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Article

4G Model of Simplified and Approximate Formulae for Estimating Nuclear Mass Radii and Charge Radii

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Abstract

Precise determination of nuclear radii and charge radii is a cornerstone of nuclear physics, essential for understanding nuclear structure, reaction dynamics, and astrophysical phenomena. Traditional empirical models often include numerous corrections to address isotopic asymmetry, shell effects, and pairing energies, resulting in complex formulae with many fitted parameters that restrict practical use. Building on the 4G model of final unification, this work introduces a simplified, physically motivated nuclear radius framework. It separately accounts for proton and neutron contributions based on their cubic root form and incorporates an adjustable mass distribution coefficient (C_{md}), empirically dependent on the fine structure ratio and the strong coupling constant. Crucially, the mass radius is formulated as the product of (C_{md}) and the nuclear charge radius—a feature that directly relates unified scaling parameters to experimentally accessible quantities. Close to the stable mass numbers of $Z = (2 \text{ to } 118)$, $C_{md} = (1.127 \text{ to } 1.382)$. It needs fine tuning. This novel approach's predictive performance is rigorously benchmarked against advanced formulae incorporating detailed nuclear structure corrections. Results show that this minimalistic method achieves accuracy comparable to complex models across a broad range of nuclei while substantially reducing computational complexity. It thus provides an efficient and physically transparent tool for rapid nuclear radius estimation, suitable for both theoretical studies and practical nuclear science applications.

Keywords: Nuclear mass radius; Nuclear charge radius; Mass distribution coefficient; Proton and neutron contributions; Fine structure ratio; strong coupling constant; 4G model of final unification

1. Introduction

The nuclear radius is a primary characteristic of atomic nuclei, influencing cross sections, decay properties, and nuclear matter distributions. Traditionally, the nuclear radius R_A is approximated by the empirical formula [1]:

$$R_A \cong A^{1/3} R_0 \tag{1}$$

where $A=Z+N$ is the mass number, and R_0 is a constant of about 1.2–1.3 fm. However, as experimental data and theoretical understanding advanced, formulas began incorporating corrections for proton-neutron asymmetry, pairing, shell closures, and surface diffuseness to improve accuracy, leading to complex multi-term models [2-8].

Despite these improvements, the complexity restricts practical use, especially in applications requiring quick estimates or large-scale computations. Motivated by this, we investigate a simple formula for mass distribution radius [9,10,11]:

$$R_{md} \cong C_{md} \times \left[Z^{1/3} + \left(Z^2 N \right)^{1/9} \right] R_p \tag{2}$$

where R_p is a 4G model of characteristic radius constant associated with proton and C_{md} is a coefficient that can be empirically tuned. This respects the separate roles of protons and neutrons in nuclear size and offers flexibility with a minimal parameter set.

2. Three Assumptions and Two Applications of Our 4G Model of Final Unification

Following our 4G model of final unification [9,10,11],

- (1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725 \text{ GeV}$. It can be considered as the zygote of all elementary particles.
- (2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n} \right)^2 \cong \alpha_s \cong 0.1152 = \text{Strong}$ coupling constant and $e_n \cong 2.9464e$.
- (3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$G_e \cong \text{Electromagnetic gravitational constant} \cong 2.374335 \times 10^{37} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}$

$G_n \cong \text{Nuclear gravitational constant} \cong 3.329561 \times 10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}$

$G_w \cong \text{Electroweak gravitational constant} \cong 2.909745 \times 10^{22} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}$

It may be noted that,

- (1) In a unified approach, most important point to be noted is that,

$$\hbar c \equiv G_w M_{wf}^2 \tag{3}$$

Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning. Following this kind of relation, there is a possibility to understand the integral nature of quantum mechanics with a relation of the form,

$$n^2 \hbar \equiv \frac{G_w \left(n M_{wf} \right)^2}{c} \text{ where } n = 1, 2, 3, \dots \text{ It needs further study with reference to EPR argument [12].}$$

- (2) Another interesting application is that, string theory [9] can be made practical with reference to the three atomic gravitational constants associated with weak, strong, and electromagnetic interaction gravitational constants. These constants provide a framework to bridge the gap between quantum mechanics and gravity at the atomic scale, offering new insights into particle interactions. By incorporating these gravitational couplings into string theory models, it becomes possible to explore unified descriptions of fundamental forces. This approach may lead to testable predictions linking microscopic string dynamics with observable nuclear phenomena, thereby advancing both theoretical and experimental physics. See Table 1. and Table 2. for sample string tensions and energies without any coupling constants.

Table 1. Charge dependent string tensions and string energies.

S.No	Interaction	String Tension	String Energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94 \times 10^{10} \text{ N}$	$\sqrt{\frac{e^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_w} \right)} \cong 24.975 \text{ GeV}$

2	Strong	$\frac{c^4}{4G_n} \cong 6.065 \times 10^4 \text{ N}$	$\sqrt{\frac{e_n^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_n} \right)} \cong 68.79 \text{ MeV}$
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505 \times 10^{-5} \text{ N}$	$\sqrt{\frac{e^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_e} \right)} \cong 874.3 \text{ eV}$

Table 2. Quantum string tensions and string energies.

S.No	Interaction	String Tension	String Energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94 \times 10^{10} \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_w} \right)} \cong 292.36 \text{ GeV}$
2	Strong	$\frac{c^4}{4G_n} \cong 6.065 \times 10^4 \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_n} \right)} \cong 273.3 \text{ MeV}$
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505 \times 10^{-5} \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_e} \right)} \cong 10234.77 \text{ eV}$

(3) Weak interaction point of view [13], following our assumptions, Fermi’s weak coupling constant can be fitted with the following relations.

$$G_F \cong \left(\frac{m_e}{m_p} \right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021 \times 10^{-62} \text{ J.m}^3$$

where, $\left\{ \begin{array}{l} R_0 \cong \frac{2G_n m_p}{c^2} \cong 1.24 \times 10^{-15} \text{ m} \\ R_w \cong \frac{2G_w M_{wf}}{c^2} \cong 6.75 \times 10^{-19} \text{ m} \end{array} \right.$

(4)

3. Formula Development and Methodology

3.1. Advanced Nuclear Mass Radius Formula

We adopt an advanced mass radius formula [14-20] used in nuclear physics literature incorporating empirical corrections for nuclear structure effects:

$$R_{md} \cong aA^{1/3} + bI + c\delta + dA^{-1/3} + e + fS$$

(5)

where

- The $aA^{1/3}$ base term originates from the classic geometric model of the nucleus, where the radius scales with the cube root of the mass number, describing the approximate constancy of nuclear density across the chart.
- The bI isospin asymmetry term $I=(N-Z)/A$ represents the correction for neutron-proton imbalance, which is known to influence nuclear size and is included in several radius fitting studies.
- The $c\delta$ pairing correction accounts for the variation in nuclear radii depending on whether the nucleus has even or odd numbers of protons and neutrons; such terms appear in both mass and radius systematics.
- The $dA^{-1/3}$ inverse scaling term and the constant e are added for additional fitting flexibility and improved regression accuracy.

- The fS shell correction term incorporates effects from nuclear shell structure, particularly close to magic numbers, as described in various empirical and theoretical studies
- a, b, c, d, e, f are coefficients fitted to experimental data.
- This formula is given by AI by considering so many references. It needs proper citation and confirmation. For the time being, we consider it as an advanced mass radius formula. For details, see the python program presented in section (6).
- It is very interesting to note that, numerically, output of this advanced mass radius relation can be compared with 4G model relation, $A^{1/3} \left(\frac{2G_n m_p}{c^2} \right) \cong A^{1/3} \times 1.24 \text{ fm}$. See Table 3, columns 4 and 5.
- This formula captures detailed nuclear structure features but involves multiple fitted parameters and terms.

3.2. Proposed Simple Formula

It may be noted that, the mass radius and charge radius of atomic nuclides are related but distinct nuclear properties, each reflecting different aspects of nuclear structure:

(A) Mass Radius:

- (1) The mass radius refers to the spatial extent of the total nuclear mass distribution, considering both protons and neutrons as massive particles within the nucleus.
- (2) It describes the radius at which the nuclear matter density drops to half its central value and is defined through gravitational form factors or nuclear density profiles.
- (3) Mass radii can be probed in specialized reactions and models but are not directly measured in typical scattering experiments.

(B) Charge Radius:

- (1) The charge radius is the root-mean-square (rms) distance of the distribution of the proton (electric charge) density inside the nucleus.
- (2) It is measured directly by high-precision experiments, such as elastic electron scattering, muonic atom spectroscopy, or optical isotope shifts, since these techniques respond to the nuclear electric field produced by protons.
- (3) The charge radius is particularly sensitive to the spatial arrangement of protons, not neutrons, in the nucleus.

(C) Key Differences:

- (1) The mass radius reflects the distribution of all nucleons (protons and neutrons), while the charge radius primarily describes the distribution of protons only.
- (2) Experimental nuclear charge radii are typically slightly smaller than mass radii in nuclei with a neutron excess, because neutron-rich nuclei have a more extended neutron distribution (sometimes called a neutron skin).

- (3) In some light nuclei (e.g., 4He), mass radius and charge radius can be nearly identical due to similar proton and neutron distributions, but differences become apparent in heavier or neutron-rich nuclei.

(D) Proposed Formulae

Close to the stable mass numbers of any Z , we propose the mass radius R_{md} as follows.

For $Z > 1$,

$$(R_{md})_{A_s} \cong C_{md} \times R_{cd} \quad (6)$$

where,

$R_{md} \cong$ Nuclear mass radius

$R_{cd} \cong$ Nuclear charge radius

$$\cong \left[Z^{1/3} + (Z^2 N_s)^{1/9} \right] 0.62 \text{ fm}$$

$A_s \cong$ Approximate stable mass number

$$\cong 2Z + 0.0064Z^2 [9,10,11]$$

$$N_s \cong A_s - Z \cong Z + 0.0064Z^2$$

Further,

$$C_{md} \cong \text{Mass distribution coefficient} \cong [1 + x + y]$$

$$\begin{cases} x \cong \alpha Z^{2/3} \cong 0.0073Z^{2/3} \\ \alpha \cong \text{Fine structure ratio [21]} \\ y \cong \left[1 + \sqrt{\frac{N_s - 2}{A}} \right] \times \alpha_s \cong \left[1 + \sqrt{\frac{N_s - 2}{A}} \right] \times 0.1152 \\ \alpha_s \cong \text{Strong coupling constant [22,23]} \end{cases}$$

For estimating the mass radii of isotopes of $Z=2$ to 118, we are working on refining this mass distribution coefficient with reference to fine structure constant [21], strong coupling constant [22,23] and the approximate stable mass numbers of Z [9-11]. With further study, things can be improved well and back ground physics can be explored in a unified approach. One such approximate relation can be expressed as,

$$C_{md} \cong \left(\frac{A}{A_s} \right)^{0.25} [1 + x + y] \quad (7)$$

where

$$\begin{cases} A \cong \text{Mass number} \\ A_s \cong 2Z + 0.0064Z^2 \\ x \cong \alpha Z^{2/3} \cong 0.0073Z^{2/3} \\ \text{and } \alpha \cong \text{Fine structure ratio} \\ y \cong \left[1 + \sqrt{\frac{N-2}{A}} \right] \times \alpha_s \cong \left[1 + \sqrt{\frac{N-2}{A}} \right] \times 0.1152 \\ \alpha_s \cong \text{Strong coupling constant} \end{cases}$$

Based on relations (6) and (7), it is possible to give some physical meaning to the proposed mass distribution factor as follows. With reference to nuclear charge radius,

- (1) Fine structure ratio helps in increasing the mass distribution radius by a factor of

$$(0.0073Z^{2/3})R_{cd}.$$

- (2) Strong coupling coefficient helps in increasing the mass distribution by a factor of

$$\left\{ \left[1 + \sqrt{\frac{N-2}{A}} \right] \times 0.1152 \right\} R_{cd}.$$

- (3) Close to stable mass numbers, mass distribution radius takes the following form,

$$\begin{aligned} (R_{md})_{A_s} &\cong R_{cd} + (0.0073Z^{2/3})R_{cd} + \left\{ \left[1 + \sqrt{\frac{N_s-2}{A_s}} \right] \times 0.1152 \right\} R_{cd} \\ &\cong R_{cd} \left(1 + (0.0073Z^{2/3}) + \left\{ \left[1 + \sqrt{\frac{N_s-2}{A_s}} \right] \times 0.1152 \right\} \right) \end{aligned}$$

- (4) Above and below the stable mass numbers, mass distribution radius takes the following form,

$$(R_{md})_A \cong \left(\frac{A}{A_s} \right)^{0.25} \left\{ \left(1 + (0.0073Z^{2/3}) + \left\{ \left[1 + \sqrt{\frac{N-2}{A}} \right] \times 0.1152 \right\} \right) \right\} R_{cd}$$

4. Results: Figures and Data Table

Considering our 4G model of unification [9-11], we assume the approximate stable mass number as,

$$A_s \cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \quad (8)$$

The key phenomenological scaling factor “0.0016 appearing in our proposed nuclear line of beta stability is the ratio of the geometric mean of the charged and neutral pion masses (~137.26 MeV) to that of the weak boson masses (~85.61 GeV), which numerically evaluates to approximately 0.0016. This dimensionless ratio encapsulates the profound hierarchical gap between the strong interaction scale and the electroweak scale and forms a cornerstone of the mass relations underlying our 585 GeV electroweak fermion. Importantly, this ratio is not merely a numerical coincidence but has substantive implications for understanding nuclear stability and nuclear binding energy. The interplay of these fundamental mass scales suggests that the dynamics governing nuclear forces and nucleon interactions may be intimately connected to electroweak-scale physics mediated by the 585 GeV fermion. For a deeper exploration of how this mass ratio informs nuclear binding mechanisms and stability criteria, interested readers are encouraged to refer our recent preprints and other peer-reviewed publications [24,25], where these connections are discussed in detail with complementary theoretical and phenomenological analyses.

$$\begin{aligned} \frac{m_p}{M_{wf}} &\cong 0.001605 \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_w c^2)^\pm (m_z c^2)^0}} \right) \\ &\cong \left(\frac{\sqrt{134.98 \times 139.57} \text{ MeV}}{\sqrt{80379.0 \times 91187.6} \text{ MeV}} \right) \cong 0.0016032 \cong \beta \dots (\text{say}) \end{aligned} \quad (9)$$

Based on this electroweak coefficient $\beta \cong 0.001605$, stability corresponding to nuclear beta decay can be understood with the following relation.

$$A_s \cong 2Z + \beta(2Z)^2 \cong 2Z + 0.00642Z^2 \quad (10)$$

$$\frac{A_s - 2Z}{(2Z)^2} \cong \frac{A_s - 2Z}{4Z^2} \cong \beta \quad (11)$$

One can find a similar relation in the literature. This relation can be well tested for $Z=21$ to 92. For example,

$$\begin{aligned} \frac{45 - (2 \times 21)}{4(21)^2} &\cong 0.00170; & \frac{63 - (2 \times 29)}{4(29)^2} &\cong 0.00149; & \frac{89 - (2 \times 39)}{4(39)^2} &\cong 0.00181; \\ \frac{109 - (2 \times 47)}{4(47)^2} &\cong 0.0017; & \frac{169 - (2 \times 69)}{4(69)^2} &\cong 0.00163; & \frac{238 - (2 \times 92)}{4(92)^2} &\cong 0.001595; \end{aligned}$$

This is one best practical and quantitative application of our proposed electroweak fermion and bosons. Following this relation and based on various semi empirical mass formulae [26,27], by knowing any stable mass number, its corresponding proton number can be estimated with,

$$Z \cong \frac{A_s}{1 + \sqrt{1 + 0.0064 A_s}} \cong \frac{A_s}{2 + 0.0153 A_s^{2/3}} \quad (12)$$

$$\text{where } \frac{a_c}{2a_{asy}} \cong \frac{0.71 \text{ MeV}}{2 \times 23.21 \text{ MeV}} \cong \frac{0.6615 \text{ MeV}}{2 \times 21.6091 \text{ MeV}} \cong 0.0153$$

Based on these relations and concepts, we have prepared Table 3 & Figure 1 for the approximate stable mass numbers of $Z=2$ to 118. See Figures 2,3,4,5,6,7,8 for the isotopes of the magic numbers 2,8,20,28,50,82 and 114.

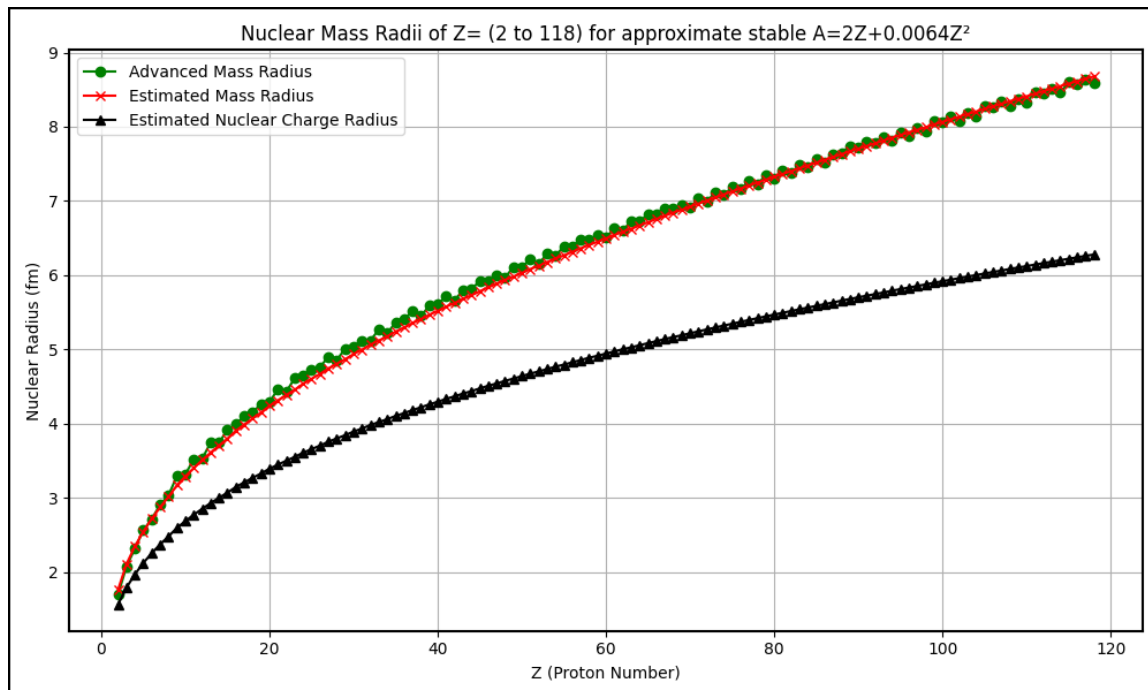


Figure 1. Estimated mass radii of $Z=2$ to 118 based on proton and neutron numbers.

The simple formula with minimal fitting achieves less than 2% error for medium to heavy nuclei and reasonable accuracy even for lighter nuclei, apart from extremely light cases like He-4 where nuclear structure effects dominate beyond mass scaling.

Table 3. Approximate fitting of nuclear mass radii based on cubic root of proton and neutron numbers.

Z	N	Approximate stable mass number (A _s)	Advanced Mass Radius (fm)	A ^(1/3) *1.24 (fm)	Estimated Nuclear Charge Radius [28] (fm)	Mass Distribution Factor	Estimated Mass Radius (fm)	Error (fm)	Percentage Error (%)
2	2	4	1.695	1.968	1.562	1.127	1.760	-0.065	3.84
3	3	6	2.071	2.253	1.788	1.177	2.106	-0.035	1.67
4	4	8	2.315	2.480	1.968	1.191	2.345	-0.03	1.28
5	5	10	2.562	2.671	2.120	1.200	2.544	0.018	0.70
6	6	12	2.712	2.839	2.253	1.206	2.717	-0.005	0.17
7	7	14	2.906	2.989	2.372	1.211	2.872	0.034	1.16
8	8	16	3.032	3.125	2.480	1.215	3.013	0.019	0.63
9	10	19	3.293	3.309	2.594	1.222	3.169	0.124	3.75
10	11	21	3.317	3.421	2.686	1.224	3.289	0.028	0.84
11	12	23	3.517	3.526	2.771	1.227	3.401	0.116	3.30
12	13	25	3.525	3.626	2.852	1.230	3.507	0.018	0.52
13	14	27	3.743	3.720	2.928	1.232	3.608	0.135	3.60
14	15	29	3.740	3.810	3.000	1.235	3.704	0.036	0.95
15	16	31	3.917	3.895	3.069	1.237	3.797	0.12	3.08
16	18	34	3.994	4.017	3.145	1.241	3.902	0.092	2.30
17	19	36	4.102	4.094	3.208	1.243	3.987	0.115	2.81
18	20	38	4.157	4.169	3.269	1.245	4.068	0.089	2.12
19	21	40	4.259	4.241	3.327	1.247	4.148	0.111	2.60
20	23	43	4.290	4.344	3.392	1.249	4.239	0.051	1.21
21	24	45	4.461	4.411	3.447	1.251	4.313	0.148	3.33
22	25	47	4.430	4.475	3.499	1.253	4.385	0.045	1.01
23	26	49	4.611	4.538	3.551	1.255	4.455	0.156	3.38
24	28	52	4.655	4.628	3.608	1.257	4.536	0.119	2.55
25	29	54	4.727	4.687	3.656	1.259	4.603	0.124	2.62
26	30	56	4.763	4.744	3.703	1.261	4.668	0.095	1.98
27	32	59	4.898	4.827	3.755	1.263	4.743	0.155	3.16
28	33	61	4.855	4.881	3.800	1.265	4.806	0.049	1.02
29	34	63	5.011	4.934	3.844	1.266	4.867	0.144	2.87
30	36	66	5.040	5.011	3.892	1.268	4.937	0.103	2.04
31	37	68	5.118	5.061	3.934	1.270	4.996	0.122	2.39
32	39	71	5.118	5.135	3.980	1.272	5.063	0.055	1.08
33	40	73	5.268	5.182	4.020	1.273	5.120	0.148	2.83
34	41	75	5.218	5.229	4.059	1.275	5.175	0.043	0.82
35	43	78	5.363	5.298	4.103	1.277	5.239	0.124	2.32
36	44	80	5.400	5.343	4.141	1.278	5.293	0.107	1.99
37	46	83	5.518	5.409	4.183	1.280	5.354	0.164	2.97
38	47	85	5.448	5.452	4.219	1.282	5.406	0.042	0.77
39	49	88	5.588	5.515	4.259	1.283	5.466	0.122	2.19
40	50	90	5.607	5.557	4.294	1.285	5.517	0.09	1.61
41	52	93	5.719	5.618	4.333	1.286	5.574	0.145	2.53
42	53	95	5.661	5.658	4.367	1.288	5.623	0.038	0.66
43	55	98	5.796	5.717	4.404	1.290	5.680	0.116	2.01
44	56	100	5.812	5.756	4.437	1.291	5.728	0.084	1.45
45	58	103	5.920	5.813	4.474	1.292	5.782	0.138	2.33
46	60	106	5.927	5.868	4.510	1.294	5.836	0.091	1.53
47	61	108	5.990	5.905	4.541	1.295	5.882	0.108	1.81
48	63	111	5.971	5.959	4.576	1.297	5.935	0.036	0.60
49	64	113	6.108	5.995	4.606	1.298	5.980	0.128	2.10
50	66	116	6.112	6.047	4.640	1.300	6.031	0.081	1.32
51	68	119	6.214	6.099	4.673	1.301	6.082	0.132	2.14

52	69	121	6.150	6.133	4.702	1.303	6.125	0.025	0.40
53	71	124	6.291	6.183	4.735	1.304	6.175	0.116	1.85
54	73	127	6.267	6.233	4.767	1.306	6.224	0.043	0.69
55	74	129	6.386	6.265	4.795	1.307	6.266	0.12	1.87
56	76	132	6.385	6.314	4.826	1.308	6.314	0.071	1.11
57	78	135	6.483	6.361	4.857	1.310	6.361	0.122	1.88
58	80	138	6.481	6.408	4.887	1.311	6.408	0.073	1.12
59	81	140	6.538	6.439	4.914	1.312	6.449	0.089	1.36
60	83	143	6.510	6.484	4.944	1.314	6.495	0.015	0.23
61	85	146	6.631	6.529	4.973	1.315	6.540	0.091	1.36
62	87	149	6.601	6.574	5.002	1.317	6.585	0.016	0.23
63	88	151	6.731	6.603	5.027	1.318	6.625	0.106	1.58
64	90	154	6.726	6.647	5.056	1.319	6.669	0.057	0.84
65	92	157	6.819	6.689	5.084	1.320	6.713	0.106	1.56
66	94	160	6.812	6.732	5.112	1.322	6.756	0.056	0.82
67	96	163	6.905	6.774	5.139	1.323	6.799	0.106	1.53
68	98	166	6.897	6.815	5.166	1.324	6.842	0.055	0.80
69	99	168	6.951	6.842	5.190	1.326	6.880	0.071	1.02
70	101	171	6.917	6.883	5.217	1.327	6.922	-0.005	0.07
71	103	174	7.033	6.923	5.243	1.328	6.963	0.07	0.98
72	105	177	6.998	6.962	5.269	1.329	7.005	-0.007	0.10
73	107	180	7.113	7.001	5.295	1.331	7.046	0.067	0.94
74	109	183	7.077	7.040	5.320	1.332	7.087	-0.01	0.13
75	111	186	7.191	7.078	5.346	1.333	7.127	0.064	0.89
76	113	189	7.155	7.116	5.371	1.334	7.167	-0.012	0.17
77	115	192	7.268	7.154	5.396	1.336	7.207	0.061	0.84
78	117	195	7.231	7.191	5.420	1.337	7.246	-0.015	0.22
79	119	198	7.343	7.227	5.445	1.338	7.286	0.057	0.78
80	121	201	7.305	7.264	5.469	1.339	7.325	-0.02	0.27
81	123	204	7.417	7.300	5.493	1.341	7.363	0.054	0.72
82	125	207	7.378	7.335	5.516	1.342	7.402	-0.024	0.32
83	127	210	7.489	7.370	5.540	1.343	7.440	0.049	0.65
84	129	213	7.450	7.405	5.563	1.344	7.478	-0.028	0.38
85	131	216	7.560	7.440	5.586	1.345	7.516	0.044	0.58
86	133	219	7.520	7.474	5.609	1.347	7.553	-0.033	0.44
87	135	222	7.629	7.508	5.632	1.348	7.590	0.039	0.51
88	138	226	7.650	7.553	5.657	1.349	7.631	0.019	0.24
89	140	229	7.733	7.586	5.679	1.350	7.668	0.065	0.85
90	142	232	7.717	7.619	5.701	1.351	7.704	0.013	0.17
91	144	235	7.800	7.652	5.723	1.352	7.740	0.06	0.77
92	146	238	7.783	7.684	5.745	1.354	7.777	0.006	0.09
93	148	241	7.866	7.717	5.767	1.355	7.812	0.054	0.68
94	151	245	7.809	7.759	5.791	1.356	7.852	-0.043	0.55
95	153	248	7.916	7.791	5.812	1.357	7.887	0.029	0.36
96	155	251	7.873	7.822	5.833	1.358	7.922	-0.049	0.63
97	157	254	7.980	7.853	5.854	1.359	7.957	0.023	0.28
98	159	257	7.936	7.884	5.875	1.360	7.992	-0.056	0.71
99	162	261	8.077	7.924	5.898	1.362	8.030	0.047	0.58
100	164	264	8.058	7.955	5.918	1.363	8.065	-0.007	0.09
101	166	267	8.138	7.985	5.939	1.364	8.099	0.039	0.48
102	169	271	8.078	8.024	5.961	1.365	8.137	-0.059	0.72
103	171	274	8.183	8.054	5.981	1.366	8.170	0.013	0.16
104	173	277	8.138	8.083	6.001	1.367	8.204	-0.066	0.81
105	176	281	8.277	8.122	6.023	1.368	8.241	0.036	0.44
106	178	284	8.257	8.151	6.042	1.369	8.274	-0.017	0.21
107	180	287	8.336	8.179	6.062	1.370	8.308	0.028	0.34
108	183	291	8.274	8.217	6.083	1.372	8.344	-0.07	0.84

109	185	294	8.378	8.245	6.103	1.373	8.377	0.001	0.01
110	187	297	8.331	8.273	6.122	1.374	8.410	-0.079	0.94
111	190	301	8.469	8.310	6.143	1.375	8.445	0.024	0.27
112	192	304	8.447	8.338	6.162	1.376	8.478	-0.031	0.37
113	195	308	8.508	8.374	6.182	1.377	8.513	-0.005	0.06
114	197	311	8.461	8.401	6.201	1.378	8.545	-0.084	1.00
115	200	315	8.597	8.437	6.221	1.379	8.580	0.017	0.20
116	202	318	8.574	8.464	6.240	1.380	8.612	-0.038	0.44
117	205	322	8.635	8.499	6.260	1.381	8.647	-0.012	0.14
118	207	325	8.587	8.525	6.278	1.382	8.678	-0.091	1.07

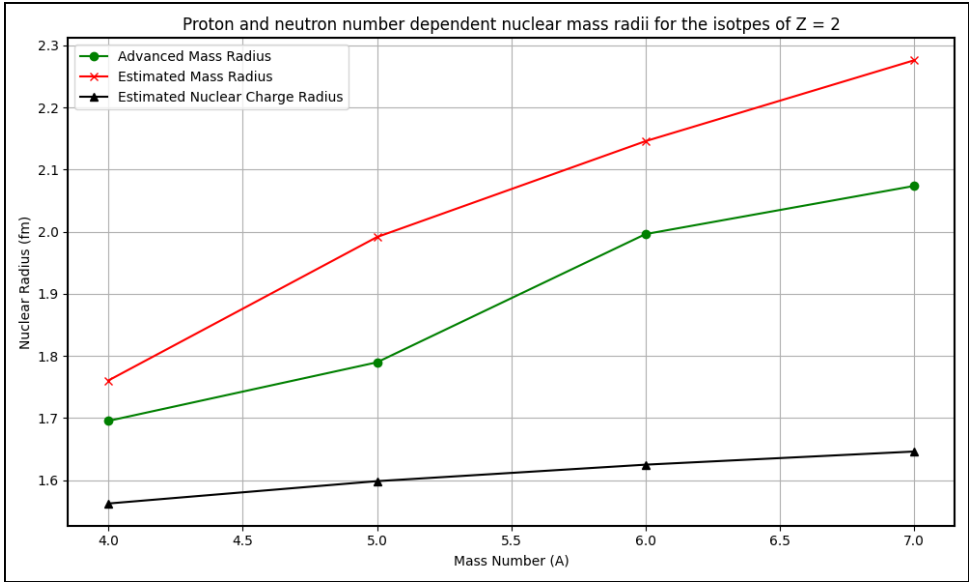


Figure 2. Estimated mass radii of isotopes of Z=2 based on cubic root of proton and neutron numbers.

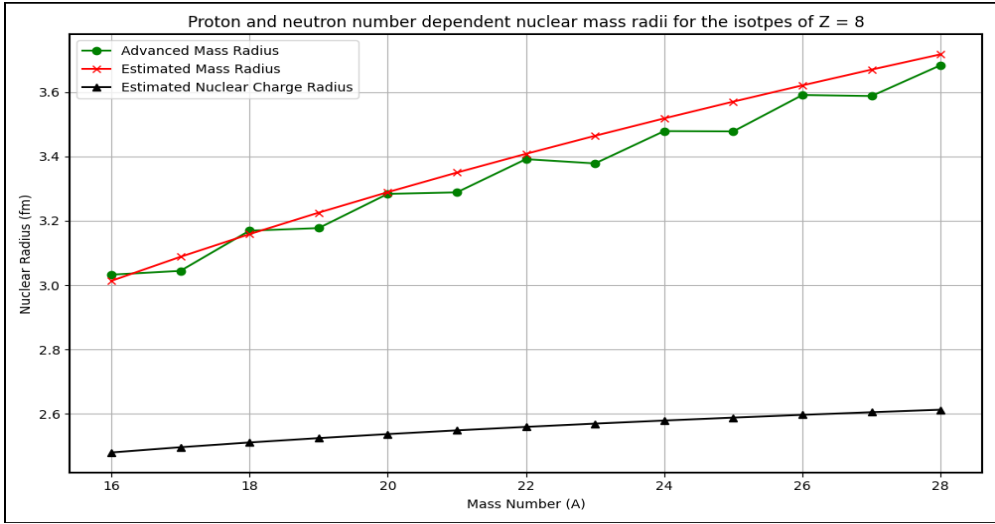


Figure 3. Estimated mass radii of isotopes of Z=8 based on cubic root of proton and neutron numbers.

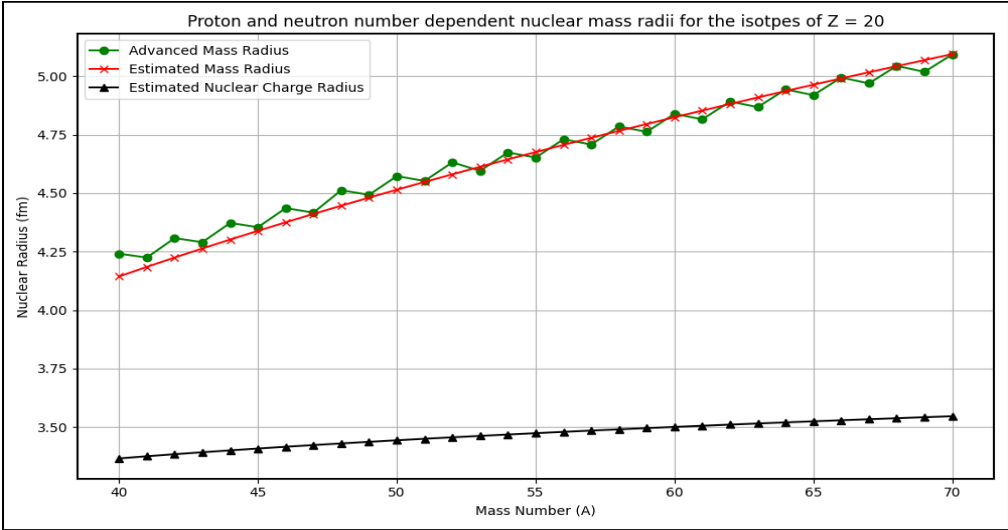


Figure 4. Estimated mass radii of isotopes of Z=20 based on cubic root of proton and neutron numbers.

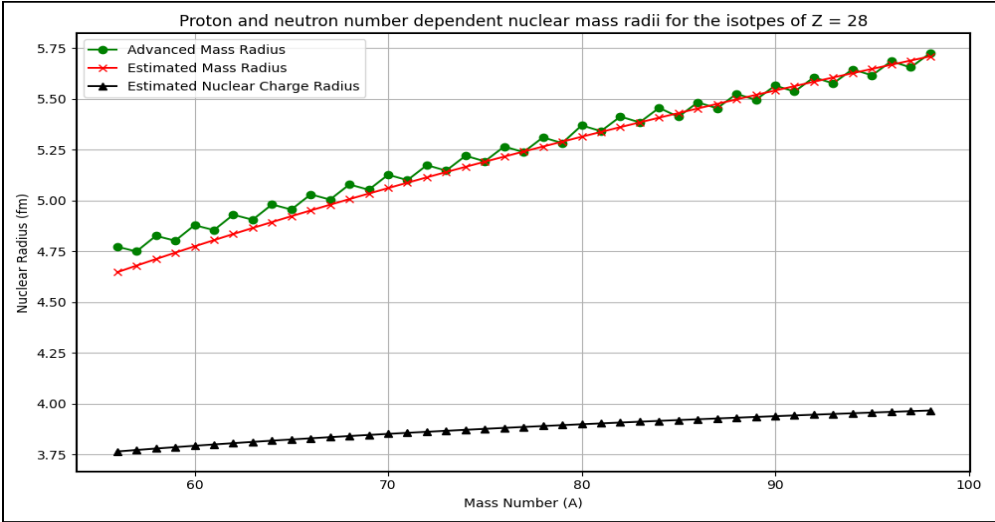


Figure 5. Estimated mass radii of isotopes of Z=28 based on cubic root of proton and neutron numbers.

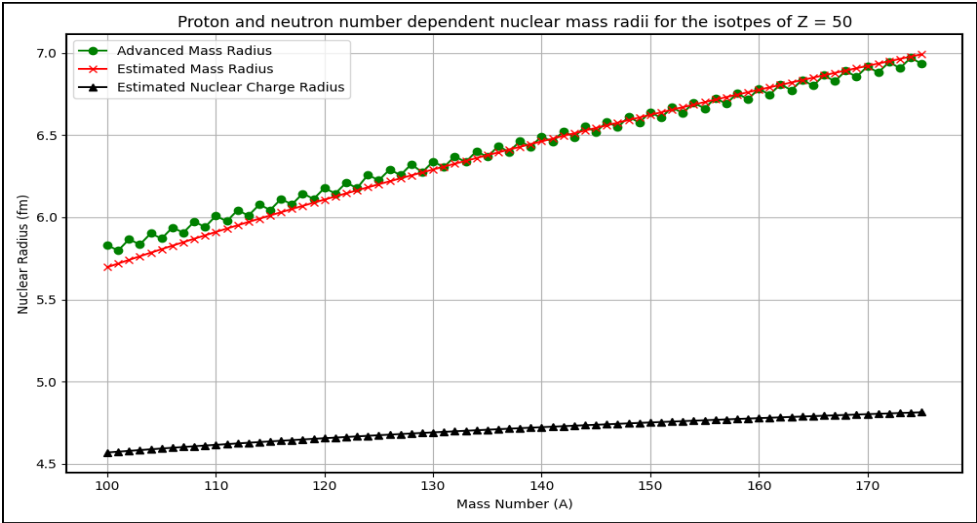


Figure 6. Estimated mass radii of isotopes of Z=50 based on cubic root of proton and neutron numbers.

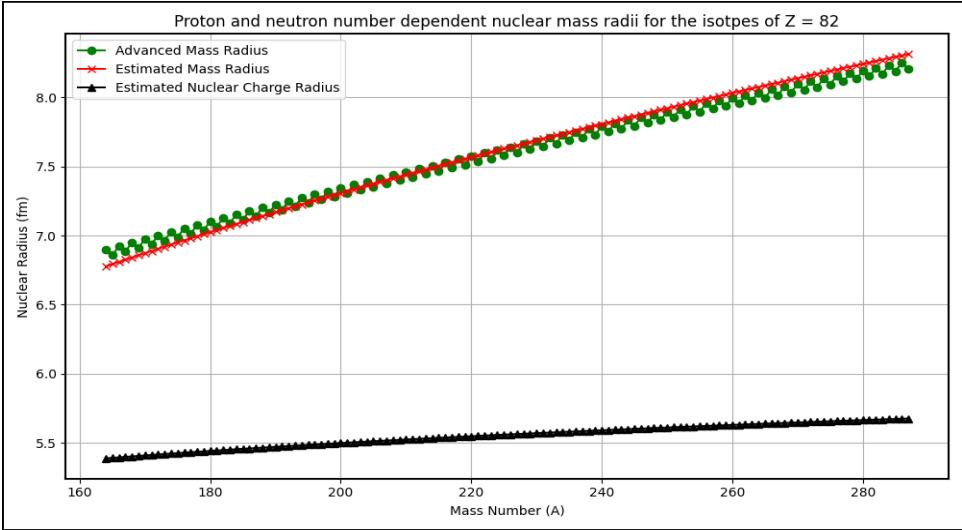


Figure 7. Estimated mass radii of isotopes of Z=82 based on cubic root of proton and neutron numbers.

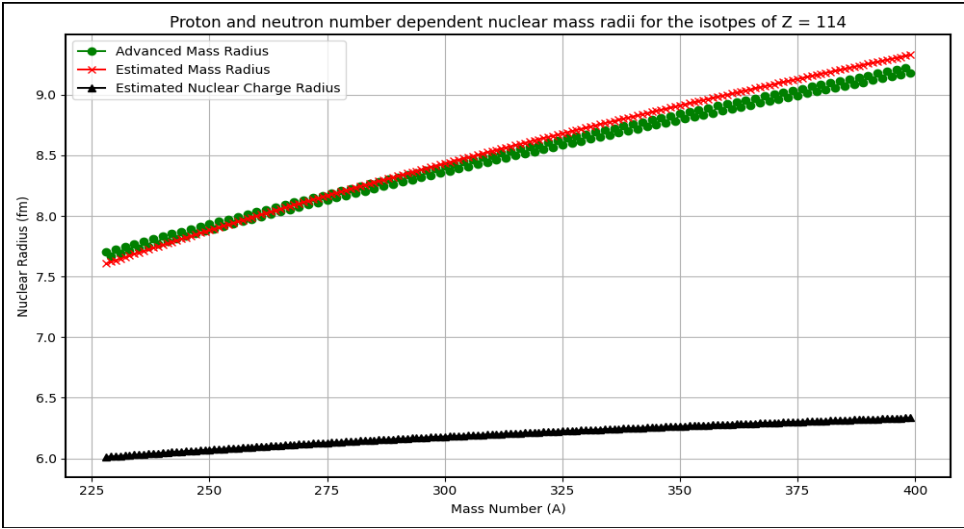


Figure 8. Estimated mass radii of isotopes of Z=114 based on cubic root of proton and neutron numbers.

4.1. Data Table Observations and analysis

- (1) The table shows data for various nuclides with proton number (Z), neutron number (N), and mass number (A).
- (2) It lists the advanced radius, estimated nuclear charge radius (using relation involving $Z^{1/3}$ and $N^{1/3}$, mass distribution factor, estimated mass radius, and percentage error between the estimated and advanced mass radii.
- (3) The percentage error is generally very low, often below 2%, especially for medium and heavy nuclei. For example, for larger nuclei like Z=50 and above, errors hover near or below 1%, indicating high accuracy of the simple formula.

- (4) This confirms the simple formula's efficacy for rapid nuclear radius estimation with minimal parameters, particularly suitable for large-scale or educational applications where complex calculations are impractical.
- (5) For $Z=2$ and its isotopes, average % error is 8.1%. Similarly for $Z=8,20,28,50,82$ and 114, average % errors are, 1.29%, 0.725%, 0.81%, 0.665%, 0.632% and 0.784% respectively. These percentage errors can be considered for further analysis positively.
- (6) Even though percentage errors are slightly larger (3 to 12 %) in very light nuclei, range of difference in radii is (0.03 to 0.14) fm. It needs further study.

4.2. Graphical Observations

- (1) The graphs depict estimated nuclear mass radii for isotopes of magic numbers $Z=2, 8,20,28,50,82,114$ as a function of mass number A .
- (2) Trends show a smooth increase in radius with increasing A , consistent with the power-law dependence on proton and neutron numbers.
- (3) The estimated radii closely track the advanced formula results across isotopes, validating the model's predictive power.
- (4) Regions near magic numbers show slight structural effects possibly reflected in minor deviations or curvature in the radii trends.

The simple formula manages to capture these global trends well despite its simplicity, further supporting its practical utility.

4.3. Overall Commentary

The data table and accompanying graphs demonstrate that the proposed simple cubic-root-based formula with a tuneable mass distribution factor performs very well across a wide range of nuclei. It achieves accuracy comparable to more complex, parameter-heavy advanced formulas while offering significant simplicity in implementation. This balance makes it valuable for nuclear physics research, applications, and educational purposes focused on nuclear size estimation. Limitations are mainly restricted to very light nuclei where more detailed nuclear structure effects need consideration.

This strong agreement between experiment-based advanced formulas and the minimalistic approach underscores the 4G model's foundational insight into nuclear size scaling using unified gravitational-electroweak parameters, serving as a promising theoretical tool.

5. Simplicity of the Proposed Formula

The minimalistic formula possesses several advantages:

- (1) It separately accounts for proton and neutron contributions through $Z^{1/3}$ and $N^{1/3}$, physically motivated by differing mass distributions.
- (2) The mass distribution coefficient C_{md} introduces a tuneable mild dependence on proton number, neutron number and mass number offering empirical flexibility without excessive complication.

- (3) Compared to the multi-parameter advanced formula, it is computationally simpler and easier to implement in broad nuclear system studies or educational settings.
- (4) Despite its simplicity, it exhibits high fidelity across a broad mass range, supporting its utility for approximate radius calculations when detailed corrections are unavailable or unnecessary.
- (5) Crucially, $R_{cd} \cong \left[Z^{1/3} + \left(Z^2 N \right)^{1/9} \right] 0.62 \text{ fm}$ reliably approximates experimental nuclear charge radii [28-31] for a wide spectrum of elements. This strong correspondence validates the formula’s capability to capture essential aspects of nuclear spatial charge distribution, underscoring its scientific robustness and practical utility. See the following Figure 9 and Table 4. For light atomic nuclides, charge radius can be approximated with, $R_{cd} \cong \left[(Z + 2)^{1/3} + \left[(Z + 2)^2 (N + 2) \right]^{1/9} \right] 0.62 \text{ fm}$.

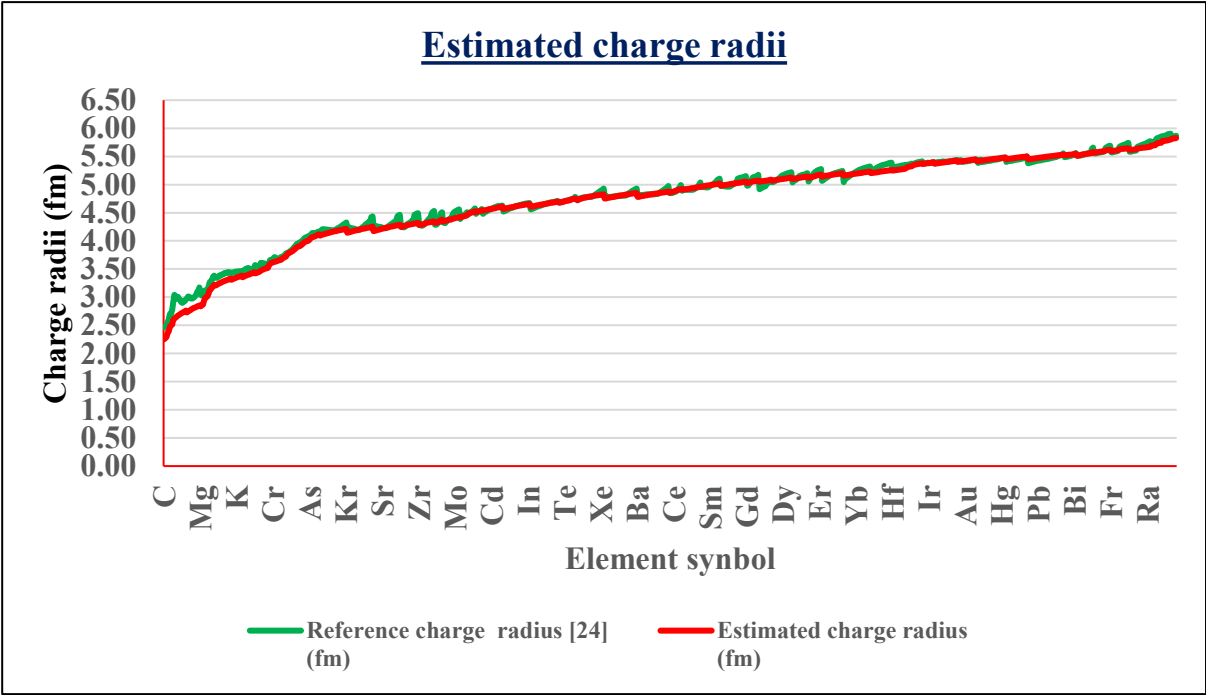


Figure 9. Estimated charge radii of Z= (6 to 96) based on cubic root of proton and neutron numbers.

Table 4. Estimated nuclear charge radii.

Symbol	Z	N	Approximate Stable Mass Number A _s	Reference Charge Radius [24] (fm)	Reference Uncertainty (fm)	Estimated Charge Radius (fm)	Difference (fm)	%Error
C	6	6	12	2.4702	0.0022	2.2532	0.2170	8.78
C	6	7	13	2.4614	0.0034	2.2727	0.1887	7.67
C	6	8	14	2.5025	0.0087	2.2898	0.2127	8.50
N	7	7	14	2.5582	0.007	2.3720	0.1862	7.28
N	7	8	15	2.6058	0.008	2.3898	0.2160	8.29
O	8	8	16	2.6991	0.0052	2.4800	0.2191	8.12
O	8	9	17	2.6932	0.0075	2.4963	0.1969	7.31

O	8	10	18	2.7726	0.0056	2.5111	0.2615	9.43
F	9	10	19	2.8976	0.0025	2.5945	0.3031	10.46
Ne	10	7	17	3.0413	0.0088	2.6196	0.4217	13.87
Ne	10	8	18	2.9714	0.0076	2.6388	0.3326	11.19
Ne	10	9	19	3.0082	0.004	2.6560	0.3522	11.71
Ne	10	10	20	3.0055	0.0021	2.6715	0.3340	11.11
Ne	10	11	21	2.9695	0.0033	2.6857	0.2838	9.56
Ne	10	12	22	2.9525	0.004	2.6988	0.2537	8.59
Ne	10	13	23	2.9104	0.0071	2.7110	0.1994	6.85
Ne	10	14	24	2.9007	0.0078	2.7224	0.1783	6.15
Ne	10	15	25	2.9316	0.0088	2.7331	0.1985	6.77
Ne	10	16	26	2.9251	0.01	2.7431	0.1820	6.22
Ne	10	18	28	2.9642	0.0134	2.7616	0.2026	6.83
Na	11	9	20	2.9718	0.042	2.7273	0.2445	8.23
Na	11	10	21	3.0136	0.0284	2.7432	0.2704	8.97
Na	11	11	22	2.9852	0.0169	2.7577	0.2275	7.62
Na	11	12	23	2.9936	0.0021	2.7711	0.2225	7.43
Na	11	13	24	2.9735	0.0169	2.7836	0.1899	6.39
Na	11	14	25	2.9769	0.0252	2.7952	0.1817	6.10
Na	11	15	26	2.9928	0.0331	2.8061	0.1867	6.24
Na	11	16	27	3.0136	0.0467	2.8164	0.1972	6.55
Na	11	17	28	3.04	0.0581	2.8261	0.2139	7.04
Na	11	18	29	3.0922	0.0723	2.8353	0.2569	8.31
Na	11	19	30	3.118	0.0884	2.8441	0.2739	8.79
Na	11	20	31	3.1704	0.0893	2.8524	0.3180	10.03
Mg	12	12	24	3.057	0.0016	2.8389	0.2181	7.13
Mg	12	13	25	3.0284	0.0022	2.8516	0.1768	5.84
Mg	12	14	26	3.0337	0.0018	2.8634	0.1703	5.61
Al	13	14	27	3.061	0.0031	2.9277	0.1333	4.35
Si	14	14	28	3.1224	0.0024	2.9886	0.1338	4.29
Si	14	15	29	3.1176	0.0052	3.0001	0.1175	3.77
Si	14	16	30	3.1336	0.004	3.0109	0.1227	3.92
P	15	16	31	3.1889	0.0019	3.0691	0.1198	3.76
S	16	16	32	3.2611	0.0018	3.1246	0.1365	4.19
S	16	18	34	3.2847	0.0021	3.1452	0.1395	4.25
S	16	20	36	3.2985	0.0024	3.1638	0.1347	4.08
Cl	17	18	35	3.3654	0.0191	3.1985	0.1669	4.96
Cl	17	20	37	3.384	0.017	3.2174	0.1666	4.92
Ar	18	14	32	3.3468	0.0062	3.2050	0.1418	4.24
Ar	18	15	33	3.3438	0.0058	3.2171	0.1267	3.79
Ar	18	16	34	3.3654	0.004	3.2286	0.1368	4.07
Ar	18	17	35	3.3636	0.0042	3.2394	0.1242	3.69
Ar	18	18	36	3.3905	0.0023	3.2497	0.1408	4.15
Ar	18	19	37	3.3908	0.0022	3.2595	0.1313	3.87
Ar	18	20	38	3.4028	0.0019	3.2689	0.1339	3.94
Ar	18	21	39	3.4093	0.0031	3.2778	0.1315	3.86
Ar	18	22	40	3.4274	0.0026	3.2864	0.1410	4.12
Ar	18	23	41	3.4251	0.003	3.2946	0.1305	3.81
Ar	18	24	42	3.4414	0.0041	3.3025	0.1389	4.04
Ar	18	25	43	3.4354	0.0039	3.3101	0.1253	3.65
Ar	18	26	44	3.4454	0.0046	3.3175	0.1279	3.71

Ar	18	28	46	3.4377	0.0044	3.3315	0.1062	3.09
K	19	19	38	3.4264	0.0051	3.3088	0.1176	3.43
K	19	20	39	3.4349	0.0019	3.3183	0.1166	3.40
K	19	21	40	3.4381	0.0028	3.3273	0.1108	3.22
K	19	22	41	3.4518	0.0055	3.3360	0.1158	3.36
K	19	23	42	3.4517	0.007	3.3443	0.1074	3.11
K	19	24	43	3.4556	0.0086	3.3523	0.1033	2.99
K	19	25	44	3.4563	0.0101	3.3600	0.0963	2.78
K	19	26	45	3.4605	0.0118	3.3675	0.0930	2.69
K	19	27	46	3.4558	0.0126	3.3747	0.0811	2.35
K	19	28	47	3.4534	0.0138	3.3817	0.0717	2.08
Ca	20	19	39	3.4595	0.0025	3.3563	0.1032	2.98
Ca	20	20	40	3.4776	0.0019	3.3659	0.1117	3.21
Ca	20	21	41	3.478	0.0019	3.3750	0.1030	2.96
Ca	20	22	42	3.5081	0.0021	3.3838	0.1243	3.54
Ca	20	23	43	3.4954	0.0019	3.3922	0.1032	2.95
Ca	20	24	44	3.5179	0.0021	3.4003	0.1176	3.34
Ca	20	25	45	3.4944	0.0021	3.4081	0.0863	2.47
Ca	20	26	46	3.4953	0.002	3.4157	0.0796	2.28
Ca	20	27	47	3.4783	0.0024	3.4229	0.0554	1.59
Ca	20	28	48	3.4771	0.002	3.4300	0.0471	1.35
Ca	20	30	50	3.5168	0.0064	3.4434	0.0734	2.09
Sc	21	21	42	3.5702	0.0238	3.4211	0.1491	4.18
Sc	21	22	43	3.5575	0.0147	3.4299	0.1276	3.59
Sc	21	23	44	3.5432	0.0016	3.4384	0.1048	2.96
Sc	21	24	45	3.5459	0.0025	3.4466	0.0993	2.80
Sc	21	25	46	3.5243	0.0089	3.4545	0.0698	1.98
Ti	22	22	44	3.6115	0.0051	3.4745	0.1370	3.79
Ti	22	23	45	3.5939	0.0032	3.4831	0.1108	3.08
Ti	22	24	46	3.607	0.0022	3.4914	0.1156	3.20
Ti	22	25	47	3.5962	0.0019	3.4994	0.0968	2.69
Ti	22	26	48	3.5921	0.0017	3.5071	0.0850	2.37
Ti	22	27	49	3.5733	0.0021	3.5145	0.0588	1.65
Ti	22	28	50	3.5704	0.0022	3.5217	0.0487	1.36
V	23	28	51	3.6002	0.0022	3.5654	0.0348	0.97
Cr	24	26	50	3.6588	0.0065	3.5928	0.0660	1.81
Cr	24	28	52	3.6452	0.0042	3.6077	0.0375	1.03
Cr	24	29	53	3.6511	0.0075	3.6148	0.0363	0.99
Cr	24	30	54	3.6885	0.0074	3.6217	0.0668	1.81
Mn	25	25	50	3.712	0.0196	3.6258	0.0862	2.32
Mn	25	26	51	3.7026	0.0212	3.6337	0.0689	1.86
Mn	25	27	52	3.6706	0.0128	3.6414	0.0292	0.80
Mn	25	28	53	3.6662	0.0076	3.6488	0.0174	0.48
Mn	25	29	54	3.6834	0.0049	3.6559	0.0275	0.75
Mn	25	30	55	3.7057	0.0022	3.6629	0.0428	1.16
Mn	25	31	56	3.7146	0.0052	3.6696	0.0450	1.21
Fe	26	28	54	3.6933	0.0019	3.6887	0.0046	0.13
Fe	26	30	56	3.7377	0.0016	3.7029	0.0348	0.93
Fe	26	31	57	3.7532	0.0017	3.7097	0.0435	1.16
Fe	26	32	58	3.7745	0.0014	3.7164	0.0581	1.54
Co	27	32	59	3.7875	0.0021	3.7554	0.0321	0.85

Ni	28	30	58	3.7757	0.002	3.7799	-0.0042	-0.11
Ni	28	32	60	3.8118	0.0016	3.7935	0.0183	0.48
Ni	28	33	61	3.8225	0.0019	3.8001	0.0224	0.59
Ni	28	34	62	3.8399	0.0021	3.8064	0.0335	0.87
Ni	28	36	64	3.8572	0.0023	3.8187	0.0385	1.00
Cu	29	34	63	3.8823	0.0015	3.8436	0.0387	1.00
Cu	29	36	65	3.9022	0.0014	3.8560	0.0462	1.18
Zn	30	34	64	3.9283	0.0015	3.8799	0.0484	1.23
Zn	30	36	66	3.9491	0.0014	3.8924	0.0567	1.44
Zn	30	37	67	3.953	0.0027	3.8984	0.0546	1.38
Zn	30	38	68	3.9658	0.0014	3.9042	0.0616	1.55
Zn	30	40	70	3.9845	0.0019	3.9155	0.0690	1.73
Ga	31	38	69	3.9973	0.0017	3.9399	0.0574	1.44
Ga	31	40	71	4.0118	0.0018	3.9513	0.0605	1.51
Ge	32	38	70	4.0414	0.0012	3.9747	0.0667	1.65
Ge	32	40	72	4.0576	0.0013	3.9862	0.0714	1.76
Ge	32	41	73	4.0632	0.0014	3.9917	0.0715	1.76
Ge	32	42	74	4.0742	0.0012	3.9971	0.0771	1.89
Ge	32	44	76	4.0811	0.0012	4.0077	0.0734	1.80
As	33	42	75	4.0968	0.002	4.0314	0.0654	1.60
Se	34	40	74	4.07	0.02	4.0537	0.0163	0.40
Se	34	42	76	4.1395	0.0016	4.0648	0.0747	1.80
Se	34	43	77	4.1395	0.0018	4.0702	0.0693	1.67
Se	34	44	78	4.1406	0.0017	4.0755	0.0651	1.57
Se	34	46	80	4.14	0.0018	4.0857	0.0543	1.31
Se	34	48	82	4.14	0.0019	4.0956	0.0444	1.07
Br	35	44	79	4.1629	0.0021	4.1084	0.0545	1.31
Br	35	46	81	4.1599	0.0021	4.1187	0.0412	0.99
Kr	36	36	72	4.1635	0.006	4.0944	0.0691	1.66
Kr	36	38	74	4.187	0.0041	4.1067	0.0803	1.92
Kr	36	39	75	4.2097	0.0041	4.1127	0.0970	2.30
Kr	36	40	76	4.202	0.0036	4.1185	0.0835	1.99
Kr	36	41	77	4.2082	0.0037	4.1242	0.0840	2.00
Kr	36	42	78	4.2038	0.0033	4.1298	0.0740	1.76
Kr	36	43	79	4.2034	0.0032	4.1352	0.0682	1.62
Kr	36	44	80	4.197	0.0029	4.1405	0.0565	1.35
Kr	36	45	81	4.1952	0.0026	4.1458	0.0494	1.18
Kr	36	46	82	4.1919	0.0025	4.1509	0.0410	0.98
Kr	36	47	83	4.1871	0.0023	4.1559	0.0312	0.74
Kr	36	48	84	4.1884	0.0022	4.1609	0.0275	0.66
Kr	36	49	85	4.1846	0.0022	4.1657	0.0189	0.45
Kr	36	50	86	4.1835	0.0021	4.1705	0.0130	0.31
Kr	36	51	87	4.1984	0.0027	4.1752	0.0232	0.55
Kr	36	52	88	4.2171	0.0043	4.1798	0.0373	0.89
Kr	36	53	89	4.2286	0.0054	4.1843	0.0443	1.05
Kr	36	54	90	4.2423	0.0069	4.1887	0.0536	1.26
Kr	36	55	91	4.2543	0.0081	4.1931	0.0612	1.44
Kr	36	56	92	4.2724	0.0099	4.1974	0.0750	1.76
Kr	36	57	93	4.2794	0.0107	4.2016	0.0778	1.82
Kr	36	58	94	4.3002	0.0129	4.2058	0.0944	2.20
Kr	36	59	95	4.3067	0.0136	4.2099	0.0968	2.25

Kr	36	60	96	4.3267	0.0158	4.2139	0.1128	2.61
Rb	37	39	76	4.2273	0.007	4.1441	0.0832	1.97
Rb	37	40	77	4.2356	0.008	4.1499	0.0857	2.02
Rb	37	41	78	4.2385	0.0083	4.1557	0.0828	1.95
Rb	37	42	79	4.2284	0.0065	4.1613	0.0671	1.59
Rb	37	43	80	4.2271	0.0061	4.1667	0.0604	1.43
Rb	37	44	81	4.2213	0.0051	4.1721	0.0492	1.17
Rb	37	45	82	4.216	0.0042	4.1774	0.0386	0.92
Rb	37	46	83	4.2058	0.0028	4.1825	0.0233	0.55
Rb	37	47	84	4.1999	0.0023	4.1876	0.0123	0.29
Rb	37	48	85	4.2036	0.0024	4.1926	0.0110	0.26
Rb	37	49	86	4.2025	0.0023	4.1975	0.0050	0.12
Rb	37	50	87	4.1989	0.0021	4.2022	-0.0033	-0.08
Rb	37	51	88	4.217	0.0038	4.2069	0.0101	0.24
Rb	37	52	89	4.2391	0.0074	4.2116	0.0275	0.65
Rb	37	53	90	4.2554	0.0102	4.2161	0.0393	0.92
Rb	37	54	91	4.2723	0.0131	4.2206	0.0517	1.21
Rb	37	55	92	4.2903	0.0163	4.2250	0.0653	1.52
Rb	37	56	93	4.3048	0.0187	4.2293	0.0755	1.75
Rb	37	57	94	4.3184	0.0211	4.2336	0.0848	1.96
Rb	37	58	95	4.3391	0.0248	4.2378	0.1013	2.34
Rb	37	59	96	4.3501	0.0267	4.2419	0.1082	2.49
Rb	37	60	97	4.4231	0.0395	4.2460	0.1771	4.00
Rb	37	61	98	4.4336	0.0414	4.2500	0.1836	4.14
Sr	38	39	77	4.2569	0.0044	4.1749	0.0820	1.93
Sr	38	40	78	4.2561	0.004	4.1808	0.0753	1.77
Sr	38	41	79	4.2586	0.0039	4.1865	0.0721	1.69
Sr	38	42	80	4.2562	0.0037	4.1922	0.0640	1.50
Sr	38	43	81	4.2547	0.0034	4.1977	0.0570	1.34
Sr	38	44	82	4.2478	0.003	4.2031	0.0447	1.05
Sr	38	45	83	4.2455	0.0027	4.2084	0.0371	0.87
Sr	38	46	84	4.2394	0.0024	4.2136	0.0258	0.61
Sr	38	47	85	4.2304	0.0021	4.2187	0.0117	0.28
Sr	38	48	86	4.2307	0.002	4.2237	0.0070	0.17
Sr	38	49	87	4.2249	0.0019	4.2286	-0.0037	-0.09
Sr	38	50	88	4.224	0.0018	4.2334	-0.0094	-0.22
Sr	38	51	89	4.2407	0.0023	4.2381	0.0026	0.06
Sr	38	52	90	4.2611	0.0037	4.2428	0.0183	0.43
Sr	38	53	91	4.274	0.0046	4.2473	0.0267	0.62
Sr	38	54	92	4.2924	0.0064	4.2518	0.0406	0.94
Sr	38	55	93	4.3026	0.0075	4.2563	0.0463	1.08
Sr	38	56	94	4.3191	0.0091	4.2606	0.0585	1.35
Sr	38	57	95	4.3305	0.0102	4.2649	0.0656	1.51
Sr	38	58	96	4.3522	0.0125	4.2691	0.0831	1.91
Sr	38	59	97	4.3625	0.0135	4.2733	0.0892	2.05
Sr	38	60	98	4.4377	0.0214	4.2774	0.1603	3.61
Sr	38	61	99	4.4495	0.0226	4.2814	0.1681	3.78
Sr	38	62	100	4.464	0.024	4.2854	0.1786	4.00
Y	39	47	86	4.2513	0.0023	4.2491	0.0022	0.05
Y	39	48	87	4.2498	0.0022	4.2542	-0.0044	-0.10
Y	39	49	88	4.2441	0.0021	4.2591	-0.0150	-0.35

Y	39	50	89	4.243	0.0021	4.2640	-0.0210	-0.49
Y	39	51	90	4.2573	0.0026	4.2687	-0.0114	-0.27
Y	39	53	92	4.2887	0.005	4.2780	0.0107	0.25
Y	39	54	93	4.3052	0.0065	4.2825	0.0227	0.53
Y	39	55	94	4.3142	0.0074	4.2870	0.0272	0.63
Y	39	56	95	4.3284	0.0087	4.2913	0.0371	0.86
Y	39	57	96	4.3402	0.0099	4.2957	0.0445	1.03
Y	39	58	97	4.358	0.0116	4.2999	0.0581	1.33
Y	39	59	98	4.3711	0.0129	4.3041	0.0670	1.53
Y	39	60	99	4.4658	0.0223	4.3082	0.1576	3.53
Y	39	61	100	4.4705	0.0228	4.3122	0.1583	3.54
Y	39	62	101	4.4863	0.0244	4.3162	0.1701	3.79
Y	39	63	102	4.4911	0.0249	4.3202	0.1709	3.81
Zr	40	47	87	4.2789	0.003	4.2791	-0.0002	0.00
Zr	40	48	88	4.2787	0.0025	4.2841	-0.0054	-0.13
Zr	40	49	89	4.2706	0.001	4.2891	-0.0185	-0.43
Zr	40	50	90	4.2694	0.001	4.2940	-0.0246	-0.58
Zr	40	51	91	4.2845	0.0013	4.2988	-0.0143	-0.33
Zr	40	52	92	4.3057	0.0013	4.3035	0.0022	0.05
Zr	40	54	94	4.332	0.0013	4.3126	0.0194	0.45
Zr	40	56	96	4.3512	0.0015	4.3215	0.0297	0.68
Zr	40	57	97	4.3792	0.0136	4.3258	0.0534	1.22
Zr	40	58	98	4.4012	0.0164	4.3301	0.0711	1.62
Zr	40	59	99	4.4156	0.0181	4.3343	0.0813	1.84
Zr	40	60	100	4.4891	0.0289	4.3385	0.1506	3.36
Zr	40	61	101	4.5119	0.0318	4.3425	0.1694	3.75
Zr	40	62	102	4.5292	0.034	4.3465	0.1827	4.03
Nb	41	49	90	4.2891	0.004	4.3186	-0.0295	-0.69
Nb	41	50	91	4.2878	0.004	4.3235	-0.0357	-0.83
Nb	41	51	92	4.3026	0.0043	4.3283	-0.0257	-0.60
Nb	41	52	93	4.324	0.0017	4.3330	-0.0090	-0.21
Nb	41	58	99	4.4062	0.0125	4.3598	0.0464	1.05
Nb	41	60	101	4.4861	0.0203	4.3682	0.1179	2.63
Nb	41	62	103	4.5097	0.0227	4.3763	0.1334	2.96
Mo	42	48	90	4.3265	0.0016	4.3425	-0.0160	-0.37
Mo	42	49	91	4.3182	0.0012	4.3475	-0.0293	-0.68
Mo	42	50	92	4.3151	0.0012	4.3524	-0.0373	-0.87
Mo	42	52	94	4.3529	0.0013	4.3620	-0.0091	-0.21
Mo	42	53	95	4.3628	0.0018	4.3667	-0.0039	-0.09
Mo	42	54	96	4.3847	0.0015	4.3713	0.0134	0.31
Mo	42	55	97	4.388	0.0015	4.3758	0.0122	0.28
Mo	42	56	98	4.4091	0.0018	4.3803	0.0288	0.65
Mo	42	58	100	4.4468	0.0025	4.3890	0.0578	1.30
Mo	42	60	102	4.4914	0.0038	4.3974	0.0940	2.09
Mo	42	61	103	4.5145	0.0046	4.4015	0.1130	2.50
Mo	42	62	104	4.5249	0.0051	4.4056	0.1193	2.64
Mo	42	63	105	4.5389	0.0057	4.4096	0.1293	2.85
Mo	42	64	106	4.549	0.0058	4.4135	0.1355	2.98
Mo	42	66	108	4.5602	0.0067	4.4213	0.1389	3.05
Ru	44	52	96	4.3908	0.0047	4.4186	-0.0278	-0.63
Ru	44	54	98	4.4229	0.0055	4.4280	-0.0051	-0.12

Ru	44	55	99	4.4338	0.0042	4.4326	0.0012	0.03
Ru	44	56	100	4.4531	0.0031	4.4371	0.0160	0.36
Ru	44	57	101	4.4606	0.002	4.4415	0.0191	0.43
Ru	44	58	102	4.4809	0.0018	4.4459	0.0350	0.78
Ru	44	60	104	4.5098	0.002	4.4544	0.0554	1.23
Rh	45	58	103	4.4945	0.0023	4.4736	0.0209	0.46
Pd	46	56	102	4.4827	0.0044	4.4921	-0.0094	-0.21
Pd	46	58	104	4.5078	0.0027	4.5009	0.0069	0.15
Pd	46	59	105	4.515	0.003	4.5053	0.0097	0.22
Pd	46	60	106	4.5318	0.0029	4.5095	0.0223	0.49
Pd	46	62	108	4.5563	0.0027	4.5179	0.0384	0.84
Pd	46	64	110	4.5782	0.003	4.5260	0.0522	1.14
Ag	47	54	101	4.4799	0.0088	4.5097	-0.0298	-0.67
Ag	47	56	103	4.5036	0.0065	4.5189	-0.0153	-0.34
Ag	47	57	104	4.5119	0.0058	4.5234	-0.0115	-0.26
Ag	47	58	105	4.5269	0.0045	4.5278	-0.0009	-0.02
Ag	47	60	107	4.5454	0.0031	4.5365	0.0089	0.20
Ag	47	62	109	4.5638	0.0025	4.5449	0.0189	0.41
Cd	48	54	102	4.481	0.0122	4.5361	-0.0551	-1.23
Cd	48	55	103	4.4951	0.0105	4.5408	-0.0457	-1.02
Cd	48	56	104	4.5122	0.0083	4.5454	-0.0332	-0.74
Cd	48	57	105	4.5216	0.007	4.5499	-0.0283	-0.63
Cd	48	58	106	4.5383	0.0036	4.5543	-0.0160	-0.35
Cd	48	59	107	4.5466	0.0039	4.5587	-0.0121	-0.27
Cd	48	60	108	4.5577	0.0031	4.5630	-0.0053	-0.12
Cd	48	61	109	4.5601	0.0035	4.5673	-0.0072	-0.16
Cd	48	62	110	4.5765	0.0026	4.5715	0.0050	0.11
Cd	48	63	111	4.5845	0.0058	4.5756	0.0089	0.19
Cd	48	64	112	4.5944	0.0024	4.5796	0.0148	0.32
Cd	48	65	113	4.6012	0.0028	4.5837	0.0175	0.38
Cd	48	66	114	4.6087	0.0023	4.5876	0.0211	0.46
Cd	48	67	115	4.6114	0.0046	4.5915	0.0199	0.43
Cd	48	68	116	4.6203	0.0059	4.5954	0.0249	0.54
Cd	48	69	117	4.6136	0.0025	4.5992	0.0144	0.31
Cd	48	70	118	4.6246	0.006	4.6029	0.0217	0.47
Cd	48	72	120	4.63	0.0069	4.6103	0.0197	0.43
In	49	55	104	4.5184	0.0117	4.5668	-0.0484	-1.07
In	49	56	105	4.5311	0.0103	4.5715	-0.0404	-0.89
In	49	57	106	4.5375	0.0095	4.5760	-0.0385	-0.85
In	49	58	107	4.5494	0.0082	4.5804	-0.0310	-0.68
In	49	59	108	4.5571	0.0071	4.5848	-0.0277	-0.61
In	49	60	109	4.5685	0.0061	4.5892	-0.0207	-0.45
In	49	61	110	4.5742	0.0056	4.5934	-0.0192	-0.42
In	49	62	111	4.5856	0.0044	4.5976	-0.0120	-0.26
In	49	63	112	4.5907	0.0041	4.6018	-0.0111	-0.24
In	49	64	113	4.601	0.0031	4.6059	-0.0049	-0.11
In	49	65	114	4.6056	0.0029	4.6099	-0.0043	-0.09
In	49	66	115	4.6156	0.0026	4.6139	0.0017	0.04
In	49	67	116	4.6211	0.0027	4.6178	0.0033	0.07
In	49	68	117	4.6292	0.0032	4.6217	0.0075	0.16
In	49	69	118	4.6335	0.0033	4.6255	0.0080	0.17

In	49	70	119	4.6407	0.004	4.6293	0.0114	0.25
In	49	71	120	4.6443	0.0042	4.6330	0.0113	0.24
In	49	72	121	4.6505	0.0047	4.6367	0.0138	0.30
In	49	73	122	4.6534	0.0051	4.6403	0.0131	0.28
In	49	74	123	4.6594	0.0056	4.6439	0.0155	0.33
In	49	75	124	4.6625	0.006	4.6474	0.0151	0.32
In	49	76	125	4.667	0.0064	4.6509	0.0161	0.34
In	49	77	126	4.6702	0.0068	4.6544	0.0158	0.34
In	49	78	127	4.6733	0.0071	4.6578	0.0155	0.33
Sn	50	58	108	4.5605	0.0029	4.6062	-0.0457	-1.00
Sn	50	59	109	4.5679	0.0027	4.6106	-0.0427	-0.93
Sn	50	60	110	4.5785	0.0025	4.6149	-0.0364	-0.80
Sn	50	61	111	4.5836	0.0024	4.6192	-0.0356	-0.78
Sn	50	62	112	4.5948	0.0022	4.6234	-0.0286	-0.62
Sn	50	63	113	4.6015	0.0021	4.6276	-0.0261	-0.57
Sn	50	64	114	4.6099	0.002	4.6317	-0.0218	-0.47
Sn	50	65	115	4.6148	0.0019	4.6358	-0.0210	-0.45
Sn	50	66	116	4.625	0.0019	4.6398	-0.0148	-0.32
Sn	50	67	117	4.6302	0.0019	4.6437	-0.0135	-0.29
Sn	50	68	118	4.6393	0.0019	4.6476	-0.0083	-0.18
Sn	50	69	119	4.6438	0.002	4.6514	-0.0076	-0.16
Sn	50	70	120	4.6519	0.0021	4.6552	-0.0033	-0.07
Sn	50	71	121	4.6566	0.0021	4.6589	-0.0023	-0.05
Sn	50	72	122	4.6634	0.0022	4.6626	0.0008	0.02
Sn	50	73	123	4.6665	0.0023	4.6663	0.0002	0.00
Sn	50	74	124	4.6735	0.0023	4.6699	0.0036	0.08
Sn	50	75	125	4.6765	0.0026	4.6735	0.0030	0.07
Sn	50	76	126	4.6833	0.0043	4.6770	0.0063	0.14
Sn	50	77	127	4.6867	0.0048	4.6805	0.0062	0.13
Sn	50	78	128	4.6921	0.0054	4.6839	0.0082	0.17
Sn	50	79	129	4.6934	0.0058	4.6873	0.0061	0.13
Sn	50	80	130	4.7019	0.0066	4.6907	0.0112	0.24
Sn	50	81	131	4.7078	0.0073	4.6940	0.0138	0.29
Sn	50	82	132	4.7093	0.0076	4.6973	0.0120	0.26
Sb	51	70	121	4.6802	0.0026	4.6808	-0.0006	-0.01
Sb	51	72	123	4.6879	0.0025	4.6883	-0.0004	-0.01
Te	52	64	116	4.6847	0.0128	4.6823	0.0024	0.05
Te	52	66	118	4.6956	0.0105	4.6904	0.0052	0.11
Te	52	68	120	4.7038	0.0088	4.6983	0.0055	0.12
Te	52	70	122	4.7095	0.0031	4.7060	0.0035	0.07
Te	52	71	123	4.7117	0.0035	4.7098	0.0019	0.04
Te	52	72	124	4.7183	0.0029	4.7135	0.0048	0.10
Te	52	73	125	4.7204	0.003	4.7172	0.0032	0.07
Te	52	74	126	4.7266	0.0032	4.7208	0.0058	0.12
Te	52	76	128	4.7346	0.0029	4.7280	0.0066	0.14
Te	52	78	130	4.7423	0.0025	4.7350	0.0073	0.15
Te	52	80	132	4.75	0.0031	4.7418	0.0082	0.17
Te	52	82	134	4.7569	0.0041	4.7484	0.0085	0.18
Te	52	84	136	4.7815	0.0089	4.7550	0.0265	0.55
I	53	74	127	4.75	0.0081	4.7458	0.0042	0.09
Xe	54	62	116	4.7211	0.0096	4.7232	-0.0021	-0.04

Xe	54	64	118	4.7387	0.007	4.7316	0.0071	0.15
Xe	54	66	120	4.7509	0.0063	4.7397	0.0112	0.23
Xe	54	68	122	4.759	0.0059	4.7477	0.0113	0.24
Xe	54	70	124	4.7661	0.0055	4.7555	0.0106	0.22
Xe	54	72	126	4.7722	0.0052	4.7630	0.0092	0.19
Xe	54	73	127	4.7747	0.0038	4.7667	0.0080	0.17
Xe	54	74	128	4.7774	0.005	4.7704	0.0070	0.15
Xe	54	75	129	4.7775	0.005	4.7740	0.0035	0.07
Xe	54	76	130	4.7818	0.0049	4.7776	0.0042	0.09
Xe	54	77	131	4.7808	0.0049	4.7811	-0.0003	-0.01
Xe	54	78	132	4.7859	0.0048	4.7846	0.0013	0.03
Xe	54	79	133	4.7831	0.0048	4.7881	-0.0050	-0.10
Xe	54	80	134	4.7899	0.0047	4.7915	-0.0016	-0.03
Xe	54	82	136	4.7964	0.0047	4.7982	-0.0018	-0.04
Xe	54	83	137	4.8094	0.0049	4.8015	0.0079	0.16
Xe	54	84	138	4.8279	0.0079	4.8048	0.0231	0.48
Xe	54	85	139	4.8409	0.01	4.8081	0.0328	0.68
Xe	54	86	140	4.8566	0.0125	4.8113	0.0453	0.93
Xe	54	87	141	4.8694	0.0147	4.8144	0.0550	1.13
Xe	54	88	142	4.8841	0.0169	4.8176	0.0665	1.36
Xe	54	89	143	4.8942	0.0187	4.8207	0.0735	1.50
Xe	54	90	144	4.9082	0.0208	4.8238	0.0844	1.72
Xe	54	92	146	4.9315	0.0245	4.8298	0.1017	2.06
Cs	55	63	118	4.7832	0.0092	4.7515	0.0317	0.66
Cs	55	64	119	4.7896	0.0089	4.7557	0.0339	0.71
Cs	55	65	120	4.7915	0.0075	4.7598	0.0317	0.66
Cs	55	66	121	4.7769	0.0078	4.7639	0.0130	0.27
Cs	55	67	122	4.7773	0.007	4.7679	0.0094	0.20
Cs	55	68	123	4.782	0.007	4.7719	0.0101	0.21
Cs	55	69	124	4.7828	0.0062	4.7758	0.0070	0.15
Cs	55	70	125	4.788	0.0062	4.7797	0.0083	0.17
Cs	55	71	126	4.7872	0.0056	4.7835	0.0037	0.08
Cs	55	72	127	4.7936	0.0055	4.7873	0.0063	0.13
Cs	55	73	128	4.7921	0.0052	4.7910	0.0011	0.02
Cs	55	74	129	4.7981	0.005	4.7947	0.0034	0.07
Cs	55	75	130	4.7992	0.0049	4.7983	0.0009	0.02
Cs	55	76	131	4.8026	0.0047	4.8019	0.0007	0.01
Cs	55	77	132	4.8002	0.0046	4.8055	-0.0053	-0.11
Cs	55	78	133	4.8041	0.0046	4.8090	-0.0049	-0.10
Cs	55	79	134	4.8031	0.0046	4.8125	-0.0094	-0.19
Cs	55	80	135	4.8067	0.0047	4.8159	-0.0092	-0.19
Cs	55	81	136	4.8059	0.0052	4.8193	-0.0134	-0.28
Cs	55	82	137	4.8128	0.005	4.8226	-0.0098	-0.20
Cs	55	83	138	4.8255	0.005	4.8260	-0.0005	-0.01
Cs	55	84	139	4.8422	0.0069	4.8293	0.0129	0.27
Cs	55	85	140	4.8554	0.0088	4.8325	0.0229	0.47
Cs	55	86	141	4.8689	0.0108	4.8357	0.0332	0.68
Cs	55	87	142	4.8825	0.0132	4.8389	0.0436	0.89
Cs	55	88	143	4.8965	0.0151	4.8421	0.0544	1.11
Cs	55	89	144	4.9055	0.0161	4.8452	0.0603	1.23
Cs	55	90	145	4.9188	0.0191	4.8483	0.0705	1.43

Cs	55	91	146	4.9281	0.0193	4.8513	0.0768	1.56
Ba	56	64	120	4.8092	0.0058	4.7795	0.0297	0.62
Ba	56	65	121	4.8176	0.0052	4.7837	0.0339	0.70
Ba	56	66	122	4.8153	0.0054	4.7878	0.0275	0.57
Ba	56	67	123	4.8135	0.0055	4.7918	0.0217	0.45
Ba	56	68	124	4.8185	0.0052	4.7958	0.0227	0.47
Ba	56	69	125	4.8177	0.0052	4.7997	0.0180	0.37
Ba	56	70	126	4.8221	0.005	4.8036	0.0185	0.38
Ba	56	71	127	4.8204	0.0051	4.8075	0.0129	0.27
Ba	56	72	128	4.8255	0.0048	4.8112	0.0143	0.30
Ba	56	73	129	4.8248	0.0049	4.8150	0.0098	0.20
Ba	56	74	130	4.8283	0.0047	4.8187	0.0096	0.20
Ba	56	75	131	4.8276	0.0048	4.8223	0.0053	0.11
Ba	56	76	132	4.8303	0.0047	4.8259	0.0044	0.09
Ba	56	77	133	4.8286	0.0047	4.8295	-0.0009	-0.02
Ba	56	78	134	4.8322	0.0047	4.8330	-0.0008	-0.02
Ba	56	79	135	4.8294	0.0047	4.8365	-0.0071	-0.15
Ba	56	80	136	4.8334	0.0046	4.8400	-0.0066	-0.14
Ba	56	81	137	4.8314	0.0047	4.8434	-0.0120	-0.25
Ba	56	82	138	4.8378	0.0046	4.8467	-0.0089	-0.18
Ba	56	83	139	4.8513	0.0049	4.8501	0.0012	0.03
Ba	56	84	140	4.8684	0.0059	4.8534	0.0150	0.31
Ba	56	85	141	4.8807	0.0069	4.8566	0.0241	0.49
Ba	56	86	142	4.8953	0.0083	4.8599	0.0354	0.72
Ba	56	87	143	4.9087	0.0096	4.8631	0.0456	0.93
Ba	56	88	144	4.9236	0.0112	4.8662	0.0574	1.17
Ba	56	89	145	4.9345	0.0123	4.8694	0.0651	1.32
Ba	56	90	146	4.9479	0.0138	4.8725	0.0754	1.52
Ba	56	92	148	4.9731	0.0167	4.8786	0.0945	1.90
La	57	78	135	4.8488	0.006	4.8568	-0.0080	-0.16
La	57	80	137	4.8496	0.0053	4.8637	-0.0141	-0.29
La	57	81	138	4.8473	0.0051	4.8671	-0.0198	-0.41
La	57	82	139	4.855	0.0049	4.8705	-0.0155	-0.32
Ce	58	78	136	4.8739	0.0018	4.8802	-0.0063	-0.13
Ce	58	80	138	4.8737	0.0018	4.8872	-0.0135	-0.28
Ce	58	82	140	4.8771	0.0018	4.8940	-0.0169	-0.35
Ce	58	84	142	4.9063	0.002	4.9007	0.0056	0.11
Ce	58	86	144	4.9303	0.0024	4.9073	0.0230	0.47
Ce	58	88	146	4.959	0.0028	4.9137	0.0453	0.91
Ce	58	90	148	4.9893	0.0035	4.9200	0.0693	1.39
Pr	59	82	141	4.8919	0.005	4.9172	-0.0253	-0.52
Nd	60	72	132	4.9174	0.0026	4.9041	0.0133	0.27
Nd	60	74	134	4.9128	0.0026	4.9117	0.0011	0.02
Nd	60	75	135	4.9086	0.0026	4.9154	-0.0068	-0.14
Nd	60	76	136	4.9111	0.0026	4.9190	-0.0079	-0.16
Nd	60	77	137	4.908	0.0026	4.9227	-0.0147	-0.30
Nd	60	78	138	4.9123	0.0026	4.9262	-0.0139	-0.28
Nd	60	79	139	4.9076	0.0025	4.9298	-0.0222	-0.45
Nd	60	80	140	4.9101	0.0026	4.9333	-0.0232	-0.47
Nd	60	81	141	4.9057	0.0026	4.9367	-0.0310	-0.63
Nd	60	82	142	4.9123	0.0025	4.9402	-0.0279	-0.57

Nd	60	83	143	4.9254	0.0026	4.9435	-0.0181	-0.37
Nd	60	84	144	4.9421	0.0027	4.9469	-0.0048	-0.10
Nd	60	85	145	4.9535	0.0028	4.9502	0.0033	0.07
Nd	60	86	146	4.9696	0.003	4.9535	0.0161	0.32
Nd	60	88	148	4.9999	0.0036	4.9600	0.0399	0.80
Nd	60	90	150	5.04	0.0044	4.9663	0.0737	1.46
Sm	62	76	138	4.9599	0.0034	4.9639	-0.0040	-0.08
Sm	62	77	139	4.9556	0.0034	4.9676	-0.0120	-0.24
Sm	62	78	140	4.9565	0.0034	4.9712	-0.0147	-0.30
Sm	62	79	141	4.9517	0.0034	4.9748	-0.0231	-0.47
Sm	62	80	142	4.9518	0.0034	4.9783	-0.0265	-0.53
Sm	62	81	143	4.9479	0.0034	4.9818	-0.0339	-0.68
Sm	62	82	144	4.9524	0.0034	4.9852	-0.0328	-0.66
Sm	62	83	145	4.9651	0.0034	4.9886	-0.0235	-0.47
Sm	62	84	146	4.9808	0.0035	4.9920	-0.0112	-0.22
Sm	62	85	147	4.9892	0.0035	4.9953	-0.0061	-0.12
Sm	62	86	148	5.0042	0.0034	4.9986	0.0056	0.11
Sm	62	87	149	5.0134	0.0035	5.0019	0.0115	0.23
Sm	62	88	150	5.0387	0.0048	5.0052	0.0335	0.67
Sm	62	89	151	5.055	0.0057	5.0084	0.0466	0.92
Sm	62	90	152	5.0819	0.006	5.0115	0.0704	1.38
Sm	62	91	153	5.0925	0.0068	5.0147	0.0778	1.53
Sm	62	92	154	5.1053	0.0067	5.0178	0.0875	1.71
Eu	63	74	137	4.9762	0.0095	4.9785	-0.0023	-0.05
Eu	63	75	138	4.9779	0.0094	4.9823	-0.0044	-0.09
Eu	63	76	139	4.976	0.0093	4.9860	-0.0100	-0.20
Eu	63	77	140	4.9695	0.0091	4.9897	-0.0202	-0.41
Eu	63	78	141	4.9697	0.0091	4.9933	-0.0236	-0.47
Eu	63	79	142	4.9607	0.0091	4.9969	-0.0362	-0.73
Eu	63	80	143	4.9636	0.0091	5.0004	-0.0368	-0.74
Eu	63	81	144	4.9612	0.0091	5.0039	-0.0427	-0.86
Eu	63	82	145	4.9663	0.0091	5.0074	-0.0411	-0.83
Eu	63	83	146	4.9789	0.0092	5.0108	-0.0319	-0.64
Eu	63	84	147	4.9938	0.0094	5.0142	-0.0204	-0.41
Eu	63	85	148	5.0045	0.0097	5.0175	-0.0130	-0.26
Eu	63	86	149	5.0202	0.0103	5.0208	-0.0006	-0.01
Eu	63	87	150	5.0296	0.0108	5.0241	0.0055	0.11
Eu	63	88	151	5.0522	0.0046	5.0274	0.0248	0.49
Eu	63	89	152	5.1064	0.0066	5.0306	0.0758	1.48
Eu	63	90	153	5.1115	0.0062	5.0338	0.0777	1.52
Eu	63	91	154	5.1239	0.0079	5.0369	0.0870	1.70
Eu	63	92	155	5.1221	0.0069	5.0400	0.0821	1.60
Eu	63	93	156	5.1264	0.0071	5.0431	0.0833	1.62
Eu	63	94	157	5.1351	0.0075	5.0462	0.0889	1.73
Eu	63	95	158	5.1413	0.0078	5.0492	0.0921	1.79
Eu	63	96	159	5.1498	0.0084	5.0522	0.0976	1.89
Gd	64	81	145	4.9786	0.0077	5.0258	-0.0472	-0.95
Gd	64	82	146	4.9801	0.014	5.0292	-0.0491	-0.99
Gd	64	84	148	5.008	0.0171	5.0361	-0.0281	-0.56
Gd	64	86	150	5.0342	0.0159	5.0428	-0.0086	-0.17
Gd	64	88	152	5.0774	0.0048	5.0493	0.0281	0.55

Gd	64	90	154	5.1223	0.004	5.0557	0.0666	1.30
Gd	64	91	155	5.1319	0.0041	5.0589	0.0730	1.42
Gd	64	92	156	5.142	0.0042	5.0620	0.0800	1.55
Gd	64	93	157	5.1449	0.0042	5.0651	0.0798	1.55
Gd	64	94	158	5.1569	0.0043	5.0682	0.0887	1.72
Gd	64	96	160	5.1734	0.0044	5.0743	0.0991	1.92
Tb	65	82	147	4.9201	0.1508	5.0509	-0.1308	-2.66
Tb	65	83	148	4.9291	0.1507	5.0543	-0.1252	-2.54
Tb	65	84	149	4.9427	0.1506	5.0577	-0.1150	-2.33
Tb	65	85	150	4.9499	0.1505	5.0611	-0.1112	-2.25
Tb	65	86	151	4.963	0.1504	5.0645	-0.1015	-2.04
Tb	65	87	152	4.9689	0.1504	5.0678	-0.0989	-1.99
Tb	65	88	153	4.995	0.1502	5.0710	-0.0760	-1.52
Tb	65	89	154	5.0333	0.1501	5.0743	-0.0410	-0.81
Tb	65	90	155	5.0391	0.15	5.0775	-0.0384	-0.76
Tb	65	92	157	5.0489	0.15	5.0838	-0.0349	-0.69
Tb	65	94	159	5.06	0.15	5.0900	-0.0300	-0.59
Dy	66	80	146	5.0438	0.2389	5.0653	-0.0215	-0.43
Dy	66	82	148	5.0455	0.2389	5.0723	-0.0268	-0.53
Dy	66	83	149	5.0567	0.2394	5.0758	-0.0191	-0.38
Dy	66	84	150	5.0706	0.2413	5.0792	-0.0086	-0.17
Dy	66	85	151	5.0801	0.2435	5.0826	-0.0025	-0.05
Dy	66	86	152	5.095	0.2482	5.0859	0.0091	0.18
Dy	66	87	153	5.1035	0.2516	5.0892	0.0143	0.28
Dy	66	88	154	5.1241	0.2618	5.0925	0.0316	0.62
Dy	66	89	155	5.1457	0.2751	5.0958	0.0499	0.97
Dy	66	90	156	5.1622	0.2869	5.0990	0.0632	1.22
Dy	66	91	157	5.1709	0.2936	5.1022	0.0687	1.33
Dy	66	92	158	5.1815	0.3023	5.1053	0.0762	1.47
Dy	66	93	159	5.1825	0.3031	5.1085	0.0740	1.43
Dy	66	94	160	5.1951	0.3139	5.1115	0.0836	1.61
Dy	66	95	161	5.1962	0.0459	5.1146	0.0816	1.57
Dy	66	96	162	5.2074	0.0172	5.1177	0.0897	1.72
Dy	66	97	163	5.2099	0.012	5.1207	0.0892	1.71
Dy	66	98	164	5.2218	0.0106	5.1236	0.0982	1.88
Ho	67	84	151	5.0398	0.0354	5.1004	-0.0606	-1.20
Ho	67	85	152	5.0614	0.0343	5.1038	-0.0424	-0.84
Ho	67	86	153	5.076	0.0339	5.1071	-0.0311	-0.61
Ho	67	87	154	5.0856	0.0333	5.1105	-0.0249	-0.49
Ho	67	88	155	5.1076	0.0326	5.1138	-0.0062	-0.12
Ho	67	89	156	5.1156	0.0326	5.1170	-0.0014	-0.03
Ho	67	90	157	5.1535	0.0316	5.1203	0.0332	0.64
Ho	67	91	158	5.1571	0.0316	5.1235	0.0336	0.65
Ho	67	92	159	5.1675	0.0314	5.1266	0.0409	0.79
Ho	67	93	160	5.1662	0.0315	5.1298	0.0364	0.71
Ho	67	94	161	5.1785	0.0313	5.1329	0.0456	0.88
Ho	67	95	162	5.1817	0.0313	5.1359	0.0458	0.88
Ho	67	96	163	5.1907	0.0313	5.1390	0.0517	1.00
Ho	67	98	165	5.2022	0.0312	5.1450	0.0572	1.10
Er	68	82	150	5.0548	0.0254	5.1144	-0.0596	-1.18
Er	68	84	152	5.0843	0.0257	5.1214	-0.0371	-0.73

Er	68	86	154	5.1129	0.0268	5.1282	-0.0153	-0.30
Er	68	88	156	5.1429	0.0285	5.1348	0.0081	0.16
Er	68	90	158	5.1761	0.0312	5.1413	0.0348	0.67
Er	68	92	160	5.2045	0.0336	5.1477	0.0568	1.09
Er	68	94	162	5.2246	0.004	5.1540	0.0706	1.35
Er	68	96	164	5.2389	0.0035	5.1601	0.0788	1.50
Er	68	98	166	5.2516	0.0031	5.1661	0.0855	1.63
Er	68	99	167	5.256	0.0031	5.1691	0.0869	1.65
Er	68	100	168	5.2644	0.0035	5.1721	0.0923	1.75
Er	68	102	170	5.2789	0.0041	5.1779	0.1010	1.91
Tm	69	84	153	5.0643	0.019	5.1421	-0.0778	-1.54
Tm	69	85	154	5.0755	0.0166	5.1456	-0.0701	-1.38
Tm	69	87	156	5.0976	0.0135	5.1523	-0.0547	-1.07
Tm	69	88	157	5.114	0.0074	5.1556	-0.0416	-0.81
Tm	69	89	158	5.1235	0.0069	5.1589	-0.0354	-0.69
Tm	69	90	159	5.1392	0.006	5.1621	-0.0229	-0.45
Tm	69	91	160	5.1504	0.0055	5.1654	-0.0150	-0.29
Tm	69	92	161	5.1616	0.005	5.1685	-0.0069	-0.13
Tm	69	93	162	5.1713	0.0048	5.1717	-0.0004	-0.01
Tm	69	94	163	5.1849	0.0042	5.1748	0.0101	0.19
Tm	69	95	164	5.1906	0.0042	5.1779	0.0127	0.24
Tm	69	96	165	5.2004	0.0038	5.1810	0.0194	0.37
Tm	69	97	166	5.2046	0.0038	5.1840	0.0206	0.40
Tm	69	98	167	5.2129	0.0036	5.1870	0.0259	0.50
Tm	69	99	168	5.217	0.0036	5.1900	0.0270	0.52
Tm	69	100	169	5.2256	0.0035	5.1930	0.0326	0.62
Tm	69	101	170	5.2303	0.0036	5.1959	0.0344	0.66
Tm	69	102	171	5.2388	0.0037	5.1988	0.0400	0.76
Tm	69	103	172	5.2411	0.0052	5.2017	0.0394	0.75
Yb	70	82	152	5.0423	0.0146	5.1557	-0.1134	-2.25
Yb	70	84	154	5.0875	0.0105	5.1627	-0.0752	-1.48
Yb	70	85	155	5.104	0.011	5.1661	-0.0621	-1.22
Yb	70	86	156	5.1219	0.0103	5.1695	-0.0476	-0.93
Yb	70	87	157	5.1324	0.01	5.1729	-0.0405	-0.79
Yb	70	88	158	5.1498	0.0088	5.1762	-0.0264	-0.51
Yb	70	89	159	5.1629	0.0084	5.1795	-0.0166	-0.32
Yb	70	90	160	5.1781	0.0076	5.1828	-0.0047	-0.09
Yb	70	91	161	5.1889	0.0072	5.1860	0.0029	0.06
Yb	70	92	162	5.2054	0.0067	5.1892	0.0162	0.31
Yb	70	93	163	5.2157	0.0064	5.1923	0.0234	0.45
Yb	70	94	164	5.2307	0.006	5.1955	0.0352	0.67
Yb	70	95	165	5.2399	0.0058	5.1986	0.0413	0.79
Yb	70	96	166	5.2525	0.0057	5.2017	0.0508	0.97
Yb	70	97	167	5.2621	0.0056	5.2047	0.0574	1.09
Yb	70	98	168	5.2702	0.0056	5.2077	0.0625	1.19
Yb	70	99	169	5.2771	0.0056	5.2107	0.0664	1.26
Yb	70	100	170	5.2853	0.0056	5.2137	0.0716	1.35
Yb	70	101	171	5.2906	0.0057	5.2166	0.0740	1.40
Yb	70	102	172	5.2995	0.0058	5.2195	0.0800	1.51
Yb	70	103	173	5.3046	0.0059	5.2224	0.0822	1.55
Yb	70	104	174	5.3108	0.006	5.2253	0.0855	1.61

Yb	70	105	175	5.3135	0.0061	5.2281	0.0854	1.61
Yb	70	106	176	5.3215	0.0062	5.2310	0.0905	1.70
Lu	71	90	161	5.2293	0.032	5.2032	0.0261	0.50
Lu	71	91	162	5.2398	0.0317	5.2064	0.0334	0.64
Lu	71	92	163	5.2567	0.0312	5.2096	0.0471	0.90
Lu	71	93	164	5.2677	0.031	5.2128	0.0549	1.04
Lu	71	94	165	5.283	0.0307	5.2159	0.0671	1.27
Lu	71	95	166	5.2972	0.0305	5.2190	0.0782	1.48
Lu	71	96	167	5.3108	0.0303	5.2221	0.0887	1.67
Lu	71	97	168	5.3227	0.0302	5.2252	0.0975	1.83
Lu	71	98	169	5.329	0.0302	5.2282	0.1008	1.89
Lu	71	99	170	5.3364	0.0302	5.2312	0.1052	1.97
Lu	71	100	171	5.3436	0.0302	5.2342	0.1094	2.05
Lu	71	101	172	5.3486	0.0302	5.2371	0.1115	2.08
Lu	71	102	173	5.3577	0.0303	5.2401	0.1176	2.20
Lu	71	103	174	5.3634	0.0303	5.2430	0.1204	2.25
Lu	71	104	175	5.37	0.0304	5.2458	0.1242	2.31
Lu	71	105	176	5.3739	0.0304	5.2487	0.1252	2.33
Lu	71	106	177	5.3815	0.0305	5.2515	0.1300	2.42
Lu	71	107	178	5.3857	0.0306	5.2543	0.1314	2.44
Lu	71	108	179	5.3917	0.0307	5.2571	0.1346	2.50
Hf	72	98	170	5.2898	0.0055	5.2485	0.0413	0.78
Hf	72	99	171	5.3041	0.0049	5.2515	0.0526	0.99
Hf	72	100	172	5.3065	0.0043	5.2545	0.0520	0.98
Hf	72	101	173	5.314	0.0038	5.2575	0.0565	1.06
Hf	72	102	174	5.3201	0.0035	5.2604	0.0597	1.12
Hf	72	103	175	5.3191	0.0036	5.2633	0.0558	1.05
Hf	72	104	176	5.3286	0.0032	5.2662	0.0624	1.17
Hf	72	105	177	5.3309	0.0031	5.2690	0.0619	1.16
Hf	72	106	178	5.3371	0.0031	5.2719	0.0652	1.22
Hf	72	107	179	5.3408	0.0031	5.2747	0.0661	1.24
Hf	72	108	180	5.347	0.0032	5.2775	0.0695	1.30
Hf	72	110	182	5.3516	0.0036	5.2830	0.0686	1.28
Ta	73	108	181	5.3507	0.0034	5.2976	0.0531	0.99
W	74	106	180	5.3491	0.0022	5.3120	0.0371	0.69
W	74	108	182	5.3559	0.0017	5.3176	0.0383	0.71
W	74	109	183	5.3611	0.002	5.3204	0.0407	0.76
W	74	110	184	5.3658	0.0023	5.3232	0.0426	0.79
W	74	112	186	5.3743	0.0026	5.3286	0.0457	0.85
Re	75	110	185	5.3596	0.0172	5.3429	0.0167	0.31
Re	75	112	187	5.3698	0.0173	5.3484	0.0214	0.40
Os	76	108	184	5.3823	0.0022	5.3570	0.0253	0.47
Os	76	110	186	5.3909	0.0017	5.3626	0.0283	0.53
Os	76	111	187	5.3933	0.0018	5.3653	0.0280	0.52
Os	76	112	188	5.3993	0.0011	5.3680	0.0313	0.58
Os	76	113	189	5.4016	0.0012	5.3708	0.0308	0.57
Os	76	114	190	5.4062	0.0013	5.3734	0.0328	0.61
Os	76	116	192	5.4126	0.0015	5.3788	0.0338	0.63
Ir	77	105	182	5.3705	0.1061	5.3678	0.0027	0.05
Ir	77	106	183	5.378	0.1061	5.3707	0.0073	0.14
Ir	77	107	184	5.3805	0.1061	5.3736	0.0069	0.13

Ir	77	108	185	5.3854	0.1061	5.3764	0.0090	0.17
Ir	77	109	186	5.39	0.1061	5.3792	0.0108	0.20
Ir	77	110	187	5.3812	0.1061	5.3820	-0.0008	-0.01
Ir	77	111	188	5.3838	0.1061	5.3848	-0.0010	-0.02
Ir	77	112	189	5.3898	0.1061	5.3875	0.0023	0.04
Ir	77	114	191	5.3968	0.1061	5.3929	0.0039	0.07
Ir	77	116	193	5.4032	0.1061	5.3982	0.0050	0.09
Pt	78	100	178	5.3728	0.0066	5.3722	0.0006	0.01
Pt	78	101	179	5.3915	0.005	5.3753	0.0162	0.30
Pt	78	102	180	5.3891	0.0049	5.3782	0.0109	0.20
Pt	78	103	181	5.3996	0.0041	5.3812	0.0184	0.34
Pt	78	104	182	5.3969	0.0041	5.3841	0.0128	0.24
Pt	78	105	183	5.4038	0.0036	5.3871	0.0167	0.31
Pt	78	106	184	5.4015	0.0036	5.3899	0.0116	0.21
Pt	78	107	185	5.4148	0.0028	5.3928	0.0220	0.41
Pt	78	108	186	5.4037	0.0036	5.3956	0.0081	0.15
Pt	78	109	187	5.4063	0.0037	5.3984	0.0079	0.15
Pt	78	110	188	5.4053	0.0034	5.4012	0.0041	0.08
Pt	78	111	189	5.406	0.0035	5.4040	0.0020	0.04
Pt	78	112	190	5.4108	0.003	5.4068	0.0040	0.07
Pt	78	113	191	5.4102	0.0031	5.4095	0.0007	0.01
Pt	78	114	192	5.4169	0.0028	5.4122	0.0047	0.09
Pt	78	115	193	5.4191	0.0027	5.4149	0.0042	0.08
Pt	78	116	194	5.4236	0.0025	5.4175	0.0061	0.11
Pt	78	117	195	5.427	0.0026	5.4202	0.0068	0.13
Pt	78	118	196	5.4307	0.0027	5.4228	0.0079	0.15
Pt	78	120	198	5.4383	0.0032	5.4280	0.0103	0.19
Au	79	104	183	5.4247	0.0043	5.4032	0.0215	0.40
Au	79	105	184	5.4306	0.0041	5.4061	0.0245	0.45
Au	79	106	185	5.4296	0.0041	5.4090	0.0206	0.38
Au	79	107	186	5.4354	0.0039	5.4118	0.0236	0.43
Au	79	108	187	5.4018	0.0058	5.4147	-0.0129	-0.24
Au	79	109	188	5.4049	0.0055	5.4175	-0.0126	-0.23
Au	79	110	189	5.4084	0.0052	5.4203	-0.0119	-0.22
Au	79	111	190	5.4109	0.0049	5.4231	-0.0122	-0.23
Au	79	112	191	5.4147	0.0046	5.4258	-0.0111	-0.21
Au	79	113	192	5.4179	0.0044	5.4286	-0.0107	-0.20
Au	79	114	193	5.4221	0.0042	5.4313	-0.0092	-0.17
Au	79	115	194	5.4252	0.004	5.4340	-0.0088	-0.16
Au	79	116	195	5.4298	0.004	5.4366	-0.0068	-0.13
Au	79	117	196	5.4332	0.0039	5.4393	-0.0061	-0.11
Au	79	118	197	5.4371	0.0038	5.4419	-0.0048	-0.09
Au	79	119	198	5.44	0.0038	5.4445	-0.0045	-0.08
Au	79	120	199	5.4454	0.0039	5.4471	-0.0017	-0.03
Hg	80	101	181	5.4364	0.0032	5.4131	0.0233	0.43
Hg	80	102	182	5.3833	0.0052	5.4161	-0.0328	-0.61
Hg	80	103	183	5.4405	0.0031	5.4191	0.0214	0.39
Hg	80	104	184	5.3949	0.0047	5.4220	-0.0271	-0.50
Hg	80	105	185	5.4397	0.0031	5.4249	0.0148	0.27
Hg	80	106	186	5.4017	0.0043	5.4279	-0.0262	-0.48
Hg	80	107	187	5.4046	0.0042	5.4307	-0.0261	-0.48

Hg	80	108	188	5.4085	0.004	5.4336	-0.0251	-0.46
Hg	80	109	189	5.41	0.004	5.4364	-0.0264	-0.49
Hg	80	110	190	5.4158	0.0037	5.4392	-0.0234	-0.43
Hg	80	111	191	5.4171	0.0037	5.4420	-0.0249	-0.46
Hg	80	112	192	5.4232	0.0035	5.4448	-0.0216	-0.40
Hg	80	113	193	5.4238	0.0035	5.4475	-0.0237	-0.44
Hg	80	114	194	5.4309	0.0033	5.4502	-0.0193	-0.36
Hg	80	115	195	5.4345	0.0032	5.4529	-0.0184	-0.34
Hg	80	116	196	5.4385	0.0031	5.4556	-0.0171	-0.31
Hg	80	117	197	5.4412	0.0031	5.4583	-0.0171	-0.31
Hg	80	118	198	5.4463	0.0031	5.4609	-0.0146	-0.27
Hg	80	119	199	5.4474	0.0031	5.4635	-0.0161	-0.30
Hg	80	120	200	5.4551	0.0031	5.4661	-0.0110	-0.20
Hg	80	121	201	5.4581	0.0032	5.4687	-0.0106	-0.19
Hg	80	122	202	5.4648	0.0033	5.4712	-0.0064	-0.12
Hg	80	123	203	5.4679	0.0035	5.4738	-0.0059	-0.11
Hg	80	124	204	5.4744	0.0036	5.4763	-0.0019	-0.03
Hg	80	125	205	5.4776	0.0038	5.4788	-0.0012	-0.02
Hg	80	126	206	5.4837	0.004	5.4813	0.0024	0.04
Tl	81	107	188	5.4017	0.0072	5.4494	-0.0477	-0.88
Tl	81	109	190	5.4121	0.0056	5.4551	-0.0430	-0.80
Tl	81	110	191	5.4169	0.0048	5.4580	-0.0411	-0.76
Tl	81	111	192	5.4191	0.0051	5.4607	-0.0416	-0.77
Tl	81	112	193	5.4243	0.0042	5.4635	-0.0392	-0.72
Tl	81	113	194	5.4259	0.0046	5.4663	-0.0404	-0.74
Tl	81	114	195	5.4325	0.0039	5.4690	-0.0365	-0.67
Tl	81	115	196	5.4327	0.0042	5.4717	-0.0390	-0.72
Tl	81	116	197	5.4388	0.0036	5.4744	-0.0356	-0.65
Tl	81	117	198	5.4396	0.0036	5.4770	-0.0374	-0.69
Tl	81	118	199	5.4479	0.0031	5.4797	-0.0318	-0.58
Tl	81	119	200	5.4491	0.0031	5.4823	-0.0332	-0.61
Tl	81	120	201	5.4573	0.0029	5.4849	-0.0276	-0.51
Tl	81	121	202	5.4595	0.0027	5.4875	-0.0280	-0.51
Tl	81	122	203	5.4666	0.0027	5.4901	-0.0235	-0.43
Tl	81	123	204	5.4704	0.0028	5.4926	-0.0222	-0.41
Tl	81	124	205	5.4759	0.0026	5.4951	-0.0192	-0.35
Tl	81	126	207	5.4853	0.0027	5.5001	-0.0148	-0.27
Tl	81	127	208	5.4946	0.0028	5.5026	-0.0080	-0.15
Pb	82	100	182	5.3788	0.0035	5.4472	-0.0684	-1.27
Pb	82	101	183	5.3869	0.003	5.4503	-0.0634	-1.18
Pb	82	102	184	5.393	0.0029	5.4533	-0.0603	-1.12
Pb	82	103	185	5.3984	0.0028	5.4563	-0.0579	-1.07
Pb	82	104	186	5.4027	0.0027	5.4592	-0.0565	-1.05
Pb	82	105	187	5.4079	0.0026	5.4622	-0.0543	-1.00
Pb	82	106	188	5.4139	0.0025	5.4651	-0.0512	-0.95
Pb	82	107	189	5.4177	0.0024	5.4680	-0.0503	-0.93
Pb	82	108	190	5.4222	0.0023	5.4709	-0.0487	-0.90
Pb	82	109	191	5.4229	0.0026	5.4737	-0.0508	-0.94
Pb	82	110	192	5.43	0.0025	5.4765	-0.0465	-0.86
Pb	82	111	193	5.431	0.0023	5.4793	-0.0483	-0.89
Pb	82	112	194	5.4372	0.0023	5.4821	-0.0449	-0.83

Pb	82	113	195	5.4389	0.0045	5.4849	-0.0460	-0.85
Pb	82	114	196	5.4444	0.0024	5.4876	-0.0432	-0.79
Pb	82	115	197	5.4446	0.0024	5.4903	-0.0457	-0.84
Pb	82	116	198	5.4524	0.0022	5.4930	-0.0406	-0.74
Pb	82	117	199	5.4529	0.0022	5.4957	-0.0428	-0.78
Pb	82	118	200	5.4611	0.002	5.4983	-0.0372	-0.68
Pb	82	119	201	5.4629	0.0019	5.5010	-0.0381	-0.70
Pb	82	120	202	5.4705	0.0017	5.5036	-0.0331	-0.60
Pb	82	121	203	5.4727	0.0017	5.5062	-0.0335	-0.61
Pb	82	122	204	5.4803	0.0014	5.5087	-0.0284	-0.52
Pb	82	123	205	5.4828	0.0015	5.5113	-0.0285	-0.52
Pb	82	124	206	5.4902	0.0014	5.5138	-0.0236	-0.43
Pb	82	125	207	5.4943	0.0014	5.5163	-0.0220	-0.40
Pb	82	126	208	5.5012	0.0013	5.5188	-0.0176	-0.32
Pb	82	127	209	5.51	0.0014	5.5213	-0.0113	-0.21
Pb	82	128	210	5.5208	0.0016	5.5238	-0.0030	-0.05
Pb	82	129	211	5.529	0.0017	5.5262	0.0028	0.05
Pb	82	130	212	5.5396	0.0019	5.5287	0.0109	0.20
Pb	82	132	214	5.5577	0.0023	5.5335	0.0242	0.44
Bi	83	119	202	5.484	0.0912	5.5194	-0.0354	-0.65
Bi	83	120	203	5.4911	0.0911	5.5220	-0.0309	-0.56
Bi	83	121	204	5.4934	0.091	5.5246	-0.0312	-0.57
Bi	83	122	205	5.5008	0.0909	5.5272	-0.0264	-0.48
Bi	83	123	206	5.5034	0.0909	5.5298	-0.0264	-0.48
Bi	83	124	207	5.5103	0.0907	5.5323	-0.0220	-0.40
Bi	83	125	208	5.5147	0.0907	5.5349	-0.0202	-0.37
Bi	83	126	209	5.5211	0.0906	5.5374	-0.0163	-0.29
Bi	83	127	210	5.53	0.0904	5.5399	-0.0099	-0.18
Bi	83	129	212	5.5489	0.0901	5.5448	0.0041	0.07
Bi	83	130	213	5.5586	0.09	5.5472	0.0114	0.20
Po	84	108	192	5.522	0.0178	5.5075	0.0145	0.26
Po	84	110	194	5.5167	0.0178	5.5132	0.0035	0.06
Po	84	112	196	5.5136	0.0178	5.5188	-0.0052	-0.09
Po	84	114	198	5.5146	0.0178	5.5243	-0.0097	-0.18
Po	84	116	200	5.5199	0.0178	5.5298	-0.0099	-0.18
Po	84	118	202	5.5281	0.0177	5.5351	-0.0070	-0.13
Po	84	120	204	5.5378	0.0177	5.5404	-0.0026	-0.05
Po	84	121	205	5.5389	0.0177	5.5430	-0.0041	-0.07
Po	84	122	206	5.548	0.0177	5.5456	0.0024	0.04
Po	84	123	207	5.5501	0.0177	5.5481	0.0020	0.04
Po	84	124	208	5.5584	0.0176	5.5507	0.0077	0.14
Po	84	125	209	5.5628	0.0176	5.5532	0.0096	0.17
Po	84	126	210	5.5704	0.0176	5.5557	0.0147	0.26
Po	84	132	216	5.6359	0.0174	5.5704	0.0655	1.16
Po	84	134	218	5.6558	0.0173	5.5752	0.0806	1.42
Rn	86	116	202	5.5521	0.0181	5.5659	-0.0138	-0.25
Rn	86	118	204	5.5568	0.018	5.5713	-0.0145	-0.26
Rn	86	119	205	5.5569	0.018	5.5739	-0.0170	-0.31
Rn	86	120	206	5.564	0.0178	5.5766	-0.0126	-0.23
Rn	86	121	207	5.5652	0.0178	5.5792	-0.0140	-0.25
Rn	86	122	208	5.5725	0.0177	5.5818	-0.0093	-0.17

Rn	86	123	209	5.5743	0.0177	5.5844	-0.0101	-0.18
Rn	86	124	210	5.5813	0.0177	5.5869	-0.0056	-0.10
Rn	86	125	211	5.585	0.0176	5.5895	-0.0045	-0.08
Rn	86	126	212	5.5915	0.0176	5.5920	-0.0005	-0.01
Rn	86	132	218	5.654	0.0187	5.6068	0.0472	0.83
Rn	86	133	219	5.6648	0.0191	5.6092	0.0556	0.98
Rn	86	134	220	5.6731	0.0194	5.6116	0.0615	1.08
Rn	86	135	221	5.6834	0.0199	5.6140	0.0694	1.22
Rn	86	136	222	5.6915	0.0203	5.6163	0.0752	1.32
Fr	87	120	207	5.572	0.0176	5.5944	-0.0224	-0.40
Fr	87	121	208	5.5729	0.0176	5.5971	-0.0242	-0.43
Fr	87	122	209	5.5799	0.0176	5.5997	-0.0198	-0.35
Fr	87	123	210	5.5818	0.0176	5.6023	-0.0205	-0.37
Fr	87	124	211	5.5882	0.0176	5.6048	-0.0166	-0.30
Fr	87	125	212	5.5915	0.0176	5.6074	-0.0159	-0.28
Fr	87	126	213	5.5977	0.0176	5.6099	-0.0122	-0.22
Fr	87	133	220	5.6688	0.0177	5.6272	0.0416	0.73
Fr	87	134	221	5.679	0.0177	5.6296	0.0494	0.87
Fr	87	135	222	5.689	0.0177	5.6319	0.0571	1.00
Fr	87	136	223	5.6951	0.0178	5.6343	0.0608	1.07
Fr	87	137	224	5.7061	0.0178	5.6367	0.0694	1.22
Fr	87	138	225	5.7112	0.0178	5.6390	0.0722	1.26
Fr	87	139	226	5.719	0.0178	5.6413	0.0777	1.36
Fr	87	140	227	5.7335	0.0179	5.6436	0.0899	1.57
Fr	87	141	228	5.7399	0.0179	5.6459	0.0940	1.64
Ra	88	120	208	5.585	0.0183	5.6122	-0.0272	-0.49
Ra	88	121	209	5.5853	0.0182	5.6148	-0.0295	-0.53
Ra	88	122	210	5.5917	0.018	5.6174	-0.0257	-0.46
Ra	88	123	211	5.5929	0.0179	5.6200	-0.0271	-0.48
Ra	88	124	212	5.5991	0.0177	5.6226	-0.0235	-0.42
Ra	88	125	213	5.602	0.0177	5.6251	-0.0231	-0.41
Ra	88	126	214	5.6079	0.0177	5.6277	-0.0198	-0.35
Ra	88	132	220	5.6683	0.0215	5.6426	0.0257	0.45
Ra	88	133	221	5.6795	0.0228	5.6450	0.0345	0.61
Ra	88	134	222	5.6874	0.0239	5.6474	0.0400	0.70
Ra	88	135	223	5.6973	0.0253	5.6498	0.0475	0.83
Ra	88	136	224	5.7046	0.0263	5.6521	0.0525	0.92
Ra	88	137	225	5.715	0.0279	5.6545	0.0605	1.06
Ra	88	138	226	5.7211	0.0288	5.6568	0.0643	1.12
Ra	88	139	227	5.7283	0.03	5.6592	0.0691	1.21
Ra	88	140	228	5.737	0.0315	5.6615	0.0755	1.32
Ra	88	141	229	5.7455	0.0329	5.6638	0.0817	1.42
Ra	88	142	230	5.7551	0.0346	5.6661	0.0890	1.55
Ra	88	144	232	5.7714	0.0375	5.6706	0.1008	1.75
Th	90	137	227	5.7404	0.0165	5.6897	0.0507	0.88
Th	90	138	228	5.7488	0.0152	5.6921	0.0567	0.99
Th	90	139	229	5.7557	0.0143	5.6944	0.0613	1.06
Th	90	140	230	5.767	0.0131	5.6967	0.0703	1.22
Th	90	142	232	5.7848	0.0124	5.7014	0.0834	1.44
U	92	141	233	5.8203	0.0049	5.7338	0.0865	1.49
U	92	142	234	5.8291	0.0052	5.7361	0.0930	1.60

U	92	143	235	5.8337	0.0041	5.7384	0.0953	1.63
U	92	144	236	5.8431	0.0038	5.7407	0.1024	1.75
U	92	146	238	5.8571	0.0033	5.7452	0.1119	1.91
Pu	94	144	238	5.8535	0.0378	5.7749	0.0786	1.34
Pu	94	145	239	5.8601	0.0378	5.7772	0.0829	1.42
Pu	94	146	240	5.8701	0.0379	5.7794	0.0907	1.54
Pu	94	147	241	5.8748	0.0379	5.7817	0.0931	1.59
Pu	94	148	242	5.8823	0.038	5.7839	0.0984	1.67
Pu	94	150	244	5.8948	0.0382	5.7883	0.1065	1.81
Am	95	146	241	5.8928	0.0042	5.7964	0.0964	1.64
Am	95	148	243	5.9048	0.0035	5.8008	0.1040	1.76
Cm	96	146	242	5.8285	0.0192	5.8132	0.0153	0.26
Cm	96	148	244	5.8429	0.0181	5.8177	0.0252	0.43
Cm	96	149	245	5.8475	0.0182	5.8199	0.0276	0.47
Cm	96	150	246	5.8562	0.0184	5.8221	0.0341	0.58
Cm	96	152	248	5.8687	0.0193	5.8265	0.0422	0.72

6. Python Program for Nuclear Mass Radius Calculation

This section provides a Python implementation of both the advanced nuclear radius formula and the proposed minimalistic formula incorporating the separate contributions of proton and neutron numbers with an empirical scaling factor. The program also calculates the percentage error between the two formulas to evaluate the accuracy of the simpler approach.

Proposed minimalistic formula reflects the intimate connection between nuclear structure and unified gravitational-electromagnetic interactions at the nuclear scale. It explicitly accounts for distinct mass distributions of protons and neutrons through terms $Z^{1/3}$ and $N^{1/3}$, providing a physically grounded representation of nuclear size. This approach, rooted in fundamental physics, demonstrates the 4G model’s potential for predictive power by bridging microscopic nuclear dimensions and universal constants.

```
import math
import csv
import os
import matplotlib.pyplot as plt

def calculate_nuclear_radius_advanced(Z, A):
    N = A - Z
    I = (N - Z) / A

    if (Z % 2 == 0) and (N % 2 == 0):
        delta = 0
    elif (Z % 2 == 1) and (N % 2 == 1):
        delta = 0.5
    elif (Z % 2 == 1) and (N % 2 == 0):
        delta = 1
    elif (Z % 2 == 0) and (N % 2 == 1):
        delta = -1
    else:
        delta = 0
```

```

magic_numbers = [2, 8, 20, 28, 50, 82, 126]
S = 0.1 if any(abs(A - m) <= 2 for m in magic_numbers) else 0

a = 1.25
b = -0.15
c = 0.05
d = -0.8
e = 0.2
f = 0.15

R = a * A ** (1/3) + b * I + c * delta + d * A ** (-1/3) + e + f * S
return R # precision kept for calculations

def calculate_estimated_radii(Z, A):
    N = A - Z
    estimated_charge_radius = 0.62 * (pow(Z,1/3)+pow(Z*N,1/9))
    sms = int((2 * Z) + (0.0064 * Z * Z))
    mass_distribution_factor = 1 + (pow(pow(Z*N,0.5), 2/3) * 0.0073) + (0.1152*(1+pow((pow(N*N,0.5)-
2)/A,0.5)))
    mass_distribution_factor = mass_distribution_factor * pow(A/sms,0.25)
    estimated_mass_radius = mass_distribution_factor * estimated_charge_radius
    return N, estimated_charge_radius, mass_distribution_factor, estimated_mass_radius

def get_positive_integer(prompt):
    while True:
        try:
            value = int(input(prompt))
            if value > 0:
                return value
            else:
                print("Please enter a positive integer greater than zero.")
        except ValueError:
            print("Invalid input. Please enter a positive integer.")

def save_results_to_csv(Z, A_values, neutrons, advanced_radii, estimated_charge_radii,
mass_distribution_factors, estimated_mass_radii, filename_csv):
    with open(filename_csv, mode='w', newline='') as file:
        writer = csv.writer(file)
        writer.writerow([
            "Z", "N", "A", "Advanced Mass Radius (fm)",
            "Estimated Nuclear Charge Radius (fm)",
            "Mass Distribution Factor",
            "Estimated Mass Radius (fm)",

```

```

        "Percentage Error (%)"
    ])

    error_sum = 0
    count = 0

    for i, A in enumerate(A_values):
        radius_adv = advanced_radrii[i]
        radius_charge = estimated_charge_radrii[i]
        mdf = mass_distribution_factors[i]
        radius_mass = estimated_mass_radrii[i]
        neutron = neutrons[i]
        percent_error = abs((radius_mass - radius_adv) / radius_adv) * 100

        writer.writerow([
            Z, neutron, A,
            round(radius_adv, 4),
            round(radius_charge, 4),
            round(mdf, 5),
            round(radius_mass, 4),
            round(percent_error, 3)
        ])

        error_sum += percent_error
        count += 1

    avg_error = error_sum / count
    writer.writerow([])
    writer.writerow(["Average % Error", "", "", "", "", "", "", round(avg_error, 3)])

    print(f'Data has been saved to '{filename_csv}'.')

def plot_nuclear_radrii(Z, A_values, advanced_radrii, estimated_charge_radrii, estimated_mass_radrii,
filename_graph):
    fig, ax = plt.subplots(figsize=(10, 6))
    ax.plot(A_values, advanced_radrii, label='Advanced Mass Radius', marker='o', color='green')
    ax.plot(A_values, estimated_mass_radrii, label='Estimated Mass Radius', marker='x', color='red')
    ax.plot(A_values, estimated_charge_radrii, label='Estimated Nuclear Charge Radius', marker='^',
color='black')

    ax.set_xlabel('Mass Number (A)')
    ax.set_ylabel('Nuclear Radius (fm)')
    ax.set_title(f'Proton and neutron number dependent nuclear mass radrii for the isotpes of Z = {Z}')
    ax.legend()

```

```

ax.grid(True)

# Set border thickness for all four axis spines
for spine in ax.spines.values():
    spine.set_linewidth(1.5)

# Add border around entire figure
fig.patch.set_linewidth(2)
fig.patch.set_edgecolor('black')
fig.patch.set_facecolor('white') # Keep background white

plt.tight_layout()
plt.savefig(filename_graph)
print(f'Graph has been saved as '{filename_graph}'.')

plt.show()

def main():
    print("Current working directory:", os.getcwd())
    Z = get_positive_integer("Enter proton number Z: ")
    A_lower = 2 * Z
    A_upper = int(3.5 * Z)

    A_values = list(range(A_lower, A_upper + 1))
    neutrons = []
    advanced_radii = []
    estimated_charge_radii = []
    mass_distribution_factors = []
    estimated_mass_radii = []

    for A in A_values:
        N, charge_r, mdf, mass_r = calculate_estimated_radii(Z, A)
        neutrons.append(N)
        estimated_charge_radii.append(charge_r)
        mass_distribution_factors.append(mdf)
        estimated_mass_radii.append(mass_r)
        advanced_radii.append(calculate_nuclear_radius_advanced(Z, A))

    filename_csv = f'nuclear_radius_output_Z{Z}.csv'
    filename_graph = f'nuclear_radius_plot_Z{Z}.png'

    save_results_to_csv(Z, A_values, neutrons, advanced_radii, estimated_charge_radii,
mass_distribution_factors, estimated_mass_radii, filename_csv)

```

```
plot_nuclear_radii(Z, A_values, advanced_radii, estimated_charge_radii, estimated_mass_radii,
filename_graph)

if __name__ == "__main__":
    main()
```

7. Discussion

The present study introduces a simplified yet physically insightful formula for estimating nuclear mass and charge radii, founded on the 4G model of final unification. This model fundamentally distinguishes itself by emphasizing the distinct contributions of protons and neutrons to the nuclear radius through separate cubic root dependencies, $Z^{1/3}$ and $N^{1/3}$. This separation aligns with the physical reality that protons and neutrons have different spatial distributions and roles within the nucleus due to their differing charges, masses, and interactions.

7.1. Physical Rationale for Separate Contributions

Traditional empirical formulas often treat the nucleus as a single entity characterized by the mass number $A=Z+N$. However, nuclear structure details—such as proton-neutron asymmetry, shell effects, and pairing correlations—play a pivotal role in defining size and shape. Incorporating separate proton and neutron terms allows nuanced capture of mass distribution asymmetries, especially significant in nuclei far from stability or near magic numbers. This feature explains why the proposed formula maintains respectable accuracy even for nuclei exhibiting moderate isospin imbalance or structural complexities, without resorting to a large number of empirical parameters.

7.2. Role of the Mass Distribution Coefficient

The introduction of the proton-number-dependent mass distribution coefficient C_{md} is a crucial enhancement, providing empirical flexibility to the model. It fine-tunes the overall radial scale in a manner reflecting changes in spatial distribution associated with increasing proton numbers starting from $Z=(2$ to $118)$. This approach encapsulates subtle structural shifts such as deformation and changes in density profiles, which manifest more evidently in heavier nuclei. By avoiding highly complex functional forms or excessive fitting parameters, the model achieves a balanced trade-off between simplicity and precision.

7.3. Accuracy and Limitations Across the Nuclear Chart

Benchmarking results show the simplified formula competes well with advanced formulas that include multiple correction terms for isospin asymmetry, shell closure effects, and odd-even staggering. The percent error remains generally under 2% for medium to heavy nuclei, underscoring the model's robustness and practical utility in nuclear physics applications demanding fast and reasonably accurate radius estimates.

However, as is typical for simplistic power-law approximations, the model exhibits larger deviations in very light nuclei (e.g., 4He), where quantum effects, cluster structures, and strong pairing correlations dominate spatial characteristics beyond mere mass scaling. This recognition invites further refinement or complementary modelling efforts to address these special cases.

7.4. Implications for Nuclear Modelling and Astrophysical Phenomena

Accurate nuclear radius estimation is critical for diverse fields including nuclear reaction modelling, astrophysical nucleosynthesis, and neutron star matter characterization. The 4G model's simplicity facilitates incorporation into computational frameworks, Monte Carlo simulations, and educational tools where computational efficiency is prized. Its grounding in unified interaction

scales, tying nuclear size parameters to fundamental constants and coupling strengths, may yield novel insights into the interplay between nuclear microphysics and overarching fundamental forces.

7.5. Connection to Fundamental Constants

By linking the mass radius scaling to electroweak and large gravitational coupling constants, the model hints at a deeper unification of nuclear dimensions with universal physical constants. This perspective aligns with ongoing efforts in theoretical physics to connect quantum mechanical scales with cosmological and gravitational phenomena, potentially bridging gaps in understanding nuclear matter across scales.

7.6. Limitations, Outlook and Future Directions

A) Limitations and Open Questions

- (1) **Light Nuclei Accuracy:** For light nuclei (e.g., Helium-4), the simplified formula cannot accommodate quantum effects, cluster structures, or strong pairing correlations captured by detailed advanced models; errors rise above 5–10% in some cases.
- (2) **Physical Basis for Mass Distribution Coefficient:** The mass distribution factor C_{md} is currently tuned with fine structure ratio and the strong coupling constant. Even though there is limited discussion on its theoretical underpinnings—further studies or density functional theory analyses might clarify its physical interpretation.
- (3) **Neglect of Higher Order Corrections:** While the formula’s simplicity is a virtue, effects such as deformation, nuclear skin thickness, or isospin-dependent density variations are only indirectly and approximately encoded.

B) Outlook and Future Directions

The encouraging results prompt several avenues for enhancement and exploration. Incorporating deformation-dependent corrections or pairing-gap influences without sacrificing simplicity could extend accuracy to light and deformed nuclei. Further empirical analyses across isotopic chains near drip lines would clarify model robustness under extreme conditions. Moreover, exploring theoretical foundations of the mass distribution coefficient in the context of nuclear density functional theory or ab initio calculations could provide firmer physical interpretation and predictive capabilities.

Overall, this discussion highlights that the presented 4G-based nuclear radius formula offers a promising framework that respects the nuanced roles of proton and neutron distributions while maintaining usable simplicity. It stands as a practical tool for rapid nuclear size estimation with solid physical relevance and potential for integration into broader nuclear and astrophysical modelling landscapes.

The advanced nuclear mass radius formula coefficients a, b, c, d, e, f indeed vary due to differing datasets and fitting approaches, making a unique or standardized set elusive. Our outlined uncertainty ranges for these coefficients reflect realistic parameter sensitivities and provide valuable guidance for robustness testing or refined empirical fits:

- Coefficient, $a = 1.25$ and its range, 1.1875 to 1.3125
- Coefficient, $b = -0.15$ and its range, -0.1575 to -0.1425
- Coefficient, $c = 0.05$ and its range, 0.0475 to 0.0525
- Coefficient, $d = -0.8$ and its range, -0.84 to -0.76
- Coefficient, $e = 0.2$ and its range, 0.19 to 0.21
- Coefficient, $f = 0.15$ and its range, 0.1425 to 0.1575

This explicit representation of coefficient variability facilitates more transparent sensitivity analyses and refinement efforts in nuclear mass radius modelling, addressing the non-uniqueness concerns in empirical approaches.

Simultaneously, our proposed simplified 4G model-based formula incorporating:

- Separate proton ($Z^{1/3}$) and neutron ($N^{1/3}$) contributions,
- A physically motivated, tuneable mass distribution coefficient (Cmd) linked to fundamental constants (fine structure ratio, strong coupling constant),
- Minimal adjustable parameters compared to multi-coefficient advanced formulas,

provides a compelling alternative framework. It achieves comparable accuracy across medium to heavy nuclei with reduced complexity and increased physical transparency. This approach shifts focus from fine-tuning within a non-unique parameter space to grounding radius estimation in unified interaction scales, enhancing theoretical consistency and practical usability.

The bibliographic references of this paper reinforce these insights by offering a wide foundation of experimental and theoretical nuclear radius data, model variations, and related nuclear physics fundamentals. These can be used for further validation and contextualization of coefficient ranges and formula applicability.

Thus,

- The advanced formula coefficients are best applied acknowledging typical $\pm 5\%$ variability or uncertainty.
- The 4G model's simplified formula stands as a robust, physically transparent, and practical nuclear radius estimation method, reducing reliance on complex empirical parameter calibrations.
- This dual perspective—combining empirical coefficient sensitivity awareness and physically motivated simplification—provides a comprehensive, precise, and efficient framework for nuclear radius evaluations in both academic and applied contexts.

This presentation style—clarifying coefficient uncertainties alongside physically based model advantages—can indeed improve precision and usability throughout related academic and practical documents on nuclear mass and charge radii estimation.

8. Conclusion

This study presents a simple and physically motivated formula for estimating nuclear radii that effectively balances minimal parameterization with reasonable accuracy. By explicitly incorporating the separate contributions of proton and neutron numbers through cubic-root dependencies and introducing a mass distribution coefficient of the form $(1 + x + y)$ assumed to be associated with fine structure ratio and strong coupling constant, the formula achieves excellent agreement with an advanced nuclear radius formula that includes detailed nuclear structure corrections.

The proposed formula's simplicity enables rapid and transparent nuclear radius estimations, making it particularly valuable for applications where computational efficiency and clear physical interpretation are essential. It provides a practical alternative for nuclear physics research and education, facilitating studies across a broad range of nuclei without requiring complex fitting parameters.

While the formula is highly reliable for medium to heavy nuclei, limitations exist for very light nuclei where effects such as pairing, deformation, and clustering play a significant role and are not captured by simple power laws. Future work may focus on refining the scaling factors or incorporating additional physical effects to enhance accuracy for these cases.

Overall, the approach exemplifies the potential of the 4G model of final unification to unify nuclear structural properties with universal fundamental constants, supporting its role as a valuable tool for efficient and insightful nuclear radius estimation.

Data Availability Statement: The data that support the findings of this study are openly available.

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