 7.3 ppm. The advantage of such a formulation lies in the use of parameters and quantities that are intrinsic to the electroweak theory.

Is this what the mass of an electron should look like? Does eq. 15 determine the mass of the electron? Despite the close value of the calculated energy scale in eq. 15 to the energy at rest of the electron, a skeptic reader might also consider that the previous heuristic reasoning leads only to a *fortunate* guess of an energy value which is very close to the mass of an electron. On the one hand, we might not exclude that such a conclusion is a pure coincidence arising from the calculation of the Bohr energy of a W^+W^-pair. On the other hand, there is substantial evidence such as:

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1) The Bohr energy of an off-shell W^+W^- pair, which is very close to the energy at rest of an e^+e^-pair,
2) The Bohr radius of the W^+W^- system, which is comparable to the classical radius of the electron.
3) The VEV of the Wilson loop calculated in the RxT rectangle for the U(1) gauge field that allows to establish a relationship between a Bohr energy of a particle-antiparticle pair and the mass of the electron-positron pair.

Therefore, independently of the possible implications, it is certain that eq. 15 provides a Bohr energy, which is the value of an energy scale comparable to the energy scale of an electron at rest formulated by means of the mass M_W of the W particle.

Moreover, this result has been presented following a logical-deductive reasoning. We started from the $U(1)$ gauge field, calculated the VEV of the Wilson loop in a rectangle $R \times T$ contour, and determined the function $g(T)$ such that the calculated limit for $T \to \infty$ assumes well-determined values, if eq. 7 holds. If this is the case, there must be a Bohr energy of a particle-antiparticle pair matching the energy of an electron-positron pair. We have shown that such an energy exists and is related to the Bohr energy of an off-shell W^+W^-pair heuristically determined. Along with the logical-deductive method we can also consider the abductive reasoning [46]. For instance, suppose we find the existence of a well-defined off shell W^+W^-pair whose Bohr energy matches the opposite value of the mass at rest of an electron-positron pair. In this case, we may wonder what the most likely explanation of this agreement with the current theory is. By recurring to the $U(1)$ VEV of the Wilson loop calculated in the $R \times T$ rectangle for a particle-antiparticle system and evaluating the correlation function, we can deduce that, if the opposite value of the Bohr energy of such a system is comparable with an opportunely chosen interval of time linked to the electron dynamics, which is determined by its mass, the correlation function assumes specific values. Finally, we hope that this work can also be exploited to the progress of the mass-gap problem extended to compact Lie groups [47].

Appendix A. Propagation of Uncertainties in Eq. 15

Let us calculate the propagation of uncertainties in equation 15.

$$m_e = \frac{g_s}{2}\left(\frac{1}{8}\alpha^2 M_W - \frac{1}{8}\alpha^2(2\delta M_W)cos\theta_W\right) \quad \text{(A1)}$$

We consider that $cos\theta_W = \sqrt{1 - sin^2\theta_W}$

Let us calculate the derivatives

$$\frac{\partial m_e}{\partial M_W} = \frac{\partial}{\partial M_W}\frac{g_s}{2}\left(\frac{1}{8}\alpha^2 M_W - \frac{1}{8}\alpha^2(2\delta M_W)cos\theta_W\right) = \frac{g_s}{2}\frac{1}{8}\alpha^2 (A2)$$

$$\frac{\partial m_e}{\partial g_s} = \frac{1}{2}\left(\frac{1}{8}\alpha^2 M_W - \frac{1}{8}\alpha^2(2\delta M_W)cos\theta_W\right)(A3)$$

$$\frac{\partial m_e}{\partial \alpha} = \frac{g_s}{2}2\alpha\left(\frac{1}{8}M_W - \frac{1}{8}(2\delta M_W)cos\theta_W\right)(A4)$$

$$\frac{\partial m_e}{\partial sin^2\theta_W} = \frac{1}{8}\frac{g_s}{2}\alpha^2(2\delta M_W)\frac{1}{\sqrt{1-sin^2\theta_W}} \quad \text{(A5)}$$

$$\frac{\partial m_e}{\partial \delta M_W} = -\frac{g_s}{8}\alpha^2\sqrt{1 - sin^2\theta_W} \quad \text{(A6)}$$

The error in the calculated value of the mass of the electron can be calculated as

$$\Delta m_e = \left|\frac{\partial m_e}{\partial M_W}\right|\Delta M_W + \left|\frac{\partial m_e}{\partial g_s}\right|\Delta g_s + \left|\frac{\partial m_e}{\partial \alpha}\right|\Delta\alpha + \left|\frac{\partial m_e}{\partial sin^2\theta_W}\right|\Delta sin^2\theta_W + \left|\frac{\partial m_e}{\partial \delta M_W}\right|\Delta\delta M_W \quad \text{(A7)}$$

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Where ΔM_W, Δg_s, $\Delta\alpha$, $\Delta sin^2\theta_W$, $\Delta\delta M_W$ are the errors on the experimental values of the M_W, g_s, α, $sin^2\theta_W$, δM_W and are reported in Table 1.

The final error on m_e can be also evaluated as the square root of the sum of the squares of each contributing error.

$$\Delta m_e = \sqrt{\left(\left(\left|\frac{\partial m_e}{\partial M_W}\right|\Delta M_W\right)^2 + \left|\frac{\partial m_e}{\partial g_s}\right|\Delta g_s\right)^2 + \left(\left|\frac{\partial m_e}{\partial \alpha}\right|\Delta\alpha\right)^2 + \left(\left|\frac{\partial m_e}{\partial sin^2\theta_W}\right|\Delta sin^2\theta_W\right)^2 + \left(\left|\frac{\partial m_e}{\partial \delta M_W}\right|\Delta\delta M_W\right)^2} \quad (A8)$$

Table A1. Table summarizing the contribution of the uncertainties in eq. 15 in determining the maximum error absolute a priori and the absolute error determined by the sum in quadrature, for the case of 0.22290(30) for s_W^2.

Experiment	**dme/dM_W x delta M_w**	**dme/dalfa deltaAlfa**	**dm_e/dg_s *delta gs**	**dme/dsin^2 x delta_sin2W_1**	**dm_e/ddeltaMw**	**Maximum a priori absolute error (MeV)**	**Absolute error, sum in quadrature (MeV)**
CMS collaboration https://arxiv.org/pdf/2412.13872	6.6E-05	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000569	0.000498
ATLAS collaboration arXiv:2403.15085	0.000106	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000609	0.000505
LHCb JHEP 01 (2022) 036, https://arxiv.org/abs/2109.01113	0.000213	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000716	0.000538
CDF Science 376 (2022) 6589	6.26E-05	1.54E-10	9.2E-14	9.46E-06	0.000493	0.000566	0.000498
D0 PRL 108 (2012) 151804	0.000153	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000656	0.000517
LEP combination Phys. Rep. 532 (2013) 11	0.00022	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000723	0.00054
Electroweak fit PRD 110 (2024) 030001, https://arxiv.org/pdf/2311.08203	4E-05	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000543	0.000495
S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024).	8.86E-05	1.54E-10	9.19E-14	9.46E-06	0.000493	0.000592	0.000501

Table A2. Table summarizing the contribution of the uncertainties in eq. 15 in determining the maximum error absolute a priori and the absolute error determined by the sum in quadrature, for the case of 0.22305(23) for s_W^2.

Experiment	**dme/dM_W x delta M_w**	**dme/dalfa deltaAlfa**	**dm_e/dg_s *delta gs**	**dme/dsin^2 x delta_sin2W_1**	**dm_e/ddeltaMw**	**Maximum a priori absolute error (MeV)**	**Absolute error, sum in quadrature (MeV)**

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CMS collaboration https://arxiv.org/pdf/2412.13872	6.5975E-05	1.54067E-10	9.188E-14	7.25122E-06	0.000493	0.000567	0.000498
ATLAS collaboration arXiv:2403.15085	0.00010596	1.5408E-10	9.18876E-14	7.25122E-06	0.000493	0.000607	0.000505
LHCb JHEP 01 (2022) 036, https://arxiv.org/abs/2109.01113	0.000213252	1.54054E-10	9.18726E-14	7.25122E-06	0.000493	0.000714	0.000538
CDF Science 376 (2022) 6589	6.26429E-05	1.54214E-10	9.19679E-14	7.25122E-06	0.000493	0.000563	0.000497
D0 PRL 108 (2012) 151804	0.000153275	1.54097E-10	9.18978E-14	7.25122E-06	0.000493	0.000654	0.000517
LEP combination Phys. Rep. 532 (2013) 11	0.000219917	1.54099E-10	9.1899E-14	7.25122E-06	0.000493	0.000721	0.000540
Electroweak fit PRD 110 (2024) 030001, https://arxiv.org/pdf/2311.08203	3.99848E-05	1.54052E-10	9.18714E-14	7.25122E-06	0.000493	0.000541	0.000495
S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024).	8.8633E-05	1.54085E-10	9.18908E-14	7.25122E-06	0.000493	0.000589	0.000501

References

1. Nakamura, K. et al. Review of Particle Physics. Journal of Physics G: *Nuclear and Particle Physics*. 3786, 075021-076443 (2010).
2. Gaillard, M. K., Grannis, P. D. & Sciulli, F. J. The standard model of particle physics. *Rev. Mod. Phys.* 71, 96-111 (1999).
3. The deconstructed Standard Model equation 07/28/16 By Rashmi Shivni https://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation?language_content_entity=und
4. Let's have a coffee with the Standard Model of particle physics! Julia Woithe et al 2017 Phys. Educ. 52 034001 https://doi.org/10.1088/1361-6552/aa5b25
5. An Introduction to the Standard Model of Particle Physics W.N. Cottingham (Bristol U.), D.A. Greenwood (Bristol U.) 2007,293 pages, Published: 2007 by Cambridge University Press, DOI: 10.1017/9781009401685
6. Sze, S. M. & Kwok K. N. *Physics of Semiconductor Devices* (3rd ed) (John Wiley & Sons, 2006).
7. M. Rudan, R. Brunetti and S. Reggiani," Springer Handbook of Semiconductor Devices", 2023.
8. Grundmann, M. (2021). The Physics of Semiconductors (4th ed.). Springer
9. Cramer, C. J. *Essentials of Computational Chemistry* 153–189 (John Wiley & Sons, 2002).
10. Pavarini, E., Koch, E., van den Brink, J., Sawatzky, G. (Eds). Quantum materials: experiments and theory. Lecture notes of the autumn school on correlated electrons, https://www.cond-mat.de/events/correl16/manuscripts/correl16.pdf (2016).
11. Kohn, W. Nobel Lecture: Electronic structure of matter - wave functions and density functionals. Rev. Mod. Phys. 71, 1253–1266 (1999).
12. Jones, R. O. Density functional theory: Its origins, rise to prominence, and future. *Rev. Mod. Phys.* 87, 897-923 (2015).

13. Hohenberg, P. & Kohn, W. Inhomogeneous Electron Gas. *Phys. Rev. B*. 136, 864–871 (1964).
14. Kohn, W. & Sham, L. J. Self-consistent equations including exchange and correlation effects. *Phys. Rev.* 140, A1133–A1138 (1965).
15. Sasaki, S. Theory of the Integer & Fractional Quantum Hall Effects (Nova Science Publishers, 2016). Preprint at https://arxiv.org/abs/1603.08625 (2016).
16. Hasnip, P. J. et al. Density functional theory in the solid state. *Philos. Trans. A. Mat. Phys. Eng. Sci.* 372, 20130270 (2014).
17. Greiner, W. & Reinhardt, J. *Quantum Electrodynamics* (Springer-Verlag, 2009).
18. Weinberg, S. *The Quantum Theory of Fields,* Vol. I (Foundations) (Cambridge University Press, 1995).
19. An Introduction to quantum field theory Michael E. Peskin (SLAC), Daniel V. Schroeder (Weber State U.) 1995, 842 pages ISBN: 9780201503975, 9780429503559 (eBook), 9780429494178 Published: 1995 in Reading, USA by Addison-Wesley, DOI: 10.1201/9780429503559, 10.1201/9780429494178
20. Srednicki, Mark. (2007). Quantum Field Theory. United Kingdom: Cambridge University Press.
21. Cabibbo, N., Maiani, L., & Benhar, O. (2017). An Introduction to Gauge Theories (1st ed.). CRC Press. https://doi.org/10.1201/9781315369723
22. Thomson, J. J. Cathode rays. *Philosophical Magazine.* **44,** 293-316 (1897).
23. Wilczek, F. The enigmatic electron. *Nature.* **498,** 31-32 (2013). Indeed, the words of the scholar Wilczeck: *"An electron is a particle and a wave; it is ideally simple, and unimaginably complex; it is precisely understood, and utterly mysterious; it is rigid, and subject to creative disassembly. No single answer does justice to reality".*
24. Weinberg, S. Models of lepton and quark masses, PHYSICAL REVIEW D 101, 035020 (2020).
25. Weinberg, S. A model of leptons, Phys. Rev. Lett., 19 (1967), pp. 1264-1266.
26. H. Georgi and S.L. Glashow, Attempts to calculate the electron mass, Phys. Rev. D 7 (1973).
27. S. M. Barr and A. Zee, Phys. Rev. D 17, 1854 (1978); Calculating the electron mass in terms of measured quantities.
28. Indeed, also from a classical reasoning, the magnetic dipole moment μ determined by a current i flowing, e.g. in a circular coil of radius r, is $\mu = i\pi r^2$. The magnetic field generated at the center of the coil is $B = \frac{2\pi i}{r}$, and the magnetic flux is approximately $\Phi = B\pi r^2$. By combining previous equations, we can express the magnetic flux in terms of the magnetic moment μ. In fact, $\Phi = B\pi r^2 = \frac{2\pi i}{r}\pi r^2 = \mu\frac{2\pi}{r}$.
29. Y. Nambu, An Empirical Mass Spectrum of Elementary Particles, *Progress of Theoretical Physics*, Volume 7, Issue 5-6, May 1952, Pages 595–596, https://doi.org/10.1143/PTP.7.5.595
30. A.O. Barut, The mass of the muon, Physics Letters B, Volume 73, Issue 3, 1978, Pages 310-312, ISSN 0370-2693, https://doi.org/10.1016/0370-2693(78)90522-1.
31. A. O. Barut, Lepton Mass Formula, Phys. Rev. Lett. **43**, 1057 (1979).
32. CODATA recommended values of the fundamental physical constants: 2022, Peter J. Mohr, David B. Newell, Barry N. Taylor, and Eite Tiesinga, Rev. Mod. Phys. 97, 025002 – Published 30 April, 2025, DOI: https://doi.org/10.1103/RevModPhys.97.025002 https://arxiv.org/pdf/2409.03787, https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=958143
33. Navas, S.; et al. (Particle Data Group) (2024). "Review of Particle Physics". Physical Review D. 110 (3): 030001. Bibcode:2018PhRvD..98c0001T. doi:10.1103/PhysRevD.110.030001. hdl:20.500.11850/695340.]
34. Revised April 2025 by M. Grünewald (University Coll. Dublin), A. Gurtu (CERN; TIFR Mumbai) and J. Kretzschmar (Liverpool U.). S. Navas et al. (Particle Data Group), "Mass and Width of the W Boson", Phys. Rev. D 110, 030001 (2024). https://pdg.lbl.gov/2025/reviews/rpp2024-rev-w-mass.pdf
35. We used the CODATA value of 2018 https://physics.nist.gov/cuu/pdf/all_2018.pdf of $sin^2(\theta_W) = s_W^2 = 0.22290(30)$, since the first version of the paper was released as a preprint in 2019 [Vinciguerra, V. (2019).

Topological Quantum Interplay between Magnetic Flux and Mass in an Electron. Preprints. https://doi.org/10.20944/preprints201910.0220.v1]. The updated value of 0.22305(23) reported in the CODATA 2022 has also been used https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=958143] see reference [32].

36. 2022 CODATA Value: electron g factor". The NIST Reference on Constants, Units, and Uncertainty. NIST. May 2024. Retrieved 18 May 2024. https://physics.nist.gov/cgi-bin/cuu/Value?gem, reported also in [32] https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=958143
37. Mass of the electron in CODATA Recommended Values of the Fundamental of the Physical Constants: 2022, https://physics.nist.gov/cuu/pdf/wall_2022.pdf, reported also in [32].
38. High-precision measurement of the W boson mass with the CMS experiment at the LHC, The CMS collaboration, 18-Dec-2024, https://arxiv.org/pdf/2412.13872
39. Measurement of the W-boson mass and width with the ATLAS detector using proton-proton collisions at $\sqrt{s}$ = 7 TeV, ATLAS collaboration, 28 Jan 2025, arXiv:2403.15085
40. The LHCb collaboration., Aaij, R., Abdelmotteleb, A.S.W. et al. Measurement of the W boson mass. J. High Energ. Phys. 2022, 36 (2022). https://doi.org/10.1007/JHEP01(2022)036**, https://arxiv.org/abs/2109.01113**
41. CDF Collaboration†‡ et al., High-precision measurement of the W boson mass with the CDF II detector. Science 376,170-176(2022). DOI:10.1126/science.abk1781
42. Measurement of the *W* Boson Mass with the D0 Detector, V. M. Abazov, B. Abbott, B. S. Acharya, M. Adams, T. Adams, G. D. Alexeev, G. Alkhazov, A. Alton, G. Alverson et al. (D0 Collaboration) Phys. Rev. Lett. 108, 151804 – Published 12 April, 2012, DOI: https://doi.org/10.1103/PhysRevLett.108.151804
43. LEP combination, Electroweak measurements in electron–positron collisions at W-boson-pair energies at LEP, Physics Reports, Volume 532, Issue 4, 2013, Pages 119-244, ISSN 0370-1573, https://doi.org/10.1016/j.physrep.2013.07.004.
44. Electroweak fit Precise prediction for the mass of the *W* boson in gauged U(1) extensions of the standard model, Zoltán Péli and Zoltán Trócsányi, Phys. Rev. D 108, L031704 – Published 28 August, 2023, DOI: https://doi.org/10.1103/PhysRevD.108.L031704, https://arxiv.org/pdf/2311.08203, also reported in [33].
45. Koide, Y. (1981), Quark and lepton masses speculated from a subquark model.
46. Dellsén, Finnur. Abductive Reasoning in Science. Cambridge: Cambridge University Press, 2024. Print. Elements in the Philosophy of Science.
47. Jaffe, A., and Witten, E. Quantum Yang-Mills Theory, http://www.claymath.org/prizeproblems/yang mills.pdf.