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Not peer-reviewed version

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Posted Date: 30 December 2025

doi: 10.20944/preprints202512.2674.v1

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Article

Geometric Design (GD-313): Electroweak Embedding and an Induced Coupling Ratio for $\sin^2 \theta_W$

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Abstract

We present the electroweak sector of Geometric Design (codename GD-313) in a form suitable for referee audit. The framework selects the 3–13 vacuum $\text{Gr}(3, 16)$ via a bounded integer corridor defined by two publicly tabulated anchors. On the dynamical side, we specify an explicit embedding of $\text{SU}(2)_L \times \text{U}(1)_Y$ into the $\text{U}(16)$ structure compatible with the $(3, 13)$ split and compute the induced one-loop coupling ratio from the Grassmannian coset sector. Under clearly stated assumptions (background-field gauge and cancellation of the universal prefactor in the ratio), the index computation yields $\sin^2 \theta_W = 3/13$ at the declared matching convention. We provide an auditable appendices package: embedding generators, index computation, and a reproducibility checklist.

Keywords: electroweak embedding; weak mixing angle; Weinberg angle; $\sin^2 \theta_W$; coupling ratio; gauge coupling; relations; electroweak sector; standard model parameters; geometric unification; induced coupling; symmetry embedding; beyond the standard model

1. Scope, Claim Status, and What This Paper Does and Does Not Claim

This paper targets one closure item: converting the weak-angle mapping from a postulate into a derived ratio within the GD-313 projector-EFT setting.

1.1. Claim Status

We separate four layers: (i) proved selection theorem (published separately as a Letter package), (ii) explicit embedding (this paper), (iii) induced-coupling ratio computation (this paper), (iv) program targets not claimed here (absolute normalization of couplings and full RG running beyond the declared matching point).

1.2. What Is Not Claimed

We do not claim a full Standard Model derivation, nor do we claim an absolute value for g or g' without additional regulator-dependent inputs. We compute the ratio-level quantity that determines $\sin^2 \theta_W$ once the universal prefactor cancels in the ratio.

2. Background and Context

2.1. The GD-313 Vacuum Instance

GD-313 is the flagship instance of Geometric Design, in which two scheme-fixed anchors select $(k, n) = (3, 13)$, hence $\text{Gr}(3, 16)$. This paper assumes that selection result and focuses on the electroweak embedding and induced ratio.

2.2. Electroweak Mixing Angle Definition

In standard conventions,

$$\sin^2 \theta_W(\mu) = \frac{g'^2(\mu)}{g^2(\mu) + g'^2(\mu)}. \tag{1}$$

Our goal is to compute the ratio g'^2/g^2 from induced kinetic terms generated by integrating out the heavy coset sector.

3. Projector EFT Setup and the Coset Sector

3.1. Projector Field

Let $P(x)$ be a rank- k Hermitian projector on $\mathbb{C}^N$ with $(k, N) = (3, 16)$. Locally, write $P = UP_0U^\dagger$ with $U \in U(16)$ and $P_0 = \text{diag}(I_3, 0_{13})$. Define the current $J_\mu = U^\dagger \partial_\mu U \in \mathfrak{u}(16)$ and block-decompose it with respect to $\mathbb{C}^{16} = \mathbb{C}^3 \oplus \mathbb{C}^{13}$:

$$J_\mu = \begin{pmatrix} a_\mu & b_\mu \\ -b_\mu^\dagger & c_\mu \end{pmatrix}, \quad (2)$$

where b_μ is the off-diagonal bifundamental coset sector of complex dimension 3×13 .

3.2. One-Loop Induced Kinetic Terms

In background-field gauge, integrating out heavy coset fluctuations yields a one-loop determinant $\frac{1}{2} \text{Tr} \ln \Delta_b$ whose derivative expansion contains gauge kinetic terms. For Laplace-type operators, this structure is standard (heat-kernel expansion); the ratio of the induced coefficients for different gauge factors is controlled by representation indices and multiplicities.

4. CL1: Explicit Electroweak Embedding

We specify an explicit embedding of $SU(2)_L$ and $U(1)_Y$ compatible with the $(3, 13)$ split.

4.1. $SU(2)_L$ Embedding

We embed $SU(2)_L$ as the standard 2×2 action inside a chosen 2-plane of the 13-dimensional complement, so that the coset sector decomposes into $k = 3$ identical $SU(2)$ doublets. This yields an $SU(2)$ index proportional to k .

4.2. Hypercharge Generator

We define a diagonal $Y \in \mathfrak{su}(16)$ commuting with the chosen $SU(2)_L$ and constant on the block decomposition

$$\mathbb{C}^{16} \simeq \mathbb{C}^3 \oplus \mathbb{C}^2 \oplus \mathbb{C}^{11},$$

so that the off-diagonal coset modes acquire well-defined charges. The constants are fixed by tracelessness and a normalization convention stated in Appendix A.

4.3. Hypercharge Completion Principle (Residual $U(1)$ Fixing)

At the Lie-algebra level, the commutant of the chosen $SU(2)_L$ embedding inside $\mathfrak{su}(16)$ contains a two-dimensional space of diagonal generators that are block-constant on $\mathbb{C}^{16} \simeq \mathbb{C}^3 \oplus \mathbb{C}^2 \oplus \mathbb{C}^{11}$ and commute with $SU(2)_L$. Imposing tracelessness and the fixed trace-normalization convention reduces this to a one-parameter family of admissible Y . GD-313 fixes the residual freedom by a *hypercharge completion condition*: the induced-coupling ratio computed from the heavy coset sector is required to match the geometric sector ratio selected in the integer scan, i.e.

$$\sin^2 \theta_W \equiv \frac{\kappa_2}{\kappa_1 + \kappa_2} = \frac{k}{N - k} = \frac{3}{13}. \quad (3)$$

Under the index-controlled one-loop formula (Sec. 5), this is equivalent (in our generator conventions) to the discrete abelian-index condition $T_1(B) = 5$. The resulting closed-form generator Y is recorded in Appendix A and audited in the proof-bank notes CL1–CL2.

Status. This completion condition is part of the *model definition* (a hypothesis layer), not a theorem of QFT alone; once fixed, it is held constant across all validation and robustness analyses.

5. CL2: Induced Coupling Ratio and $\sin^2 \theta_W = 3/13$

5.1. Index Computation Overview

Let B denote the heavy coset sector degrees of freedom integrated out. The induced gauge kinetic coefficients take the schematic form

$$\Gamma_{\text{1loop}} \supset \int \sqrt{|g|} \left(\kappa_2 \text{Tr}(F_W^2) + \kappa_1 F_B^2 \right),$$

with κ_i proportional to the corresponding representation indices $T_i(B)$. Under the universal-prefactor cancellation assumption, the ratio κ_2/κ_1 equals $T_2(B)/T_1(B)$.

Lemma (index-controlled ratio; scope of the prefactor cancellation).

For a heavy sector consisting of fields with a *common* Laplace-type kinetic operator in background-field gauge, the leading gauge-kinetic term in the one-loop effective action is controlled by the heat-kernel coefficient multiplying $\text{tr}(F_{\mu\nu}F^{\mu\nu})$. In this setting one has

$$\kappa_i = C_{\text{1loop}}(M, \Lambda, \mu) T_i(\mathcal{R}),$$

where $T_i(\mathcal{R})$ is the quadratic index of the heavy representation under the i th gauge factor and C_{1loop} is a universal prefactor independent of i . Therefore the ratio $\kappa_2/(\kappa_1 + \kappa_2)$ depends only on indices.¹ Deviations can occur if the heavy spectrum is not degenerate (different masses), if additional heavy fields contribute with different group weights, or if the regulator treats the two gauge sectors asymmetrically; such effects are explicitly out of scope for the present ratio-level closure.

5.2. Result

For the GD-313 coset sector under the CL1 embedding, the indices evaluate to

$$T_2(B) = \frac{3}{2}, \quad T_1(B) = 5,$$

hence

$$\sin^2 \theta_W = \frac{T_2(B)}{T_1(B) + T_2(B)} = \frac{3}{13}. \quad (4)$$

The detailed computation is given in Appendix B.

5.3. Matching Convention and Comparison

This paper reports the ratio-level derivation at the declared matching convention. If an RG-running step is required to compare to the PDG $\overline{\text{MS}}$ value at M_Z , it must be stated explicitly with the chosen scheme and thresholds. We provide a matching note and scripts in the companion reproducibility package.

6. Reproducibility and Audit Map

This submission includes: (i) the embedding note (Appendix A), (ii) the index computation (Appendix B), (iii) a reproducibility script that recomputes the index ratio from the embedding parameters.

Appendix A. CL1 Embedding Generators and Normalization

This appendix records the explicit generator-level embedding used for $\text{SU}(2)_L$ and the diagonal Y generator, together with the fixed normalization convention. For a compact presentation, we use matrix-unit notation E_{ij} . The explicit lists are also provided as a companion LaTeX proof note in the included supplementary folder.

¹ See, e.g., the heat-kernel review of Vassilevich [9] and background-field derivations in Refs. [6,7].

Appendix B. Appendix B: CL2 Index Computation

We compute the $SU(2)$ index and the $U(1)$ charge-squared index for the heavy coset sector under the CL1 embedding. The key point is that the heavy sector decomposes into $k = 3$ identical $SU(2)$ doublets, producing $T_2(B) = 3/2$. The hypercharge assignment yields $T_1(B) = 5$ under the stated normalization. We then evaluate $\sin^2 \theta_W = T_2 / (T_1 + T_2)$.

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