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Emergence of Gauge Structure, Confinement, and Gravity from Admissible Continuation

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Abstract

We develop a general framework for fundamental physics in which spacetime geometry, gauge structure, and effective field dynamics arise from admissible continuation of pre-geometric configurations, rather than from dynamical evolution on a fixed background. In this class of generative frameworks, physical configurations are constructed iteratively under structural feasibility constraints, without assuming a priori spacetime or temporal ordering. External degrees of freedom—spacetime geometry, causal structure, and propagating fields—emerge through coarse-graining over ensembles of admissible continuation histories. Smooth continuum fields appear as stable limits of this process, and their governing equations arise as compatibility conditions required for coherent continuation. In particular, we show that admissible continuum equations must possess hyperbolic principal symbols, ensuring finite propagation domains for local perturbations. Internal degrees of freedom encode closure data transported along continuation paths, giving rise to gauge structures. Abelian sectors permit unsuppressed long-range transport, while for non-Abelian closure algebras we establish a general buffer saturation mechanism: finite interface capacity together with noncommutativity enforces confinement and implies a mass gap as structural consequences of admissibility. Gravity lies outside the gauge universality class and instead encodes global feasibility conditions governing the persistence of extended spacetime configurations. In four dimensions the unique leading-order continuum feasibility functional compatible with admissibility is the Einstein–Hilbert functional, yielding Einstein's field equations as stationarity conditions of structural consistency. The Real-Now-Front (RNF) construction provides a concrete realization of this framework, but the results derived here depend only on general admissibility principles and therefore apply broadly across generative physical models.

Keywords: admissibility; reconstruction; emergent spacetime; continuum limits; gauge theory; confinement; mass gap; non-Abelian gauge theory; gravity; Einstein–Hilbert action; Lorentz symmetry; universality classes

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1. Introduction

Modern theoretical physics achieves extraordinary quantitative success, yet its foundational picture of reality remains conceptually fragmented. Quantum field theory (QFT) describes matter and interactions as fields evolving on a prescribed spacetime background, while general relativity (GR) treats spacetime geometry itself as dynamical [59,63]. Both frameworks are internally consistent and experimentally validated, but their coexistence exposes persistent structural tensions: the ontological status of spacetime, the origin and rigidity of symmetry principles, the universality of gravitational coupling, the special role of gauge structure, and the emergence of confinement and mass gaps in non-Abelian sectors [52,54].

Rather than assuming a completed spacetime that subsequently evolves, one may consider a broader class of frameworks in which physical configurations are generated through successive admissible continuation steps. In such settings, spacetime structure, fields, and physical events are interpreted as elements of an iteratively constructed configuration sequence, rather than as entities evolving within a pre-existing temporal background. This perspective does not require commitment

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to a specific ontology; it may be understood either as a fundamental description of physical reality or as an abstract framework for analyzing consistency conditions governing sequential configuration generation.

From this viewpoint, physical structure is identified with the ongoing generation of admissible configurations rather than with the time evolution of states within a fixed background. At each stage, a new spacetime slice—with physical structure already present—is generated through the coherent assembly of local patches. This slice is not embedded in an external temporal parameter; it constitutes the realized physical configuration at that stage of continuation.

Continuation proceeds locally but must remain globally compatible: every local patch must fit consistently with all others to form a coherent global configuration. Structures that cannot be extended indefinitely without accumulating incompatibility are progressively suppressed, while stable patterns persist. Physical laws may then be interpreted as structural regularities that survive this indefinite continuation process.

This perspective raises a logically prior question:

> *Why are physical configurations continuously extendable at all, and which structural constraints select the universality classes in which continuum field equations, symmetries, and particles persist?*

The present work develops a reconstruction-based answer to this question. The Real-Now-Front (RNF) framework studied here provides a concrete realization of such a generative structure, in which admissibility acts as a selection principle governing viable continuation.

1.1. RNF Reconstruction and Feasibility-First Physics

Within this general setting, we introduce the *Real-Now-Front* (RNF) construction as a specific implementation of generative continuation. Physical reality is modeled not as the evolution of states inside a completed spacetime, but as the admissible extension of structure across a continually advancing reconstruction front.

At the fundamental level no manifold, metric, Hilbert space, or scale is assumed. Instead we postulate a metascopic configuration space $\mathcal{X}$ consisting of pre-geometric configurations $\Phi \in \mathcal{X}$ together with a directed admissibility relation

$$\Phi \prec \Phi'.$$

Equivalently, admissible continuation may be described by a set-valued reconstruction map

$$R : \mathcal{X} \to \mathcal{P}(\mathcal{X}), \qquad R(\Phi) = \{\Phi' \mid \Phi \prec \Phi'\}.$$

The relation $\prec$ does not represent temporal evolution. Rather, it encodes *structural feasibility*: which candidate configurations can be coherently extended to form a globally admissible configuration without accumulating irreducible inconsistency [23,55].

Admissibility is governed by local feasibility constraints summarized by the *Temporal Coherence Principle* (TCP). TCP specifies when local extensions remain compatible under composition. Local success alone is insufficient: continuation must remain globally consistent across all neighboring patches. This simultaneous enforcement of local coherence and global compatibility drives the emergence of nontrivial physical structure.

Spacetime, locality, causal order, and differential equations are therefore not fundamental inputs. They emerge only in regimes where admissible continuation patterns stabilize under coarse-graining. Physical laws should thus be interpreted not as imposed microscopic axioms, but as persistent residues of structural feasibility, consistent with general principles of universality and hydrodynamic emergence [33,37,67].

The RNF admissibility framework is developed here at an implementation-independent level. Concrete realizations—such as discrete, lattice-based, or chronon-field substrates—may supply explicit

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reconstruction maps and constraint functionals, but the central results of the present work depend only on structural admissibility requirements common to this class of generative frameworks.

1.2. Main Results

Within this reconstruction-based framework we establish the following results.

1. **External continuum laws as stability limits.**
   External degrees of freedom—metric structure, causal relations, and propagating fields—arise as equivalence classes of metascopic configurations under reconstruction-based coarse-graining. Smooth fields emerge as ensemble expectation values over admissible reconstruction histories. The governing continuum equations appear as stability conditions required for reconstruction ensembles to remain coherent under arbitrarily fine coarse-graining.
2. **Hyperbolic propagation as a reconstruction stability condition.**
   For external continuum fields to represent stable reconstruction channels, local perturbations must propagate with finite domain of influence. We show that this requirement implies that admissible continuum equations must possess hyperbolic principal symbols. Hyperbolicity therefore emerges as a structural stability condition of continuation rather than a fundamental dynamical assumption.
3. **Gauge fields as constraint transport.**
   Gauge structure arises as an interface mechanism for redistributing internal reconstruction constraints across spacetime patches. Internal degrees of freedom appear as relational book-keeping variables whose transport along continuation paths produces connection and holonomy structures closely related to standard gauge theory [5,16,31]. Abelian closure algebras permit unsuppressed long-range transport, yielding Maxwell-type behavior in the continuum limit.
4. **Non-Abelian buffer saturation, confinement, and mass gap.**
   Under minimal assumptions of locality, compositionality, and finite interface buffering capacity, we establish a *Non-Abelian Buffer Saturation Theorem*. Because non-Abelian closure transport is noncommutative, long-range transport of internal mismatch accumulates irreducible obstruction. Finite interface buffering capacity therefore forces progressive saturation of non-Abelian transport, suppressing unscreened long-range modes. This mechanism yields confinement and implies a nonzero mass gap as structural consequences of admissibility [26,48,67].
5. **Gravity as global reconstruction feasibility.**
   Gravity lies outside the gauge universality class. While gauge fields implement local constraint transport within spacetime, gravity encodes global feasibility conditions governing whether extended spacetime reconstruction remains admissible at all. We show that in four dimensions the unique leading-order continuum feasibility functional compatible with admissibility is the Einstein–Hilbert functional, consistent with known uniqueness results for diffeomorphism-invariant gravity [40,61].
6. **Symmetry as an admissibility attractor.**
   Lorentz symmetry emerges as a stable attractor of admissible continuation patterns, consistent with general constraints on relativistic symmetry structures [12,60].

1.3. Existence of Einstein–Maxwell Realizations

Although the present work is formulated at a structural level, the framework is non-empty in a strong sense. Explicit RNF implementations exist in which the admissibility axioms are satisfied and the stabilized continuum limit reproduces the Einstein field equations coupled to Maxwell electrodynamics [39]. These constructions serve as existence proofs that standard gravitational and electromagnetic dynamics lie within an admissibility attractor of RNF reconstruction. The present paper does not rely on these realizations for its central results. Instead it identifies the universal admissibility principles that make such realizations possible across distinct microscopic implementations.

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1.4. Scope and Organization

This paper focuses on structural physics and restricts attention to feasibility principles and their mathematical consequences. Detailed microscopic realizations are deferred to separate work and are invoked only as constructive demonstrations of existence.

The scope of the present work is intentionally broad. The admissibility-based reconstruction framework is proposed as a structural perspective, and its internal consistency is most clearly assessed by examining how it connects multiple domains of theoretical physics within a unified setting. Accordingly, the paper develops several interrelated consequences—spanning continuum propagation, gauge structure, and gravitational dynamics—in parallel.

At the same time, the primary aim is not to provide a complete microscopic theory, but to identify general feasibility constraints and their leading-order implications for continuum physics. The breadth of coverage is therefore intended to demonstrate structural coherence and explanatory scope, while the central claims remain limited to implementation-independent results derived from admissibility principles.

Section 2 formalizes RNF reconstruction as a directed admissibility map. Section 3 derives external continuum fields as stable limits. Sections 4 and 5 analyze gauge transport and non-Abelian saturation. Section 6 clarifies why gravity does not belong to the gauge universality class. Section 7 derives general relativity as the unique leading-order feasibility theory of RNF reconstruction. Sections 8– 9 discuss symmetry attractors and observational constraints. We conclude in Section 12.

2. RNF Reconstruction as a Directed Admissibility Map

We now formalize a class of generative frameworks in which physical configurations are constructed through admissible continuation on a pre-geometric configuration space. The construction is deliberately minimal: no spacetime, metric, or dynamical assumptions are introduced. All subsequent physical structure arises from the selective persistence of configuration patterns under admissibility constraints, in close analogy with feasibility- and constraint-based formulations in mathematics and physics [23,55].

The Real-Now-Front (RNF) construction studied here provides a concrete realization of this general structure.

2.1. Metascopic Configuration Space

Definition 2.1 (Metascopic (pre-geometric) configuration space)**.** Let $\mathcal{X}$ denote the *metascopic (pre-geometric) configuration space*. Elements $\Phi \in \mathcal{X}$ represent configurations prior to the emergence of spacetime structure, metric notions, causal ordering, or externally observable variables.

No algebraic, topological, or metric structure is assumed on $\mathcal{X}$. In particular, $\mathcal{X}$ is not required to be a manifold, vector space, or Hilbert space. The sole structural requirement is that $\mathcal{X}$ supports a notion of admissible continuation, which allows configurations to be compared and composed while selectively suppressing incompatible combinations. This stance is consistent with other pre-geometric and relational approaches in which spacetime structure is treated as emergent rather than fundamental [52,56], while remaining agnostic about the specific underlying microscopic realization.

At this level, configurations are not interpreted as physical states evolving in time. They are abstract candidates for admissible continuation, acquiring physical meaning only through sequencing, stabilization, and coarse-graining under reconstruction.

2.2. RNF Admissibility Relation and Reconstruction Map

Definition 2.2 (Admissibility relation)**.** Admissible continuation is defined by a directed relation

$$\prec \subset \mathcal{X} \times \mathcal{X},$$

where $\Phi \prec \Phi'$ indicates that Φ' represents an admissible extension of Φ.

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In the RNF realization, this relation corresponds to admissible RNF extension.

Equivalently, the admissibility relation can be represented by a set-valued *reconstruction map*

$$R : \mathcal{X} \to \mathcal{P}(\mathcal{X}),$$

where

$$R(\Phi) = \{\Phi' \in \mathcal{X} \mid \Phi \prec \Phi'\}$$

denotes the set of admissible continuations of configuration Φ. Here $\mathcal{P}(\mathcal{X})$ denotes the power set of $\mathcal{X}$.

The relation $\prec$ encodes structural feasibility rather than temporal evolution. Its directionality represents admissible continuation: only configurations related by $\prec$ can coherently persist as successive stages under admissibility filtering. Similar directed or partially ordered structures appear in causal, operational, and constraint-based formulations, though no underlying spacetime or causal order is assumed [7,25].

We impose the following structural axioms:

1. *Irreflexivity*: $\Phi \not\prec \Phi$, ensuring nontrivial extension.
2. *Local admissibility*: whether $\Phi \prec \Phi'$ holds depends only on local features of the pair (Φ, Φ') under the admissibility rules.
3. *Conditional composability*: if $\Phi \prec \Phi'$ and $\Phi' \prec \Phi''$, then $\Phi \prec \Phi''$ holds provided no additional incompatibility is introduced under composition.

The relation $\prec$ is not assumed to be total or globally transitive. Branching, competition, and partial ordering are generic at the metascopic level. Deterministic evolution, when it appears, is therefore an emergent property of selectively stable coarse-graining rather than a fundamental axiom, consistent with general results on universality and renormalization [33,67].

2.3. Reconstruction Paths and Obstruction Accumulation

Definition 2.3 (Reconstruction path)**.** A reconstruction path is a directed sequence

$$\gamma : \quad \Phi_0 \prec \Phi_1 \prec \Phi_2 \prec \cdots$$

representing successive admissible continuations.

A reconstruction path defines an ordered extension history, but not a trajectory in time: there is no associated metric duration, rate, or continuous parameter labeling the sequence. This distinction mirrors earlier insights that ordering and duration need not coincide in foundational formulations of time [6,51].

Distinct reconstruction paths may coexist at the metascopic level while differing in their long-range compatibility. Under coarse-graining, paths that accumulate structural incompatibilities are progressively suppressed, while mutually coherent families persist. This selective survival underlies the emergence of effective probabilistic and statistical descriptions without assuming any probability measure *a priori*.

In dense admissibility regimes, combinatorial multiplicities of admissible continuation patterns induce sharply peaked statistical distributions over reconstruction histories. These distributions provide the effective probabilistic weights used in coarse-grained external descriptions.

2.4. Temporal Coherence Principle as a Selective Constraint

The admissibility relation $\prec$ is governed by a constraint enforcing coherence across composed continuation steps.

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Definition 2.4 (Temporal Coherence Principle)**.** The Temporal Coherence Principle (TCP) is specified by a nonnegative constraint functional

$$\mathcal{C} : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_{\geq 0}$$

such that

$$\Phi \prec \Phi' \quad \Longleftrightarrow \quad \mathcal{C}(\Phi, \Phi') = 0.$$

A positive value of $\mathcal{C}$ quantifies structural mismatch introduced by a candidate extension. Rather than causing immediate failure, such mismatch accumulates under continued extension and leads to progressive suppression of incompatible continuation patterns. This accumulative role of constraint violation parallels obstruction-theoretic mechanisms in topology and geometry [55].

TCP enforces:

- *Local feasibility*: each elementary extension must satisfy $\mathcal{C} = 0$.
- *Global coherence*: extended sequences are selectively retained only if composed mismatches do not accumulate beyond admissible bounds.

Obstruction accumulation and saturation are therefore central mechanisms by which admissible continuation sculpts the space of viable external structures.

2.5. Emergence of Temporal Ordering

The directed nature of $\prec$ induces an ordering on reconstruction paths. After external coarse-graining, this ordering yields an effective notion of temporal succession and causal structure. Time thus emerges as an ordering relation derived from admissible continuation, not as a fundamental parameter, in contrast with formulations that presuppose an external time coordinate [52,54].

Continuous time coordinates and differential equations arise only in regimes where dense, stable families of continuation paths admit smooth approximations. Their validity reflects the presence of admissibility attractors rather than the primacy of microscopic dynamics.

2.6. Summary

Admissible continuation is formalized as a directed relation on a metascopic configuration space $\mathcal{X}$. Governed by the Temporal Coherence Principle, this structure replaces fundamental dynamics with selective feasibility constraints. Time ordering, spacetime reconstruction, and effective physical laws emerge through the suppression of incompatible extensions and the persistence of coherent continuation patterns under coarse-graining. The RNF construction provides a concrete realization of this general framework.

Figure 1 provides a schematic representation of RNF reconstruction as a directed admissibility structure on the metascopic configuration space $\mathcal{X}$. Each node denotes a pre-geometric configuration Φ, not a physical state at a moment in time. Solid arrows indicate admissible RNF extensions—pairs of configurations that can be coherently composed under the Temporal Coherence Principle—while dashed arrows represent extensions that generate obstruction and are therefore suppressed under continued reconstruction. Importantly, the diagram is not embedded in spacetime and does not depict temporal evolution. Directionality arises solely from feasibility of extension: RNF reconstruction proceeds by selectively retaining admissible continuations and sculpting away inconsistent ones, without assuming a background metric, clock, or dynamical law.

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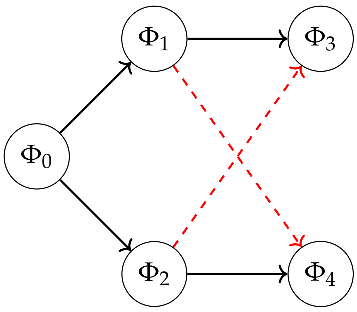

RNF admissibility:
solid arrows = feasible extension
dashed arrows = obstruction

Figure 1. RNF reconstruction as a directed admissibility relation on the metascopic configuration space. Nodes represent pre-geometric configurations $\Phi \in \mathcal{X}$. Solid arrows indicate admissible RNF extensions, while dashed arrows denote obstructed transitions. No background spacetime, metric, or time parameter is assumed; ordering arises solely from feasibility of reconstruction.

3. Emergence of External Fields as Continuum Limits

This section provides a controlled structural account of how external spacetime fields and their governing partial differential equations arise as continuum limits of admissible RNF reconstruction. No spacetime structure, metric, or field equations are assumed *a priori*. All results follow from admissibility, locality, and stability under coarse-graining, in accordance with general principles underlying hydrodynamic limits and universality [33,67].

3.1. Reconstruction Framework

Let $\mathcal{X}$ denote the space of metascopic configurations. Elements $\Phi \in \mathcal{X}$ represent complete admissibility-relevant distinction structures at the reconstruction level.

As introduced in Section 2, RNF reconstruction is defined by a directed admissibility relation

$$\Phi \prec \Phi'$$

or equivalently by the set-valued reconstruction map

$$\mathcal{R}(\Phi) = \{\Phi' \in \mathcal{X} \mid \Phi \prec \Phi'\}.$$

An *admissible reconstruction history* is a chain

$$\gamma = (\Phi_0 \prec \Phi_1 \prec \cdots \prec \Phi_n)$$

such that

$$\Phi_{i+1} \in \mathcal{R}(\Phi_i).$$

For two configurations Φ_i and Φ_f we denote by

$$\Gamma(\Phi_i, \Phi_f)$$

the set of admissible histories connecting them.

RNF reconstruction therefore defines a partially ordered structure of configurations linked by admissible continuations. Continuum spacetime, fields, and causal relations emerge only after coarse-graining over ensembles of such reconstruction histories.

3.2. Standing Assumptions

Throughout this section we assume the following structural conditions:

(A1) **Local admissibility.** The constraint functional $C(\Phi, \Phi')$ depends only on bounded local features of the extension (Φ, Φ').

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(A2) **Finite sensitivity.** Admissible perturbations of reconstruction paths induce continuous changes in external equivalence classes.
(A3) **Dense admissibility regime.** Reconstructed spacetime admits arbitrarily fine coverings by admissible local patches.
(A4) **Stability under coarse-graining.** External observables are defined only when limits exist and are stable under admissible perturbations.

These assumptions formalize the conditions under which continuum descriptions arise as emergent summaries of admissible reconstruction, rather than as fundamental microscopic descriptions [37].

Under assumptions (A1)–(A4), admissible reconstruction imposes strong structural constraints on any continuum description that persists under coarse-graining. In particular, stable external fields must satisfy local continuum compatibility conditions, and the corresponding continuum equations must respect finite propagation of admissibility disturbances.

Theorem 3.1 (Hyperbolicity Dominance)**.** *Let ψ be a stable external field arising as a continuum limit of admissible RNF reconstruction, and let $\mathcal{D}$ denote its effective continuum differential operator. Under assumptions (A1)–(A4), the principal symbol of $\mathcal{D}$ must be hyperbolic.*

Equivalently, admissible continuum limits cannot be governed by elliptic or parabolic propagation laws, since such laws violate finite propagation of reconstruction perturbations and are therefore incompatible with stable RNF continuation.

Sketch. Local admissibility implies that reconstruction disturbances can influence only bounded neighborhoods at each continuation step. Stable coarse-grained fields must therefore possess finite domains of dependence. Elliptic and parabolic operators violate this condition by permitting instantaneous or arbitrarily rapid propagation across the reconstructed domain. Hence only hyperbolic principal symbols are compatible with admissible continuum stability.

A rigorous derivation is given in Appendix B. □

The following subsections develop the emergence of external and internal degrees of freedom, smooth continuum fields, and the compatibility conditions that govern them. Together these results provide the structural basis for the continuum effective laws used in the remainder of the paper.

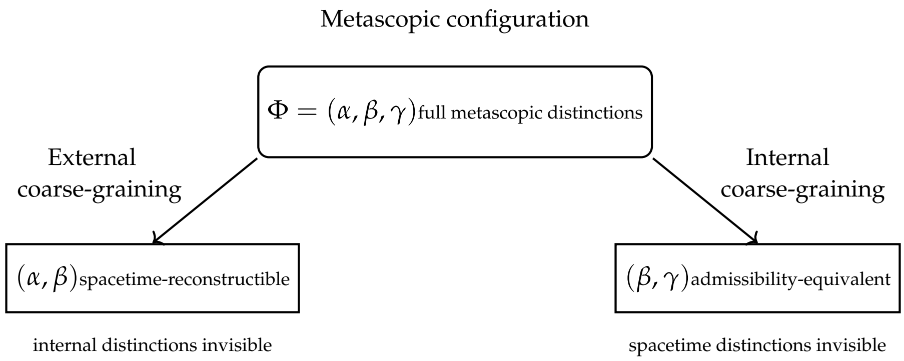

Figure 2. Distinction-based coarse-graining. A metascopic configuration Φ carries multiple independent distinctions. External coarse-graining projects onto those distinctions relevant for spacetime reconstruction, while internal coarse-graining projects onto admissibility and closure structure. Coarse-graining is defined by selective loss of distinguishability, not by scale, averaging, or resolution.

3.3. Origin of External and Internal Degrees of Freedom

The RNF framework distinguishes between *external* and *internal* degrees of freedom. This distinction is not assumed *a priori* but arises naturally from the structure of reconstruction coarse-graining.

A metascopic configuration $\Phi \in \mathcal{X}$ encodes multiple independent admissibility-relevant distinctions. RNF continuation acts locally on these distinctions through admissible extensions

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$$\Phi \to \Phi' \in \mathcal{R}(\Phi).$$

External observables arise only after coarse-graining over ensembles of such admissible continuations.

External Degrees of Freedom

A distinction in Φ is *external* if modifying it changes the adjacency relations among admissible configurations. Such distinctions therefore affect which local continuations remain feasible under RNF extension.

Under coarse-graining these distinctions determine the effective geometric relations among reconstructed regions and therefore define the spacetime structure of the reconstructed manifold $\mathcal{M}$. The resulting external variables appear as smooth fields such as the metric $g_{\mu\nu}$ or other spacetime-propagating quantities.

Internal Degrees of Freedom

A distinction in Φ is *internal* if its variation does not modify local admissible adjacency relations. Such distinctions leave the reconstructed external geometry unchanged but influence closure conditions governing compositional continuation of reconstruction.

Internal degrees of freedom therefore behave as relational bookkeeping data attached to reconstruction histories. They become observable only through comparison of reconstruction paths and therefore appear in the continuum limit as fiber variables transported along curves in spacetime.

This transport structure naturally yields internal gauge bundles

$$\mathcal{U} : \Pi_1(\mathcal{M}) \to G.$$

Bundle Interpretation

External degrees of freedom determine the base manifold $\mathcal{M}$, while internal degrees of freedom form fibers attached to each spacetime point. Transport along reconstruction paths produces the bundle structures characteristic of gauge theories.

Relation to Earlier Work

A detailed derivation of external geometric degrees of freedom from RNF reconstruction was presented in [39]. The present work focuses instead on the admissibility constraints governing internal closure transport and their consequences for gauge structure and confinement.

3.4. Emergence of Smooth Fields and Continuum Equations

We now describe how smooth external fields and continuum equations arise from ensembles of admissible reconstruction histories.

Ensembles of Reconstruction Histories

Fix configurations Φ_i and Φ_f and consider the set $\Gamma(\Phi_i, \Phi_f)$ of admissible reconstruction histories connecting them. Each history

$$\gamma = (\Phi_0 \prec \Phi_1 \prec \cdots \prec \Phi_n)$$

represents one admissible continuation sequence consistent with TCP.

External observables correspond not to individual histories but to statistical summaries over large ensembles of admissible histories. Suppression of incompatible continuations by TCP generically produces sharply peaked statistical distributions over reconstruction patterns.

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Probability Densities from Admissibility

Let $\mathcal{O}(\Phi)$ denote any observable depending only on the external equivalence class of a configuration. Coarse-grained external fields arise as ensemble expectations

$$\langle \mathcal{O} \rangle(x) = \sum_{\gamma \in \Gamma_x} w(\gamma)\, \mathcal{O}(\gamma)$$

where $w(\gamma)$ represents the statistical weight induced by admissibility filtering. These weights emerge from the combinatorial multiplicity of admissible reconstruction histories rather than from any fundamental stochastic law.

In dense admissibility regimes the resulting expectation values vary continuously across neighboring reconstruction patches, giving rise to smooth fields on the reconstructed manifold $\mathcal{M}$.

Continuum Equations

Smooth fields arise only when reconstruction ensembles remain stable under arbitrarily fine coarse-graining. In this regime neighboring reconstruction patches must satisfy compatibility conditions expressing consistency of local admissible continuations.

These compatibility relations take the form of partial differential equations governing the continuum fields. The equations therefore do not represent microscopic dynamical laws but stability conditions that coarse-grained reconstruction ensembles must satisfy.

Hydrodynamic universality implies that these continuum equations depend primarily on locality, symmetry, and conservation constraints rather than on microscopic details [33,67].

Hyperbolic Stability

For external fields to represent stable reconstruction channels, local perturbations must propagate with finite domain of influence. As discussed earlier in this section (Theorem 3.1) and in Appendix B, this requirement implies that the principal symbol of admissible continuum equations must be hyperbolic.

Hyperbolicity therefore appears not as an imposed dynamical principle but as a structural stability condition required for coherent RNF reconstruction.

Interpretation

External spacetime fields should thus be understood as collective variables summarizing ensembles of admissible reconstruction histories. Their governing differential equations encode compatibility and stability constraints of RNF continuation rather than fundamental microscopic dynamics.

In the following sections we analyze how internal reconstruction degrees of freedom behave under such continuum limits and show that their transport structure naturally gives rise to gauge connections, curvature, and confinement phenomena.

4. Gauge Fields as Constraint Transport

Given the distinction between external and internal degrees of freedom established in Section 3, we now analyze how internal closure data must be transported under RNF continuation. Internal degrees of freedom are not microscopic states, hidden variables, or observable quantities. Rather, they function as *bookkeeping structures* that parameterize how local reconstruction constraints are satisfied. Two configurations may be externally equivalent yet differ internally in the manner in which admissibility conditions are realized. Such differences do not affect the reconstructed spacetime directly but become relevant under further RNF continuation.

Because admissibility is compositional, internal reconstruction data cannot be eliminated pointwise. Instead they must be compared, reconciled, or transported across interfaces between reconstructed regions. Gauge structure arises precisely from this requirement, in direct analogy with connection-based formulations of parallel transport [31].

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4.1. Internal Closure Mismatch

When RNF reconstruction is restricted to a bounded spacetime region, internal reconstruction constraints need not close exactly within that region. The resulting discrepancy appears as a residual internal mismatch at the boundary.

Definition 4.1 (Closure algebra)**.** An internal closure algebra $\mathfrak{g}$ is a Lie algebra whose elements generate admissibility-preserving reparametrizations of internal reconstruction data that leave external equivalence classes invariant.

Let $R \subset \mathcal{M}$ be a reconstructed spacetime region. We associate to R a closure mismatch

$$\Delta(R) \in \mathfrak{g},$$

which quantifies the residual internal inconsistency across the reconstruction boundary ∂R.

RNF admissibility does not require such mismatch to vanish locally. Instead, mismatch must be consistently transported, redistributed, or neutralized under further continuation.

4.2. Constraint Transport as a Groupoid Functor

Because internal reconstruction data are defined only through relational comparison, their transport is inherently path dependent.

Let $\Pi_1(\mathcal{M})$ denote the path groupoid of reconstructed spacetime, whose objects are points of $\mathcal{M}$ and whose morphisms are equivalence classes of admissible paths under endpoint fixing [8].

Definition 4.2 (Constraint transport)**.** Constraint transport is a functor

$$\mathcal{U} : \Pi_1(\mathcal{M}) \longrightarrow G,$$

where G is the Lie group generated by the closure algebra $\mathfrak{g}$.

Functoriality requires

$$\mathcal{U}(\gamma_1 \circ \gamma_2) = \mathcal{U}(\gamma_1)\mathcal{U}(\gamma_2), \tag{1}$$

$$\mathcal{U}(\mathrm{id}_x) = \nVdash. \tag{2}$$

These relations guarantee that redistribution of internal mismatch remains composable under successive RNF reconstruction steps.

Gauge redundancy arises as equivalence of transport functors under pointwise conjugation,

$$\mathcal{U}(\gamma) \;\mapsto\; g(x)\,\mathcal{U}(\gamma)\,g^{-1}(y),$$

which reflects freedom in the representation of internal reconstruction data rather than a physical transformation of reconstructed spacetime.

4.3. Curvature as Obstruction Accumulation

Closed loops play a distinguished role because they compare two nominally identical reconstructions of the same region.

Definition 4.3 (Transport obstruction)**.** For a loop $\ell \in \Pi_1(\mathcal{M})$ based at x, define the obstruction

$$\Omega(\ell) := \mathcal{U}(\ell) \in G.$$

A nontrivial value $\Omega(\ell) \neq \mathbf{1}$ indicates accumulated incompatibility under compositional reconstruction.

In dense admissibility regimes the transport functor admits a continuum representation

$$\mathcal{U}(\gamma) = \mathcal{P}\exp\left(\int_\gamma A_\mu(x)\,dx^\mu\right),$$

where A_μ is an emergent connection one-form. The associated curvature two-form

$$F = dA + A \wedge A$$

measures the density of obstruction accumulation in the continuum limit [31].

4.4. Abelian Transport

If the closure algebra $\mathfrak{g}$ is Abelian, transport operators commute and curvature contributions combine linearly.

Proposition 4.4 (Unsuppressed Abelian transport)**.** For Abelian closure algebras, redistribution of internal mismatch remains bounded across arbitrarily large reconstructed regions.

Consequently long-range constraint transport remains admissible. In the continuum limit the obstruction density reduces to

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and Maxwell-type equations arise as effective conditions governing stable redistribution of mismatch [27].

4.5. Non-Abelian Transport and the Onset of Saturation

For non-Abelian closure algebras, curvature contributions combine nonlinearly through the term $A \wedge A$. As a consequence, successive transport operations can generate new obstruction components that cannot be eliminated by simple recombination.

When reconstruction interfaces possess finite buffering capacity, continued transport of closure mismatch leads to progressive accumulation of obstruction load along extended transport channels. Beyond a critical scale this load cannot be redistributed indefinitely, and admissible continuation requires the formation of confined flux structures.

This qualitative transition from unsuppressed transport to saturation is a structural feature of non-Abelian closure sectors. The precise conditions under which saturation occurs are established in the following section, where a general buffer saturation theorem is proved.

4.6. Low-Rank Closure Algebras and Physical Gauge Groups

The admissibility-based transport construction imposes strong structural constraints on closure algebras that can persist as stable reconstruction channels. Among compact Lie algebras, low-rank structures provide the simplest realizations of bounded buffering, local composability, and stable continuum limits.

In this sense the gauge groups of the Standard Model arise as minimal nontrivial closure algebras whose transport properties remain within admissibility basins under RNF continuation [20].

4.7. Summary

Gauge fields arise as emergent bookkeeping structures governing the redistribution of internal reconstruction constraints under RNF continuation. Functorial constraint transport yields connection-like objects and curvature obstruction in the continuum limit. Abelian transport permits unsuppressed propagation, while non-Abelian transport generically leads to accumulation of obstruction load. The structural consequences of this accumulation—including confinement and mass-gap behavior—are analyzed in Section 5.

Figure 3 schematically illustrates gauge fields as constraint-transport structures: internal closure mismatch $\delta\chi$ is carried locally across admissible interfaces by transport operators, while nontrivial holonomy—and hence curvature—emerges only from the composition of these transports around closed loops.

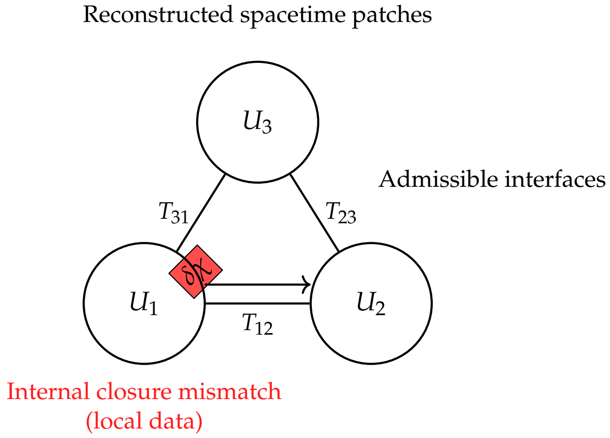

Figure 3. Gauge fields as constraint-transport structures. Each reconstructed spacetime patch U_i carries internal closure mismatch data $\delta\chi$. Transport operators T_{ij} move internal mismatch across admissible interfaces. The mismatch itself is not the holonomy. Nontrivial holonomy arises only from the composition of transport operators around closed loops, signaling obstruction and curvature.

5. Non-Abelian Buffer Saturation and Confinement

This section establishes a structural constraint on the transport of non-Abelian closure mismatch under RNF continuation. We show that when interface buffering capacity is finite, long-range transport of non-Abelian closure mismatch becomes generically incompatible with admissible continuation.

The result is implementation independent: it follows solely from locality, noncommutativity of the closure algebra, and bounded interface capacity. While the external effective consequences parallel confinement phenomena familiar from non-Abelian gauge theory, the mechanism derived here arises purely from admissibility constraints rather than from microscopic dynamics [26,67].

5.1. Finite Interface Capacity

RNF reconstruction proceeds by local extension across interfaces separating neighboring reconstructed regions. At each interface, admissibility requires that internal closure data associated with adjacent regions be made mutually compatible.

Definition 5.1 (Interface buffering)**.** Let R be a reconstructed region with boundary ∂R, and let $\Delta(R) \in \mathfrak{g}$ denote the net internal closure mismatch associated with R, valued in a closure algebra $\mathfrak{g}$. An *interface buffering scheme* assigns to each local element of ∂R a finite set of internal degrees of freedom capable of compensating transported closure mismatch.

Assumption 5.2 (Finite interface capacity)**.** There exists a finite bound on the number of algebraically independent, noncommuting closure transport contributions that can be buffered at any local interface element without generating irreducible obstruction.

This assumption is purely structural. It does not impose a numerical value, does not presuppose a metric notion of area or energy, and does not depend on any specific microscopic implementation of RNF reconstruction. It asserts only that admissibility cannot be preserved if arbitrarily many independent noncommuting transport contributions must be absorbed locally, consistent with general obstruction-theoretic limits on compositional extension [23,55].

5.2. Buffering Load and Obstruction Accumulation

To characterize the onset of saturation we introduce a monotone measure of buffering demand.

Definition 5.3 (Buffering load)**.** Let R be a reconstructed region whose boundary ∂R supports a collection of loop transport operators $\{U(\ell_i)\} \subset G$, where G is the transport group generated by

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the closure algebra $\mathfrak{g}$. The *buffering load* $\mathcal{B}(R)$ is defined as the minimal number of algebraically independent noncommuting transport contributions that must be simultaneously buffered along ∂R in order for R to remain admissible.

The buffering load is monotone under RNF extension and additive under composition of independent loops. It therefore provides a structural diagnostic of admissibility under continued reconstruction.

5.3. Non-Abelian Buffer Saturation Theorem

We now state the central result.

Theorem 5.4 (Non-Abelian Buffer Saturation)**.** *Let $\mathfrak{g}$ be a non-Abelian closure algebra with transport group G. Let R be a reconstructed region whose boundary ∂R supports loop transport operators generating noncommuting elements of G.*

Then the buffering load $\mathcal{B}(R)$ grows at least linearly with the number of independent noncommuting loop contributions. Under Assumption 5.2, there exists a finite threshold $\mathcal{B}_{\max}$ such that admissible RNF continuation fails whenever $\mathcal{B}(R) > \mathcal{B}_{\max}$.

Consequently unsaturated long-range transport of non-Abelian closure mismatch is generically inadmissible.

Structural proof. Decompose ∂R into elementary loops $\{\ell_i\}$. Each loop ℓ_i induces a transport operator $U(\ell_i) \in G$ that must be buffered to compensate closure mismatch.

Because $\mathfrak{g}$ is non-Abelian, the operators $U(\ell_i)$ do not generally commute. Noncommuting transport contributions cannot be merged or canceled without generating additional mismatch, so each independent contribution requires a distinct buffering channel.

As RNF extension increases the size of R, the number of independent loops required to maintain admissibility grows. The buffering load $\mathcal{B}(R)$ therefore increases monotonically with extension.

By Assumption 5.2, there exists a maximal buffering threshold $\mathcal{B}_{\max}$ beyond which no admissible compensation is possible. RNF continuation therefore suppresses reconstruction patterns requiring unsaturated long-range non-Abelian transport.

□

5.4. Physical Interpretation: Confinement and Mass Gap

The buffer saturation theorem has two immediate consequences at the level of the external effective description.

Confinement.

Because non-Abelian closure mismatch cannot be transported arbitrarily far without exceeding finite interface capacity, isolated long-range non-Abelian charges become inadmissible. Closure mismatch must instead neutralize locally through the formation of composite, closure-balanced structures. Under external coarse-graining these appear as color-neutral bound states.

Mass gap.

Any excitation carrying nontrivial non-Abelian closure mismatch incurs a minimal buffering load in order to remain admissible. Because noncommuting contributions cannot be continuously diluted, this buffering requirement implies a finite lower bound on admissible excitation cost in the effective continuum description, corresponding to a mass gap.

5.5. Structural Basis of Saturation

The saturation mechanism derived above follows from three structural features of admissible RNF reconstruction:

1. local composability of closure transport,

2. noncommutativity of the closure algebra,
3. finite interface buffering capacity.

Appendix G provides an explicit implementation-independent construction demonstrating that these features enforce irreducible obstruction accumulation and area-law scaling.

This result should be interpreted as a structural mechanism for confinement within the admissibility framework rather than as a proof of the Yang–Mills mass gap problem.

5.6. Summary

Non-Abelian closure transport generically leads to buffer saturation under RNF continuation when interface capacity is finite. This saturation forbids unscreened long-range transport, enforcing confinement and implying a mass gap. These results arise as structural consequences of admissibility and noncommutativity rather than from specific microscopic gauge dynamics.

This result should be interpreted as a structural mechanism for confinement and the emergence of a mass gap within the RNF admissibility framework. It does not constitute a rigorous proof of the Yang–Mills mass gap problem, but rather identifies a class of reconstruction dynamics in which confinement arises naturally.

Figure 4 illustrates the structural origin of non-Abelian saturation in the RNF framework. Transport of closure mismatch around a loop generates irreducible obstruction whose minimal buffering cost grows with the enclosed area, whereas Abelian transport does not accumulate such obstruction.

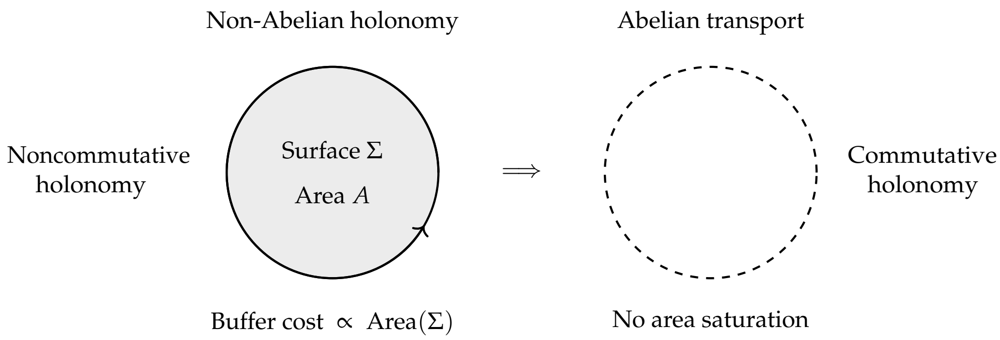

Figure 4. Non-Abelian buffer saturation and area law. Left: transport of internal closure mismatch around a loop generates irreducible obstruction whose minimal buffering cost grows with the area of the surface Σ bounded by the loop. Right: in Abelian sectors, commuting holonomies do not accumulate such obstruction, permitting unscreened long-range transport.

6. Why Gravity Is Not a Gauge Field

This section establishes that gravity does not belong to the gauge universality class identified in Section 4. The distinction is structural rather than phenomenological. Gauge fields implement local transport of internal reconstruction constraints *within* an already reconstructed spacetime, whereas gravity encodes global feasibility conditions that determine whether spacetime itself can be coherently reconstructed under RNF continuation. This distinction parallels, but is not reducible to, the traditional conceptual separation between Yang–Mills theory and general relativity [17,61,71].

6.1. Local Transport versus Global Feasibility

As shown in Section 4, gauge fields arise as admissible mechanisms for transporting internal closure mismatch across interfaces between neighboring reconstructed regions. They are characterized by: (i) locality, (ii) path-dependent transport operators, (iii) buffering or neutralization at interfaces, and (iv) representation by connections acting within a fixed reconstructed spacetime [43].

Gravity does not satisfy these criteria. Gravitational effects cannot be interpreted as the transport of an internal closure mismatch along spacetime paths, nor can they be screened, neutralized, or localized by interface structure. Instead, gravity constrains whether extended regions of reconstructed

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spacetime are admissible under RNF continuation at all, consistent with its role as a theory of dynamical geometry rather than internal symmetry [52].

Proposition 6.1 (Categorical distinction between gauge fields and gravity)**.** Gauge fields operate *within* a reconstructed spacetime by redistributing internal closure constraints across interfaces. Gravity operates *on* the space of admissible reconstructions by constraining their global feasibility under RNF continuation.

This distinction is invariant under external coarse-graining and does not depend on the choice of coordinates, variables, or field representations. It reflects a difference in *functional role* within reconstruction, not merely a difference in mathematical form [61].

6.2. Metric Structure as a Feasibility Functional

Within the RNF framework, the spacetime metric does not represent an independently propagating transport degree of freedom. Instead, it summarizes global consistency conditions required for admissible RNF reconstruction.

Let $(\mathcal{M}, g)$ denote the external spacetime associated with an admissibility attractor. The metric g encodes how local reconstruction steps can be coherently stacked across extended regions without accumulating obstruction. In this sense, g functions as a *feasibility functional*: it assigns admissibility weights to infinitesimal neighborhoods rather than carrying an internal transport charge.

Variations of g therefore correspond to changes in global reconstruction capacity, not to the exchange of a locally conserved quantity. In particular, there is no notion of gravitational charge analogous to internal closure mismatch transported by gauge fields.

This interpretation explains several characteristic features of gravity that distinguish it from gauge interactions [45]:

- *Universality of coupling*: all external structures contribute to global reconstruction feasibility, hence all forms of energy–momentum influence g.
- *Absence of screening*: global feasibility constraints cannot be locally buffered or neutralized without altering the admissibility class of the reconstruction.
- *Geometric character*: gravitational phenomena manifest as relations among metric structures encoding reconstruction consistency, rather than as force-mediated exchanges.

6.3. Emergence of Gravitational Field Equations

In dense admissibility basins, small variations of reconstruction feasibility induce smooth variations of the effective metric. Requiring RNF continuation to remain admissible under such variations yields continuum consistency conditions relating curvature to external energy–momentum content.

Proposition 6.2 (Einstein equations as feasibility conditions)**.** Einstein-type field equations arise as necessary consistency conditions for stable RNF reconstruction in the continuum limit, rather than as fundamental equations of motion for an independently propagating transport field.

Accordingly, gravitational field equations encode the requirement that reconstructed spacetime remain globally admissible under RNF continuation. They are structural conditions on feasible reconstruction, not dynamical transport laws of the same type as Yang–Mills equations [17,45].

6.4. Structural Scope of Non-Quantizability

Standard quantization procedures presuppose independently propagating degrees of freedom defined on a fixed or kinematically specified background. In the RNF framework, the metric defines that background by summarizing admissibility itself. Promoting the metric to an independent quantum operator would therefore amount to quantizing a feasibility condition with respect to itself, echoing longstanding conceptual tensions in canonical and covariant approaches to quantum gravity [34,52].

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Proposition 6.3 (Structural non-quantizability of gravity)**.** Because the spacetime metric encodes global reconstruction feasibility rather than local constraint transport, direct quantization of gravity *in analogy with gauge fields* is structurally ill-posed within the RNF framework.

This claim is deliberately scoped. It does not exclude the existence of quantized *effective* gravitational excitations (e.g. linearized spin–2 modes) in suitable external continuum limits [15]. Rather, it asserts that the metric itself is not a fundamental quantum transport degree of freedom.

Remark on a Common Misconception: "Gravity must be quantized"

It is often asserted that consistency with quantum matter requires gravity itself to be a fundamental quantum gauge field. The RNF framework does not make this assumption. The claim advanced here is narrower and structural: gravity does not function as a local constraint-transport sector and therefore does not admit quantization in direct analogy with Yang–Mills fields.

This does not deny the existence of quantum gravitational phenomena in effective descriptions, nor does it exclude semiclassical or effective-field-theory treatments. It asserts only that the spacetime metric encodes global reconstruction feasibility and is not an independently propagating internal degree of freedom subject to canonical or path-integral quantization [34].

6.5. Existence of Einstein–Maxwell Realizations

The results of Sections 2–6 are formulated at the level of RNF admissibility and do not assume a specific microscopic realization. Consequently, the general framework yields structural conclusions—such as hyperbolic continuum limits, gauge transport, and non-Abelian saturation—while the identification of particular continuum equations (Einstein, Maxwell, etc.) requires a concrete realization of the reconstruction map and TCP constraint functional.

Nevertheless, the framework is non-empty in a strong sense: explicit RNF implementations exist in which the admissibility axioms are satisfied and the stabilized continuum limit reproduces the standard Einstein–Maxwell system.

Proposition 6.4 (Einstein–Maxwell existence)**.** There exists a class of concrete RNF reconstruction models satisfying the assumptions of Sections 2–3 and the transport construction of Section 4, such that in the stabilized continuum limit the emergent external fields obey the Einstein field equations coupled to Maxwell electrodynamics, up to higher-order corrections outside the admissibility basin.

Proof by construction (summary). Concrete realizations are provided by chronon-field RNF models developed in prior work, in which an explicit RNF extension map and TCP constraint functional are defined, an emergent metric is extracted from reconstruction feasibility, and the continuum limit yields Einstein curvature relations together with Maxwell-type equations for unsaturated Abelian constraint transport [39]. A concise synopsis of the construction is given in Appendix F. □

Proposition 6.4 is an existence result. The present paper identifies which features are universal consequences of RNF admissibility, while explicit models demonstrate that standard gravitational and electromagnetic dynamics lie within at least one admissibility attractor.

6.6. Summary

Gauge fields and gravity occupy distinct universality classes. Gauge fields implement local transport of internal constraints within spacetime, while gravity encodes global feasibility conditions for spacetime reconstruction itself. This distinction explains the universality, geometric character, absence of screening, and the limited sense in which gravity resists quantization, and it reinterprets general relativity as an effective theory governing admissible reconstruction rather than a fundamental gauge dynamics.

As shown in Figure 5, gauge interactions act on internal degrees of freedom within an already reconstructed geometry, while gravity arises from feasibility constraints on the global gluing of reconstructed spacetime patches.

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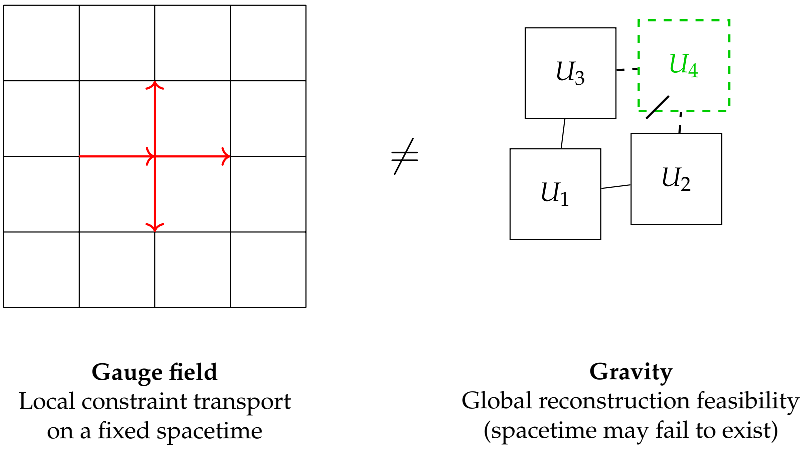

Figure 5. Why gravity is not a gauge field. Left: gauge interactions implement local transport of internal closure constraints on an already reconstructed spacetime. The background structure is fixed and unaffected by transport. Right: gravity encodes global feasibility conditions for spacetime reconstruction itself. Local patches may exist, but global gluing can fail. Gravity therefore governs whether a spacetime can be coherently reconstructed at all, rather than transporting constraints within it.

7. Emergence of General Relativity from Reconstruction Feasibility

In this section we derive general relativity as the unique leading-order continuum theory governing global reconstruction feasibility under RNF admissibility. Unlike gauge fields, which arise from local transport of internal closure mismatch, gravity encodes constraints on the admissibility of spacetime reconstruction itself. As a result, its emergence is variational rather than transport-based.

The derivation assumes locality, second-order stability, and diffeomorphism invariance of the continuum feasibility functional.

The derivation proceeds in four steps. First, we show that admissible continuum limits must belong to a hyperbolic causal universality class. Second, we identify the metric as the unique order parameter capable of encoding global reconstruction feasibility. Third, we characterize the admissible class of feasibility functionals compatible with RNF continuation. Finally, we show that the Einstein equations arise as the Euler–Lagrange conditions of the unique admissible functional in four spacetime dimensions.

7.1. Hyperbolic Continuum Geometry

From Theorem 3.1 established in Section 3, admissible continuum descriptions of RNF reconstruction must possess a hyperbolic principal structure in order to preserve finite propagation of admissibility constraints.

Hyperbolicity implies the existence of finite characteristic cones governing information propagation. In continuum descriptions this structure naturally admits representation by a Lorentzian metric $g_{\mu\nu}$ whose null cones define the causal propagation of reconstruction compatibility.

Thus any admissible large-scale RNF reconstruction basin admits an effective description in terms of a causal metric geometry.

7.2. Global Feasibility Functionals

Let $(\mathcal{M}, g)$ denote an external spacetime emerging as a stable admissibility basin of RNF reconstruction.

The metric does not represent a transported internal degree of freedom. Instead it summarizes how local reconstruction steps can be coherently composed over extended regions.

We formalize this by introducing a *global feasibility functional*

$$\mathcal{F}[g] = \int_{\mathcal{M}} \mathcal{L}(g, \partial g, \partial^2 g)\, dV_g,$$

which assigns a reconstruction feasibility cost to a candidate external geometry.

Definition 7.1 (Admissible feasibility functional)**.** A functional $\mathcal{F}[g]$ is admissible if it satisfies

1. **Locality:** $\mathcal{L}$ depends only on g and finitely many derivatives.
2. **Extensivity:** $\mathcal{F}$ is additive under gluing of reconstructed regions.
3. **Diffeomorphism invariance:** $\mathcal{F}$ depends only on intrinsic geometry.
4. **Stability:** small perturbations of g do not generate unbounded feasibility cost under RNF continuation.

These conditions arise directly from RNF admissibility and do not depend on any particular microscopic realization of reconstruction dynamics.

7.3. Metric as the Feasibility Order Parameter

We now establish that the metric tensor is the unique geometric object capable of encoding global reconstruction feasibility.

Proposition 7.2 (Metric feasibility parameter)**.** Under RNF admissibility, the external metric $g_{\mu\nu}$ is the unique local tensor field capable of encoding global reconstruction feasibility while remaining compatible with locality, compositional extension, and coarse-grained stability.

Sketch. Scalar fields cannot encode directional propagation constraints, while vector fields introduce preferred directions that destabilize reconstruction under RNF continuation.

Higher-rank geometric structures introduce anisotropies or nonlocal dependencies incompatible with coarse-grained stability.

A symmetric rank–2 tensor with Lorentzian signature uniquely encodes local reconstruction capacity while preserving compositional extension.

□

Thus the metric emerges naturally as the order parameter governing reconstruction feasibility.

7.4. Structural Constraints on Feasibility Functionals

RNF admissibility imposes strong restrictions on the functional $\mathcal{F}[g]$.

Lemma 7.3 (Derivative order bound)**.** *Admissible feasibility functionals depend on at most second derivatives of the metric.*

Sketch. Higher-derivative terms introduce additional boundary data at each reconstruction step. Small perturbations therefore amplify under continuation, violating RNF stability.

□

Consequently admissible functionals belong to the Lovelock class of geometric actions [40].

7.5. Lovelock Uniqueness and Einstein Gravity

A classical result due to Lovelock establishes that in four spacetime dimensions the only local diffeomorphism-invariant action built from the metric and its derivatives up to second order is the Einstein–Hilbert functional with a cosmological constant.

Theorem 7.4 (Lovelock uniqueness)**.** *In four spacetime dimensions, the only nontrivial local gravitational functional satisfying the admissibility conditions above is*

$$\mathcal{F}[g] = \int_{\mathcal{M}} (R - 2\Lambda)\, dV_g.$$

All higher Lovelock terms either vanish identically or reduce to topological boundary contributions and therefore do not influence feasibility stationarity.

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7.6. Einstein Equations as Feasibility Stationarity

Admissible reconstruction requires stationarity of the feasibility functional.

Proposition 7.5 (Einstein equations)**.** Stationarity of $\mathcal{F}[g]$ yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

Sketch. Variation of the Einstein–Hilbert functional produces the Einstein tensor. Coupling to external reconstruction content follows from extensivity under gluing of reconstructed regions.

□

Thus the Einstein equations represent the condition that reconstructed spacetime remains globally admissible under RNF continuation.

7.7. Interpretation and Scope

The derivation above establishes general relativity as the unique leading-order continuum theory governing global reconstruction feasibility in four dimensions.

This result does not determine the numerical values of coupling constants nor exclude higher-curvature corrections outside the admissibility basin. Instead it explains why any stable continuum description of RNF reconstruction must reduce to Einstein gravity at large scales.

The argument depends only on structural admissibility requirements and therefore holds across broad classes of RNF realizations admitting a smooth continuum limit.

8. Symmetry as an Admissibility Attractor

In the RNF Admissibility Framework, symmetry is neither postulated as a kinematic axiom nor imposed as an invariance principle at the metascopic level. Instead, symmetry arises as a *structural property of reconstruction patterns that persist* under indefinite RNF continuation. In this sense, symmetries characterize *admissibility attractors*: stable equivalence classes of reconstruction whose local feasibility constraints remain coherent and unsuppressed across arbitrarily extended continuation. This perspective parallels, but is distinct from, renormalization-group and effective-field-theory notions of universality [47,68].

The purpose of this section is to formalize this notion and to show that familiar physical symmetries—most notably Lorentz invariance and gauge redundancy—arise as unavoidable structural features of admissible reconstruction, rather than as fundamental postulates.

8.1. Standing Assumptions

Throughout this section we assume the following minimal structural conditions, all of which are intrinsic to RNF reconstruction:

1. **Locality of feasibility:** admissibility constraints act locally and compose under RNF extension.
2. **Finite interface capacity:** the amount of internal mismatch that can be buffered or repaired across any reconstruction interface is bounded.
3. **Coarse-graining stability:** reconstruction patterns are identified up to external equivalence, and only coarse-grained feasibility properties are physically relevant.
4. **Indefinite continuation:** RNF reconstruction proceeds without halting; patterns that fail admissibility are suppressed rather than terminating reconstruction.

No assumption of spacetime, metric structure, or symmetry is made *a priori*.

8.2. Admissibility Attractors

Let $\mathcal{R}$ denote the space of admissible reconstruction patterns modulo external equivalence. RNF continuation induces a (generally non-dynamical) extension operator

$$\mathcal{E} : \mathcal{R} \to \mathcal{R},$$

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which maps a reconstruction pattern to the class of patterns obtained by admissible extension and coarse-graining.

Definition 8.1 (Admissibility attractor)**.** A reconstruction class $[R] \in \mathcal{R}$ is an *admissibility attractor* if there exists a neighborhood $\mathcal{N}([R])$ such that any admissible perturbation $R' \in \mathcal{N}([R])$ remains within $\mathcal{N}([R])$ under repeated RNF continuation, while perturbations outside this basin accumulate feasibility obstruction and are progressively suppressed.

Admissibility attractors therefore represent reconstruction patterns whose feasibility constraints are self-consistent under indefinite extension. Symmetry arises as an invariance property of these basins, not as a microscopic assumption, mirroring how universality classes emerge in statistical and quantum field theories [68].

8.3. Lorentz Symmetry as a Causal Attractor

RNF reconstruction does not presuppose spacetime or causal structure. Instead, causal organization emerges as a structural property of admissible continuum limits.

Theorem 3.1 in Section 3 establishes that stable external fields arising from admissible RNF reconstruction satisfy hyperbolic continuum equations. The rigorous derivation of this result is given in Appendix B. Hyperbolic differential operators possess well-defined characteristic cones that determine the domain of influence of disturbances [13]. These cones define an invariant causal structure in the emergent continuum description.

Lemma 8.2 (Frame-independence of feasibility)**.** *If an admissibility basin admits multiple inertial descriptions, feasibility constraints must be invariant under changes of inertial frame within the basin. Reconstruction patterns that privilege a foliation or spatial direction generate anisotropic obstruction under RNF continuation and are therefore unstable.*

Combining hyperbolicity with frame-independence restricts admissible continuum descriptions to those whose characteristic cones are preserved under inertial transformations.

Proposition 8.3 (Lorentz symmetry as an admissibility attractor)**.** Consider an admissible continuum limit whose governing equations are hyperbolic and whose feasibility constraints admit no preferred inertial frame.

Under indefinite RNF continuation, the Lorentz group constitutes the unique non-singular symmetry group preserving the causal cone structure. The Galilean group arises only as a singular limiting case corresponding to infinite signal speed.

Proof. Hyperbolic equations define invariant characteristic cones. If no preferred inertial frame exists within an admissibility basin, the transformations relating inertial descriptions must preserve these cones.

Among continuous transformation groups, only Lorentz transformations preserve a finite invariant causal cone under composition [2]. Transformations outside this class distort the cone structure, introducing anisotropy or preferred directions that accumulate feasibility obstruction under RNF continuation.

Therefore Lorentz symmetry defines the stable admissibility attractor for continuum causal structure.

□

A detailed classification of admissible causal symmetry groups under RNF feasibility assumptions is provided in Appendix E.

8.4. Gauge Symmetry as Redundancy of Closure Transport

Gauge symmetry arises not as a spacetime symmetry, but as a redundancy in the representation of internal closure transport, consistent with the geometric interpretation of gauge theories [43].

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Let $\mathcal{U}$ and $\mathcal{U}'$ be two constraint-transport functors related by pointwise conjugation:

$$\mathcal{U}'(\gamma) = g(x)\,\mathcal{U}(\gamma)\,g(y)^{-1}, \qquad \gamma : x \to y.$$

Proposition 8.4 (Gauge redundancy)**.** Transport functors related by local conjugation define the same admissibility basin and yield identical reconstructed spacetime under external coarse-graining.

Gauge symmetry therefore reflects representational freedom in internal closure bookkeeping, not a physical transformation acting on reconstructed spacetime, in agreement with the standard fiber-bundle formulation of gauge theories [43].

8.5. Constraints on Admissible Gauge Groups

Finite interface capacity and obstruction accumulation impose strong restrictions on closure algebras that can persist under RNF continuation.

Proposition 8.5 (Admissible gauge limitation)**.** Only closure algebras whose associated transport admits indefinite RNF continuation without unbounded saturation define admissibility attractors. Closure structures outside this class accumulate irreducible obstruction and are suppressed under extension.

These admissibility constraints strongly favor compact, low-rank closure algebras whose transport properties remain stable under indefinite reconstruction. Excessively large or highly non-Abelian algebras typically accumulate obstruction faster than it can be redistributed across reconstruction interfaces.

8.6. Admissibility and the Standard Model Gauge Structure

The admissibility framework constrains the class of internal symmetry structures that can persist under indefinite RNF continuation.

As shown in Section 4, internal degrees of freedom arise as closure data attached to reconstruction histories, and their transport is described by a closure algebra $\mathfrak{g}$ whose integrated group G governs constraint transport. Admissibility therefore requires internal symmetries to form Lie algebras generating connection-based transport structures.

Additional structural constraints arise from reconstruction stability. Abelian closure algebras permit unsuppressed long-range transport, producing Maxwell-type continuum dynamics. The electromagnetic $U(1)$ sector of the Standard Model therefore lies naturally within the class of admissible long-range closure algebras.

By contrast, the Non-Abelian Buffer Saturation Theorem (Section 5) shows that non-Abelian closure transport generically leads to saturation when interface buffering capacity is finite. In the continuum limit this produces confinement and a nonzero mass gap. These admissibility properties match the behavior of the strong interaction described by the $SU(3)$ gauge sector of quantum chromodynamics.

More generally, admissibility constraints favor compact low-rank closure algebras whose transport properties remain stable under indefinite reconstruction. The gauge group

$$SU(3) \times SU(2) \times U(1)$$

of the Standard Model lies within this admissible class. While the present work does not attempt a full derivation of the Standard Model gauge group, it identifies structural admissibility principles that constrain the space of viable gauge symmetries and naturally accommodate the observed gauge structure of particle physics.

8.7. Universality Classes and Symmetry Breaking

Admissibility attractors define universality classes of reconstruction.

Definition 8.6 (Universality class)**.** A universality class is an equivalence class of reconstruction patterns that converge to the same admissibility attractor under RNF continuation.

Symmetry breaking corresponds to transitions between admissibility basins rather than to violations of fundamental laws. Apparent symmetry violation reflects a change in persistence structure under RNF continuation, analogous to phase transitions in condensed matter systems [21].

8.8. Universality Classes and Particle Spectra

Particle-like entities arise as stable excitation types supported by admissibility attractors.

Definition 8.7 (Excitation type)**.** Fix an admissibility attractor $[R]$. An excitation type is an equivalence class of localized admissible defects or composites that persist under RNF continuation, remain composable within the basin, and induce stable external signatures under coarse-graining.

Let $\mathrm{Spec}([R])$ denote the set of excitation types supported by $[R]$.

Theorem 8.8 (Spectral invariance of universality classes)**.** *If two reconstruction patterns converge to the same admissibility attractor, then their excitation spectra coincide:*

$$R_1 \sim_{\mathrm{univ}} R_2 \quad \Longrightarrow \quad \mathrm{Spec}([R_1]) = \mathrm{Spec}([R_2]).$$

Proof. Convergence to the same admissibility attractor implies identical basin-level feasibility constraints. Persistence, composability, and stability of excitation types depend only on these constraints, not on microscopic realization. Hence excitation spectra coincide under external coarse-graining [68]. □

8.9. Summary

Symmetries arise as consequences of admissibility selection. Lorentz invariance emerges as the unique causal organization compatible with finite-speed, frame-independent RNF continuation, while gauge symmetries reflect redundancy in closure transport constrained by interface capacity and noncommutativity. Admissibility constraints favor compact low-rank closure algebras and naturally accommodate the Standard Model gauge symmetry $SU(3) \times SU(2) \times U(1)$. Together, these features define admissibility basins and universality classes that determine the observed symmetry structure and particle content of physical law.

As depicted in Figure 6, among candidate causal cone structures, Lorentz-invariant cones uniquely support stable admissible continuation under RNF reconstruction, whereas non-Lorentzian cones accumulate incompatibilities and fail to persist in the external description.

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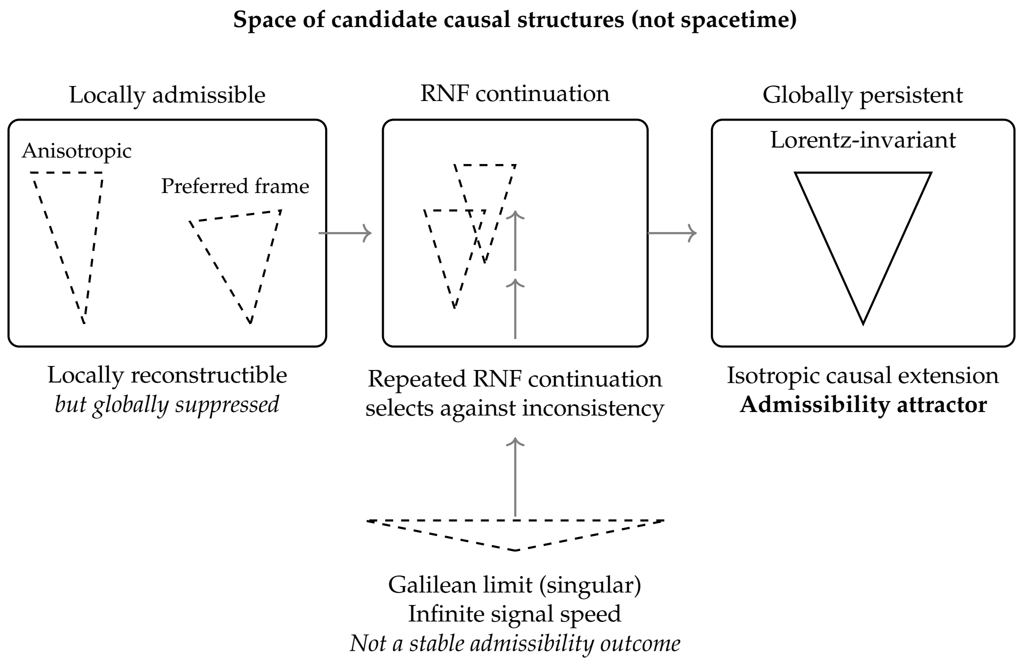

Figure 6. Lorentz symmetry as an admissibility attractor under RNF continuation. The figure depicts the space of candidate causal structures, not spacetime itself. Anisotropic and preferred-frame causal cones are locally reconstructible and may arise under finite RNF extension. However, under repeated RNF continuation they accumulate global incompatibilities and are progressively suppressed. Only Lorentz-invariant causal cones permit isotropic, obstruction-free continuation and thus persist as stable external descriptions. The Galilean cone represents a singular boundary case with infinite signal speed and does not arise as a generic outcome of admissibility selection. Arrows denote structural selection under RNF continuation, not temporal evolution or dynamical flow.

9. Constraints and Predictions

The RNF Admissibility Framework constrains physical theory at a structural level. Rather than predicting numerical parameters or detailed spectra, it restricts the space of reconstruction patterns that can persist under indefinite RNF continuation. Physical predictions therefore take the form of *structural exclusions, stability requirements, and suppression mechanisms*. These predictions are qualitative but falsifiable: entire classes of phenomena are either admissible or systematically sculpted away by reconstruction, in a manner analogous to universality constraints in effective field theory [47,68].

9.1. Admissibility Constraints on Theory Space

Let $\mathcal{R}$ denote the space of reconstruction patterns modulo external equivalence, and let $\mathcal{E}$ be the RNF extension operator introduced in Section 8. RNF continuation acts not as a dynamical evolution but as a filtering process that progressively suppresses incompatible structure.

Proposition 9.1 (Structural constraint on extensions)**.** Any viable physical theory must correspond to a reconstruction pattern that lies within the basin of an admissibility attractor of $\mathcal{E}$. Patterns outside all such basins are not forbidden outright, but are progressively sculpted away under RNF continuation and cannot support stable large-scale external descriptions.

This criterion sharply limits beyond–Standard–Model proposals. Additional degrees of freedom, interactions, or symmetry structures must be compatible with extended admissibility; otherwise their effects are suppressed, confined, or rendered non-propagating at macroscopic scales. This structural constraint complements, but is independent of, conventional naturalness or fine-tuning arguments [62].

9.2. Predictions for Gauge Structure

The admissibility analysis of Sections 4 and 5 yields the following predictions:

- *Universality of confinement*. Non-Abelian closure transport generically produces irreducible obstruction under RNF continuation. Long-range non-Abelian mismatch is therefore progressively suppressed, forcing neutralization into composite structures. Observation of a genuinely unscreened long-range non-Abelian interaction would falsify the framework, consistent with the empirical absence of free color charge in QCD [22].
- *Restricted gauge algebra complexity*. Only closure algebras whose transport can be buffered without rapid saturation define admissibility attractors. Highly noncommutative or large-rank gauge groups are not excluded *a priori*, but their long-range effects are structurally suppressed unless they decouple from reconstruction interfaces, providing a structural explanation for the limited gauge content observed in nature [64].
- *Uniqueness of unsaturated Abelian sectors*. Abelian closure transport uniquely avoids obstruction accumulation and defines the sole unsaturated admissibility channel. This explains the persistence of electromagnetism as the only long-range gauge interaction without invoking special dynamical assumptions, in contrast to non-Abelian confinement mechanisms [46].

These conclusions depend only on noncommutativity, locality, and finite interface capacity, and are independent of renormalization-group flow or specific Lagrangian dynamics.

9.3. Predictions for Spacetime Structure

The emergence of spacetime geometry as an admissibility attractor yields further constraints:

- *Robustness of Lorentz symmetry*. Lorentz-invariant causal organization defines a stable admissibility fixed point. Causal structures that privilege anisotropy or preferred frames are not prohibited locally, but are progressively suppressed under RNF continuation. Persistent macroscopic Lorentz violation would therefore signal departure from the standard reconstruction basin, consistent with stringent experimental bounds [36,42].
- *Absence of gravitational screening*. Because gravity encodes global feasibility rather than local transport, no admissibility basin supports screened, short-range, or neutralized gravitational interaction, in agreement with the universality of gravitational coupling in general relativity [65].
- *Non-propagating character of gravitational degrees of freedom*. Gravity does not correspond to an independent constraint-transport channel. Observations requiring freely propagating, gauge-like quantum gravitational modes would contradict the structural assumptions of the framework, though effective graviton descriptions may still arise in suitable perturbative limits [15].

These predictions do not deny quantum effects in gravitational contexts, but constrain their origin to matter-induced changes in admissibility rather than independent gravitational quanta.

9.4. Breakdown Regimes and Observable Deviations

Effective field theories correspond to interior regions of admissibility basins where reconstruction is far from saturation. Near basin boundaries, where obstruction accumulates, continuum descriptions must progressively fail.

Proposition 9.2 (Predictive breakdown)**.** Observable deviations from standard continuum physics are expected in regimes approaching admissibility saturation, including extreme curvature, density, or non-Abelian excitation. Such deviations are not abrupt failures but smooth departures reflecting structural suppression.

These regimes provide concrete experimental and observational tests. Confirmation that continuum descriptions degrade precisely where admissibility predicts saturation would support the reconstruction-first framework, while sharp violations in admissible regimes would falsify it. Similar breakdown expectations appear in effective quantum gravity and high-energy EFT analyses [3].

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9.5. Summary

Predictions of the RNF Admissibility Framework take the form of structural selection rules and suppression mechanisms. Only reconstruction patterns corresponding to admissibility attractors can support stable large-scale physics. Confinement, limited gauge structure, robust Lorentz symmetry, and the distinctive role of gravity emerge as necessary features of admissible reconstruction rather than as contingent properties of specific dynamical models.

10. Discussion

10.1. Relation to Standard Quantum Field Theory

Standard quantum field theory (QFT) provides an extraordinarily successful effective description of particle physics, formulated in terms of fields, symmetries, and operator algebras defined on a spacetime background [46,63]. The present framework does not modify QFT at the level of empirical predictions. Instead, it proposes a structural reinterpretation of why QFT provides a stable and universal description across a wide range of physical systems.

Within the admissibility framework, gauge fields can be viewed as effective representations of internal constraint transport, while quantum states encode coarse-grained statistical summaries over ensembles of admissible reconstruction histories. The success of perturbative methods, lattice formulations, and nonperturbative techniques reflects the stability of these summaries under coarse-graining, rather than implying that Hilbert space or operator algebras are necessarily fundamental.

Renormalization-group flow admits a natural interpretation in this setting as the systematic organization of admissible reconstruction patterns under successive coarse-graining. Fixed points correspond to admissibility attractors, while irrelevant operators represent distinctions that do not persist under continued reconstruction, consistent with standard universality arguments [47,69].

From this perspective, phenomena such as confinement and mass gaps may be understood as structural consequences of admissibility constraints in non-Abelian sectors. This does not replace existing QFT descriptions, but provides an alternative route to understanding why such phenomena arise generically in consistent continuum limits.

10.2. Relation to General Relativity

General relativity (GR) describes gravity as the dynamics of spacetime geometry governed by Einstein's field equations [9,18]. Within the admissibility framework, these equations arise as effective conditions ensuring global feasibility of spacetime reconstruction, rather than as fundamental dynamical laws for an independently propagating metric field.

This viewpoint provides a structural interpretation of several characteristic features of gravity, including the universality of coupling and the absence of screening. Gravitational curvature encodes constraints on admissible global reconstruction rather than the exchange of gauge-mediated forces. In this sense, gravity occupies a distinct universality class from gauge interactions.

It should be emphasized that this interpretation is fully compatible with the experimentally verified predictions of GR within its domain of validity. The contribution of the present framework is not to alter these predictions, but to offer a structural explanation for why gravitational dynamics take the observed form and why gravity differs qualitatively from other interactions.

10.3. Relation to Other Foundational and Emergent Frameworks

The admissibility-based reconstruction framework is conceptually aligned with approaches that treat spacetime and physical law as emergent rather than fundamental [38,44]. However, it differs in both assumptions and mechanisms.

Many emergent-spacetime programs rely on coarse-graining of systems defined on a pre-existing Hilbert space or information-theoretic structure [57,58]. In contrast, the present framework does not assume Hilbert space, entanglement measures, or predefined information structures. Coarse-graining is instead defined operationally through indistinguishability under admissible reconstruction.

Constraint- and causality-based approaches, including causal set theory and axiomatic reconstructions of spacetime symmetry, emphasize partially ordered structures [7,41]. The present framework shares this emphasis on ordering, but differs in that spacetime causality itself is emergent: the directed admissibility relation is defined prior to any spacetime interpretation.

Approaches that quantize geometry or treat spacetime degrees of freedom as fundamental quantum variables adopt a different starting point [4]. In the present framework, spacetime geometry instead summarizes global feasibility conditions and is not assumed to belong to the same class of fundamental variables as transport degrees of freedom.

Concrete realizations of admissibility-based reconstruction, including chronon-field models, have been developed in prior work [39]. These should be understood as specific implementations of a broader structural class.

10.4. Advantages of a Feasibility-First Perspective

A central feature of the admissibility framework is the shift from dynamics-first to feasibility-first reasoning. Rather than postulating microscopic dynamical laws on a fixed background, the framework identifies constraints under which extended structures can be consistently constructed and maintained.

This perspective provides a unified way to understand several persistent features of known physics:

- the stability of continuum field descriptions across scales,
- the robustness of symmetry structures,
- the generic emergence of confinement in non-Abelian sectors,
- the distinct role of gravity relative to gauge interactions.

Importantly, these features are not introduced as independent axioms, but arise as structural consequences of admissibility. This suggests that aspects of known physical law may reflect general consistency conditions rather than arbitrary dynamical choices.

At the same time, the framework is intended to complement existing theories rather than replace them. It provides constraints on the space of viable continuum descriptions, while leaving detailed dynamical modeling to established formalisms.

10.5. Sharp Falsifiers

The admissibility framework makes structural claims that restrict the space of physically realizable theories. These restrictions lead to testable exclusions and therefore render the framework falsifiable.

The framework would be challenged or falsified by any of the following:

- **Unscreened long-range non-Abelian interactions.** Observation of freely propagating, unscreened long-range non-Abelian forces would contradict the predicted suppression of such modes under finite admissibility constraints.
- **Macroscopic gravitational screening.** Experimental evidence for gravitational shielding or Yukawa-type suppression at large scales would be inconsistent with the identification of gravity as a global feasibility constraint.
- **Robust Lorentz violation in inertial regimes.** Persistent Lorentz violation not attributable to cutoff or environmental effects would contradict the emergence of Lorentz symmetry as a stable admissibility attractor.

These falsifiers distinguish the present framework from purely interpretive approaches by excluding classes of possible effective theories.

10.6. Limitations and Open Questions

The present work is intentionally structural and abstract. While this generality allows broad applicability, it leaves open important questions concerning explicit realizations, numerical simulation, and quantitative prediction. In particular, the connection between admissibility-based reconstruction and

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full quantum dynamics, including interference and measurement structure, remains to be developed in detail.

10.7. Summary

Within the admissibility framework, quantum field theory and general relativity can be interpreted as effective descriptions associated with stable reconstruction regimes. The feasibility-first perspective provides a unifying structural viewpoint while preserving the empirical success of existing theories, and offers a framework for identifying which features of known physics are likely to persist in more fundamental descriptions.

11. Scope and Limitations of the Reconstruction Framework

The present work establishes structural consequences of admissible reconstruction in the continuum limit. The results should therefore be interpreted as constraints on possible effective physical theories compatible with reconstruction feasibility, rather than as a complete microscopic theory of nature or a replacement for established frameworks such as quantum field theory or general relativity.

The analysis applies to a broad class of reconstruction-based or generative frameworks, of which the RNF formulation introduced here provides one concrete realization. The conclusions rely only on general admissibility, locality, and compositionality assumptions, and are therefore intended to be implementation-independent.

Several limitations should be emphasized.

Microscopic realization.

The framework does not assume a specific microscopic model of metascopic configurations or reconstruction dynamics. Instead it derives continuum consequences from general admissibility principles. Explicit microscopic realizations capable of reproducing the continuum structures described here remain an important direction for future work. Existing constructions should be regarded as existence proofs rather than complete physical models.

Coupling constants.

The derivations presented here determine the structural form of the leading-order continuum equations but do not fix numerical values of coupling constants such as the gravitational constant or gauge couplings. These parameters depend on details of the underlying reconstruction mechanism and renormalization structure, and therefore lie outside the scope of the present analysis.

Higher-curvature and non-leading corrections.

The emergence results apply to the leading continuum limit within admissibility basins admitting smooth coarse-graining. Near reconstruction saturation, or outside the stability basin, higher-order curvature terms and nonlocal corrections may become relevant. The present work does not attempt to classify these corrections in detail, but only establishes the leading-order admissible structure.

Matter sector and particle content.

The present work treats external matter content through an effective stress–energy tensor summarizing coarse-grained reconstruction structure. While the framework provides structural constraints on gauge sectors and transport, it does not constitute a full derivation of the Standard Model, particle spectra, or interaction hierarchies. Such results would require additional assumptions about admissibility classes and microscopic realization.

Non-Abelian confinement and mass gap.

The Non-Abelian buffer saturation mechanism derived here provides a structural route to confinement and the suppression of long-range non-Abelian modes under admissibility constraints. However, it should not be interpreted as a formal solution of the Yang–Mills mass gap problem in the mathemati-

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cal sense. Rather, it identifies a general mechanism by which a mass scale can emerge in admissible continuum limits.

Gravity and uniqueness.

The identification of the Einstein–Hilbert functional as the leading-order admissible continuum action in four dimensions should be understood as a structural compatibility result within the reconstruction framework. It does not exclude the presence of subleading corrections, alternative phases, or additional degrees of freedom in more general admissibility regimes.

Quantum reconstruction amplitudes.

Although reconstruction histories naturally suggest a path-sum structure for transition amplitudes, the present paper focuses on the classical continuum limit of admissible reconstruction. The precise relation between reconstruction ensembles and quantum mechanical amplitudes, including interference structure and Hilbert-space representations, remains to be developed.

Interpretational status.

The framework is best understood as a structural selection principle for admissible continuum physics. It does not assume that reconstruction is the unique or fundamental ontology of nature. Instead, it identifies general constraints under which continuum field theories, gauge structures, and gravitational dynamics can emerge as stable, coarse-grained descriptions. In this sense, the results complement rather than replace existing theoretical frameworks.

Despite these limitations, the analysis demonstrates that a wide class of continuum structures—including causal propagation, gauge transport, and Einstein gravity—can arise as feasibility conditions of admissible reconstruction. This suggests that key features of known physics may be understood as stability properties of a broader class of generative structures rather than as fundamental dynamical postulates.

12. Conclusion

This work has developed a feasibility-first framework for fundamental physics based on reconstruction and admissibility principles. In this perspective, physical law is not formulated as dynamical evolution on a pre-existing spacetime background. Instead, the observable world is associated with the persistent organization of reconstruction patterns that remain coherent under indefinite admissible continuation. Reconstruction does not halt or fail in a dynamical sense; rather, incompatible structures accumulate obstruction and are progressively suppressed, while structurally stable configurations persist as admissibility attractors.

Within this framework several structural results follow.

First, external spacetime fields arise as stable order parameters of admissible reconstruction. Their continuum descriptions emerge as coarse-grained limits of reconstruction ensembles. Locality, finite signal speed, and hyperbolic field equations appear not as imposed kinematic principles but as stability conditions required for admissible continuation. The Hyperbolicity Dominance Theorem (Appendix B) shows that stable continuum descriptions must possess hyperbolic principal symbols, ensuring finite domains of influence and consistent causal structure.

Second, internal degrees of freedom arise as closure data transported along reconstruction paths. Gauge structure can therefore be interpreted as an effective representation of constraint transport across reconstructed spacetime. Abelian closure algebras define unsaturated transport channels and yield long-range interactions, while non-Abelian closure transport generically accumulates obstruction under finite interface capacity. The resulting Non-Abelian Buffer Saturation Theorem identifies a structural mechanism by which confinement and a nonzero mass scale can emerge in admissible continuum limits, consistent with known nonperturbative features of gauge theory.

Third, gravity is structurally distinct from gauge interactions. The spacetime metric does not represent transport of internal closure data but instead encodes global feasibility conditions governing

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which spacetime reconstructions remain admissible. In the continuum limit, these conditions select Einstein-type consistency relations at leading order, in agreement with known uniqueness results for diffeomorphism-invariant gravitational theories. In this sense, gravity can be viewed as a constraint on global reconstruction rather than a conventional gauge interaction.

Symmetry in this framework is not postulated a priori but emerges as a property of stable reconstruction regimes. Lorentz invariance arises as a robust organization compatible with finite propagation and admissibility constraints, while gauge symmetry reflects redundancy in the transport of internal closure data. These considerations suggest a structural basis for the emergence and stability of symmetry in effective continuum theories.

Taken together, these results support a reorganization of foundational physics around structural feasibility rather than fundamental dynamics. The admissibility framework imposes nontrivial constraints on the space of viable continuum descriptions while remaining fully compatible with established empirical results. Explicit constructions demonstrating Einstein–Maxwell emergence provide evidence that familiar gravitational and electromagnetic dynamics can arise within admissible reconstruction regimes.

Future work lies in the systematic exploration of admissibility basins and their boundaries. Near such boundaries, continuum descriptions are expected to break down, potentially revealing new universality classes of reconstruction and new forms of physical organization beyond the domain of standard field-theoretic descriptions.

Appendix A. A Toy Reconstruction Model

To illustrate how continuum equations can arise from admissible reconstruction, we present a minimal toy model. The purpose of this example is not to represent a realistic microscopic model of RNF reconstruction but to demonstrate that reconstruction rules satisfying local admissibility can generate familiar continuum behavior.

Appendix A.1. Discrete Configuration Space

Consider a one-dimensional lattice with sites $i \in \mathbb{Z}$. At each reconstruction step the configuration is specified by a scalar field

$$\Phi_t = \{\phi_i(t)\}_{i\in\mathbb{Z}}.$$

A reconstruction step consists of generating a new configuration Φ_{t+1} from Φ_t according to a local admissibility rule.

Appendix A.2. Local Reconstruction Rule

We impose a local constraint that favors smooth continuation of the field across neighboring lattice sites. A simple admissible rule is

$$\phi_i(t+1) = 2\phi_i(t) - \phi_i(t-1) + c^2\big(\phi_{i+1}(t) - 2\phi_i(t) + \phi_{i-1}(t)\big),$$

where c is a constant controlling propagation speed.

This rule represents a discrete reconstruction constraint: the value at site i at step $t+1$ is chosen so that local mismatch with neighboring sites is minimized.

Appendix A.3. Continuum Limit

Let lattice spacing be Δx and reconstruction step size be Δt. Expanding $\phi_{i\pm1}(t)$ and $\phi_i(t\pm1)$ in Taylor series gives

$$\phi(x,t+\Delta t) = 2\phi(x,t) - \phi(x,t-\Delta t) + c^2\Delta x^2\frac{\partial^2\phi}{\partial x^2}.$$

Rearranging and taking the limit $\Delta t, \Delta x \to 0$ yields

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \frac{\partial^2 \phi}{\partial x^2},$$

which is the classical wave equation.

Appendix A.4. Interpretation

In this toy example the continuum field $\phi(x,t)$ emerges as an order parameter summarizing admissible reconstruction of the discrete configuration Φ_t.

The wave equation arises as the stability condition for local reconstruction feasibility: configurations that violate the wave equation amplify mismatch under continuation and therefore lie outside the admissibility basin.

This simple model illustrates the general mechanism proposed in the present work: continuum field equations appear as feasibility conditions for stable reconstruction rather than as fundamental microscopic laws.

Appendix A.5. Limitations of the Toy Model

The model presented here is intentionally minimal. It does not include internal closure structure, gauge transport, or metric reconstruction. Its sole purpose is to demonstrate how local reconstruction rules can generate continuum equations in the appropriate limit.

More sophisticated reconstruction models incorporating internal closure algebras and compatibility transport may generate richer continuum structures such as gauge fields and curved spacetime geometry.

Appendix B. Hyperbolicity as a Structural Consequence of RNF Admissibility

This appendix provides a structural derivation of hyperbolicity for continuum field equations emerging from admissible RNF reconstruction. The goal is to show that hyperbolic propagation is not assumed but follows necessarily from locality, compositional admissibility, and stability under coarse-graining.

Throughout this appendix we adopt the standing assumptions (A1)–(A4) introduced in Section 3.

Appendix B.1. Local Reconstruction Influence

RNF reconstruction proceeds by admissible extension of configurations across local interfaces. By assumption (A1), admissibility constraints depend only on bounded neighborhoods of the extension.

Definition B.1 (Reconstruction influence radius)**.** For any admissible extension (Φ, Φ'), define the *reconstruction influence radius* r as the maximal distance over which local changes in Φ can modify the admissibility of the extension.

Local admissibility implies that r is finite.

Lemma B.2 (Finite propagation of admissibility perturbations)**.** *Let $\delta\Phi$ be a localized perturbation of a configuration Φ. Under RNF continuation, the region whose admissibility may be affected by $\delta\Phi$ expands at a bounded rate determined by the reconstruction influence radius.*

Proof. Because admissibility constraints depend only on bounded local neighborhoods, perturbations cannot influence arbitrarily distant regions in a single reconstruction step. Any propagation of the perturbation must occur through successive local extensions, producing a bounded growth of the affected region under RNF continuation.
□

Thus admissible reconstruction imposes a fundamental finite-propagation property.

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Appendix B.2. Continuum Limit and Differential Operators

In the dense admissibility regime (A3), coarse-graining of reconstruction histories produces smooth external fields $\psi(x)$ governed by effective differential operators.

Let

$$\mathcal{D}[\psi] = 0$$

denote the effective continuum equation satisfied by ψ. The principal symbol of $\mathcal{D}$ determines the propagation properties of disturbances.

Definition B.3 (Principal symbol)**.** Let $\mathcal{D}$ be a linear differential operator of order k. The *principal symbol* $\sigma_{\mathcal{D}}(x,\xi)$ is the homogeneous polynomial obtained from the highest-order derivative terms of $\mathcal{D}$.

Characteristic propagation cones are determined by the vanishing of the principal symbol.

Appendix B.3. Incompatibility of Elliptic Propagation

We now show that elliptic operators violate the finite propagation property established above.

Lemma B.4 (Elliptic operators imply instantaneous influence)**.** *Let $\mathcal{D}$ be elliptic. Then perturbations of ψ at a single point generically influence the solution everywhere in the domain.*

Proof. For elliptic operators, the principal symbol is non-vanishing for all nonzero covectors ξ. Classical results in partial differential equations show that solutions of elliptic equations are analytic under broad conditions, and perturbations propagate instantaneously across the entire domain [13].
□

Instantaneous influence contradicts the bounded propagation of admissibility perturbations derived in Lemma B.2. Therefore elliptic continuum equations are incompatible with admissible RNF reconstruction.

Appendix B.4. Parabolic Degeneracy

Parabolic operators exhibit one-sided propagation but still violate reconstruction locality.

Lemma B.5 (Parabolic diffusion instability)**.** *Parabolic operators generate diffusion processes with arbitrarily rapid spatial spreading of perturbations. Under RNF continuation, such diffusion produces unbounded accumulation of admissibility mismatch across distant regions.*

Proof. Diffusion equations possess Green functions with non-compact spatial support at arbitrarily small times. Consequently, perturbations spread across arbitrarily large regions, contradicting the finite propagation property implied by reconstruction locality.
□

Thus parabolic continuum equations are also incompatible with RNF admissibility.

Appendix B.5. Hyperbolicity Dominance Theorem

We now establish the central result.

Theorem B.6 (Hyperbolicity dominance)**.** *Let $\psi(x)$ be an external field arising as a stable continuum limit of admissible RNF reconstruction. The effective differential operator governing ψ must possess a hyperbolic principal symbol.*

Proof. From Lemma B.2, admissible reconstruction requires bounded propagation of perturbations. Lemma B.4 shows that elliptic operators violate this requirement, while the parabolic diffusion lemma demonstrates that parabolic operators also produce unbounded spatial influence.

The remaining class of operators compatible with finite propagation cones are hyperbolic operators. Therefore the principal symbol of the effective continuum operator must be hyperbolic. □

Hyperbolicity therefore emerges as a structural requirement of admissible reconstruction rather than a kinematic postulate.

Appendix B.6. Interpretation

The result above establishes that causal propagation and finite signal speed arise directly from the locality of admissible reconstruction. Continuum causal cones are therefore emergent features of admissibility structure rather than fundamental spacetime primitives.

In particular, Lorentzian causal structure can be understood as a stable attractor of admissible continuum limits. This observation connects the hyperbolicity theorem to the Lorentz symmetry attractor discussed in Appendix E.

Appendix C. Proof of the Non-Abelian Buffer Saturation Theorem

This appendix gives a self-contained proof of Theorem 5.4 under explicit structural assumptions. The proof is intentionally implementation-independent: it uses only (i) locality of admissibility, (ii) the existence of a transport group G with a non-Abelian subgroup, and (iii) a finite interface capacity bound. The argument is purely structural and does not invoke dynamics, renormalization, or action principles, in contrast to standard confinement proofs in gauge theory [22,67].

Appendix C.1. Preliminaries and Assumptions

We work in a reconstructed regime where a continuum approximation of external structure is available (Section 3). Let $R \subset \mathcal{M}$ be a simply connected reconstructed region with piecewise smooth boundary ∂R. The internal closure transport along a path γ is represented by an element $U(\gamma) \in G$, and for a loop ℓ we write $U(\ell) \in G$, following standard holonomy constructions in gauge theory and differential geometry [43].

Definition C.1 (Loop obstruction)**.** For a loop ℓ based at $p \in \partial R$, define its obstruction element as

$$\Omega(\ell) \ := \ U(\ell) \in G.$$

The loop is *externally contractible* if it is contractible in R as a curve in $\mathcal{M}$. A contractible loop is *internally trivial* if $\Omega(\ell) = \mathbf{1}$.

In an admissible reconstruction without mismatch injection, contractible loops are internally trivial (up to buffering). Nontrivial $\Omega(\ell) \neq \mathbf{1}$ indicates an internal closure mismatch that must be buffered on (or near) the interface. This mirrors the interpretation of holonomy as obstruction in non-Abelian gauge theories, though here no gauge field or action is assumed [46].

We now state the structural assumptions used in the proof.

1. **(A1) Locality and compositionality.** Transport satisfies path concatenation:

$$U(\gamma_1 \circ \gamma_2) = U(\gamma_1)U(\gamma_2),$$

   whenever the concatenation is admissible, and depends only on local data along the path.
2. **(A2) Non-Abelianity.** There exist $a, b \in G$ such that $[a, b] \neq \mathbf{1}$, where $[a, b] := aba^{-1}b^{-1}$ is the commutator.

3. **(A3) Finite interface capacity.** There exists a nonnegative *buffer cost* functional $\mathfrak{B}$ assigning to each finite set of loop obstructions $\{\Omega_i\} \subset G$ a cost $\mathfrak{B}(\{\Omega_i\}) \in [0, \infty)$, with the properties:
   - (Normalization) $\mathfrak{B}(\{\mathbf{1}\}) = 0$ and $\mathfrak{B}(\emptyset) = 0$.
   - (Monotonicity) If $S \subseteq T$ then $\mathfrak{B}(S) \leq \mathfrak{B}(T)$.
   - (Non-Abelian irreducibility) There exists $\beta_0 > 0$ such that for any $x, y \in G$ with $[x, y] \neq \mathbf{1}$,
   $$\mathfrak{B}(\{x, y\}) \geq \beta_0.$$
   - (Capacity bound) There exists a finite $B_{\max}$ such that admissible reconstruction across ∂R requires $\mathfrak{B}(\{\Omega_i\}) \leq B_{\max}$.

Assumption (A3) abstracts the physically familiar fact that noncommuting constraints cannot be freely merged or screened, a principle underlying area-law behavior in non-Abelian gauge theories [22,67], but here expressed without reference to energy, fields, or dynamics.

Appendix C.2. Loop Tiling and Independent Commutator Sources

The key step is to show that non-Abelian mismatch generically generates a number of *independent* nontrivial commutator obstructions that grows with the number of elementary tiles spanning R.

Choose a tiling of R by N simply connected cells $\{C_k\}_{k=1}^N$ whose interiors are disjoint and whose union is R up to measure-zero boundaries. Such decompositions are standard in lattice and discrete gauge formulations [67].

For each cell, define the cell holonomy

$$\Omega_k := \Omega(\partial C_k) = U(\partial C_k) \in G.$$

By compositionality and cancellation of shared internal edges, the product of cell boundary loops equals the outer boundary loop up to conjugation by transport between basepoints. One convenient way to formalize this is to choose a spanning tree of basepoint transports from a global basepoint $p \in \partial R$ to each cell boundary and define *based* cell loops $\tilde{\Omega}_k$ by conjugation:

$$\tilde{\Omega}_k := T_k \, \Omega_k \, T_k^{-1},$$

where T_k is the transport from p to the basepoint of ∂C_k along the spanning tree. Then $\tilde{\Omega}_k$ are all based at the same point p and satisfy

$$\Omega(\partial R) = \prod_{k=1}^{N} \tilde{\Omega}_k, \tag{A1}$$

with an ordering determined by the chosen tiling traversal. Equation (A1) is the discrete non-Abelian Stokes-type identity familiar from lattice gauge theory, but here derived solely from admissibility compositionality [43].

Lemma C.2 (Commutator production)**.** *For any adjacent cells C_i, C_j with $[\tilde{\Omega}_i, \tilde{\Omega}_j] \neq \mathbf{1}$, buffering admissible transport requires buffering at least one nontrivial commutator contribution.*

Appendix C.3. Growth with Region Size: Saturation Threshold

To obtain saturation, we show that in a genuinely non-Abelian regime the number of such independent commutator sources grows with the number of tiles N. This requires a mild genericity condition: that the non-Abelian mismatch does not localize to finitely many tiles as R grows.

Definition C.3 (Non-Abelian density condition)**.** A family of regions $\{R_L\}$ (indexed by an external size parameter L) is said to carry *non-Abelian mismatch with positive density* if there exists $\eta > 0$ such

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that, for sufficiently large L, at least $\eta N(L)$ of the tile holonomies $\{\tilde{\Omega}_k\}$ can be partitioned into disjoint adjacent pairs (i,j) with

$$[\tilde{\Omega}_i, \tilde{\Omega}_j] \neq \mathbf{1},$$

where $N(L)$ is the number of tiles in a tiling of R_L.

This condition states that noncommutativity occurs throughout the region rather than on a set of vanishing fraction. It is the natural analogue of "finite non-Abelian curvature density" in continuum gauge theory, but is stated purely combinatorially.

Lemma C.4 (Linear lower bound on buffering cost)**.** *Under Definition C.3 and assumptions (A1)–(A3), the required buffer cost satisfies, for sufficiently large L,*

$$\mathfrak{B}\big(\{\Omega_i\}\big) \;\geq\; \eta N(L)\,\beta_0,$$

where $\{\Omega_i\}$ denotes any set of independent obstructions that must be buffered on the interface to admit $\Omega(\partial R_L)$.

Proof. By Definition C.3, there exist at least $\eta N(L)$ disjoint adjacent pairs with nontrivial commutator. By Lemma C.2, each such pair forces buffering of at least one nontrivial commutator contribution at cost $\geq \beta_0$. Because the pairs are disjoint, these contributions are independent in the sense required by (A3): they arise from distinct local incompatibilities that cannot be eliminated by buffering a single localized defect. By monotonicity of $\mathfrak{B}$, buffering all such independent contributions costs at least $\eta N(L)\beta_0$. □

Theorem C.5 (Non-Abelian Buffer Saturation)**.** *Assume (A1)–(A3) and suppose a family of reconstructed regions $\{R_L\}$ carries non-Abelian mismatch with positive density in the sense of Definition C.3. Then there exists L_* such that for all $L \geq L_*$, admissible RNF reconstruction across ∂R_L fails unless the net non-Abelian mismatch is neutralized within R_L. Equivalently, unsaturated long-range transport of non-Abelian closure mismatch is inadmissible.*

Proof. By Lemma C.4,

$$\mathfrak{B}\big(\{\Omega_i\}\big) \;\geq\; \eta N(L)\beta_0.$$

Since $N(L) \to \infty$ as the region grows, choose L_* such that $\eta N(L_*)\beta_0 > B_{\max}$. Then for all $L \geq L_*$ we have $\mathfrak{B} > B_{\max}$, violating the capacity bound in (A3). Hence RNF continuation across ∂R_L is obstructed unless the mismatch is neutralized so that the effective non-Abelian density condition fails (i.e. the region becomes closure-balanced into composites). This establishes saturation and the inadmissibility of long-range non-Abelian mismatch transport. □

Theorem C.5 implies Theorem 5.4 in the main text. The latter states the conclusion in physical language, while the appendix states the precise structural conditions.

Appendix C.4. Area Growth and Mass-Gap Interpretation

The argument above gives a *combinatorial* growth bound in terms of the number of tiles $N(L)$. In reconstructed regimes where a metric is well-defined, $N(L)$ scales with the boundary-enclosed area rather than boundary length, reproducing the familiar area-law scaling associated with confinement in non-Abelian gauge theories [22,67].

To make the physics interpretation explicit, define an effective excitation cost

$$E_{\rm buf}(R) \;\propto\; \mathfrak{B}\big(\{\Omega_i\}\big).$$

This interpretation aligns with standard nonperturbative results while arising here as a purely structural consequence of noncommutativity and finite buffering capacity, independent of

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renormalization-group flow or microscopic dynamics. Explicit RNF implementations realizing this mechanism are given in [39].

Appendix D. Structural Derivation of Einstein Gravity

This appendix provides rigorous proofs for the main claims of Section 7. The goal is to derive, without committing to a specific microscopic RNF realization, that in four dimensions the unique leading-order continuum feasibility functional compatible with RNF admissibility is the Einstein–Hilbert functional (with cosmological constant), and that its stationarity yields Einstein's equations as feasibility consistency conditions.

Throughout this appendix, all statements are purely continuum and structural. They do not assume a particular RNF substrate. The RNF content enters only through the admissibility requirements formalized below. The mathematical results used here are standard in differential geometry and classical field theory [40,43,59].

Appendix D.1. Axioms for Continuum Feasibility Functionals

Let $(\mathcal{M}, g)$ be a smooth four-dimensional Lorentzian manifold representing an external continuum limit inside an admissibility basin. A *feasibility functional* assigns a scalar value to g measuring global RNF reconstructibility cost:

$$\mathcal{F}[g] \in \mathbb{R}.$$

The admissibility requirements of RNF impose the following axioms.

Assumption D.1 (Local action form)**.** $\mathcal{F}$ is local and extensive: for any region $U \subset \mathcal{M}$,

$$\mathcal{F}_U[g] = \int_U \mathcal{L}(x, g, \partial g, \dots, \partial^k g)\, d^4x,$$

where $\mathcal{L}$ is a scalar density built from finitely many derivatives of g and is additive under gluing of regions with compatible boundary data.

Assumption D.2 (Diffeomorphism invariance)**.** For any diffeomorphism $\varphi : \mathcal{M} \to \mathcal{M}$,

$$\mathcal{F}[\varphi^* g] = \mathcal{F}[g].$$

Equivalently, $\mathcal{L}$ is a scalar density under coordinate changes.

Assumption D.3 (Second-order stability)**.** The Euler–Lagrange equations obtained from $\delta\mathcal{F} = 0$ are at most second order in derivatives of g (i.e. contain no derivatives of g higher than second order).

Assumption D.4 (Non-degeneracy)**.** The functional is not purely topological and contributes nontrivially to the local feasibility stationarity conditions in the bulk.

RNF interpretation. Assumption D.1 encodes locality and extensivity of reconstruction feasibility under RNF continuation; Assumption D.2 encodes external coarse-graining independence of coordinates; Assumption D.3 encodes the RNF requirement that admissible continuation not require unbounded additional interface data per step. Assumption D.4 excludes purely boundary or topological functionals, consistent with standard treatments of gravitational actions [59].

We now prove that these axioms force the Einstein–Hilbert form in $4D$.

Appendix D.2. From Diffeomorphism Invariance to Curvature Dependence

We first show that any local diffeomorphism-invariant scalar density depending on g and up to second derivatives can be written (up to a total divergence) as a function of curvature invariants.

Lemma D.5 (Normal-coordinate reduction)**.** *Let $\mathcal{L}(x, g, \partial g, \partial^2 g)$ be a scalar density satisfying Assumption D.2. Then at any point $p \in \mathcal{M}$ there exist coordinates (Riemann normal coordinates) in which*

$$g_{\mu\nu}(p) = \eta_{\mu\nu}, \qquad \partial_\alpha g_{\mu\nu}(p) = 0,$$

and the dependence of $\mathcal{L}$ on $\partial^2 g$ at p can be expressed entirely via the Riemann curvature tensor $R_{\mu\nu\rho\sigma}(p)$ and the metric $\eta_{\mu\nu}$.

Proof. Standard normal-coordinate arguments apply [43,59].

Fix $p \in \mathcal{M}$. Choose Riemann normal coordinates at p, so that

$$g_{\mu\nu}(p) = \eta_{\mu\nu} \quad \text{and} \quad \Gamma^\lambda_{\mu\nu}(p) = 0,$$

hence

$$\partial_\alpha g_{\mu\nu}(p) = 0.$$

In such coordinates, any coordinate-invariant scalar density evaluated at p cannot depend on ∂g, since $\partial g(p) = 0$, and can only depend on the equivalence class of $\partial^2 g(p)$ under the remaining linear coordinate freedom preserving $\eta_{\mu\nu}$ at p, namely the Lorentz group $O(1,3)$.

The $GL(4)$-covariant part of $\partial^2 g$ that survives coordinate changes is precisely the Riemann curvature tensor. In normal coordinates,

$$R_{\mu\nu\rho\sigma}(p) = \tfrac{1}{2}\left(\partial_\rho \partial_\nu g_{\mu\sigma} + \partial_\sigma \partial_\mu g_{\nu\rho} - \partial_\sigma \partial_\nu g_{\mu\rho} - \partial_\rho \partial_\mu g_{\nu\sigma}\right)\Big|_p.$$

Thus any diffeomorphism-invariant scalar density depending on $(g, \partial g, \partial^2 g)$ can, at p, be expressed as a scalar constructed from $\eta_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}(p)$. Since p is arbitrary, the statement holds pointwise on $\mathcal{M}$. □

Lemma D.6 (Curvature-invariant form up to a divergence)**.** *Under Assumptions D.1–D.2, any admissible $\mathcal{L}$ can be rewritten as*

$$\mathcal{L} = \sqrt{-g}\, f(R_{\mu\nu\rho\sigma}, g_{\mu\nu}) + \partial_\mu V^\mu,$$

where the divergence term does not contribute to bulk field equations.

Because $\mathcal{L}$ is diffeomorphism invariant and depends on the metric and finitely many derivatives, the scalar density can depend only on curvature invariants and their contractions. Terms involving explicit derivatives of curvature can be converted to curvature-only terms plus a total divergence by integration by parts.

Proof. This follows from standard results on diffeomorphism-invariant local actions [40,59]. □

Appendix D.3. Second-Order Stability Forces Linearity in Curvature

We now prove that Assumption D.3 forces f to be *at most linear* in curvature. Any nonlinear dependence on curvature generates higher-than-second derivative terms in the Euler–Lagrange equations.

Lemma D.7 (Nonlinearity produces higher derivatives)**.** *Let*

$$\mathcal{F}[g] = \int_{\mathcal{M}} \sqrt{-g}\, f(R_{\mu\nu\rho\sigma}, g)\, d^4x$$

with f smooth in its curvature arguments. If f depends nonlinearly on curvature (i.e. if the second derivative of f with respect to curvature components is not identically zero), then the Euler–Lagrange equations obtained from $\delta\mathcal{F} = 0$ contain derivatives of g of order strictly greater than two.

Proof. The curvature tensor contains second derivatives of the metric schematically: $R \sim \partial^2 g + (\partial g)^2$. Under a metric variation $\delta g_{\mu\nu}$, the variation of curvature takes the form

$$\delta R_{\mu\nu\rho\sigma} = \nabla_\rho \delta\Gamma_{\mu\nu\sigma} - \nabla_\sigma \delta\Gamma_{\mu\nu\rho}, \quad \text{with} \quad \delta\Gamma \sim \nabla(\delta g).$$

Hence δR contains second derivatives of δg:

$$\delta R \sim \nabla\nabla(\delta g) + \text{lower-order terms}.$$

Compute the first variation:

$$\delta\mathcal{F} = \int \sqrt{-g}\left(\frac{\partial f}{\partial R_{\mu\nu\rho\sigma}}\,\delta R_{\mu\nu\rho\sigma} + \cdots\right) d^4x,$$

where "$\cdots$" denotes terms involving δg but not δR.

Substituting $\delta R \sim \nabla\nabla(\delta g)$ and integrating by parts to isolate δg, we obtain bulk terms containing two derivatives acting on $\frac{\partial f}{\partial R_{\mu\nu\rho\sigma}}$:

$$\delta\mathcal{F} \supset \int \sqrt{-g}\,(\nabla\nabla \frac{\partial f}{\partial R_{\mu\nu\rho\sigma}})\,\delta g_{\alpha\beta}\,d^4x + \text{boundary terms}.$$

If f is nonlinear in curvature, then $\frac{\partial f}{\partial R_{\mu\nu\rho\sigma}}$ depends on R, hence on $\partial^2 g$. Therefore $\nabla\nabla(\frac{\partial f}{\partial R})$ contains derivatives of g up to fourth order. Consequently the Euler–Lagrange equations contain derivatives of g of order greater than two.

Thus, to satisfy Assumption D.3, f must be at most linear in curvature. □

Therefore, admissible feasibility functionals must have the form

$$\mathcal{F}[g] = \int \sqrt{-g}\left(a^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + b\right) d^4x \;+\; (\text{boundary/topological terms}),$$

where the tensor $a^{\mu\nu\rho\sigma}$ is built algebraically from g.

The only exception occurs for special Lovelock combinations whose variational derivatives cancel higher-order terms. In four dimensions the only such combination quadratic in curvature is the Gauss–Bonnet density, which is topological and therefore excluded by the non-degeneracy assumption.

Lemma D.8 (Reduction to Ricci scalar in $4D$)**.** *Any scalar linear in the Riemann tensor must contract its indices using the metric tensor. Up to overall constants and index permutations, the only independent contraction is*

$$g^{\mu\rho}g^{\nu\sigma}R_{\mu\nu\rho\sigma} = R.$$

All other contractions reduce to the Ricci scalar by symmetries of the Riemann tensor. In four dimensions the only additional curvature scalar yielding second-order field equations is the Gauss–Bonnet combination, which is topological and does not contribute to the bulk Euler–Lagrange equations.

Proof. Any scalar linear in $R_{\mu\nu\rho\sigma}$ must contract indices using $g_{\mu\nu}$. Up to overall constants, the only such contraction is

$$g^{\mu\rho}g^{\nu\sigma}R_{\mu\nu\rho\sigma} = R.$$

Other linear contractions differ by permutations and symmetries of the Riemann tensor and reduce to the same scalar. Any additional admissible term in four dimensions that is diffeomorphism-invariant and second-order is either a boundary term or the Gauss–Bonnet density, which is topological in $4D$ and does not contribute to bulk field equations. □

We can now prove the main theorem.

Appendix D.4. Einstein–Hilbert Uniqueness in Four Dimensions

Theorem D.9 (Einstein–Hilbert feasibility functional is unique at leading order)**.** *Assume $\mathcal{F}[g]$ satisfies Assumptions D.1–D.4 in four spacetime dimensions. Then, up to boundary terms and topological terms that do not affect bulk stationarity, $\mathcal{F}$ must take the Einstein–Hilbert form with cosmological constant:*

$$\mathcal{F}[g] \equiv \alpha \int_{\mathcal{M}} \sqrt{-g}\,(R - 2\Lambda)\,d^4x \quad + \quad \textit{(boundary/topological terms)},$$

for constants $\alpha \neq 0$ and Λ.

Proof. By Lemma D.6, up to a divergence we may write $\mathcal{F} = \int \sqrt{-g}\, f(\text{Riemann}, g)\, d^4x$. By Lemma D.7 and Assumption D.3, f must be linear in curvature. Therefore, up to constants,

$$\mathcal{F}[g] = \int \sqrt{-g}\,(\alpha\, R + \beta)\, d^4x \quad + \quad \text{(boundary/topological terms)}.$$

Writing $\beta = -2\alpha\Lambda$ yields the stated form. The non-degeneracy assumption excludes the case $\alpha = 0$ (pure cosmological constant only) as the sole bulk term, and excludes the case where only topological/boundary terms remain. Hence the Einstein–Hilbert term is required at leading order. □

This completes the realization-independent derivation: RNF admissibility constraints select Einstein–Hilbert as the unique stable leading-order feasibility functional in $4D$.

Appendix D.5. Einstein Equations as Feasibility Stationarity

We now derive the field equations from feasibility stationarity and show how coupling to external content enters.

Definition D.10 (Matter feasibility contribution and stress tensor)**.** Let $\mathcal{F}_{\text{m}}[g, \psi]$ denote the feasibility contribution of external content (matter fields, effective degrees of freedom, etc.), with ψ denoting collectively those fields. Define the stress–energy tensor by

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta \mathcal{F}_{\text{m}}}{\delta g^{\mu\nu}}.$$

Proposition D.11 (Feasibility stationarity yields Einstein equations)**.** Let $\mathcal{F}[g]$ be the Einstein–Hilbert feasibility functional from Theorem D.9, and let $\mathcal{F}_{\text{m}}[g, \psi]$ be any diffeomorphism invariant matter feasibility contribution. Stationarity of

$$\mathcal{F}_{\text{tot}}[g, \psi] := \mathcal{F}[g] + \mathcal{F}_{\text{m}}[g, \psi]$$

with respect to variations of g yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\, T_{\mu\nu},$$

with $\kappa := (2\alpha)^{-1}$.

Proof. The standard variation identity for the Einstein–Hilbert functional gives

$$\delta \int \sqrt{-g}\, R\, d^4x = \int \sqrt{-g}\, G_{\mu\nu}\, \delta g^{\mu\nu}\, d^4x + \text{boundary terms}.$$

Also,

$$\delta \int \sqrt{-g}\,(-2\Lambda)\, d^4x = \int \sqrt{-g}\,(-\Lambda g_{\mu\nu})\, \delta g^{\mu\nu}\, d^4x.$$

Therefore,

$$\delta \mathcal{F}[g] = \alpha \int \sqrt{-g}\,(G_{\mu\nu} + \Lambda g_{\mu\nu})\, \delta g^{\mu\nu}\, d^4x + \text{boundary terms}.$$

For the matter part, by definition of $T_{\mu\nu}$,

$$\delta\mathcal{F}_{\rm m}[g,\psi] = -\tfrac{1}{2}\int \sqrt{-g}\, T_{\mu\nu}\,\delta g^{\mu\nu}\, d^4x \quad + \quad (\text{terms proportional to } \delta\psi).$$

Stationarity with respect to $\delta g^{\mu\nu}$ (holding ψ fixed) implies

$$\alpha\,(G_{\mu\nu} + \Lambda g_{\mu\nu}) - \tfrac{1}{2}\,T_{\mu\nu} = 0,$$

which is equivalent to the claimed equation with $\kappa = (2\alpha)^{-1}$. □

Proposition D.12 (Covariant conservation)**.** If $\mathcal{F}_{\rm m}[g,\psi]$ is diffeomorphism invariant, then on-shell

$$\nabla^{\mu} T_{\mu\nu} = 0.$$

Proof. Diffeomorphism invariance implies invariance under infinitesimal diffeomorphisms generated by a vector field X^{μ}. The induced metric variation is $\delta g^{\mu\nu} = \nabla^{\mu}X^{\nu} + \nabla^{\nu}X^{\mu}$. Using $\delta\mathcal{F}_{\rm m} = 0$ and the definition of $T_{\mu\nu}$ yields

$$0 = \delta\mathcal{F}_{\rm m} = -\tfrac{1}{2}\int \sqrt{-g}\, T_{\mu\nu}(\nabla^{\mu}X^{\nu} + \nabla^{\nu}X^{\mu})\, d^4x.$$

Integrating by parts and using arbitrariness of X^{μ} gives $\nabla^{\mu}T_{\mu\nu} = 0$. □

RNF interpretation.

Equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$ is not interpreted here as fundamental "metric dynamics." It expresses feasibility stationarity: small deformations of the reconstructed external geometry must not decrease global reconstructibility within the admissibility basin. The universality of coupling arises because any external content that contributes to feasibility enters through $T_{\mu\nu}$.

Appendix D.6. Comment on Higher-Curvature Corrections

Theorem D.9 is a *leading-order* statement under the second-order stability requirement, which encodes RNF admissibility constraints on interface data. If one relaxes Assumption D.3 (e.g. allowing effective higher-derivative terms in a controlled regime), then higher-curvature corrections may appear as subleading contributions outside the strict interior of the admissibility basin. This does not affect the conclusion that any stable large-scale continuum limit must reduce to Einstein gravity at leading order in four dimensions.

Appendix D.7. Summary

Under RNF-motivated admissibility axioms—locality/extensivity, diffeomorphism invariance, and second-order stability—the only nontrivial leading-order feasibility functional in $4D$ is the Einstein–Hilbert functional with cosmological constant. Stationarity yields Einstein's equations with covariant stress–energy conservation. This establishes general relativity as a realization-independent structural consequence of RNF admissibility, in direct parallel with the realization-independent emergence of electromagnetism as unsaturated Abelian constraint transport [39].

Appendix E. A Scoped Derivation of Lorentz Symmetry as an Admissibility Attractor

Appendix E.1. Purpose and Scope

This appendix provides a scoped, rigorous result linking RNF admissibility to Lorentz symmetry in the emergent external description. We emphasize what is (and is not) proven.

- We *do not* classify all possible admissibility universality classes.

- We *do* prove that, within any external regime admitting (i) stable inertial charts, (ii) a finite invariant signal speed enforced by admissibility, and (iii) no preferred inertial frame at the external level, the admissible inertial coordinate transformations are forced into the Lorentz–Galilei dichotomy, with the Galilean case arising as the singular $c \to \infty$ limit.

Thus Lorentz symmetry appears as an *attractor*: once an admissible reconstruction supports stable inertial charts and an invariant causal cone, the residual freedom of admissible coordinate changes collapses to the Lorentz/Galilei alternatives [50,59].

Appendix E.2. External Inertial Charts and Hyperbolic Cone Structure

We work entirely at the external (coarse-grained) level defined in Section 3. Fix an admissibility basin $\mathfrak{U}$ in which reconstruction is dense and stable, so that effective continuum fields and smooth charts are well-defined.

Section 3 establishes that continuum fields emerging from admissible RNF reconstruction satisfy hyperbolic field equations. A rigorous proof of this statement is provided in Appendix B. Hyperbolic differential operators possess well-defined characteristic cones that determine the domain of influence of disturbances [13].

These characteristic cones provide the causal structure of the emergent external spacetime.

Definition E.1 (External inertial chart)**.** An *external inertial chart* is a coordinate chart $(t, \mathbf{x})$ on a reconstructed domain $\mathcal{M}$ in which: (i) free (unforced) effective excitations follow straight worldlines in the chart, (ii) admissible composition of reconstruction steps is homogeneous in t and $\mathbf{x}$ (no preferred origin), and (iii) admissibility constraints are stationary (time-translation invariant) within the basin $\mathfrak{U}$.

Definition E.2 (Admissible causal cone and invariant speed)**.** An *admissible causal cone* in an inertial chart is the characteristic cone determined by the hyperbolic continuum equations governing stable external fields.

Equivalently, there exists a constant $c \in (0, \infty]$ such that admissible propagation increments satisfy

$$\|\Delta\mathbf{x}\| \le c\,\Delta t \quad \text{for all admissible increments with } \Delta t \ge 0,$$

where $\|\cdot\|$ denotes the Euclidean norm in the inertial chart.

The existence of finite c is therefore not an independent assumption but a structural consequence of hyperbolicity in admissible RNF continuum limits.

Appendix E.3. Assumptions (Admissibility Axioms in External Form)

We now state the minimal assumptions needed for a rigorous group-theoretic conclusion.

Assumption E.3 (Linearity of inertial transformations)**.** Between any two inertial charts $(t, \mathbf{x})$ and $(t', \mathbf{x}')$ in the basin $\mathfrak{U}$, the coordinate change is affine, and after aligning origins may be taken linear:

$$\begin{pmatrix} t' \\ \mathbf{x}' \end{pmatrix} = A \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}, \qquad A \in GL(1+d, \mathbb{R}).$$

Assumption E.4 (Relativity principle at the external level)**.** The set of admissible inertial transformations forms a group $\mathcal{G}$ acting transitively on inertial frames, and the admissible cone $\mathcal{C}$ is invariant under this action:

$$A(\mathcal{C}) = \mathcal{C}.$$

Assumption E.5 (Spatial isotropy)**.** In each inertial chart, the stabilizer subgroup of $\mathcal{G}$ contains $SO(d)$ acting on spatial coordinates, so that no spatial direction is preferred within $\mathfrak{U}$.

Assumptions E.3–E.5 parallel classical derivations of relativistic kinematics [1,50]. Linearity follows from homogeneity and inertial straightness; the group property encodes frame-independence

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of admissibility laws within a basin; isotropy expresses the direction-independence of admissible reconstruction at macroscopic scales.

Appendix E.4. Theorem: Lorentz/Galilei Dichotomy from Cone Preservation

Theorem E.6 (Lorentz symmetry as cone-preserving inertial equivalence)**.** *Let $d \geq 2$ and suppose a stable admissibility basin $\mathfrak{U}$ admits external inertial charts and an admissible causal cone with invariant speed $c \in (0, \infty]$. Under Assumptions E.3–E.5, the group $\mathcal{G}$ of admissible inertial transformations is, up to conjugacy, either:*

1. *the Lorentz group (or the inhomogeneous Lorentz/Poincaré group when translations are included), preserving the causal cone and the associated quadratic form*

$$q(\Delta t, \Delta \mathbf{x}) = c^2(\Delta t)^2 - \|\Delta \mathbf{x}\|^2,$$

2. *or the Galilean group obtained as the singular limit $c \to \infty$.*

In particular, if $c < \infty$ then Lorentz symmetry is the unique nontrivial cone-preserving inertial equivalence compatible with isotropy.

Proof. The proof follows the classical cone-preservation argument of Alexandrov and related derivations of relativistic kinematics [1,50].

Hyperbolic continuum equations define characteristic cones that determine admissible signal propagation. Under Assumption E.4, the inertial transformation group must preserve this cone structure.

Analyzing linear boosts along a fixed spatial axis and imposing preservation of the cone boundary yields invariance of the quadratic form

$$c^2t^2 - \|\mathbf{x}\|^2$$

up to an overall scale. Compatibility with isotropy and group composition then fixes the standard Lorentz boost relations

$$\gamma(v) = (1 - v^2/c^2)^{-1/2},$$

recovering the Lorentz group for finite c. When $c \to \infty$, the cone degenerates to simultaneity hyperplanes, yielding the Galilean group. □

Appendix E.5. Interpretation as an Admissibility Attractor

Theorem E.6 does not assert that RNF admissibility uniquely forces a Lorentzian basin in all realizations. Rather, it establishes the implication:

> *If* an admissible RNF reconstruction admits stable external inertial charts and hyperbolic continuum dynamics enforcing a finite signal speed, *then* the admissible inertial equivalence group is necessarily Lorentz (with Galilei as a degenerate limit).

Lorentz symmetry therefore functions as an attractor: once reconstruction stabilizes into a basin with a well-defined causal cone and no preferred inertial frame, the residual coordinate freedom collapses to the Lorentz/Galilei dichotomy.

Appendix E.6. Remark on Concealment Mechanisms

Even within a Lorentz-attractor basin, concrete realizations may contain microscopic structures that are not manifestly Lorentz invariant. Mechanisms such as co-moving concealment can suppress observable Lorentz-violation signals within the basin. The present theorem constrains the emergent inertial equivalence of the external description, not the detailed microstructure of specific RNF realizations.

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Appendix F. Relation to Concrete RNF Implementations

The RNF Admissibility Framework developed in the main text is formulated at a deliberately abstract and implementation-independent level. All results rely only on structural features of admissible reconstruction—local feasibility, compositionality, finite interface capacity, and the algebraic properties of internal closure transport—and do not presuppose any specific microscopic substrate or update rule.

Nevertheless, it is essential to demonstrate that the framework is non-empty and that its structural consequences are realized in explicit models. This appendix summarizes how concrete RNF implementations instantiate the abstract principles and reproduce known physical laws in appropriate continuum limits [39].

Appendix F.1. Purpose of Implementation Independence

The separation between admissibility principles and concrete realizations serves two complementary purposes.

First, it isolates which features of physical law follow universally from admissibility constraints rather than from model-specific assumptions. Second, it sharpens falsifiability: if a proposed realization fails to reproduce the structural consequences derived in the main text, the failure lies in the realization rather than in the admissibility framework itself.

Accordingly, the present work does not rely on any particular choice of metascopic degrees of freedom, reconstruction variables, or evolution rules beyond those required to define RNF extension and the Temporal Coherence Principle.

Appendix F.2. General Structure of RNF Realizations

Concrete RNF implementations introduce additional structure on the metascopic configuration space $\mathcal{X}$ to make admissibility and reconstruction explicitly computable. Typical features include:

- a discrete, partially ordered, or field-based substrate supporting RNF extension steps,
- explicit local rules implementing TCP as feasibility or consistency constraints,
- reconstruction maps generating effective spacetime adjacency and causal ordering,
- internal variables encoding closure, alignment, or admissibility margins.

Within such realizations, RNF reconstruction paths can be constructed explicitly, external coarse-graining can be performed algorithmically, and effective metric, field, and excitation structures can be extracted and compared with known physics. These constructions serve as existence proofs that the abstract RNF framework admits physically relevant instantiations.

Appendix F.3. Emergence of Classical and Quantum Field Equations

Specific RNF realizations developed in prior work demonstrate that the admissibility framework supports the emergence of standard physical laws in stabilized continuum limits. In particular, explicit constructions show [39]:

- *Lorentzian causal structure*. Admissibility constraints on RNF stacking select a Lorentzian signature as a stable causal organization, yielding relativistic causal cones and invariant signal speed.
- *Einstein field equations*. The reconstructed spacetime metric, defined via global feasibility of RNF continuation, satisfies Einstein-type relations between curvature and effective energy–momentum content. These equations arise as consistency conditions for admissible reconstruction, not as fundamental dynamical postulates.
- *Maxwell electrodynamics*. Unsaturated Abelian closure transport yields long-range gauge fields whose continuum limit obeys Maxwell-type equations. Electromagnetic fields appear as gradients of admissible constraint transport rather than as primitive entities.

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- *Schrödinger dynamics*. Effective wave equations emerge as continuum descriptions of admissible reconstruction amplitudes, with Schrödinger-type evolution arising in appropriate nonrelativistic limits.
- *Born rule*. Probabilistic measurement statistics arise as stability weights associated with the relative frequency of admissible reconstruction paths, providing a structural interpretation of the Born rule without postulating probability as a fundamental axiom.

These results are realization-dependent in their technical details, but they establish that the admissibility principles developed in this paper are sufficient to reproduce both classical and quantum dynamics in explicit models.

Appendix F.4. Relation to the Present Work

The main text explains *why* such structures arise generically in admissible RNF reconstructions, independent of microscopic detail. Gauge transport, non-Abelian confinement, mass gaps, gravitational universality, and symmetry attractors are derived as structural consequences of admissibility alone.

Concrete realizations demonstrate *that* these consequences are realized in practice. They therefore function as constructive existence proofs, not as additional assumptions. The abstract results of the present paper constrain all viable realizations, while realizations provide explicit arenas for testing admissibility boundaries, breakdown regimes, and numerical behavior.

Appendix F.5. Outlook for Realizations and Simulations

Concrete RNF implementations play an essential role in extending and testing the framework. They enable:

- numerical exploration of reconstruction stability and saturation thresholds,
- verification of continuum limits and their domains of validity,
- extraction of particle spectra and composite excitation structure,
- investigation of transitions between admissibility universality classes.

Future work may therefore proceed along two complementary tracks: refinement of abstract admissibility principles and systematic exploration of explicit RNF realizations. The framework developed here provides the structural criteria against which all such models must be judged.

Appendix F.6. Summary

The RNF Admissibility Framework is intentionally realization-agnostic. Nevertheless, explicit RNF implementations exist in which Lorentzian causal structure, Einstein–Maxwell dynamics, quantum evolution, and probabilistic measurement emerge from admissible reconstruction. These realizations confirm that the framework is both internally consistent and physically productive, while preserving a clear separation between universal structural principles and model-dependent details [39].

Appendix G. Minimal Interface Model and Non-Abelian Saturation

This appendix provides a minimal constructive argument supporting the Non-Abelian Buffer Saturation Theorem stated in Section 5. The goal is not to introduce a microscopic RNF implementation but to demonstrate that saturation follows from structural admissibility requirements alone.

The derivation assumes only locality of reconstruction, compositional closure transport, and finite interface buffering capacity. No assumptions are made regarding background geometry, dynamics, or quantization.

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Appendix G.1. Local Interfaces and Finite Capacity

RNF reconstruction proceeds by local extension across interfaces separating neighboring reconstructed regions. At each interface, admissibility requires reconciliation of internal closure data associated with adjacent regions.

Definition G.1 (Interface state space)**.** An *interface* is a codimension-one boundary across which RNF reconstruction is locally extended. Each interface carries an internal state space

$$\mathcal{I} \cong \mathbb{R}^N, \qquad N < \infty,$$

representing the internal degrees of freedom available to encode admissible matching conditions for closure data.

The finite dimensionality of $\mathcal{I}$ expresses the locality of reconstruction: admissible extension rules must be implementable using finitely many independent parameters.

Remark. If interface capacity were infinite, arbitrarily many independent closure contributions could be canceled locally, eliminating the possibility of obstruction accumulation. Finite capacity is therefore a minimal structural requirement for nontrivial admissibility constraints.

Appendix G.2. Closure Transport and Buffer Cost

As described in Section 4, closure mismatch is represented by a groupoid functor

$$\mathcal{U} : \Pi_1(\mathcal{M}) \to G,$$

where G is the Lie group generated by the closure algebra $\mathfrak{g}$.

To quantify interface capacity usage we introduce an abstract measure of buffering demand.

Definition G.2 (Buffer cost functional)**.** Let

$$B : G \to \mathbb{R}_{\geq 0}$$

be a nonnegative functional satisfying

1. $B(e) = 0$,
2. $B(g^{-1}) = B(g)$,
3. $B(g_1 g_2) \leq B(g_1) + B(g_2)$, with equality for independent non-canceling contributions,
4. $B(g) \leq B_{\max} < \infty$ for admissible interface transport.

The functional B measures irreducible consumption of interface capacity required to buffer closure mismatch. It does not represent energy or action and is introduced solely as a monotone diagnostic of interface usage.

Appendix G.3. Independent Contributions and Non-Abelian Transport

Lemma G.3 (Independent non-canceling contributions)**.** *Let $\{\ell_i\}$ be admissibly independent loops with holonomies $g_i = \mathcal{U}(\ell_i)$. Suppose the corresponding closure contributions are pairwise non-canceling in the sense that no finite subproduct of the g_i can be trivialized by local conjugation or reparametrization within a single interface patch.*

Then there exists a constant $\beta_0 > 0$ such that each additional independent loop increases the minimal buffer demand by at least β_0 until interface saturation is reached.

Proof. By admissible independence, each loop contributes a closure component that cannot be absorbed into previously buffered components within the same local interface data. Because interface state space is finite-dimensional, each independent closure contribution consumes a nonzero minimal fraction of interface capacity. Denote the infimum of this nonzero consumption over admissible independent loop contributions by $\beta_0 > 0$. Hence each additional independent loop increases the minimal buffer demand by at least β_0 until the finite interface capacity is exhausted.

□

Lemma G.4 (Non-Abelian irreducibility)**.** *Let G be non-Abelian. Then there exist elements $g_1, g_2 \in G$ such that*

$$[g_1, g_2] \neq e.$$

Transport contributions generated by such elements cannot, in general, be simultaneously trivialized by conjugation or local reparametrization. Independent loop transports therefore produce generically irreducible closure mismatch.

Proof. Non-Abelianity implies the existence of generators whose commutator is nontrivial. Conjugation preserves the commutator structure, so local reparametrization cannot generally eliminate both generators simultaneously. Transport around independent loops therefore produces closure contributions that cannot be mutually canceled.

□

Proposition G.5 (Irreducible buffer accumulation)**.** Let $\{\ell_i\}$ be admissibly independent loops with holonomies $g_i = \mathcal{U}(\ell_i)$. If G is non-Abelian and the corresponding closure contributions are non-canceling in the sense of Lemma G.3, then

$$B\left(\prod_{i=1}^{k} g_i\right) \geq k\,\beta_0$$

for some $\beta_0 > 0$, until the interface capacity $B_{\max}$ is reached.

Proof. By Lemma G.4, independent loops in a non-Abelian closure group generate generically irreducible mismatch components. By Lemma G.3, each such independent component consumes a minimal nonzero fraction β_0 of interface buffering capacity. Consequently, the minimal buffering demand grows at least linearly with the number of independent loop transports until the finite interface capacity is saturated.

□

Appendix G.4. RNF Directionality and Area Scaling

RNF reconstruction introduces a preferred continuation direction along which admissible configurations are stacked. Closure mismatch therefore accumulates transversely to this direction.

Lemma G.6 (Codimension-two obstruction support)**.** *Independent closure obstructions under RNF continuation localize on interfaces transverse to the reconstruction direction and therefore form codimension-two defect structures in the reconstructed external geometry.*

Proof. RNF continuation enforces compatibility along the reconstruction direction. Obstruction sources cannot extend along this direction without violating admissibility constraints. They therefore localize on interfaces transverse to RNF continuation. In a d-dimensional reconstructed geometry this yields defect structures of codimension two.

□

Theorem G.7 (Area-law saturation)**.** *Let a loop ℓ enclose a transverse region of area A in the reconstructed external geometry. The minimal buffer cost required to neutralize closure mismatch generated by ℓ scales proportionally to the enclosed area:*

$$B_{\min} \sim A.$$

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Proof. Independent obstruction sources tile the RNF-transverse region enclosed by the loop. Each source contributes irreducible buffering demand according to Proposition G.5. The number of such independent sources therefore scales with the enclosed transverse area, yielding area-law scaling of the minimal buffer cost.
□

This establishes saturation and confinement as structural consequences of admissibility rather than dynamical gauge interactions.

Appendix G.5. Abelian Transport as the Unique Unsaturated Case

If the closure algebra is Abelian, all transport elements commute and loop holonomies cancel pairwise.

Proposition G.8 (Absence of saturation in the Abelian case)**.** For Abelian closure algebras, buffer cost does not accumulate irreducibly under loop composition. Admissible transport can therefore remain unsaturated over arbitrarily large reconstructed regions.

This explains the unique status of Abelian gauge sectors as long-range interactions within the RNF admissibility framework.

Appendix G.6. Relation to Section 5

The constructive arguments above establish that non-Abelian closure transport generically produces irreducible obstruction accumulation under RNF continuation. Combined with finite interface buffering capacity, this leads to saturation and area-law scaling.

Section 5 interprets these structural results at the level of external effective physics, where saturation manifests as confinement and a mass gap. The appendix therefore provides the admissibility-theoretic foundation for the physical consequences discussed in the main text.

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Abbreviations

The following abbreviations are used in this manuscript:

RNF	Real-Now-Front

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