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Article

A Grouping and Aggregation Modeling Method of Induction Motors for Transient Voltage Stability Analysis

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Abstract: Induction motors are the most numerous and significant dynamic loads in power systems. In transient voltage stability analysis, dynamic equivalent models are commonly used to simplify the representation of a group of induction motors. This paper presents a method for the grouping and aggregation of induction motors at a common bus. Firstly, grouping rules were provided for clustering induction motors into several subgroups based on the mechanical principles of rotor force and motion, and Aggregation Rules were provided for aggregating a motor subgroup into a single-unit model based on the relationship between voltage drop and power transmission in distribution networks. Secondly, guided by the grouping rules, high-speed remaining electromagnetic torque and low-speed remaining electromagnetic torque were defined as new clustering indicators, and an adaptive K-means clustering method using silhouette coefficient verification was provided to obtain the optimal motor subgroups. Thirdly, guided by the Aggregation Rules, a dynamic equivalent method was further provided to obtain the equivalent single-unit model from a motor subgroup. Lastly, a transient voltage stability simulation in a typical distribution network was presented to illustrate that the proposed clustering and equivalent methods are more reasonable, accurate, and effective than traditional methods, as the obtained model has better dynamic characteristics and can more accurately reproduce the process of voltage collapse.

Keywords: dynamic load modeling; induction motor; model reduction; grouping and aggregation method; transient voltage stability

1. Introduction

The purpose of dynamic load modeling research is to develop dynamic load models with performance characteristics of both *sufficient accuracy* and *simple usability* for power system simulations. In a actual power system, induction motors are the predominant dynamic loads, observing 70% to 90% of the total active power supplied by the power sources. Load modeling researchers refer to the collection of all induction motors (a total of N units) in a distribution system as an induction motor group. They use a component-based approach [9,11] to aggregate the induction motor group into a dynamic equivalent model with the correct structure and accurate parameters, aiming to reduce the model order and improve the accuracy of transient stability calculations.

The basic concept of the component-based approach is to treat the load as a set of various electrical devices. These electrical devices are first classified according to their typical mathematical models or characteristics, then the proportions of each type are statistically analyzed, and finally, a series of equivalent calculation formulas are used to derive an aggregated model that represents the actual devices. In the 1980s, the Electric Power Research Institute (EPRI) and General Electric Company (GE) conducted in-depth research on the aggregation of electrical devices such as induction motors [12–18]. The implementation steps are as follows: 1) Survey and collect technical data of each device, including their types, characteristics, and parameters, and calculate the proportions of static and dynamic loads. 2) Aggregate the characteristics of electrical devices of the same type to derive equivalent load

characteristic parameters. 3) Convert these equivalent parameters into aggregated load models that are compatible with computer simulation software.

Early studies on the component-based approach employed a single-unit induction motor model to represent a group of induction motors [16,17,19,20]. These approaches had some drawbacks: the dynamic load model's structure was overly simplified, resulting in "distortion" that could not accurately describe the actual induction motor group's dynamic behavior. This could lead to overly conservative or risky voltage stability simulation results [21–23]. In subsequent studies [19,24–32], the subdivision and equivalent modeling of induction motor groups were further researched. Motors on the same bus may exhibit significant differences in dynamic characteristics such as mechanical motion, active power, and reactive power. To better preserve these differences' impact on bus voltage recovery, some studies [1,32,33] proposed clustering the induction motors into several sub-groups before they are aggregated into equivalent models. Current grouping methods for the induction motors use various indicators: 1) Damping ratio and attenuation rate [26,34–36]. 2) Critical slip [25]. 3) Impedance parameters, inertia time constant, and slip at steady state [28,37].

After dividing the induction motor group into several sub-groups, all motors in a sub-group need to be equivalently modeled as a single motor. Currently, weighted averaging methods are mainly used to aggregate induction motor sub-groups. These methods use different weighting factors: 1) Rated power weighting factor [16,17,34]. 2) Power weighting factor adjusted by motor locked-rotor reactance and open-circuit time constant [27]. 3) Power weighting factor adjusted by load factor and critical slip [29,38].

To achieve higher modeling accuracy, the above methods still need improvement [39]:

- 1) For the grouping methods of induction motors, during the entire voltage recovery process, individuals within a sub-group should exhibit similar patterns of resultant torque (electromagnetic torque and mechanical load torque) changes. However, studies [26,34,35] considered local linearization near the actual operating point without accounting for the integral effect of resultant torque over time. Studies [25,37] focused only on electromagnetic torque parameters, neglecting mechanical torque parameters.
- 2) For the aggregation methods of each induction motor sub-group, the single-unit models should reflect the dynamic processes of active power, reactive power, and voltage exhibited by the sub-groups during voltage recovery. However, studies [16,17] used weighting factors based on rated power rather than the active power under actual operating conditions. Studies [27,29,38] used weighting factors based on the power adjusted by load factors under actual operating conditions but still lacked consideration of the relationships among active power transmission, reactive power transmission, and voltage reduction.

To address the aforementioned shortcomings, this paper aims to improve the methods for grouping and aggregating induction motors:

- 1) It first defines and uses high-speed remaining torque and low-speed remaining torque as indicators to measure the similarity of rotor acceleration and deceleration processes. An adaptive K-means clustering method is introduced to identify sub-groups of induction motors, which facilitates a more reasonable reduction in the order of the dynamic model while ensuring the simplicity and usability of the load model.
- 2) It proposes using equal total power, equal total torque, and equal rotor stored kinetic energy as aggregation rules. An improved single-unit equivalent method for each induction motor sub-group is introduced, which enhances the accuracy of the reduced-order model and helps prevent overly conservative or risky transient voltage stability simulation results.
- 3) Through transient voltage stability simulations in a typical distribution network, it is demonstrated that the induction motor model obtained using the proposed methods exhibits dynamic characteristics that are closer to those of the actual induction motor group compared to similar methods. Additionally, this confirms that the proposed methods more accurately reproduce the dynamic processes of transient voltage collapse.

2. Grouping and Aggregation Rules for Induction Motor Group

2.1. Model and Parameters of Induction Motor

2.1.1. Parameters of Induction Motor Model

Dynamic loads primarily consist of a large number of induction motors. The T-shaped equivalent circuit of induction motor is shown in Figure 1:

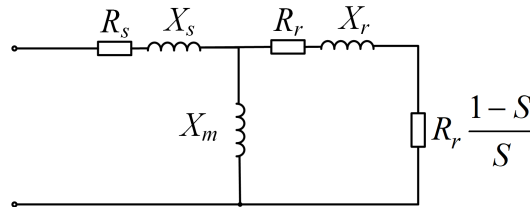


Figure 1. T-Equivalent Circuit of Induction Motor.

In Figure 1, R_s represents the stator winding resistance and R_r represents the rotor winding resistance. S denotes the rotor slip. X_s, X_r, X_m denote the stator leakage reactance, rotor leakage reactance, and magnetizing reactance, respectively. Here, $X_s = \omega_0 L_{0s}$, $X_r = \omega_0 L_{0r}$, $X_m = \omega_0 L_{ms}$, where ω_0 is the synchronous angular velocity, and L_{0s}, L_{0r}, L_{ms} are the stator leakage inductance, rotor leakage inductance, and magnetizing inductance, respectively. When using the dq-axis stator currents i_{sd}, i_{sq} , rotor currents i_{rd}, i_{rq} , and slip S as state variables, the T-shaped equivalent circuit in Figure 1 can be expressed as an electromagnetic transient model in the synchronous dq coordinates:

$$\begin{cases} \left(L_{ss} \frac{di_{sd}}{dt} + L_m \frac{di_{rd}}{dt} \right) = -R_s i_{sd} + \omega_0 (L_{ss} i_{sq} + L_m i_{rq}) + u_{sd} \\ \left(L_{ss} \frac{di_{sq}}{dt} + L_m \frac{di_{rq}}{dt} \right) = -R_s i_{sq} - \omega_0 (L_{ss} i_{sd} + L_m i_{rd}) + u_{sq} \\ \left(L_m \frac{di_{sd}}{dt} + L_{rr} \frac{di_{rd}}{dt} \right) = -R_r i_{rd} + \omega_0 S (L_m i_{sq} + L_{rr} i_{rq}) \\ \left(L_m \frac{di_{sq}}{dt} + L_{rr} \frac{di_{rq}}{dt} \right) = -R_r i_{rq} - \omega_0 S (L_m i_{sd} + L_{rr} i_{rd}) \end{cases} \quad (1)$$

$$\begin{cases} \frac{dS}{dt} = -\frac{1}{J\omega_0} (T_e - T_m) = -\frac{\omega_0}{2HP_N} (T_e - T_m) \\ T_e = L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) \\ T_m = T_{m0} [(1-a)(1-S)^2 + a] \end{cases} \quad (2)$$

In Equation (1), $L_{ss} = L_{0s} + L_m$, $L_{rr} = L_{0r} + L_m$, $L_m = \frac{3}{2} L_{ms}$. In Equation (2), T_e is the electromagnetic torque, T_m is the mechanical torque, J is the rotor's moment of inertia, H is the rotor's inertia time constant, P_N is the rated power, and T_{m0} and a are mechanical load coefficients.

2.1.2. Parameters of the Induction Motor Group

Assume that all induction motors connected to a power supply bus in an actual distribution network form a motor group. The parameters for each induction motor can be obtained by surveying the electrical devices or equipment in manufacturing enterprises [16] or by employing identification techniques [40–42]. These parameters serve as the basic data for this study. If the induction motors in the group are indexed by the serial number i , then the basic parameters for the i -th motor in the group are shown in Table 1.

Based on the basic parameters provided in Table 1, the operational parameters necessary for the grouping and aggregation methods can be calculated as detailed in Table 2.

Table 1. Basic Parameters Obtained from Statistical and Parameter Identification Methods.

Category	Details
Rated Parameters	Rated Voltage U_{Ni} , Rated Power P_{Ni}
T-Equivalent Circuit Parameters	R_{si} , R_{ri} , X_{si} , X_{ri} , X_{mi}
Rotor Parameters	Rotor Inertia J_i and Time Constant H_i
Mechanical Load Parameters	Mechanical Torque Coefficients T_{m0i} , a_i

Table 2. Operational Parameters for the Subsequent Grouping and Aggregating Modeling Method.

Category	Details
Starting Torque Parameters	Electromagnetic Torque T_{ei_st} , Mechanical Torque T_{mi_st}
Maximum Electromagnetic Torque Parameters	Critical Slip S_{mi} , Maximum Electromagnetic Torque T_{ei_max}
Steady-State Torque Parameters	Slip Rate S_{oi} , Electromagnetic Torque T_{eio} , Mechanical Torque T_{mio}
Steady-State Electrical Parameters	Stator and Rotor Currents I_{si} and I_{ri} , Absorbed Active and Reactive Power $P_i + jQ_i$, Electromagnetic Power P_{ei}

The parameters listed in Table 2 can be calculated using Equations (3)–(8). Assume that the actual voltage phasor of motor i under steady-state operating condition is:

$$\dot{U} = \frac{U_N}{\sqrt{3}} + j0 \quad (3)$$

The actual electromagnetic torque and mechanical torque are expressed as follows:

$$\begin{cases} T_{ei}(S_i) = \frac{3U^2}{\omega_0} \cdot \frac{\frac{R_{ri}}{S_i}}{\left(R_{si} + \frac{R_{ri}}{S_i}\right)^2 + (X_{si} + X_{ri})^2} \\ T_{mi}(S_i) = T_{m0i} \left[(1 - a_i)(1 - S_i)^2 + a_i \right] \end{cases} \quad (4)$$

Note that both $T_{ei}(S_i)$ and $T_{mi}(S_i)$ are functions defined on the interval $S_i \in (0, 1]$. By using numerical methods, the parameters listed in Table 2 can be obtained:

- 1) $T_{ei_st} = T_{ei}(1)$ and $T_{mi_st} = T_{mi}(1)$;
- 2) $T_{ei_max} = \max[T_{ei}(S_i)]$ and its corresponding critical slip ratio S_{mi} ;
- 3) The intersection of $T_{ei}(S_i)$ and $T_{mi}(S_i)$ within the interval $S_i \in (S_{mi}, 1)$. This intersection, where $T_{eio} = T_{mio}$, represents the actual steady-state operating point S_{oi} for motor i .

After determining the actual steady-state operating point S_{oi} , we can further calculate the stator current phasor, rotor current phasor, voltage phasor, active power, reactive power, and electromagnetic power of motor i using the following Equations (5)–(8):

$$\dot{I}_{si} = \frac{\dot{U}}{R_{si} + jX_{si} + \frac{1}{\left(\frac{1}{\left(\frac{R_{ri}}{S_{oi}} + jX_{ri}\right)} + \frac{1}{jX_{mi}}\right)}} \quad (5)$$

$$\dot{I}_{ri} = \dot{I}_{si} \frac{jX_{mi}}{\frac{R_{ri}}{S_{oi}} + jX_{ri} + jX_{mi}} \quad (6)$$

$$P_i + jQ_i = \dot{U} \dot{I}_{si}^* \quad (7)$$

$$P_{ei} = I_{ri}^2 \frac{R_{ri}}{S_{oi}} \quad (8)$$

The above basic and operational parameters provide the essential information for deriving the grouping and aggregation methods, and their symbols (as denoted in Tables 1 and 2) will be utilized in the following sections.

2.2. Grouping Rules and Aggregation Rules

2.2.1. Rotor Motion Characteristics and Grouping Rules

An induction motor group can be regarded as a set of parallel nonlinear impedances. These impedances respond to the voltage excitation from the power source by injecting current back into the circuit system through the feeder lines. Even under the same voltage, motors with different mechanical loads or different electromagnetic torque characteristics exhibit different dynamic impedance characteristics. For example, during a transient period when the supply bus voltage gradually recovers from a low level, the rotors of motors driving fans may easily return to their pre-transient speeds, resulting in a rapid increase in their impedance. In contrast, the rotors of motors driving pumps may struggle to return to their pre-transient speeds due to insufficient electromagnetic torque, leading to relatively low impedance being maintained for an extended period. If a larger proportion of motors can easily recover their speeds, the total impedance of the power load will increase more rapidly, aiding the recovery of the supply bus voltage. Conversely, if a larger proportion of motors struggle to recover their speeds, the total impedance of the power load will remain low for a longer period, making voltage recovery more difficult.

Therefore, voltage stability is primarily determined by the stability of the induction motor group, which, in turn, is governed by the rotor stability of each individual motor. In summary, if all motors connected to the same bus are indiscriminately represented by a single equivalent motor without distinguishing their individual characteristics, the dynamic features of different motors will be conflated. This can cause significant discrepancies between the simulation results by using the reduced-order equivalent model and the actual measured instability data of the actual motor group, leading to fundamentally incorrect assessments of transient voltage instability.

Therefore, to accurately assess the voltage stability of the supply bus, it is essential to group the induction motors based on the similarity of their rotor acceleration and deceleration processes before applying equivalent reduction to each group. From this, we can derive the following grouping rules (GR) for induction motor groups:

Grouping Rule 1 (GR1): Induction motors with similar rotor motion characteristics during the entire deceleration process following a sudden voltage drop should be grouped together.

Grouping Rule 2 (GR2): Induction motors with similar rotor motion characteristics during the entire acceleration process as the voltage gradually recovers should be grouped together.

2.2.2. Power Transmission Characteristics and Aggregation Rules

In power systems, the transmission of active power causes a lateral voltage drop along the line, while the transmission of reactive power causes a longitudinal voltage drop. With constant line impedance, the voltage deviation at the distribution bus depends on the active and reactive power drawn by the loads connected to the bus. Therefore, to study the transient voltage stability of the distribution system, it is essential to understand the active and reactive power characteristics of dynamic loads. To ensure that the transient voltage stability is neither too optimistic nor too conservative when using aggregated models, the instantaneous active and reactive characteristics of the actual motor group and the aggregated model must be as similar as possible [43].

More specifically, for the study of transient voltage stability, the aggregation of induction motor groups should follow these rules (Aggregation Rules, AR):

Aggregation Rule 1 (AR1): The active power, reactive power, stator and rotor losses, electromagnetic power, and mechanical power of the aggregated model should be equal to the total active power, total reactive power, total stator and rotor losses, total electromagnetic power, and total mechanical power of the actual induction motor group.

Aggregation Rule 2 (AR2): The mechanical load torque of the aggregated model should be equal to the total mechanical load torque of the actual induction motor group.

Aggregation Rule 3 (AR3): The stored kinetic energy of the aggregated model's rotor should be equal to the actual total stored kinetic energy of all the rotors in the actual induction motor group.

3. Grouping Method for the Induction Motors

3.1. Grouping Indicators

To accurately assess the transient voltage stability of a supply bus using a simplified dynamic load model, the primary challenge is effectively and rationally grouping the induction motors. Grouping motors with known parameters is essentially a multidimensional data classification problem. Therefore, it is necessary to establish grouping indicators that meet **GR1** and **GR2**.

GR1 and **GR2** focus on the principles of rotor dynamics during voltage drops and recovery. For clarity, we rewrite Equations (2) and (4) as the rotor motion Equation (9) and provide the electromagnetic torque and mechanical characteristic curves shown in Figure 2. From the form of the rotor motion Equation (9), we see that:

- 1) The internal factors determining the acceleration and deceleration rate of the motor rotor are its mechanical inertia and the resultant torque.
- 2) The external factors are the extent of the voltage drop and its duration.

To further illustrate the mechanisms of these internal and external factors, we use Equation (9) and Figure 2 to analyze the three periods: before the grid fault, during the fault, and after the fault is cleared:

- 1) Before the fault occurs in the power system, motor i operates normally with a constant speed. Therefore, in the interval $S_i \in [S_{oi}, 1]$, $T_{ei}(S_i, U^2) \geq T_{mi}(S_i)$ always holds true.
- 2) During a power system fault period, the voltage drops suddenly within a brief moment (less than 0.02 seconds). Therefore, in the interval $S_i \in [S_{oi}, 1]$, $T_{ei}(S_i, U^2)$ drops suddenly. As shown in Figure 2, the solid line $T_{ei}(S_i, U^2)$ drops to the dashed line $T'_{ei}(S_i, U_{drop}^2)$. Since the mechanical torque $T_{mi}(S_i)$ depends on the resisting moment or inertia of mechanical load (such as pump or fan impeller) and cannot change suddenly, it can easily result in $T'_{ei}(S_i, U_{drop}^2) < T_{mi}(S_i)$, causing the rotor to enter a braking and deceleration period. According to equation , the greater the voltage drop and the smaller the moment of inertia J_i , the faster the rotor decelerates. Conversely, the smaller the voltage drop and the greater the moment of inertia J_i , the slower the rotor decelerates.
- 3) After the power system fault is cleared, the voltage gradually recovers, causing $T_{ei}(S_i, U^2)$ to rise gradually. In the interval $S_i \in [S_{oi}, 1]$, the relationship between $T_{ei}(S_i, U^2)$ and $T_{mi}(S_i)$ depends on the voltage change. According to equation , the greater the voltage rise and the smaller the moment of inertia J_i , the faster the rotor accelerates. Conversely, the smaller the voltage rise and the greater the moment of inertia J_i , the slower the rotor accelerates.

$$\begin{cases} \frac{dS_i}{dt} = -\frac{1}{J_i\omega_0} [T_{ei}(S_i, U^2) - T_{mi}(S_i)] \\ T_{ei}(S_i, U^2) = \frac{3U^2}{\omega_0} \frac{R_{rj}/S_i}{(R_{si} + \frac{R_{rj}}{S_i})^2 + (X_{si} + X_{ri})^2} \\ T_{mi}(S_i) = T_{m0i} [(1 - a_i)(1 - S_i)^2 + a_i] \end{cases} \quad (9)$$

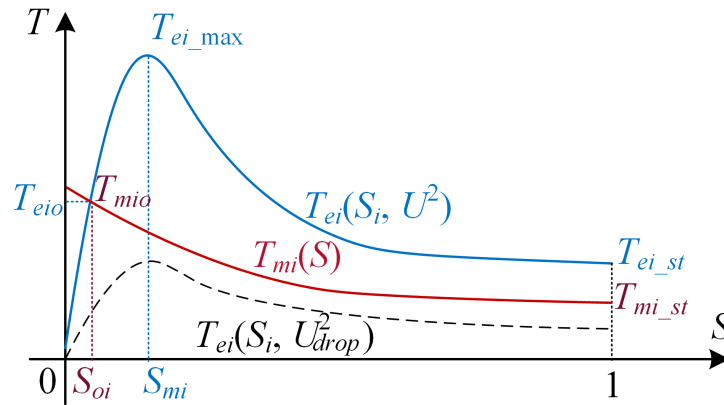


Figure 2. Torque-Slip Curves of an Induction Motor at various values of constant terminal voltage.

Based on the mechanical analysis results of the three periods above, the internal factors' influence can be quantified using the inertia time constant H_i , high-speed remaining torque K_{HSi} , and low-speed remaining torque K_{LSi} as grouping indicators:

$$K_{HSi} = 1 - \frac{T_{mi_HS}}{T_{ei_max}} \quad (10)$$

$$K_{LSi} = 1 - \frac{T_{mi_LS}}{T_{ei_st}} \quad (11)$$

In Equations (10) and (11):

K_{HSi} represents the high-speed remaining torque, indicating the residual electromagnetic torque that motor i can output at high-speed operating points.

K_{LSi} represents the low-speed remaining torque, indicating the residual electromagnetic torque that motor i can output at low-speed operating points.

T_{mi_HS} is the mechanical torque at the actual operating point $S_i = S_{oi}$, $T_{mi_HS} = T_{mi_o}$, as shown in Table 2.

T_{mi_LS} is the mechanical torque at $S_i = 1$, $T_{mi_LS} = T_{mi_st}$, as shown in Table 2.

T_{ei_max} and T_{ei_st} represent the maximum electromagnetic torque and starting torque, respectively, as shown in Table 2.

Under significant voltage variations, a larger K_{HSi} indicates that the motor's torque output performance is more robust near the actual high-speed operating point $S_i = S_{oi}$. Conversely, a smaller K_{HSi} means that the motor's torque output performance is less robust near $S_i = S_{oi}$. Similarly, a larger K_{LSi} implies that the motor's torque output performance is more robust at low speed point $S_i = 1$. Conversely, a smaller K_{LSi} means that the motor's torque output performance is less robust at low speed point $S_i = 1$.

3.2. Grouping Method Based on Adaptive K-Means Clustering

Based on the above analysis, using the inertia time constant H_i , high-speed remaining torque K_{HSi} , and low-speed remaining torque K_{LSi} as grouping indicators can yield more rational sub-groups of the induction motor group. The following is an adaptive clustering algorithm with the steps outlined below:

1) Initialize dataset. For an induction motor group with N individuals, where the basic parameters of individual i in the group are known (see Table 1) along with the operating parameters (see Table 2), the K_{HSi} and K_{LSi} can be calculated using Equations (10) and (11), respectively. This allows for the construction of a three-dimensional data point set Y :

$$Y = \{y_i | y_i = (K_{HSi}, K_{LSi}, H_i), i = 1, \dots, N\} \quad (12)$$

Set a constant $Iterations_max$, and initialize an empty set $S_{evaluation} = \{\}$ to record the evaluation coefficients of clustering effectiveness.

- 2) Initialize the number of clusters by setting $K = 2$.
- 3) Initialize the cluster centers by randomly selecting K points from the dataset Y as the centers $\mu_K = \{\mu_k | k = 1, \dots, K\}$
- 4) Assign each data point to its nearest cluster center. Calculate the Euclidean distance from data point i to the cluster center μ_K :

$$d_{iK} = \|y_i - \mu_K\| \quad (13)$$

In Equation (13), d_{iK} has the same dimensions as μ_K . Calculate the index of the minimum value in the vector d_{iK} :

$$I_i^K = \arg \min_k (d_{iK}) \quad (14)$$

Apply equations (13) and (14) to all points in the dataset Y to obtain the data point assignments $I_{labels}^K = \{I_i^K | i = 1, \dots, N\}$, which in turn provides the cluster assignment results $C_K = \{C_k | C_k = Y(I_{labels}^K = k), k = 1, \dots, K\}$.

- 5) Update Cluster Centers. Calculate the mean of all data points y_i in each sub-cluster C_k , and use this mean as the new cluster center:

$$\mu_k = \frac{1}{|C_k|} \sum_{y_j \in C_k} y_j \quad (15)$$

Here, $|C_k|$ represents the total number of points in sub-cluster C_k .

- 6) Check Convergence Criteria. Verify if the cluster centers μ_K have stabilized, if the data point assignments I_{labels}^K have ceased to change, and if the number of iterations has reached the maximum $Iterations_max$. If convergence is achieved, proceed to step 7) to evaluate the clustering results C_K ; otherwise, return to steps 4) and 5).

- 7) Calculate the Validity of Clustering Results. Compute the silhouette coefficient $s_K = s(C_K)$ according to reference [44] and add (k, s_K, C_K) to the evaluation set $S_{evaluation} = \{(K, s_K, C_K)\}$.

- 8) Increment the Number of Clusters. Set $K = K + 1$. If $K > N$, proceed to step 9); otherwise, return to steps 3) through 7).

- 9) Determine the Optimal Clustering. Find the maximum silhouette coefficient in the evaluation set $S_{evaluation}$ and output the optimal clustering result $C_K = \{C_k | k = 1, \dots, K\}$. The adaptive clustering process is complete.

4. Aggregation Method for the Induction Motor Sub-Group

4.1. Calculation of Equivalent Impedance Parameters

After obtaining the optimal grouping result $C_K = \{C_k | k = 1, \dots, K\}$ for the induction motor group, each motor in the sub-group C_k needs to be represented as a single-unit machine. Given the basic and operational parameters of each motor (as shown in Tables 1 and 2), the equivalent results for each motor sub-group should satisfy the following relationship according to **AR1**:

$$P_{e\Sigma} = \sum_{i=1}^n P_{ei} \quad (16)$$

$$P_{cu1\Sigma} = 3 \left| \sum_{i=1}^n \dot{I}_{si} \right|^2 R_{sE} = \sum_{i=1}^n \left[3(I_{si})^2 R_{si} \right] \quad (17)$$

$$P_{cu2\Sigma} = 3I_{rE}^2 R_{rE} = \sum_{i=1}^n (P_{ei} S_{oi}) \quad (18)$$

$$Q_{xs\Sigma} = 3 \left| \sum_{i=1}^n i_{si} \right|^2 X_{0sE} = \sum_{i=1}^n [3(I_{si})^2 X_{0si}] \quad (19)$$

For the equivalent T-circuit of the single-unit machine., let the excitation branch voltage be $\dot{U}_{mE}$. According to Kirchhoff's Voltage Law (KVL), Kirchhoff's Current Law (KCL), and Ohm's Law, the following relationships hold:

$$\dot{U}_{mE} = \dot{U} - \left(\sum_{i=1}^n i_{si} \right) (R_{sE} + jX_{0sE}) \quad (20)$$

$$\dot{U}_{mE} = \dot{I}_{rE} (R_{rE}/S_{oE} + jX_{0rE}) \quad (21)$$

$$\dot{I}_{mE} = \left(\sum_{i=1}^n i_{si} \right) - \dot{I}_{rE} \quad (22)$$

$$G_{mE} + jB_{mE} = \frac{1}{jX_{mE}} + \frac{1}{R_{rE}/S_{oE} + jX_{0rE}} = \frac{\left(\sum_{i=1}^n i_{si} \right)}{\dot{U}_{mE}} \quad (23)$$

The electromagnetic power of the single-unit machine can also be expressed as:

$$P_{e\Sigma} = 3U_{mE}^2 G_{mE} = 3I_{rE}^2 \frac{R_{rE}}{S_{oE}} \quad (24)$$

The equality between the rotor leakage reactance and the stator leakage reactance is used as an additional condition:

$$X_{0rE} = X_{0sE} \quad (25)$$

Research [45,46] indicated that the proportional relationship between rotor and stator leakage reactances has a minimal effect on both the dynamic and steady-state characteristics of the motor. Therefore, it is both necessary and reasonable to assume that rotor and stator leakage reactances are equal.

By combining Equations (23), (24), and (25), we obtain:

$$P_{e\Sigma} = \frac{3U_{mE}^2 (R_{rE}/S_{oE})}{(R_{rE}/S_{oE})^2 + X_{0rE}^2} \quad (26)$$

Equation (26) can be simplified to a quadratic equation in R_{rE}/S_{oE} :

$$(R_{rE}/S_{oE})^2 - \frac{U_{mE}^2}{P_{e\Sigma}/3} (R_{rE}/S_{oE}) + X_{0rE}^2 = 0 \quad (27)$$

Solving Equation (27) using the quadratic formula yields R_{rE}/S_{oE} :

$$\frac{R_{rE}}{S_{oE}} = \frac{(3U_{mE}^2/P_{e\Sigma}) \pm \sqrt{(3U_{mE}^2/P_{e\Sigma})^2 - 4X_{0rE}^2}}{2} \quad (28)$$

Since the actual operating slip rate S_o of each motor is usually less than 0.05, the value of R_{rE}/S_{oE} and X_m should be close to each other, and fall within the same range (tens to hundreds of ohms).

Therefore, in Equation (28), the " $\pm$ " on the right side should be taken as "+", ensuring that R_{rE}/S_{oE} yields the correct value.

Based on Equations (20) to (28), the excitation branch voltage $\dot{U}_{mE}$, excitation branch current $\dot{I}_{mE}$, and rotor branch current $\dot{I}_{rE}$ of the single-unit machine can be determined. This allows for the calculation of the equivalent machine's stator resistance, rotor resistance, stator and rotor leakage reactance, excitation reactance, and operating slip:

$$R_{sE} = \frac{\sum_{i=1}^n [(I_{si})^2 R_{si}]}{\left| \sum_{i=1}^n \dot{I}_{si} \right|^2} \quad (29)$$

$$R_{rE} = \frac{\sum_{i=1}^n (P_{ei} S_{oi})}{3I_{rE}^2} \quad (30)$$

$$X_{0sE} = X_{0rE} = \frac{\sum_{i=1}^n [(I_{si})^2 X_{0si}]}{\left| \sum_{i=1}^n \dot{I}_{si} \right|^2} \quad (31)$$

$$X_{mE} = \text{Im} \left[\frac{\dot{U}_{mE}}{\dot{I}_{mE}} \right] \quad (32)$$

$$S_{oE} = \frac{2 \sum_{i=1}^n (P_{ei} S_{oi})}{3I_{rE}^2 \left[(3U_{mE}^2 / P_{e\Sigma}) + \sqrt{(3U_{mE}^2 / P_{e\Sigma})^2 - 4X_{0rE}^2} \right]} \quad (33)$$

Here, $\text{Im}[\cdot]$ denotes the imaginary part of a complex number. Using Equations (29) to (30), the T-equivalent circuit parameters R_{sE} , X_{0sE} , R_{rE} , X_{0rE} , X_{mE} of the equivalent machine, as well as the slip rate S_{oE} under steady-state operating conditions, can be determined.

4.2. Calculation of Equivalent Mechanical Parameters

According to **AR1**, under the actual steady-state operating conditions with nearly constant mechanical load, the aggregated mechanical load power of the single-unit machine should equal the sum of the mechanical load powers of all individual motors in the group:

$$\begin{aligned} T_{m0E} \omega_0 \left[(1 - a_E) \omega_0 (1 - S_{oE})^3 + a_E \omega_0 (1 - S_{oE}) \right] \\ = \sum_{i=1}^n T_{m0i} \left[(1 - a_i) (1 - S_{oi})^3 + a_i (1 - S_{oi}) \right] \end{aligned} \quad (34)$$

Since relationship (34) holds even when the speeds of the induction motors in the group are very close to the synchronous speed ($S_{oi} = 0$), the mechanical load coefficient T_{m0E} can be calculated as follows:

$$T_{m0E} = \sum_{i=1}^n T_{m0i} \quad (35)$$

According to **AR2**, the mechanical load coefficient a_E can be calculated as follows:

$$a_E = \frac{\sum_{i=1}^n T_{m0i} a_i (1 - S_{oi})}{T_{0E} (1 - S_{oE})} \quad (36)$$

According to **AR3**, the stored kinetic energy of the single-unit machine's rotor should be equal to the actual total stored kinetic energy of all the rotors in the sub-group:

$$\frac{1}{2}J_E(\omega_r)^2 = \sum_{i=1}^n \left[\frac{1}{2}J_i(\omega_{ri})^2 \right] \quad (37)$$

Therefore, the rotor moment of inertia J_E and time constant H_E of the single-unit machine can be estimated as follows:

$$J_E = \frac{\sum_{i=1}^n [J_i(\omega_{ri})^2]}{(\omega_r)^2} = \frac{\sum_{i=1}^n [J_i(1 - S_{oi})^2]}{(1 - S_o)^2} \quad (38)$$

$$H_E = \frac{J_E(\omega_r)^2}{2 \sum_{i=1}^n P_i} \quad (39)$$

Using Equations (16) through (39), the mechanical load torque coefficients T_{m0E} , a_E , moment of inertia J_E , and inertia time constant H_E for the single-unit machine of the sub-group under the actual steady-state operating condition can be determined.

5. Simulation and Discussion

5.1. Simulation Setup

5.1.1. Distribution Network and Induction Motor Parameters

In this section, a typical 10 kV distribution system is used for transient voltage stability simulation. This distribution system has one incoming line and three feeders on the bus. The circuit structure and parameters are shown in Figure 3:

1) Converting all parameters to the 10 kV side, the equivalent impedance of the transmission line in power supply side is $x_l = j0.4644\Omega$, and the short-circuit impedance parameter of the transformer (model SFZ10-40000/110kV) is $Z_T = 0.0112 + j1.0145\Omega$.

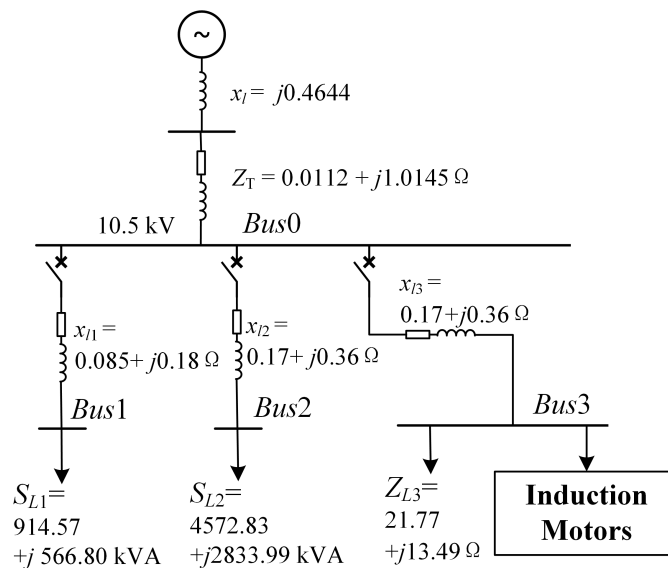


Figure 3. Single-Line Diagram of a Typical 10kV Distribution Network.

2) The line impedance of feeder 1 is $x_{l1} = 0.085 + j0.18\Omega$, and the load at terminal bus Bus1 (converted to the 10 kV side) is $S_{L1} = 914.57 + j566.80kVA$.

3) The line impedance of feeder 2 is $x_{l2} = 0.17 + j0.36\Omega$, and the load at terminal bus Bus2 (converted to the 10 kV side) is $S_{L2} = 4572.83 + j2833.99kVA$.

4) The line impedance of feeder 3 is $x_{l3} = 0.17 + j0.36\Omega$. The load at terminal bus Bus3 (converted to the 10 kV side) includes a linear impedance $Z_{L3} = 21.77 + j13.49\Omega$ and an induction motor group (parameters detailed in Table 3).

Using the component-based approach from references [17,18,47] or the online parameter identification technique from reference [40], the parameters of the induction motors can be obtained, as shown in Table 3. Table 3 provides the impedance parameters and inertia time constants for 25 induction motors, as reported in reference [47]. In Table 3, K_L represents the actual loading rate, and a denotes the mechanical coefficient in Equation (2).

Table 3. Impedance parameters and inertia time constants for 25 induction motors.

No.	P_N (kW)	U_N (kV)	Impedance (p.u.)					Mechanical Load		
			R_s	X_s	R_r	X_r	X_m	H (s)	K_L (%)	a
1	1000	10	0.0379	0.1139	0.0079	0.1139	1.4411	0.6593	40	0.53
2	900	10	0.0176	0.122	0.0108	0.122	1.9507	0.7294	70	0.53
3	1000	10	0.0299	0.1172	0.0107	0.1172	1.6506	0.7706	50	0.53
4	2000	10	0.0351	0.1221	0.0086	0.1221	5.7783	0.8939	70	0.53
5	800	10	0.0159	0.123	0.0109	0.123	2.0896	0.9001	60	0.95
6	200	10	0.0129	0.1245	0.0109	0.1245	2.2618	0.9334	40	0.55
7	450	10	0.0485	0.0886	0.0116	0.0886	2.4915	1.1508	50	0.53
8	2000	10	0.0127	0.1281	0.0057	0.1281	3.7853	1.381	80	0.82
9	450	10	0.0728	0.0761	0.0173	0.0761	2.3504	1.4572	60	0.53
10	750	10	0.0334	0.0957	0.0117	0.0957	2.958	1.5721	70	0.87
11	900	10	0.0836	0.0634	0.0203	0.0634	1.9818	1.6141	60	0.53
12	450	10	0.0671	0.0793	0.0144	0.0793	2.4316	1.644	60	0.53
13	2000	10	0.0213	0.1251	0.0057	0.1251	3.5895	2.1137	40	0.82
14	750	10	0.0236	0.1231	0.0033	0.1231	2.9099	2.2128	80	0.95
15	750	10	0.0252	0.1226	0.0033	0.1226	2.8846	2.2852	70	0.87
16	1000	10	0.0232	0.1227	0.0111	0.1227	2.6789	2.3327	50	0.69
17	450	10	0.0218	0.1243	0.0034	0.1243	3.2272	2.3427	90	0.95
18	315	10	0.0231	0.1239	0.0056	0.1239	3.2025	2.532	90	0.82
19	750	10	0.0242	0.1229	0.0033	0.1229	2.9003	2.6363	70	0.87
20	1000	10	0.0239	0.1225	0.0111	0.1225	2.6691	2.7026	40	0.69
21	1000	10	0.0209	0.1235	0.0111	0.1235	2.712	2.8616	40	0.69
22	800	10	0.0099	0.1243	0.0032	0.1243	2.0193	3.1924	80	0.95
23	900	10	0.0123	0.123	0.0048	0.123	1.884	3.1972	60	0.9
24	900	10	0.0082	0.1242	0.0048	0.1242	1.9184	3.4254	60	0.9
25	800	10	0.0049	0.1258	0.0033	0.1258	2.0657	3.7413	80	0.95

5.1.2. Simulation Arrangement

To demonstrate the advantages of the methods proposed in Section 3 and Section 4, a simulation model is built in MATLAB/Simulink using the network parameters in Figure 3 and the induction motor group parameters in Table 3. The transient voltage recovery process at the Bus0 is observed under the same large disturbance for different induction motor models.

The specific simulation tests' arrangements are shown in Table 4. As listed in Table 4, the purpose of the No.1 simulation test is to verify that the motor model established in this paper has dynamic characteristics closer to the actual induction motor group. And the purpose of the No.2 simulation test is to verify that the motor model established in this paper can accurately reproduce the transient voltage collapse process caused by the actual motor group. The modeling methods from References

[32,47] are replicated to obtain their aggregated models (Model 1 and Model 2), which will serve as references for the subsequent simulations.

Table 4. Arrangement of Simulation Tests.

No.	Disturbance Settings	Induction Motor Model on Bus3	
		Model description	Parameters
1	Disturbance 1: A temporary ground fault occurs on Bus0, lasting for 0.1 seconds.	Model 1: Single-unit model based on Ref. [32].	See Table 6
		Model 2: Two-machine model based on Ref. [47].	See Table 7
		Model 3: Models based on the proposed method in this paper.	See Table 5
		Motor Group: The actual group with 25 induction motors.	See Table 3
2	Disturbance 2: A temporary ground fault occurs on Bus0, lasting for 0.2 seconds.	Model 1: Single-unit model based on Ref. [32].	See Table 6
		Model 2: Two-machine model based on Ref. [47].	See Table 7
		Model 3: Models based on the proposed method in this paper.	See Table 5
		Motor Group: The actual group with 25 induction motors.	See Table 3

5.2. Simulation Results and Discussion

5.2.1. Grouping Results

For the 25 induction motors listed in Table 2, we applied the grouping method from Section 3 to calculate the remaining torque values K_{HSi} , K_{LSi} . Additionally, we normalized the inertia constant H_i by taking its maximum value and rounding it up to the nearest ten as the base value, resulting in the dataset $\{(\bar{H}_i, K_{HSi}, K_{LSi})\}$. We then performed adaptive K-means clustering, producing the silhouette coefficient curve shown in Figure 4 and the clustering scatter plot depicted in Figure 5.

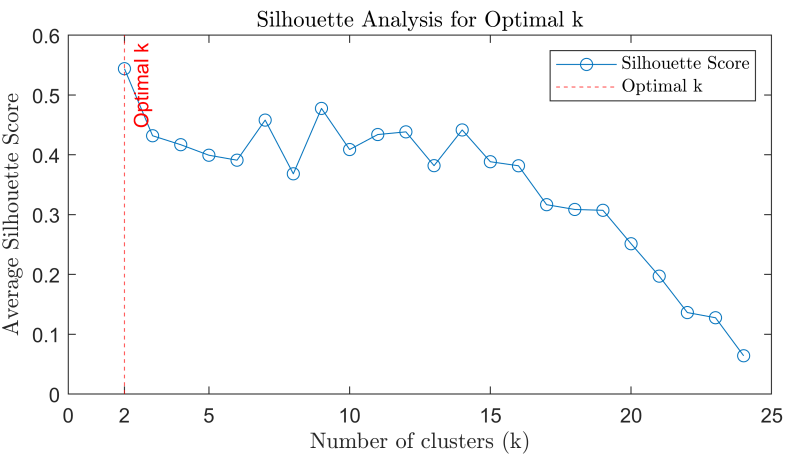


Figure 4. Clustering Validity Evaluation Curve of Silhouette Coefficient.

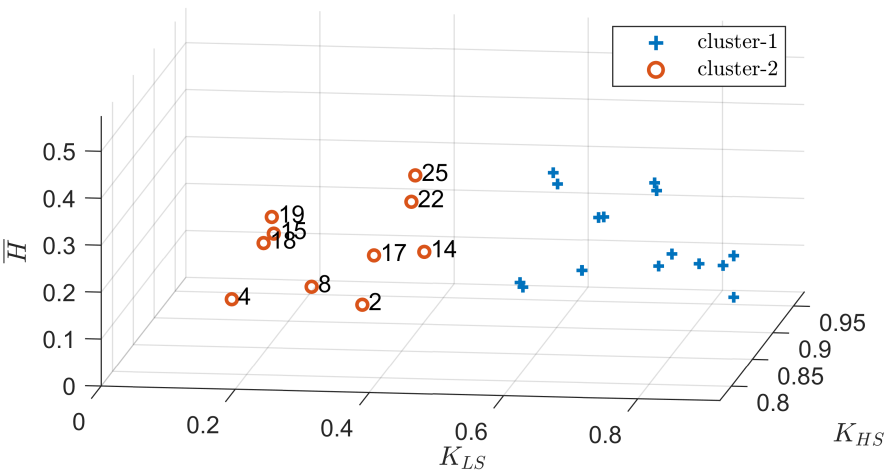


Figure 5. Scatter Plot of Clustering Results for Induction Motor Group.

From the silhouette coefficient curve in Figure 4, it is clear that the optimal number of clusters, corresponding to the maximum silhouette coefficient, is 2. Thus, we applied the clustering method in Section 3.2 and obtained two clusters ($K = 2$). Figure 5 shows that the motors marked with a "+" belong to Cluster C1, which represents the motors less likely to become unstable. The motors marked with an "o" belong to Cluster C2, which includes the more likely unstable motors: specifically, motors numbered 2, 4, 8, 14, 15, 17, 18, 19, 22, and 25.

5.2.2. Equivalent Results

Based on the series of Equations provided in Section 4 of this paper, the parameters of aggregation models for sub-groups (converted to the 10 kV side) are shown in Table 5:

Table 5. Parameters of the Models (Model-3) Based on the Proposed Grouping and Aggregation Method.

No.	P_N (kW)	U_N (kV)	Impedance (Ω)					Mechanical Load		
			R_s	X_s	R_r	X_r	X_m	H (s)	K_L (%)	a
C1	12800	10	0.0346	0.1175	0.0111	0.1175	2.2653	1.9345	50.55	0.72
C2	9515	10	0.0190	0.1327	0.0055	0.1327	3.0524	1.9084	76.18	0.79

Based on the method from reference [32], the parameters of the single-unit model is shown in Table 6.

Table 6. Parameters of the Single-unit Model (Model-1) Based on Reference [32].

No.	P_N (kW)	U_N (kV)	Impedance (Ω)					Mechanical Load		
			R_s	X_s	R_r	X_r	X_m	H (s)	K_L (%)	a
1	22315	10	0.0300	0.1207	0.0093	0.1262	2.5387	1.9235	61.48	0.74

Based on the method from reference [47], the parameters of the two-machine model are shown in Table 7.

Table 7. Parameters of the Two-machine Model (Model-2) Based on Reference [47].

No.	P_N (kW)	U_N (kV)	Impedance (Ω)					Mechanical Load		
			R_s	X_s	R_r	X_r	X_m	H (s)	K_L (%)	a
1	20515	10	0.0236	0.1205	0.0071	0.1205	2.5802	1.9534	61.61	0.77
2	1800	10	0.0784	0.0696	0.0182	0.0696	2.1665	1.5824	60.00	0.53

5.2.3. Transient Voltage Stability Result and Discussion

As shown in Figure 6, once disturbance 1 is cleared at 0.6 seconds, the distribution system undergoes a transient process of gradual voltage recovery. Figure 6a–c illustrate the voltage curve at Bus0, the active power curve, and the reactive power curve of the transformer, respectively, throughout the entire transient voltage recovery period. The black solid line in Figure 6a shows that during the grounding fault, the voltage drops to zero at 0.5 seconds and remains there for 0.1 seconds. By 0.6 seconds, the system begins to recover rapidly. The enlarged sub-figures in Figure 6a–c demonstrate that Model-3, developed using the proposed method, more accurately replicates the dynamic behavior of the actual motor group. In contrast, the single-unit model from reference [32] (Model-1) and the two-machine model from reference [47] (Model-2) exhibit greater deviations from the actual motor group’s dynamic behavior.

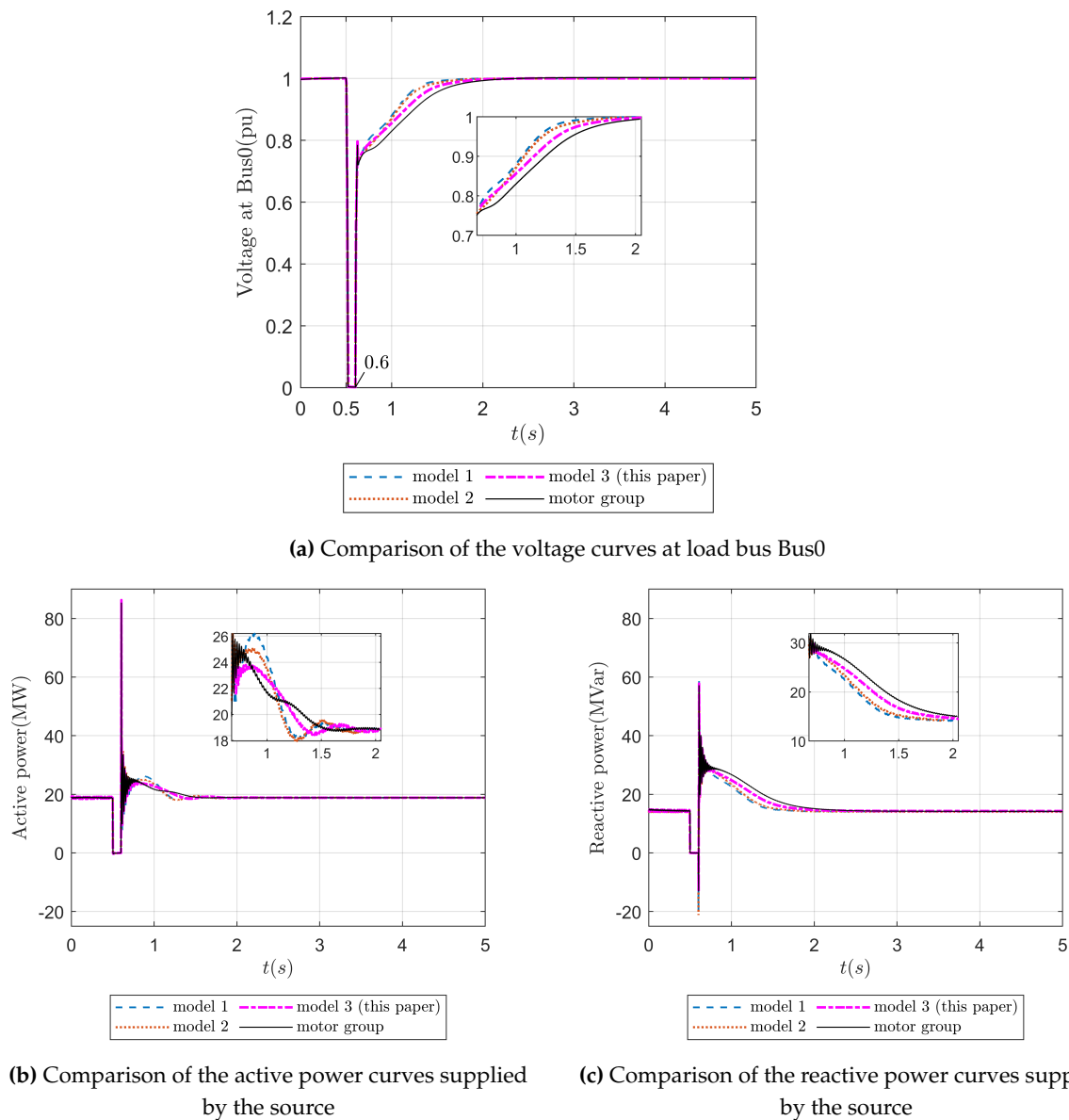
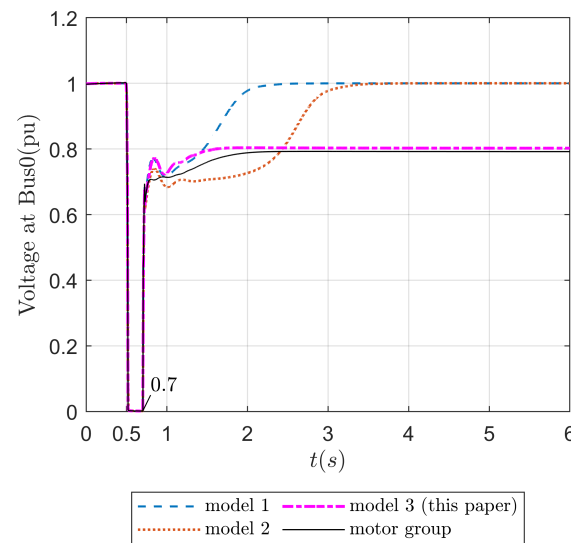


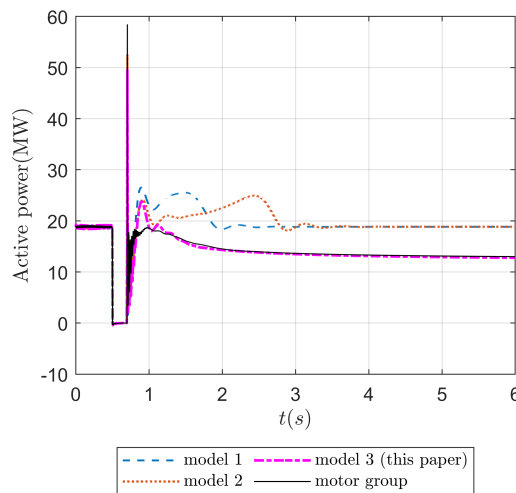
Figure 6. Reproduction of the transient voltage recovery process in a typical distribution network under three dynamic models.

Therefore, compared to Model-1 from reference [32] and Model-2 from reference [47], Model-3 presented in this paper has higher precision, as its dynamic characteristic more closely resembles that of the actual motor group under the given simulation conditions.

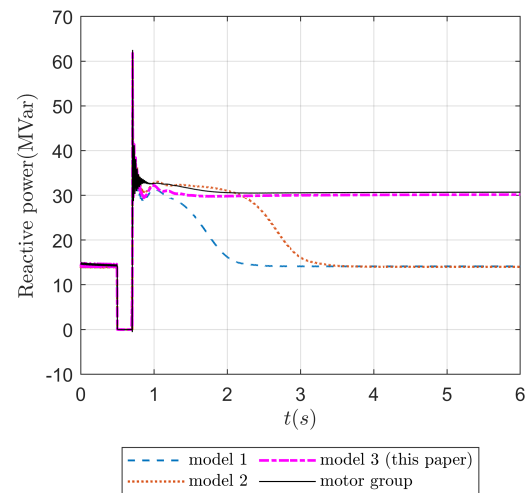
As depicted in Figure 7, once disturbance 2 is cleared at 0.7 seconds, the distribution system undergoes a transient period of gradual voltage collapse. Figure 7a–c illustrate the voltage curve at Bus0, the active power curve, and the reactive power curve of the transformer, respectively, throughout the entire transient voltage collapse period.



(a) Comparison of the voltage curves at load bus Bus0



(b) Comparison of the active power curves supplied by the source



(c) Comparison of the reactive power curves supplied by the source

Figure 7. Reproduction of the transient voltage collapse process in a typical distribution network under three different dynamic models.

The black solid line in Figure 7a shows that during the grounding fault, the voltage at Bus0 drops to zero at 0.5 seconds and stays at zero for 0.2 seconds. By 0.7 seconds, the system enters a transient recovery period. However, after 1 second, the voltage at Bus0 does not recover to its expected value, resulting in transient voltage instability. Additionally, a visual comparison of Figure 7a–c reveals the differences among Model-1, Model-2, and Model-3:

1) The dynamic processes of Model-1 (single-unit model from reference [32]) and Model-2 (two-machine model from reference [47]) are represented by blue dashed lines and orange dotted lines, respectively, in Figure 7. In Figure 7b,c, the active and reactive power curves of both models diverge from those of the actual motor group, gradually returning to levels before the fault happened. This results in their voltage curves in Figure 7a failing to accurately reproduce the voltage collapse at Bus0, constituting a significant misjudgment.

2) The dynamic process of the model developed in this paper (Model-3) is represented by a magenta dotted line in Figure 7. In Figure 7b,c, its voltage, active power, and reactive power curves successfully replicate the voltage collapse at Bus0.

Therefore, under the given simulation conditions, the motor model established by the proposed method (Model-3) provides a more accurate assessment of transient voltage stability in the distribution system compared to Model-1 and Model-2.

From the above simulation results, we can conclude the following:

- 1) The grouping rules (**GR1** and **GR2**) and Aggregation Rules (**AR1** to **AR3**) presented in Section 2.2 are correct and reasonable.
- 2) The grouping indicators defined in Section 3.1, specifically the high-speed remaining torque and low-speed remaining torque, along with the adaptive clustering method proposed in Section 3.2, are effective. The introduction of the silhouette coefficient has enhanced the adaptive capability of the K-means clustering algorithm, resulting in more reasonable induction motor grouping.
- 3) The impedance parameter aggregation method derived in Section 4.1 and the mechanical parameter aggregation method derived in Section 4.2, for motor subgroups, are both accurate and effective.

6. Conclusions

This paper first analyzes the operating principles of induction motors and the mechanism of voltage drop caused by power transmission, providing specific grouping and Aggregation Rules. Guided by the grouping rules, the inertia constant H , high-speed remaining torque K_{HS} , and low-speed remaining torque K_{LS} are used as grouping indicators. An adaptive K-means clustering method incorporating the silhouette coefficient is introduced to divide the induction motors into sub-groups rationally. Under the Aggregation Rules, impedance parameter and mechanical parameter equivalent methods are proposed for the sub-groups of induction motors. Through simulations in typical distribution systems, compared with traditional single-unit and two-machine equivalent methods, the following conclusions can be drawn:

- 1) the dynamic characteristic of the induction motor model from the proposed method are closer to that of the actual induction motor group. This demonstrates that the proposed method can ensure a reasonable reduction in the order of the dynamic load model while maintaining simplicity and usability.
- 2) the induction motor model from the proposed method can more accurately reproduce the bus voltage collapse process. This proves that the proposed method improves the accuracy of the dynamic load model, avoiding overly conservative or risky transient voltage stability simulation results. This is crucial for the security and stable operation of distribution systems.

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